

Measuring lepton number violation via heavy neutrino-antineutrino oscillations

based on work together with Stefan Antusch and Bruno Oliveira

Jan Hajer

Centro de Física Teórica de Partículas, Instituto Superior Técnico, Universidade de Lisboa

Physics studies — Muon collider



Funded by
the European Union

Standard Model neutrinos

Standard Model particle content

0	1/2			1
h	u <small>right</small> <small>left</small>	c <small>right</small> <small>left</small>	t <small>right</small> <small>left</small>	g
	d <small>right</small> <small>left</small>	s <small>right</small> <small>left</small>	b <small>right</small> <small>left</small>	γ
	e <small>right</small> <small>left</small>	μ <small>right</small> <small>left</small>	τ <small>right</small> <small>left</small>	Z
	ν_e <small>left</small>	ν_μ <small>left</small>	ν_τ <small>left</small>	W
	I	II	III	

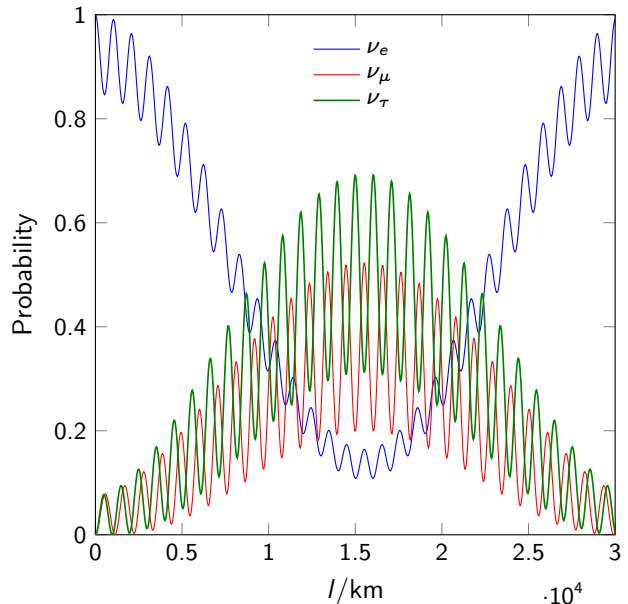
Neutrinos ν_α stand out

purely left-chiral and massless

Right-chiral or sterile Neutrinos

neutral under SM symmetries

Observed neutrino flavour oscillations



Flavour oscillations are explained by

right-chiral neutrinos allowing mass terms

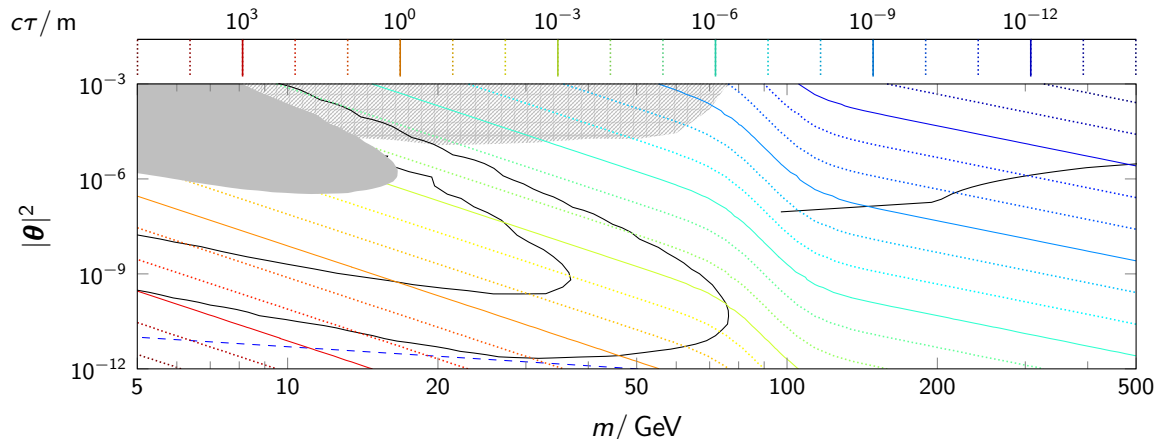
Simplest model

Interactions of a Majorana or Dirac heavy neutral lepton (HNL)

$$\mathcal{L}_N = -\frac{m_W}{v} \bar{N} \boldsymbol{\theta}^* \gamma^\mu e W_\mu^+ - \frac{m_Z}{\sqrt{2}v} \bar{N} \boldsymbol{\theta}^* \gamma^\mu \nu Z_\mu - \frac{m}{\sqrt{2}v} \boldsymbol{\theta} h \bar{\nu} N + \text{H.c.}$$

Seesaw mass

$$M_\nu = m_M \boldsymbol{\theta} \otimes \boldsymbol{\theta}$$



Dirac

- No massive light neutrino
- No LNV

Majorana

- Single massive light neutrino
- Generated mass is correct for small coupling or GUT scale
- Decay width wrong by factor of 2

Seesaw model regimes

Dirac mass

$$\mathcal{L}_D = -m_{D\alpha} \bar{\nu}_\alpha N + \text{h.c.}, \quad \mathbf{m}_D = \mathbf{v} \mathbf{y}$$

Majorana mass

$$\mathcal{L}_M = -\frac{1}{2} m_M \bar{N} N^c + \text{h.c.}$$

Coupling strength is determined by

$$\boldsymbol{\theta} = \mathbf{m}_D / m_M$$

Majorana mass introduces

Lepton number violation (LNV)

Majorana mass vanishes if

lepton-number L is conserved

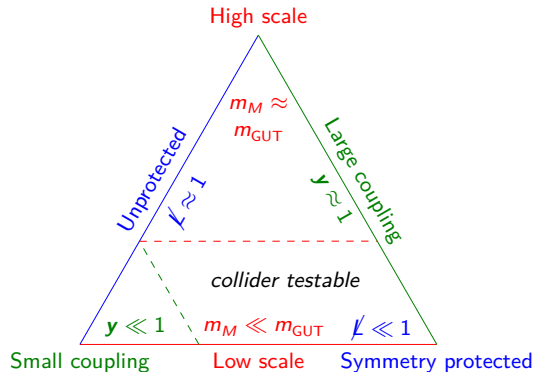
Neutrino oscillation pattern requires

at least two massive neutrinos

Neutrino mass matrix from two sterile neutrinos

$$M_\nu = \frac{\mathbf{m}_D^{(1)} \otimes \mathbf{m}_D^{(1)}}{m_M^{(1)}} + \frac{\mathbf{m}_D^{(2)} \otimes \mathbf{m}_D^{(2)}}{m_M^{(2)}}$$

Viable seesaw models



Neutrino masses are small for

- small \mathbf{y}
- large m_M
- symmetry protected cancellation

Particle content of benchmark model candidates

Number of Majorana degrees of freedom (DOFs)

DOF	Particles	Properties	
1	Majorana	One massive light neutrino	⚡
2	Dirac	No massive light neutrino	⚡
	pseudo-Dirac	Minimal linear seesaw / pSPSS	✓
3	2 Majorana	Light neutrinos too heavy	⚡
	pseudo-Dirac + Majorana	ν MSM (Dark Matter) Majorana active (no Dark Matter)	✓ ✓
4	2 pseudo-Dirac	Minimal inverse seesaw	✓
5	2 pseudo-Dirac + Majorana	...	
6	3 pseudo-Dirac	...	

Good benchmark model

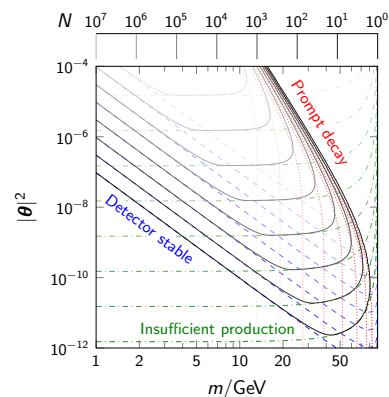
- Reproduces neutrino mass scale
- Captures dominant collider effects
- Minimal possible number of parameters

Minimal set of parameters for single pseudo-Dirac

- Mass m
- Coupling vector θ
- Mass splitting Δm

HNLs can be long-lived particles

$$P_{\text{decay}}(\tau) = -\frac{d}{d\tau} \exp(-\Gamma\tau) = \Gamma \exp(-\Gamma\tau)$$



HNLs can be long-lived particles

$$P_{\text{decay}}(\tau) = -\frac{d}{d\tau} \exp(-\Gamma\tau) = \Gamma \exp(-\Gamma\tau)$$

Since they are pseudo-Dirac they oscillate

$$P_{\text{osc}}^{\text{LNC/LNV}}(\tau) = \frac{1 \pm \cos(\Delta m\tau)}{2}$$

Collider signature: Decaying oscillations

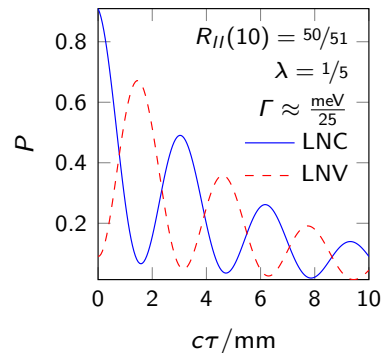
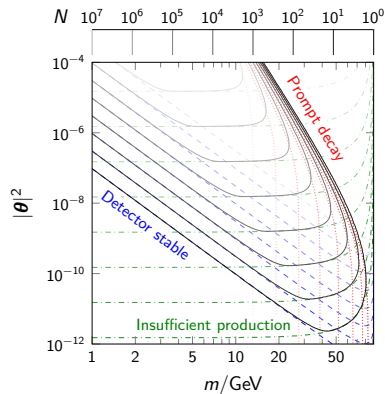
$$P_{II}^{\text{LNC/LNV}}(\tau) = P_{\text{osc}}^{\text{LNC/LNV}}(\tau) P_{\text{decay}}(\tau)$$

Time-integrated oscillations

$$P_{II}^{\text{LNC/LNV}} = \frac{1}{2} \pm \frac{1}{2} \frac{\Gamma^2}{\Gamma^2 + \Delta m^2}$$

Charged lepton ratio

$$R_{II} = \frac{P_{II}^{\text{LNV}}}{P_{II}^{\text{LNC}}} = \frac{\Delta m^2}{\Delta m^2 + 2\Gamma^2}$$



Problems measuring R_{II}

Integration limits correspond to

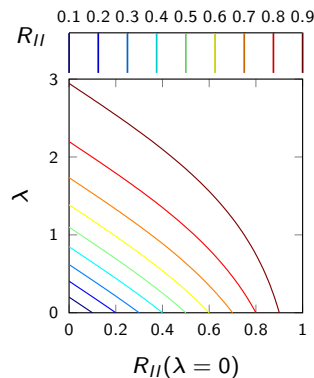
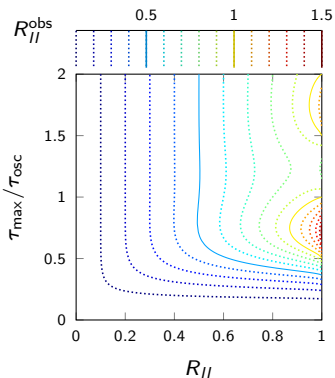
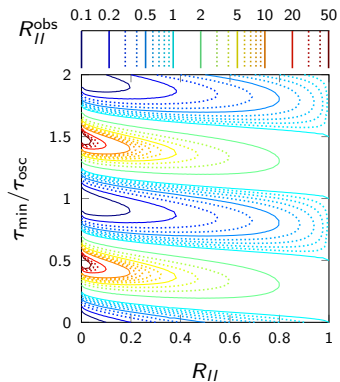
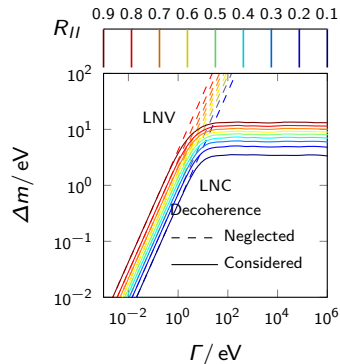
[2210.10738]

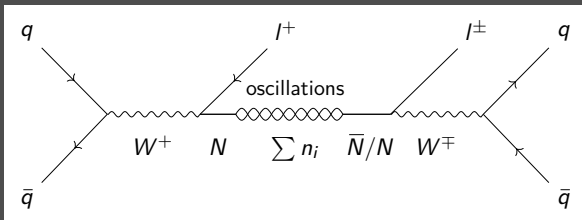
- Minimal distance cut
- Maximal measurable vertex distance

Decoherence

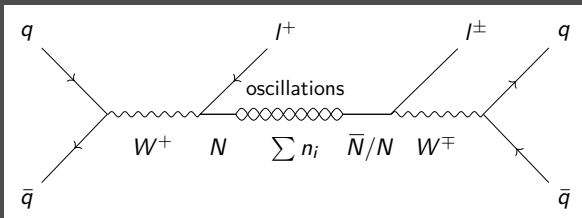
[2307.06208]

- Quantum mechanical oscillations can suffer from decoherence
- Calculation in external wave packet formalism
- Can increase measurable LNV drastically
- Captured by single parameter λ

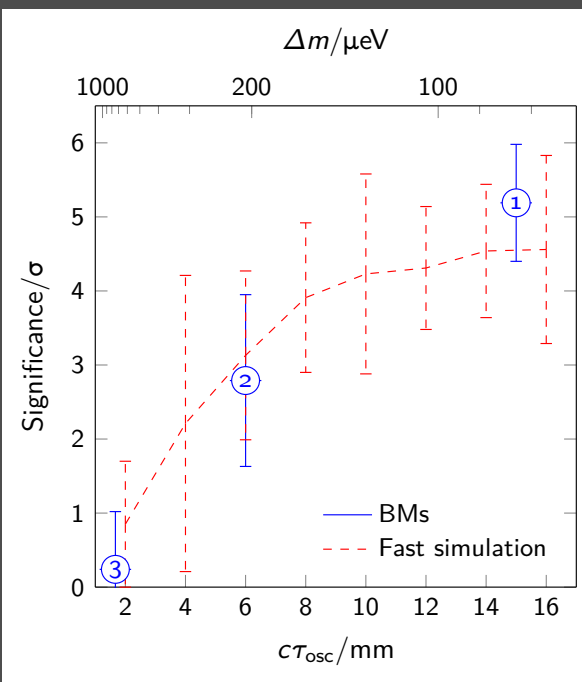
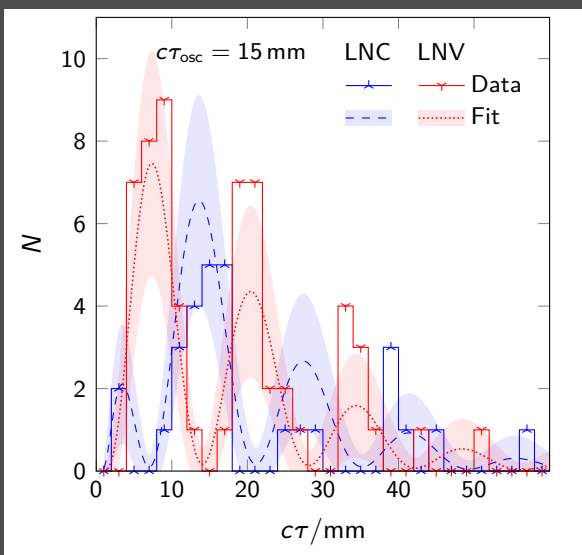


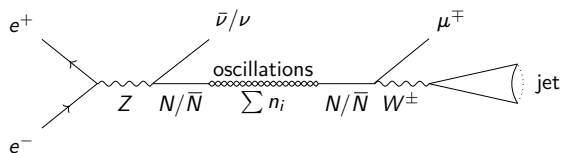


LNV is measured
by comparing the charges of the two leptons



LNV is measured by comparing the charges of the two leptons





LNV cannot be measured using two different charges

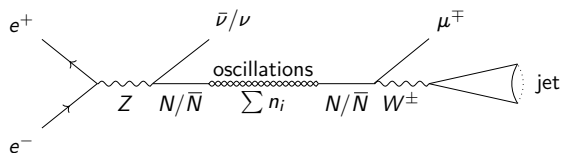
One can still measure angular distributions

Angular dependent probability

$$P(\cos \theta) := \frac{1}{\sigma} \frac{d\sigma(\cos \theta)}{d\cos \theta}$$

Probability for HNLs

$$P^{M/D}(\cos \theta) = \frac{3 m_Z^2 f^{M/D}(\theta) + m^2 \sin^2 \theta}{4 (2 m_Z^2 + m^2)}$$



LNV cannot be measured using two different charges

One can still measure angular distributions

Angular dependent probability

$$P(\cos\theta) := \frac{1}{\sigma} \frac{d\sigma(\cos\theta)}{d\cos\theta}$$

Probability for HNLs

$$P^{M/D}(\cos\theta) = \frac{3}{4} \frac{m_Z^2 f^{M/D}(\theta) + m^2 \sin^2\theta}{2m_Z^2 + m^2}$$

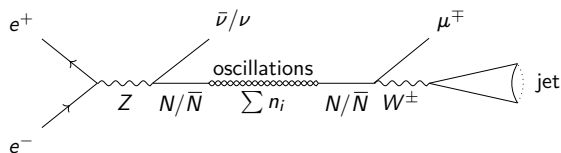
Z-boson polarisation due to P-violation

$$P_Z = -\Delta\gamma, \quad \Delta\gamma = \gamma_L - \gamma_R \approx 0.1494$$

$$\gamma_L = \frac{g_L^2}{g_L^2 + g_R^2}, \quad \gamma_R = \frac{g_R^2}{g_L^2 + g_R^2}$$

couplings of the charged leptons to the Z boson

$$g_L = 1 - 2\sin^2\theta_W, \quad g_R = 2\sin^2\theta_W$$



LNV cannot be measured using two different charges

One can still measure angular distributions

Angular dependent probability

$$P(\cos\theta) := \frac{1}{\sigma} \frac{d\sigma(\cos\theta)}{d\cos\theta}$$

Probability for HNLs

$$P^{M/D}(\cos\theta) = \frac{3 m_Z^2 f^{M/D}(\theta) + m^2 \sin^2\theta}{4 (2m_Z^2 + m^2)}$$

Majorana

$$f^M(\theta) = 1 + \cos^2\theta$$

Symmetric charge distribution

$$P_{I^-}^M(\cos\theta) = P_{I^+}^M(\cos\theta) = \frac{1}{2} P^M(\cos\theta)$$

Z-boson polarisation due to P-violation

$$P_Z = -\Delta\gamma, \quad \Delta\gamma = \gamma_L - \gamma_R \approx 0.1494$$

$$\gamma_L = \frac{g_L^2}{g_L^2 + g_R^2}, \quad \gamma_R = \frac{g_R^2}{g_L^2 + g_R^2}$$

couplings of the charged leptons to the Z boson

$$g_L = 1 - 2 \sin^2\theta_W, \quad g_R = 2 \sin^2\theta_W$$

Dirac

$$f_{N/\bar{N}}^D(\theta) = \gamma_{R/L}(1 - \cos\theta)^2 + \gamma_{L/R}(1 + \cos\theta)^2$$

Asymmetric charge distribution

$$P_{I^\mp}^D(\cos\theta) = P_{N/\bar{N}}^D(\cos\theta)$$

Forward-backward asymmetry (FBA)

(Anti-)symmetrised Dirac HNL probability

$$P_N^\pm(\cos\theta) := \frac{P_N^D(\cos\theta) \pm P_{\bar{N}}^D(\cos\theta)}{2},$$

Symmetrization corresponds to Majorana

$$P_N^+(\cos\theta) = P^M(\cos\theta)$$

Asymmetric contribution contains polarisation

$$P_N^-(\cos\theta) = \frac{3}{2} \frac{m_Z^2}{2m_Z^2 + m^2} \Delta\gamma \cos\theta$$

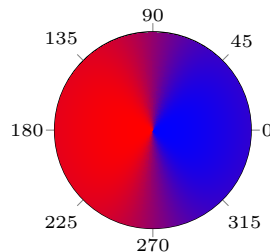
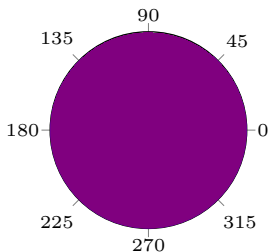
Forward backward symmetric

$$S^{\text{FB}} := P_N^+(\pm 1) = \frac{3}{2} \frac{m_Z^2}{2m_Z^2 + m^2}$$

Forward backward asymmetry

[2105.06576]

$$A_{N/\bar{N}}^{\text{FB}} := P_N^-(\pm 1) = \pm S^{\text{FB}} \Delta\gamma$$



LNv corresponds to symmetric distribution \rightarrow Not possible to measure LNv

What about pseudo-Dirac HNL?

Majorana and Dirac HNLs can only be considered as limiting cases of the pseudo-Dirac HNL

Probability to measure an (anti-)lepton

$$P_{l^\mp}(\tau, \cos \theta) = P_{\nu l}^{\text{LNC}}(\tau) P_{N/\bar{N}}(\cos \theta) + P_{\nu l}^{\text{LNV}}(\tau) P_{\bar{N}/N}(\cos \theta)$$

l^- from non-oscillating N or from oscillating \bar{N} (similar for l^+)

What about pseudo-Dirac HNL?

Majorana and Dirac HNLs can only be considered as limiting cases of the pseudo-Dirac HNL

Probability to measure an (anti-)lepton

$$P_{l^\mp}(\tau, \cos \theta) = P_{\nu l}^{\text{LNC}}(\tau) P_{N/\bar{N}}(\cos \theta) + P_{\nu l}^{\text{LNV}}(\tau) P_{\bar{N}/N}(\cos \theta)$$

l^- from non-oscillating N or from oscillating \bar{N} (similar for l^+)

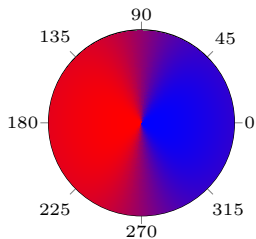
Probability to measure an (anti-)lepton

$$P_{l^\mp}(\tau, \cos \theta) = P_{\text{decay}}(\tau) [P_N^+(\cos \theta) \pm P_N^-(\cos \theta) \Delta P_{\text{osc}}(\tau)]$$

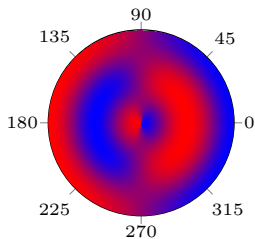
Oscillation probability difference

$$\Delta P_{\text{osc}}(\tau) = \cos(\Delta m \tau)$$

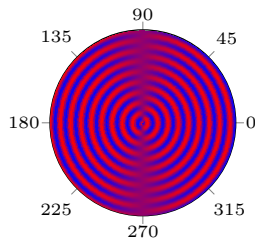
'Dirac BM'-like



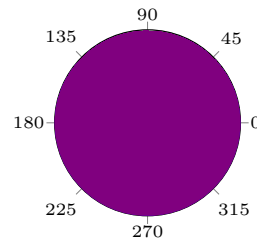
Slow oscillation



Fast oscillation



'Majorana BM'-like



Time integrated observable

Time integrated probability

$$P_{I\mp}(\cos\theta) := \int_0^\infty P_{I\mp}(\tau, \cos\theta) d\tau$$

$$P_{I\mp}(\cos\theta) = P_N^+(\cos\theta) \pm P_N^-(\cos\theta) \Delta P_{\nu I}$$

Difference of time-integrated probabilities

$$\Delta P_{\nu I} := \int_0^\infty P_{\nu I}^{\text{LNC}}(\tau) - P_{\nu I}^{\text{LNV}}(\tau) d\tau$$

Is a function of decay width and mass splitting

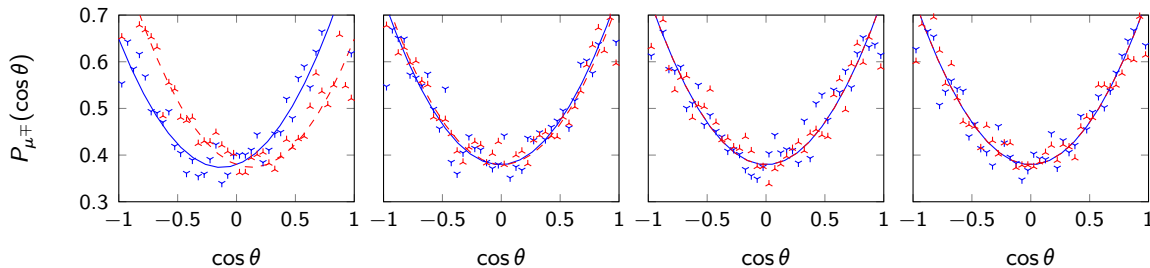
$$\Delta P_{\nu I} = \frac{\Gamma^2}{\Gamma^2 + \Delta m^2}$$

'Dirac BM'-like

Slow oscillation

Fast oscillation

'Majorana BM'-like



Charged lepton ratio

$$R_I(\cos\theta) := \frac{P_{I-}(\cos\theta)}{P_{I+}(\cos\theta)}$$

has the form

$$R_I(\cos\theta) = 1 + 2 \frac{P_N^-(\cos\theta)}{P_N^+(\cos\theta) \Delta P_{\nu I}^{-1} - P_N^-(\cos\theta)}$$

Angular-integrated distributions

Angular integrated probability

$$P_{I^\mp}^{[\theta_{\min}, \theta_{\max}]}(\tau) := \int_{\cos \theta_{\min}}^{\cos \theta_{\max}} P_{I^\mp}(\tau, \cos \theta) d \cos \theta$$

Forward-backward probability

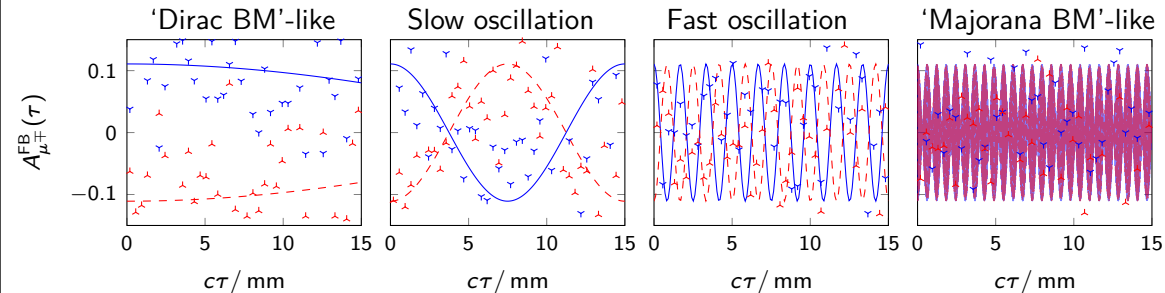
$$P_{I^\mp}^{[\pi/2, 0]}(\tau) = P_{I^\pm}^{[\pi, \pi/2]}(\tau) = \frac{1 + A_{I^\mp}^{\text{FB}}(\tau)}{2} P_{\text{decay}}(\tau)$$

Time dependent FBAs

$$A_{I^\mp}^{\text{FB}}(\tau) = A_{N/N}^{\text{FB}} \Delta P_{\text{osc}}(\tau)$$

Limit recovers non-oscillating result

$$A_{I^\mp}^{\text{FB}}(\tau = 0) = A_{N/N}^{\text{FB}}$$



Forward lepton ratio

$$R_I^{[\pi/2, 0]}(\tau) = \frac{P_{I^-}^{[\pi/2, 0]}(\tau)}{P_{I^+}^{[\pi/2, 0]}(\tau)} = \frac{1 + A_{I^-}^{\text{FB}}(\tau)}{1 + A_{I^+}^{\text{FB}}(\tau)}$$

Backward lepton ratio

$$R_I^{[\pi, \pi/2]}(\tau) = \frac{P_{I^-}^{[\pi, \pi/2]}(\tau)}{P_{I^+}^{[\pi, \pi/2]}(\tau)} = \frac{1 - A_{I^-}^{\text{FB}}(\tau)}{1 - A_{I^+}^{\text{FB}}(\tau)}$$

Combined observable

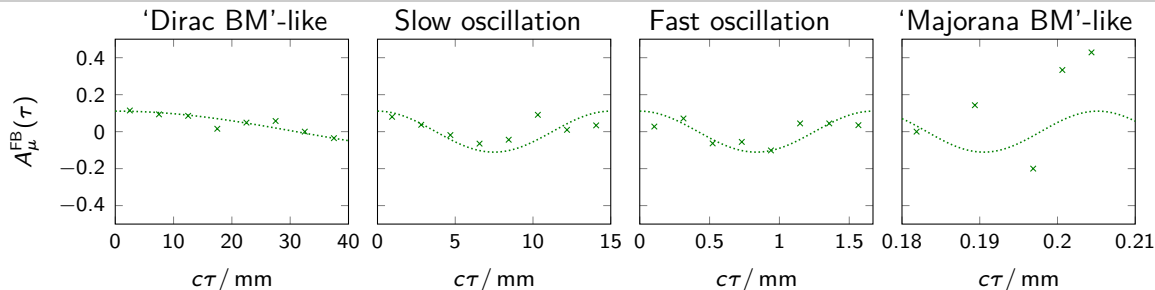
Combined lepton FBA

$$A_i^{\text{FB}}(\tau) := \frac{A_{i^-}^{\text{FB}}(\tau) - A_{i^+}^{\text{FB}}(\tau)}{2}$$

has the simple form

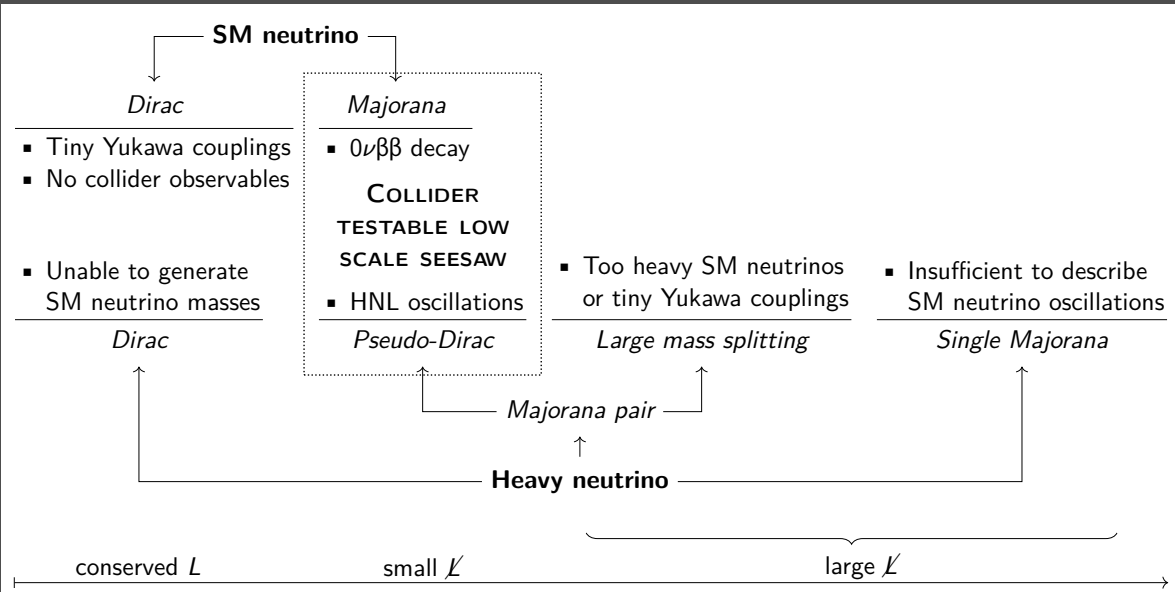
$$A_i^{\text{FB}}(\tau) = A_N^{\text{FB}} \Delta P_{\text{osc}}(\tau)$$

Focusing on the first oscillation



- Low-scale seesaw models predict pseudo-Dirac HNLs
- LNV can be measured via heavy neutrino-antineutrino oscillations
- At the Z -pole of the FCC- ee LNV can only be observed in distributions
- Heavy neutrino-antineutrino oscillations appear in these distributions
- Observation of LNV is possible at the FCC- ee

- S. Antusch, J. Hajer, and J. Roskopp (2023a). 'Simulating lepton number violation induced by heavy neutrino-antineutrino oscillations at colliders'. In: *JHEP* 03, p. 110. DOI: 10.1007/JHEP03(2023)110. arXiv: 2210.10738 [hep-ph]
- S. Antusch, J. Hajer, and J. Roskopp (July 2023b). 'Decoherence effects on lepton number violation from heavy neutrino-antineutrino oscillations'. arXiv: 2307.06208 [hep-ph]
- S. Antusch, J. Hajer, O. Bruno, and J. Roskopp (Oct. 2022a). 'pSPSS: Phenomenological symmetry protected seesaw scenario'. FeynRules model file. DOI: 10.5281/zenodo.7268418. GitHub: heavy-neutral-leptons/pSPSS. URL: feynrules.irmp.ucl.ac.be/wiki/pSPSS
- S. Antusch, J. Hajer, and J. Roskopp (Dec. 2022b). 'Beyond lepton number violation at the HL-LHC: Resolving heavy neutrino-antineutrino oscillations'. arXiv: 2212.00562 [hep-ph]
- A. Blondel, A. de Gouvêa, and B. Kayser (2021). 'Z-boson decays into Majorana or Dirac heavy neutrinos'. In: *Phys. Rev. D* 104.5, p. 55027. DOI: 10.1103/PhysRevD.104.055027. arXiv: 2105.06576 [hep-ph]. №: FERMILAB-PUB-21-227-T
- S. Antusch, J. Hajer, and B. M. S. Oliveira (Oct. 2023c). 'Heavy neutrino-antineutrino oscillations at the FCC-ee'. In: 10, p. 129. DOI: 10.1007/JHEP10(2023)129. arXiv: 2308.07297 [hep-ph]



Single Majorana and Dirac HNLs are

- not predicted by low-scale seesaw models

Unique phenomenology of pseudo-Dirac HNLs

- Heavy neutrino-antineutrino oscillations
- $0 < R_{II} = \frac{M_{LNV}}{M_{LNC}} < 1$
- Governed by mass splitting Δm

Single pseudo-Dirac symmetry protected seesaw scenario (SPSS) [2210.10738]

Exact limit

$$\mathcal{L}_{\text{SPSS}}^L = -m_M \bar{N}_1 N_2^c - y_1 \tilde{H}^\dagger \bar{\ell} N_1^c + \text{h.c.}$$

Small breaking terms $v y_2 \approx \mu_M \approx \mu'_M \ll m_M$

$$\mathcal{L}_{\text{SPSS}}^L = -y_2 \tilde{H}^\dagger \bar{\ell} N_2^c - \mu'_M \bar{N}_1 N_1^c - \mu_M \bar{N}_2 N_2^c + \text{h.c.}$$

Lepton number-like symmetry
generalises accidental SM lepton number L

One simple choice of charges

	ℓ	N_1	N_2
L	+1	-1	+1

Other new fields

further terms in Lagrangian

Neutrino mass matrix M_n

contains seesaw information

Basis

$$n = (\nu, n_4, n_5)$$

Dirac masses

$$\mathbf{m}_D = \mathbf{y}_1 v, \quad \boldsymbol{\mu}_D = \mathbf{y}_2 v$$

Symmetric limit

$$M_n^L = \begin{pmatrix} 0 & \mathbf{m}_D & 0 \\ \mathbf{m}_D^T & 0 & m_M \\ 0 & m_M & 0 \end{pmatrix}$$

Mild symmetry breaking

$$M_n^{L \ll 1} = \begin{pmatrix} 0 & \mathbf{m}_D & \boldsymbol{\mu}_D \\ \mathbf{m}_D^T & \boldsymbol{\mu}'_M & m_M \\ \boldsymbol{\mu}_D^T & m_M & \boldsymbol{\mu}_M \end{pmatrix}$$

Large symmetry breaking

$$M_n^{L \gg 0} = \begin{pmatrix} 0 & \mathbf{m}_D & \hat{\mathbf{m}}_D \\ \mathbf{m}_D^T & \hat{\mathbf{m}}'_M & m_M \\ \hat{\mathbf{m}}_D^T & m_M & \hat{\mathbf{m}}_M \end{pmatrix}$$

- Massless neutrinos $M_\nu = 0$
- Dirac HNL

- Pseudo-Dirac HNL (small Δm Majorana pair)
- Phenomenology governed by small parameters μ

- Large Δm Majorana pair
- Requires large m_M or tiny θ

	Linear seesaw μ_D	Inverse seesaw μ_M	Seesaw independent μ'_M
$M_n =$	$\begin{pmatrix} 0 & \mathbf{m}_D & \mu_D \\ \mathbf{m}_D^T & 0 & m_M \\ \mu_D^T & m_M & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \mathbf{m}_D & 0 \\ \mathbf{m}_D^T & 0 & m_M \\ 0 & m_M & \mu_M \end{pmatrix}$	$\begin{pmatrix} 0 & \mathbf{m}_D & 0 \\ \mathbf{m}_D^T & \mu'_M & m_M \\ 0 & m_M & 0 \end{pmatrix}$
$M_\nu =$	$\mu_D \otimes \theta$	$\mu_M \theta \otimes \theta$	0 (at tree level)
$\Delta m =$	Δm_ν	$m_\nu \theta ^{-2}$	$ \mu'_M $

Benchmark models

Seesaw	Hierarchy	BM
Linear	Normal	$\Delta m_\nu = 42.3 \text{ meV}$
	Inverted	$\Delta m_\nu = 748 \mu\text{eV}$
Inverse		$m_\nu = 0.5 \text{ meV}$
		$m_\nu = 5 \text{ meV}$
		$m_\nu = 50 \text{ meV}$

Generic seesaw

All small parameter μ are nonzero

