Nuclear Contributions to Two-Photon Exchange in Muonic Deuterium

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Two-Photon Exchange in (Muonic) Atoms

Bohr radius

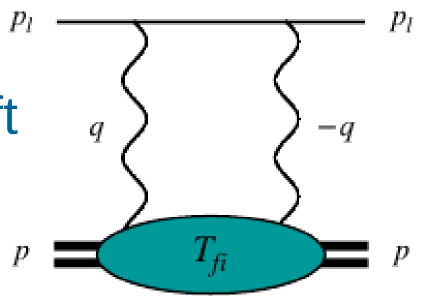
$a = (Z\alpha m_r)^{-1}$

- Muonic atoms: greater sensitivity to charge radii
- But also greater sensitivity to subleading nuclear response

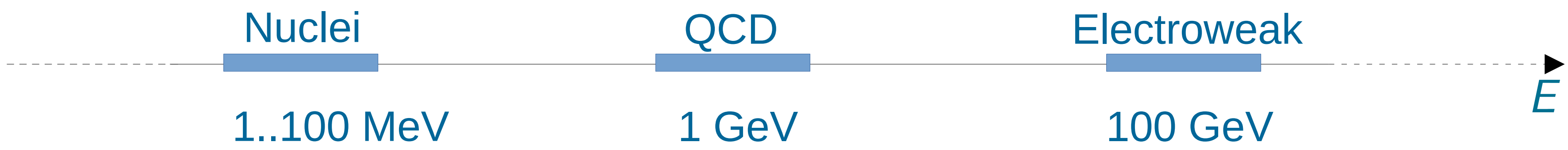
Lamb Shift: $$\Delta E_{nS} = \frac{2\pi Z\alpha}{3}\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R_F^3\right] + \dots$$

Friar radius: $$R_{\mathrm{F}}^3 = \frac{48}{\pi}\int_0^\infty \frac{dQ}{Q^4}\left[G_C^2(Q^2) - 1 - 2G_C'(0)\,Q^2\right]$$

- The dominant correction: Two-Photon Exchange (TPE) $\propto \alpha^5$
- At current precision already significant in ordinary atoms
- Doubly virtual forward Compton scattering: VVCS
  - spin-independent (unpolarised nucleus): Lamb shift
  - spin-dependent (polarised nucleus): HFS
- Nuclear polarisabilities – two-photon nuclear response

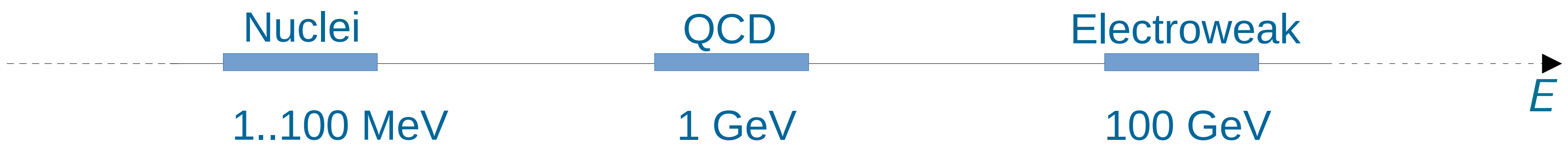


Dealing with Nuclei: Effective Field Theories



- QCD is the theory of strong interactions
- At low energies, however:
  - QCD is non-perturbative, calculations still forbiddingly difficult
- Can we still investigate nuclear interactions?

Dealing with Nuclei: Effective Field Theories

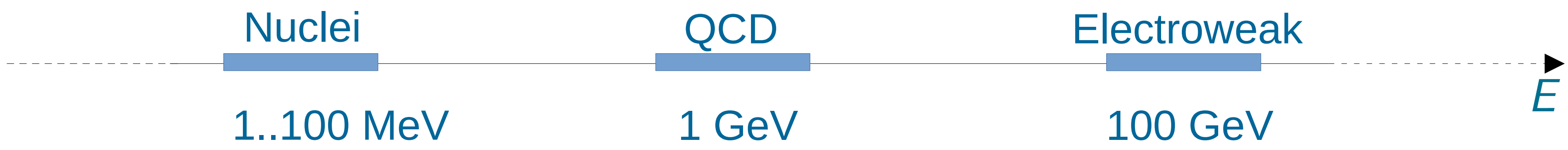


- QCD is the theory of strong interactions
- At low energies, however:
  - QCD is non-perturbative, calculations still forbiddingly difficult

- Can we still investigate nuclear interactions?

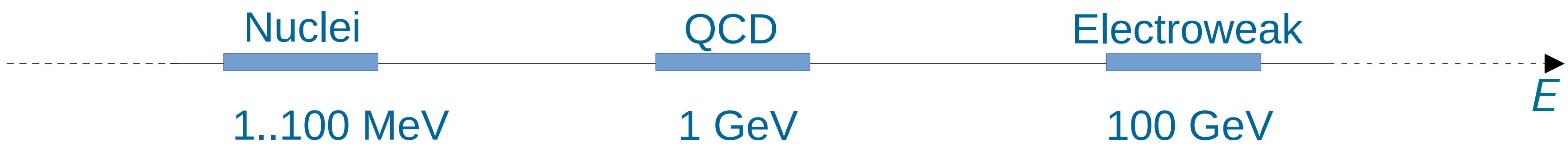
- Yes, using effective field theories (EFTs)!

Dealing with Nuclei: Effective Field Theories



- General ideas underlying EFTs:
  - at low energies we cannot distinguish the details of whatever happens at high energies (even the high-energy d.o.f.’s)
  - still, we want a theory that describes the same physics as the high-energy theory: symmetries must be preserved!
  - separation of scales: high-energy d.o.f.’s have to remain unresolved!
- Interactions: anything allowed by the underlying symmetries
  - coupling constants (LECs) not always constrained by the symmetry
  - they are fitted to data or obtained from the underlying theory
- Expansion in low momenta, interactions with high derivatives or diagrams with many loops are suppressed*
  - controlled evaluation order-by-order and error estimate

Dealing with Nuclei: Chiral EFT



F. Hagelstein's talk today
A. Filin's talk@µASTI tomorrow

- Chiral EFT:
  - the pions enjoy a very special role: Goldstone bosons of the spontaneously broken $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry of QCD
  - isospin symmetry (only slightly broken by quark masses) remains
  - interactions are ordered by increasing powers of momenta
  - expansion in powers of $\chi \simeq m_\pi/\Lambda_{\text{high}}$ $\quad \Lambda_{\text{high}} \simeq 600..800\ \text{MeV}$
  - to describe nucleon-nucleon interaction, expand the NN (NNN, …) potentials and solve the Lippmann-Schwinger equation

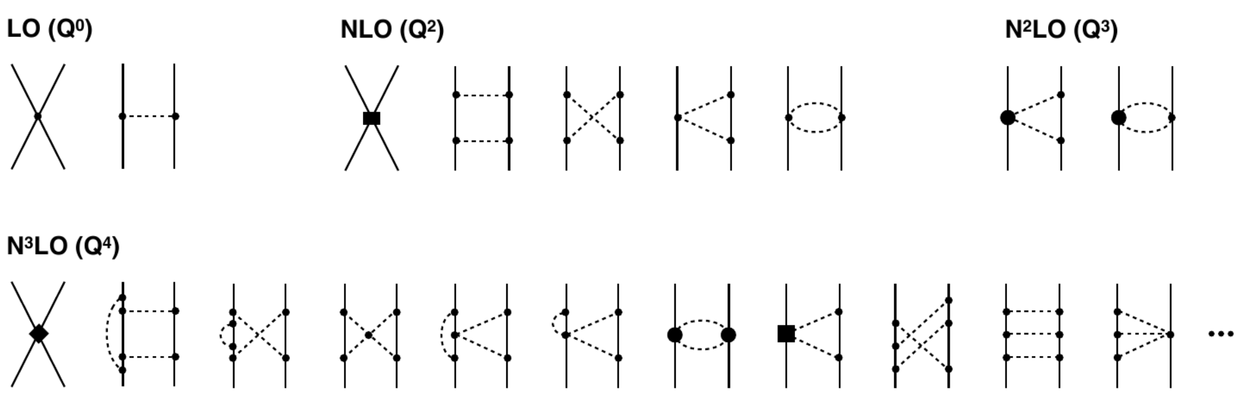
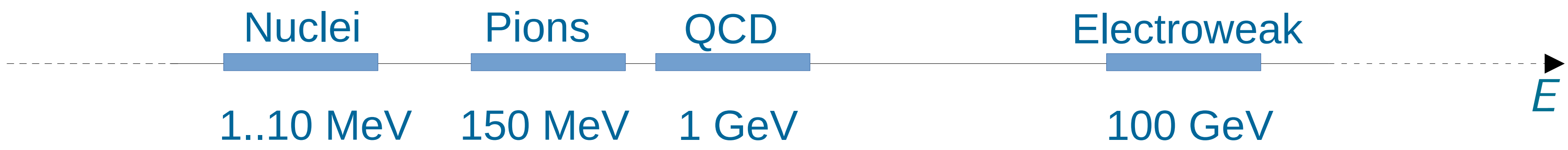
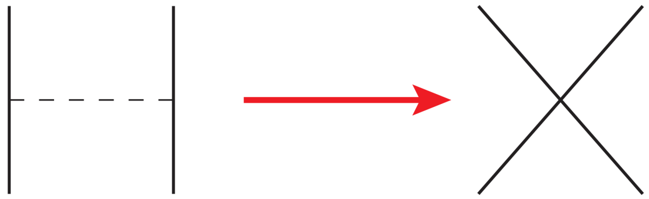


figure from Epelbaum, Krebs, Meißner, 2014

Dealing with Nuclei: Pionless EFT

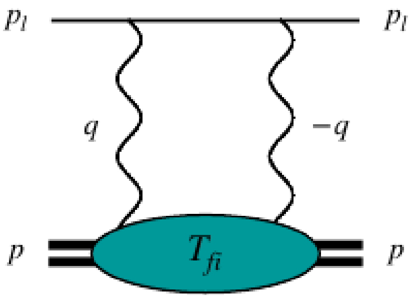


- At very low energies [such as typical energies in (muonic) atoms] one can treat the pions as heavy d.o.f.'s and use only nucleons
  - Contact interactions between nucleons
  - Expansion in powers of $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$
  - LO loops are resummed to reproduce the bound/virtual states [equivalent to solving the Lippmann-Schwinger equation]



- Advantages:
  - analytic results (at least for two nucleons)
  - relatively easy to solve and analyse
  - explicit renormalisation and gauge invariance
- Potentially slower convergence than Chiral EFT
- Let's calculate!

Back to VVCS



- Forward unpolarised VVCS amplitude

$$\alpha_{em} M^{\mu\nu}(\nu, Q^2) = -\left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) + \frac{1}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) T_2(\nu, Q^2) \right\}$$

$Q^2 = -q^2, \ \nu = p \cdot q / M_{\text{target}}$ photon virtuality and lab frame energy

Lamb Shift: $$E_{nS}^{2\gamma} = -8i\pi\alpha m \, [\phi_n(0)]^2 \int \frac{d^4 q}{(2\pi)^4} \frac{(Q^2 - 2\nu^2)\, T_1(\nu, Q^2) - (Q^2 + \nu^2)\, T_2(\nu, Q^2)}{Q^4 (Q^4 - 4m^2\nu^2)}$$

Point-like and finite size contributions need to be subtracted!

- Need to know both the elastic and inelastic parts of the amplitude

$$T_{1,2}(\nu, Q^2) = T_{1,2}^{\text{elastic}}(\nu, Q^2) + T_{1,2}^{\text{inel}}(\nu, Q^2)$$

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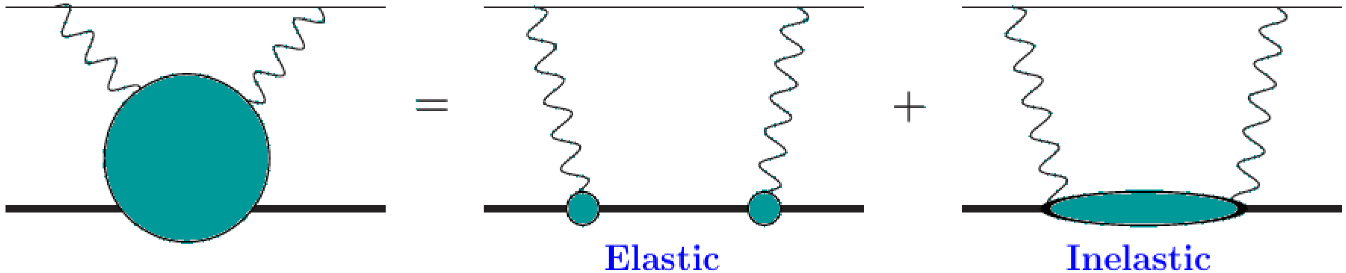


Counting for VVCS and TPE: Predictive Powers

- Longitudinal and Transverse amplitudes

$$f_L(\nu, Q^2) = -T_1(\nu, Q^2) + \left(1 + \frac{\nu^2}{Q^2}\right) T_2(\nu, Q^2), \qquad f_T(\nu, Q^2) = T_1(\nu, Q^2)$$

Lamb Shift:

$$\Delta E_{nl} = -8i\pi m\, [\phi_{nl}(0)]^2 \int \frac{d^4 q}{(2\pi)^4} \frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2) f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

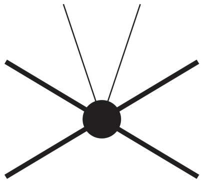
$$f_L = O(p^{-2}), \qquad f_T = O(p^0)$$ in the VVCS amplitude

$\alpha_{E1} = 0.64\ \text{fm}^3$

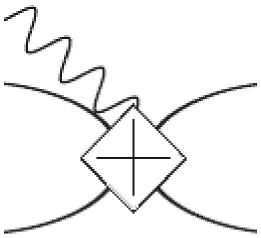
$\beta_{M1} = 0.07\ \text{fm}^3$

$$\text{longitudinal} = O(p^{-2}), \qquad \text{transverse} = O(p^2)$$ in TPE

- Transverse contribution to TPE starts only at N4LO
- N4LO: ΔE_{nl} needs to be regularised, an unknown lepton-NN LEC



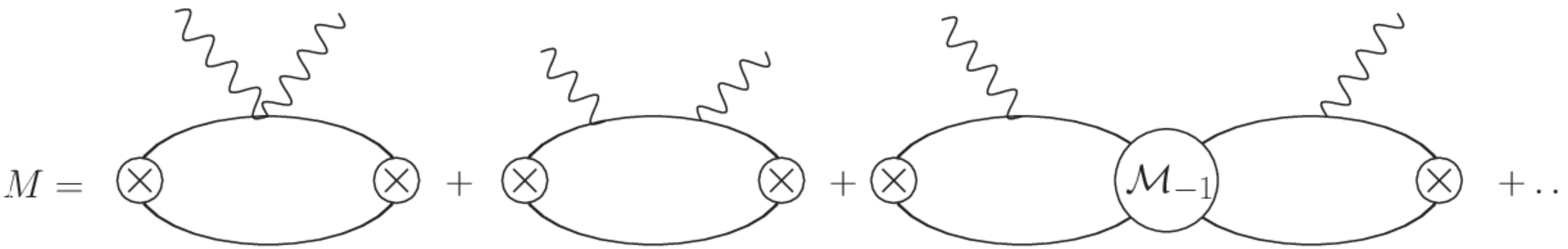
- We go up to N3LO in f_L, and up to (relative) NLO in f_T [cross check]
- One unknown LEC at N3LO in f_L
  - important for the charge form factor
  - extracted from the H-D isotope shift and proton R_E



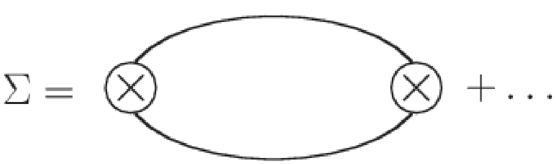
Amplitude with Deuterons

- The reaction amplitude is given by the LSZ reduction

$$T = M \left[\left. \frac{\mathrm{d}\Sigma(E)}{\mathrm{d}E} \right|_{E=E_d} \right]^{-1}$$



– irreducible VVCS graphs (here full LO for f_L; crossed not shown)



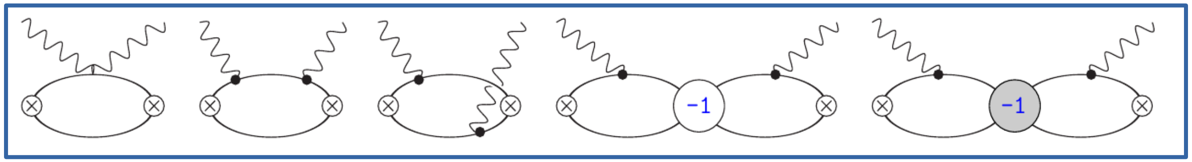
– deuteron self-energy (here at LO)

- The expression for the residue is very simple up to N3LO:

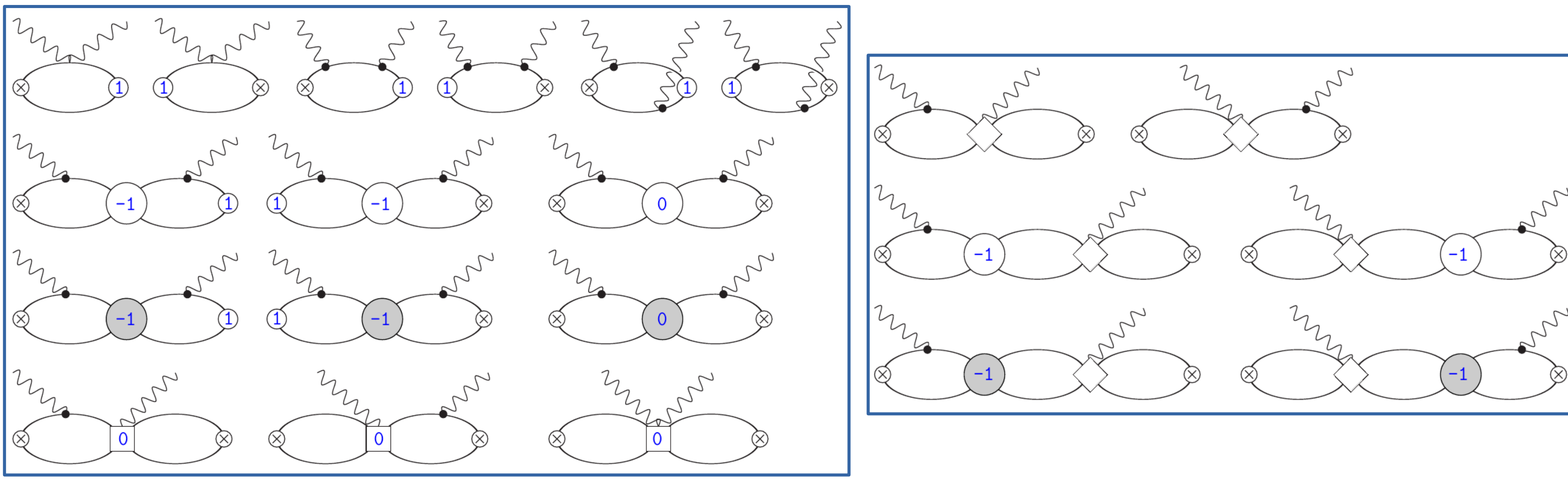
$$\left[\left. \frac{\mathrm{d}\Sigma(E)}{\mathrm{d}E} \right|_{E=E_d} \right]^{-1} = \frac{8\pi\gamma}{M^2} \left[1 + (Z-1) + 0 + 0 + \ldots\right]$$

Deuteron VVCS: Feynman Graphs

LO



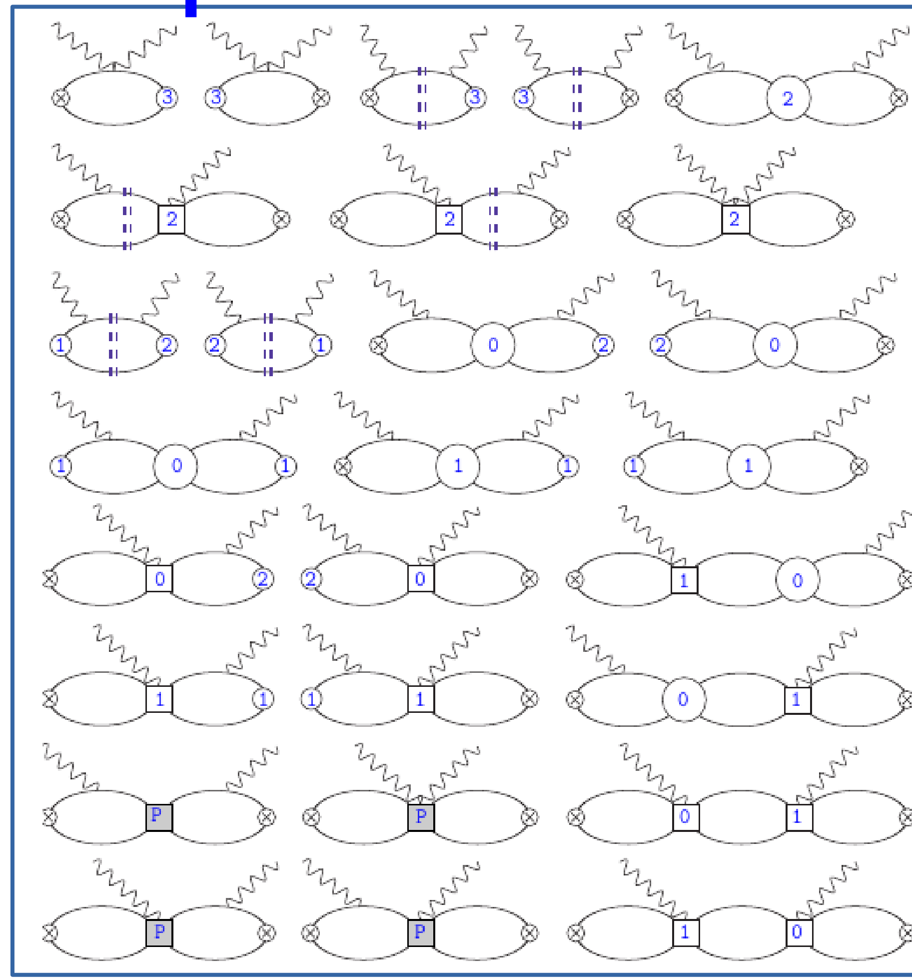
NLO



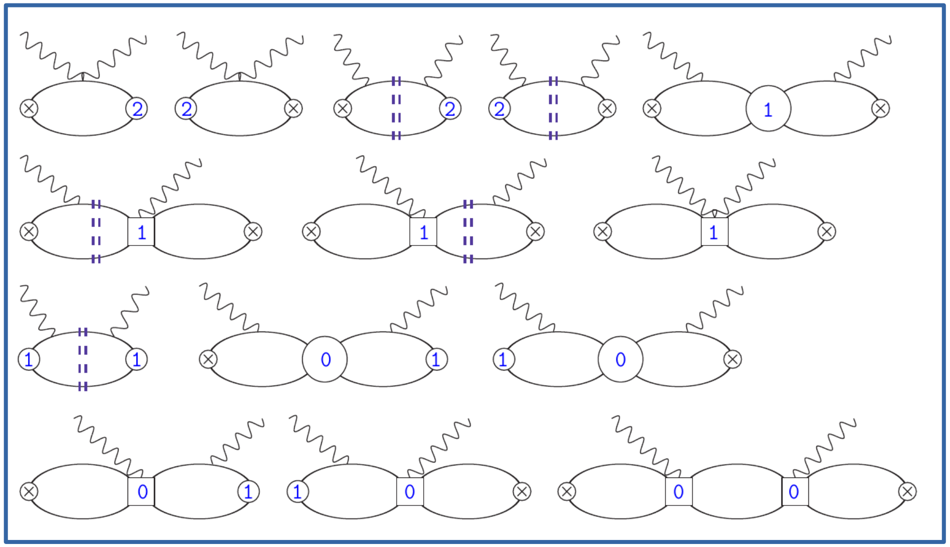
- Amplitudes are calculated analytically (dimreg+PDS) Kaplan, Savage, Wise (1998)
- Checks:
  - ➔ the sum of each subgroup (+ respective crossed graphs) is gauge invariant
  - ➔ regularisation scale dependence has to vanish

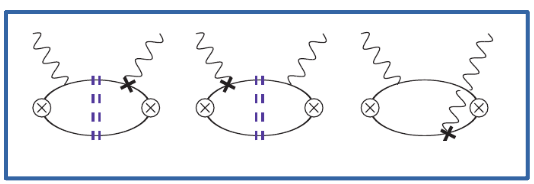
Deuteron VVCS: Feynman Graphs

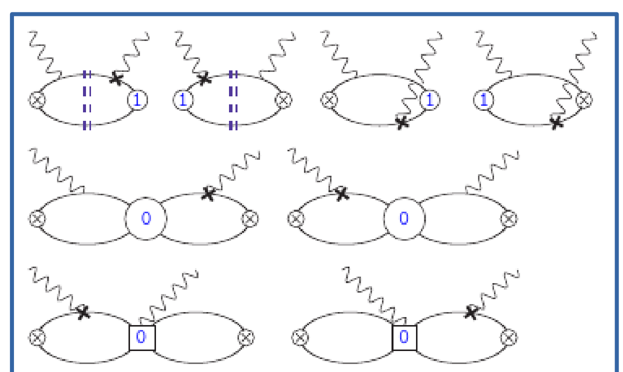
N3LO

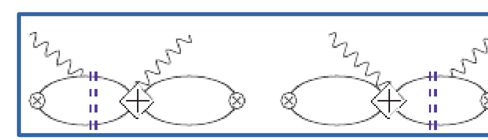


NNLO



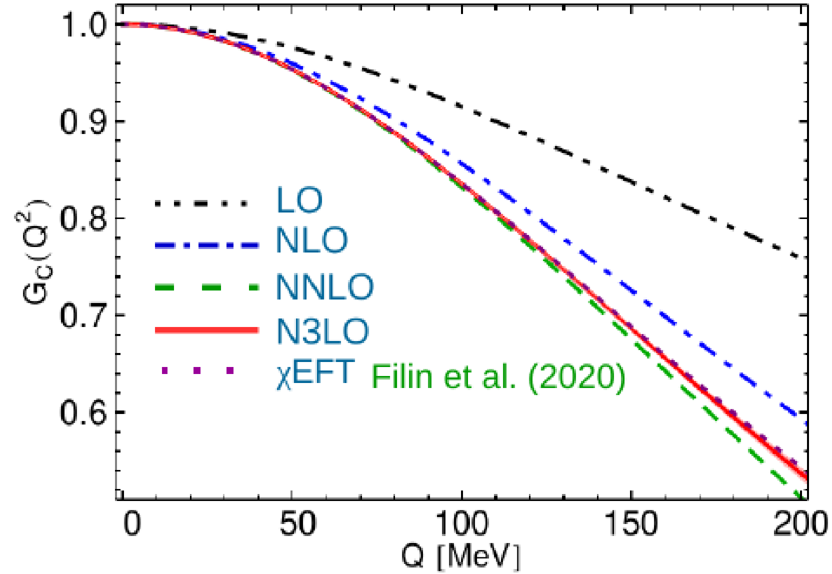




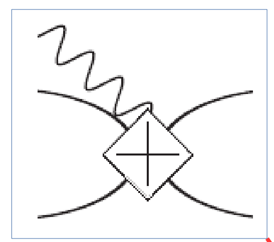


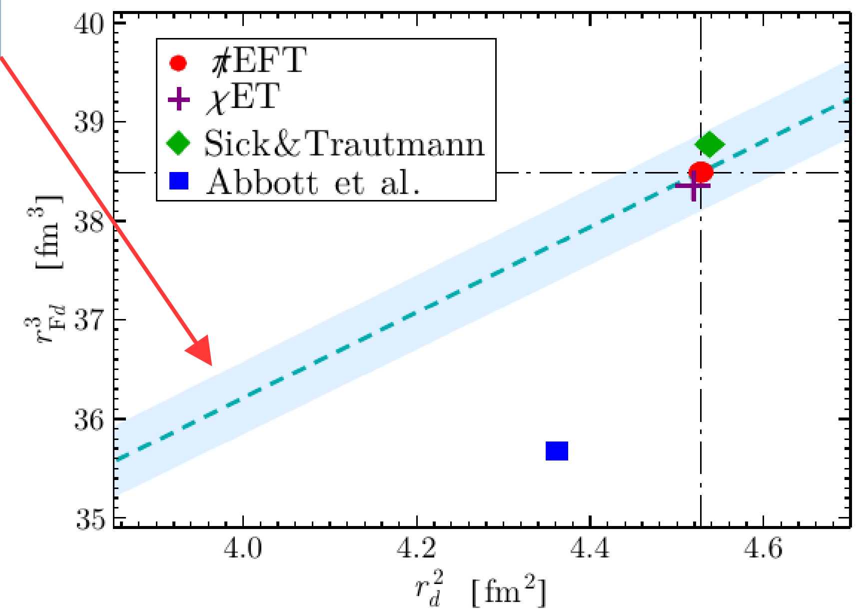
Deuteron Charge Form Factor and TPE in μD

- The deuteron charge form factor obtained from the residue of the VVCS amplitude
- The result is consistent with χEFT
- Correlation between R_F and R_E
  - generated by the N3LO LEC



VL, Hiller Blin, Pascalutsa (2021)





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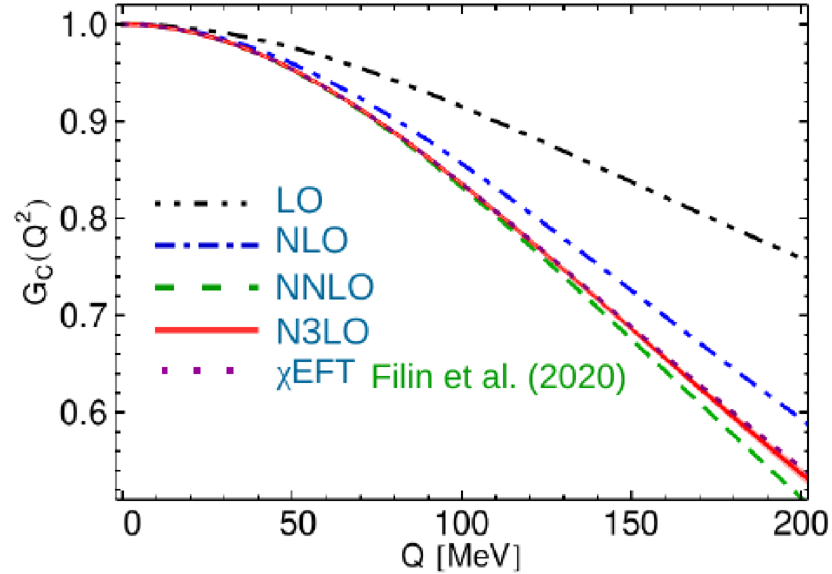

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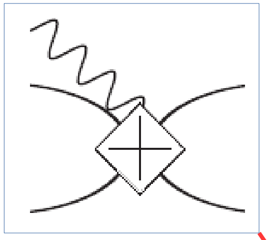


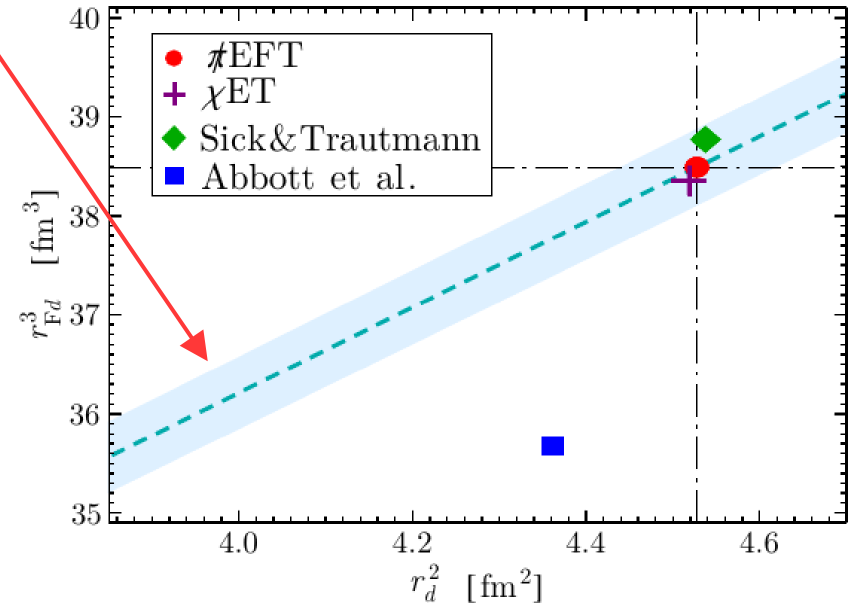
$$R_F^3 = \frac{48}{\pi} \int_0^\infty \frac{dQ}{Q^4} \left[G_C^2(Q^2) - 1 - 2G_C'(0)\, Q^2 \right]$$

$$= \frac{3}{80\gamma^3} \{ Z\,[5 - 2Z(1 - 2\ln 2)]$$

$$-320/9\ r_0^2\gamma^2\,[Z(1 - 4\ln 2) - 2 + 2\ln 2]$$

$$+80(Z-1)^3\ l_1^{C0_S} \}$$

$$R_E^2 = \frac{1}{8\gamma^2} + \frac{Z-1}{8\gamma^2} + 2r_0^2 + \frac{3(Z-1)^3}{\gamma^2}\ l_1^{C0_S}$$



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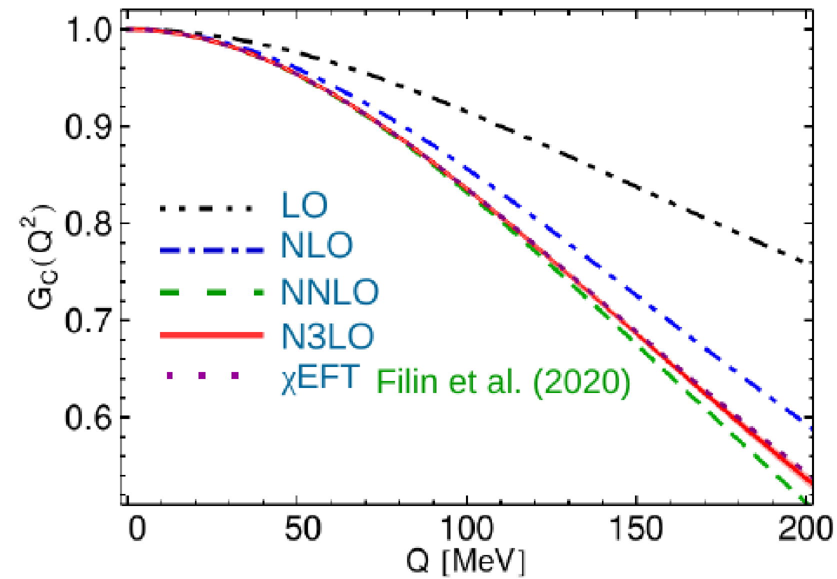

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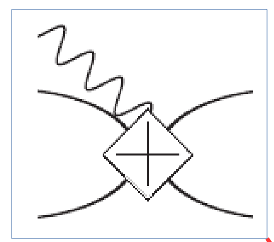

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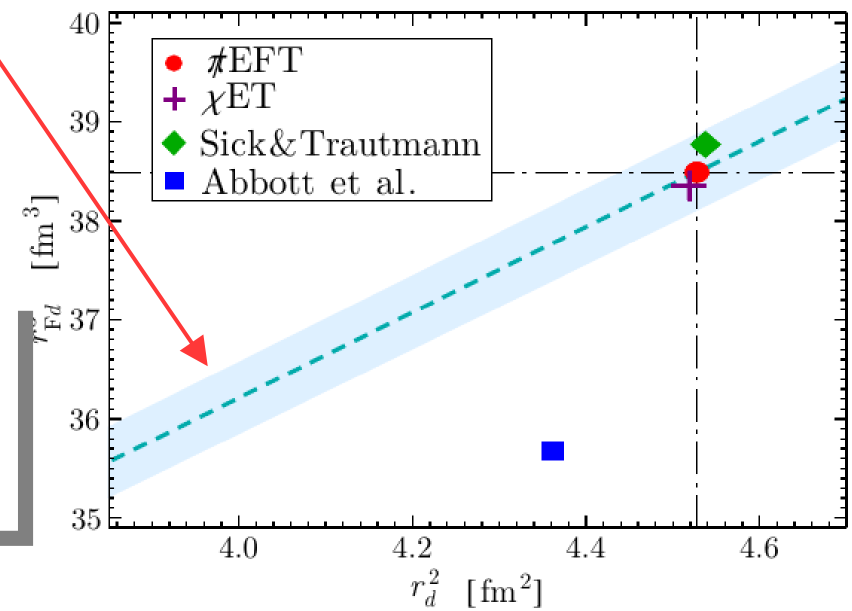
$R_E^2 =$

- Benchmark: some parametrisations fail to describe the low-Q properties
- Agreement with χEFT vindicates both EFTs



VL, Hagelstein, Pascalutsa (2022)

TPE in μD: Higher-Order Corrections

$$\Delta E_{2S} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} = -1.955(19)\text{meV}$$

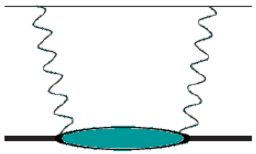
- Higher-order in α terms are important in D
  - Coulomb $[\mathcal{O}(\alpha^6 \log \alpha)]$

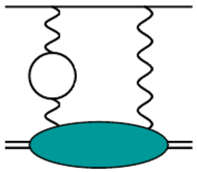
    taken from elsewhere $\Delta E_{2S}^{\text{Coulomb}} = 0.2625(15)$ meV

  - eVP $[\mathcal{O}(\alpha^6)]$ Kalinowski (2019)

    reproduced in pionless EFT $\Delta E_{2S}^{\text{eVP}} = -0.027$ meV

non-forward





- Single-nucleon terms at N4LO in pionless EFT and higher
  - insert empirical FFs in the LO+NLO VVCS amplitude
  - polarisability contribution (inelastic+subtraction)
    - inelastic: *ed* scattering data Carlson, Gorchtein, Vanderhaeghen (2013)
    - subtraction: nucleon subtraction function from χEFT

      VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)
  - in total: small but sizeable: $\Delta E_{2S}^{\text{hadr}} = -0.032(6)$ meV

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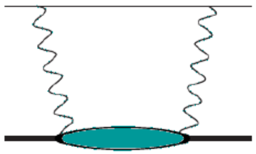

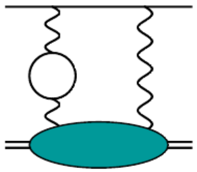

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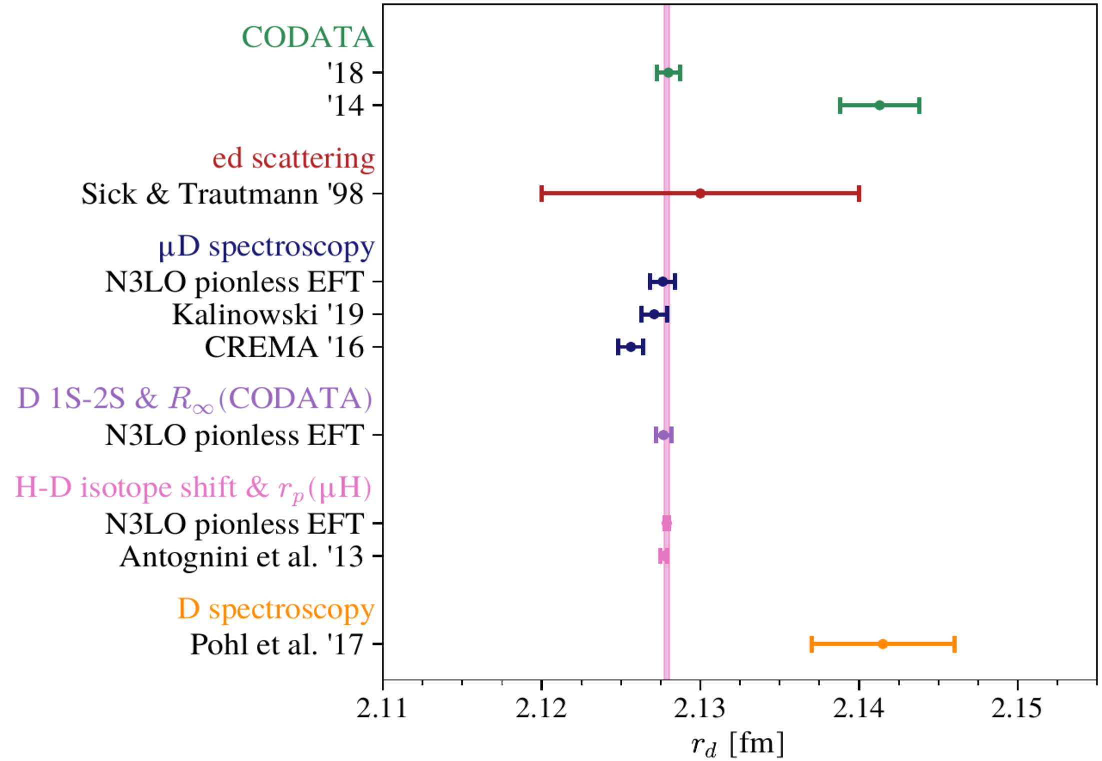
$$\Delta E_{2S}^{2\gamma} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} + \Delta E_{2S}^{\text{hadr}} + \Delta E_{2S}^{\text{eVP}} + \Delta E_{2S}^{\text{Coulomb}} = -1.752(20)\ \text{meV}$$

Deuteron Charge Radius and TPE in μD

- μD, D, and H-D isotope shift all consistent with one another
- Agreement with the very precise empirical value of 2γ exchange

	$E_{2S}^{2\gamma}$ [meV]
Theory prediction	
Krauth et al. '16 [5]	−1.7096(200)
Kalinowski '19 [6, Eq. (6) + (19)]	−1.740(21)
$\not{\pi}$EFT (this work)	−1.752(20)
Empirical (μH + iso)	
Pohl et al. '16 [3]	−1.7638(68)
This work	−1.7585(56)



VL, Hagelstein, Pascalutsa (2022)

- Agreement with other calculations

Pachucki, VL, Hagelstein, Li Muli, Bacca, Pohl – theory review (2022)
[a] experiment: CREMA (2013-2023)

	Correction	μH	μD	μ^3He$^+$	μ^4He$^+$
$E_{\rm QED}$	point nucleus	206.034 4(3)	228.774 0(3)	1644.348(8)	1668.491(7)
$\mathcal{C}\, r_C^2$	finite size	$-5.225\,9\, r_p^2$	$-6.107\,4\, r_d^2$	$-103.383\, r_h^2$	$-106.209\, r_\alpha^2$
$E_{\rm NS}$	nuclear structure	0.028 9(25)	1.750 3(200)	15.499(378)	9.276(433)
E_L(exp)	experiment[a]	202.370 6(23)	202.878 5(34)	1258.612(86)	1378.521(48)

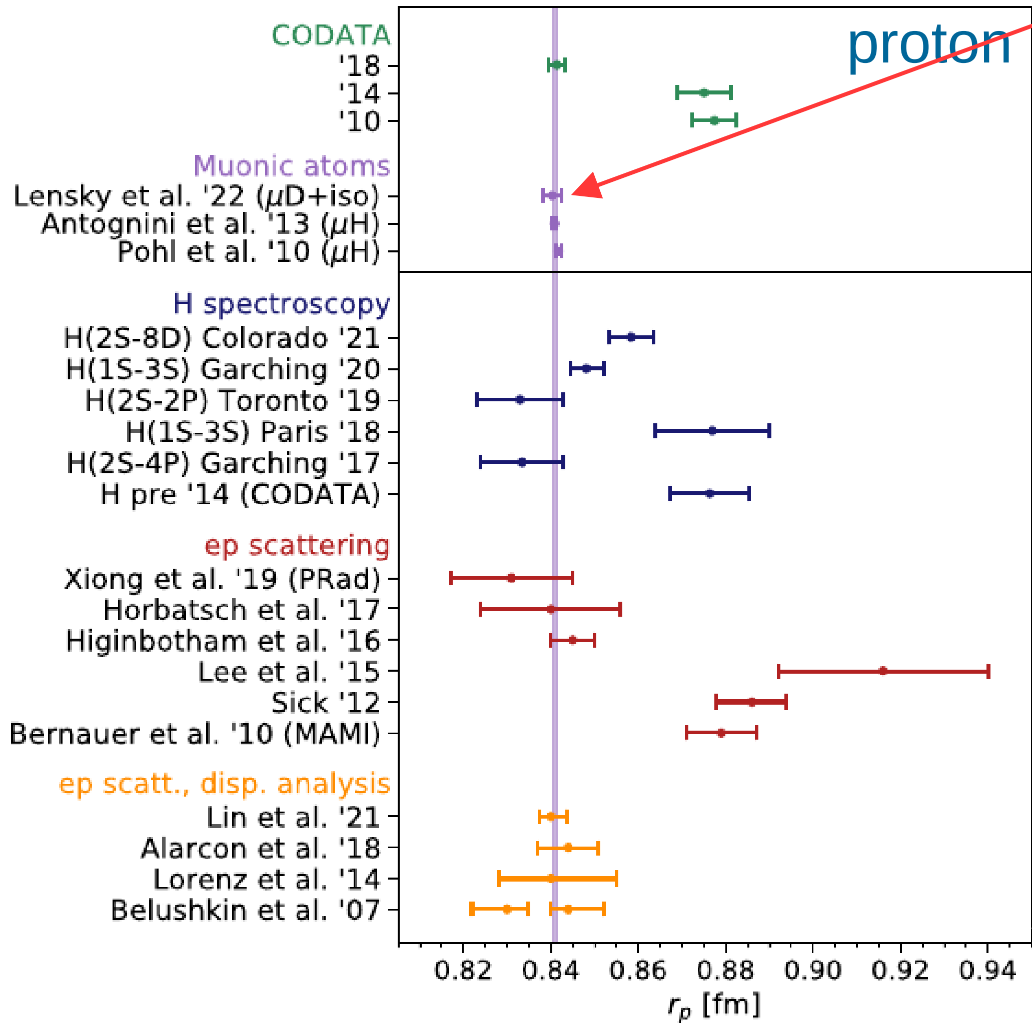
- Agreement between μD and μH+iso: small proton radius

Proton and Deuteron Radii and Isotope Shift

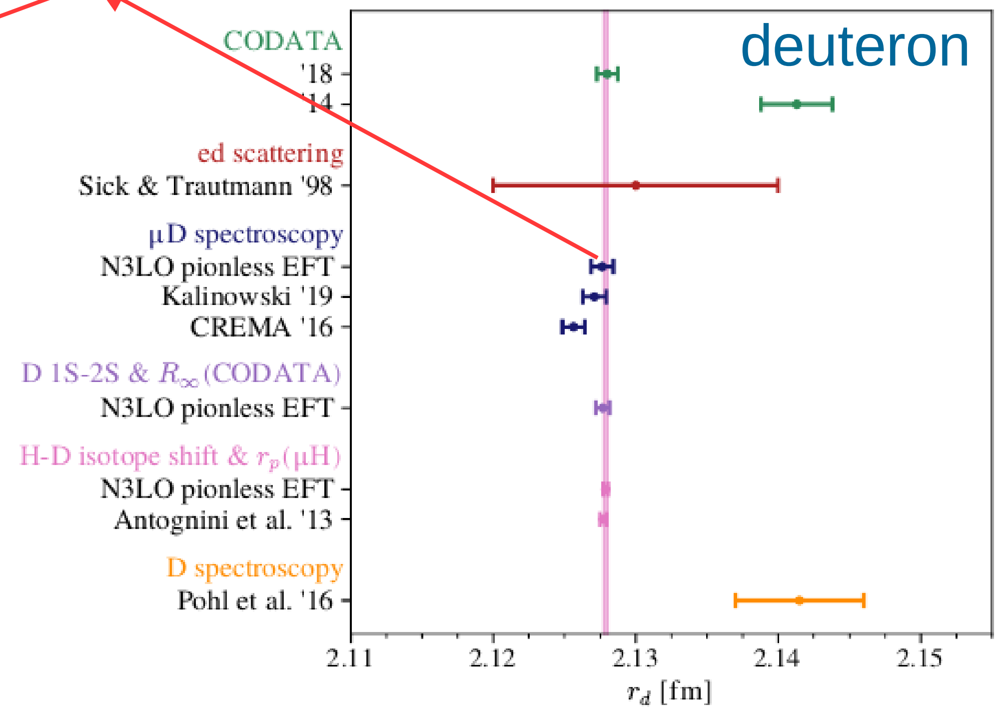
- H-D isotope shift: $E(H, 1S-2S) - E(D, 1S-2S)$

$$r_d^2 - r_p^2 = 3.820\,61(31)\ \mathrm{fm}^2$$

Jentschura et al. (2011)
VL, Hagelstein, Pascalutsa (2022)



Antognini, Hagelstein, Pascalutsa (2022)



VL, Hagelstein, Pascalutsa (2022)

HFS in Muonic Deuterium: Work in Progress

- Nuclear contribution to TPE in HFS is about 10 times smaller

$\Delta E^{2\gamma}_{2S,\mathrm{HFS}} = 0.0966(73)\ \mathrm{meV}$ Pohl et al. (2016), Pachucki, Kalinowski, Yerokhin (2018)

- Existing recent theoretical evaluations disagree

$\Delta E^{2\gamma,\mathrm{theor}}_{2S,\mathrm{HFS}} = 0.0383(86)\ \mathrm{meV}$ Pachucki, Kalinowski, Yerokhin (2018)

$\Delta E^{2\gamma,\mathrm{theor}}_{2S,\mathrm{HFS}} = 0.1180(90)\ \mathrm{meV}$ Ji, Zhang, Platter (2023)

- The smallness of the nuclear HFS contribution is the result of cancellations between different contributions
- Furthermore, there are cancellations at the VVCS amplitude level making its spin-dependent part suppressed
- There are also possible cancellations between nuclear and single-nucleon terms
- An alternative higher-order calculation (possibly accounting for relativistic corrections) is needed; in progress

Summary and Outlook

- μD and H-D isotope shift in pionless EFT consistent with each other
  - small proton radius
- Agreement with the very precise empirical value of 2γ exchange
  - experimental precision: both a challenge and a benchmark for theory
- Correlation between charge and Friar radius
  - another benchmark to check form factor parametrizations
- Single-nucleon effects are starting to be sizeable
  - more importaint in heavier nuclei

- HFS in μD: work in progress, more difficult (cancellations!)
- 3H, 3He: can pionless EFT shed light on discrepancies?

Summary and Outlook

- μD and H-D isotope shift in pionless EFT consistent with each other
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Thank you for your attention!

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Slighly More Details on Pionless EFT

- Nucleons are non-relativistic → $E \simeq p^2/M = O(p^2)$
- Loop integrals $dE\, d^3p = O(p^5)$
- Nucleon propagators $(E - p^2/2M)^{-1} = O(p^{-2})$
- Typical momenta $p \sim \gamma = \sqrt{ME_d} \simeq 45$ MeV
- Expansion parameter $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$
- *NN* system has a low-lying bound/virtual state → enhance S-wave coupling constants, resum the LO *NN* S-wave scattering amplitude

More Details on the Counting for VVCS and TPE

$$\Delta E_{nl} = -8i\pi m\,[\phi_{nl}(0)]^2 \int \frac{d^4q}{(2\pi)^4} \frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2) f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

- Transverse contribution starts at N4LO in TPE

$\alpha_{E1} = 0.64\ \mathrm{fm}^3$

$\beta_{M1} = 0.07\ \mathrm{fm}^3$

$$f_L(\nu, Q^2) = 4\pi\alpha_{E1} Q^2 + \ldots$$

$$f_T(\nu, Q^2) = -\frac{e^2}{M_d} + 4\pi\beta_{M1} Q^2 + 4\pi(\alpha_{E1} + \beta_{M1})\nu^2 + \ldots$$

$$\alpha_{E1} = \frac{\alpha M}{32\pi\gamma^4} + \ldots$$

$$\beta_{M1} = -\frac{\alpha}{32M\gamma^2}\left[1 - \frac{16}{3}\mu_1^2 + \frac{32}{3}\mu_1^2 \frac{\gamma}{\gamma_s - \gamma}\right] + \ldots$$

$f_L = O(p^{-2}),\qquad f_T = O(p^0)$ in VVCS

$\text{longitudinal} = O(p^{-2}),\qquad \text{transverse} = O(p^2)$ in TPE