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Probing New Physics through Entanglement in Diboson Production

Rafael Aoude, E.M., Fabio Maltoni, Luca Mantani – arXiv:2307.09675 [hep-ph]

Eric Madge

LHC EFT WG – October 9, 2023

Entanglement at Colliders

- Entanglement is **the** characteristic trait of Quantum Mechanics. [Schrödinger, MPCPS '35]

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- High Energy Physics $\Longrightarrow$ new path to test entanglement

  first proposals: Tornqvist, FoP '81; Abel, Dittmar, Dreiner, PLB '92

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How would new physics affect entanglement at high-energies?

for $t\bar{t}$: Aoude, E.M., Maltoni, Mantani, PRD 2022; Severi, Vryonidou, JHEP 2023; Fabbrichesi, Floreani, Gabrielli, EPJC 2023
for $h \to VV^*$: Fabbrichesi, Floreani, Gabrielli, Marzola, arXiv:2304.02403 [hep-ph]

$\Longrightarrow$ SMEFT

The LHC is not a QM experiment!

- We cannot prepare the state

  observation of many pp collision events $\Longrightarrow$ mixed state

- We cannot measure spin directly

  spin reconstruction from decay products [for massive spin-1: Ashby-Pickering, Barr, Wierzchucka, JHEP 2023]

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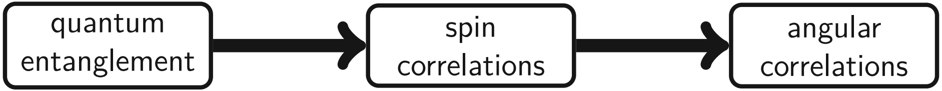

quantum entanglement → spin correlations → angular correlations

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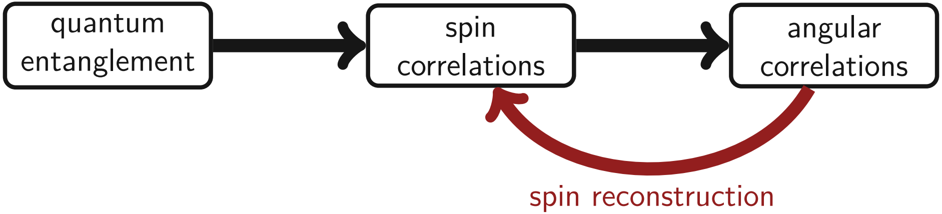

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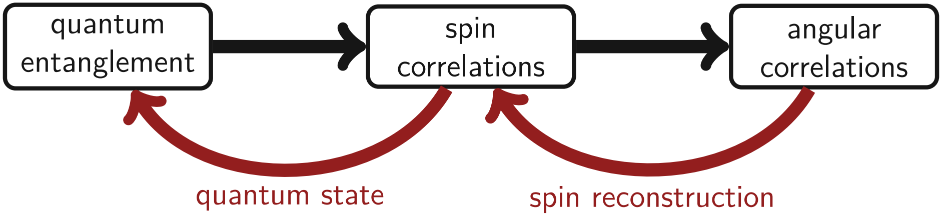

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Entanglement in Weak Boson Pairs

massive spin-1 particle ⟹ 3 polarizations ⟹ qutrit bipartite system

- **bipartite system:** $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
- **separable state:** $|\Psi\rangle = |\Psi\rangle_A \otimes |\Psi\rangle_B \implies$ **not entangled**

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- **separable state:** $|\Psi\rangle = |\Psi\rangle_A \otimes |\Psi\rangle_B \quad \Longrightarrow \quad$ **not entangled**

  otherwise $\Longrightarrow$ **entangled**
- **for statistical ensemble:** $$\rho = \sum_k \underset{\uparrow}{p_k} |\Psi_k\rangle \langle \Psi_k| = \sum_k p_k \, \rho_k$$

  $p_k \geq 0, \ \sum_k p_k = 1, \ \mathrm{tr}\, \rho_k = 1$

  entangled if $\rho_k \neq \rho_A^k \otimes \rho_B^k$

Spin Density Matrix and Fano Decomposition

- R matrix:

$$R_{\alpha_1\alpha_2,\beta_1\beta_2} = \sum_{\text{IS}} L_{\text{IS}}(\hat{s}) \overline{\sum}_{\substack{\text{DOFs (excl.} \\ \text{FS spin)}}} \mathcal{M}_{\alpha_1\beta_1}\mathcal{M}^*_{\alpha_2\beta_2} \qquad \mathcal{M}_{\alpha\beta} = \langle V(k_1,\alpha)\bar{V}(k_2,\beta)|\mathcal{T}|\text{IS}\rangle$$

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Quantum Witnesses

[Horodecki, Horodecki, Horodecki, Horodeck, RMP 2009]
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- **Concurrence:** $\mathcal{C}(\rho) = \inf_{\{|\Psi\rangle\}} \left[\sum_i p_i \, \mathcal{C}(|\Psi_i\rangle) \right], \quad \mathcal{C}(|\Psi\rangle) = \sqrt{2 \left(1 - \mathrm{tr}_A \left[\left(\mathrm{tr}_B \, |\Psi\rangle\langle\Psi| \right)^2 \right] \right)}$
  - $0 \leq \mathcal{C}(\rho) \leq \frac{2}{\sqrt{3}}, \quad \mathcal{C}(\rho) > 0 \implies$ entangled
  - for qutrits: analytically calculable only for pure states

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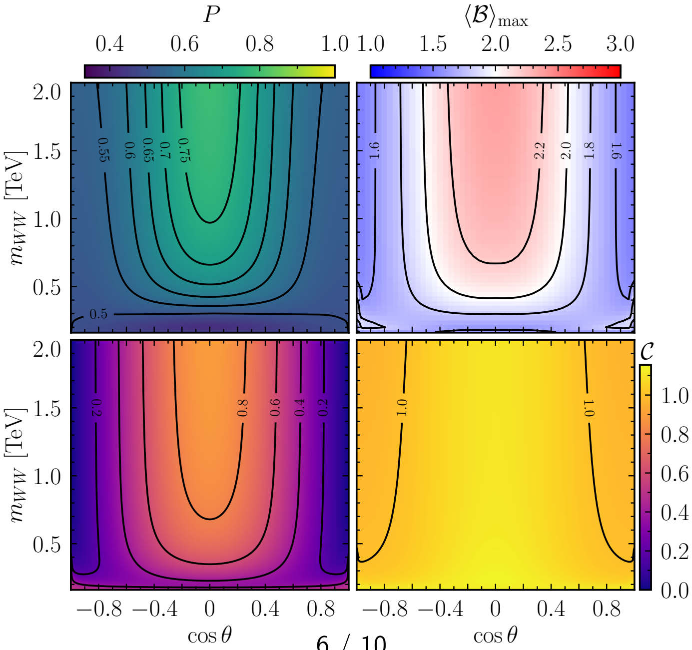
- **Bell inequality:** $\langle \mathcal{B} \rangle_{\text{max}} = \max_{U,V} \left(\mathrm{tr} \left(\rho \, (U^\dagger \otimes V^\dagger) \, \mathcal{B} \, (U \otimes V) \right) \right) \leq 2$ (CGLMP inequality)

  $\mathcal{B} = -\frac{2}{\sqrt{3}} \left(S_x \otimes S_x + S_y \otimes S_y \right) + \lambda_4 \otimes \lambda_4 + \lambda_5 \otimes \lambda_5$ [Collins et al., PRL 2002, Kaszlikowski et al., PRA 2002]

$pp \to W^+W^-$ in the SM



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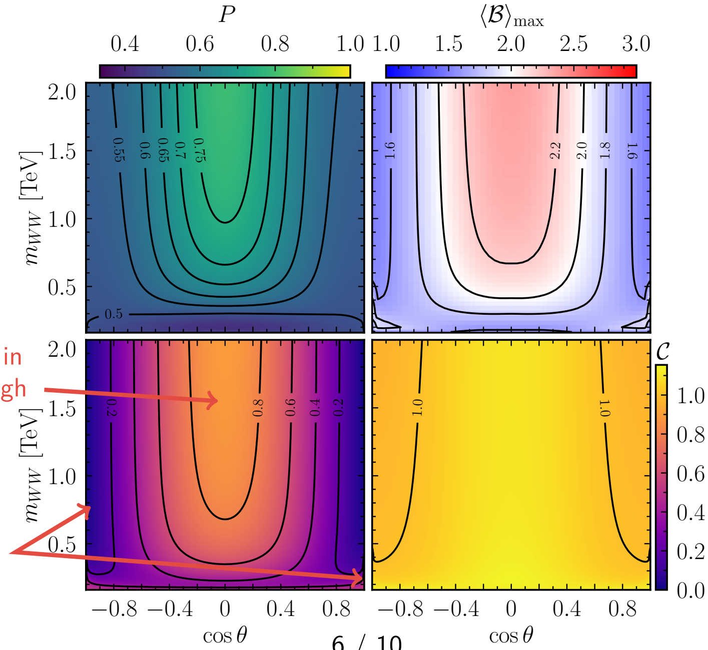

high entanglement in central region at high energy

no entanglement in collinear region

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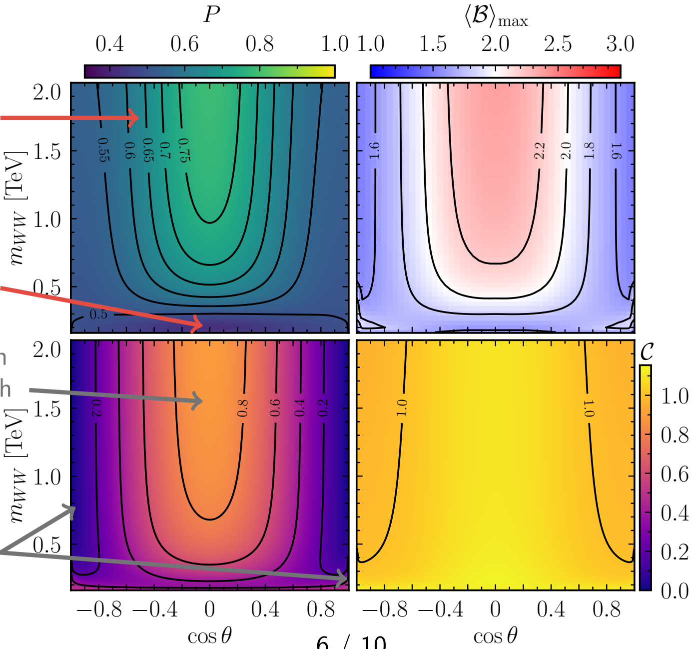

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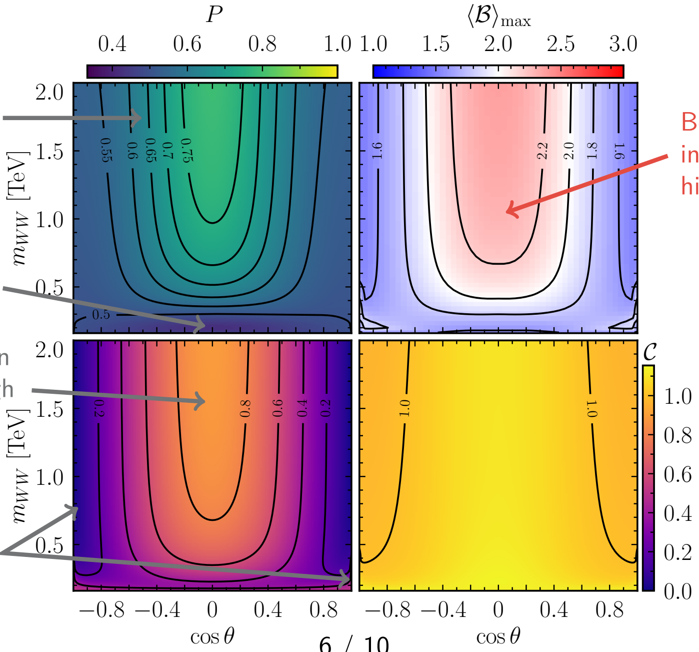

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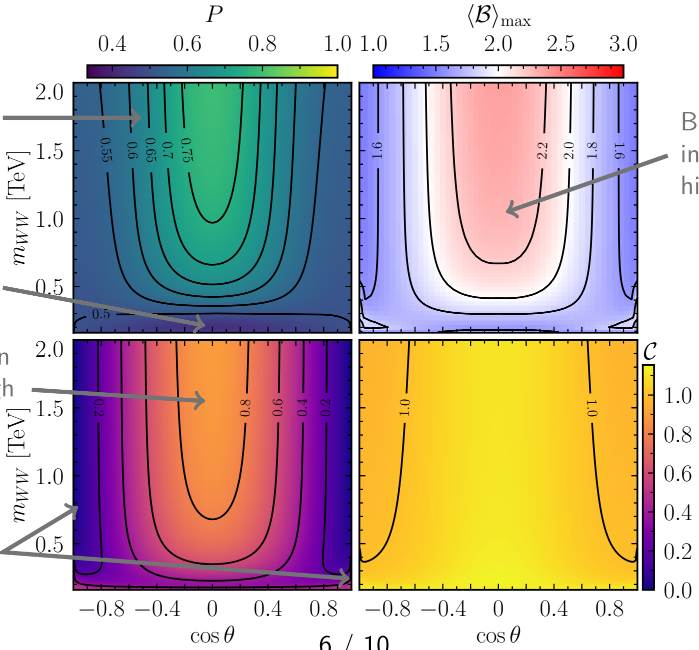

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SMEFT corrections

SM $+$ dim-6 operators:

$$\mathcal{L}_{\textsf{SMEFT}} = \mathcal{L}_{\textsf{SM}} + \sum_n c_n^{(5)} \mathcal{O}_n^{(5)} + \sum_n c_n^{(6)} \mathcal{O}_n^{(6)} + \dots \qquad c_n^{(d)} \propto \frac{1}{\Lambda^{d-4}}$$

We include dim-6 and dim-6 squared contributions to $a_i, b_j, c_{ij}, \mathcal{C}, P, \langle \mathcal{B} \rangle_{\textsf{max}}$, etc.

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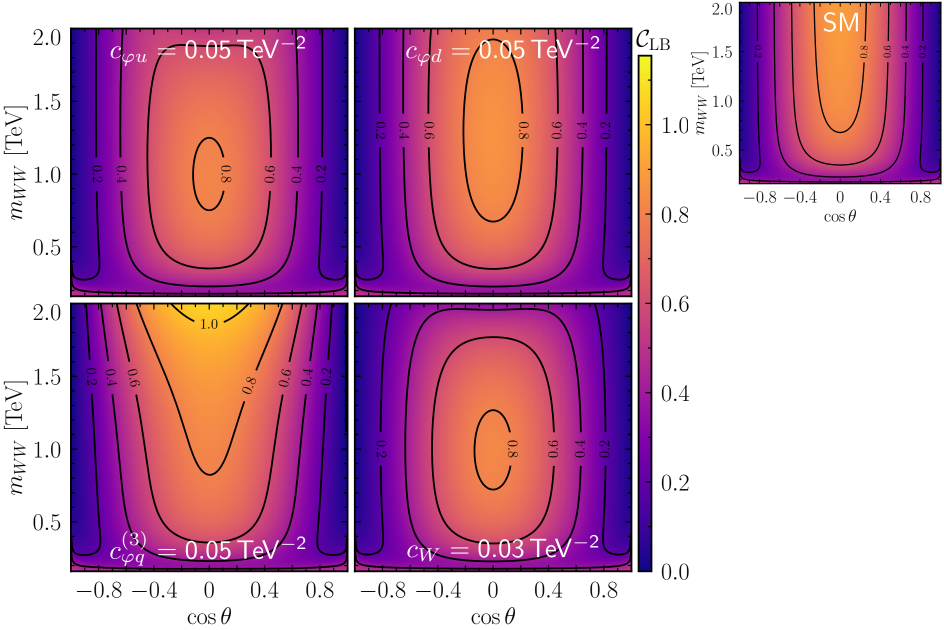
Operator	Coefficient	Definition	95 % CL bounds
$\mathcal{O}_{\varphi u}$	$c_{\varphi u}$	$i(\varphi^\dagger \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{u}\gamma^\mu u)$	$[-0.17, 0.14]$
$\mathcal{O}_{\varphi d}$	$c_{\varphi d}$	$i(\varphi^\dagger \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{d}\gamma^\mu d)$	$[-0.07, 0.09]$
$\mathcal{O}_{\varphi q}^{(3)}$	$c_{\varphi q}^{(3)}$	$i(\varphi^\dagger \overset{\leftrightarrow}{D}_\mu \,\tau_I \varphi)(\bar{q}\,\gamma^\mu\,\tau^I q)$	$[-0.21, 0.05]$
$\mathcal{O}_W$	c_W	$\varepsilon_{IJK} W_{\mu\nu}^I W^{J,\nu\rho} W_\rho^{K,\mu}$	$[-0.18, 0.22]$

95 % CL bounds in TeV^{-2} from [SMEFIT, JHEP 2021]

SMEFT effects in $pp \to W^+W^-$



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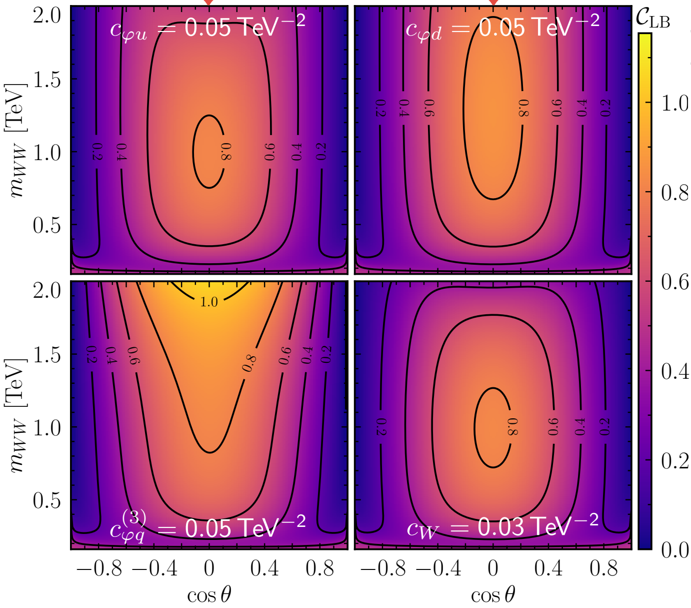

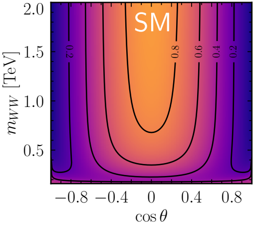

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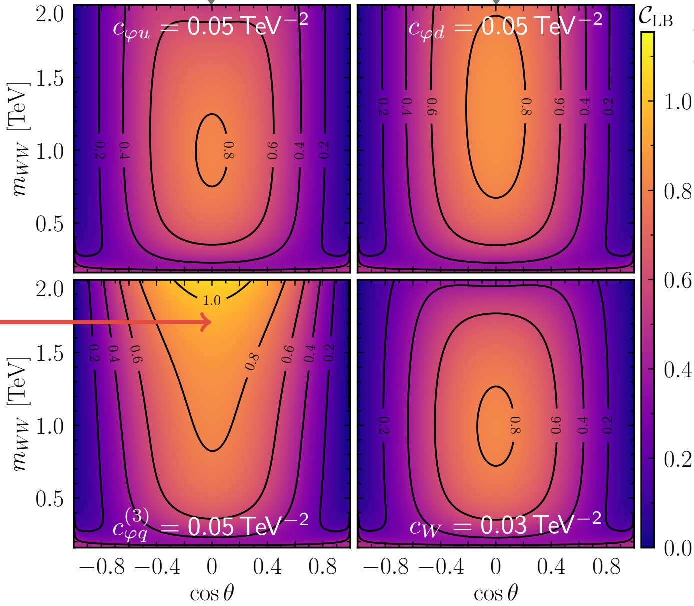

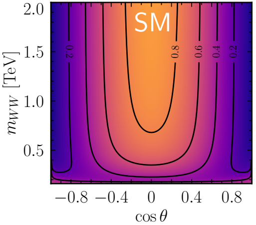

- modifies $W\bar{q}_L q'_L$ and $Z\bar{q}_L q_L$
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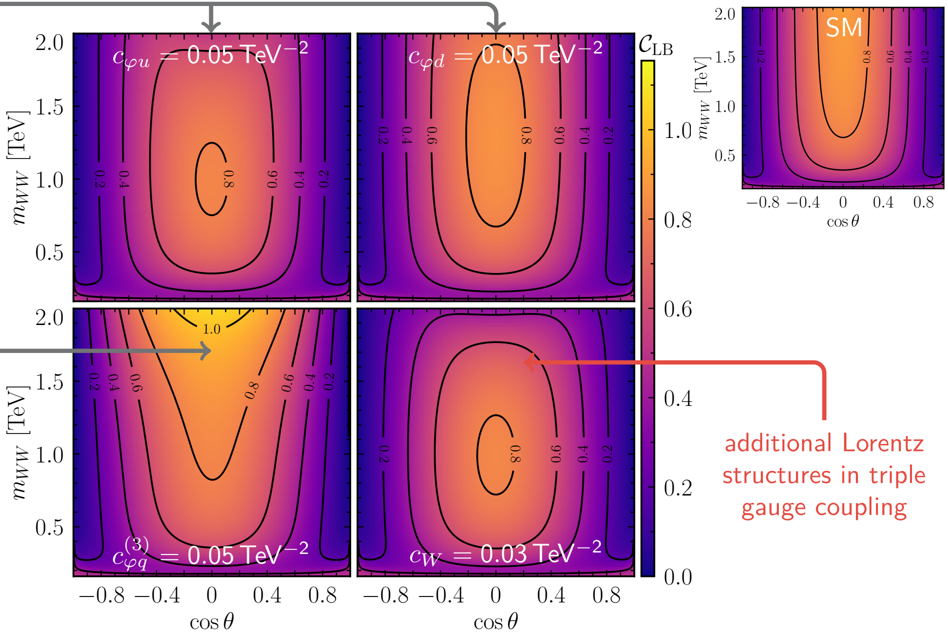

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additional Lorentz structures in triple gauge coupling

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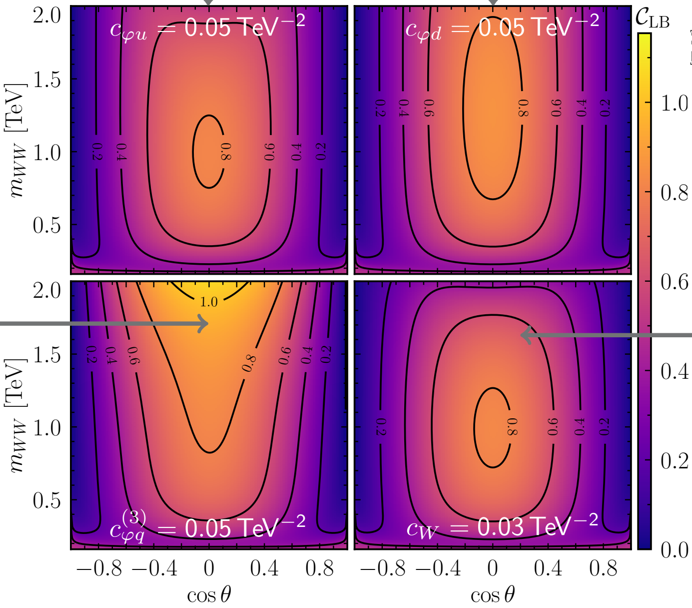

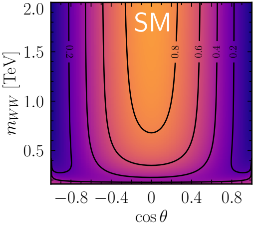

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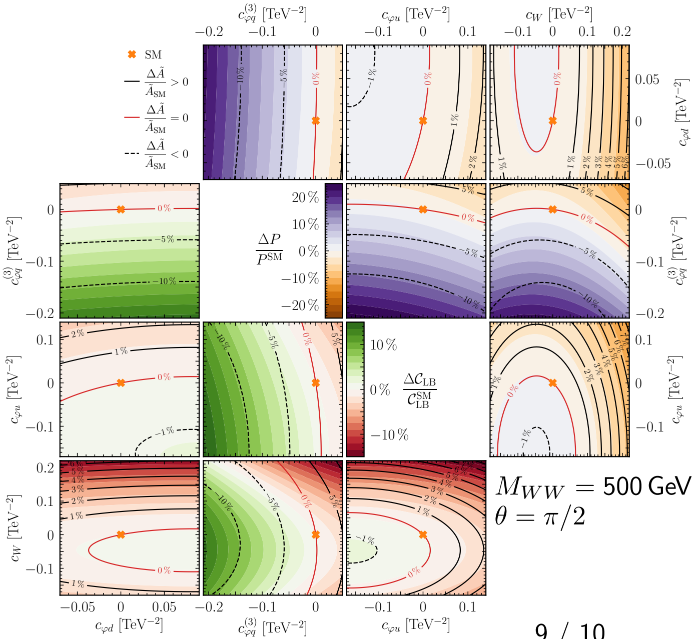
Central High-Energy Region ($pp \to W^+W^-$)



$M_{WW} = 500$ GeV
$\theta = \pi/2$

- Modification of entanglement mostly due to $\mathrm{tr}\, R$, i.e. balance of the IS channels
- Different directions can be probed at a lepton collider (or if we could single out one IS channel)

Conclusion

- weak boson pairs produced at the LHC are entangled in large part of the phase space
- highest entanglement: high-energy forward region
- EFT effects can modify the entanglement pattern

  $\Longrightarrow$ entanglement-related observables can be used to probe new physics

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Rafael Aoude, E.M., Fabio Maltoni, Luca Mantani – arXiv:2307.09675 [hep-ph]

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LHC EFT WG – October 9, 2023

Concurrence

[Horodecki, Horodecki, Horodecki, Horodeck, RMP 2009]

[Ashby-Pickering, Barr, Wierzchucka, JHEP 2022]

$$\mathcal{C}(\rho) = \inf\left[\sum_i p_i\, \mathcal{C}(|\Psi_i\rangle)\right], \qquad \mathcal{C}(|\Psi\rangle) = \sqrt{2\left[1 - \mathrm{tr}_A(\,\mathrm{tr}_B\, |\Psi\rangle\langle\Psi|\,)\right]}$$

- $0 \leq \mathcal{C}(\rho) \leq \frac{2}{\sqrt{3}}$, $\quad \mathcal{C}(\rho) > 0 \Longrightarrow$ entangled

- for 3×3: analytically calculable only for pure states

  $\Longrightarrow$ provide lower and upper bound: $(\mathcal{C}_{\mathsf{LB}}(\rho))^2 \leq (\mathcal{C}(\rho))^2 \leq (\mathcal{C}_{\mathsf{UB}}(\rho))^2$

$$(\mathcal{C}_{\mathsf{LB}}(\rho))^2 = 2\max\Big(0,\ \mathrm{tr}[\rho^2] - \mathrm{tr}[(\rho_A)^2],\ \mathrm{tr}[\rho^2] - \mathrm{tr}[(\rho_B)^2]\Big) \qquad \rho_{A/B} = \mathrm{tr}_{B/A}[\rho]$$

$$= -\frac{4}{9} + \frac{4}{3}\max\left(-2\sum_{i=1}^{8} a_i^2 + \sum_{j=1}^{8} b_j^2,\ \sum_{i=1}^{8} a_i^2 - 2\sum_{j=1}^{8} b_j^2\right) + 8\sum_{i,j=1}^{8} c_{ij}^2$$

$$(\mathcal{C}_{\mathsf{UB}}(\rho))^2 = 2\min\Big(1 - \mathrm{tr}[(\rho_A)^2],\ 1 - \mathrm{tr}[(\rho_B)^2]\Big) = \frac{4}{3} - 4\min\left(\sum_{i=1}^{8} a_i^2,\ \sum_{j=1}^{8} b_j^2\right)$$

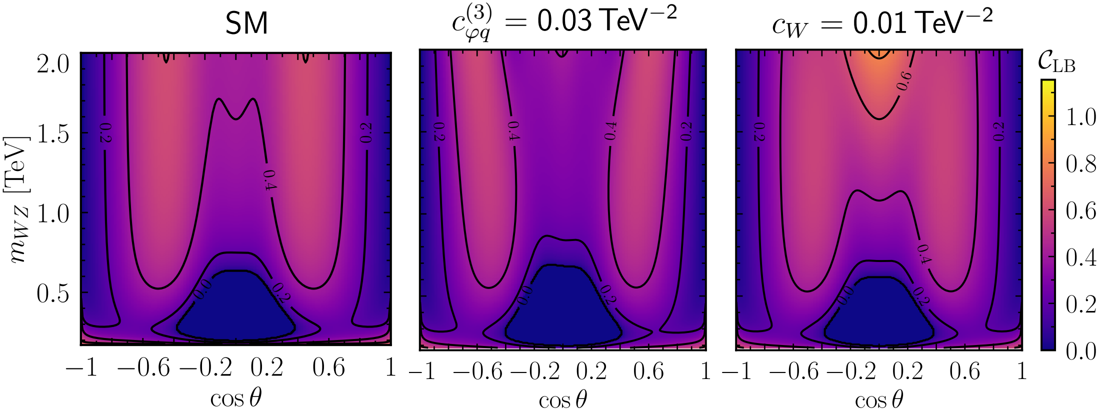
Dimension-6 Operators

	Definition	95 % CL
two-fermion operators		
$c_{\varphi u}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{u}\gamma^\mu u)$	$[-0.17, 0.14]$
$c_{\varphi d}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{d}\gamma^\mu d)$	$[-0.07, 0.09]$
$c_{\varphi q}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \, \varphi)(\bar{q} \, \gamma^\mu \, q)$	$[-0.06, 0.22]$
$c_{\varphi q}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \, \tau_I \varphi)(\bar{q} \, \gamma^\mu \, \tau^I q)$	$[-0.21, 0.05]$
$c_{\varphi e}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{e}\gamma^\mu e)$	$[-0.21, 0.26]$
$c_{\varphi l}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{l}\gamma^\mu l)$	$[-0.11, 0.13]$
$c_{\varphi l}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \tau_I \varphi)(\bar{l}\gamma^\mu \tau^I l)$	$[-0.21, 0.05]$

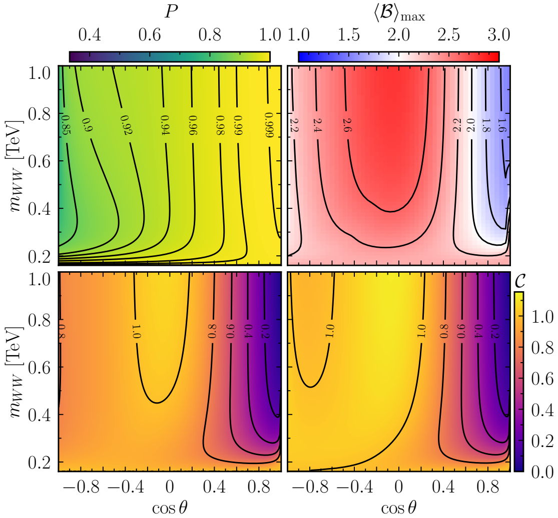
	Definition	95 % CL
bosonic operators		
c_W	$\varepsilon_{IJK} W^I_{\mu\nu} W^{J,\nu\rho} W^{K,\mu}_\rho$	$[-0.18, 0.22]$
$c_{\varphi W}$	$\left(\varphi^\dagger \varphi - \frac{v^2}{2}\right) W_I^{\mu\nu} W^I_{\mu\nu}$	$[-0.15, 0.30]$
$c_{\varphi B}$	$\left(\varphi^\dagger \varphi - \frac{v^2}{2}\right) B_{\mu\nu} B^{\mu\nu}$	$[-0.11, 0.11]$
$c_{\varphi WB}$	$(\varphi^\dagger \tau_I \varphi) B^{\mu\nu} W^I_{\mu\nu}$	$[-0.17, 0.27]$
$c_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^\dagger (\varphi^\dagger D_\mu \varphi)$	$[-0.52, 0.43]$
four-fermion operator		
c_{ll}	$(\bar{l}\gamma_\mu l)(\bar{l}\gamma^\mu l)$	$[-0.16, 0.02]$

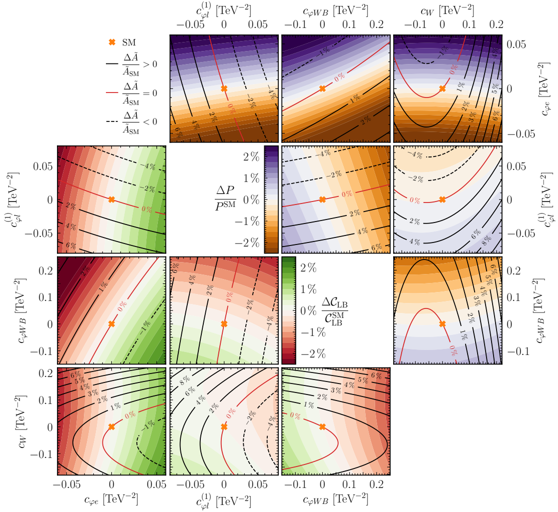
95 % CL bounds in TeV^{-2} from [SMEFIT, JHEP 2021]

WZ


SM
$c_{\varphi q}^{(3)} = 0.03$ TeV^{-2}
$c_W = 0.01$ TeV^{-2}
m_{WZ} [TeV]
2.0
1.5
1.0
0.5
$\cos\theta$
−1
−0.6
−0.2
0.2
0.6
1
0.0
0.2
0.4
0.6
$\mathcal{C}_{\mathrm{LB}}$
1.0
0.8
0.6
0.4
0.2
0.0

Electron-Positron Collider





Quantum Tomography for Massive Spin-1 (W^+W^-)

[Ashby-Pickering, Barr, Wierzchucka, JHEP 2022]

$$\langle \Phi(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) \rangle = \iint \mathrm{d}\Omega_{\hat{\mathbf{n}}_1} \mathrm{d}\Omega_{\hat{\mathbf{n}}_2}\, p(\ell^+_{\hat{\mathbf{n}}_1}, \ell^-_{\hat{\mathbf{n}}_2}; \rho)\, \Phi(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2)$$

$$\hat{a}_i = \frac{1}{2}\left\langle \Phi_i^{P+}(\hat{\mathbf{n}}_1) \right\rangle_{\text{av}} \qquad \hat{b}_j = \frac{1}{2}\left\langle \Phi_j^{P-}(\hat{\mathbf{n}}_2) \right\rangle_{\text{av}} \qquad \hat{c}_{ij} = \frac{1}{4}\left\langle \Phi_i^{P+}(\hat{\mathbf{n}}_1)\Phi_j^{P-}(\hat{\mathbf{n}}_2) \right\rangle_{\text{av}}$$

Wigner-P symbols:

$$\Phi_1^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\cos\phi \qquad \Phi_5^{P\pm} = 5\sin^2\theta\ \sin 2\phi$$

$$\Phi_2^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\sin\phi \qquad \Phi_6^{P\pm} = \sqrt{2}(\pm 1 - 5\cos\theta)\sin\theta\cos\phi$$

$$\Phi_3^{P\pm} = \frac{1}{4}(\pm 4\cos\theta + 15\cos 2\theta + 5) \qquad \Phi_7^{P\pm} = \sqrt{2}(\pm 1 - 5\cos\theta)\sin\theta\sin\phi$$

$$\Phi_4^{P\pm} = 5\sin^2\theta\ \cos 2\phi \qquad \Phi_8^{P\pm} = \frac{1}{4\sqrt{3}}(\pm 12\cos\theta - 15\cos 2\theta - 5)$$

Entanglement in $t\bar{t}$

- top has spin $\frac{1}{2} \Longrightarrow$ qubit

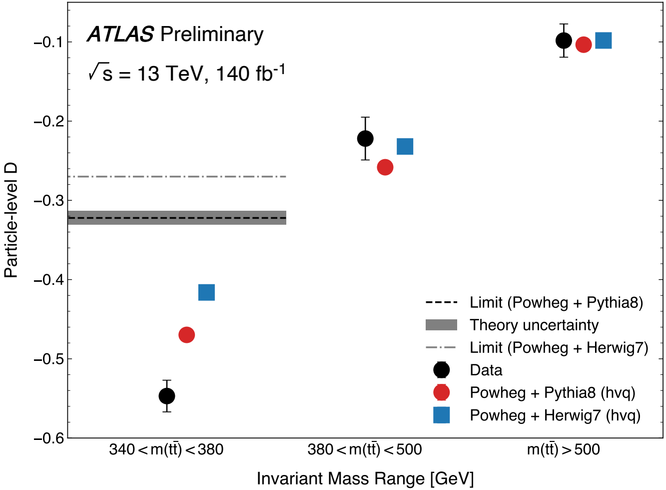
  $\Longrightarrow$ concurrence can be calculated analytically

- concurrence directly related to differential cross-section

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$

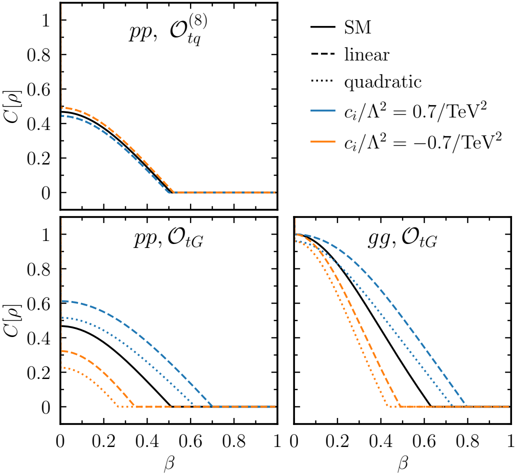
$$D = -3\langle\cos\varphi\rangle = \frac{\mathrm{tr}[C]}{3}$$

[Afik, de Nova, EPJ+ 2020, Quantum 2022, PRL 2022]

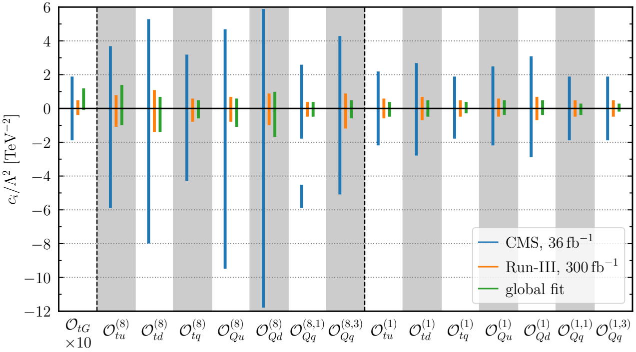


[ATLAS-CONF-2023-069]

SMEFT effects in $t\bar{t}$



[Aoude, E.M., Maltoni, Mantani, JHEP 2023]



[Sirunyan et al. (CMS), PRD 2019]
[Severi, Vryonidou, JHEP 2023]
[SMEFIT, JHEP 2021]