Lectures 2 & 3

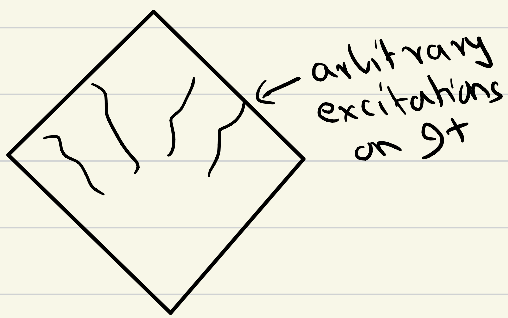
Hilbert space

We focus on the Hilbert space

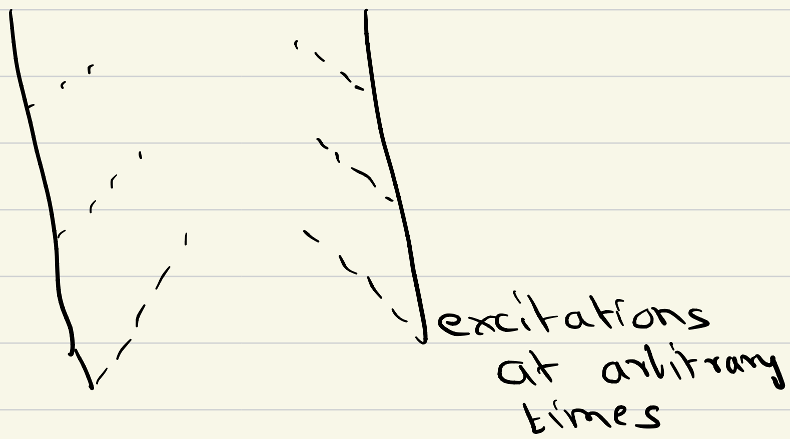
$$\mathcal{H} = \mathcal{A}|0\rangle$$

This forms a superselection sector since it is closed under time-evolution.

Flat space picture



AdS picture



This is a rich H-space that contains black holes and all sorts of complex excitations.

Lemma:

Any state in H can be approximated arbitrarily well by

(to arbitrary precision)

$$|n\rangle \doteq X_n|0\rangle$$

where $X_n \in A_\varepsilon$

Proof (by contradiction)

arbitrary operator insertions

say $\exists\, |n\rangle$ s.t. $\langle n|\, O(u_1) \dots O(u_n)|0\rangle = C(u_i) = 0$

spherical coordinate suppressed.

$\forall\, u_i \in (-\infty, -\frac{1}{\varepsilon})$

by inserting eigenstates of H

[Here we present the argument in flat space. Similar reasoning applies in AdS.]

$$\sum_{E_i} \langle n|E_1\rangle\langle E_1|O(u_1)|E_2\rangle\langle E_2|O(u_2)| \;\ldots\ldots\; |0\rangle = \sum_{E_i} e^{iE_1u_1}\, e^{iE_2(u_2-u_1)}\, e^{iE_3(u_3-u_2)}$$

$$\ldots \quad \times \quad \langle n|E_1\rangle\langle E_1|O(0)|E_2\rangle \ldots$$

In terms of the variables

$$z_1 = u_1$$

$$z_2 = u_2 - u_1$$

$$z_3 = u_3 - u_2$$

$$\vdots$$

$$z_n = u_n - u_{n-1}$$

$C(z_i)$ is analytic when z_i are extended in the upper half plane and vanishes on a segment of real axis.

$\Rightarrow C(z_i) = 0.$

[See eg. Witten 1803.04993 or 2-5 of PCT, Spin Statistics & all that]

But this means
$$\langle n|a|0\rangle = 0 \quad \forall\, a \in A$$

But this is impossible since $\langle n|$ is in $A|0\rangle$

Comments:

1) We used positivity of H here.

2) This result is true in QFT without gravity. It does not imply holography of information by itself.

Proof of main result

Any operator: $\mathcal{H} \to \mathcal{H}$ can be written as

$$Q = \sum_{n,m} c_{nm} |n\rangle\langle m|$$

$$\doteq \sum_{n,m} c_{nm} X_n |0\rangle\langle 0| X_m^\dagger$$ ← Using Lemma. Also true in QFT

$$= \sum_{n,m} c_{nm} X_n P_0 X_m^\dagger$$ ← Using assumption 2

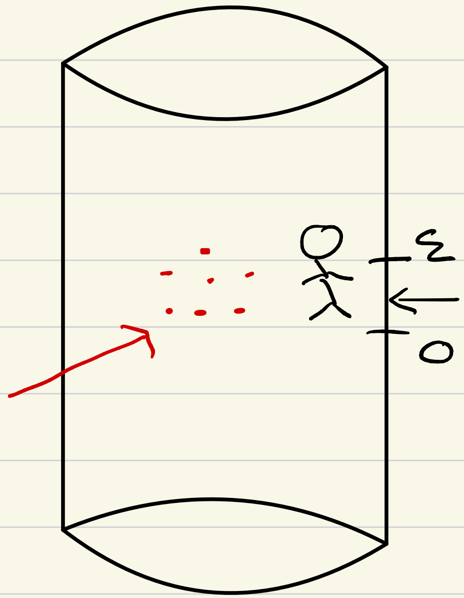
Since $P_0 \in A_\varepsilon$; $X_m, X_n \in A_\varepsilon$;

so Q can be approximated arbitrarily well by an element of A_ε

Perturbative checks [2008.01740]

We now want to show that this is not simply an abstract result.

Let us play the following game



Astrophysicist near bdry with detectors that work only in $[0, \varepsilon]$

Low energy excitation in middle of AdS

More precisely. The bulk is in some low-energy state $|g\rangle$

This means

$$1 - |P_{E<\Lambda}|g\rangle| \ll 1$$

so that "most components" of the state have energy below Λ, where $\Lambda \ll N$.

This is important since, even in LQFT, localized states have some high-energy component.

<u>Abilities of observer</u>

a) Act with unitaries e^{iJX}, where X is a simple operator in $[0,\varepsilon]$

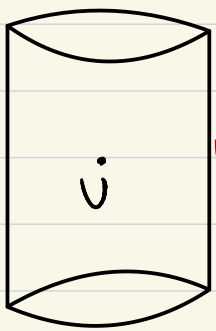
b) Measure energy and determine prob. of finding 0

Textbook QM!

Warmup task 1

Determine if $|g\rangle = |0\rangle$ or not

In LQFT, this is impossible. Consider $U|0\rangle$, where U is a localized unitary



In LQFT, all observations here are unchanged

In gravity, the answer is simple. Measure H and determine prob. of getting 0.

$$\langle g | P_0 | g\rangle = |\langle g|0\rangle|^2 \, !$$

Warmup task 2

Say we are now given that $\langle 0|g\rangle = 0$.

With X a simple Hermitian operator in $[0, \varepsilon]$ determine if

$$|g\rangle = |x\rangle$$ or not, where $|x\rangle \equiv X|0\rangle$

Again, note this is impossible in LQFT.

Here we do the following

a) act with unitary e^{iJX}

b) measure H and determine prob. of getting 0 to second order in J.

$$\langle g | e^{-iJx} P_0 e^{iJx} | g \rangle$$

$$= \langle g | (1 - iJx - \frac{J^2 x^2}{2}) |0\rangle \langle 0 | (1 + iJx + \frac{J^2 x^2}{2}) | g \rangle$$

$$= J^2 |\langle g | x \rangle|^2$$

[Recall $\langle g | 0 \rangle = 0$, by assumption.]

So the observers succeed in this task as well.

Note that states of the form

$$|x\rangle \equiv X |0\rangle$$

Form a basis for the H-space.

Full protocol

Given a state $|g\rangle$, we first act with a simple $U: |g\rangle \to U|g\rangle$ to ensure that $\langle g|0\rangle = 0$.

[This is simple; rotating 1 qubit is sufficient for orthogonality]

We now choose a set of operators X_i so that $|X_i\rangle$ forms a basis.

Determine $|\langle g|X_0\rangle|^2$ using previous step and set

$\langle g|X_0\rangle = |\langle g|X_0\rangle|$ [Overall phase of $|g\rangle$ irrelevant]

Now get $|\langle g|X_i\rangle|^2$ and $|\langle g|X_i\rangle + \langle g|X_0\rangle|^2$ using previous step.

$$|\langle g|x_i\rangle + \langle g|x_0\rangle|^2$$

$$= \underbrace{|\langle g|x_i\rangle|^2}_{\text{known}} + \underbrace{|\langle g|x_0\rangle|^2}_{\text{known}} + 2\left[\mathrm{Re}(\langle g|x_i\rangle)\right] \underbrace{\langle g|x_0\rangle}_{\text{real}}$$

can be solved for $\mathrm{Re}\langle g|x_i\rangle$

This data fixes $\mathrm{Im}\langle g|x_i\rangle$ upto a sign.

[See 2008-01740 on how to fix sign.]

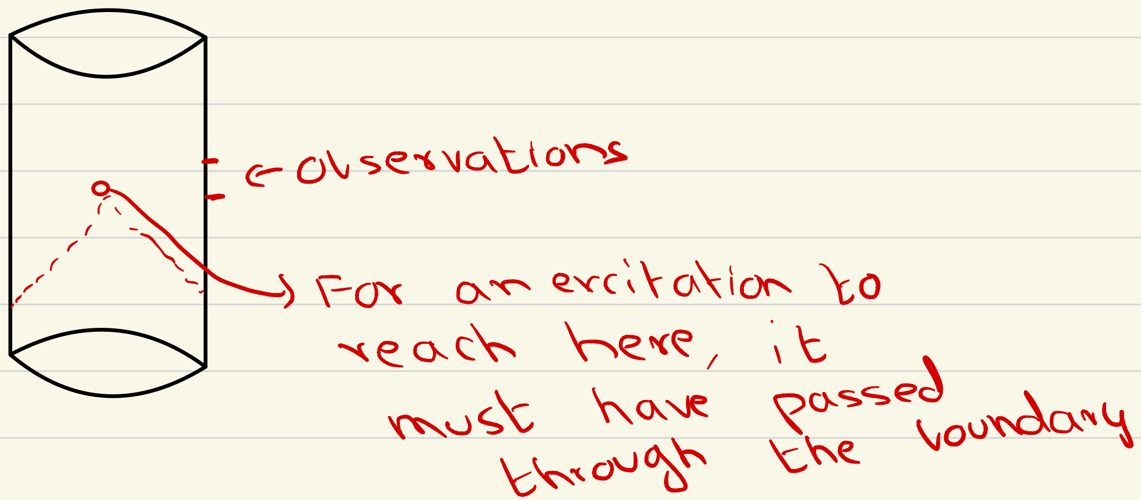
Once we know $\langle g|x_i\rangle$ for a basis, we know $|g\rangle$!

Discussion

Causality:

You might worry that information is "jumping" from the bulk to the boundary.

But this is not correct: The information had to go <u>through</u> the boundary to reach the bulk.



So the statement is:

"In a LQFT, we can prepare a bulk state, and erase all traces from the boundary.
In QG, correlators of H and other observables at ∞ always retain information about the state."

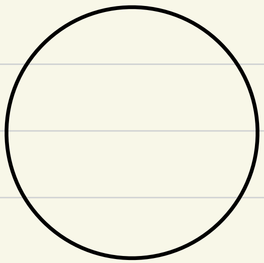
If you are ever confused about causality, just think of the Gauss law. When we measure the mass of an object by integrating its gravitational field, over a Gaussian surface, information does not "jump"

If the mass inside the Gaussian surface changes, matter will have to cross the Gaussian surface.

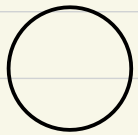
Difference with classical gravity

It is important that we are not measuring only the energy, which can also be done in the classical theory.

eg. Birkhoff's theorem tells us that two spherically symmetric wavepackets with different densities could have the same $\langle H \rangle$ and so be indistinguishable classically



$\langle H^2 \rangle$ smaller



$\langle H^2 \rangle$ larger

There is no analogue of this classically where $\langle H^2 \rangle = \langle H \rangle^2$

Global symmetry

Relatedly imagine we have two fields $\phi, \tilde{\phi}$ related by a global symmetry.

we have shown we can distinguish

$$|\psi\rangle = e^{i\int f(r)\,\phi(r,t=0)\,dr}|0\rangle$$ and

$$|\tilde{\psi}\rangle = e^{i\int f(r)\,\tilde{\phi}(r,t=0)\,dr}|0\rangle$$

This is because

U insertion breaks symmetry

$$\langle\psi|U^{+}P_0\,U|\psi\rangle \quad \text{and} \quad \langle\tilde{\psi}|U^{+}P_0\,U|\tilde{\psi}\rangle$$

are different when U contains ϕ or $\tilde{\phi}$

Difference with gauge theory

This procedure does not work in a nongravitational gauge theory.

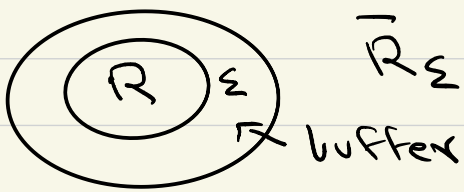
In such a theory, we can act with

$$e^{i\int tr(F^2)(t=0,r)f(r)} |0\rangle$$

and if $f(r)$ has compact support, this is indistinguishable from the vacuum near the boundary.

a) We have no analogue of the projector on the vacuum in a nongrav. gauge theory

b)



An observer in $\bar{R}_\epsilon$ cannot even tell the charge in R, since we can put a countercharge in ε.

Difficulty of reconstruction

Our two step procedure requires:

a) action with e^{iJx_i}

b) measurement of H to determine

$$|\langle g | x_i \rangle|^2$$

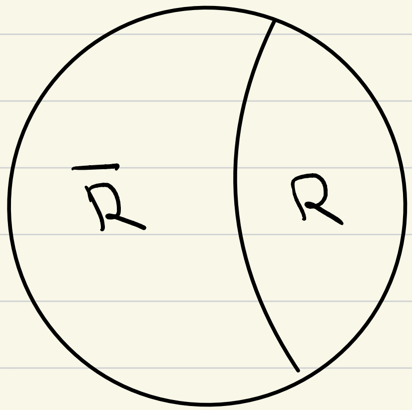
If the state $|g\rangle$ has support on very "complex" $|x_i\rangle$, this becomes difficult.

More below.

Mundane Limits

We would now like to discuss those cases where gravity does localize information like an ordinary LQFT

One simple case where information from a region is unavailable in the complement is



[P_0 inaccessible in $\bar{R}$]

setting relevant for the RT - Formula.

Heavy background

Another important case is that of a heavy background state.

1) Consider a state $|\psi\rangle$, with

$$\langle\psi|H|\psi\rangle = E$$

There are $e^{S(E)}$ states with this energy.

If $E \sim N = l_{ads}/l_{pl}$, we have $e^{N^{\#}}$ states.

2) Say we limit the abilities of the observer outside, so that they do not have access to $e^{S(E)}$ distinct unitaries. [Coarse-graining.]

Then it is clear this coarse-grained observer will not be able to distinguish $|\psi\rangle$ from nearly states.

Some of these states might be easy to distinguish in the bulk.

[Ref: 2301.08753 Bahiru, Belin, Papadodimas, Sarosi, Vardian]

We now show how to construct such states in a heavy background [easy to distinguish in bulk; difficult to distinguish on bdry.]

Let

$$A^{\text{bdry}}_{\text{coarse}} = \text{span}\{ O(t_1),\ O(t_1)O(t_2),\ \ldots\ldots\ O(t_1)O(t_2)\ldots O(t_Q)\}$$

boundary operators including H.
in time-band.

We limit Q so that

$$\dim(A_{coarse}) \ll e^{S(E)}$$

Then, for a generic state $|\psi\rangle$, we have

$$a|\psi\rangle \neq 0 \quad \left[|\psi\rangle \text{ is separating w.r.t. } A_{coarse} \right]$$

For some special states, this might fail. eg. if $|E\rangle$ is an energy eigenstate

$$H|E\rangle = E|E\rangle \Rightarrow (H-E)|E\rangle = 0$$

Next, let

$$\phi_{HKLL}(r=0, t=0)$$

be an operator corresponding to a field in the bulk, as constructed using the HKLL prescription.

In general $[\Phi_{HKLL}(0,0), H] \neq 0$

However we can <u>improve</u> the operator so that it commutes not just with H but with all elements of A_{coarse}.

We define

$$\hat{\Phi}\,|\Psi\rangle = \Phi_{HKLL}(0,0)\,|\Psi\rangle$$

but

$$\hat{\Phi}\,a\,|\Psi\rangle = a\,\Phi_{HKLL}(0,0)\,|\Psi\rangle \quad \forall\, a \in A_{coarse}$$

These =ns define $\hat{\Phi}$

These =ns can be solved because $|\Psi\rangle$ is separating.

Now, by construction

$$[\hat{\phi}, a]\, b|\psi\rangle = (\hat{\phi}\, a b - a \hat{\phi}\, b)|\psi\rangle = 0.$$

elements of A_{coarse}

[Here we are ignoring edge effects]

$\therefore$ the action of $\hat{\phi}$ is easy to detect in the bulk but difficult on the bdry.

This is called <u>dressing to the state</u>.

Related to ideas about dressing to the background but more general)

This explains why we don't see holography around us!

--- End of Lecs 2 & 3 ------