Searching for Ultralight Scalar Dark Matter with Muonium and Muonic Atoms

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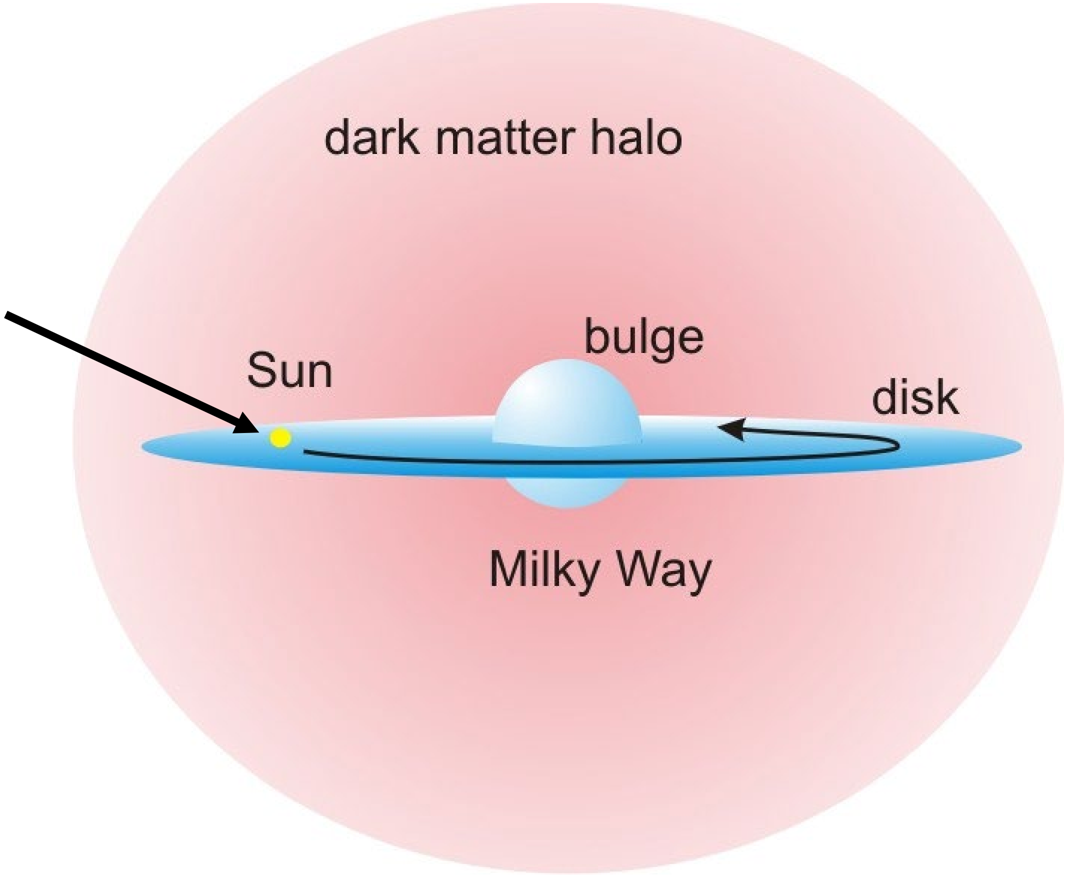
Dark Matter

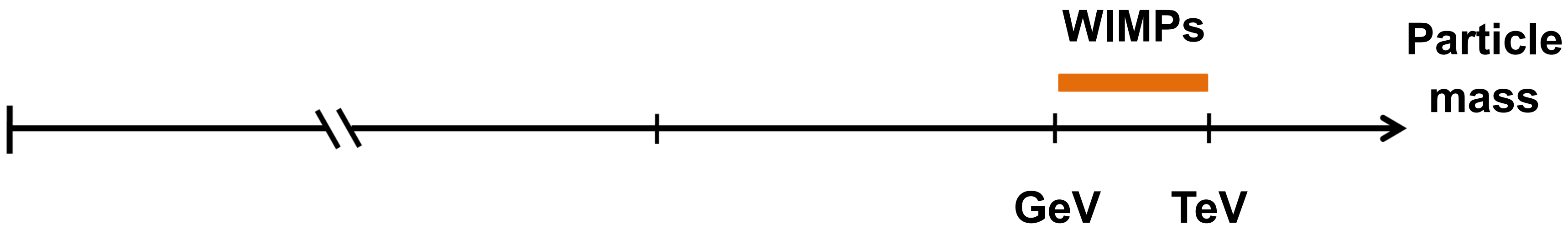
Strong astrophysical evidence for existence of **dark matter** (~5 times more dark matter than ordinary matter)

$$\rho_{\mathrm{DM}} \approx 0.4\ \mathrm{GeV/cm^3}$$

$$v_{\mathrm{DM}} \sim 300\ \mathrm{km/s}$$



Dark Matter

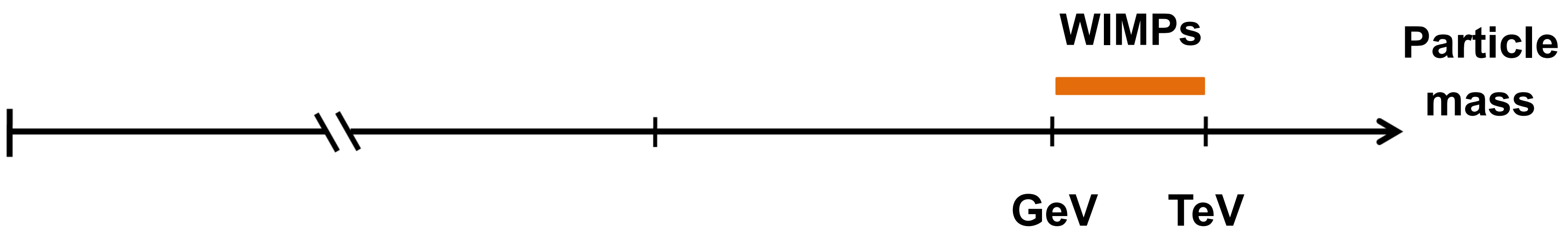


WIMPs

Particle mass

GeV **TeV**

Dark Matter

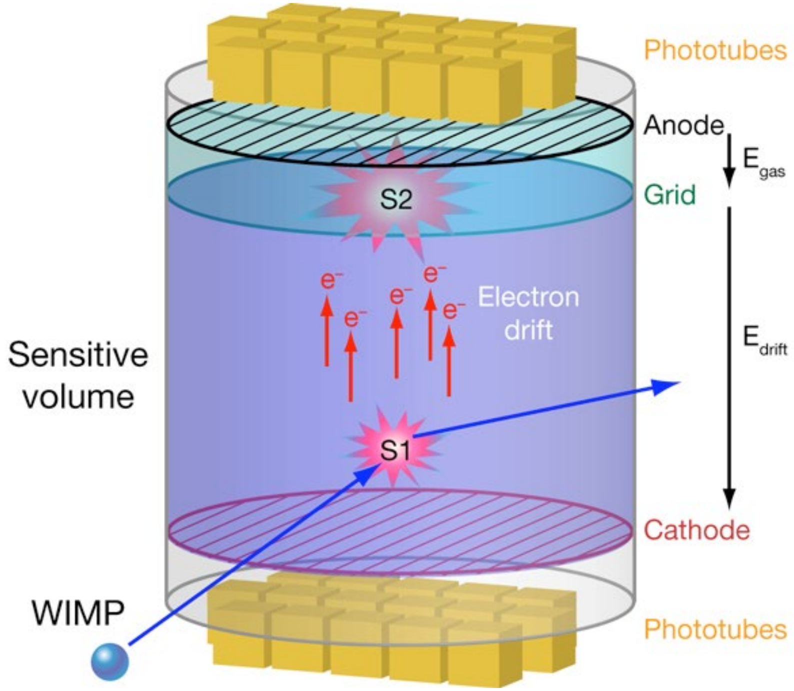


WIMPs

Particle mass

GeV **TeV**

Particle-like signatures



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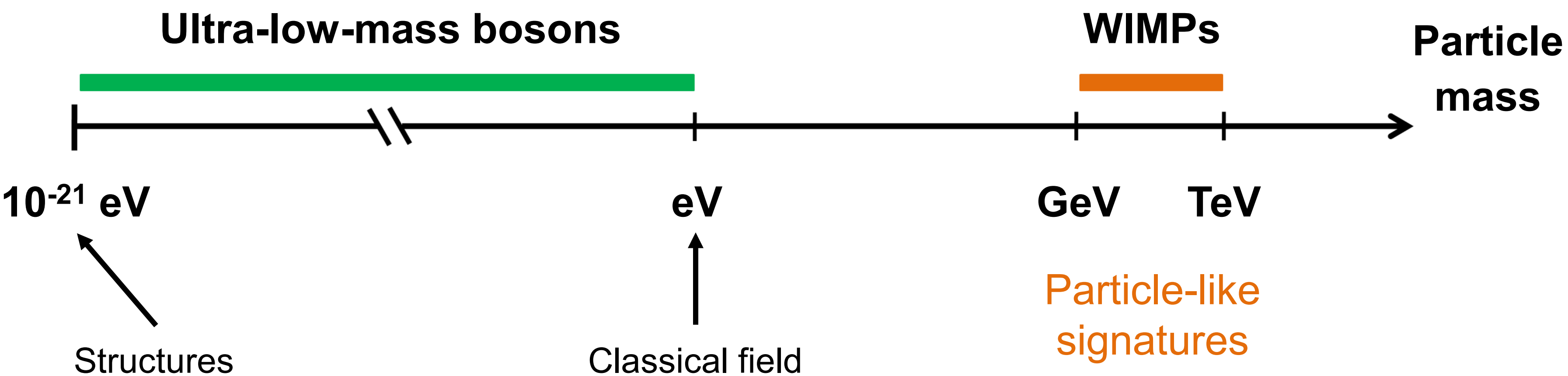


Ultra-low-mass bosons

WIMPs

Particle mass

10^{-21} eV

eV

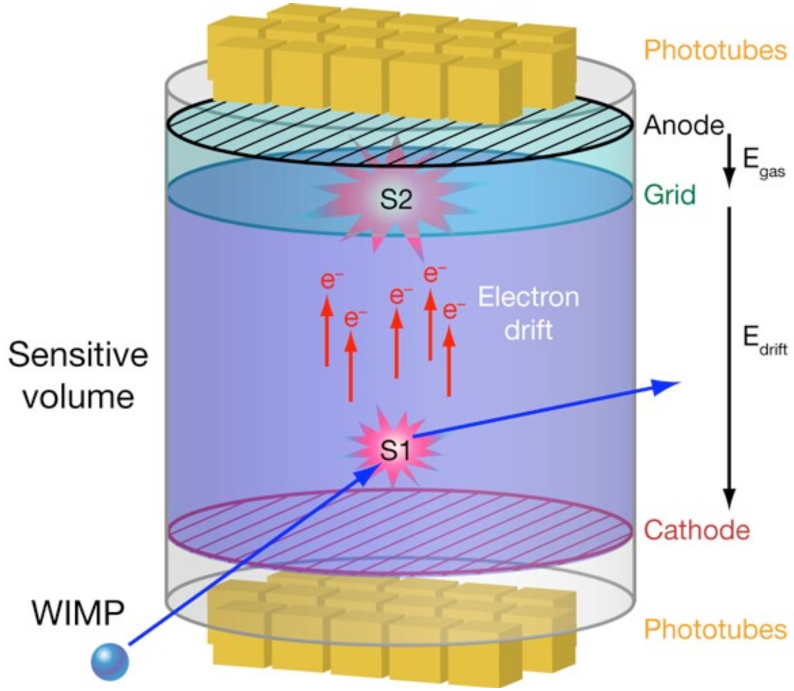
GeV

TeV

Particle-like signatures

Classical field

Structures (Lyman-α forest, dwarf galaxies)



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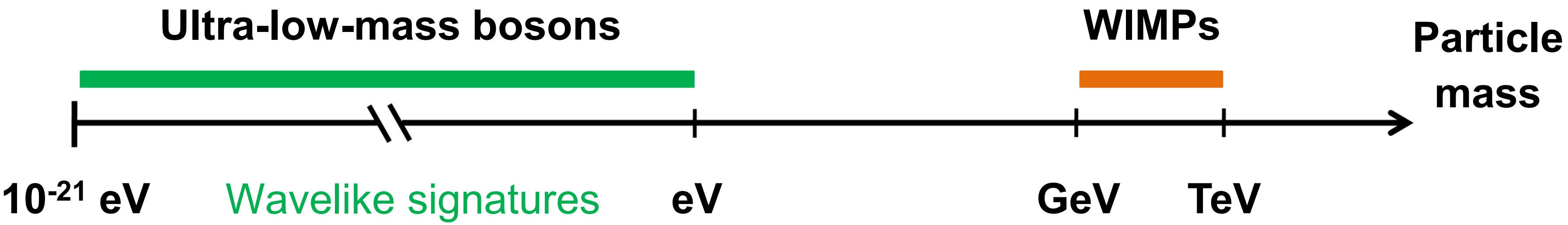

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Wavelike signatures

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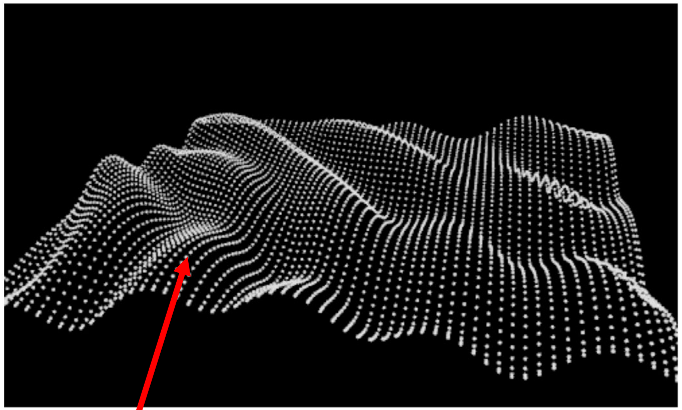
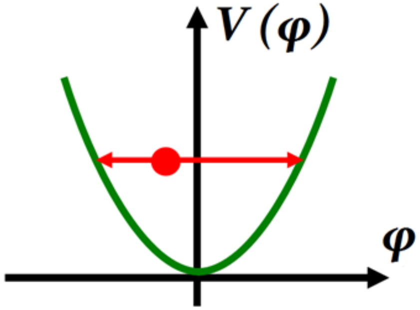
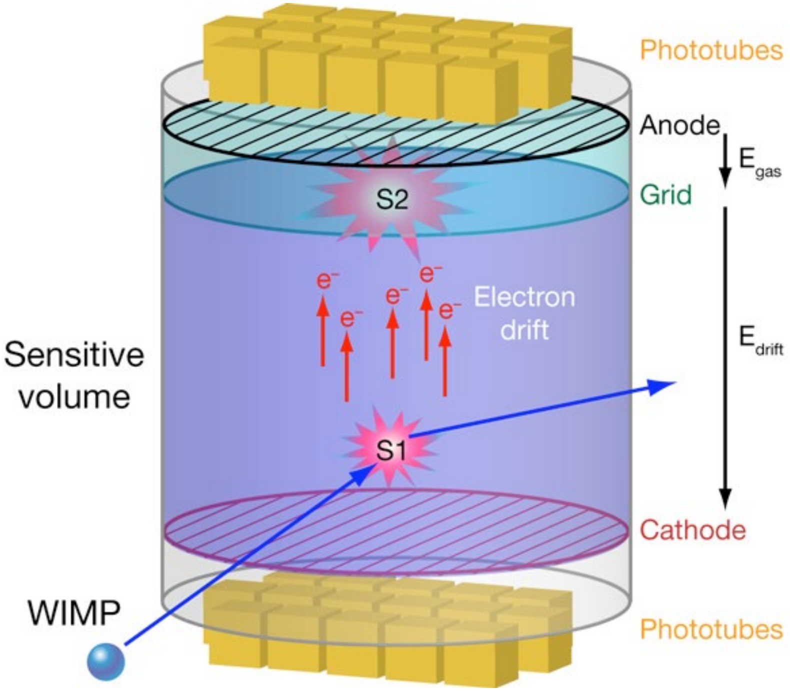


$$\varphi(t) \sim \varphi_0 \cos\left(m_\varphi c^2 t/\hbar\right)$$



Particle-like signatures



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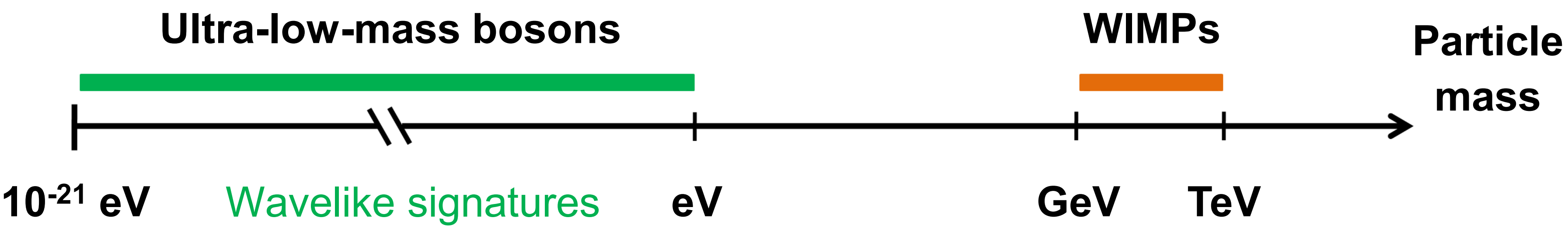

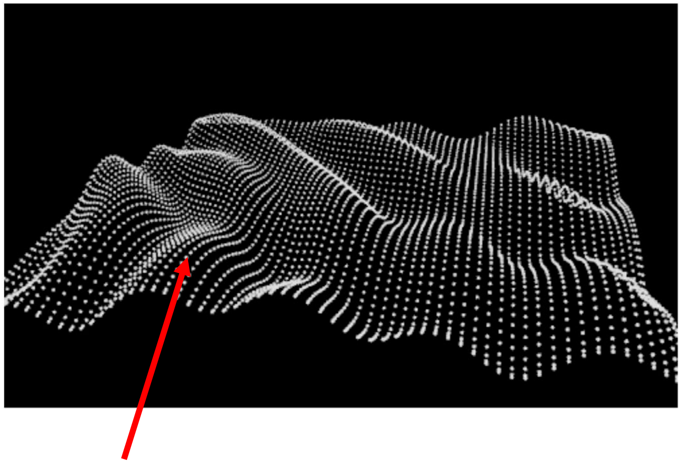

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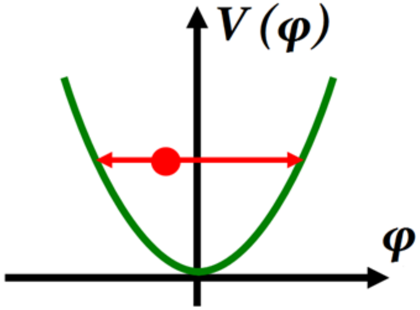
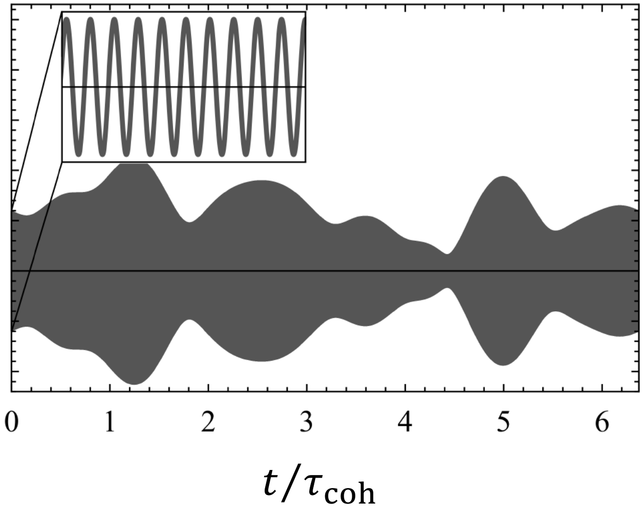


$$\Delta E_\varphi / E_\varphi \sim \Delta v_\varphi^2 / c^2 \sim 10^{-6}$$

$$\Rightarrow \ \tau_{\mathrm{coh}} \sim 2\pi / \Delta E_\varphi \sim 10^6 T_{\mathrm{osc}}$$

Evolution of φ_0 with time



Dark-Matter-Induced Variations of the Fundamental Constants

[Stadnik, Flambaum, *PRL* **114**, 161301 (2015); *PRL* **115**, 201301 (2015)],
[Hees, Minazzoli, Savalle, Stadnik, Wolf, *PRD* **98**, 064051 (2018)]

$$\mathcal{L}_\gamma = \frac{\varphi}{\Lambda_\gamma}\frac{F_{\mu\nu}F^{\mu\nu}}{4} \approx \frac{\varphi_0\cos(m_\varphi t)}{\Lambda_\gamma}\frac{F_{\mu\nu}F^{\mu\nu}}{4} \quad \Rightarrow \quad \frac{\delta\alpha}{\alpha} \approx \frac{\varphi_0\cos(m_\varphi t)}{\Lambda_\gamma}$$

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$$\mathcal{L}_f = -\frac{\varphi}{\Lambda_f} m_f \bar{f} f \approx -\frac{\varphi_0\cos(m_\varphi t)}{\Lambda_f} m_f \bar{f} f \quad \Rightarrow \quad \frac{\delta m_f}{m_f} \approx \frac{\varphi_0\cos(m_\varphi t)}{\Lambda_f}$$

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φ^2 interactions also exhibit the same oscillating-in-time signatures as above (except at frequency $2m_\varphi$), as well as …

* φ^2 interactions may arise in models with a Z_2 symmetry ($\varphi \rightarrow -\varphi$)

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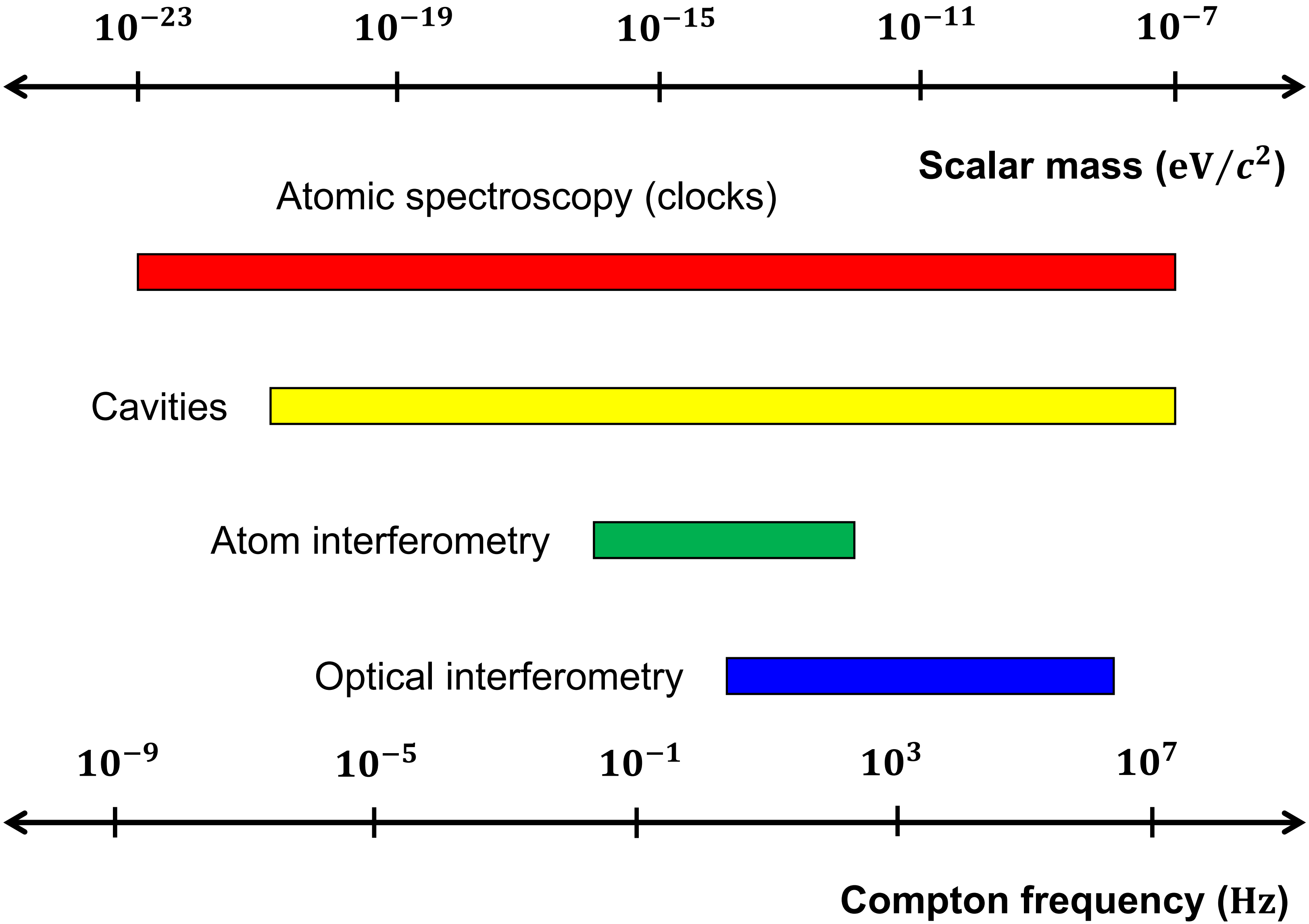
$$\Downarrow$$

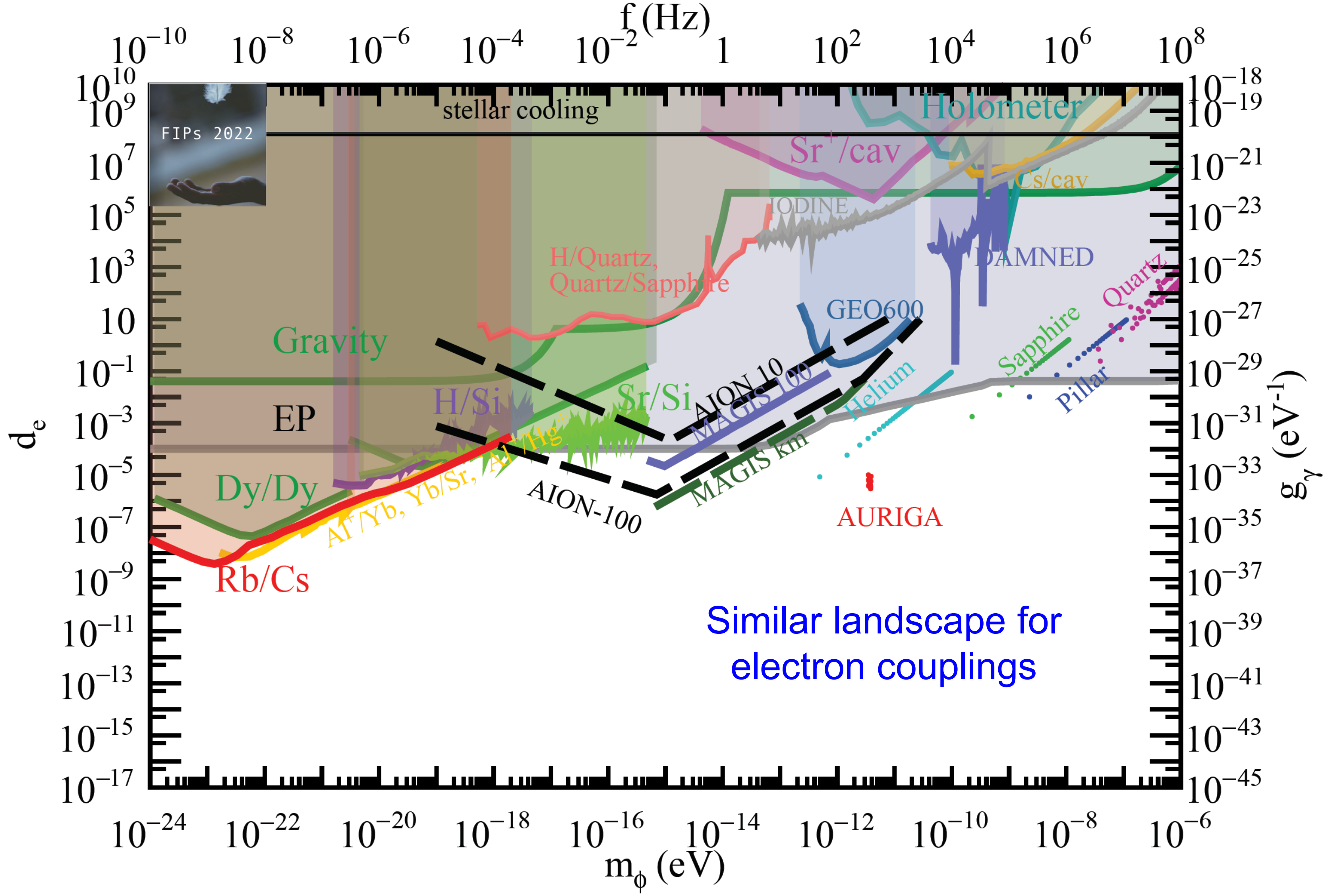
Screening of φ field in and around matter if $\delta m_\varphi > 0$

Probes of Oscillating Fundamental Constants



Constraints on Linear Scalar-Photon Coupling

Summary plot from FIPs 2022 workshop report: [Antel *et al.*, *EPJ C* **83**, 1122 (2023)]



Similar landscape for electron couplings

Muonic Probes of Ultralight Scalar DM

- *What about searching for ultralight scalar DM via other possible couplings (e.g., scalar-muon couplings)?*

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- Extra potential motivation from persistence of anomalies in muon physics, such as:
    - Proton radius puzzle
    - $(g-2)_\mu$ puzzle

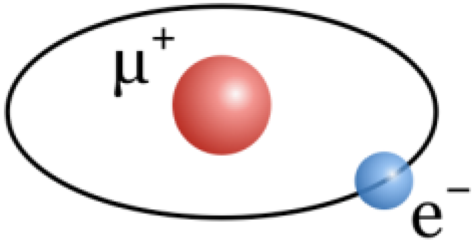
Muonic Probes of Ultralight Scalar DM

- *What about searching for ultralight scalar DM via other possible couplings (e.g., scalar-muon couplings)?*
- Possible flavour/generational dependence of scalar couplings in the lepton sector
- Extra potential motivation from persistence of anomalies in muon physics, such as:
    - Proton radius puzzle
    - $(g-2)_\mu$ puzzle
- No stable terrestrial sources of muons (unlike electrons), offering a qualitatively different phenomenology as compared to, e.g., scalar-electron couplings

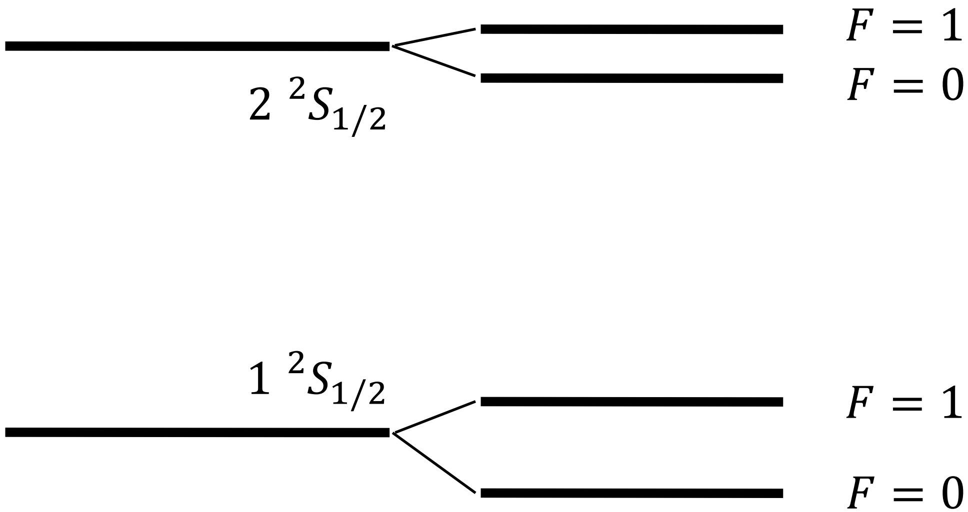
Probing Oscillations of m_μ with Muonium Spectroscopy

[Stadnik, *PRL* **131**, 011001 (2023)]

Muonium = $e^- \mu^+$ bound state, $m_r = \frac{m_e m_\mu}{m_e + m_\mu} \approx m_e(1 - m_e/m_\mu)$



$\tau_\mu \approx 2.2\ \mu s$



See Patrick Strasser's talk (later today), as well as posters (Wednesday) of Svenja Geissmann, Evans Javary, Marcus Mähring and Edward Thorpe-Woods

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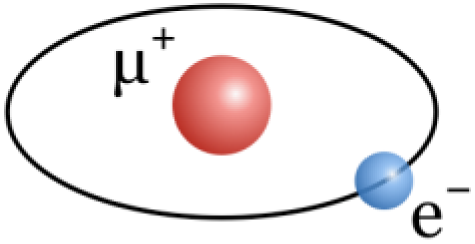

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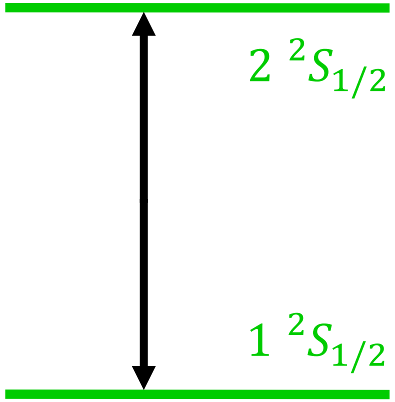

$$E_n^{\text{Rydberg}} = -\frac{m_r \alpha^2}{2n^2} \Rightarrow \frac{\Delta\nu_{1S-2S}}{\nu_{1S-2S}} \approx 2\frac{\Delta\alpha}{\alpha} + \frac{\Delta m_e}{m_e} + \frac{m_e}{m_\mu}\frac{\Delta m_\mu}{m_\mu}$$

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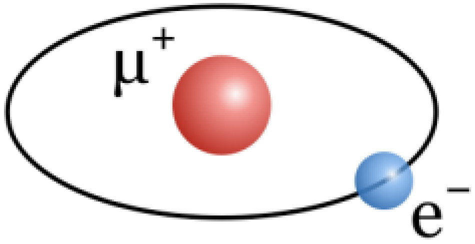
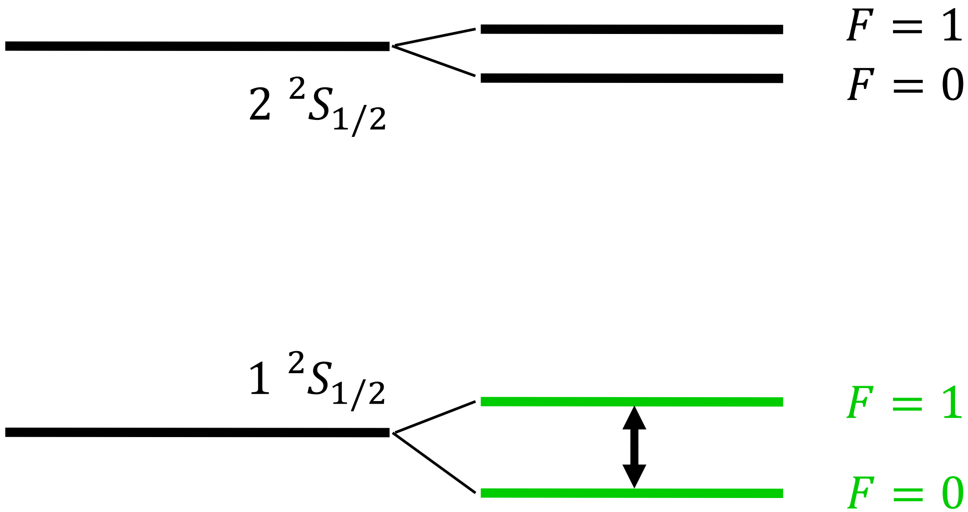


$\tau_\mu \approx 2.2$ μs



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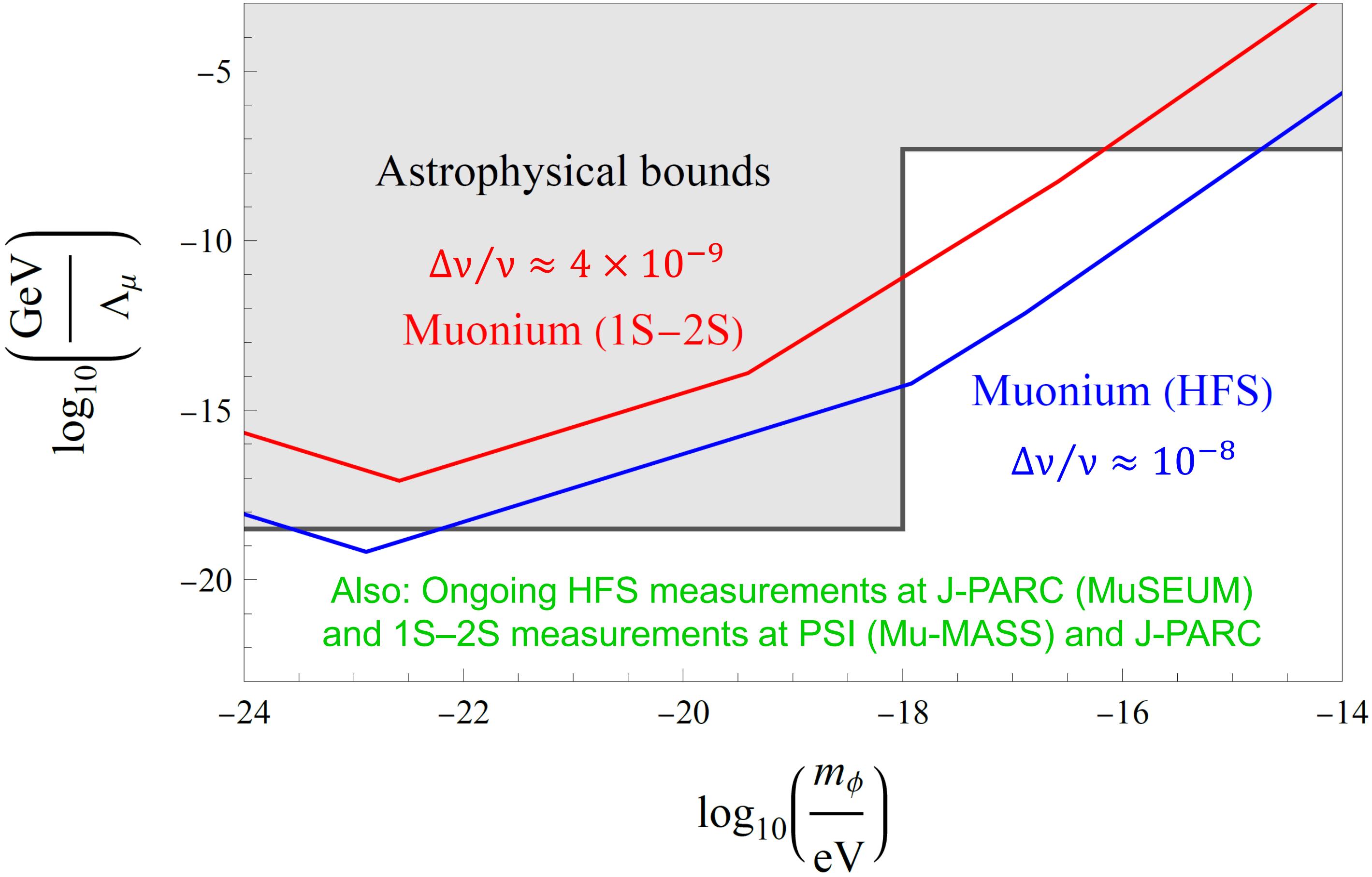
$$\Delta E_{\mathrm{Fermi}} = \frac{8 m_r^3 \alpha^4}{3 m_e m_\mu} \Rightarrow \frac{\Delta\nu_{\mathrm{HFS}}}{\nu_{\mathrm{HFS}}} \approx 4\frac{\Delta\alpha}{\alpha} + 2\frac{\Delta m_e}{m_e} - \frac{\Delta m_\mu}{m_\mu}$$

Estimated Sensitivities to Scalar Dark Matter with $\varphi\bar{\mu}\mu/\Lambda_\mu$ Coupling

[Stadnik, *PRL* **131**, 011001 (2023)]

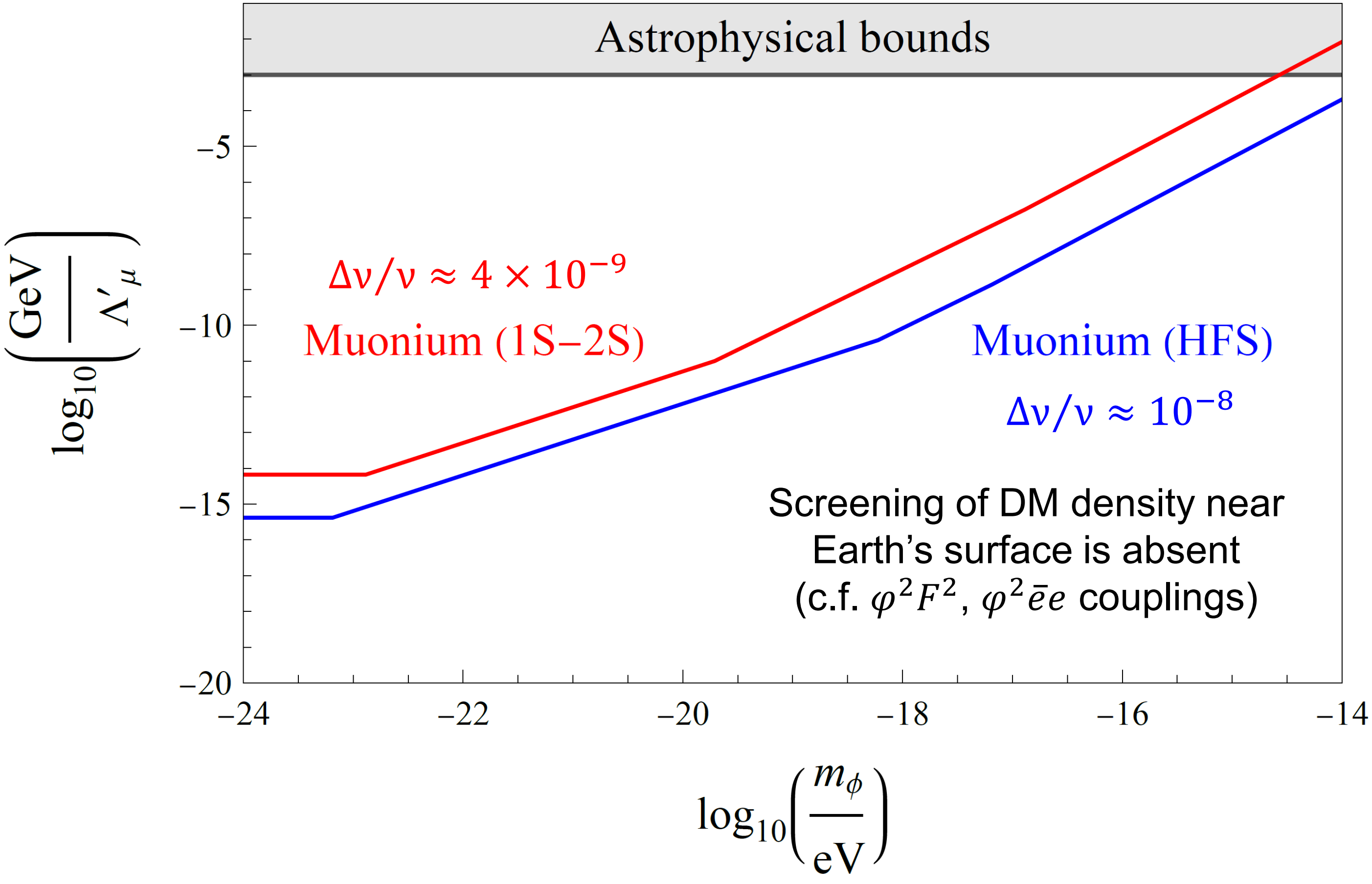
Up to 7 orders of magnitude improvement possible with existing datasets!
(Best existing datasets from muonium experiments at LAMPF and RAL in 1990s)



Estimated Sensitivities to Scalar Dark Matter with $\varphi^2\bar{\mu}\mu/(\Lambda'_\mu)^2$ Coupling

[Stadnik, *PRL* **131**, 011001 (2023)]

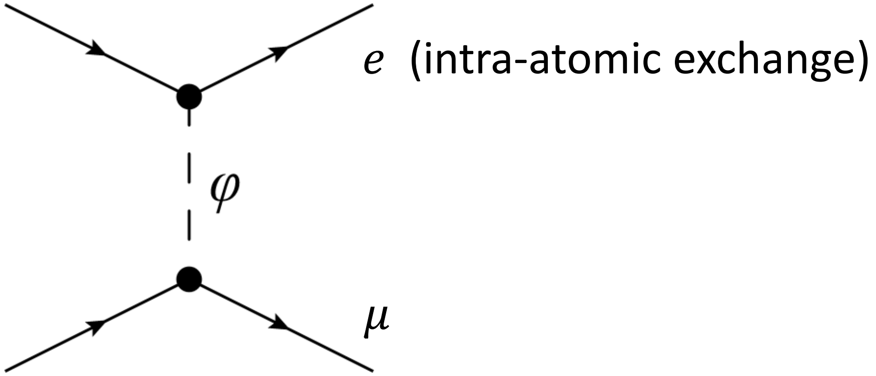
Up to 12 orders of magnitude improvement possible with existing datasets!
(Best existing datasets from muonium experiments at LAMPF and RAL in 1990s)



Scalar-Mediated New Forces (Not necessarily DM)

[Karshenboim, *PRL* **104**, 220406 (2010)], [Frugiuele, Perez-Rios, Peset, *PRD* **100**, 015010 (2019)]

$$\mathcal{L}_{\text{lin}} = -\frac{\varphi}{\Lambda_e} m_e \bar{e} e - \frac{\varphi}{\Lambda_\mu} m_\mu \bar{\mu} \mu \quad \Rightarrow \quad V_{e\mu}(r) \approx -\frac{m_e}{\Lambda_e} \frac{m_\mu}{\Lambda_\mu} \frac{e^{-m_\varphi r}}{4\pi r}$$



See Lei Cong's poster (Wednesday)

Probing Scalar-Muon Coupling with Muonium Free-fall

[Stadnik, *PRL* **131**, 011001 (2023)]

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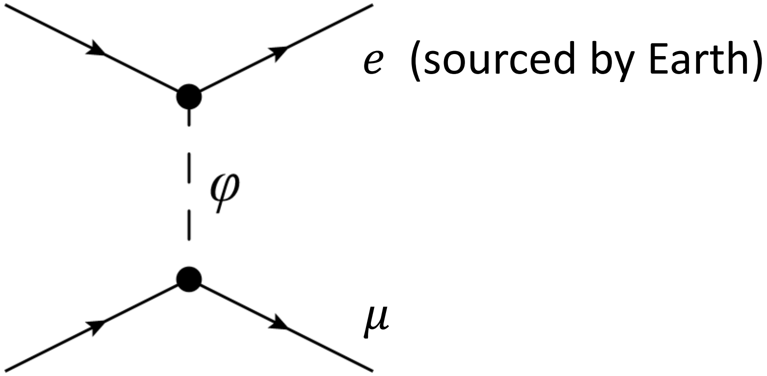

Local value of g measured in free-fall experiments using muonium would differ from experiments using non-muon-based test masses

Weak equivalence principle violation

(lepton flavour dependent)

Probing Scalar-Muon Coupling with Muonium Free-fall

[Stadnik, *PRL* **131**, 011001 (2023)]

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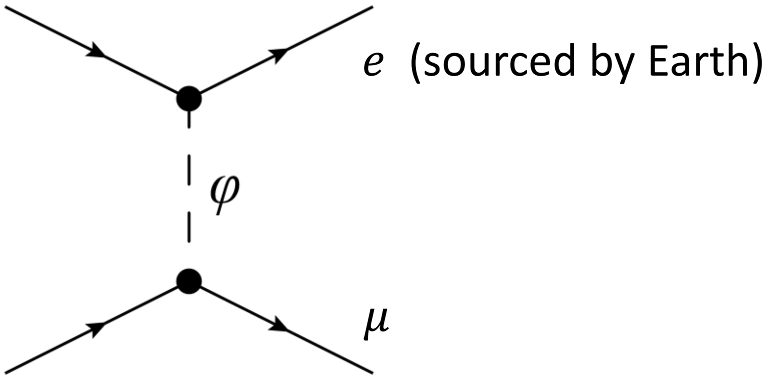

Local value of g measured in free-fall experiments using muonium would differ from experiments using non-muon-based test masses

Recently started LEMING experiment at PSI aims to measure g with a precision of $\Delta g/g \sim 10^{-2} - 10^{-1}$ using muonium

Probing Scalar-Muon Coupling with Muonium Free-fall

[Stadnik, *PRL* **131**, 011001 (2023)]

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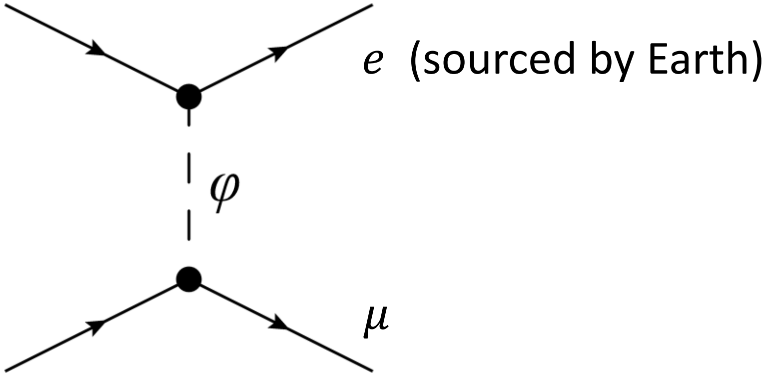
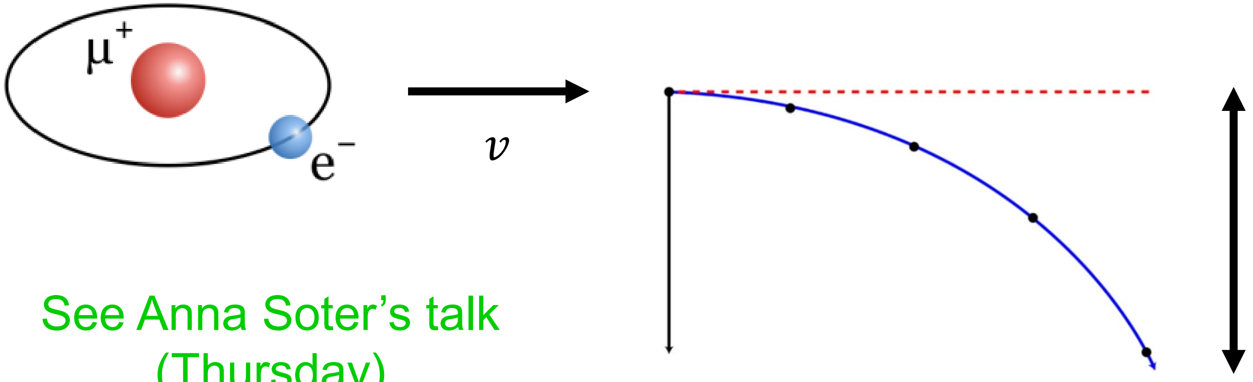




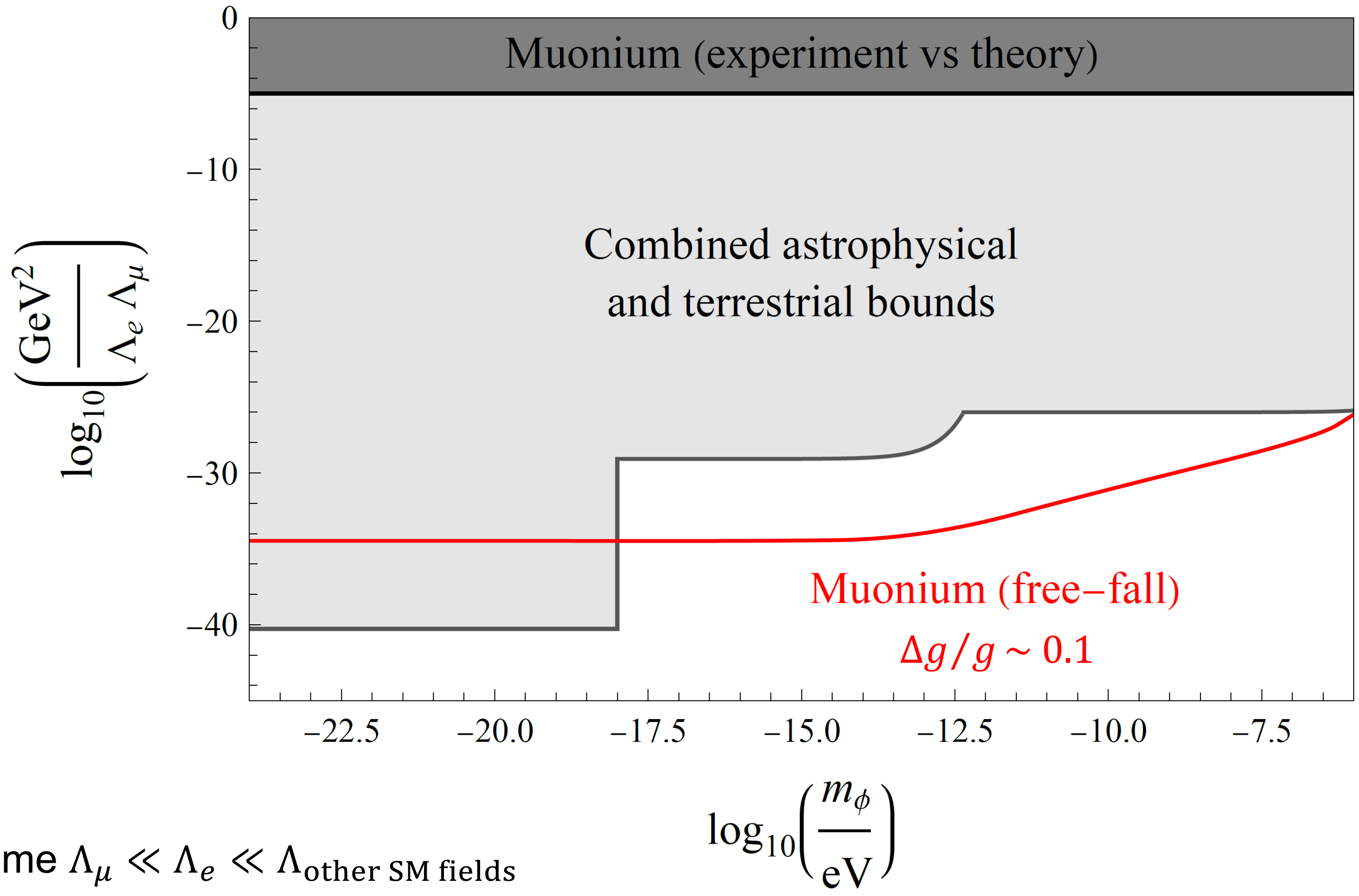
$$\Delta h \sim \frac{g\tau_\mu^2}{2} \approx 24 \text{ pm}$$

See Anna Soter's talk
(Thursday)

Probing Scalar-Muon Coupling with Muonium Free-fall

[Stadnik, *PRL* **131**, 011001 (2023)]

Up to 5 orders of magnitude improvement possible with ongoing measurements!
(Recently started LEMING experiment at PSI targets a precision of Δ*g*/*g* ~ 0.1)



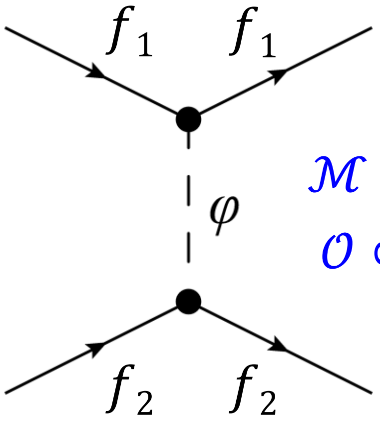
Assume $\Lambda_\mu \ll \Lambda_e \ll \Lambda_{\text{other SM fields}}$

Summary

- Recent searches for ultralight scalar DM have focused on the electromagnetic (photon) and electron couplings
- Muonium spectroscopy offers a powerful probe of ultralight scalar dark matter via interactions with muons leading to apparent oscillations of muon mass
  – With existing datasets, up to $\sim 10^7$ improvement possible for $\varphi \bar{\mu} \mu$ coupling (up to $\sim 10^{12}$ improvement for the $\varphi^2 \bar{\mu} \mu$ coupling over an even broader range of scalar DM masses)
- Ongoing muonium free-fall experiments to measure g offer up to $\sim 10^5$ improvement in sensitivity for the combination of $\varphi \bar{\mu} \mu$ and $\varphi \bar{e} e$ couplings by searching for φ-mediated forces

Back-Up Slides

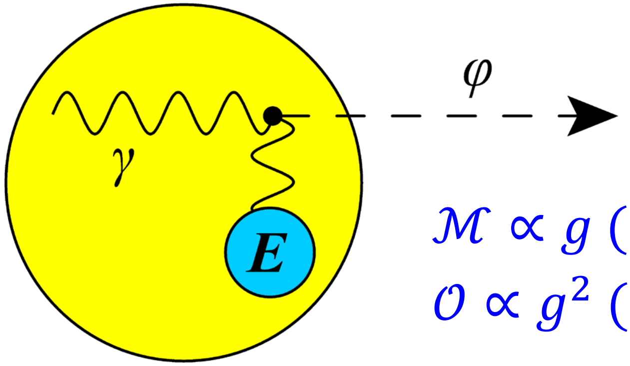
Complementary Probes of Low-mass Scalars



$\mathcal{M} \propto g^2$

$\mathcal{O} \propto g^2$

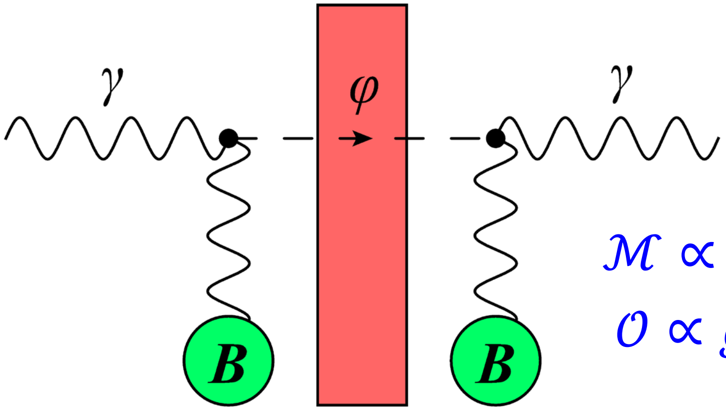
New forces



$\mathcal{M} \propto g \text{ (or } g^2\text{)}$

$\mathcal{O} \propto g^2 \text{ (or } g^4\text{)}$

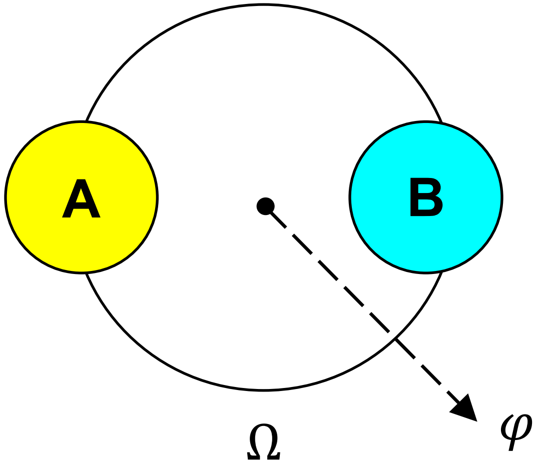
Stellar emission



$\mathcal{M} \propto g^2$

$\mathcal{O} \propto g^4$

Interconversion with ordinary particles

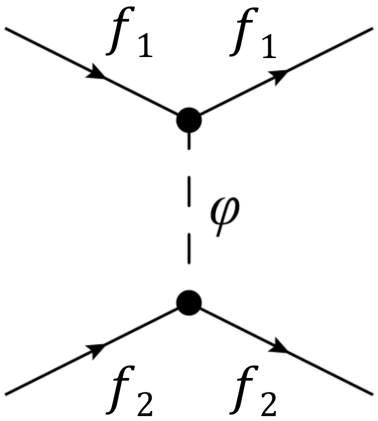


$\mathcal{M} \propto g$

$\mathcal{O} \propto g^2$

Larmor radiation

Equivalence-Principle-Violating Forces



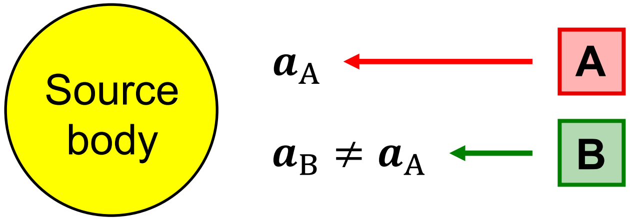
$$\mathcal{L}_f = -\frac{\varphi}{\Lambda_f} m_f \bar{f} f$$

$$\Rightarrow V_\varphi(r) = -\frac{m_1}{\Lambda_1}\frac{m_2}{\Lambda_2}\frac{e^{-m_\varphi r}}{4\pi r}$$

- Different mass-energy components of an atom generally scale differently with proton number Z and atomic number $A = Z + N$:

$$M_\mathrm{atom} \approx (A - Z)m_n + Zm_p + Zm_e + 100Z(Z-1)\alpha/A^{1/3}\ \mathrm{MeV} + \cdots$$

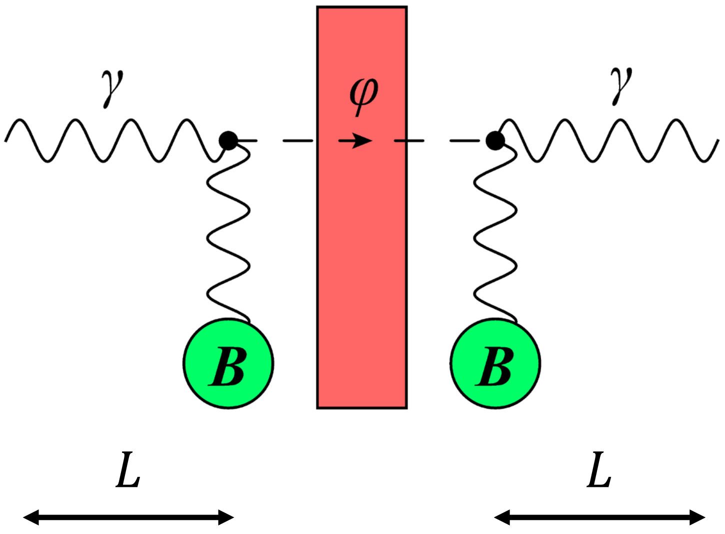
- Different atoms and isotopes would generally experience different accelerations, implying violation of the equivalence principle



Eötvös ratio: $\eta_\mathrm{AB} = 2\frac{|\boldsymbol{a}_\mathrm{A} - \boldsymbol{a}_\mathrm{B}|}{|\boldsymbol{a}_\mathrm{A} + \boldsymbol{a}_\mathrm{B}|}$

Light-shining-through-a-wall Experiments

[ALPS Collaboration, *PLB* **689**, 149 (2010)], [OSQAR Collaboration, *PRD* **92**, 092002 (2015)]



$$\mathcal{L}_\gamma = \frac{\varphi}{\Lambda_\gamma} \frac{F_{\mu\nu} F^{\mu\nu}}{4}$$

$$\Rightarrow \; p_{\gamma\to\varphi\to\gamma} = \left(\frac{B}{\Lambda_\gamma}\right)^4 \left[\frac{1}{q} \sin\left(\frac{qL}{2}\right)\right]^4$$

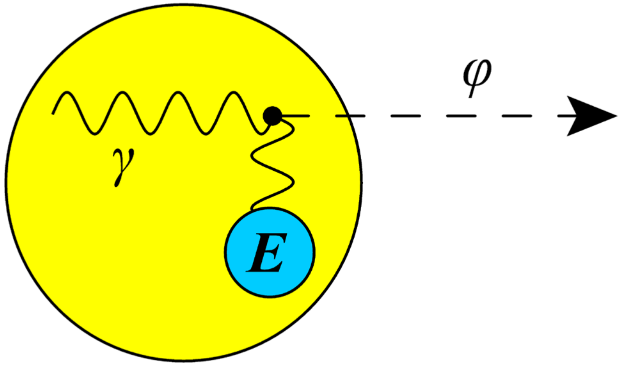
$q = \left|\boldsymbol{k}_\gamma - \boldsymbol{k}_\varphi\right|$ is the momentum transfer during interconversion

$\mathcal{L}_{\text{scalar}} \propto \varphi F_{\mu\nu} F^{\mu\nu} \propto \varphi\left(\boldsymbol{B}^2 - \boldsymbol{E}^2\right) \; \Rightarrow$ Need $\boldsymbol{E}_\gamma^{\text{lin}} \perp \boldsymbol{B}$

$\mathcal{L}_{\text{pseudoscalar}} \propto \varphi F_{\mu\nu} \tilde{F}^{\mu\nu} \propto \varphi \boldsymbol{E} \cdot \boldsymbol{B} \; \Rightarrow$ Need $\boldsymbol{E}_\gamma^{\text{lin}} \parallel \boldsymbol{B}$

Astrophysical Emission (Hot Media)

[Raffelt, *Phys. Rept.* **198**, 1 (1990)]

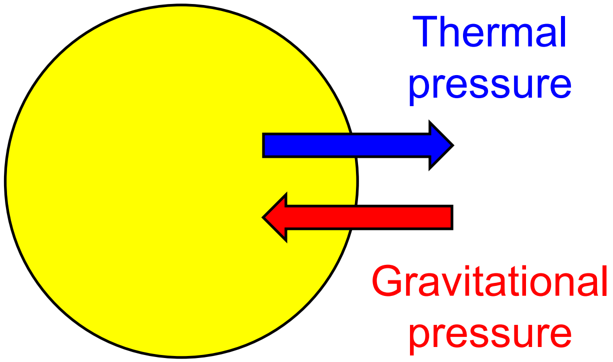


$$\mathcal{L}_\gamma = \frac{\varphi}{\Lambda_\gamma}\frac{F_{\mu\nu}F^{\mu\nu}}{4} \Rightarrow \varepsilon_{\gamma\gamma\to\varphi} \sim \frac{T^7}{\Lambda_\gamma^2}$$

Emission possible for $m_\varphi \lesssim \mathcal{O}(T)$

Primakoff-type conversion

Excessive energy loss via additional channels would contradict stellar models and observations



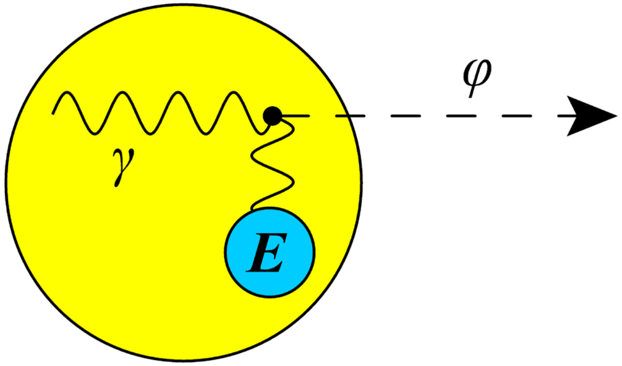
Increased heating in active stars (e.g., Sun and main sequence stars, HB stars, red giants)

$$\langle E_\mathrm{mech}\rangle = \langle E_\mathrm{kin}\rangle + \langle E_\mathrm{grav}\rangle = \langle E_\mathrm{grav}\rangle/2 < 0$$

$$\langle E_\mathrm{mech}\rangle \downarrow \;\Rightarrow\; -\langle E_\mathrm{grav}\rangle/2 = \langle E_\mathrm{kin}\rangle \uparrow$$

Astrophysical Emission (Hot Media)

[Raffelt, *Phys. Rept.* **198**, 1 (1990)]



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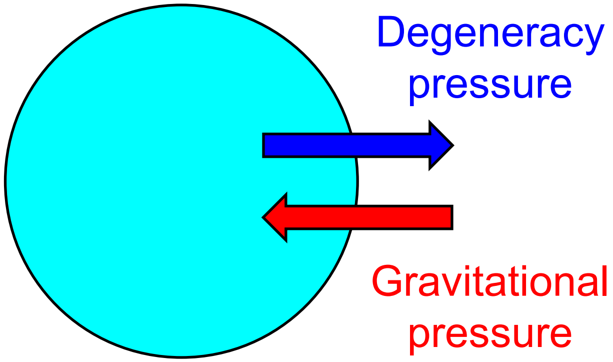
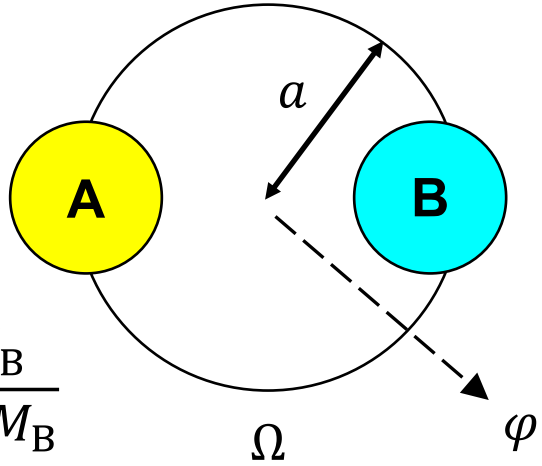


Increased cooling in dead stars
(e.g., white dwarves, neutron stars)

Astrophysical Emission (Compact Binaries)

[Kumar Poddar *et al.*, *PRD* **100**, 123923 (2019)], [Dror *et al.*, *PRD* **102**, 023005 (2020)]



$$M = \frac{M_\mathrm{A} M_\mathrm{B}}{M_\mathrm{A} + M_\mathrm{B}}$$

$$\mathcal{L}_f = -\frac{\varphi}{\Lambda_f} m_f \bar{f} f$$

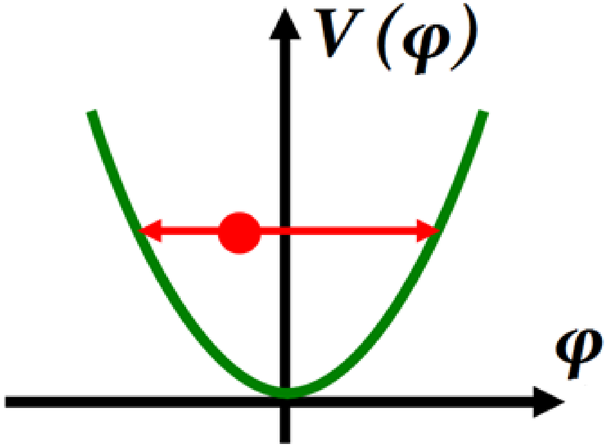
- Scalar Larmor radiation possible if $m_\varphi < \Omega$ (higher-order modes also possible for an elliptical orbit if $m_\varphi < n\Omega$, $n = 2, 3, \ldots$):

$$\frac{dE_\varphi}{dt} \sim \left(\frac{m_f}{\Lambda_f}\right)^2 (aM)^2 \Omega^4 \left(\frac{Q_\mathrm{A}}{M_\mathrm{A}} - \frac{Q_\mathrm{B}}{M_\mathrm{B}}\right)^2, \text{for } \Omega a \ll 1$$

- Dipole nature requires $Q_\mathrm{A}/M_\mathrm{A} \neq Q_\mathrm{B}/M_\mathrm{B}$, which is readily satisfied, e.g., for neutron-star/white-dwarf binary systems in the case of $f = n, e, \mu$

Low-mass Spin-0 Dark Matter

- Low-mass spin-0 particles form a coherently oscillating classical field $\varphi(t) \approx \varphi_0 \cos\left(m_\varphi c^2 t/\hbar\right)$, with energy density $\rho_\varphi \approx m_\varphi^2 \varphi_0^2/2 \;\; (\rho_{\mathrm{DM,local}} \approx 0.4\ \mathrm{GeV/cm^3})$



$$V(\varphi) = \frac{m_\varphi^2 \varphi^2}{2}$$

$$\ddot{\varphi} + m_\varphi^2 \varphi \approx 0$$

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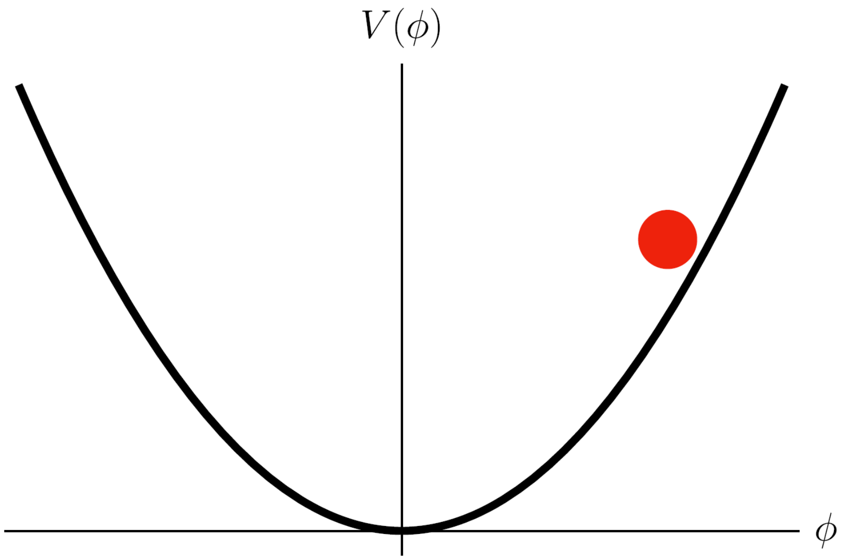

$$\ddot{\varphi} + 3H(t)\dot{\varphi} + m_\varphi^2 \varphi \approx 0 \longleftarrow$$

$$H(t) = \dot{a}(t)/a(t)$$

Damped harmonic oscillator with a time-dependent frictional term

$(H = \text{Hubble parameter},\ a = \text{scale factor})$

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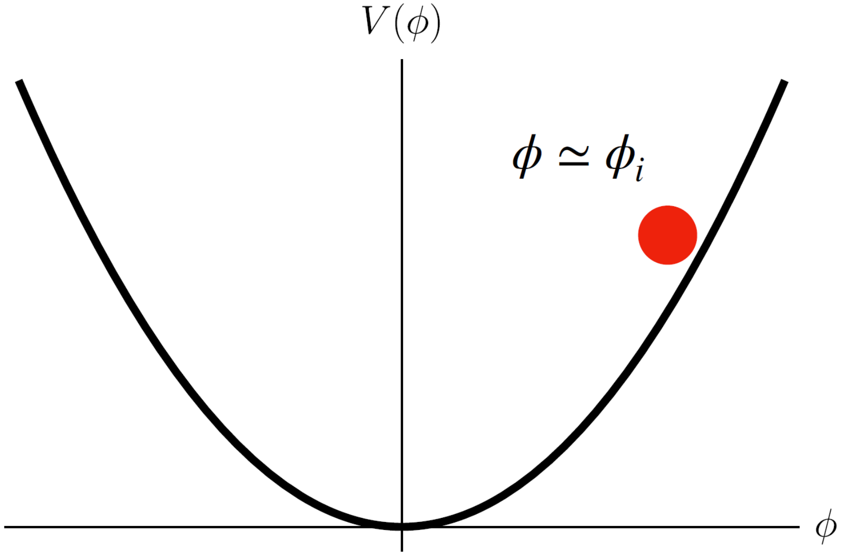
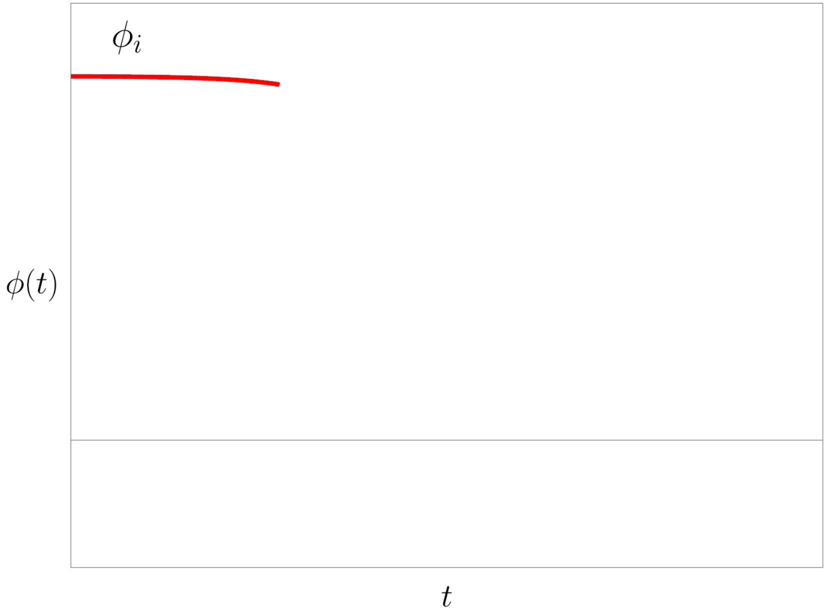


$$\ddot{\varphi} + 3H(t)\dot{\varphi} + m_\varphi^2 \varphi \approx 0$$

$$m_\varphi \ll 3H(t) \sim 1/t$$



Overdamped regime

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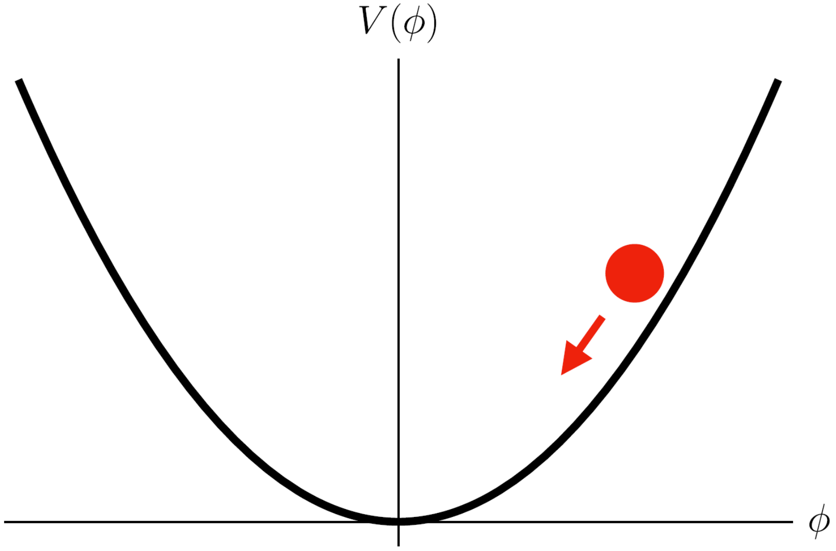

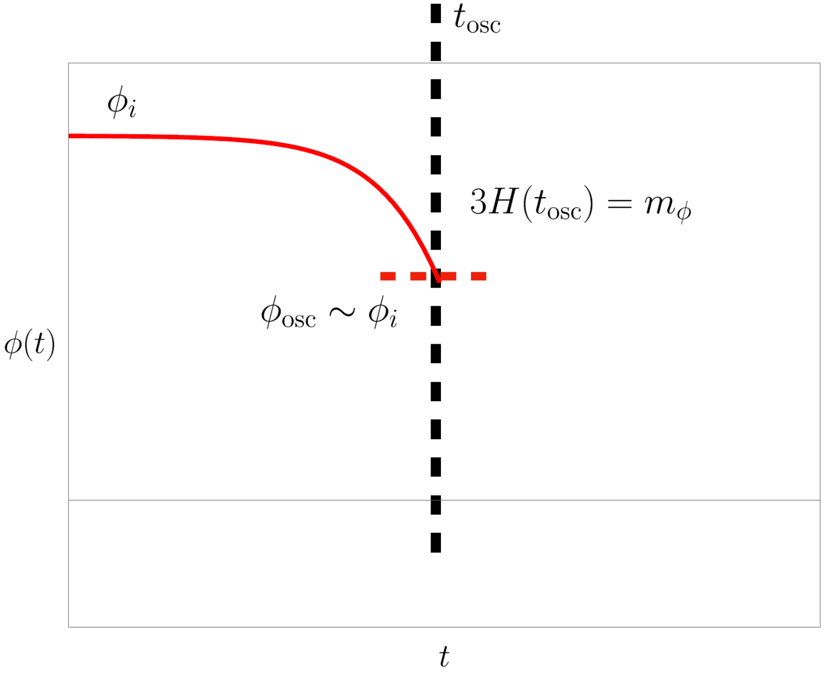

$$\ddot{\varphi} + 3H(t)\dot{\varphi} + m_\varphi^2 \varphi \approx 0$$

$$m_\varphi \sim 3H(t) \sim 1/t$$

Critically damped regime

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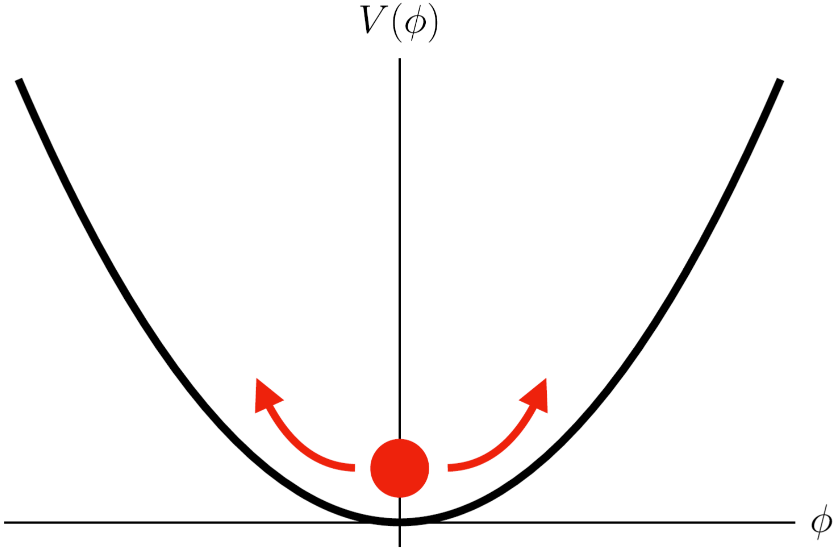
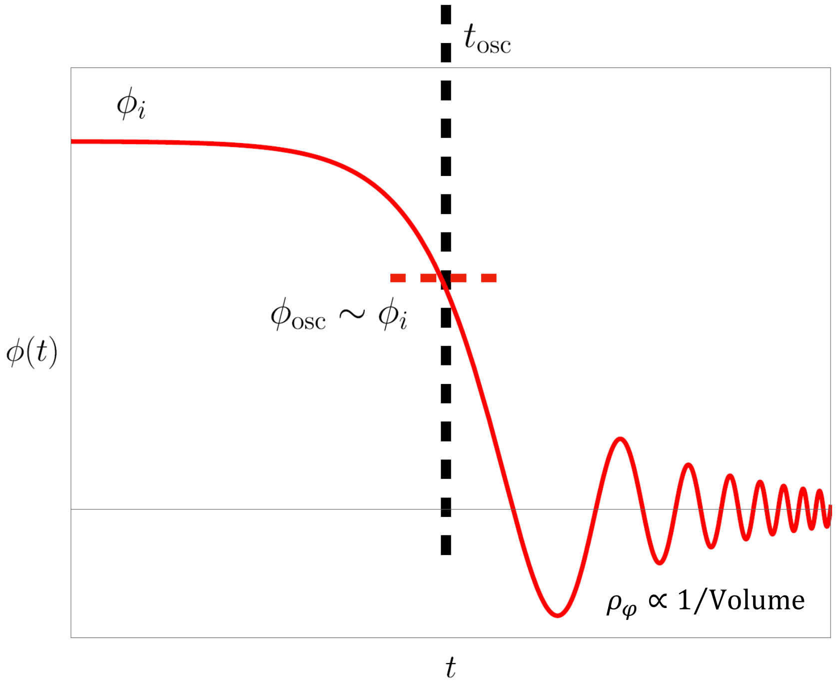


$$\ddot{\varphi} + 3H(t)\dot{\varphi} + m_\varphi^2 \varphi \approx 0$$

$$m_\varphi \gg 3H(t) \sim 1/t$$



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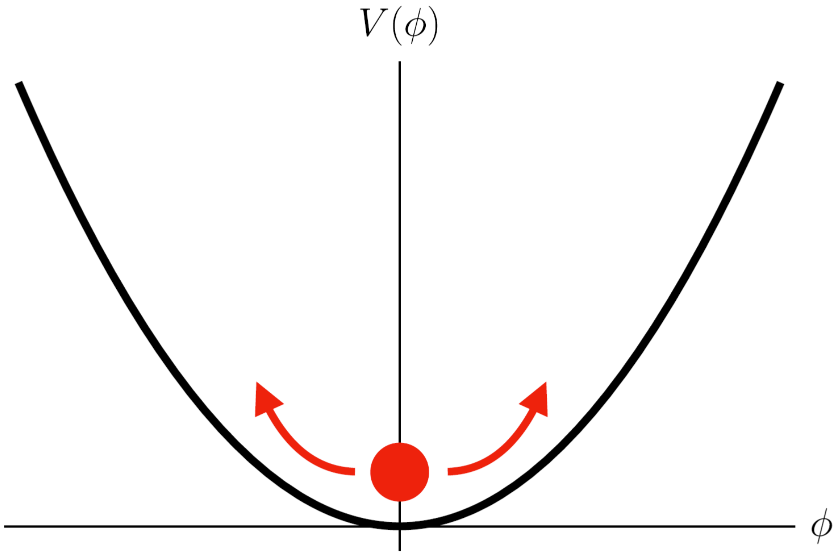

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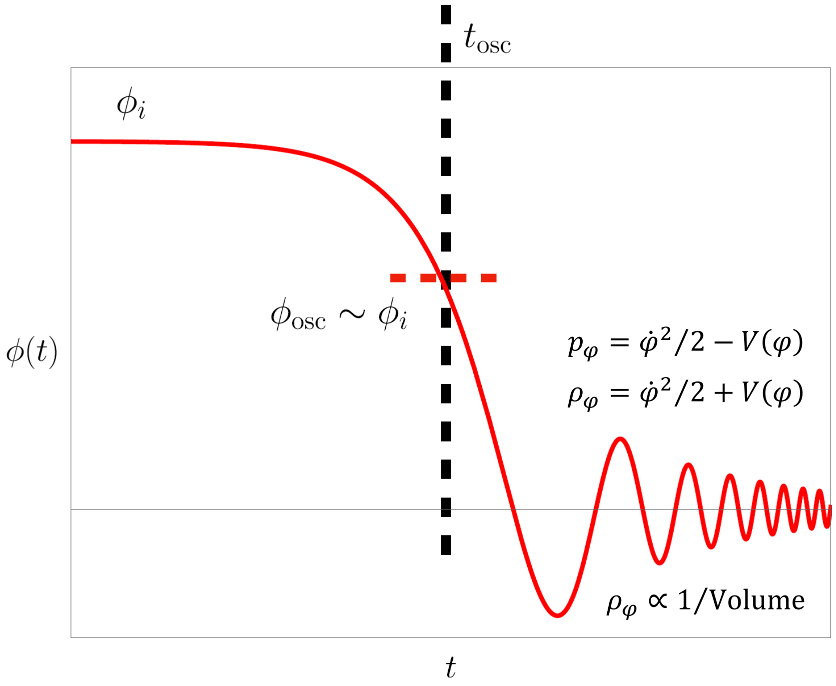

"Vacuum misalignment" mechanism – non-thermal production, ρ_φ governed by initial conditions (φ_i), redshifts as $\rho_\varphi \propto 1/\mathrm{Volume}$, with $\langle p_\varphi \rangle \ll \rho_\varphi$

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- $\Delta E_\varphi / E_\varphi \sim \langle v_\varphi^2 \rangle / c^2 \sim 10^{-6} \quad \Rightarrow \quad \tau_{\mathrm{coh}} \sim 2\pi/\Delta E_\varphi \sim 10^6 T_{\mathrm{osc}}$

$$\underset{\Big\uparrow}{} \; v_{\mathrm{DM}} \sim 300\ \mathrm{km/s} \qquad\qquad \underset{\Big\downarrow}{} \; Q_{\mathrm{DM}} \sim 10^6$$

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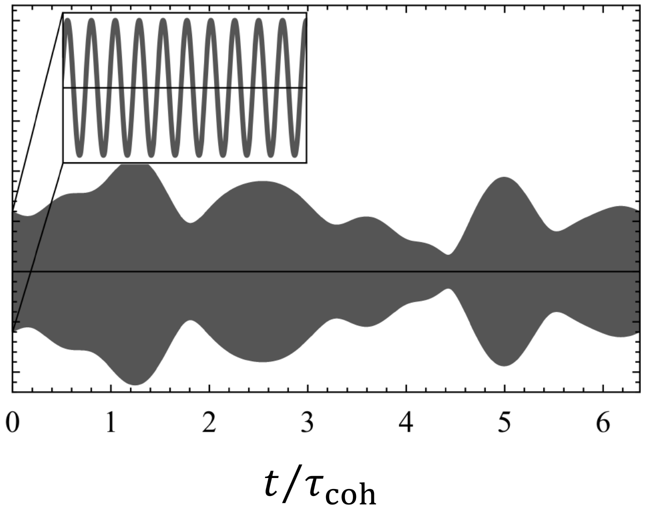
Evolution of φ_0 with time



$$\varphi(t) \sim \sum_{i=1}^{N} \frac{\varphi_0}{\sqrt{N}} \cos\left(m_\varphi t + \frac{m_\varphi v_i^2 t}{2} + \theta_i\right)$$

$\boldsymbol{v}_i$ follow quasi-Maxwell-Boltzmann distribution (in the standard halo model)

Low-mass Spin-0 Dark Matter

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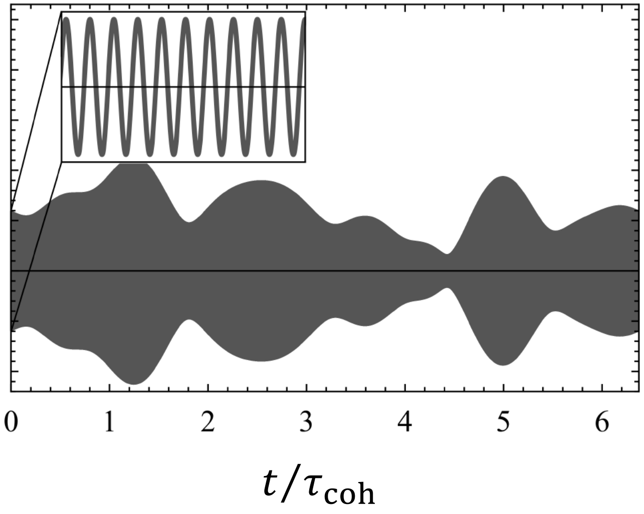
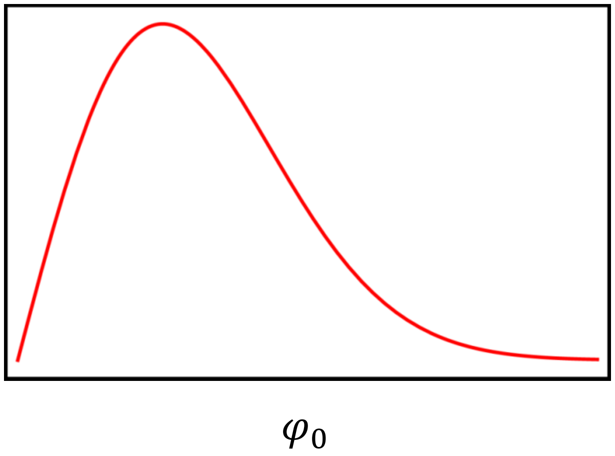


Probability distribution function of φ_0 (Rayleigh distribution)



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- *Classical* field for $m_\varphi \lesssim 1\ \mathrm{eV}$, since $n_\varphi \left(\lambda_{\mathrm{dB},\varphi}/2\pi\right)^3 \gg 1$

* Pauli exclusion principle rules out sub-eV *fermionic* dark matter

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- $10^{-21} \ \mathrm{eV} \lesssim m_\varphi \lesssim 1 \ \mathrm{eV} \;\; \Leftrightarrow \;\; 10^{-7} \ \mathrm{Hz} \lesssim f_{\mathrm{DM}} \lesssim 10^{14} \ \mathrm{Hz}$

$T_{\mathrm{osc}} \sim 1 \ \mathrm{month}$

IR frequencies

↑

Lyman-α forest measurements [suppression of structures for $L \lesssim \mathcal{O}(\lambda_{\mathrm{dB},\varphi})$]

[Related figure-of-merit: $\lambda_{\mathrm{dB},\varphi}/2\pi \leq L_{\mathrm{dwarf\,galaxy}} \sim 100 \ \mathrm{pc} \;\; \Rightarrow \;\; m_\varphi \gtrsim 10^{-21} \ \mathrm{eV}$]

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$\uparrow$

Lyman-α forest measurements [suppression of structures for $L \lesssim \mathcal{O}(\lambda_{\mathrm{dB},\varphi})$]

- *Wave-like* signatures [cf. *particle-like* signatures of WIMP DM]

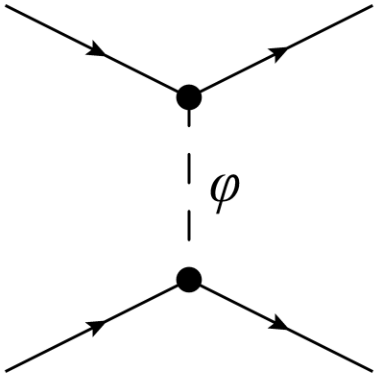
Fifth Forces: Linear vs Quadratic Couplings

[Hees, Minazzoli, Savalle, Stadnik, Wolf, *PRD* **98**, 064051 (2018)]

Consider the effect of a massive body (e.g., Earth) on the scalar DM field

Linear couplings ($\varphi\bar{X}X$)

$\Box\varphi + m_\varphi^2\varphi = \pm\kappa\rho$ **Source term**

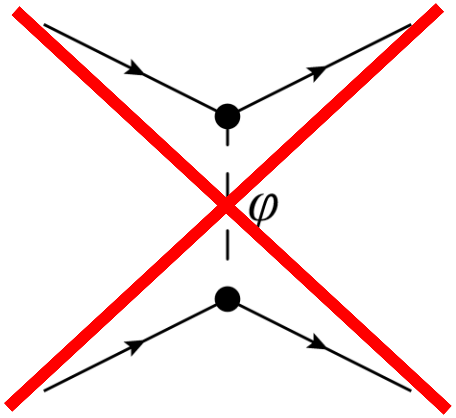


$$\varphi = \varphi_0\cos(m_\varphi t) \pm A\frac{e^{-m_\varphi r}}{r}$$

Profile outside of a spherical body

Quadratic couplings ($\varphi^2\bar{X}X$)

$\Box\varphi + m_\varphi^2\varphi = \pm\kappa'\rho\varphi$ **Effective mass**



$$m_{\text{eff}}^2(\rho) = m_\varphi^2 \mp \kappa'\rho$$

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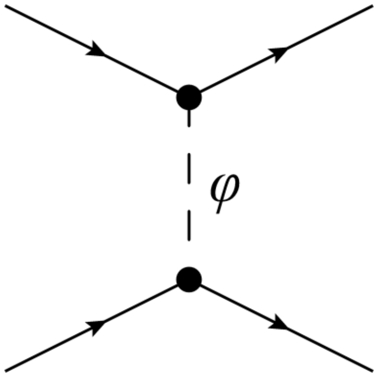

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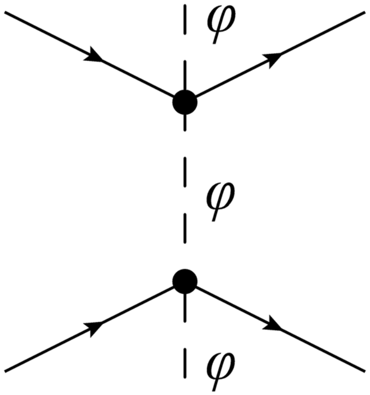

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$$\varphi = \varphi_0\cos(m_\varphi t)\left(1 \pm \frac{B}{r}\right)$$

Profile outside of a spherical body

Gradients + amplification/screening

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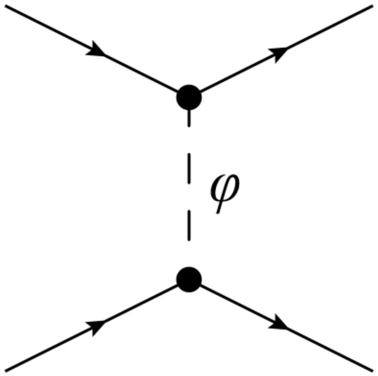

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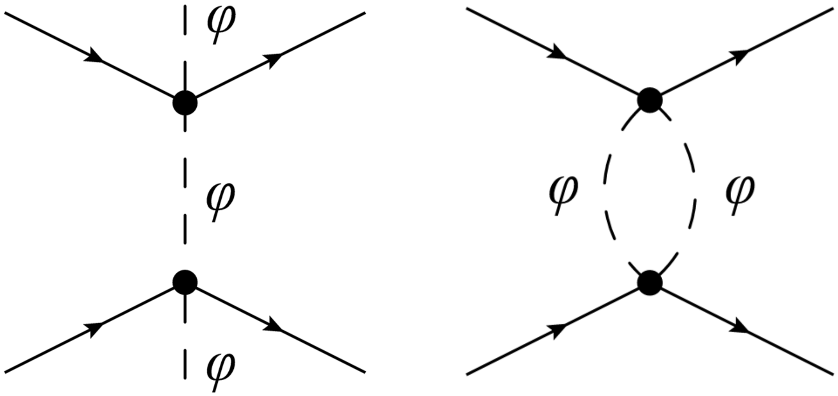

$$\varphi = \varphi_0\cos(m_\varphi t) \pm A\frac{e^{-m_\varphi r}}{r}$$

$$\varphi = \varphi_0\cos(m_\varphi t)\left(1 \pm \frac{B}{r}\right) - \hbar C\frac{e^{-2m_\varphi r}}{r^3}$$

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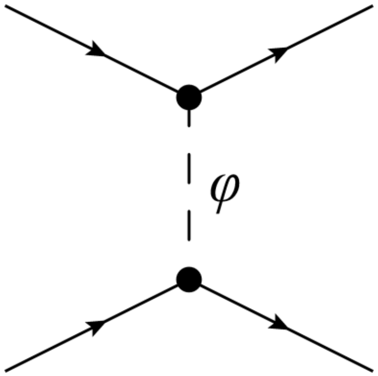
Source term



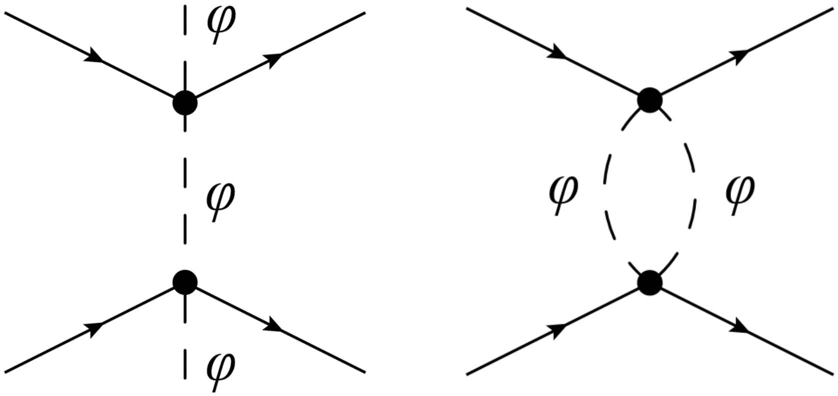
$$\varphi = \underline{\varphi_0 \cos(m_\varphi t)} \pm A\frac{e^{-m_\varphi r}}{r}$$

Motional gradients: $\varphi_0 \cos(m_\varphi t - \boldsymbol{p}_\varphi \cdot \boldsymbol{x})$

Quadratic couplings ($\varphi^2 \bar{X} X$)

$$\Box\varphi + m_\varphi^2 \varphi = \pm\kappa' \rho\varphi$$

Effective mass



$$\varphi = \underline{\varphi_0 \cos(m_\varphi t)} \left(1 \pm \frac{B}{r}\right) - \hbar C \frac{e^{-2m_\varphi r}}{r^3}$$

$\downarrow$

Gradients + amplification/screening

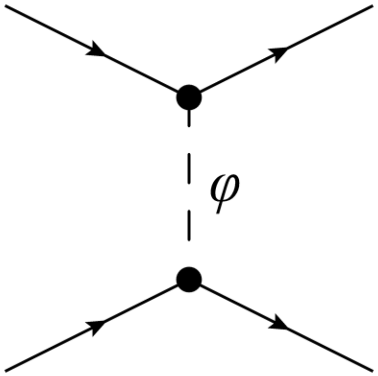
Fifth Forces: Linear vs Quadratic Couplings

[Hees, Minazzoli, Savalle, Stadnik, Wolf, *PRD* **98**, 064051 (2018)]

Consider the effect of a massive body (e.g., Earth) on the scalar DM field

Linear couplings ($\varphi\bar{X}X$)

$\Box\varphi + m_\varphi^2\varphi = \pm\kappa\rho$ **Source term**



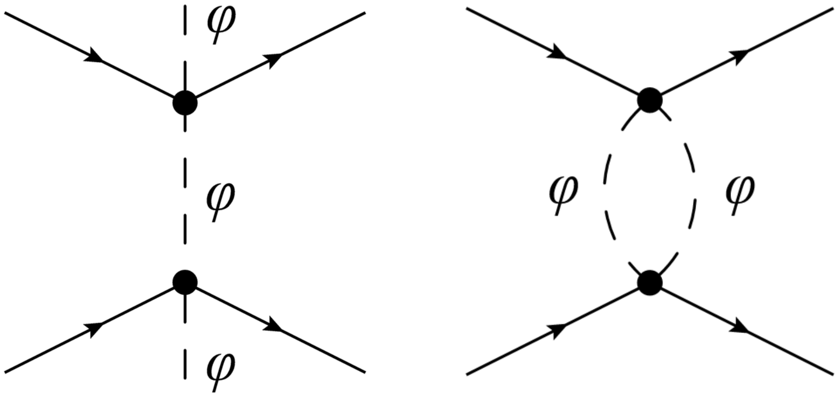
$$\varphi = \underline{\varphi_0\cos(m_\varphi t)} \pm A\frac{e^{-m_\varphi r}}{r}$$

Motional gradients: $\varphi_0\cos(m_\varphi t - \boldsymbol{p}_\varphi\cdot\boldsymbol{x})$

“Fifth-force” experiments: torsion pendula, atom interferometry

Quadratic couplings ($\varphi^2\bar{X}X$)

$\Box\varphi + m_\varphi^2\varphi = \pm\kappa'\rho\varphi$ **Effective mass**

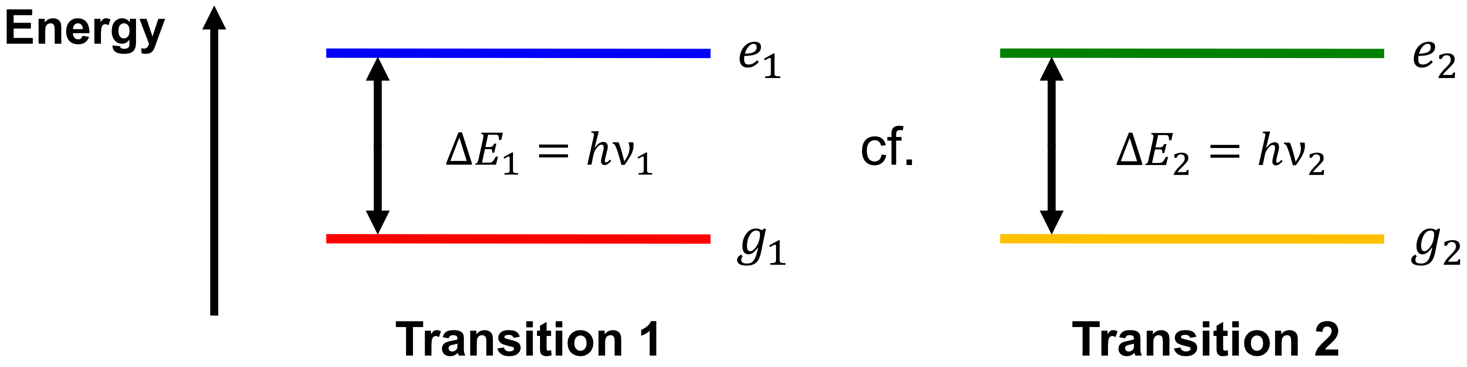


$$\varphi = \underline{\varphi_0\cos(m_\varphi t)}\left(1 \pm \frac{B}{r}\right) - \hbar C\frac{e^{-2m_\varphi r}}{r^3}$$

↓

Gradients + amplification/screening

Atomic Spectroscopy Searches for Oscillating Variations of Fundamental Constants induced by Dark Matter



$$\boxed{\frac{\delta(\nu_1/\nu_2)}{\nu_1/\nu_2} = \left(K_{X,1} - K_{X,2}\right)\frac{\delta X}{X}\ ;\ \ X = \alpha, m_e/m_N\,, \ldots}$$

Atomic spectroscopy (including clocks) has been used for decades to search for "slow drifts" in fundamental constants

Recent overview: [Ludlow, Boyd, Ye, Peik, Schmidt, *Rev. Mod. Phys.* **87**, 637 (2015)]

"Sensitivity coefficients" K_X required for the interpretation of experimental data have been calculated extensively by Flambaum group

Reviews: [Flambaum, Dzuba, *Can. J. Phys.* **87**, 25 (2009); *Hyperfine Interac.* **236**, 79 (2015)]

Effects of Varying Fundamental Constants on Atomic Transitions

[Dzuba, Flambaum, Webb, *PRL* **82**, 888 (1999); *PRA* **59**, 230 (1999);
Dzuba, Flambaum, Marchenko, *PRA* **68**, 022506 (2003); Angstmann, Dzuba, Flambaum,
PRA **70**, 014102 (2004); Dzuba, Flambaum, *PRA* **77**, 012515 (2008)]

- Atomic optical transitions:

$$\nu_{\mathrm{opt}} \propto \left(\frac{m_e e^4}{\hbar^3}\right) F_{\mathrm{rel}}^{\mathrm{opt}}(Z\alpha)$$

Non-relativistic atomic unit of frequency

Relativistic factor

Effects of Varying Fundamental Constants on Atomic Transitions

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- Atomic optical transitions:

$$\nu_{\mathrm{opt}} \propto \left(\frac{m_e e^4}{\hbar^3}\right) F_{\mathrm{rel}}^{\mathrm{opt}}(Z\alpha)$$

$$K_\alpha(\mathrm{Sr}) = 0.06, K_\alpha(\mathrm{Yb}) = 0.3, K_\alpha(\mathrm{Hg}) = 0.8$$

Increasing *Z*

$$|\boldsymbol{p}_e|_{\text{near nucleus}} \sim Z\alpha m_e c$$

Effects of Varying Fundamental Constants on Atomic Transitions

[Dzuba, Flambaum, Webb, *PRL* **82**, 888 (1999); *PRA* **59**, 230 (1999);
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Increasing *Z*

For transitions between closely spaced energy levels that arise due to the near cancellation of contributions of different nature, the K_α sensitivity coefficients can be greatly enhanced, e.g.:

- $\left|K_\alpha\left(\mathrm{Cf}^{15+}\right)\right| \approx 50$ [Dzuba *et al.*, *PRA* **92**, 060502(R) (2015)]
- $\left|K_\alpha\left({}^{229}\mathrm{Th}\right)\right| \sim 10^4$ [Flambaum, *PRL* **97**, 092502 (2006)]

Effects of Varying Fundamental Constants on Atomic Transitions

[Dzuba, Flambaum, Webb, *PRL* **82**, 888 (1999); *PRA* **59**, 230 (1999);
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Increasing *Z*

- Atomic hyperfine transitions:

$$\nu_{\mathrm{hf}} \propto \left(\frac{m_e e^4}{\hbar^3}\right) \left[\alpha^2 F_{\mathrm{rel}}^{\mathrm{hf}}(Z\alpha)\right] \left(\frac{m_e}{m_N}\right) \mu$$

Effects of Varying Fundamental Constants on Atomic Transitions

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$$K_\alpha(\mathrm{H}) = 2.0, K_\alpha(\mathrm{Rb}) = 2.3, K_\alpha(\mathrm{Cs}) = 2.8$$

Increasing *Z*

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[Dzuba, Flambaum, Webb, *PRL* **82**, 888 (1999); *PRA* **59**, 230 (1999);
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Increasing *Z*

- Atomic hyperfine transitions:

$K_{m_e/m_N} = 1$

$$\nu_{\mathrm{hf}} \propto \left(\frac{m_e e^4}{\hbar^3}\right) \left[\alpha^2 F_{\mathrm{rel}}^{\mathrm{hf}}(Z\alpha)\right] \left(\frac{m_e}{m_N}\right) \mu$$

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Increasing *Z*

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Increasing *Z*

- Atomic hyperfine transitions:

$$K_{m_e/m_N} = 1$$

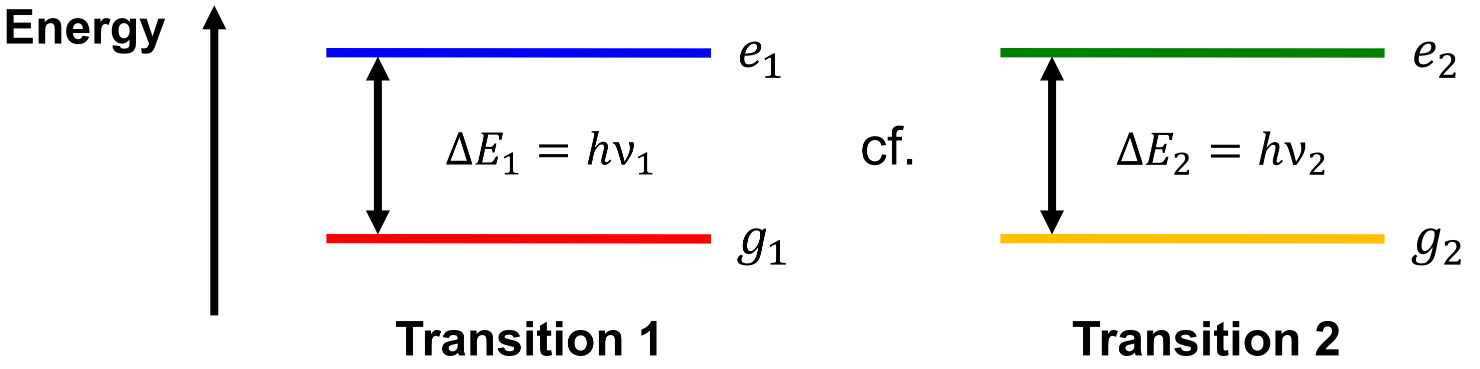
$$\nu_{\mathrm{hf}} \propto \left(\frac{m_e e^4}{\hbar^3}\right) \left[\alpha^2 F_{\mathrm{rel}}^{\mathrm{hf}}(Z\alpha)\right] \left(\frac{m_e}{m_N}\right) \mu \longleftarrow K_{m_q/\Lambda_{\mathrm{QCD}}} \neq 0$$

$$K_\alpha(\mathrm{H}) = 2.0, K_\alpha(\mathrm{Rb}) = 2.3, K_\alpha(\mathrm{Cs}) = 2.8$$

Increasing *Z*

Atomic Spectroscopy Searches for Oscillating Variations of Fundamental Constants induced by Dark Matter

[Arvanitaki, Huang, Van Tilburg, *PRD* **91**, 015015 (2015)], [Stadnik, Flambaum, *PRL* **114**, 161301 (2015)]



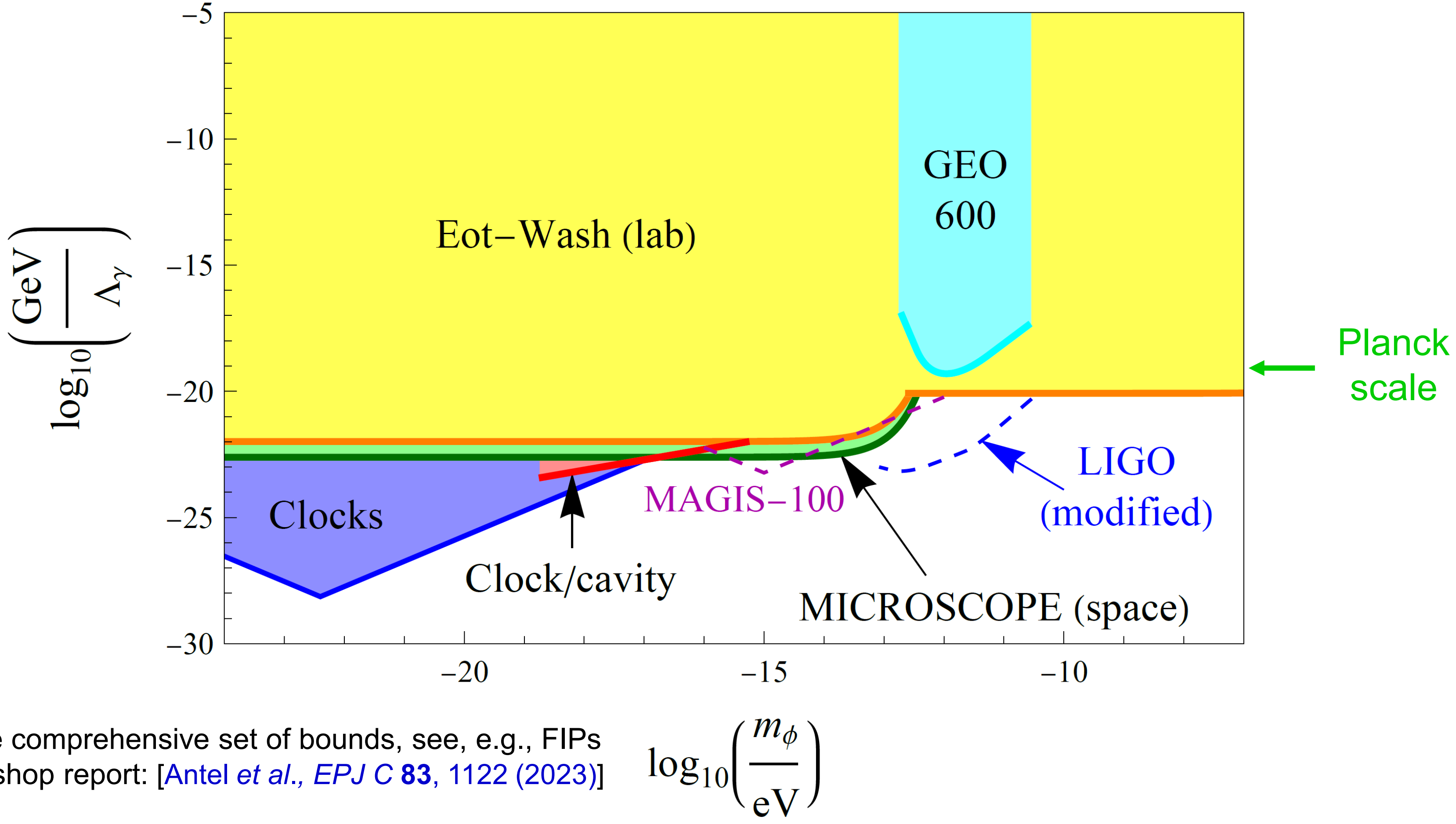
$$\frac{\delta(\nu_1/\nu_2)}{\nu_1/\nu_2} \propto \sum_{X=\alpha, m_e/m_N,\dots} \left(K_{X,1}-K_{X,2}\right)\cos(2\pi f_{\mathrm{DM}} t)\ ;\ 2\pi f_{\mathrm{DM}} = \underset{\text{(lin.)}}{m_\varphi} \text{ or } \underset{\text{(quad.)}}{2m_\varphi}$$

- **Dy/Cs [Mainz]:** [Van Tilburg *et al.*, *PRL* **115**, 011802 (2015)], [Stadnik, Flambaum, *PRL* **115**, 201301 (2015)]
- **Rb/Cs [SYRTE]:** [Hees *et al.*, *PRL* **117**, 061301 (2016)], [Stadnik, Flambaum, *PRA* **94**, 022111 (2016)]
- **Al$^+$/Yb, Yb/Sr, Al$^+$/Hg$^+$ [NIST + JILA]:** [BACON Collaboration, *Nature* **591**, 564 (2021)]
- **Yb/Cs [NMIJ]:** [Kobayashi *et al.*, *PRL* **129**, 241301 (2022)]
- **Yb$^+$(E3)/Sr [PTB]:** [Filzinger *et al.*, *PRL* **130**, 253001 (2023)]

Constraints on Scalar Dark Matter with $\varphi F_{\mu\nu}F^{\mu\nu}/4\Lambda_\gamma$ Coupling

Clock/clock: [*PRL* **115**, 011802 (2015)], [*PRL* **117**, 061301 (2016)], [*Nature* **591**, 564 (2021)], [*PRL* **130**, 253001 (2023)]; **Clock/cavity:** [*PRL* **125**, 201302 (2020)]; **GEO600:** [*Nature* **600**, 424 (2021)]

5 orders of magnitude improvement!



For a more comprehensive set of bounds, see, e.g., FIPs 2022 workshop report: [Antel *et al.*, *EPJ C* **83**, 1122 (2023)]

Constraints on Scalar Dark Matter with $\varphi F_{\mu\nu} F^{\mu\nu}/4\Lambda_\gamma$ Coupling

Clock/clock: [*PRL* **115**, 011802 (2015)], [*PRL* **117**, 061301 (2016)], [*Nature* **591**, 564 (2021)], [*PRL* **130**, 253001 (2023)]; **Clock/cavity:** [*PRL* **125**, 201302 (2020)]; **GEO600:** [*Nature* **600**, 424 (2021)]

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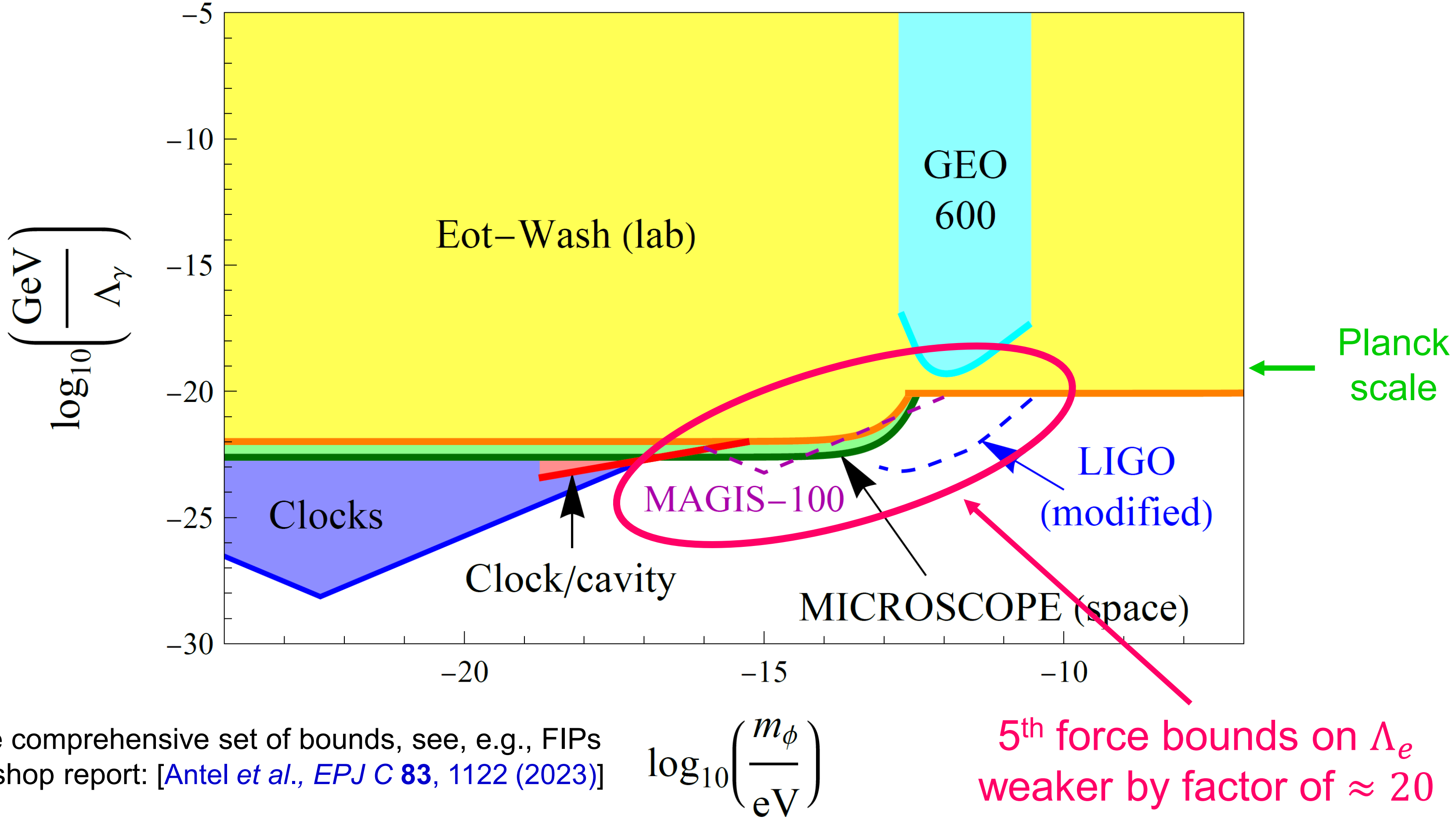


5th force bounds on Λ_e weaker by factor of ≈ 20

For a more comprehensive set of bounds, see, e.g., FIPs 2022 workshop report: [Antel *et al.*, *EPJ C* **83**, 1122 (2023)]

BBN Constraints on 'Slow' Drifts in Fundamental Constants due to Dark Matter

[Stadnik, Flambaum, *PRL* **115**, 201301 (2015)]

- Largest effects of DM in early Universe (highest ρ_DM)
- Big Bang nucleosynthesis ($t_\text{weak} \approx 1\ \text{s} - t_\text{BBN} \approx 3\ \text{min}$)
- Primordial ^{4}He abundance sensitive to n/p ratio (almost all neutrons bound in ^{4}He after BBN)

$$\frac{\Delta Y_p(^4\text{He})}{Y_p(^4\text{He})} \approx \frac{\Delta(n/p)_\text{weak}}{(n/p)_\text{weak}} - \Delta\left[\int_{t_\text{weak}}^{t_\text{BBN}} \Gamma_n(t)dt\right]$$

$$p + e^- \rightleftharpoons n + \nu_e$$

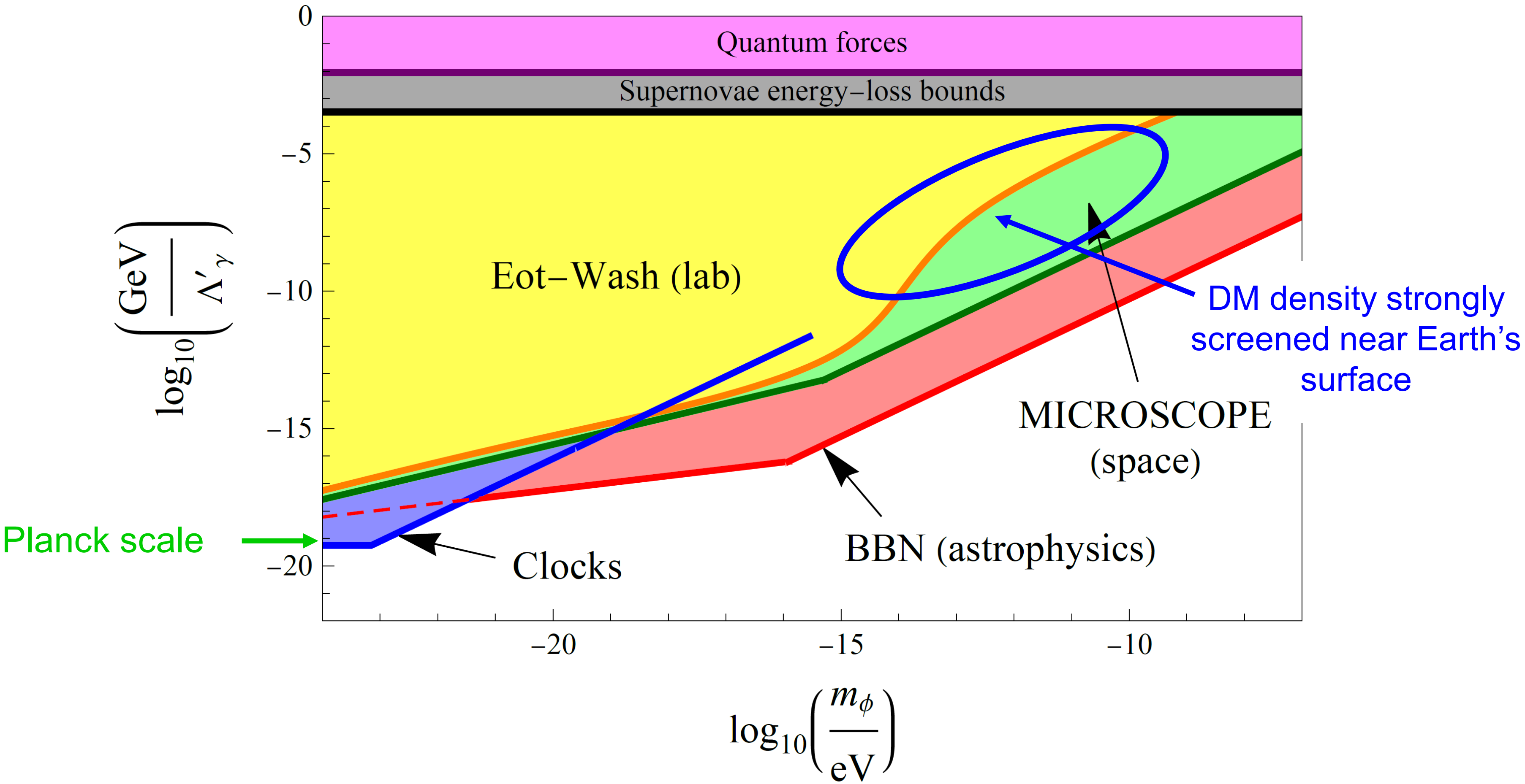
$$n + e^+ \rightleftharpoons p + \bar{\nu}_e$$

$$n \to p + e^- + \bar{\nu}_e$$

Constraints on Scalar Dark Matter with $\varphi^2 F_{\mu\nu}F^{\mu\nu}/4(\Lambda'_\gamma)^2$ Coupling

Clock/clock + BBN constraints: [Stadnik, Flambaum, *PRL* **115**, 201301 (2015); *PRA* **94**, 022111 (2016)]; **MICROSCOPE + Eöt-Wash constraints:** [Hees *et al*., *PRD* **98**, 064051 (2018)]

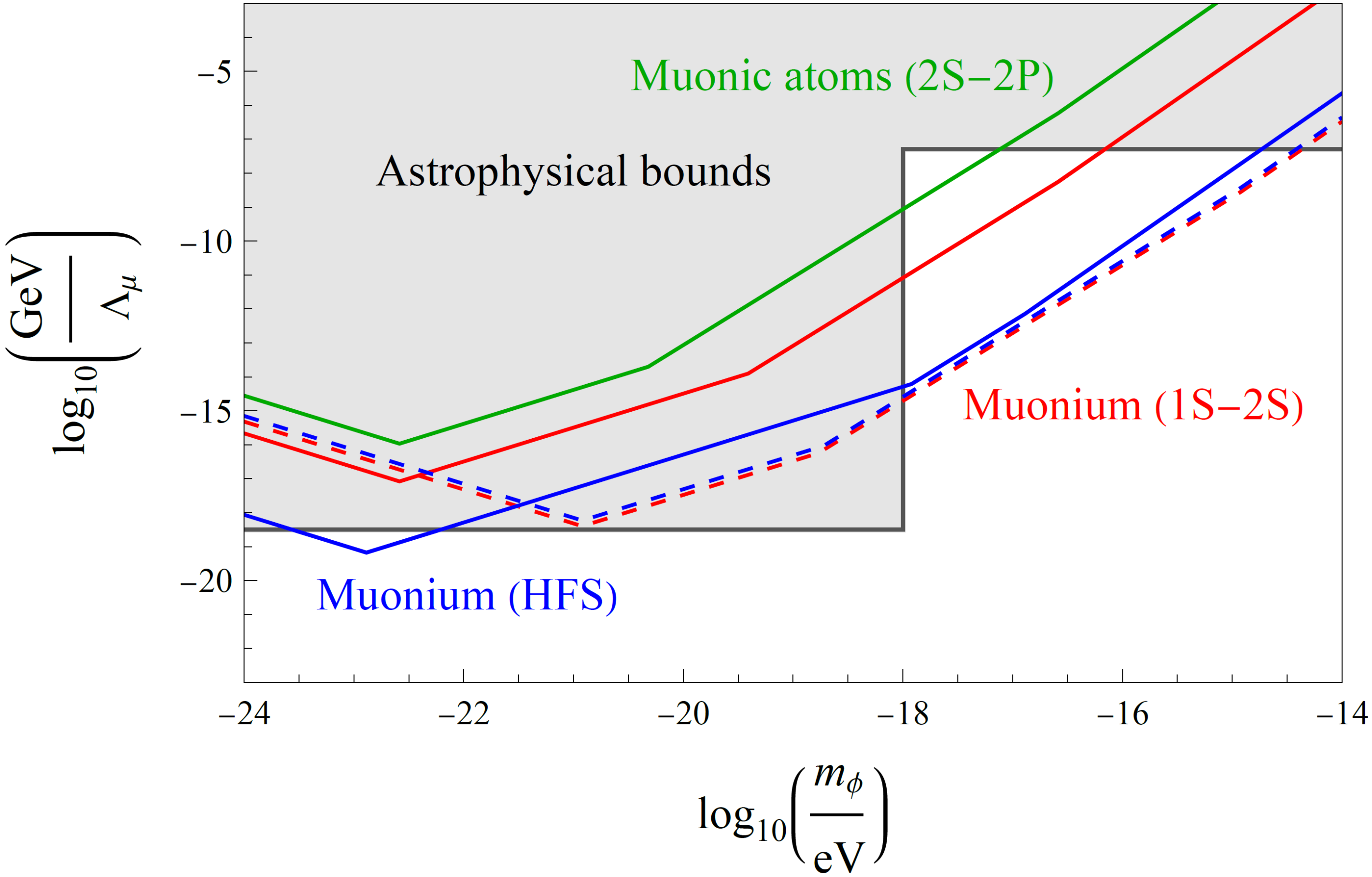
15 orders of magnitude improvement!



Estimated Sensitivities to Scalar Dark Matter with $\varphi\bar{\mu}\mu/\Lambda_\mu$ Coupling

[Stadnik, *PRL* **131**, 011001 (2023)]

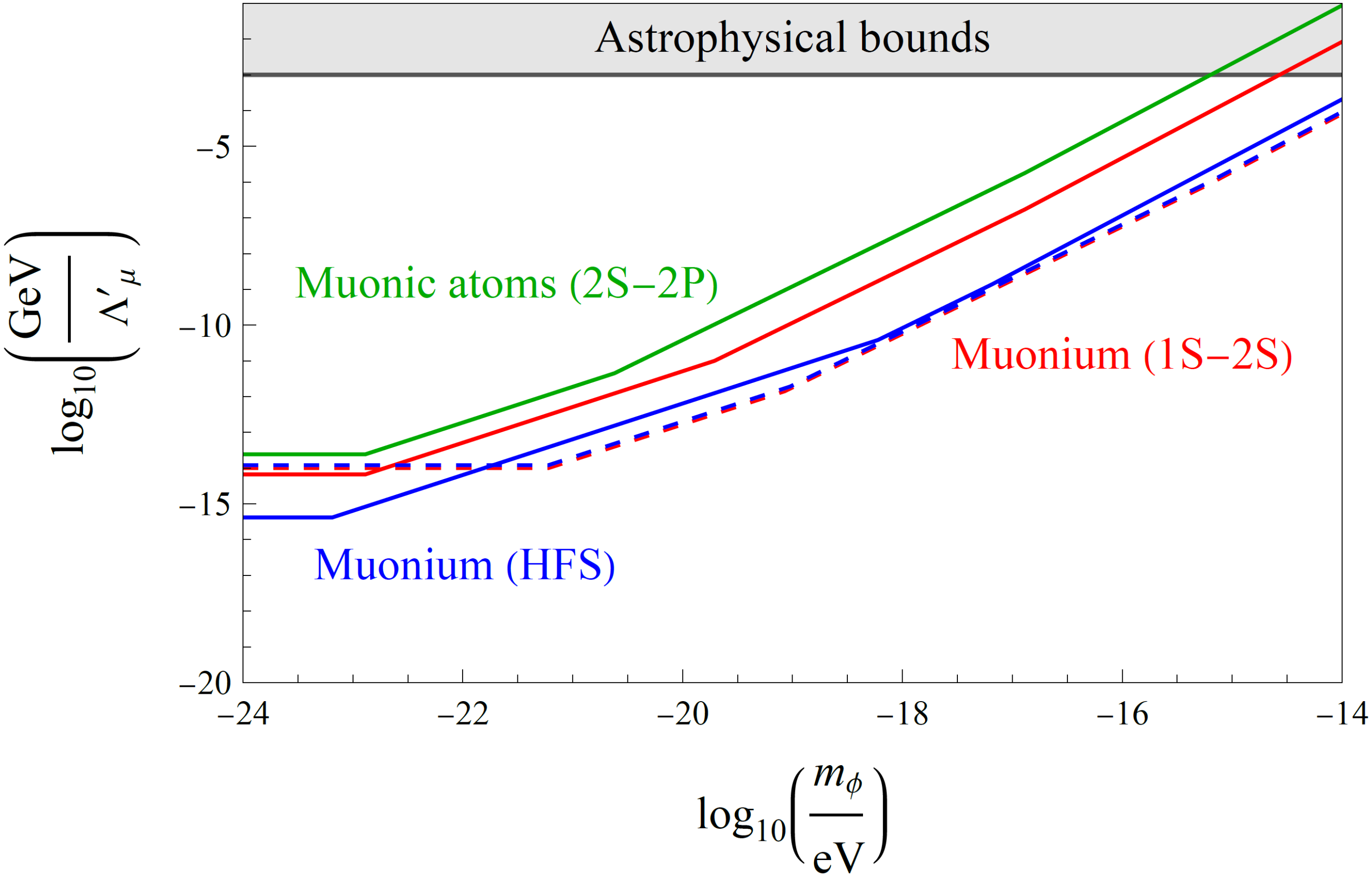
Up to 7 orders of magnitude improvement possible with existing datasets!
(Best existing datasets from muonium experiments at LAMPF and RAL in 1990s)



Estimated Sensitivities to Scalar Dark Matter with $\varphi^2\bar{\mu}\mu/(\Lambda'_\mu)^2$ Coupling

[Stadnik, *PRL* **131**, 011001 (2023)]

Up to 12 orders of magnitude improvement possible with existing datasets!
(Best existing datasets from muonium experiments at LAMPF and RAL in 1990s)



Screening Mechanisms and Environmental Effects

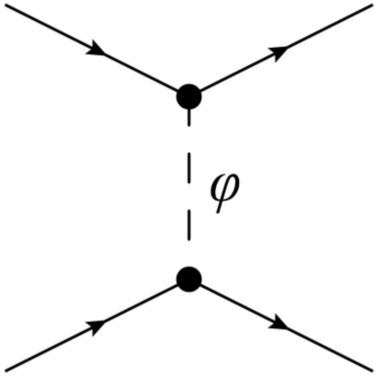
Fifth Forces: Linear vs Quadratic Couplings

[Hees, Minazzoli, Savalle, Stadnik, Wolf, *PRD* **98**, 064051 (2018)]

Consider the effect of a massive body (e.g., Earth) on the scalar DM field

Linear couplings ($\varphi \bar{X} X$)

$\Box\varphi + m_\varphi^2 \varphi = \pm\kappa\rho$ **Source term**



$$\varphi = \varphi_0 \cos(m_\varphi t) \pm A\frac{e^{-m_\varphi r}}{r}$$

Profile outside of a spherical body

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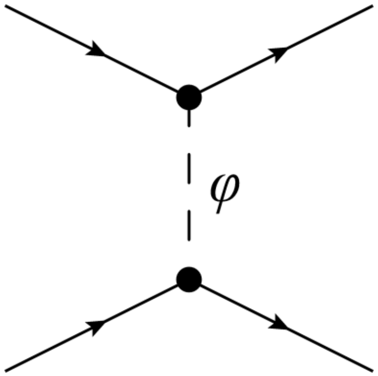


$$\varphi = \varphi_0 \cos(m_\varphi t) \pm A\frac{e^{-m_\varphi r}}{r}$$

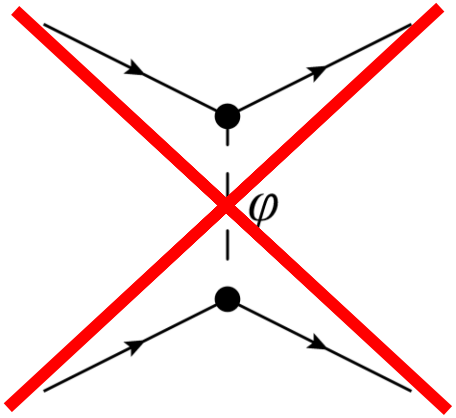
↑

Profile outside of a spherical body

Quadratic couplings ($\varphi^2 \bar{X} X$)

$$\Box\varphi + m_\varphi^2 \varphi = \pm\kappa'\rho\varphi$$

Effective mass



$$m_{\mathrm{eff}}^2(\rho) = m_\varphi^2 \mp \kappa'\rho$$

Fifth Forces: Linear vs Quadratic Couplings

[Hees, Minazzoli, Savalle, Stadnik, Wolf, *PRD* **98**, 064051 (2018)]

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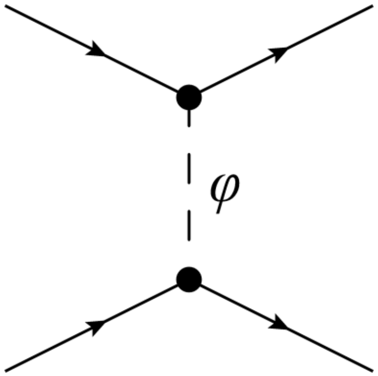

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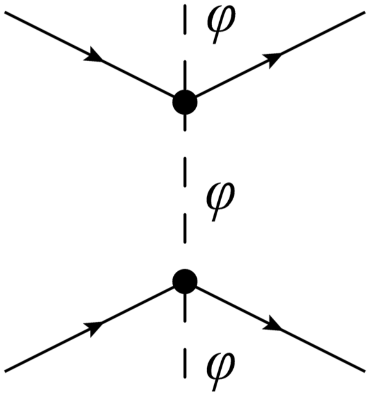


$$\varphi = \varphi_0\cos(m_\varphi t) \pm A\frac{e^{-m_\varphi r}}{r}$$

$$\varphi = \varphi_0\cos(m_\varphi t)\left(1 \pm \frac{B}{r}\right)$$

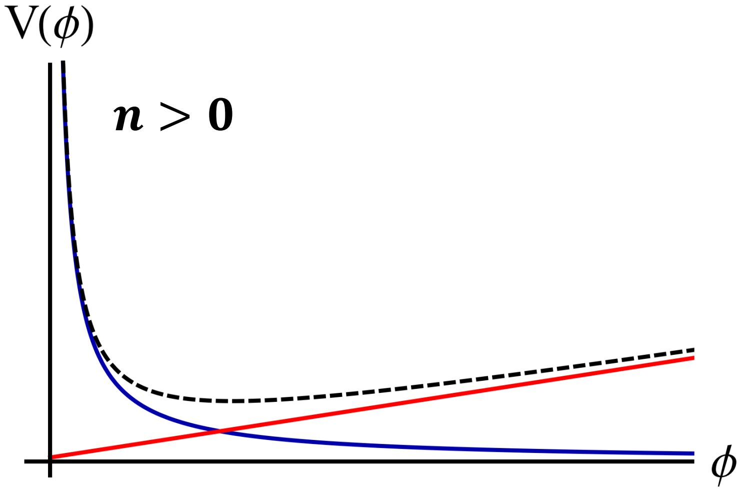
Profile outside of a spherical body

Gradients + amplification/screening

Chameleon Mechanism

[Khoury, Weltmann, *PRL* **93**, 171104 (2004); *PRD* **69**, 044026 (2004)]

$$V_{\mathrm{eff}}(\varphi) = \frac{\Lambda^{n+4}}{\varphi^n} + \frac{\rho\varphi}{M_*}$$



low ρ ⇒ low $m_{\mathrm{eff},\varphi}$

Chameleon Mechanism

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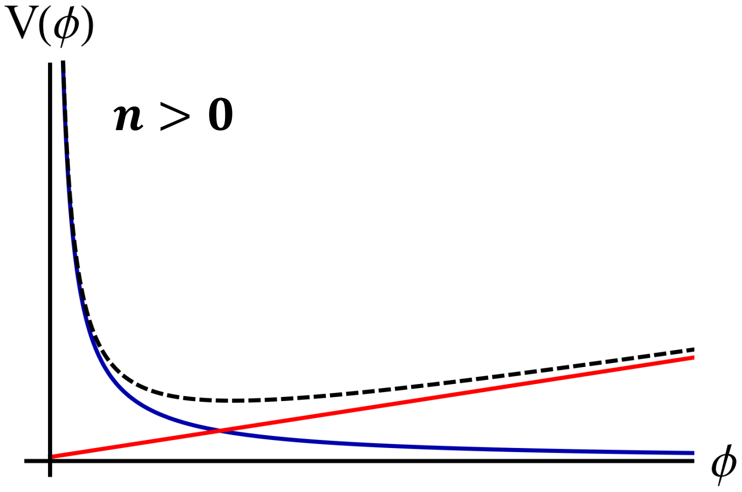
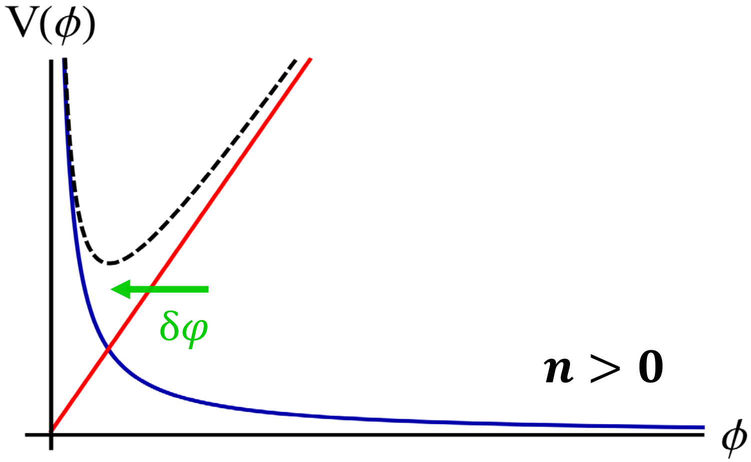


low ρ ⇒ low $m_{\text{eff},\varphi}$



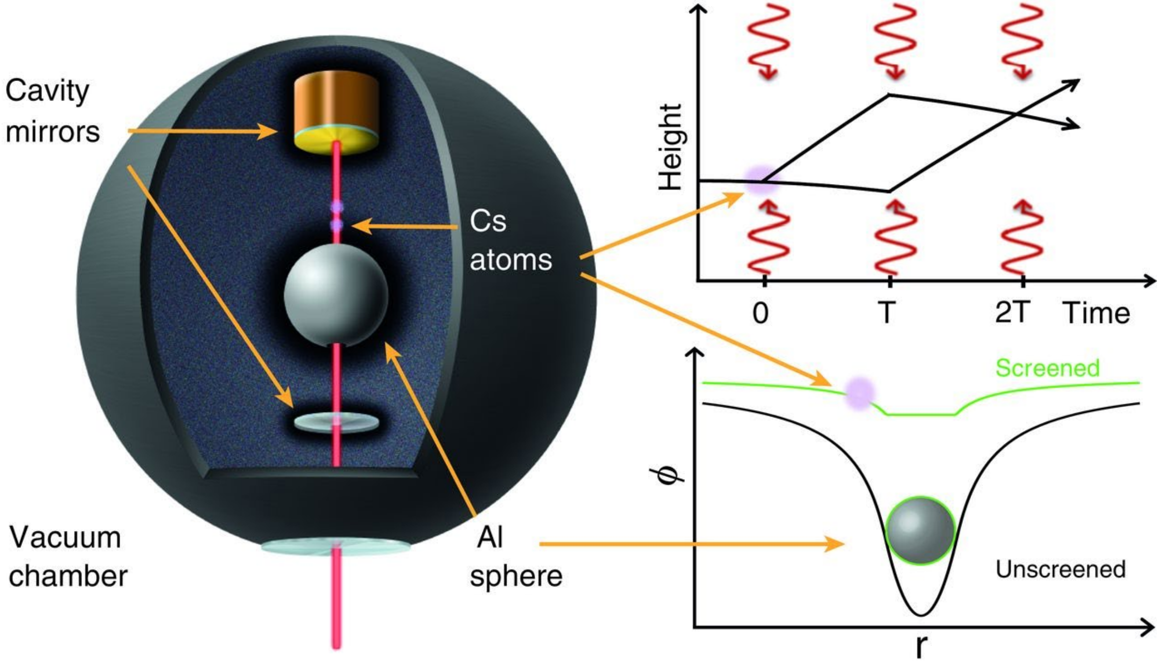
high ρ ⇒ high $m_{\text{eff},\varphi}$

- Going from a low-density environment (e.g., vacuum) to a high-density environment (e.g., terrestrial), the effective scalar mass increases, effectively reducing the interaction range $\lambda_{\text{eff}} \sim 1/m_{\text{eff},\varphi}$, and the value of φ diminishes
- Scalar field φ tends to be screened inside of dense bodies

Atom Interferometry Probes of Chameleons

[Burrage *et al*., *JCAP* **03** (2015) 042], [Hamilton *et al*., *Science* **349**, 849 (2015)], [Jaffe *et al*., *Nat. Phys.* **13**, 938 (2017)], [Sabulsky *et al*., *PRL* **123**, 061102 (2019)]

- Need $\lambda_{\mathrm{eff}} \lesssim R_{\mathrm{body}}$ inside a dense body for strong screening to occur
- Small test bodies (e.g., atoms) can evade strong screening



$$V_{\mathrm{eff}}(\varphi) = \frac{\Lambda^{n+4}}{\varphi^n} + \frac{\rho\varphi}{M_*}$$

$$\Rightarrow \ \delta\boldsymbol{a} = \frac{\boldsymbol{\nabla}\varphi}{M_*}$$

Atom Interferometry Probes of Chameleons

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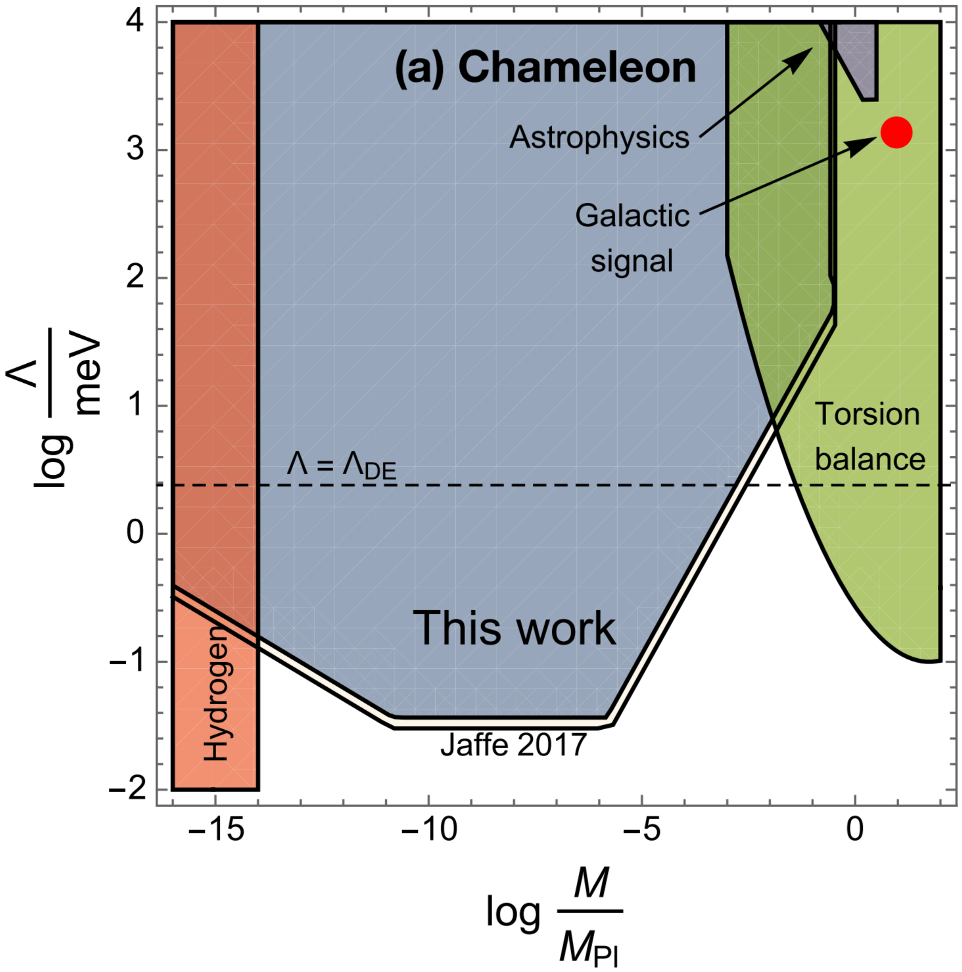
[Jaffe *et al.*, *Nat. Phys.* **13**, 938 (2017)], [Sabulsky *et al.*, *PRL* **123**, 061102 (2019)]



$$V_{\text{eff}}(\varphi) = \frac{\Lambda^5}{\varphi} + \frac{\rho\varphi}{M}$$

Symmetron Mechanism

[Olive, Pospelov, *PRD* **77**, 043524 (2008)], [Stadnik, *PRD* **102**, 115016 (2020)]

$$V_{\mathrm{eff}}(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - \varphi_0^2\right)^2 + \sum_{X=\gamma,e,N} \frac{\rho_X \varphi^2}{\left(\Lambda'_X\right)^2}$$

Effective potential

Bare potential

Ordinary matter (non-relativistic) contribution

$$\mathcal{L}'_\gamma = \frac{\varphi^2}{\left(\Lambda'_\gamma\right)^2}\frac{F_{\mu\nu}F^{\mu\nu}}{4} - \frac{\varphi^2}{\left(\Lambda'_f\right)^2} m_f \bar{f} f$$

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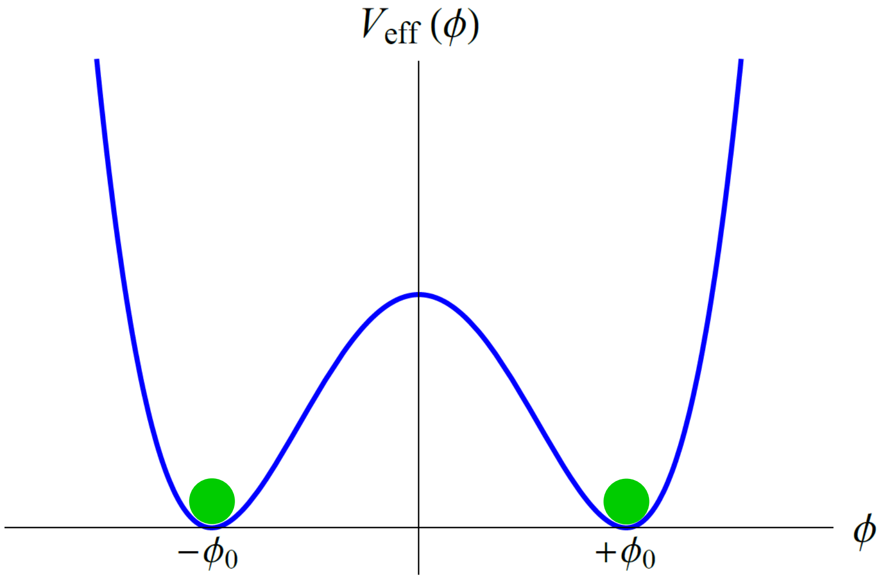
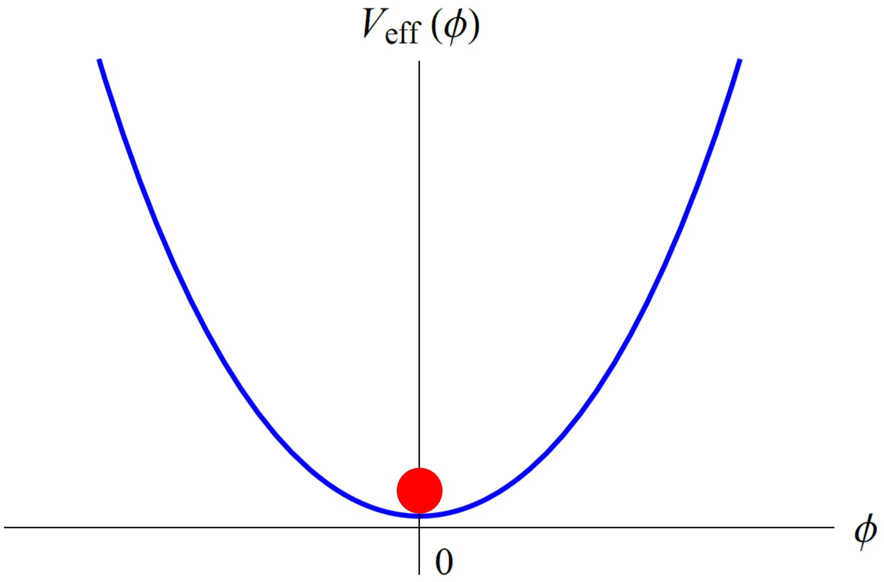


Low ρ (vacuum breaks Z_2 symmetry)



High ρ (vacuum respects Z_2 symmetry)

Z_2 symmetry: $\varphi \rightarrow -\varphi$

Symmetron Mechanism

[Olive, Pospelov, *PRD* **77**, 043524 (2008)], [Stadnik, *PRD* **102**, 115016 (2020)]

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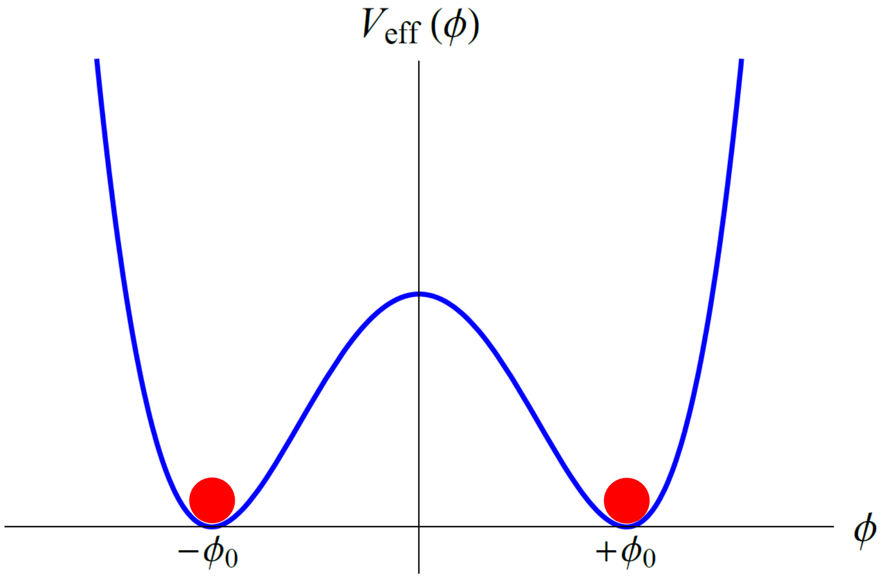

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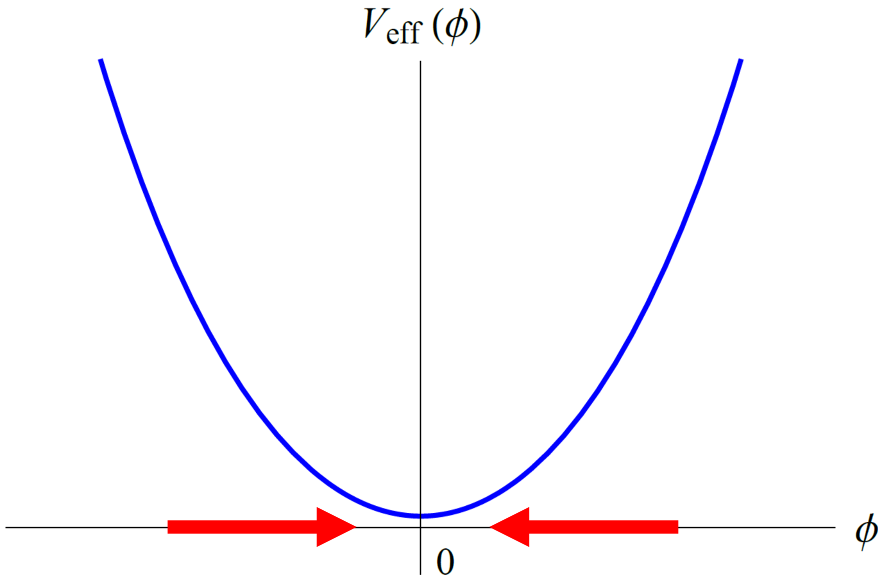


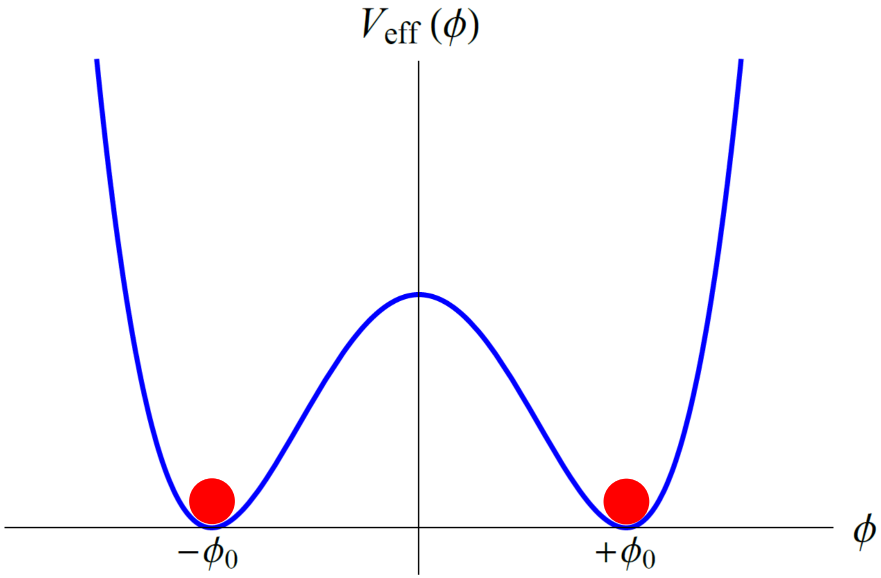
High ρ (vacuum respects Z_2 symmetry)

- Scalar field φ tends to be screened in dense environments

Symmetron Mechanism

[Olive, Pospelov, *PRD* **77**, 043524 (2008)], [Stadnik, *PRD* **102**, 115016 (2020)]

$$\mathcal{L}_\gamma = \frac{\varphi^2}{\left(\Lambda'_\gamma\right)^2} \frac{F_{\mu\nu}F^{\mu\nu}}{4} \quad \text{cf.} \quad \mathcal{L}_\gamma^{\text{SM}} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} - eJ_\mu A^\mu \quad \Rightarrow \quad \alpha\left(\varphi^2\right) \approx \alpha_0 \left[1 + \left(\frac{\varphi}{\Lambda'_\gamma}\right)^2\right]$$



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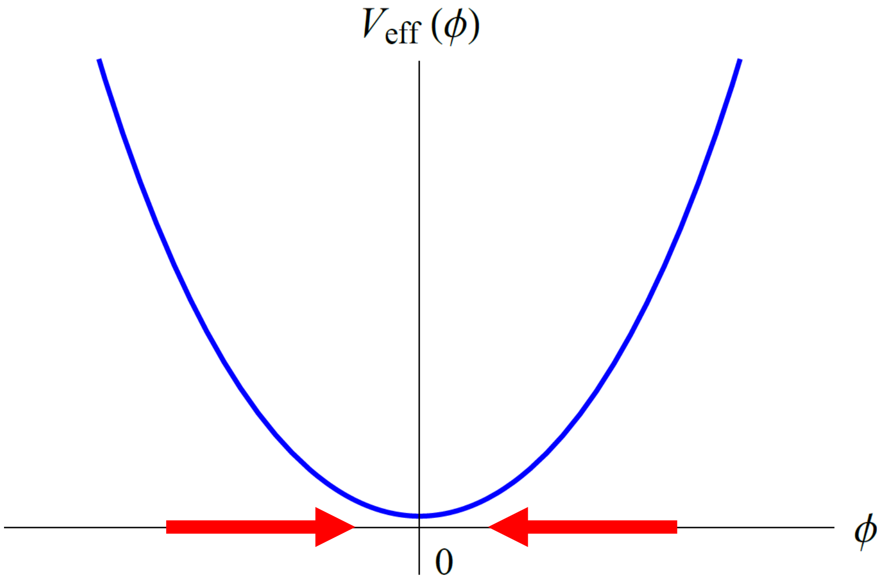
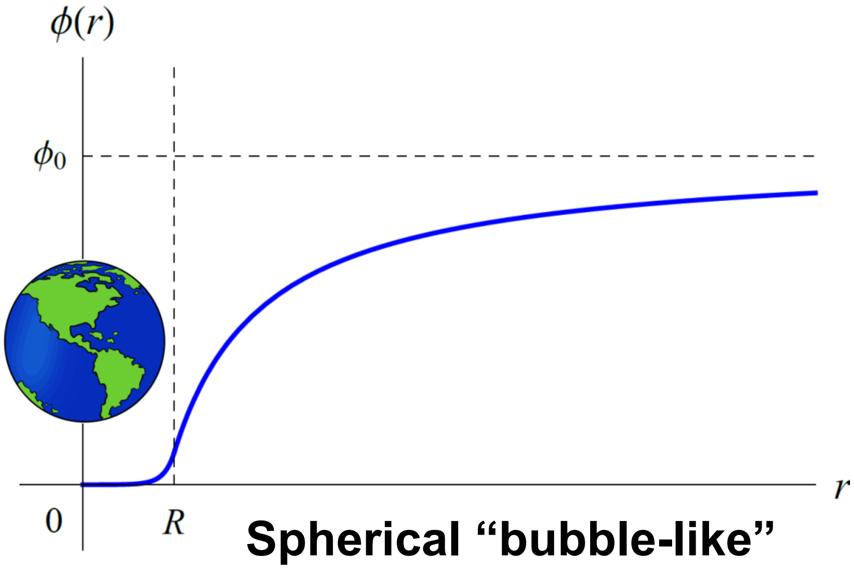


High ρ (vacuum respects Z_2 symmetry)

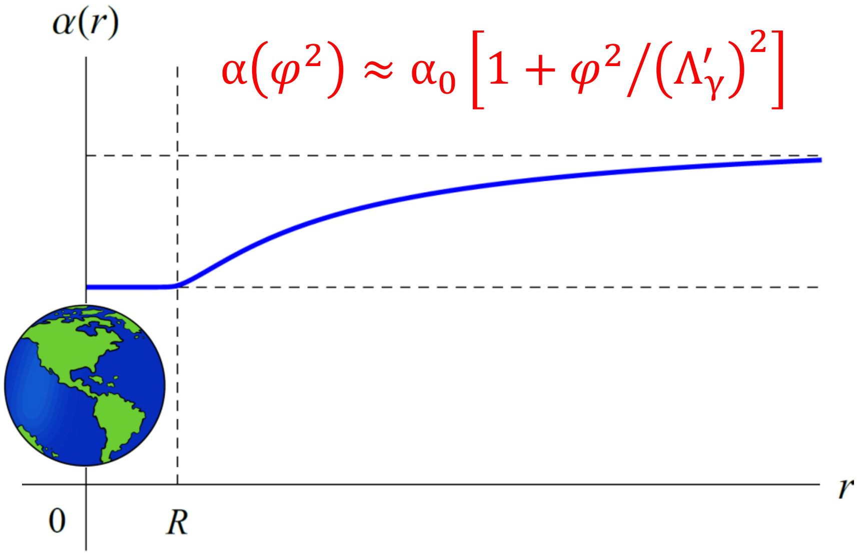
- Scalar field φ tends to be screened in dense environments
- $\alpha[\varphi^2(\rho)], m_f[\varphi^2(\rho)]$ ⇒ **Environmental dependence of "constants"**

Environmental Dependence of "Constants"

[Stadnik, *PRD* **102**, 115016 (2020)]



Spherical "bubble-like" defect around Earth



$$\alpha(\varphi^2) \approx \alpha_0 \left[1 + \varphi^2/(\Lambda'_\gamma)^2\right]$$

- **Variations of "constants" with height** above a dense body

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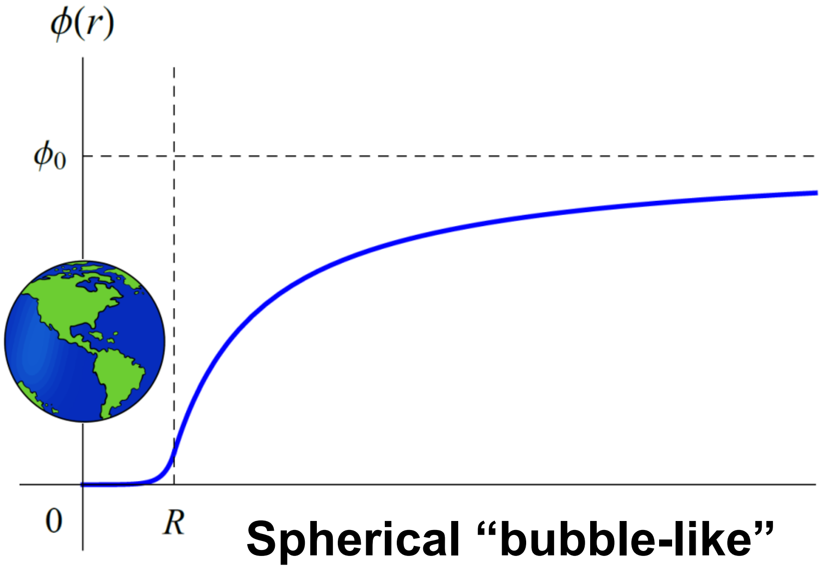

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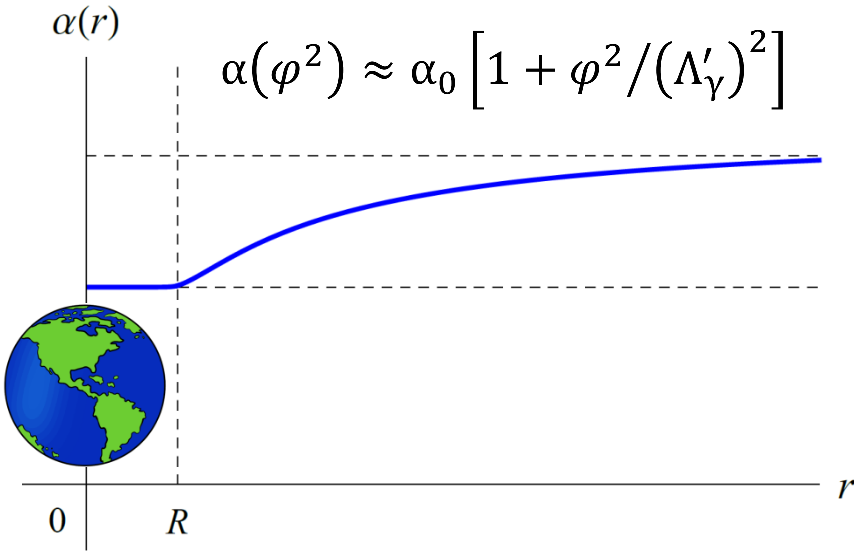
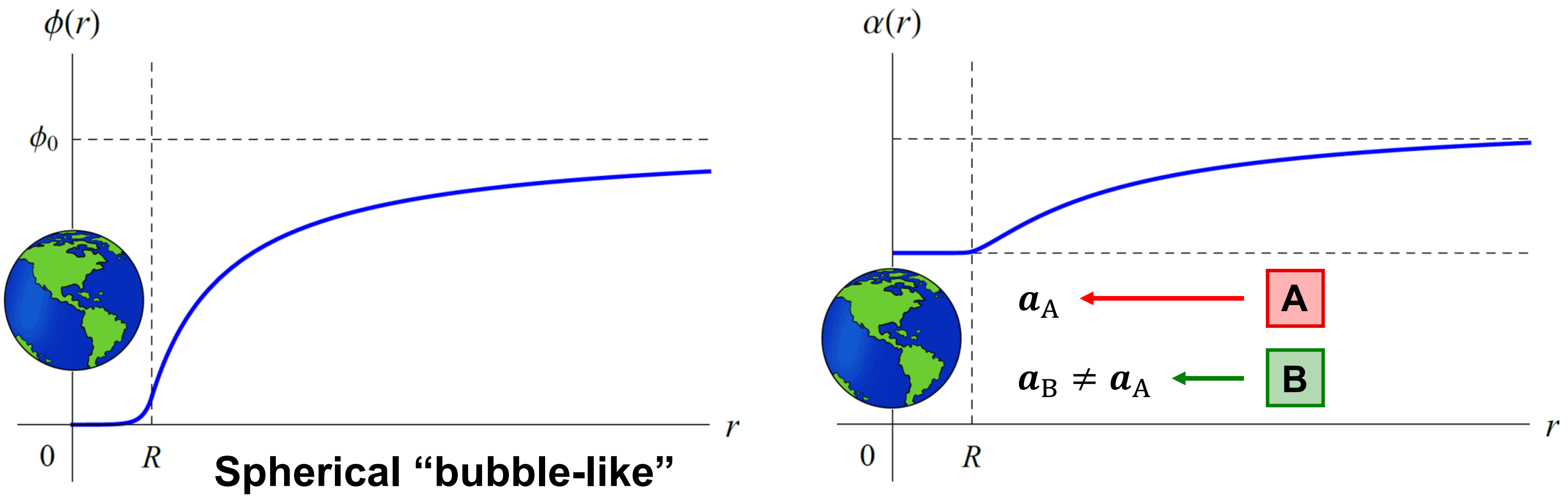


$$\alpha(\varphi^2) \approx \alpha_0 \left[1 + \varphi^2/(\Lambda'_\gamma)^2\right]$$

- **Variations of “constants” with height** above a dense body
- Can search for these spatial variations with:
  1. Equivalence-principle-violating forces: $\delta \boldsymbol{a}_{\mathrm{test}} = -\boldsymbol{\nabla}\left[m_{\mathrm{test}}(\alpha)\right]/m_{\mathrm{test}}$
  2. Compare clocks at different heights: $\Delta\nu/\nu \propto \Delta\alpha/\alpha$
  3. Compare laboratory and low-density ($\sim 10^{-3}\mathrm{cm}^{-3}$) astrophysical spectra

Environmental Dependence of "Constants"

[Stadnik, *PRD* **102**, 115016 (2020)]



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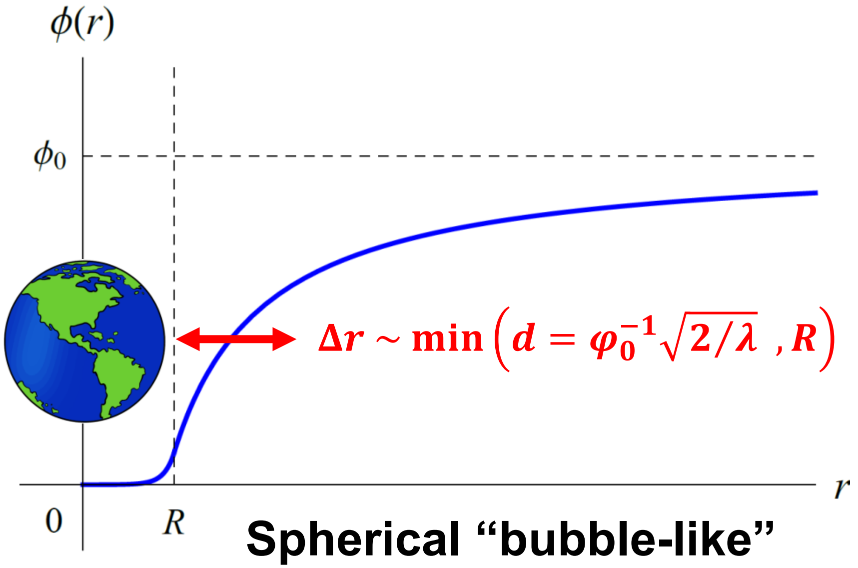

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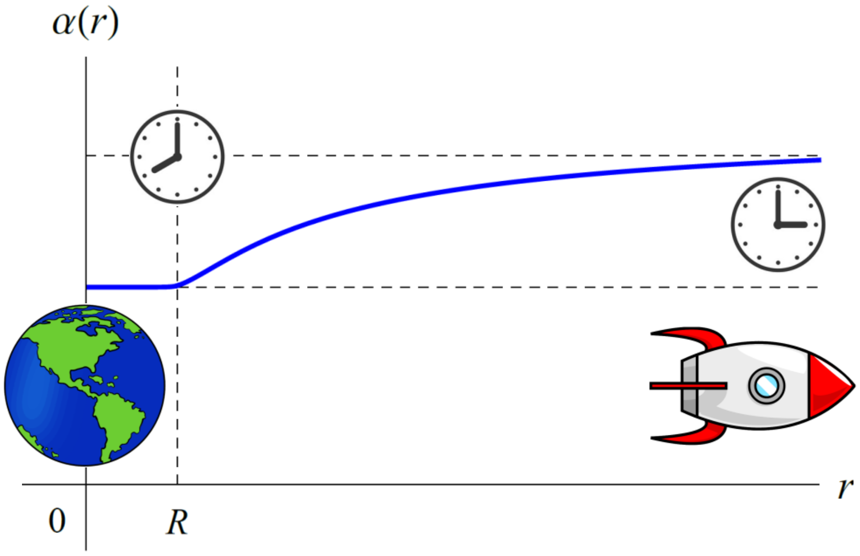

In space $(\Delta h \sim 10^4\ \mathrm{km})$

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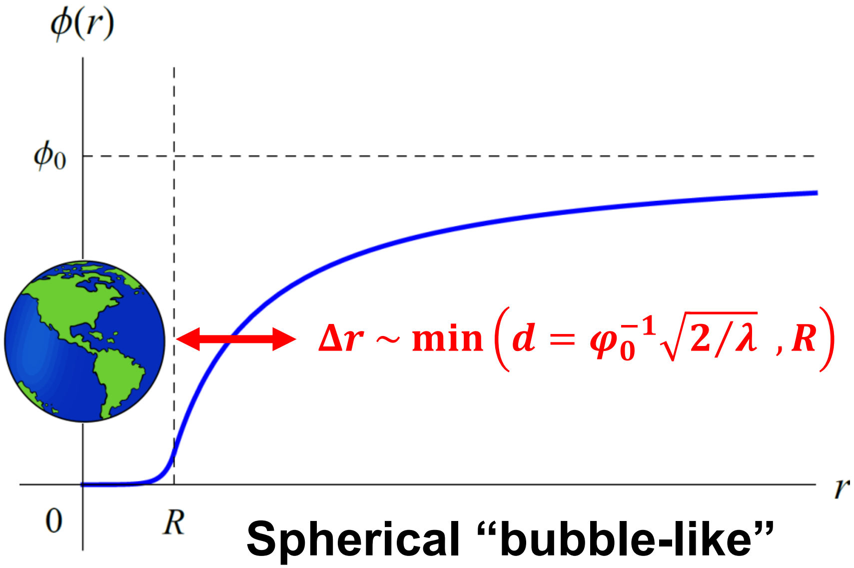

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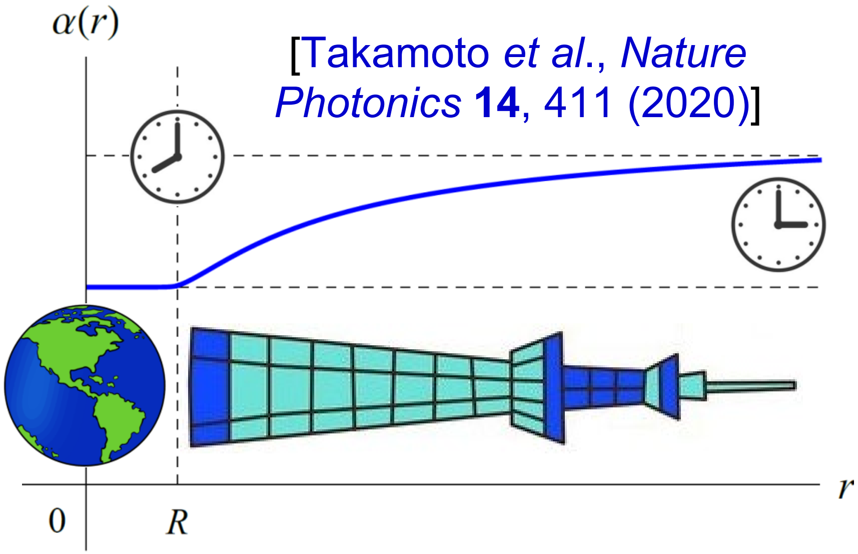

[Takamoto *et al*., *Nature Photonics* **14**, 411 (2020)]

TOKYO SKYTREE $(\Delta h \approx 0.5 \text{ km})$

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Interpretation of Single-Clock-Type Experiments

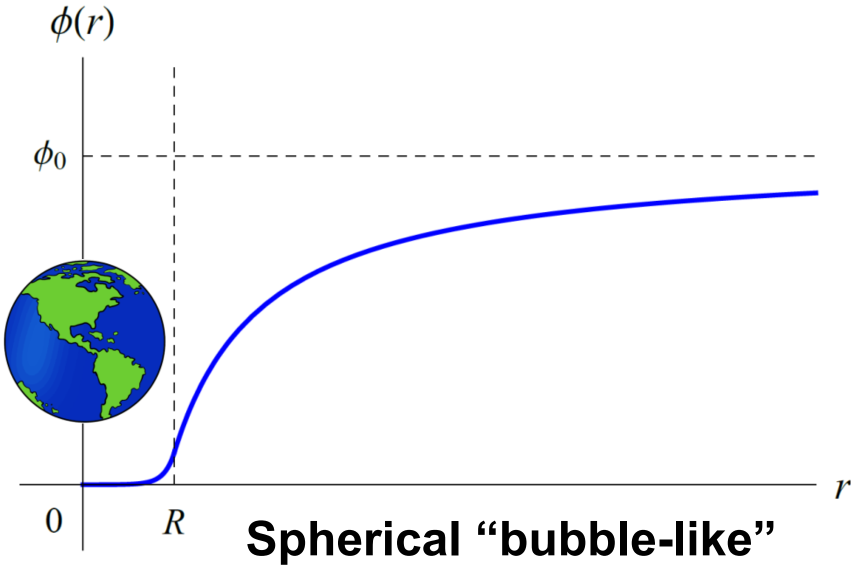
[Stadnik, *PRD* **102**, 115016 (2020)]

- The Tokyo Skytree experiment measured the gravitational potential difference between a pair of Sr optical clocks separated by a height difference of $\Delta h \approx 450\,\mathrm{m}$; for details, see: [Takamoto *et al*., *Nature Photonics* **14**, 411 (2020)]
- To distinguish the effects of a scalar field φ from the usual gravitational redshift effect, can "reference" a single pair of clocks against a combination of laser-ranging and gravimeter measurements (which provide an independent prediction of the clock frequency shift within the framework of relativity)

$$\left.\begin{array}{c} \Delta\nu_{\mathrm{Sr}} \propto \Delta\left(m_e \alpha^2\right) \\ \Delta \boldsymbol{a}_{\mathrm{grav}} \propto \boldsymbol{\nabla} m_N \end{array}\right\} \Rightarrow \left(\frac{\Delta\nu_{\mathrm{Sr}}}{\nu_{\mathrm{Sr}}}\right)_{\mathrm{eff}} \approx \Delta\left(\varphi^2\right)\left[\frac{2}{\left(\Lambda'_\gamma\right)^2} + \frac{1}{(\Lambda'_e)^2} - \frac{1}{\left(\Lambda'_N\right)^2}\right]$$

Environmental Dependence of “Constants”

[Stadnik, *PRD* **102**, 115016 (2020)]



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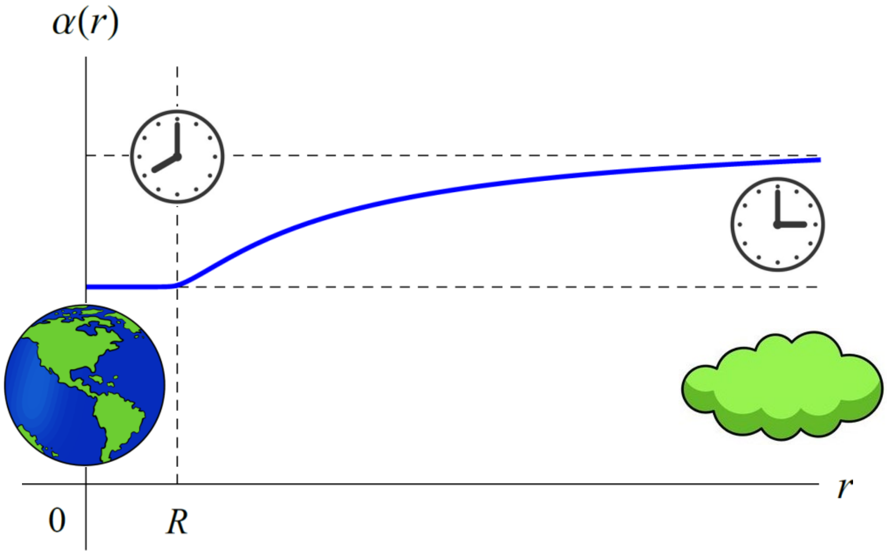

Distant gas clouds

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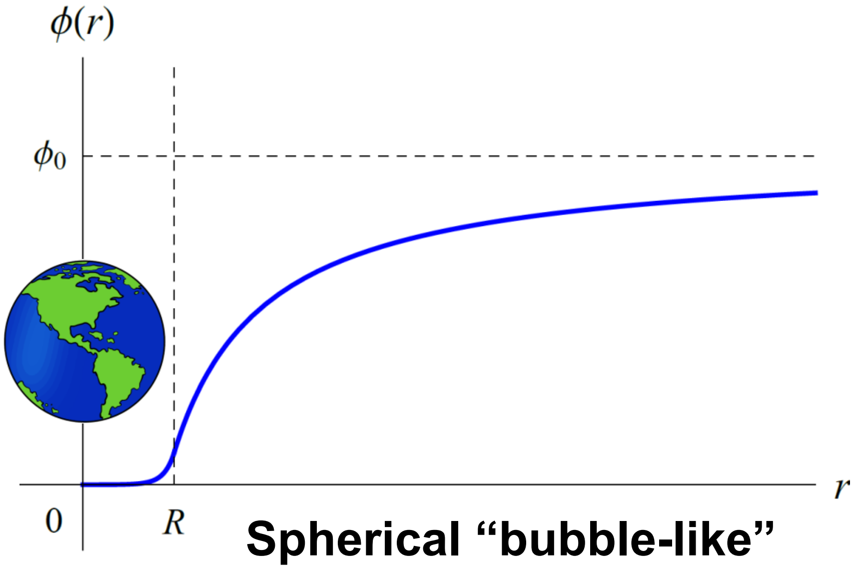

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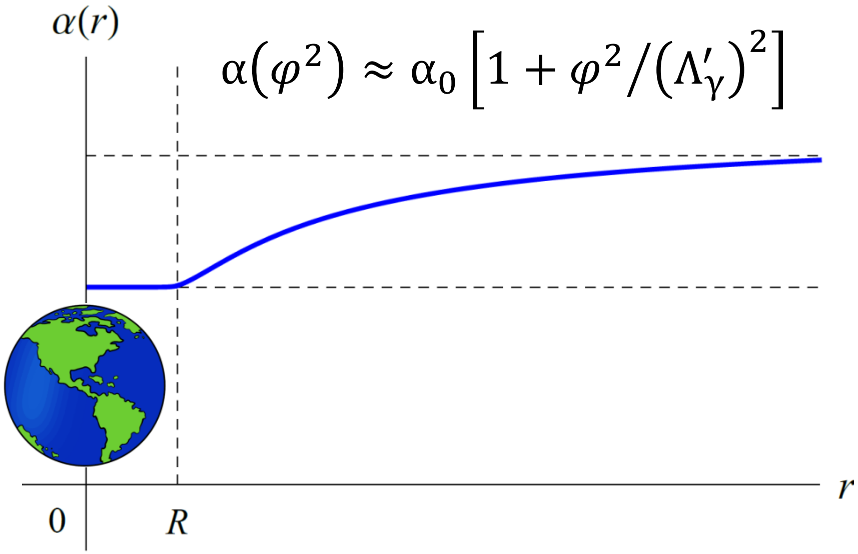

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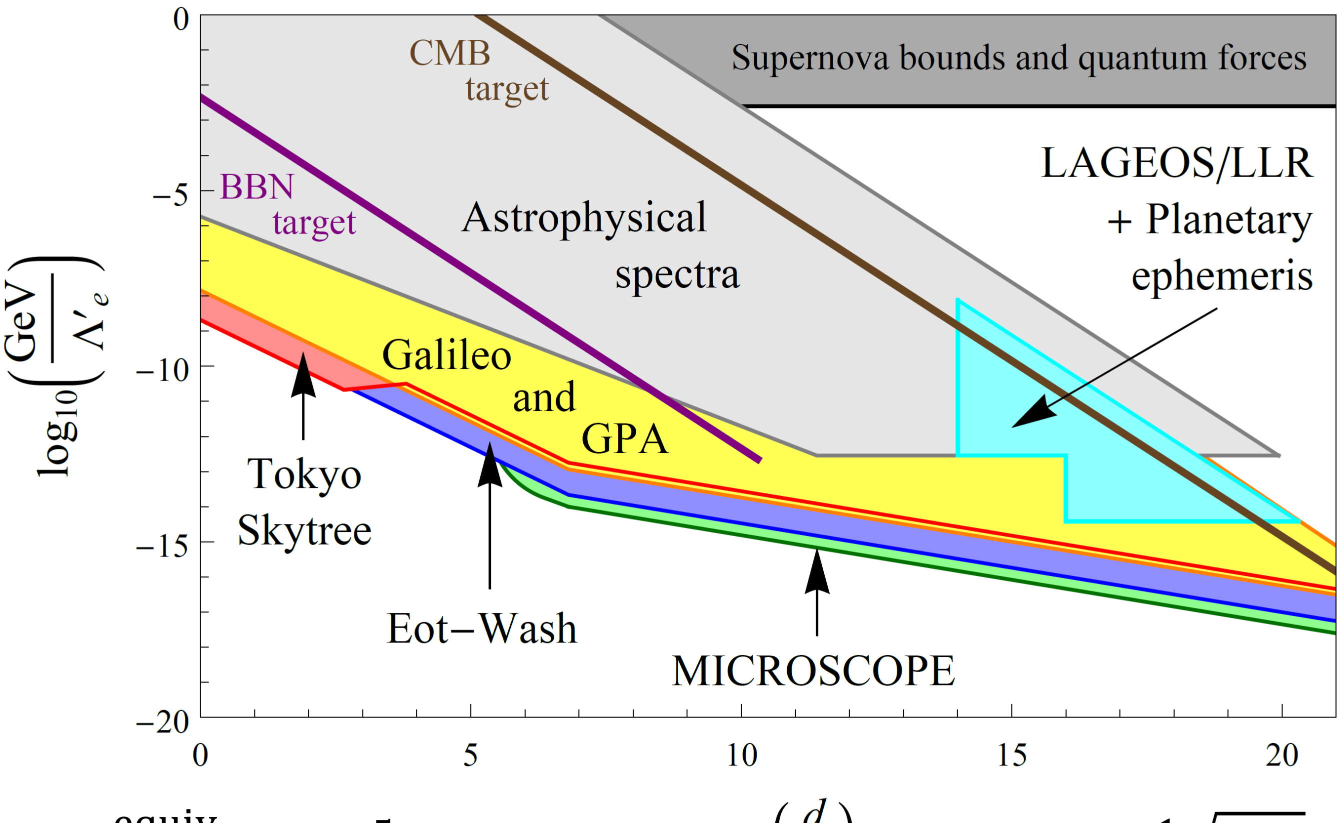
Bounds on Symmetron Model

[Stadnik, *PRD* **102**, 115016 (2020)]

$$V_{\mathrm{eff}}(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - \varphi_0^2\right)^2 + \frac{\rho_e \varphi^2}{(\Lambda'_e)^2}$$

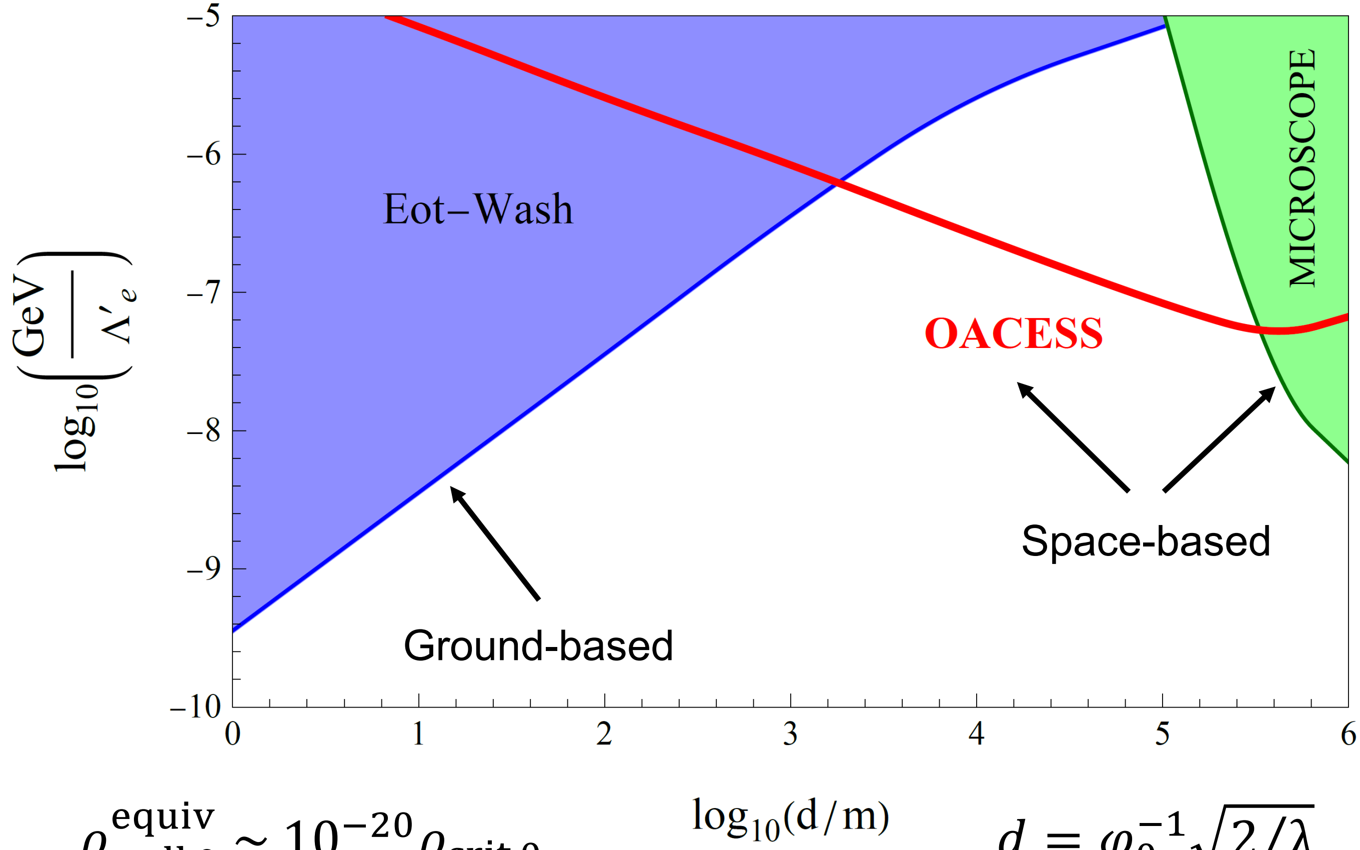


$\rho_{\mathrm{wall},0}^{\mathrm{equiv}} \sim 10^{-5}\rho_{\mathrm{crit},0}$ $\log_{10}\left(\frac{d}{\mathrm{m}}\right)$ $d = \varphi_0^{-1}\sqrt{2/\lambda}$

Opportunities for Space-Based Experiments

[Schkolnik *et al.*, *Quantum Sci. Technol.* **8**, 014003 (2023)]

$$V_{\text{eff}}(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - \varphi_0^2\right)^2 + \frac{\rho_e \varphi^2}{(\Lambda'_e)^2}$$



$\rho_{\text{wall},0}^{\text{equiv}} \sim 10^{-20}\rho_{\text{crit},0}$

$d = \varphi_0^{-1}\sqrt{2/\lambda}$