Lecture 2: Axion Dark Matter

Francesca Chadha-Day

Durham University

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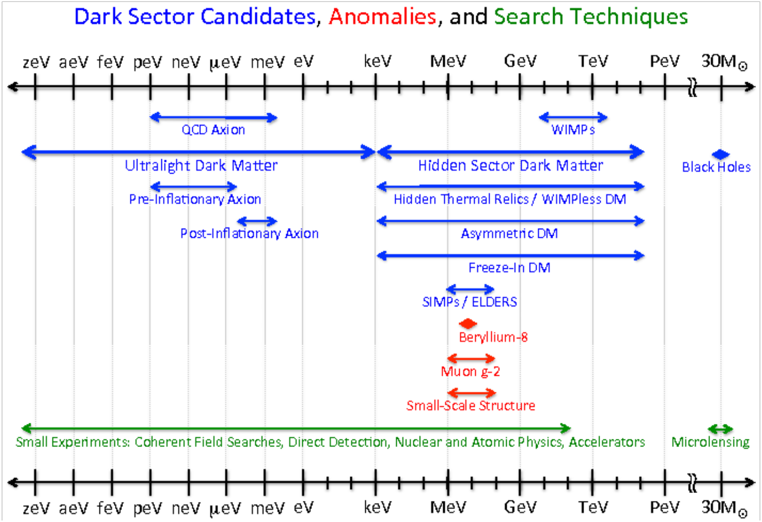
Plan

Dark Matter mass



Reproduced from Battaglieri *et al*, 1707.04591

Production

- Ultralight DM must be non-thermally produced.
- Decay of topological defects
- Misalignment mechanism

Primordial axion production

Axions may be produced in the early universe by:

- Particle decay (dark radiation)

Primordial axion production

Axions may be produced in the early universe by:

- Particle decay (dark radiation)
- Misalignment production (dark matter and dark energy):

Peccei-Quinn symmetry spontaneously broken. Massless axion field created.

$\downarrow$

Axion follows random walk in field space.

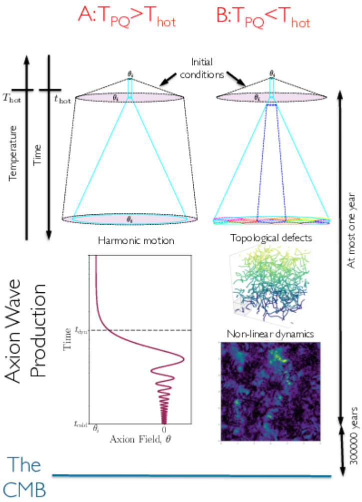
$\downarrow$

Non-perturbative effects generate axion mass. Axion field is now displaced from its minimum.

Axion Dark Matter

- Coherently oscillating scalar field: $\ddot{a} + 3H\dot{a} + m_a^2 a = 0$
- Oscillations are damped by the expansion of the universe
- Energy density redshifts like dark matter

Axion Dark Matter



Some images adapted from Vaquero, A., Redondo, J. & Stadler, J. 1809.09241 and Armengaud, E. *et al.*, 1904.09155.

Classical field description

The ultra-light DM field is the coherent state:

$$|\phi\rangle = \exp\left[\int \frac{dq^3}{(2\pi)^3}\tilde{\phi}(q)\hat{a}^\dagger(q)\right]|0\rangle\,,$$

such that:

$$\langle\phi|\,\hat{\phi}\,|\phi\rangle = \phi(x),$$

where $\phi(x)$ is the classical field.

Classical field description

$$\phi \sim A(x,t)\cos(mt - \alpha(x,t))$$

If we write $\psi = A\mathrm{e}^{i\alpha}$, ψ obeys a Schrodinger-Poisson equation:

$$i\partial_t \psi = \left(-\frac{1}{2m}\nabla^2 + m\Phi\right)\psi$$

$$\nabla^2\Phi = 4\pi G m |\psi|^2$$

We cannot use this framework for cold DM, as A and α would not be well defined.

Classical field description

$$i\partial_t\psi = \left(-\frac{1}{2m}\nabla^2 + m\Phi\right)\psi$$

- Ultralight DM is well approximated by a classical field limit of quantum field theory. Large occupation numbers lead to a low fractional uncertainty in the amplitude and phase dispersion.
- ψ is not a wavefunction.

Classical field description

$$\psi(t, \mathbf{x}) = \sqrt{n(t, \mathbf{x})}\mathrm{e}^{i\hbar S(t,\mathbf{x})}$$

$$\nabla S(t, \mathbf{x}) = m\mathbf{v}(t, \mathbf{x})$$

Number density

$$\partial_t n + \nabla \cdot \mathbf{j} = 0$$

$$\mathbf{j} = \frac{N}{2\mathrm{i}m}(\psi^* \nabla \psi - \psi \nabla \psi^*)$$

Velocity

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \nabla(Q + \Phi) = 0$$

$$Q = -\frac{1}{2m^2}\frac{\nabla^2\sqrt{n}}{\sqrt{n}}$$

Quantum Pressure

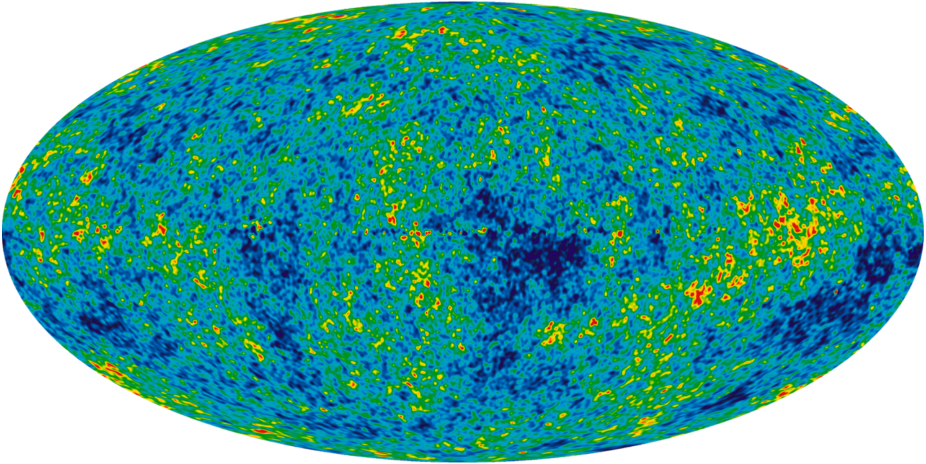
$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \nabla(Q + \Phi) = 0$$

- Ultralight DM does not behave like a perfect fluid.
- The ‘quantum pressure’ Q is a repulsive term that counteracts the gravitational potential.
- Q can be understood as arising from the zero point motion of the ultra-light particles.

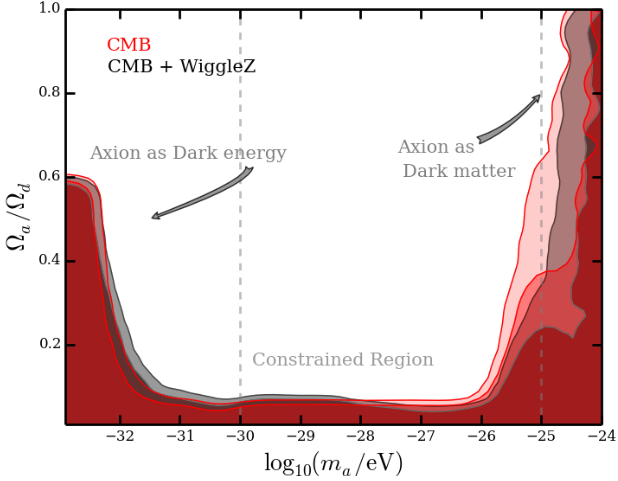
Ultralight DM structure

- Ultralight DM possess a natural scale, the Jeans scale, equal to the de Broglie wavelength of the ground state.
- Stability below the Jeans scale is guaranteed by the Uncertainty Principle.
- Power on scales below the Jeans is suppressed.

Ultralight DM in the CMB



Ultralight DM in the CMB

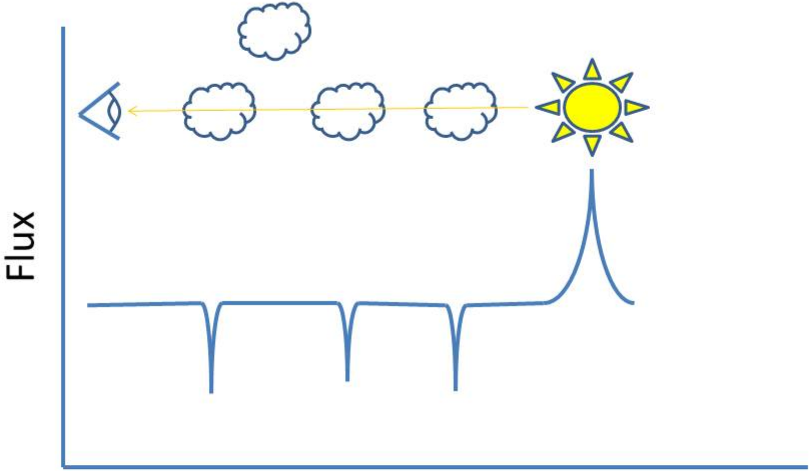


Reproduced from Hlozek *et al* (1410.2896)

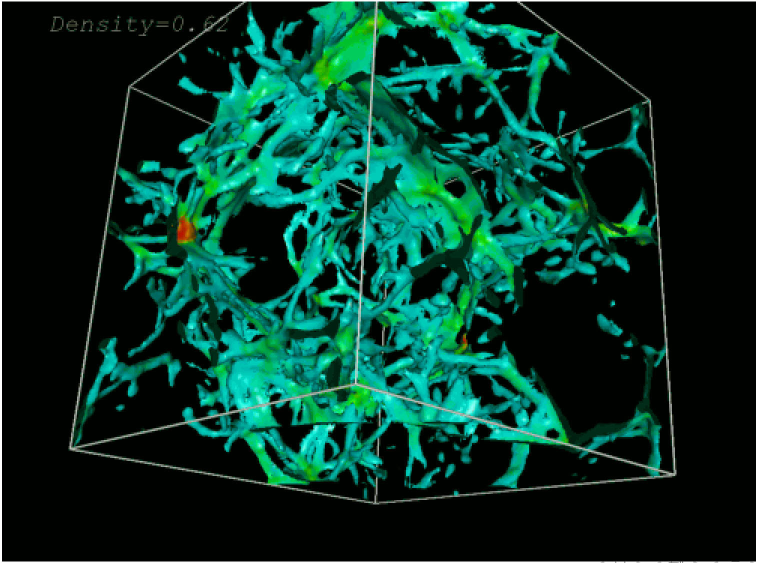
Ultralight DM in the Lyman-α forest

- Light from distant galaxies and quasars is absorbed by intergalactic gas clouds.
- We observe the Lyman-α absorption line from the ground state to the first excited state of neutral hydrogen.
- The absorption line is redshifted.
- From the gas distribution, we infer the DM distribution.

Ultralight DM in the Lyman-α forest



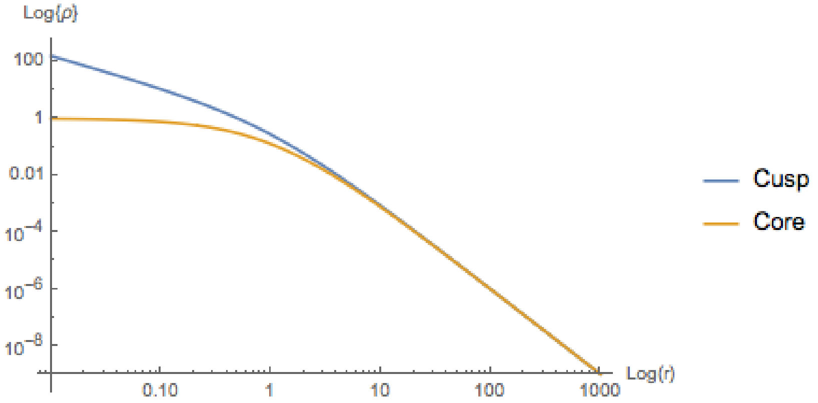
Ultralight DM in the Lyman-α forest



Ultralight DM in the Lyman-α forest

- Lyman-α forest data rules out ultralight DM with $m = 1 - 10 \times 10^{-22}$ eV. (Iršič *et al*, 1703.04683)
- Recent work using machine learning to emulate the power spectrum improves the bound to $m > 2 \times 10^{-20}$ eV. (Rogers & Peiris, 2007.12705)

The Cuspy Halo Problem



Axion Haloscopes

- Axions are much too light to detect with WIMP dark matter detectors.
- Axion haloscopes use the axion's interaction with the photon to search for the local axion dark matter density.

Axion-Maxwell equations

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 - \frac{g_{a\gamma\gamma}}{4} aF\tilde{F}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$
$\frac{1}{4}F\tilde{F} = \mathbf{E} \cdot \mathbf{B}$

In a background magnetic field, axions and photons can interconvert.

Axion-Maxwell equations

The Euler-Lagrange equations give us the axion-Maxwell equations:

$$\nabla \cdot \mathbf{E} = \rho - g_{a\gamma\gamma} \mathbf{B} \cdot \nabla a$$

$$\nabla \cdot \mathbf{B} = 0$$

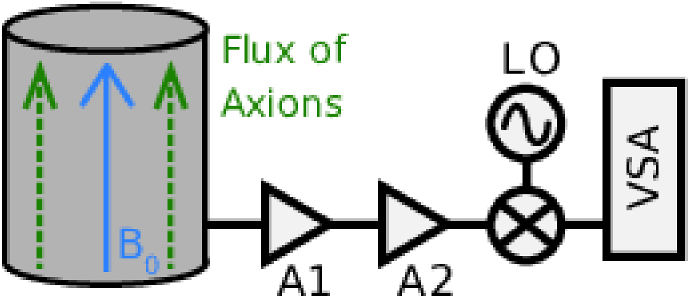
$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$$

$$\nabla \times \mathbf{B} = \dot{\mathbf{E}} + \mathbf{J} - g_{a\gamma\gamma} (\mathbf{E} \times \nabla a - \dot{a} \mathbf{B})$$

Axion Dark Matter eXperiment

- Axion dark matter oscillates with period $\tau \sim 1/m_a$.
- Conducting cavity of length $L \sim 1/m_a$ sources resonant conversion of DM axions to photons.
- Background magnetic field
- Scan over different values of L
- Experimental tests of the invisible axion, Sikivie 1983, PRL 51

Axion haloscopes



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Axion haloscopes

- Neglect gradients in a: $\nabla \times \mathbf{B} = \dot{\mathbf{E}} + \mathbf{J} + g_{a\gamma\gamma}\dot{a}\mathbf{B}$
- For complete results, we should integrate the axion-Maxwell equations in the cavity with appropriate boundary conditions.
- Use an equivalent circuit to approximate the power converted from axion dark matter to EM waves.

Axion haloscopes

Assume field changes due to axion-photon coupling are small:

$$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$$

$$\mathbf{E} = \mathbf{E}_0 + \delta\mathbf{E}$$

$$\mathbf{J} = \mathbf{J}_0 + \delta\mathbf{J}$$

$$a = a_0 + \delta a$$

Axion haloscopes

0th order in $g_{a\gamma\gamma}$:

$$\nabla \times \mathbf{B}_0 = 0$$

1st order $g_{a\gamma\gamma}$:

$$\nabla \times \delta\mathbf{B} = \delta\dot{\mathbf{E}} + g_{a\gamma\gamma}\dot{a}\mathbf{B}_0 + \delta\mathbf{J}$$

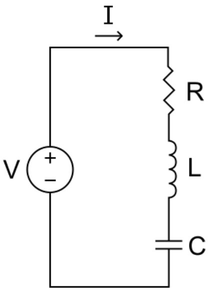
An electric field cavity mode is induced by the axion dark matter.

Axion haloscopes

Trial solution:

$$\dot{\mathbf{E}} = -g_{a\gamma\gamma}\mathbf{B}_0\dot{a}$$

Model the cavity mode as an equivalent RLC circuit.



Axion haloscopes

Equate energy stored in cavity mode and capacitor:

$$\int \frac{1}{2} E^2 dV = \frac{q^2}{2C},$$

where q is the charge on the capacitor.
Circuit equation of motion:

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = V(t)$$

Axion haloscopes

Equate driving force terms in Lagrangians:

$$V(t) = \frac{g_{a\gamma\gamma} a(t) B_0 \int \mathbf{E} \cdot \hat{\mathbf{z}} dV}{\sqrt{C \int E^2 dV}} = g_{a\gamma\gamma} a(t) B_0 \sqrt{\frac{f_{nlm} V}{C}},$$

where f_{nlm} is a form factor for a given cavity mode.

Axion haloscopes

Power delivered to equivalent circuit on resonance:

$$P = \frac{< V(t)^2 >}{R} = g_{a\gamma\gamma}^2 B_0^2 V f_{nlm} \frac{1}{RC} < a(t)^2 >$$

This is the power transferred from the axion dark matter to EM waves in the cavity.

Axion haloscopes

What are R and C for our detector? Express in terms of measureable cavity mode parameters:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Measure Q and ω_0 by driving the cavity mode (without axions!).

Axion haloscopes

$$P = g_{a\gamma\gamma}^2 B_0^2 < a(t)^2 > V f_{nlm} \omega_0 Q$$

when $\omega_0 \simeq m_a$.

We use the dark matter density distribution in the Milky Way to estimate the local DM density. Equate this to the energy density in a:

$$\rho_{DM} = m_a^2 < a(t)^2 >$$

Axion haloscopes

This gives us:

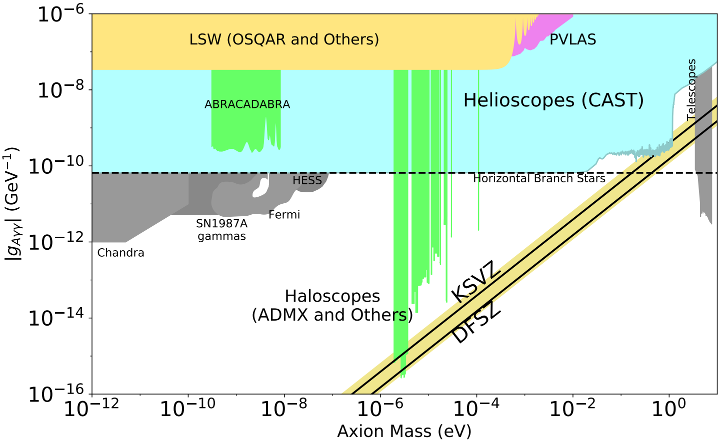
$$P = g_{a\gamma\gamma}^2 B_0^2 \frac{\rho_{DM}}{m_a^2} V f_{nlm} \omega_0 Q$$

when $\omega_0 \simeq m_a$.

Axion haloscopes

- Need to scan over resonant frequencies as m_a is unknown.
- Resonant condition requires $L \sim 1/m_a$ (i.e. one wavelength fits in cavity)
- If m_a is too low, we can't build a cavity big enough to achieve resonance.
- If m_a is too high, the resonant cavity is too small for measurable power output.
- This motivates more sophisticated axion haloscopes.

Axion haloscopes



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