2/2/24 Lecture 4

Information Paradox

In its simplest form, the information paradox is as follows

Star $\longrightarrow$ Black hole $\longrightarrow$ Thermal Hawking rad^n

which seems to violate unitarity because a pure state with density matrix

$$|\psi\rangle\langle\psi| \longrightarrow \frac{1}{Z(\beta)} e^{-\beta H}$$

Initial density matrix

Claim for final density matrix if final state is thermal.

However, Hawking computed low-point correlators

$$\langle a_\omega^\dagger a_{\omega'} \rangle = \frac{e^{-\beta\omega}}{1-e^{-\beta\omega}} \delta(\omega-\omega')$$

Here a_ω, $a_\omega^\dagger$ corresponds to modes on $\mathscr{g}^+$

But this is insufficient to estallish a paradox because typical pure states are exponentially close to mixed states.

More precisely, consider states

$$|\psi\rangle = \sum a_i |E_i\rangle$$

drawn from $E_i \in [E-\Delta, E+\Delta]$

The maximally mixed density matrix is

$$\rho_{m.c.} = \frac{1}{e^S} \sum_i |E_i\rangle\langle E_i|$$

Then

$$\int \langle\psi|A|\psi\rangle \, d\mu_\psi = \mathrm{tr}(\rho_{m.c.} A)$$

$$\int \left(\langle\psi|A|\psi\rangle - \langle A\rangle\right)^2 d\mu_\psi \leq \frac{1}{(e^S+1)} \sigma_{ens}^2$$

$$\sigma_{ens}^2 = \mathrm{tr}(\rho_{m.c.} A^2) - \left[\mathrm{tr}(\rho_{m.c.} A)\right]^2$$

[Ref: 1804.10616]

So typical pure states are exponentially close to mixed states.

This tells us:

1) The computation of Hawking radiation is far from sufficient to set up a paradox, since we do not keep track of exponentially suppressed corrections.

2) If we adopt the perspective:

"exponentially suppressed terms can never be discussed in the bulk theory, even at asymptotic infinity",

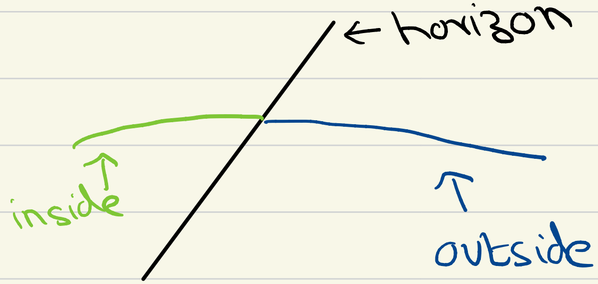
then there is no point discussing the bulk info problem

To discuss this paradox, we <u>cannot</u> be in the regime where we coarse-grain the asymptotic algebra.

Hawking was aware of point 1!

If one reads the paper on "breakdown of predictability..." the argument is not given in terms of Hawking radiation.

Rather, Hawking postulated a "<u>principle of ignorance</u>"



In modern language, this states:

the state outside [i.e. arbitrary correlator evaluated outside] is independent of the state inside except for information about the total charge/mass/angular momentum.

Therefore the observer outside must trace over all possibilities inside, and will obtain a mixed state.

So Hawking assumed the split property in gravity

His point was:

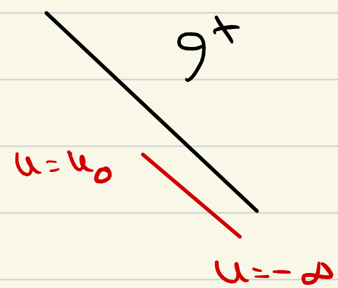
"even if the precise form of Hawking radiation receives corrections, this property relies on the causal structure and is valid even to $O(e^{-S/2})$"

Holography of information tells us the precise opposite!

All correlators outside know <u>everything</u> about all correlators inside! [opposite of being independent of corelators inside]

Identifies the precise error in Hawking's argument for information loss.

Can be made more precise in terms of entanglement entropy at $\mathscr{I}^+$



We can define an algebra of operators up to u_0, A_{u_0}

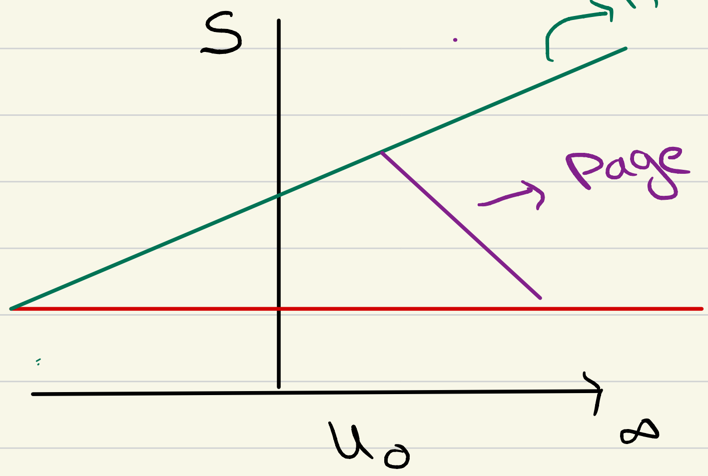
then define

$$\rho_{u_0} \in A_{u_0} \quad \text{so that} \quad \forall\, a \in A_{u_0} \quad \mathrm{tr}(\rho_{u_0} a) = \langle a \rangle$$

and study

$$S_{u_0} = -\mathrm{tr}(\rho_{u_0} \ln \rho_{u_0})$$

But recall that any element of A_{u_0} can be approximated arbitrarily well by $\rho_\Sigma \in A_\Sigma$. So we can pick ρ once and for all as an element of A_Σ



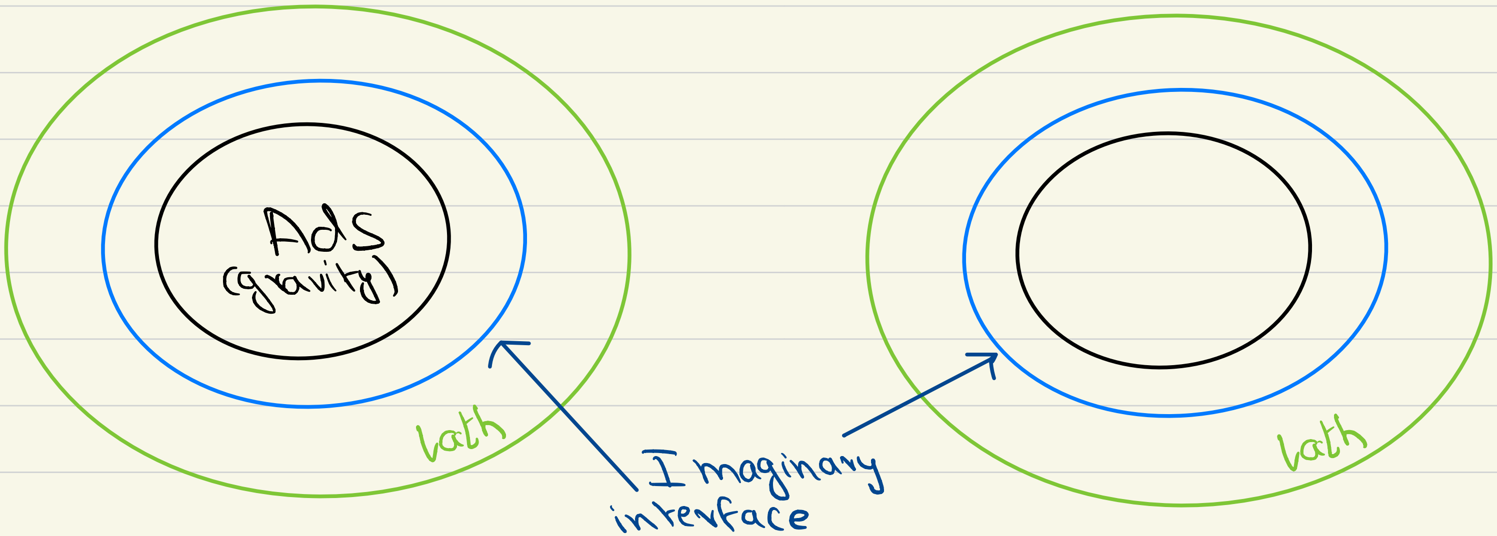
Hawking prediction $\left[\frac{\partial S}{\partial u_0} > 0\right]$

Page prediction $\left[\frac{\partial S}{\partial u_0}\right.$ switches sign at Page time$\left.\right]$

$\frac{\partial S}{\partial u_0} = 0$ [Holography of information]

The Page curve also relies on the split property

How to get a Page curve



Gravity description

Dual BCFT description

If we compute the E.E. of two parts of the nongravitational bath, we get a Page curve.

Even here, holography of information helps understand how information enters the bath.

Consider a 2-step process:

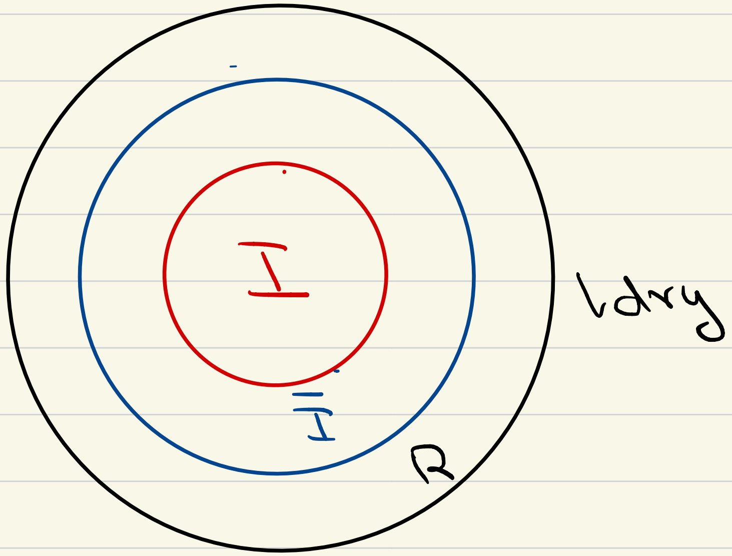
a) prepare a black hole in AdS

b) couple it to a bath.

HoI tells us that after step (a), information is already available at the AdS-boundary.

So in step (b), it just flows into the bath.

Inconsistency of islands in standard gravity



Island suggest that correlators in R do not have information about $\bar{I}$.

How can that happen, given our arguments.

In theories with a bath (for $d > 2$), gravity is massive.

So the Hamiltonian is not a boundary term.

Here is a quick argument. The bdry stress-tensor does not satisfy

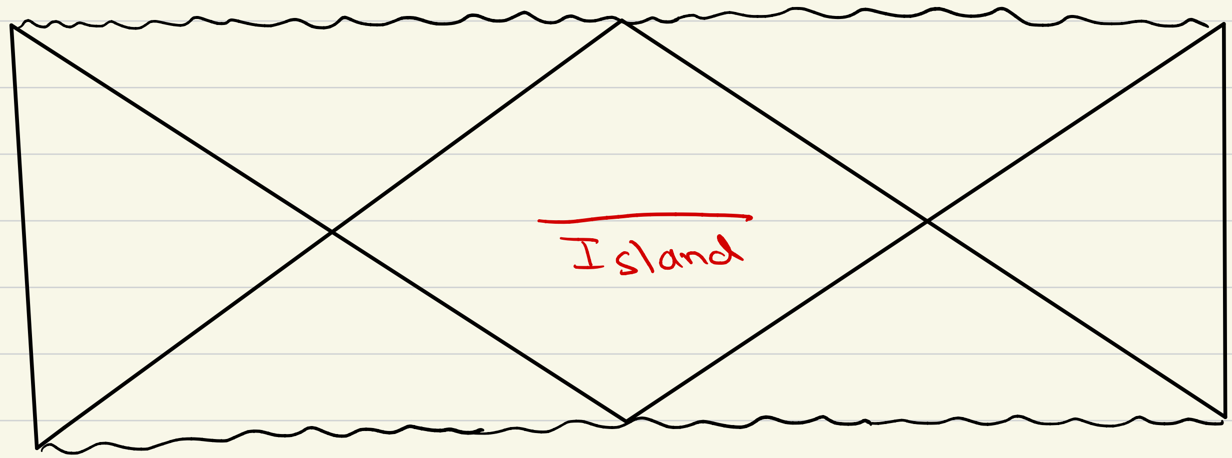
$$\partial_\mu T^{\mu\nu} = 0$$

so it is <u>not</u> in a short rep. of conformal grp

$$\Rightarrow \quad \Delta [T^{\mu\nu}] = d + \varepsilon \quad \Rightarrow \text{ bulk graviton is massive.}$$

Computations of islands are generally performed in this setting

We can get islands in standard gravity in two cases

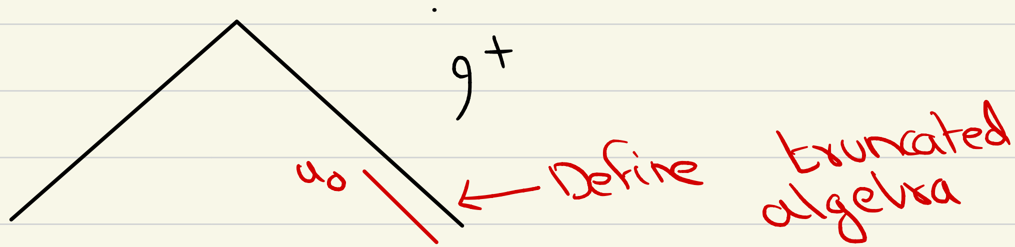


HoI tells us all information on the boundary is available in a time band in standard gravity.

but some information (behind a double horizon) might never be available.

This happens when bdry is in a mixed state.

By truncating the bdry algebra



eg. on $\mathscr{I}^+$ we can discard the ADM Hamiltonian from the algebra.

Specifically, in the expansion from lecture 1, we keep C_{AB} but not m_B.

Unnatural since we keep some components of the metric and throw away others but mathematically consistent.

This produces a Page curve for $S(u_0)$