1/2 Feb 2024

Holography of Information

In a nongravitational theory, the state inside and outside a bounded region can be specified independently

This idea is key to our notion that information is localized in a region

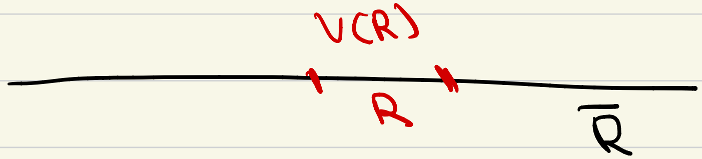
E.g.

pictures at one instant of time

Qubits: $\uparrow\uparrow\uparrow \downarrow\downarrow\downarrow \uparrow\uparrow\uparrow \ldots$

state of red qubits can be specified independently of black qubits

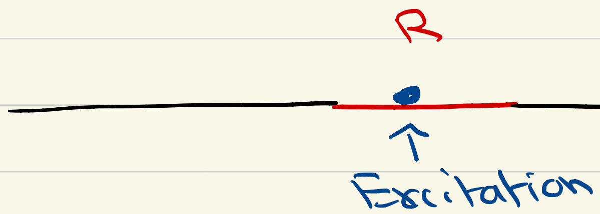
LQFT



$|\psi\rangle \to U(R)\,|\psi\rangle$

undetectable outside R.

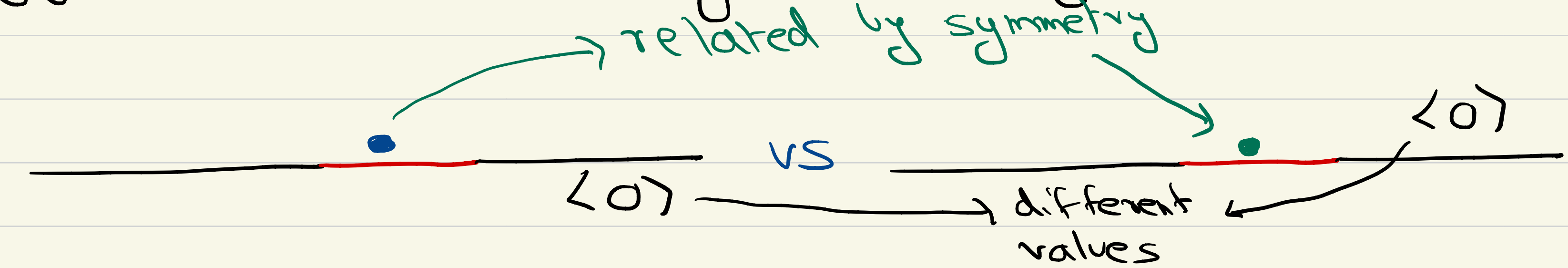
The broad theme of these lectures is that gravity localizes information differently.



$\langle O \rangle$ can detect its presence.

combination of metric and other fields

Even true if excitations have the same average energy or are related by global symmetry*



* gravity might not have global symms but that is a separate issue.

Plan of lectures

1) This unusual (holographic) localization of information can be inferred from a gravitational analysis

2) For simple states, this effect can be seen perturbatively

3) In some regimes, this effect becomes unimportant and gravity behaves like an ordinary QFT

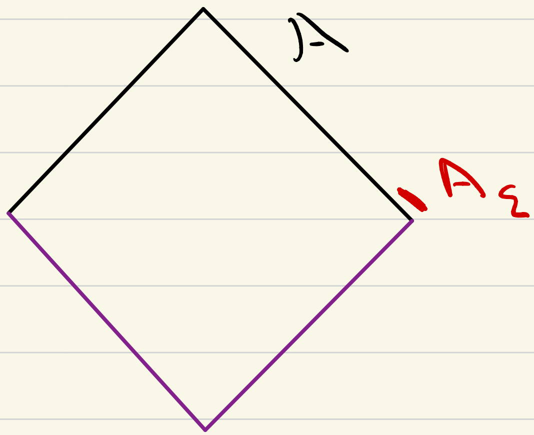
4) This effect is important for the black-hole information problem

These results pertain to information and not a full holographic dual. So we call this effect "the holography of information."

Results for $\Lambda < 0$ and $\Lambda = 0$

Consider asymptotically flat space

Penrose diagram



every operator on $\mathcal{I}^+$ can be represented arbitrary accurately using an operator near $\mathcal{I}^+_-$

algebra $\mathcal{A}$ algebra $\mathcal{A}_\varepsilon$

Consider asymptotically AdS space

Every operator on the asymptotic timelike bdry (algebra A) can be represented in a time band $[0, \varepsilon]$ (algebra A_ε)

[Bondi-gauge: $g_{rr} = 0 = g_{rA}$
$\partial_r \left(\det \frac{g_{AB}}{r^2}\right) = 0$]

Step 1: Asymptotic algebra

In Bondi gauge, the metric near $\mathscr{I}^+$ looks like

$$ds^2 \to \underbrace{-du^2 - 2du\,dr + r^2 \gamma_{AB}\, d\Omega^A d\Omega^B}_{\text{4d Minkowski metric with } u = t - r}$$

$$+ \underbrace{r\, C_{AB}\, d\Omega^A d\Omega^B + \frac{2m_B}{r} du^2}_{\text{Fluctuations}}$$

$$+ \underbrace{\gamma^{DA} D_D C_{AB}\, du\, d\Omega^B}_{\text{subleading terms}} + \cdots$$

A massless scalar field has falloff

$$\Phi(r, u, \Omega) = \frac{1}{r} O(u, \Omega) + \cdots$$

The operators
$$C_{AB}(u,\Omega), \quad m_B(u,\Omega), \quad O(u,\Omega)$$
can be thought of as intrinsic to $\mathscr{I}^+$

They give rise to the asymptotic algebra at $\mathscr{I}^+$

$$\mathcal{A} = \text{span}\{ C_{AB}(u,\Omega), m_B(u,\Omega), C_{AB}(u,\Omega)C_{CD}(u',\Omega'), C_{AB}(u,\Omega)\, m_B(u',\Omega'), O(u,\Omega), O(u,\Omega)\, m_B(u',\Omega') \ldots\ldots \text{higher products}\}$$

We also define
$$\mathcal{A}_\varepsilon = \text{span}\{ C_{AB}(u,\Omega), m_B(u,\Omega), C_{AB}(u,\Omega)C_{CD}(u',\Omega'), C_{AB}(u,\Omega)\, m_B(u',\Omega'), O(u,\Omega), O(u,\Omega)\, m_B(u',\Omega') \ldots\ldots \text{higher products}\}, \quad u \in \left(-\infty, -\frac{1}{\varepsilon}\right)$$

Assumption: This algebra continues to make sense in the full UV-complete theory of Q.G.

Justification: Even in a theory of Q.G., we keep the asymptotic geometry fixed. This is what allows us to define the theory.

In string theory, we compute the S-matrix.

The S-matrix is defined precisely in terms of radiative data at $\mathcal{I}^+$ and $\mathcal{I}^-$.

A similar construction holds in AdS

$$ds^2 = -(r^2+1)dt^2 + \frac{dr^2}{r^2+1} + r^2 d\Omega^2 + h_{\mu\nu} dx^\mu dx^\nu$$

$$h_{r\mu} = 0$$

We can define operators intrinsic to the boundary through

$$h_{ij}(r,t,\Omega) \to t_{ij}(t,\Omega) \frac{1}{r^{d-2}} + \dots$$

For a massive field

$$\phi(r,t,\Omega) \longrightarrow \frac{O(t,\Omega)}{r^\Delta} + \dots \quad , \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2}$$

[Difference with flat space: we can also study massive fields. In flat space, massive fields do not live on $\mathcal{I}^+$]

We can again define

$$\mathcal{A} = \text{span}\{ t_{ij}(t, \Omega) , \; t_{ij}(t, \Omega)\, t_{i'j'}(t', \Omega') \ldots \}$$

and

$$\mathcal{A}_{\varepsilon} = \text{span}\{ t_{ij}(t, \Omega) , \; t_{ij}(t, \Omega)\, t_{i'j'}(t', \Omega') \ldots \}$$

with $t \in [0, \varepsilon]$

We again assume the algebra is well-defined in the full UV-theory

From AdS/CFT, this is justified. This algebra is what becomes the CFT algebra.

However, as earlier, the stability of the asymptotic algebra is essential for defining the bulk theory.

Hamiltonian

In both AdS and flat space, we see that the Hamiltonian is part of the algebra A_Σ.

In flat space [4d]

$$H = \frac{1}{4\pi G} \int \sqrt{\gamma}\, d^2\Omega\, m_B(u \to -\infty, \Omega)$$

In AdS,

$$H = \frac{d}{16\pi G} \int d^{d-1}\Omega\, t_{tt}$$

This is a manifestation of the Gauss law.

In gravity, these observables define the energy.

Assumption 2: H remains bounded below in the full UV-complete theory.

We cannot really prove this, although it seems reasonable.

So there is a vacuum state, $H|0\rangle = 0$.

By the Born rule, "measuring" an operator provides information about its spectral projectors.

So, we conclude

$$P_0 = |0\rangle\langle 0| \in A_\varepsilon$$

[tells us the probability of getting "0" if we measure H.]

Assumption 3: the projector on the vacuum remains an element of A_ε in the full UV-complete theory.

One check is that this is true in AdS/CFT. Also note that this is a statement about the low-energy structure of the theory.

[In 4d flat space, the vacuum is degenerate see arXiv: 2002.02448 for discussion of the soft sectors.]

End of Lecture 1