Proton Structure In And Out Of Muonic Hydrogen

— Status of the Proton Radius Puzzle —

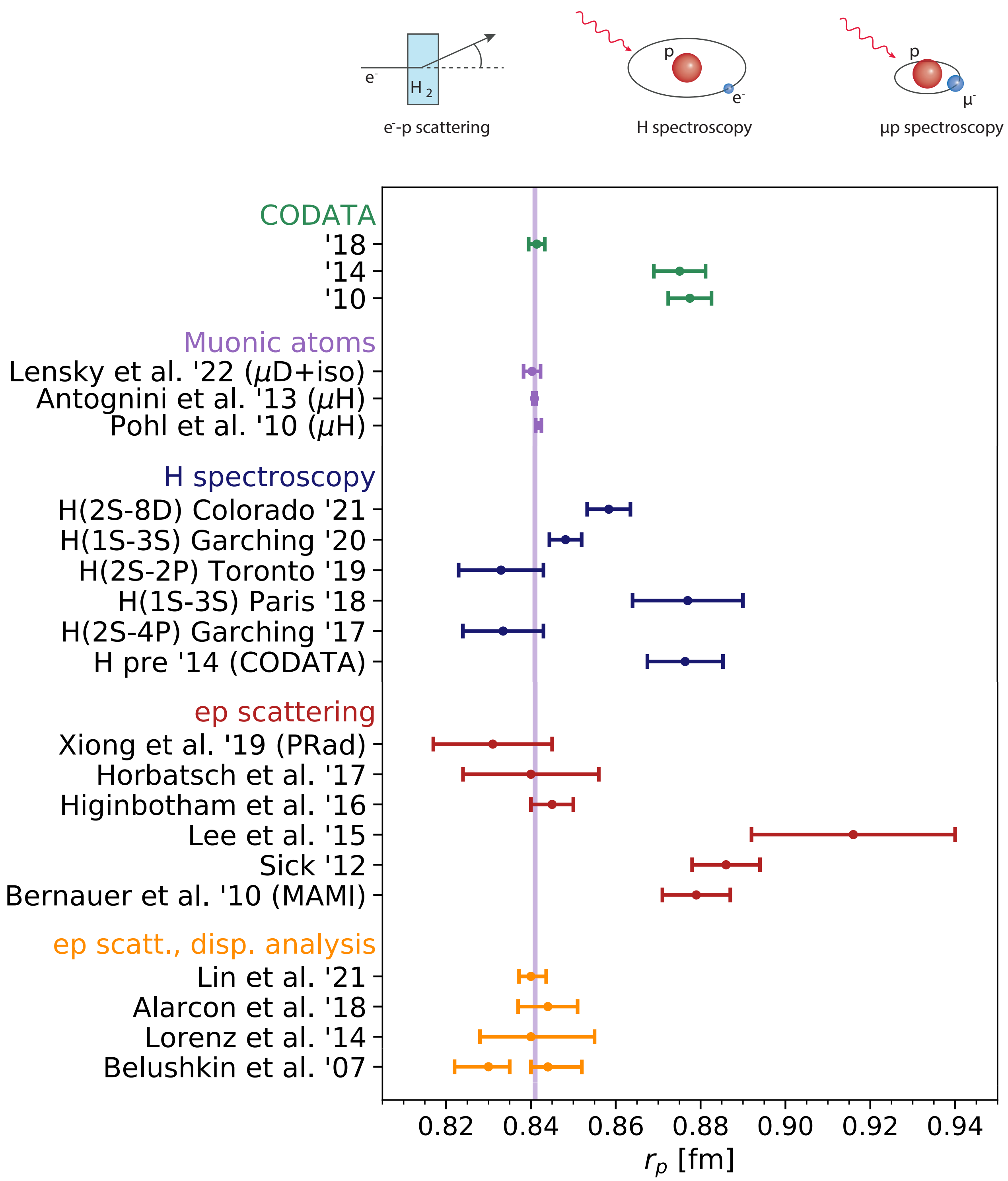
Franziska Hagelstein (JGU Mainz & PSI Villigen)

in collaboration with

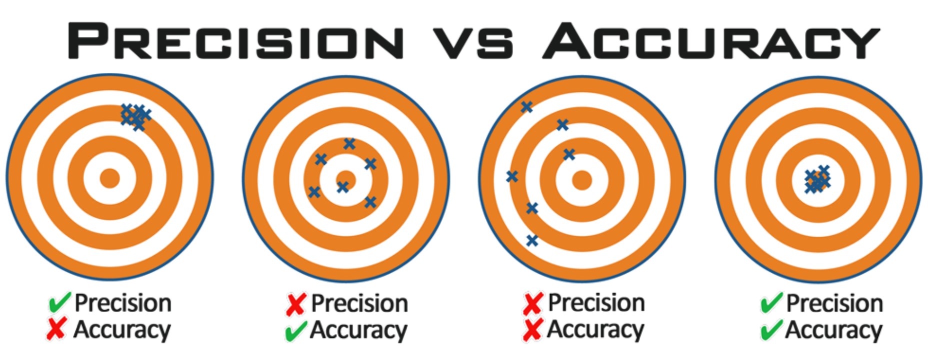
V. Biloshytskyi, T. Esser, V. Lensky, V. Pascalutsa, S. Pitelis (JGU)

and V. Sharkovska (PSI, UZH)

PROTON CHARGE RADIUS



- Muonic atoms allow for PRECISE extractions of nuclear charge and Zemach radii
- CODATA since 2018 included the μH result for r_p
- Still open issues: H(2S-8D) and H(1S-3S)
- Question:



FROM PUZZLE TO PRECISION

- Several experimental activities ongoing and proposed:
    - 1S hyperfine splitting in μH and μHe (CREMA, FAMU, J-PARC)
    - Improved measurement of Lamb shift in μH, μD and μHe^+ possible ($\times 5$)
    - Medium- and high-Z muonic atoms

- **Theory support** is needed!

Muonic Atom Spectroscopy Theory Initiative

- Initials objectives:
  - Accurate theory predictions for light muonic atoms to test fundamental interactions by comparing to electronic atoms
  - Community consensus on SM predictions
  - First emphasis on the hyperfine splitting in μH

Join us this afternoon and Saturday morning !

https://indico.him.uni-mainz.de/event/201/overview

"PREN & μASTI" workshop @ JGU, 06/23

Comprehensive theory of the Lamb shift in light muonic atoms

K. Pachucki,[1] V. Lensky,[2] F. Hagelstein,[2,3] S. S. Li Muli,[2] S. Bacca,[2,4] and R. Pohl[5]
[1] *Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*
[2] *Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, 55128 Mainz, Germany*
[3] *Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland*
[4] *Helmholtz-Institut Mainz, Johannes Gutenberg Universität Mainz, 55099 Mainz, Germany*
[5] *Institut für Physik, Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany*

(Dated: May 19, 2023) Rev. Mod. Phys. **96** (2024) 1, 015001

$E_{\rm QED}$	point nucleus	206.034 4(3)	228.774 0(3)	1644.348(8)	1668.491(7)
$\mathcal{C}\, r_C^2$	finite size	$-5.225\,9\, r_p^2$	$-6.107\,4\, r_d^2$	$-103.383\, r_h^2$	$-106.209\, r_\alpha^2$
$E_{\rm NS}$	nuclear structure	0.028 9(25)	1.750 3(200)	15.499(378)	9.276(433)
$E_L(\rm exp)$	experiment[a]	202.370 6(23)	202.878 5(34)	1258.598(48)	1378.521(48)
r_C	this work	0.840 60(39)	2.127 58(78)	1.970 07(94)	1.678 6(12)
r_C	previous[a]	0.840 87(39)	2.125 62(78)	1.970 07(94)	1.678 24(83)

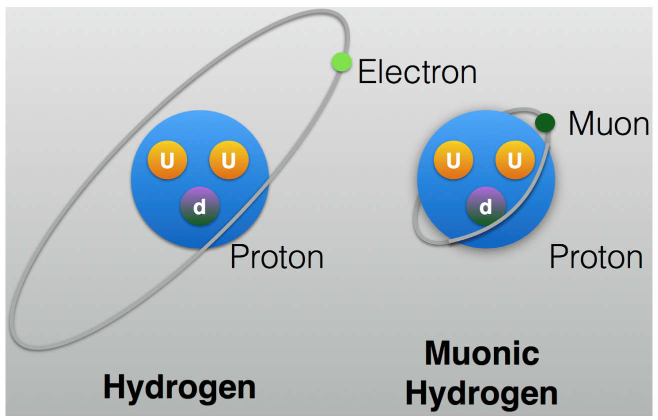
μH: present accuracy comparable with experimental precision

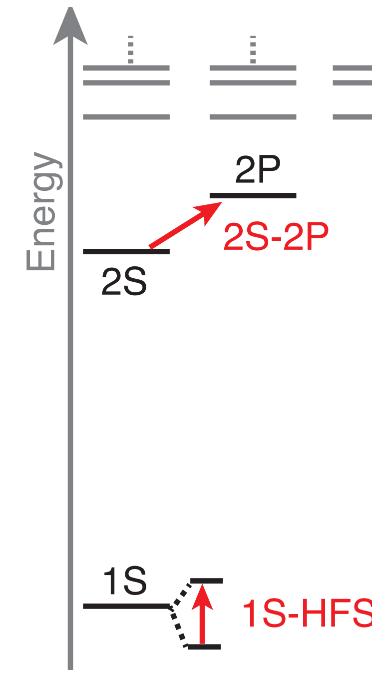
μD, μ³He⁺, μ⁴He⁺: present accuracy factor 5-10 worse than experimental precision

- Experiments will improve by up to a factor of 5
- Theoretical improvement needed for nuclear/nucleon 2- and 3-photon exchange

NUCLEAR STRUCTURE EFFECTS

Why muonic atoms ?





Lamb shift
μH, μD, μ^3He$^+$, μ^4He$^+$
⇒ Charge radii

Hyperfine splitting (HFS)
μH, μ^3He$^+$
⇒ Zemach radii, magnetic properties

- **Lamb shift:**

wave function at the origin

$$\Delta E_{nl}(\mathrm{LO{+}NLO}) = \delta_{l0}\,\frac{2\pi Z\alpha}{3}\,\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R^3_{E(2)}\right]$$

NLO becomes appreciable in μH

- **HFS:**

$$\Delta E_{nS}(\mathrm{LO}+\mathrm{NLO}) = E_F(nS)\,[1 - 2\,Z\alpha m_r\,R_Z]$$

Fermi energy:

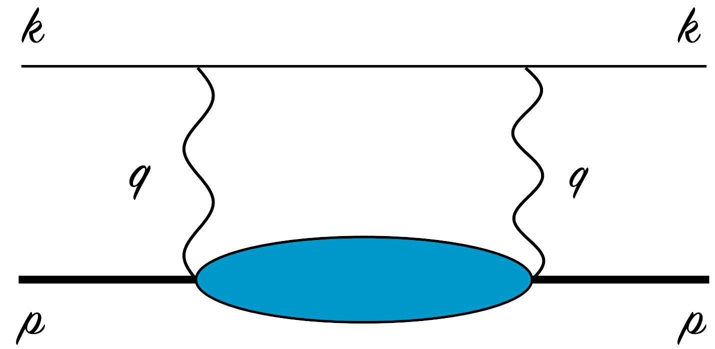
$$E_F(nS) = \frac{8}{3}\frac{Z\alpha}{a^3}\frac{1+\kappa}{mM}\frac{1}{n^3}$$

with Bohr radius $a = 1/(Z\alpha m_r)$

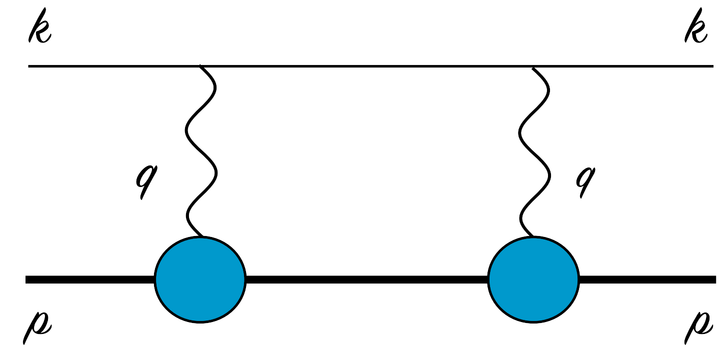
STRUCTURE EFFECTS THROUGH 2γ

- Proton-structure effects at subleading orders arise through multi-photon processes

forward
two-photon exchange (2γ)



polarizability contribution
(non-Born VVCS)



elastic contribution:
finite-size recoil,
3rd Zemach moment (Lamb shift),
Zemach radius (Hyperfine splitting)

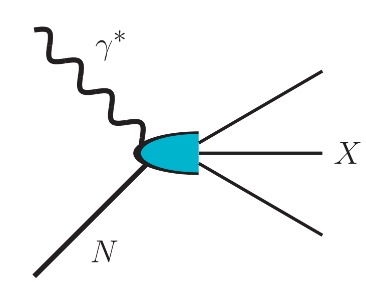
- "Blob" corresponds to doubly-virtual Compton scattering (VVCS):

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \boxed{T_1(\nu,Q^2)} + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right)\boxed{T_2(\nu,Q^2)}$$
$$-\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha \boxed{S_1(\nu,Q^2)} - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right)\boxed{S_2(\nu,Q^2)}$$

- Proton structure functions: $\boxed{f_1(x,Q^2),\ f_2(x,Q^2)}$, $\boxed{g_1(x,Q^2),\ g_2(x,Q^2)}$

Lamb shift

Hyperfine splitting (HFS)



2γ EFFECT IN THE LAMB SHIFT

wave function at the origin

$$\Delta E(nS) = 8\pi\alpha m\, \phi_n^2\, \frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\frac{\left(Q^2-2\nu^2\right)T_1(\nu,Q^2)-\left(Q^2+\nu^2\right)T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

dispersion relation & optical theorem:

$$T_1(\nu,Q^2) = T_1(0,Q^2) + \frac{32\pi Z^2\alpha M\nu^2}{Q^4}\int_0^1 \mathrm{d}x\, \frac{x f_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$T_2(\nu,Q^2) = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\, \frac{f_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

- Caution: in the data-driven dispersive approach the $T_1(0,Q^2)$ subtraction function is modelled!

low-energy expansion:

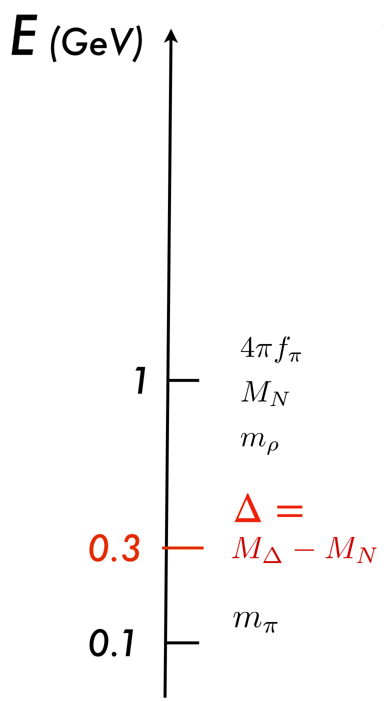
$$\lim_{Q^2\to 0}\overline{T}_1(0,Q^2)/Q^2 = 4\pi\beta_{M1}$$

modelled Q^2 behavior:

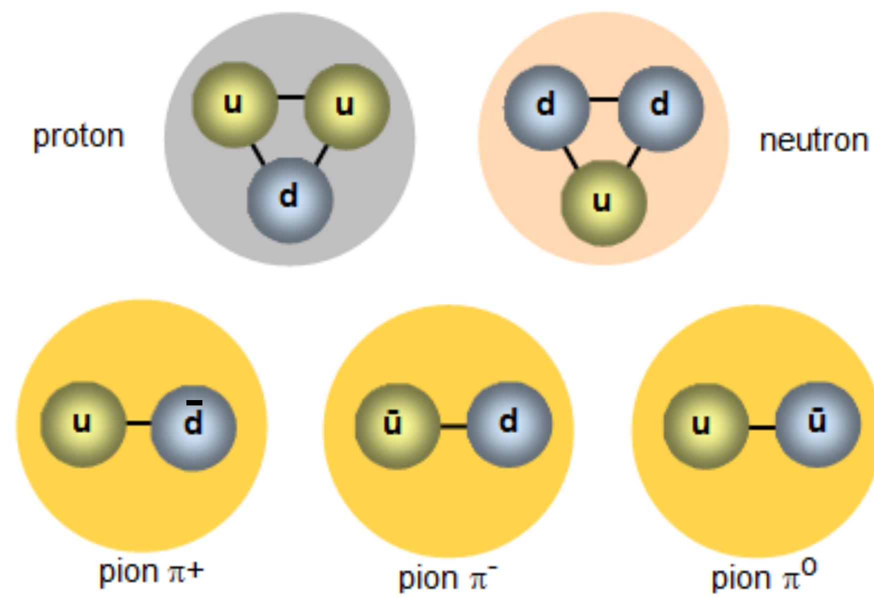
$$\overline{T}_1(0,Q^2) = 4\pi\beta_{M1}\, Q^2/\left(1+Q^2/\Lambda^2\right)^4$$

CHIRAL PERTURBATION THEORY

- ChPT — an effective field theory of QCD at low energies
- Pion, nucleon (and other hadronic) degrees of freedom
- Systematic expansion in powers of momentum over the scale of spontaneous chiral symmetry breaking ($\Lambda_{\chi SB}$~ 1 GeV)
- BChPT with $\Delta(1232)$ has two small parameters: $\epsilon = m_\pi/\Lambda_{\chi SB}$ and $\delta = \Delta/\Lambda_{\chi SB}$. Power counting:
  - δ-expansion: $\epsilon \sim \delta^2$ V. Pascalutsa, D. Phillips, Phys. Rev. C 67 (2003) 055202.
  - ϵ-expansion: $\epsilon \sim \delta$ T. Hemmert, B. Holstein, J. Kambor, Phys. Lett. B (1997) 89.
- HBChPT: additional expansion $1/M_B$



Seminal papers:
[1] S. Weinberg, Physica A **96**, 327 (1979).
[2] J. Gasser, H. Leutwyler, Ann. Phys. **158**, 142 (1984).
[3] J. Gasser, et al., Nucl. Phys. B **307**, 779 (1988).



POLARIZABILITY EFFECT IN μH LAMB SHIFT

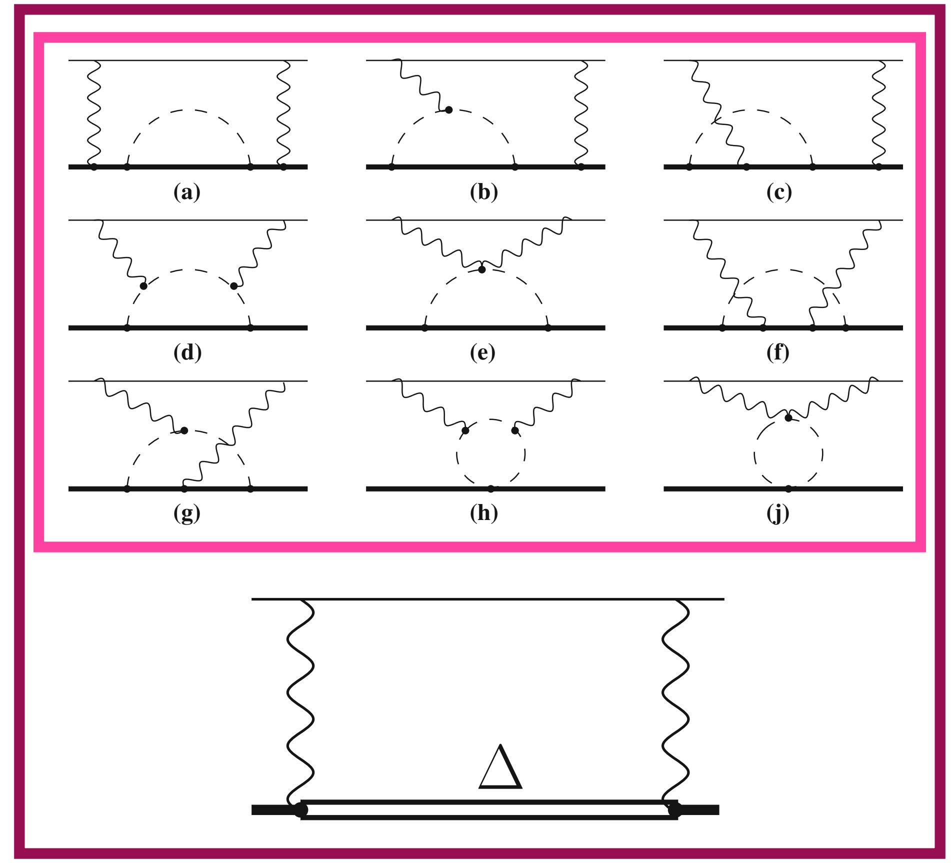
$\propto \beta_{M1}$ $\propto \alpha_{E1}$

Assuming ChPT is working, it should be best applicable to atomic systems, where the energies are very small !

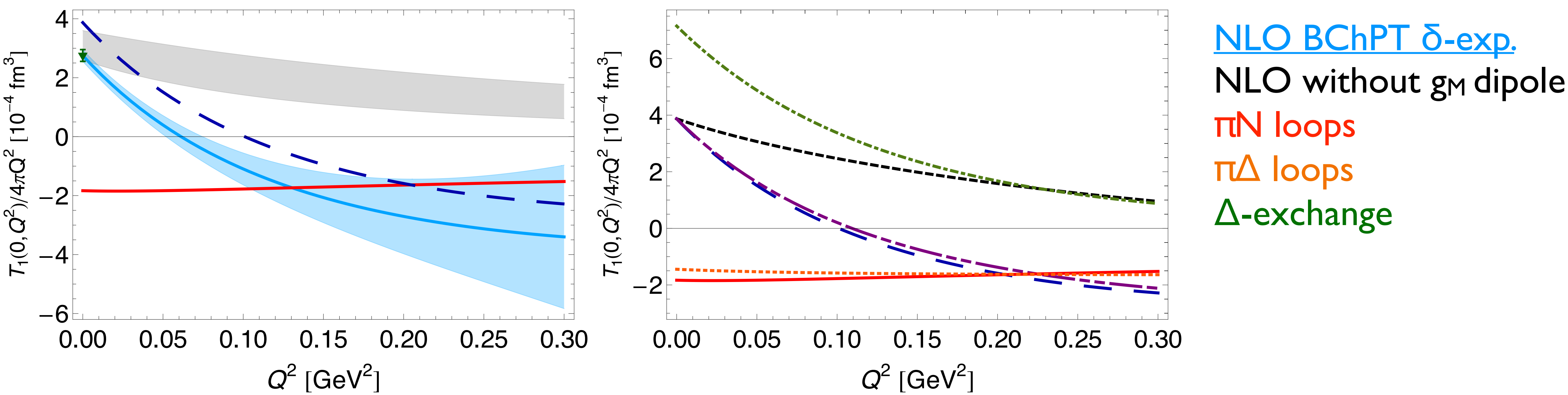
Table 1 Forward 2γ-exchange contributions to the 2S-shift in μH, in units of μeV.

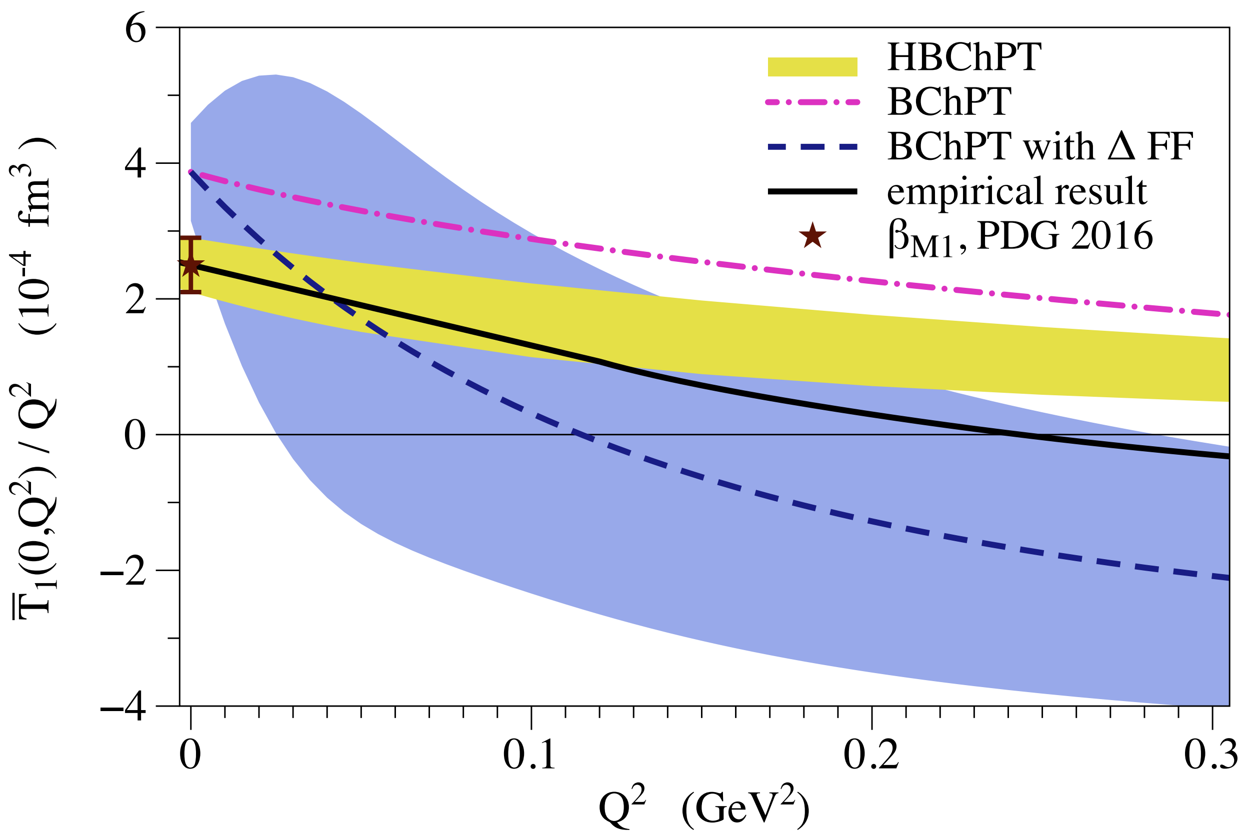
Reference	$E_{2S}^{(\mathrm{subt})}$	$E_{2S}^{(\mathrm{inel})}$	$E_{2S}^{(\mathrm{pol})}$	$E_{2S}^{(\mathrm{el})}$	$E_{2S}^{\langle 2\gamma \rangle}$
DATA-DRIVEN					
(73) Pachucki '99	1.9	−13.9	−12(2)	−23.2(1.0)	−35.2(2.2)
(74) Martynenko '06	2.3	−16.1	−13.8(2.9)		
(75) Carlson *et al.* '11	5.3(1.9)	−12.7(5)	−7.4(2.0)		
(76) Birse and McGovern '12	4.2(1.0)	−12.7(5)	−8.5(1.1)	−24.7(1.6)	−33(2)
(77) Gorchtein et al.'13 [a]	−2.3(4.6)	−13.0(6)	−15.3(4.6)	−24.5(1.2)	−39.8(4.8)
(78) Hill and Paz '16					−30(13)
(79) Tomalak'18	2.3(1.3)		−10.3(1.4)	−18.6(1.6)	−29.0(2.1)
LEADING-ORDER BχPT					
(80) Alarcòn *et al.* '14			$-9.6^{+1.4}_{-2.9}$		
(81) Lensky *et al.* '17 [b]	$3.5^{+0.5}_{-1.9}$	−12.1(1.8)	$-8.6^{+1.3}_{-5.2}$		
LATTICE QCD					
(82) Fu *et al.* '22					−37.4(4.9)

[a] Adjusted values due to a different decomposition into the elastic and polarizability contributions.
[b] Partially includes the Δ(1232)-isobar contribution.



SUBTRACTION FUNCTION





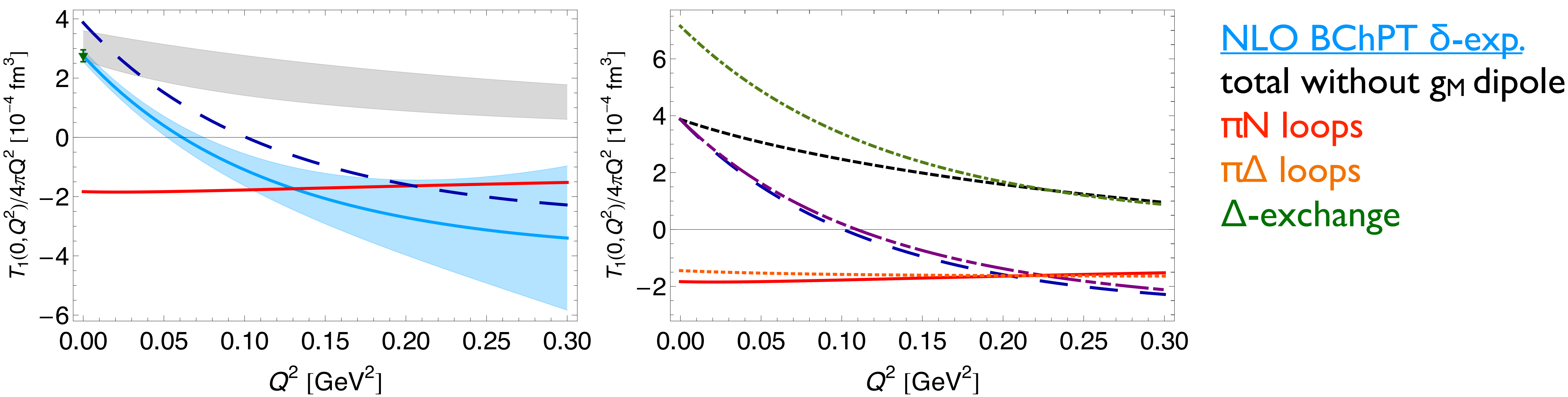
V. Lensky, FH, V. Pascalutsa and M. Vanderhaeghen
Phys. Rev. D **97** (2018) 074012

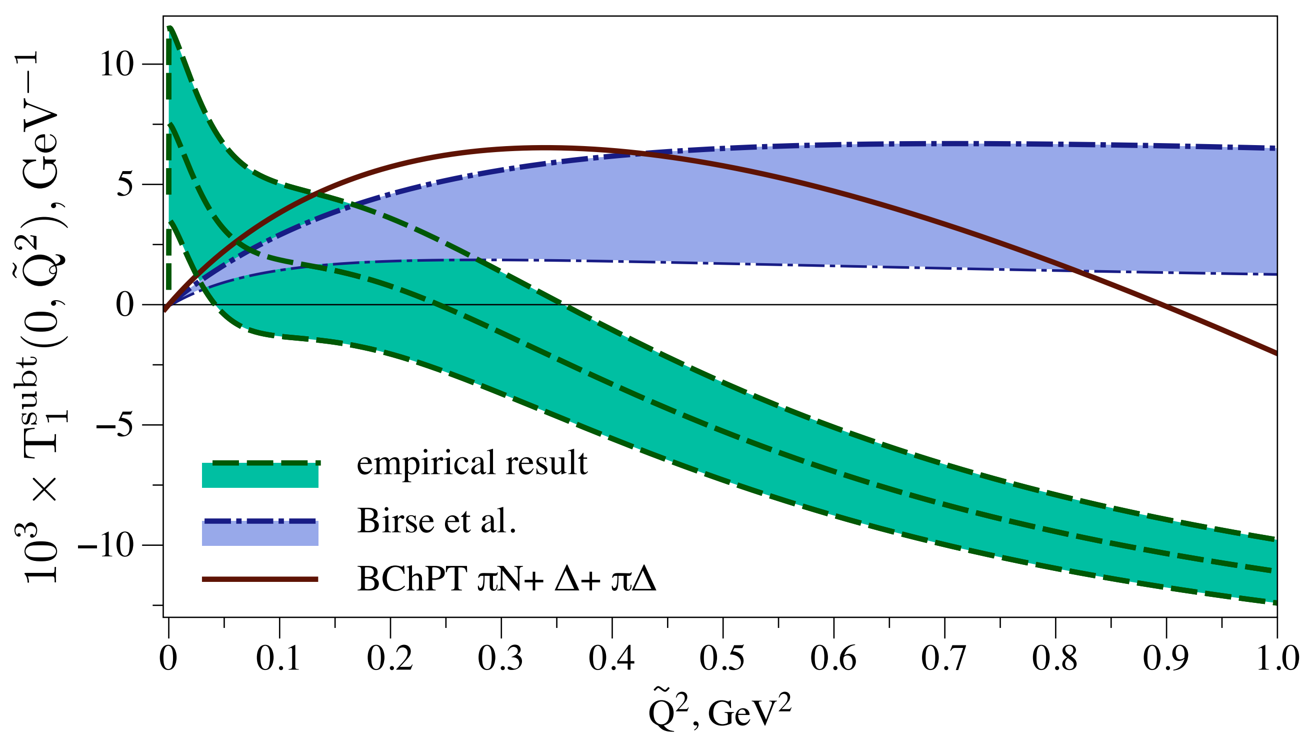
- Related to magnetic dipole polarizability:

$$\lim_{Q^2\to 0} \overline{T}_1(0,Q^2)/Q^2 = 4\pi\beta_{M1}$$

- Dominated by the Δ-exchange contribution:
  - Dipole FF on the magnetic coupling is important → zero crossing

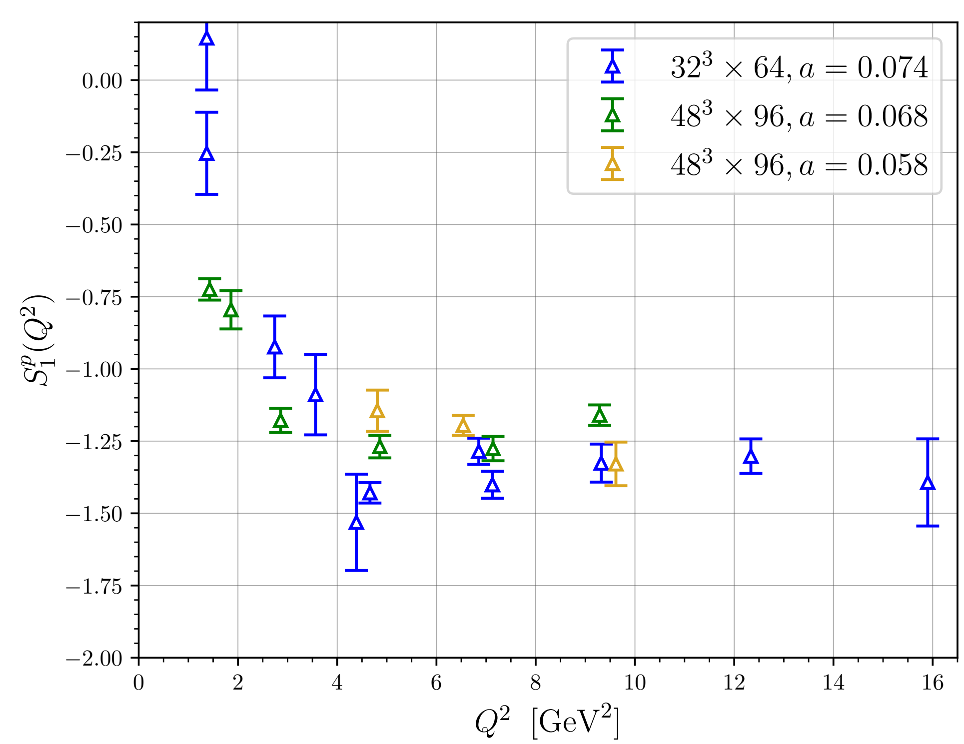
SUBTRACTION FUNCTION





O. Tomalak and M. Vanderhaeghen, EPJ C 76 (2016) 125.

First lattice results!



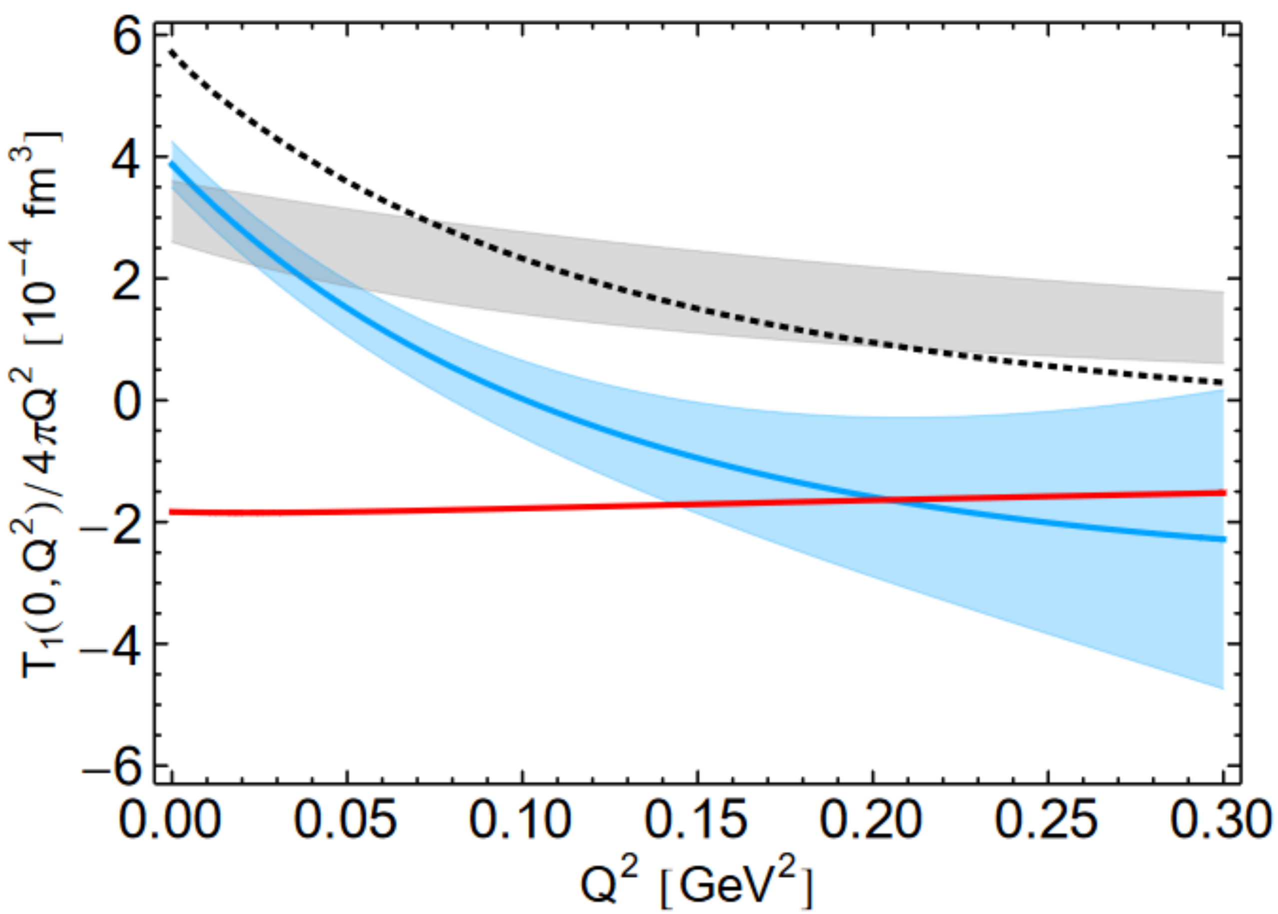
CSSM-QCDSF-UKQCD Collaboration, 2207.03040.

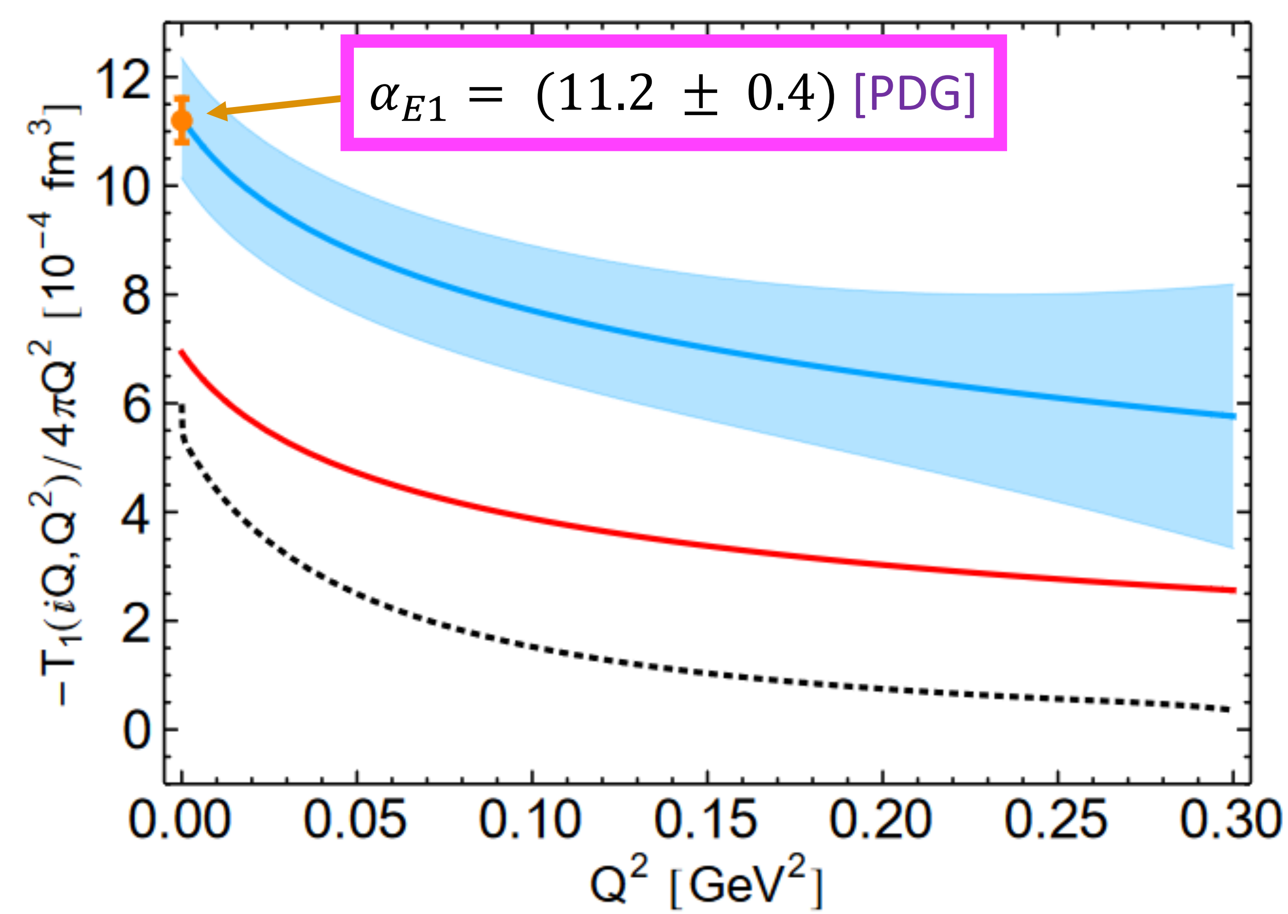
DATA-DRIVEN EVALUATION

- New integral equations for data-driven evaluation of subtraction functions
- High-quality parametrization of σ_L at $Q \to 0$ needed

$$T_1(0,Q^2) = \frac{2Q^2}{\pi}\int_{\nu_0}^{\infty}\frac{\mathrm{d}\nu}{\nu^2+Q^2}\left[\sigma_T - \frac{\nu^2}{Q^2}\sigma_L\right](\nu,Q^2)$$

$$T_L(iQ,Q^2) = \frac{2}{\pi}\int_{\nu_0}^{\infty}\mathrm{d}\nu\,\nu^2\,\frac{\sigma_L(\nu,Q^2)}{\nu^2+Q^2}$$





MAID

NLO χPT [Lensky et al., PRC (2014)] [Alarcón et al., PRD (2020)]

LO χPT: πN-loops

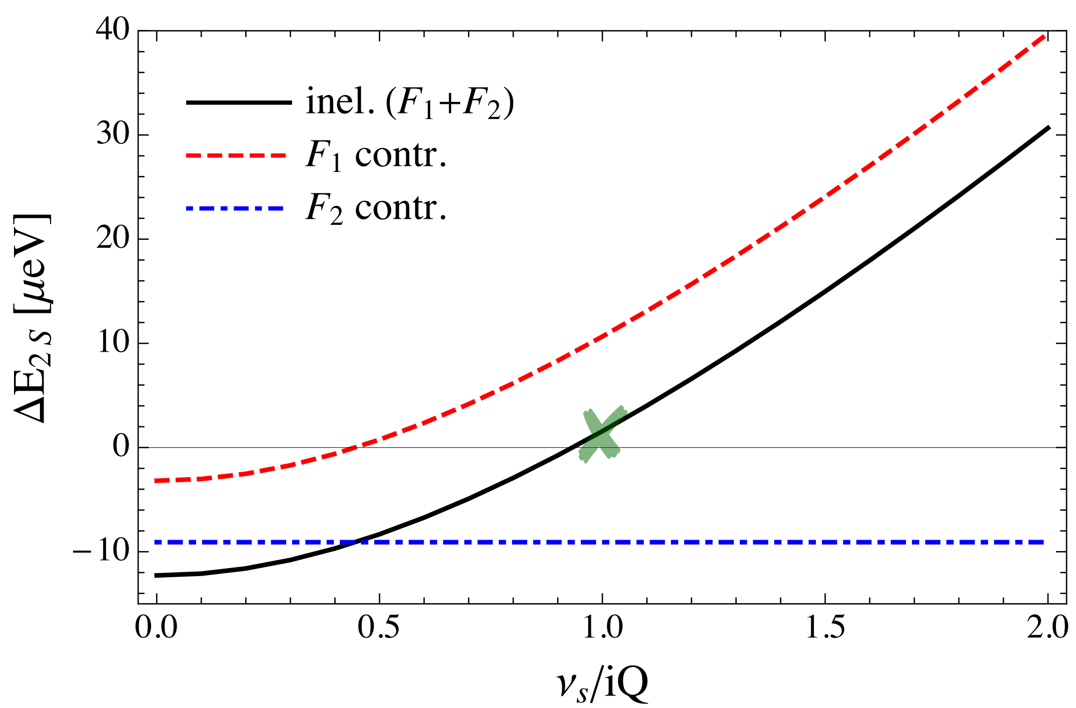
HBχPT [Birse and McGovern, EPJA, (2012)]

EUCLIDEAN SUBTRACTION FUNCTION

- Once-subtracted dispersion relation for $\overline{T}_1(\nu, Q^2)$ with subtraction at $\nu_s = iQ$
- Dominant part of polarizability contribution:

$$\Delta E_{nS}^{'\,(\mathrm{subt})} = \frac{2\alpha m}{\pi} \phi_n^2 \int_0^\infty \frac{\mathrm{d}Q}{Q^3} \frac{2+v_l}{(1+v_l)^2} \overline{T}_1(iQ, Q^2) \text{ with } v_l = \sqrt{1+4m^2/Q^2}$$

- Inelastic contribution for $\nu_s = iQ$ is order of magnitude smaller than for $\nu_s = 0$
- Prospects for future lattice QCD and EFT calculations



FH, V. Pascalutsa, Nucl. Phys. A **1016** (2021) 122323

based on Bosted-Christy parametrization:

$$\Delta E_{2S}^{(\mathrm{inel})}(\nu_s = 0) \simeq -12.3\,\mu\mathrm{eV}$$

$$\Delta E_{2S}^{'\,(\mathrm{inel})}(\nu_s = iQ) \simeq 1.6\,\mu\mathrm{eV}$$

HYPERFINE SPLITTING IN μH

$$\Delta E_{\mathrm{HFS}}(nS) = \left[1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}\right] E_F(nS)$$

with $\Delta_{\mathrm{structure}} = \boxed{\Delta_Z} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$

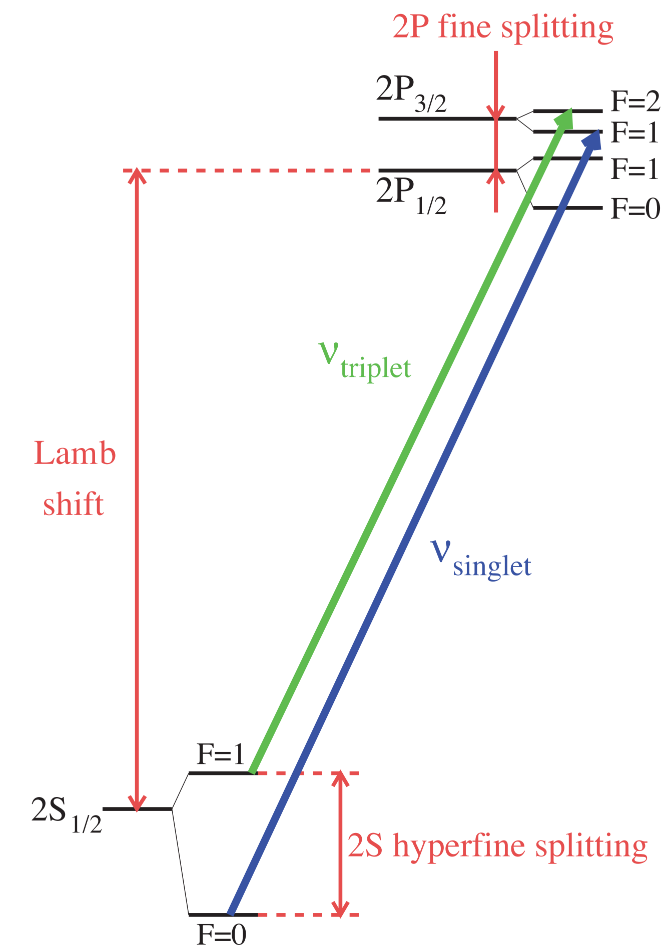
↓

Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left[\frac{G_E(Q^2) G_M(Q^2)}{1+\kappa} - 1\right] \equiv -2Z\alpha m_r R_{\mathrm{Z}}$$

experimental value: $R_Z = 1.082(37)\ \mathrm{fm}$

A. Antognini, et al., Science **339** (2013) 417–420



Measurements of the μH ground-state HFS planned by the CREMA, FAMU and J-PARC / Riken-RAL collaborations

- Very precise input for the 2γ effect needed to narrow down frequency search range for experiment
- Zemach radius can help to pin down the magnetic properties of the proton

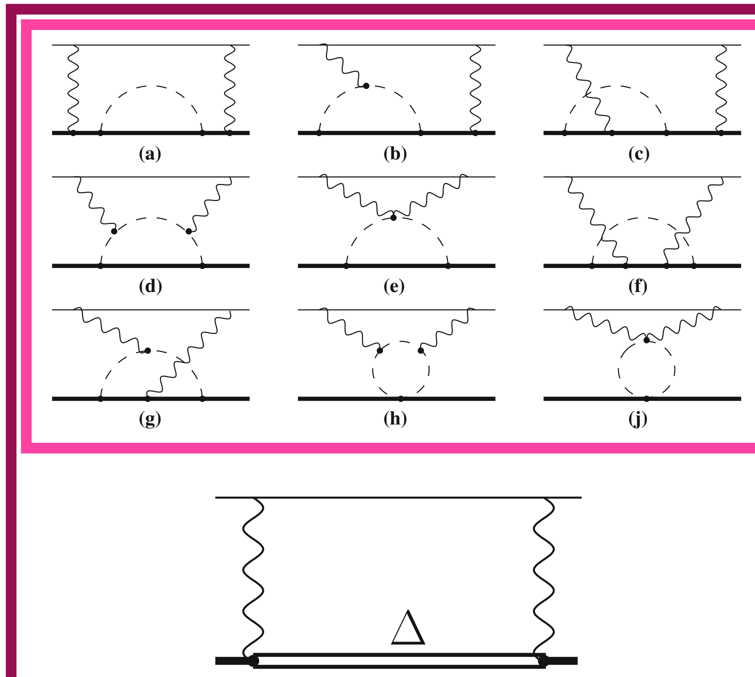
2γ EFFECT IN THE μH HFS

Table 1 Forward 2γ-exchange contribution to the HFS in μH.

Reference	Δ_Z [ppm]	Δ_{recoil} [ppm]	Δ_{pol} [ppm]	Δ_1 [ppm]	Δ_2 [ppm]	$E^{\langle 2\gamma\rangle}_{1S\text{-hfs}}$ [meV]
DATA-DRIVEN						
Pachucki '96 (1)	−8025	1666	0(658)			−1.160
Faustov et al. '01 (9)[a]	−7180		410(80)	468	−58	
Faustov et al. '06 (10)[b]			470(104)	518	−48	
Carlson et al. '11 (11)[c]	−7703	931	351(114)	370(112)	−19(19)	−1.171(39)
Tomalak '18 (12)[d]	−7333(48)	846(6)	364(89)	429(84)	−65(20)	−1.117(19)
HEAVY-BARYON χPT						
Peset et al. '17 (13)						−1.161(20)
LEADING-ORDER χPT						
Hagelstein et al. '16 (14)			37(95)	29(90)	9(29)	
+ $\Delta(1232)$ EXCIT.						
Hagelstein et al. '18 (15)			−13	84	−97	

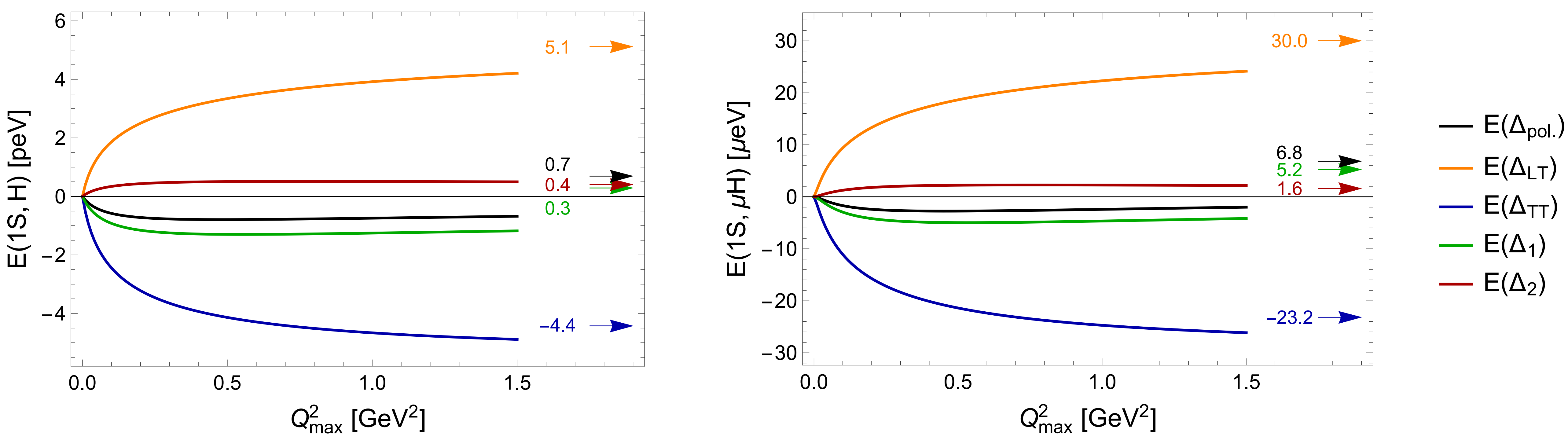
[a] Adjusted values: Δ_{pol} and Δ_1 corrected by −46 ppm as described in Ref. 16.
[b] Different convention was used to calculate the Pauli form factor contribution to Δ_1, which is equivalent to the approximate formula in the limit of $m = 0$ used for H in Ref. 11.
[c] Elastic form factors from Ref. 17 and updated error analysis from Ref. 16. Note that this result already includes radiative corrections for the Zemach-radius contribution, $(1+\delta_Z^{\text{rad}})\Delta_Z$ with $\delta_Z^{\text{rad}} \sim 0.0153$ (18, 19), as well as higher-order recoil corrections with the proton anomalous magnetic moment, cf. (11, Eq. 22) and (18).
[d] Uses r_p from μH (20) as input.

Assuming ChPT is working, it should be best applicable to atomic systems, where the energies are very small !



POLARIZABILITY EFFECT FROM BCHPT

- Low-Q region is very important!
- LO BChPT result is compatible with zero
  - Contributions from σ_{LT} and σ_{TT} are sizeable and largely cancel each other



POLARIZABILITY EFFECT IN HFS

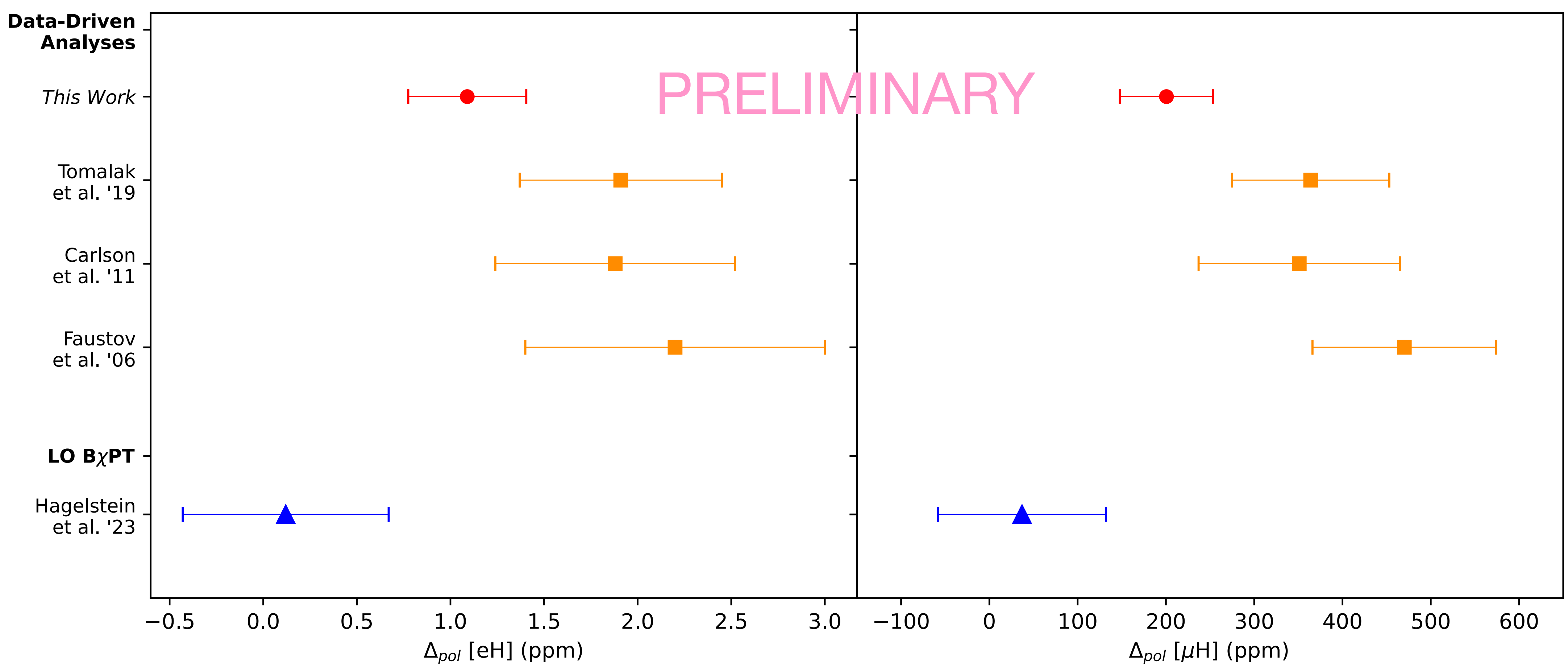
- Polarizability effect on the HFS is completely constrained by empirical information

$$\Delta_{\text{pol.}} = \Delta_1 + \Delta_2 = \frac{\alpha m}{2\pi(1+\kappa)M}(\delta_1+\delta_2)$$

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left\{\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)+F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\frac{1}{(v_l+v_x)(1+v_x)(1+v_l)}\left(4+\frac{1}{1+v_x}+\frac{1}{v_l+1}\right)\right\}$$

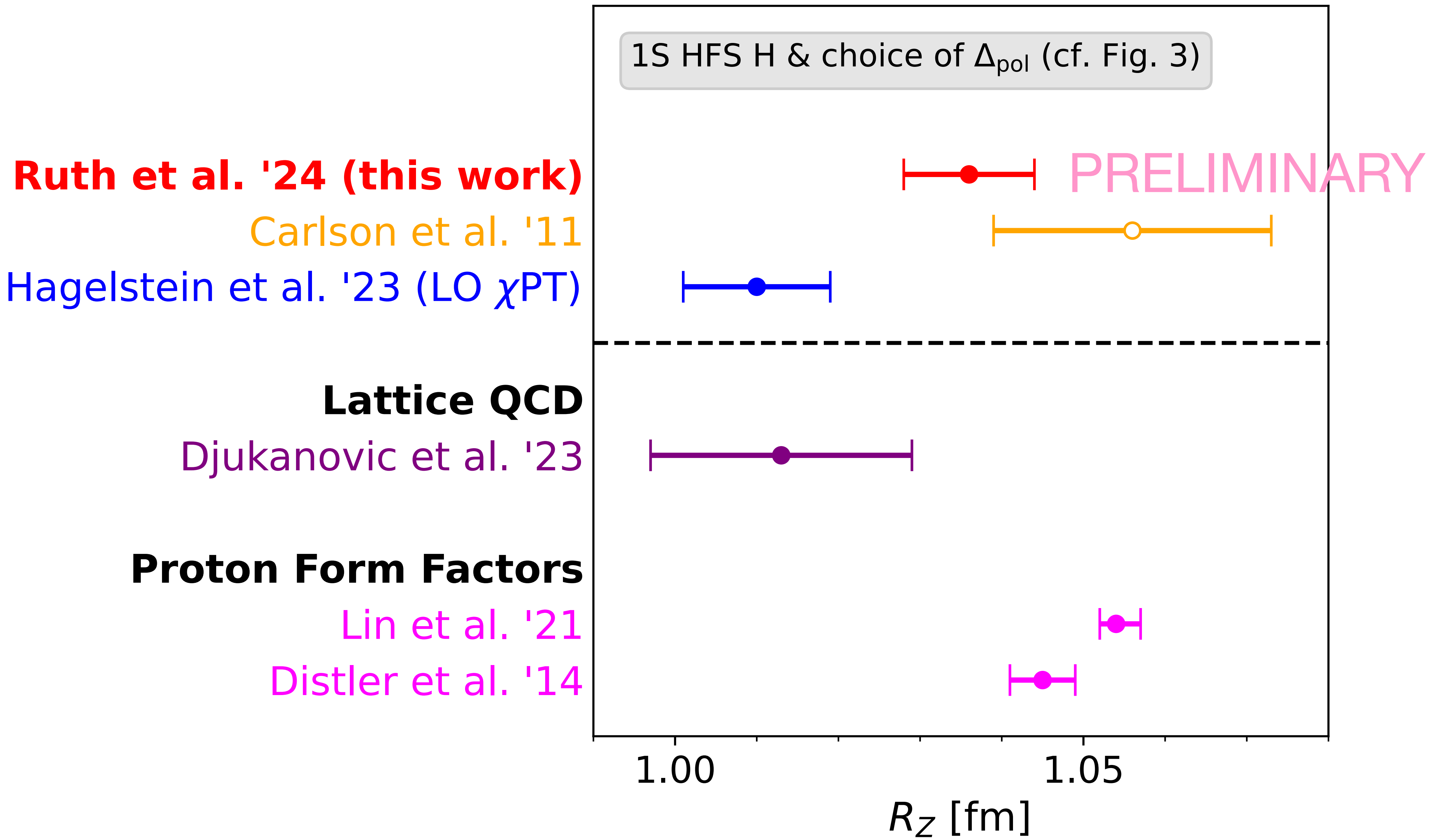
$$\delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left(\frac{1}{v_l+v_x}-\frac{1}{v_l+1}\right)$$

with $v_l = \sqrt{1+\frac{1}{\tau_l}}$, $v_x = \sqrt{1+x^2\tau^{-1}}$, $\tau_l = \frac{Q^2}{4m^2}$ and $\tau = \frac{Q^2}{4M^2}$

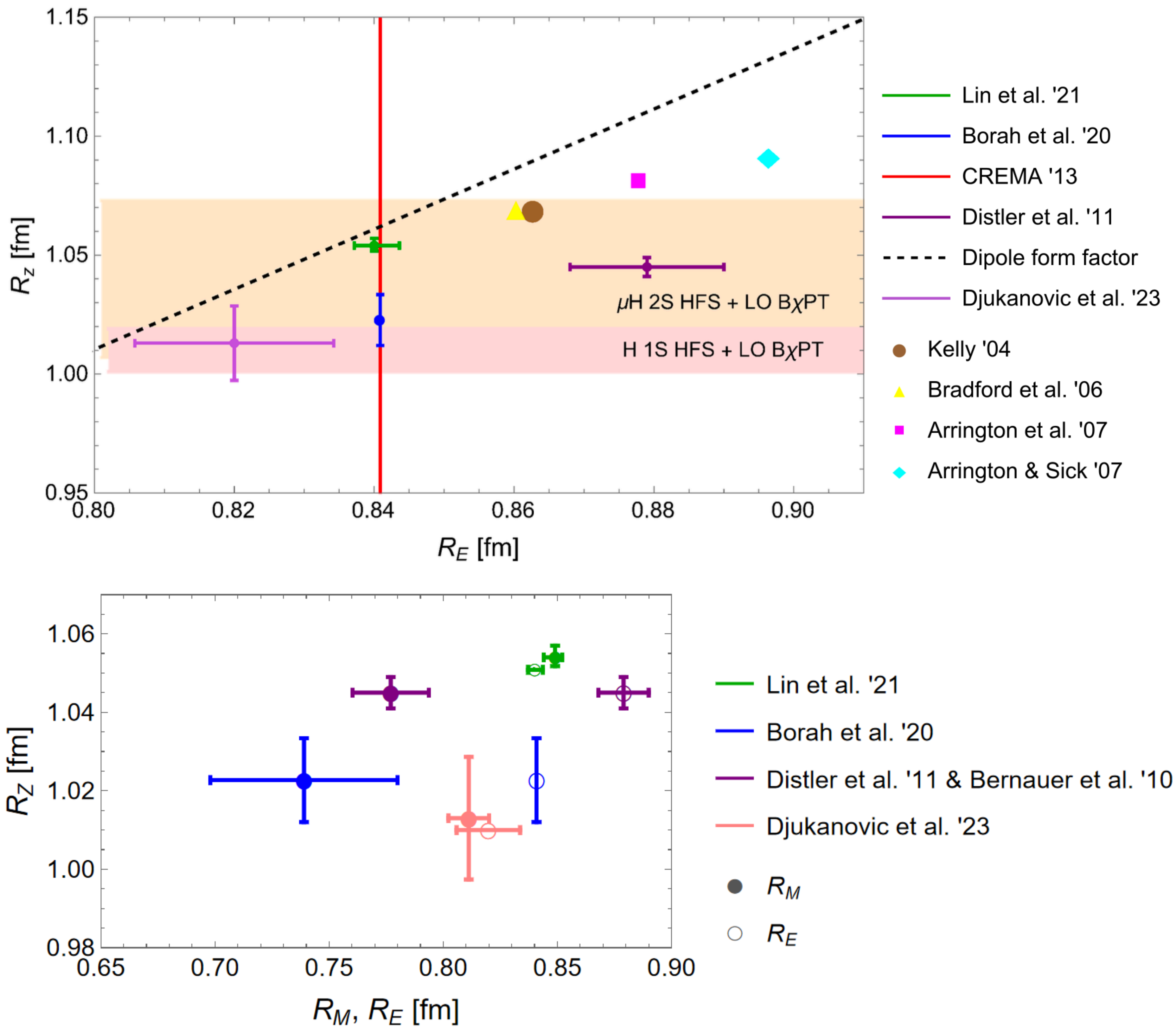


PROTON ZEMACH RADIUS

- BChPT polarizability prediction implies smaller Zemach radius (smaller, just like r_p)



CORRELATION OF PROTON RADII



HYPERFINE SPLITTING

Theory: QED, ChPT, data-driven dispersion relations, ab-initio few-nucleon theories

Experiment: HFS in μH, μHe$^+$, …

Testing the theory

- discriminate between theory predictions for polarizability effect
  - disentangle R_Z & polarizability effect by combining HFS in H & μH
- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

extract E^{TPE}, $E^{\mathrm{pol.}}$ or R_Z

Determine fundamental constants

Zemach radius R_Z

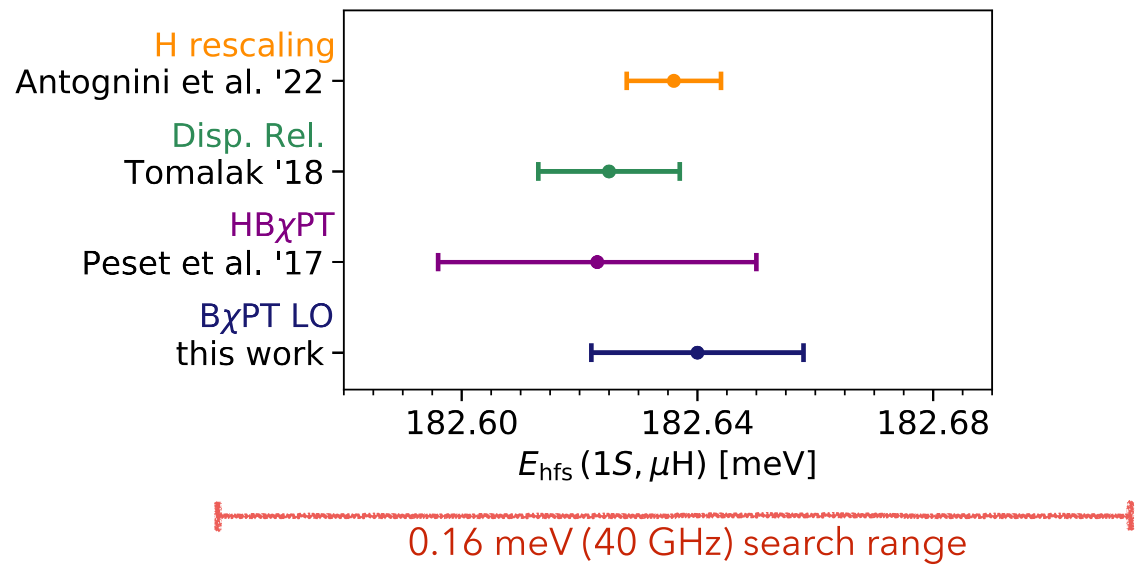
Input for data-driven evaluations

form factors, structure functions, polarizabilities

Spectroscopy of ordinary atoms (H, He$^+$)

Electron and Compton Scattering

PLITTING



Predictions for the 1S HFS in μH are driven by the 1S HFS in H

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022)

Experiment: HFS in μH, μHe$^+$, …

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

extract $E^{\rm TPE}$, $E^{\rm pol.}$ or R_Z

Input for data-driven evaluations

form factors, structure functions, polarizabilities

esting the theory

inate between theory predictions for polarizability effect

- disentangle R_Z & polarizability effect by combining HFS in H & μH

▸ test HFS theory

- combining HFS in H & μH with theory prediction for polarizability effect

▸ test nuclear theories

Determine fundamental constants

Zemach radius R_Z

Spectroscopy of ordinary atoms (H, He$^+$)

Electron and Compton Scattering

Join us this afternoon and Saturday morning !

JG|U
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Atomic Spectroscopy Theory Initiative

HOME SEARCH SITEMAP

HOME AIMS AND SCOPE MAILING LIST WORKING GROUPS PAST AND FUTURE WORKSHOPS PUBLICATIONS NEWS

- **Home**
- Aims and Scope
- Mailing List
- Working Groups
- Past and Future Workshops
- Publications
- News

Muonic Atom Spectroscopy Theory Initiative

Inspired by the success of the Muon g-2 Theory Initiative we are launching the Muonic Atom Spectroscopy Theory Initiative (μASTI).

The initiative aims to support the experimental effort on the spectroscopy of light muonic atoms by improving the Standard Model theory predictions for the Lamb shift and hyperfine splitting in muonic hydrogen, deuterium, and helium, in order to match the anticipated accuracy of future measurements. An initial focus will be on the ground state hyperfine splitting in muonic hydrogen.

The **upcoming kick-off event** for the Theory Initiative is organized as a joint meeting with the Proton Radius European Network (PREN) at the Johannes Gutenberg University Mainz (June 26-30, 2023).

from PUZZLE
ASTI
to PRECISION

Homepage and mailing list → https://asti.uni-mainz.de

Thank you for your attention!

FNSNF

Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

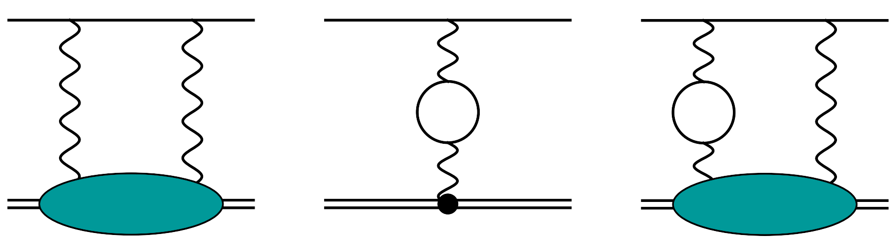
THEORY OF HYPERFINE SPLITTING

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022) 389-418

The hyperfine splitting of μH (theory update):

$$E_{1S\text{-hfs}} = \Big[\underbrace{182.443}_{E_\mathrm{F}} \underbrace{+1.350(7)}_{\text{QED+weak}} \underbrace{+0.004}_{\text{hVP}} \underbrace{-1.30653(17)\left(\frac{r_{Zp}}{\text{fm}}\right) + E_\mathrm{F}\left(1.01656(4)\,\Delta_\text{recoil} + 1.00402\,\Delta_\text{pol}\right)}_{2\gamma\text{ incl. radiative corr.}}\Big]\,\text{meV}$$

- 2γ + radiative corrections $\Longrightarrow$ differ for H vs. μH and 1S vs. 2S



+ …

The hyperfine splitting of H (theory update):

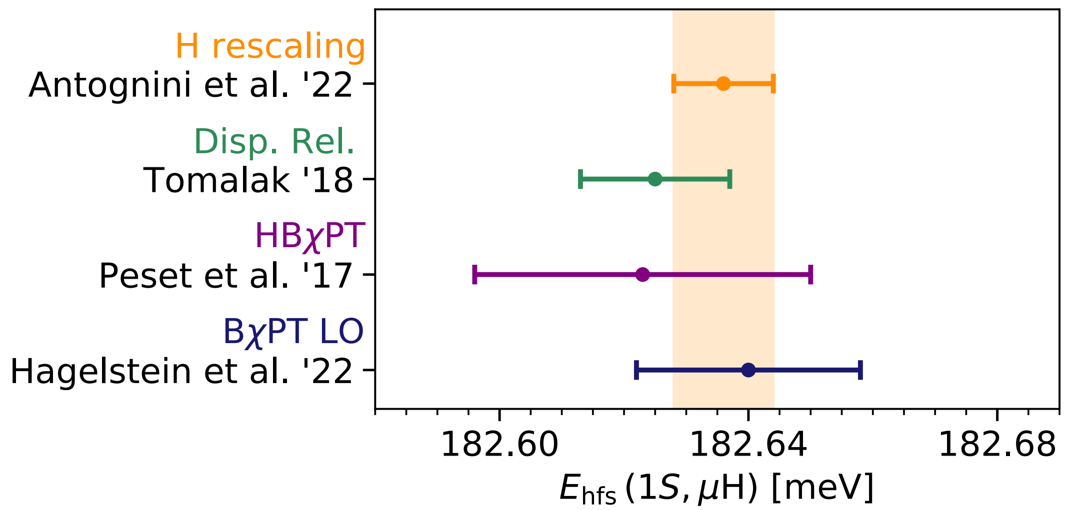
$$E_{1S\text{-hfs}}(\text{H}) = \Big[\underbrace{1\,418\,840.082(9)}_{E_\mathrm{F}} \underbrace{+1\,612.673(3)}_{\text{QED+weak}} \underbrace{+0.274}_{\mu\text{VP}} \underbrace{+0.077}_{\text{hVP}}$$
$$\underbrace{-54.430(7)\left(\frac{r_{Zp}}{\text{fm}}\right) + E_\mathrm{F}\left(0.99807(13)\,\Delta_\text{recoil} + 1.00002\,\Delta_\text{pol}\right)}_{2\gamma\text{ incl. radiative corr.}}\Big]\,\text{kHz}$$

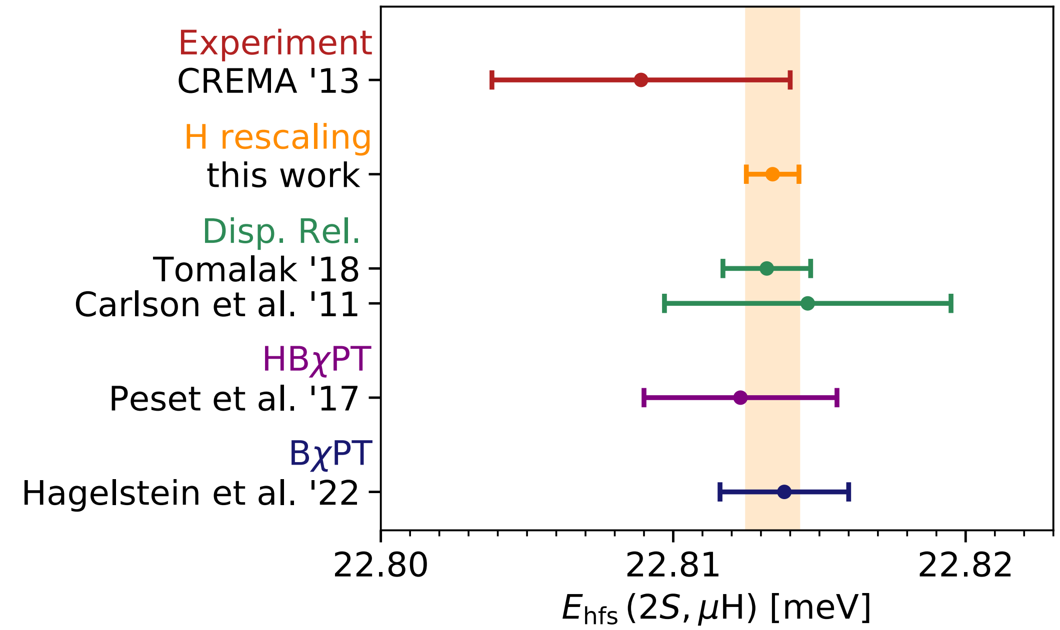
High-precision measurement of the "21cm line" in H:

$$\delta\left(E^{\text{exp.}}_{1S\text{–hfs}}(\text{H})\right) = 10\times 10^{-13}$$

Hellwig et al., 1970

IMPACT OF H 1S HFS





- Leverage radiative corrections $E^{\rm Z+pol}_{1S-\rm hfs}(\mathrm{H}) = E_{\rm F}(\mathrm{H})\Big[b_{1S}(\mathrm{H})\,\Delta_{\rm Z}(\mathrm{H}) + c_{1S}(\mathrm{H})\,\Delta_{\rm pol}(\mathrm{H})\Big] = -\,54.900(71)\,\mathrm{kHz}$ and assume the non-recoil $\mathcal{O}(\alpha^5)$ effects have simple scaling $\dfrac{\Delta_i(\mathrm{H})}{m_r(\mathrm{H})} = \dfrac{\Delta_i(\mu\mathrm{H})}{m_r(\mu\mathrm{H})}, \quad i = \mathrm{Z, pol}$

1. Prediction for μH HFS from empirical 1S HFS in H

$$E^{\rm Z+pol}_{nS-\rm hfs}(\mu\mathrm{H}) = \frac{E_{\rm F}(\mu\mathrm{H})\,m_r(\mu\mathrm{H})\,b_{nS}(\mu\mathrm{H})}{n^3 E_{\rm F}(\mathrm{H})\,m_r(\mathrm{H})\,b_{1S}(\mathrm{H})}E^{\rm Z+pol}_{1S-\rm hfs}(\mathrm{H}) - \frac{E_{\rm F}(\mu\mathrm{H})}{n^3}\,\Delta_{\rm pol}(\mu\mathrm{H}) \quad \underbrace{\left[c_{1S}(\mathrm{H})\frac{b_{nS}(\mu\mathrm{H})}{b_{1S}(\mathrm{H})} - c_{nS}(\mu\mathrm{H})\right]}_{= -6\times10^{-5}\ \text{for n = 1} \quad = -5\times10^{-5}\ \text{for n = 2}}$$

2. Disentangle Zemach radius and polarizability contribution
3. Testing the theory