Lecture 1: Axions and ALPs

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Axions

- String theory
- Null measurements of the neutron electric dipole moment
- Dark matter
- Dark energy

Plan

- This lecture: Theory of Axions and Axion-Like Particles
- Lecture 2: Axion Dark Matter
- Lecture 3: Astrophysical searches for axions

Further Reading

- Anson Hook TASI lectures: 1812.02669
- Axion dark matter: What is it and why now?: 2105.01406
- Axion dark matter: How to see it?: 2104.14831
- Ciaran O'Hare on Axion Cosmology: 2403.17697

Lecture plan

The QCD Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\theta - \mathcal{L}_M$$

$$\mathcal{L}_0 = -\frac{1}{2g^2}\,\mathrm{Tr}\left(G^{\mu\nu}G_{\mu\nu}\right) + i\overline{\psi}(\partial\!\!\!/ + iA\!\!\!/)\psi,$$
$$G = \partial_\mu A_\nu - \partial_\nu A_\mu + i\left[A_\mu, A_\nu\right],$$
$$\mathcal{L}_\theta = \theta\frac{1}{32\pi^2}\,\mathrm{Tr}\left(G_{\mu\nu}G_{\rho\delta}\right)\epsilon^{\mu\nu\rho\delta} = \theta\frac{1}{32\pi^2}G\tilde{G}$$
$$\mathcal{L}_M = \overline{\psi}_R M\psi_L + \text{ h.c.}$$

The QCD Lagrangian

- We can rotate to a basis with a diagonal quark mass matrix: $M_{ab} = m_a \delta_{ab} e^{i\rho}$
- The phase ρ cannot be rotated away.
- This phase of the quark mass matrix is CP-violating.

Charge transformation - exchange particles and antiparticles
Parity transformation - $\mathbf{x} \to -\mathbf{x}$
Time transformation - $t \to -t$
Every relativistic quantum field theory is invariant under CPT.

The QCD Lagrangian

The CP-violating term $\mathcal{L}_\theta = \theta \frac{1}{32\pi^2} \mathrm{Tr}\left(G_{\mu\nu} G_{\rho\delta}\right) \epsilon^{\mu\nu\rho\delta}$ is a *total derivative*:

$$\mathcal{L}_\theta = \partial_\mu K^\mu,$$

$$K^\mu = \theta \frac{1}{32\pi^2} \epsilon^{\mu\alpha\beta\gamma} A_{a\alpha} \left[G_{a\beta\gamma} - \frac{g}{3} f_{abc} A_{b\beta} A_{c\gamma} \right].$$

$$S_\theta = \int d^4x \mathcal{L}_\theta = \int_S d^3x \, n_\mu K^\mu$$

The QCD Lagrangian

$\mathcal{L}_\theta$ contributes as a *boundary term* in the action:

$$\int d^4x \frac{1}{32\pi^2} G\tilde{G} = n_1 - n_2. \tag{1}$$

n_1 = winding number at infinity n_2 = winding number at origin

The $U(1)_A$ Anomoly

At the classical level, QCD with massless quarks is symmetric under a $U(1)_A$ transformation:

$$\psi \to \mathrm{e}^{i\epsilon\gamma^5}\psi$$

This symmetry is spontaneously broken by quark condensates $<\psi\overline{\psi}>\neq 0$.

The $U(1)_A$ Anomoly

The $U(1)_A$ symmetry is anomalous i.e. it is broken at the quantum level as the path integral measure is not invariant under $U(1)_A$.

Under a $U(1)_A$ transformation $\psi \to \mathrm{e}^{i\epsilon\gamma^5}\psi$, we must change our effective Lagrangian as:

$$\mathcal{L} \to \mathcal{L} + \epsilon \frac{Ng^2}{32\pi^2} G\tilde{G}$$

The $U(1)_A$ Anomoly

The QCD Lagrangian is invariant under the *spurious* symmetry:

$$\psi \to \mathrm{e}^{i\epsilon\gamma^5}\psi,$$

$$\theta \to \theta + N\epsilon,$$

$$\rho \to \rho - \epsilon,$$

where θ is the strong CP angle and ρ is the phase of the quark mass matrix.

The physical combination $\overline{\theta} = \theta + \arg\det M$ is invariant under the spurious symmetry.

The Chiral Lagrangian

At low energies, QCD is described by chiral perturbation theory:

$$U(x) = \exp\left(\frac{2i\pi^a(x)\sigma^a}{f_\pi}\right)$$

$$\mathcal{L} = f_\pi^2 \,\mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger + a f_\pi^3 \,\mathrm{Tr}\, MU + b f_\pi^4 \det U + \text{ h.c.}$$

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The CPT Lagrangian must obey the spurious $U(1)_A$ symmetry:

$$U \to \mathrm{e}^{i\epsilon} U$$

$$M \to \mathrm{e}^{-i\epsilon} M$$

$$\theta \to \theta + N\epsilon$$

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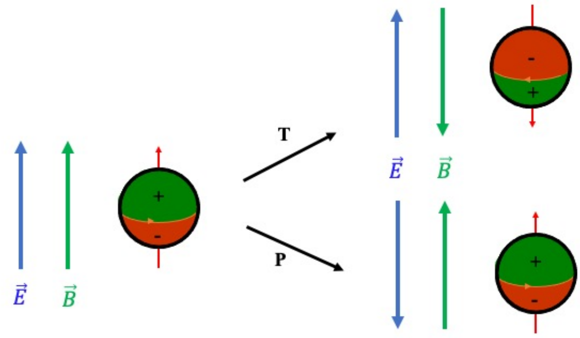
The spurion symmetry requires $b = |b| e^{-i\theta}$ *or* $b = |b| e^{i\rho}$.

- $b = |b| e^{-i\theta}$ - neutron EDM
- $b = |b| e^{-i\rho}$ - no neutron EDM?
- See Ai, Cruz, Garbrecht & Tamarit, 2001.07152

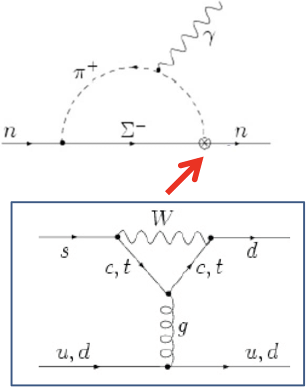
The Neutron Electric Dipole Moment

$$\mathcal{H} = -\mathbf{d} \cdot \mathbf{E}$$



The Neutron Electric Dipole Moment

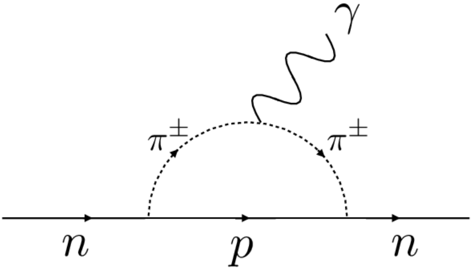
Aside: Neutron EDM from CP violating in the CKM matrix - $d_n \sim 10^{-32}\,\mathrm{ecm}$.



(Image by Peter Fierlinger.)

The Neutron Electric Dipole Moment

Neutron EDM from Strong CP Violation:



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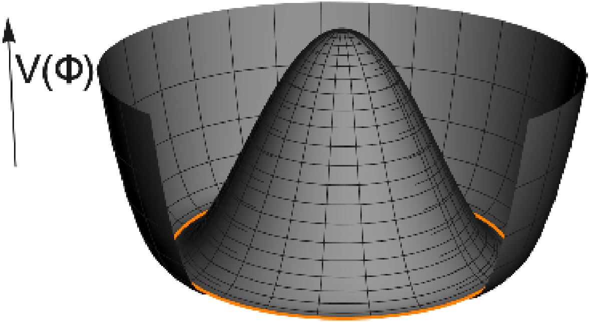
- 'Axial' - the symmetry transformation acts differently on left-handed and right-handed particles.
- $U(1)$ - the symmetry transformation is mathematically equivalent to a rotation about a single axis.

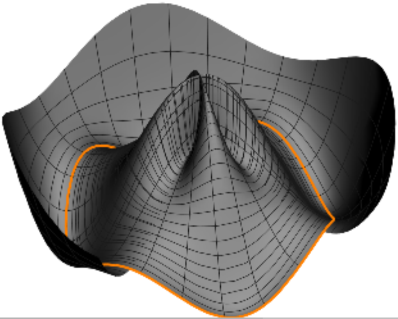
The Peccei-Quinn Solution

- The PQ symmetry is **spontaneously broken** at low enough temperatures.
- The axion is the Nambu-Goldstone boson of the spontaneously broken PQ symmetry.



The Peccei-Quinn Solution

- The PQ symmetry is *also* explicitly broken by the QCD phase transition.
- This gives a small mass to the axion, and ensures the axion field takes a value leading to a null neutron EDM.
- The axion is therefore a naturally light pseudo-scalar particle.



The Vafa-Witten Theorem

"In parity-conserving vector-like theories such as QCD, parity conservation is not spontaneously broken."

Dynamical parity violating terms have zero vacuum expectation value.
(Vafa and Witten, 1984)

The Peccei-Quinn solution

- Promote θ to a dynamical variable - the QCD axion:
  $\mathcal{L} \supset (\theta + \frac{\xi a}{f_a})\frac{g^2}{32\pi^2} G^{\mu\nu}\tilde{G}_{\mu\nu}$
- The Vafa-Witten theorem guarantees that the *total* θ term is zero in the ground state.
- A potential is generated for the axion such that the total coefficient of $G^{\mu\nu}\tilde{G}_{\mu\nu}$ is zero.

String ALPs

- Axion-like particles (ALPs) are light pseudo-scalar particles.
- ALPs do not necessarily couple to gluons or solve the neutron EDM problem.
- String theory compactificiations typically give rise to many ALPs at a range of masses.

Interactions

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 + g_{agg} a G\tilde{G} - \frac{g_{a\gamma\gamma}}{4} a F\tilde{F} + g_{aff}\overline{\Psi}_f \gamma^\mu \gamma_5 \Psi_f \partial_\mu a$$

- $g \sim \frac{1}{f_a}$
- QCD axion: $m_a f_a \sim m_\pi f_\pi$
- String ALP: m_a and f_a are free parameters.

Interactions

$$\mathcal{H} = g_{a\gamma\gamma} \int a\mathbf{E} \cdot \mathbf{B} dV + g_{aff} \nabla a \cdot \hat{\mathbf{S}} + g_{EDM} a \hat{\mathbf{S}} \cdot \mathbf{E}$$

- $g \sim \frac{1}{f_a}$
- QCD axion: $m_a f_a \sim m_\pi f_\pi$
- String axion: m_a and f_a are free parameters.
- These pseudo-scalar couplings are much harder to detect than scalar couplings to the masses of matter particles and to $\mathbf{E}^2 - \mathbf{B}^2$.

Interactions

We know $f_a \gtrsim 10^{10}\,\mathrm{GeV}$. To detect such small couplings, we can:

- Measure things very carefully
- Exploit resonance
- Arrange for very big numbers