NNLL Resummation in Z+jet Events with the ARES Method

Darcy Peake, Andrea Banfi, Matthew Lim

UNIVERSITY OF SUSSEX

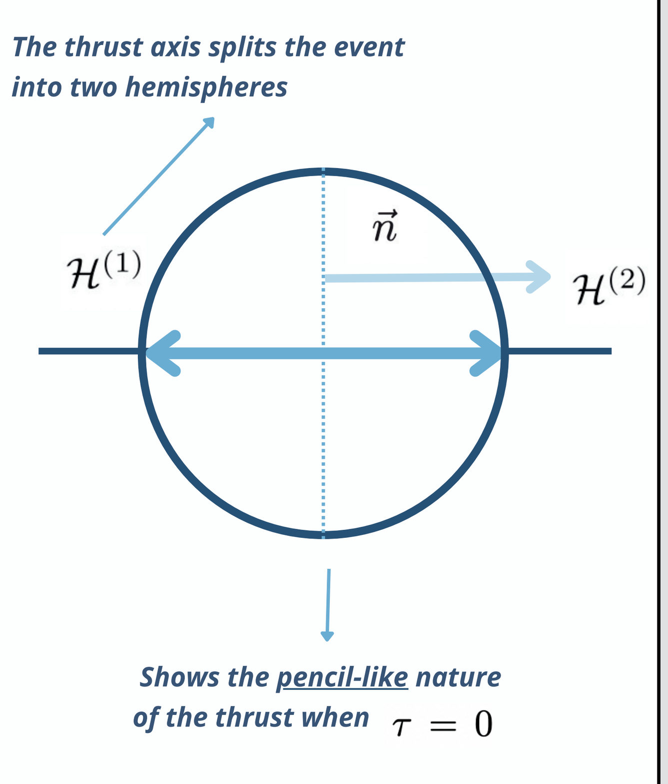
Motivation

IRC Safety and Event-shapes

Singularities arise when emissions are soft or collinear to their emitter, occurring at every order in pQCD [1], [2].

IRC Safe Observable:
- Cancels soft and collinear divergences from real and virtual diagrams at every order in pQCD

Event Shapes:
- A class of IRC safe observables measuring the hadronic energy flow in an event
- Designed to examine specific features of the 'shape' of a hadronic final state
- A common example which occurs in an electron-positron collisions is **thrust** [3]:



$$T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}, \qquad \tau \equiv 1 - T$$

The unit vector which maximises the sum

The momenta of the final-state hadrons

The centre-of-mass energy

Resummation

Fixed-order calculations of event shapes produce logarithms in the form of $L = \ln(1/v)$.

As $v \ll 1$, the logarithm becomes large thus spoiling the convergent FO series in α_s.

Resummation reorders the FO expansion, prioritising the largest powers of L opposed to the fewest powers of α_s:

$$\Sigma(v) \simeq \exp\{Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\}$$

Resums the leading logarithmic (LL) terms

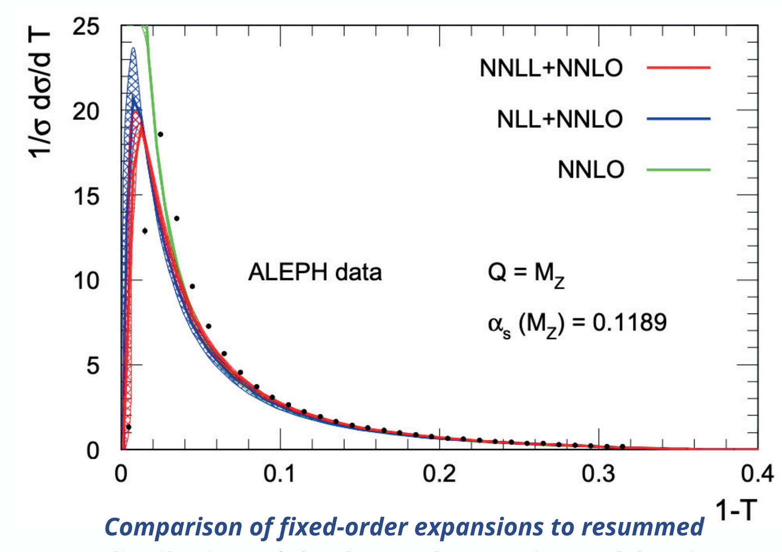
Resums the next-to-leading logarithmic (NLL) terms

Resums the NNLL terms etc.

Fixed-order predictions diverge in the small observable region, unlike experimental data, which shows a local maximum and turning point.

Fixed-order calculations are inadequate here. Resummed expansion captures the downward nature of the data, aligning theory with experiment.

Note: hadronisation corrections give a rigid shift to the right



Comparison of fixed-order expansions to resummed distributions of the thrust. The experimental data is taken from the ALEPH collaboration [4], [5].

Methodology

The ARES Formalism

Observables in the ARES formalism are required to be **continuously global** and **rIRC safe** [6].

Continuously global: Sensitive to emissions in all regions of phase space, where the scaling of the observable with respect to the energy of the emission is the same everywhere.

rIRC safety: Extends IRC safety such that within a "sea" of soft and collinear emissions, the oberservable does not change significantly with a much softer or more collinear emission.

The ARES method recognises that any rIRC safe observable V, with a single soft emission collinear to leg ℓ can be parameterised as follows:

$$V(\{p\},k) \simeq V_{\rm sc}(k) \equiv d_\ell \left(\frac{k_t^{(\ell)}}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi^{(\ell)}),$$

where $k_t^{(\ell)}, \eta^{(\ell)}$ and $\phi^{(\ell)}$ are the transverse momentum, rapidity, and azimuthal angle, while $a, b_\ell, d_\ell, g_\ell$ are the observable-dependent parameters.

The NNLL Master Formula

The ARES formalism uses the idea of a 'sea' of soft and/or collinear gluons with ***one*** extra **special emission** to obtain the **NNLL master formula** [7]:

$$\Sigma(\{p_1,p_2,p_3\},v) = e^{-R_{\rm NNLL}(v)}\left(\mathcal{F}_{\rm NLL}(\lambda)\left(1+\frac{\alpha_s(Q)}{2\pi}H^{(1)}+\frac{\alpha_s(Qv^{\frac{1}{a+b_\ell}})}{2\pi}\sum_\ell C^{(1)}_{{\rm hc},\ell}\right)+\frac{\alpha_s(Q)}{\pi}\delta\mathcal{F}_{\rm NNLL}(\lambda)\right)$$

Coefficient due to virtual emission

Sum over all outgoing legs

The NNLL radiator: includes all virtual corrections of soft and/or collinear origin. More information found [8]

The 'special' emission

All soft and collinear emissions widely separated in angle, and their corresponding virtual corrections, build up this NLL function

The **special emission** is given by the following:

$$\delta\mathcal{F}_{\rm NNLL} = \delta\mathcal{F}_{\rm sc} + \delta\mathcal{F}_{\rm hc} + \delta\mathcal{F}_{\rm rec} + \Delta\mathcal{F}_{\rm rec} + \delta\mathcal{F}_{\rm wa} + \Delta\mathcal{F}_{\rm wa} + \delta\mathcal{F}_{\rm correl},$$

where each contribution corresponds to an emission from a particular region in the phase space.

Preliminary results

Z + Jet Events

We consider the production of a Z boson and a hard jet in a **hadronic collision**:

This indicates we need initial and final-state resummation

$$h_1(p_1) + h_2(p_2) \to Z(q) + {\rm jet}(p_3) + \ldots$$

At Born level, such events results from two hadrons producing a Z boson and a hard parton

These dots represent the accompanying hadrons

The observables we examine are:

- Transverse thrust: $\tau_t \equiv 1 - T_t$, $\quad T_t \equiv \frac{\sum_i |\vec{q}_{t,i} \cdot \vec{q}_T|}{H_T q_T}$.
- One-jettiness: $\tau_1 \equiv \mathcal{T}_1 / H_T$, $\quad \mathcal{T}_1 \equiv \sum_i \min\{n_1 \cdot q_i, n_2 \cdot q_i, n_3 \cdot q_i\}$
- Thrust minor: $T_m \equiv \frac{\sum_i |\vec{q}_{t,i} \times \vec{q}_T|}{H_T q_T}$

Transverse Thrust and the One-jettiness

These are **additive observables**, meaning they satisfy the following:

$$V_{\rm sc}(\{\tilde{p}\},k_1,\ldots,k_n) = \sum_{i=1}^n V_{\rm sc}(\{\tilde{p}\},\{k_i\})$$

The hard partons recoiling against the soft emission

The soft emissions

These observables have purely analytic solutions for $\delta\mathcal{F}_{\rm NNLL}(\lambda)$, their general formula found in [6], [7].

Thrust minor

This is not an additive observable and thus we rely on semi-analytical techniques.

Numerically we use a Monte Carlo simulation of multiple soft and collinear emissions, with *one* "special" emission that triggers NNLL corrections.

Analytically we use techniques such as Mellin and Laplace transforms, for example:

$$\Theta\left(2 - \sum_{i=1}^n \zeta_i - |x|\right) = \int_{c-i\infty}^{c+i\infty} \frac{d\nu}{2\pi i\,\nu} e^{2\nu} e^{-\nu|x|} \prod_i e^{-\nu\zeta_i}$$

$$\delta\left(x - \sum_{i=1}^n \zeta_i \frac{\sin\phi_i}{|\sin\phi_i|}\right) = \nu\int_{-\infty}^{\infty} \frac{d\beta}{2\pi} e^{i\beta\nu x} \prod_i e^{-i\beta\nu\frac{\sin\phi_i}{|\sin\phi_i|}\zeta_i}$$

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[6] A. Banfi, H. McAslan, P. F. Monni, G. Zanderighi, "A general method for the resummation of event-shape distributions in e+e− annihilation," JHEP 2015, 05, 102.

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