

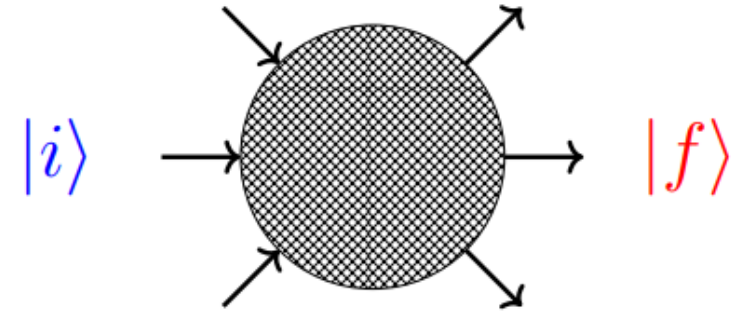
The Classical Double Copy: A Duality Between Exact solutions in Gauge and Gravity theories

Kymani Armstrong-Williams

Centre for Theoretical Physics, Queen Mary University of London

apw796@qmul.ac.uk

Introduction



- Theories of particle physics (such as the standard model) are best described by a special type of quantum field theory called *non-abelian gauge theories*.
- Recently, a new relationship between scattering amplitudes for *non-abelian gauge theories* and *gravity* has been discovered, known as the **Double Copy** (*Bern, Carrasco, and Johansson*).
- The Double Copy has allowed us to calculate previously unobtainable scattering amplitudes results in gravity, by “building” them out of analogous results in non-abelian gauge theories. (used as a tool in *gravitational scattering problems and gravitational waveform corrections*)

The Classical Double Copy

- It was soon realized that this duality at the level of scattering amplitudes, existed at the level of *classical physics* for **certain exact** solutions between **General Relativity** and **Classical** Non-abelian Gauge theories (e.g *Yang-Mills Theory*).
- This became known as the **Classical Double Copy** (*Luna, Monteiro, Nicholson, O'Connell, White*).
- The best-known case of the Classical Double Copy is the **Weyl Double Copy**, which relates *certain vacuum* solutions in **Classical Electromagnetism** and **General Relativity**. (*Nicholson, O'Connell, Godazgar, Godazgar, Peinador Veiga, Pope*)
- This relationship relies on rewriting our solutions in the language of **two-component spinors**.

Weyl Double Copy in Practice

ELECTROMAGNETISM: We can write the electromagnetic field strength $F_{\mu\nu}$ tensor in terms of **spinors**:

$$F_{\mu\nu} \rightarrow F_{AA'BB'} = \phi_{AB} \varepsilon_{A'B'} + \tilde{\phi}_{A'B'} \varepsilon_{AB}$$

2-D Levi Civita Symbol

Maxwell Spinor
Conjugate Maxwell Spinor

GRAVITY: For vacuum solutions, the *Riemann Curvature Tensor* $R_{\mu\nu\rho\lambda}$ is reduced to the **Weyl Tensor** $W_{\mu\nu\rho\lambda}$

The Weyl Tensor translated in terms of spinors is given as:

$$W_{\mu\nu\rho\lambda} \rightarrow W_{AA'BB'CC'DD'} = \Psi_{ABCD} \varepsilon_{A'B'} \varepsilon_{C'D'} + \tilde{\Psi}_{A'B'C'D'} \varepsilon_{AB} \varepsilon_{CD}$$

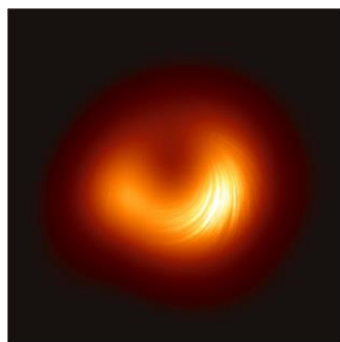
Weyl Spinor
Conjugate Weyl Spinor

Weyl Double Copy in Practice

For vacuum solutions in General Relativity that are of Petrov type **D** or **N**, we can express them in terms of analogous solutions in electromagnetism:

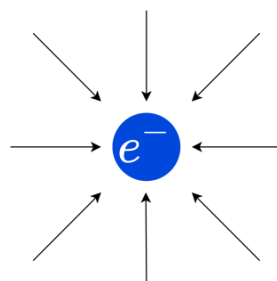
Gravity $\Psi_{ABCD} = \frac{\phi_{(AB}\phi_{CD)}}{S}$ **Electromagnetism**

S is some scalar which is a harmonic function.



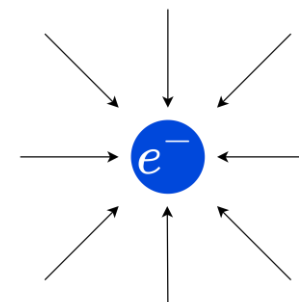
Schwarzschild Solution

=



Coulomb Solution

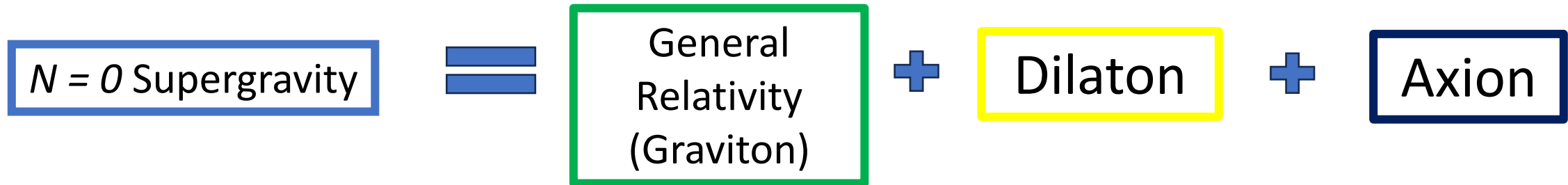
×



Coulomb Solution

Weyl Double Copy in $N = 0$ Supergravity

The Weyl Double Copy has been extended to work for more *Exotic* theories of gravity such as $N = 0$ Supergravity. (KAW, White)



For $N=0$ Supergravity, the Riemann Curvature tensor in the language of 2-components is *no longer* given by just the Weyl Tensor:

$$R_{\mu\nu\rho\lambda} \rightarrow \mathbb{X}_{ABCD}\epsilon_{A'B'}\epsilon_{C'D'} + \tilde{\mathbb{X}}_{A'B'C'D'}\epsilon_{AB}\epsilon_{CD} + \Phi_{ABC'D'}\epsilon_{A'B'}\epsilon_{CD} + \tilde{\Phi}_{A'B'CD}\epsilon_{AB}\epsilon_{C'D'}$$

$$\Psi_{ABCD} = \mathbb{X}_{(ABCD)}$$

Weyl Double Copy in $N = 0$ Supergravity

For $N=0$ Supergravity, in addition to the usual Weyl Double Copy statements, we can write down a Double Copy relationship for the so-called *mixed indexed fields* from the Riemann curvature spinor:

$$\Phi_{ABC'D'} = U^C{}_{C'} U^D{}_{D'} \left(\frac{\Phi_{(AB} \Phi_{CD)}}{S} \right)$$

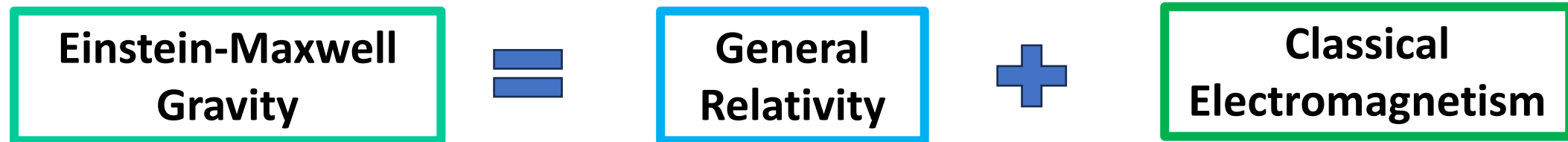
$$U_A{}^{A'} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$U^A{}_{A'} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\tilde{\Phi}_{A'B'CD} = U_C{}^{C'} U_D{}^{D'} \left(\frac{\tilde{\Phi}_{(A'B'} \tilde{\Phi}_{C'D')}}{\tilde{S}} \right)$$

Weyl Double Copy with Sources

The Weyl Double Copy has now been extended to work with non-vacuum solutions for Einstein-Maxwell Gravity. *(KAW, Moynihan, White, Manton, Easson, Svesko)*



These results were derived using methods inspired by methods from Twistor Theory and Quantum Field Theory. *(KAW, Moynihan, White)*

Kinematic Algebras

- One consequence of the Double Copy, was that gauge theories had a much *richer algebraic structure* than previously thought.
- Gauge theories are now known to possess so-called **Kinematic Algebras**.
- We currently believe that kinematics algebras are in general **not** Lie algebras, but some more general mathematical structures such as **homotopy algebras**. *(Reiterer; then Borsten, Jurco, Kim, Macrelli, Saemann, Wolf; Bonezzi, Chiafrino, Diaz-Jaramilo, Hohm, Plefka)*
- Kinematic algebras were formerly only associated with Quantum Field Theories, but it has been shown that they exist for classical physics as well. *(KAW, Nagy, Wikeley, White)*

Diffeomorphisms and Kinematic Algebras

The Kinematic Algebra of a linear theory (e.g. electromagnetism) is associated with special transformations called *diffeomorphisms* (Fu, Krasnov).

Diffeomorphism = simultaneous translation along all integral curves (field lines) of the vector field.

Recalling that $F_{\mu\nu}$ can be written in terms of so-called gauge (vector) fields $A_\mu(x)$:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

We can think about this field as being “valued” in terms of diffeomorphism and so-called gauge (local symmetries) symmetries:

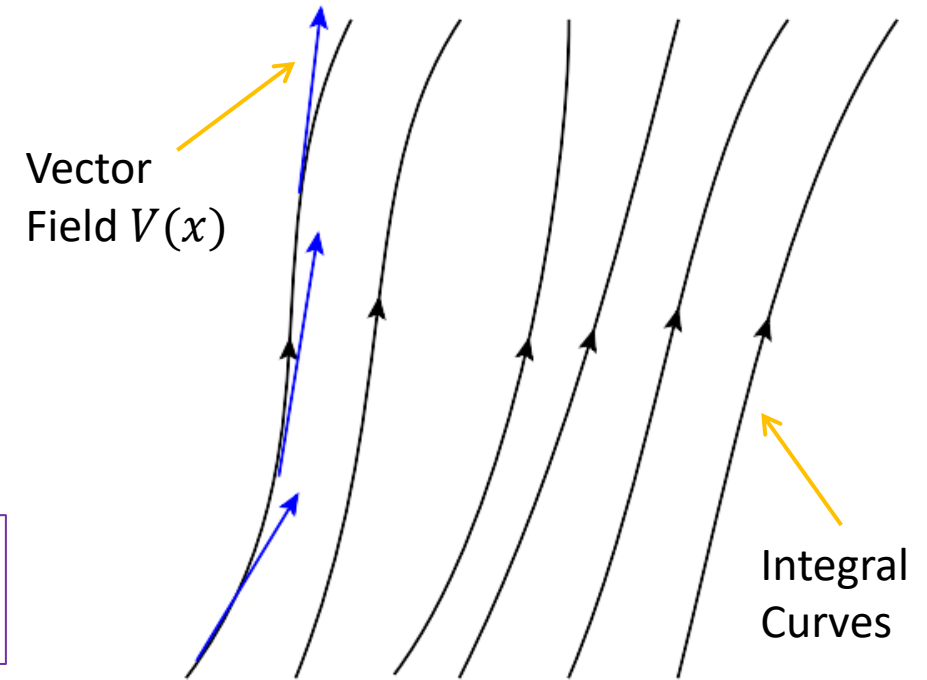
Infinitesimal diffeomorphism

$$\mathbf{A} = A^{\mu a} \partial_\mu \mathbf{T}^a$$

Infinitesimal gauge transformation



$$A^\mu = A^{\mu a} \mathbf{T}^a$$



Electromagnetism in Light Cone Gauge

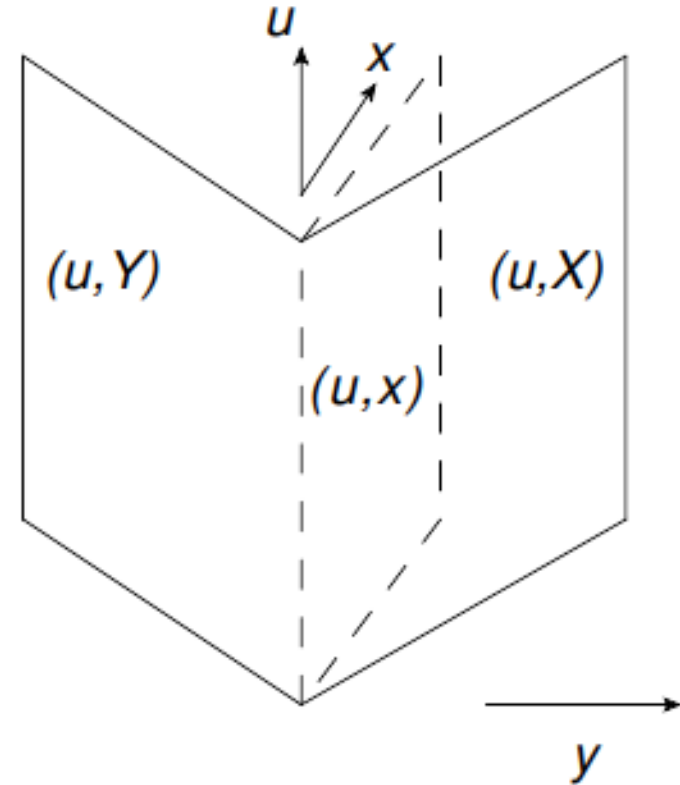
We can **fix** the “*local symmetries*” that arise in our gauge field, via something called a ***gauge transformation***.

For electromagnetism, we make a gauge choice that *restricts* our gauge field to a special gauge choice called **Light Cone gauge**.

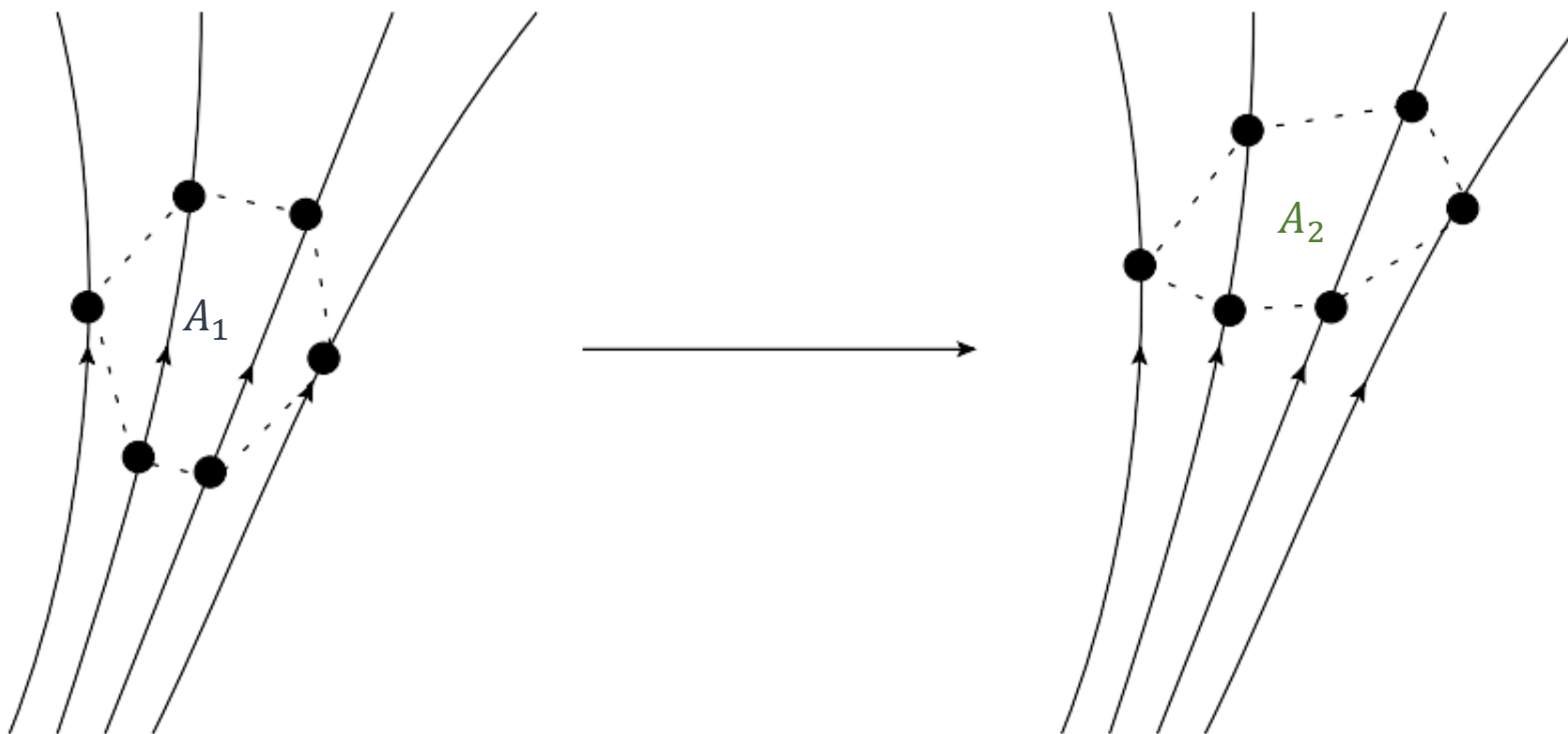
We describe Light cone gauge using a *special* set of coordinates (u, v, x, y) .

In Light Cone gauge, we can directly **geometrically visualise** the kinematic algebra for the theory.

The kinematic algebra we see is a special type of diffeomorphism known as **area-preserving diffeomorphisms** (in particular *symplectomorphism*), which act **in 2-D planes** in either (u, x) or (u, y) planes.



Area Preserving Diffeomorphisms



$$A_1 = A_2$$

Open Questions

- It turns out we can use our understanding of kinematic algebras in linear theories, to **construct** the *Lagrangians* of **non-linear** gauge theories (*QED coupled to scalar matter*) (KAW, Nagy, Wikeley, White) . Can we make this process more general and how is it related to the Double Copy?
- We can derive kinematic algebras for theories relevant to condensed matter physics (**non-abelian Chern-Simons theory** (Ben-Shahar & Johansson)) and beyond (**fluid mechanics**) (KAW, Nagy, Wikeley, White). Can we gain any physical insight from the kinematic algebras ?
- Can we find (Classical) Double Copies for more exotic theories (CFT, AdS, Supergravity, de sitter, cosmology)?
- So far, every example of the Classical Double Copy has been **exact** solutions that **linearise** the *equations of motion* for both gravity and gauge theory. Can we find a Classical Double Copy for truly non-linear solutions?

Conclusions

- The Double Copy is a duality between **Non-abelian Gauge Theories** and **Gravity** for both scattering amplitudes in quantum field theory and exact solutions in Classical Physics.
- The Classical Double Copy has been recently extended to work in **non-vacuum solutions** (Einstein-Maxwell Gravity) and exotic theories such as **$N=0$ supergravity**.
- Kinematic algebras are new and exciting structures present in gauge theories.
- There is plenty more to explore on the Double Copy!

The Double Copy NEEDS you!

