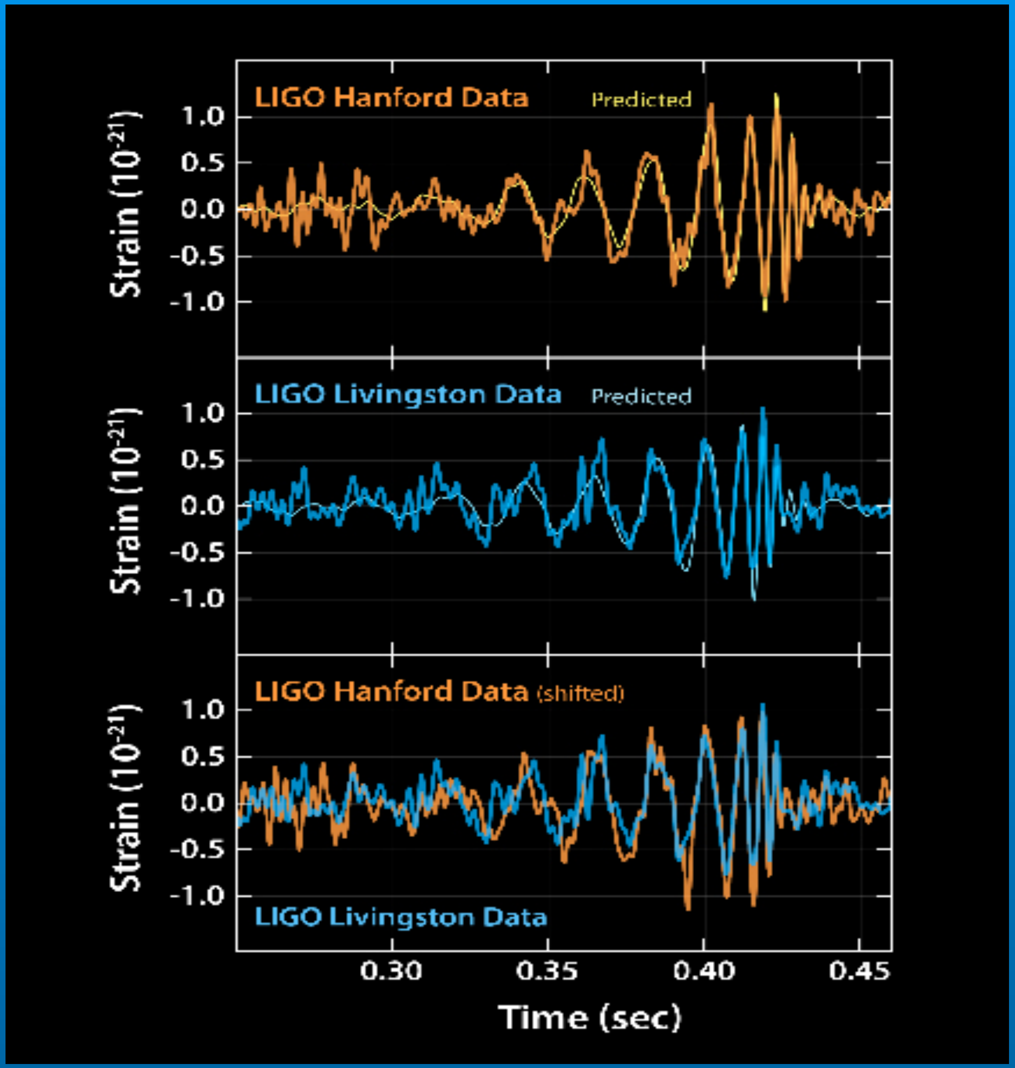
Gravitational wave archaeology

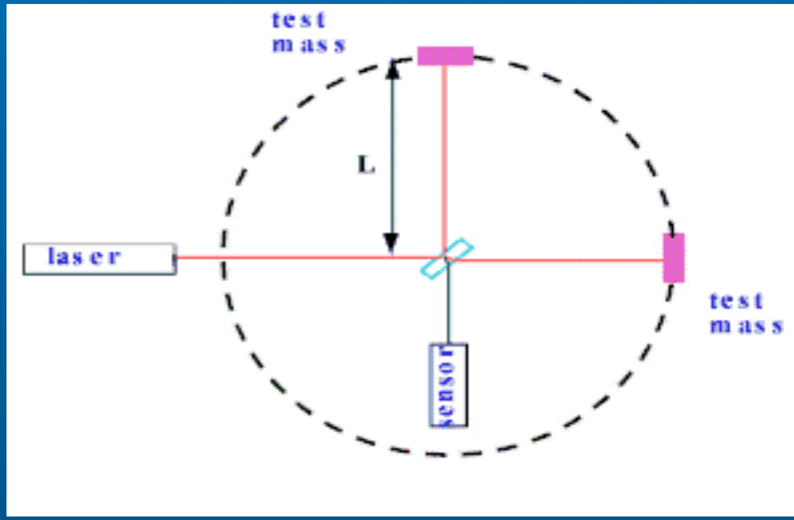
Graham White

University of Southampton

SHEP Southampton High Energy Physics

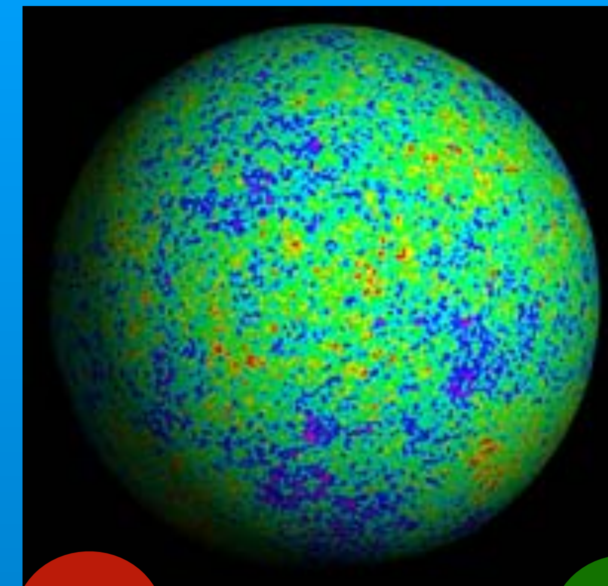
Discovery of GWs

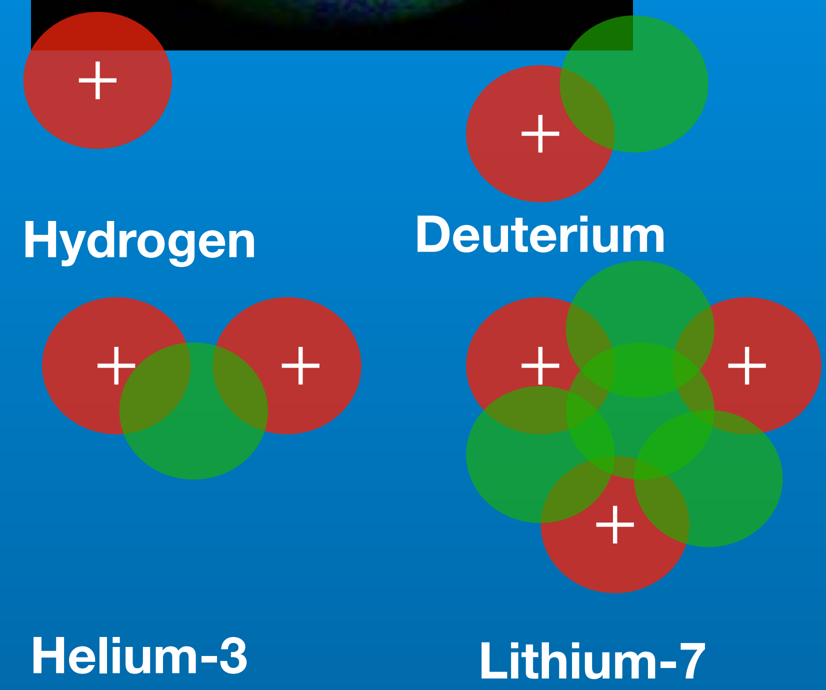




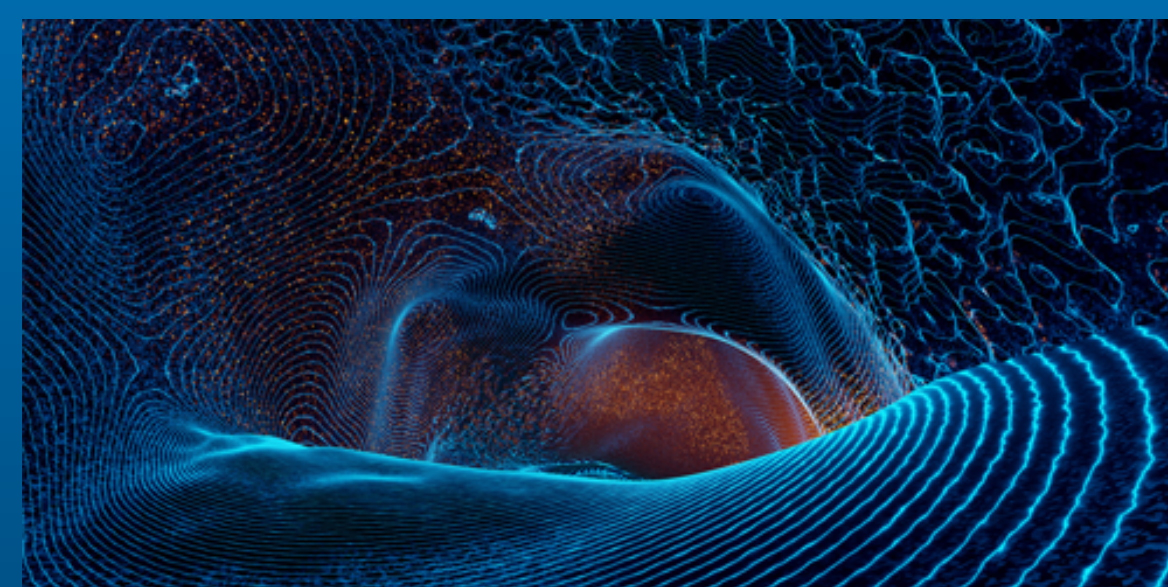
Why Gravitational waves are amazing for cosmology

$t = 10^6$ years





$t = 1$ min



$t = 0!$

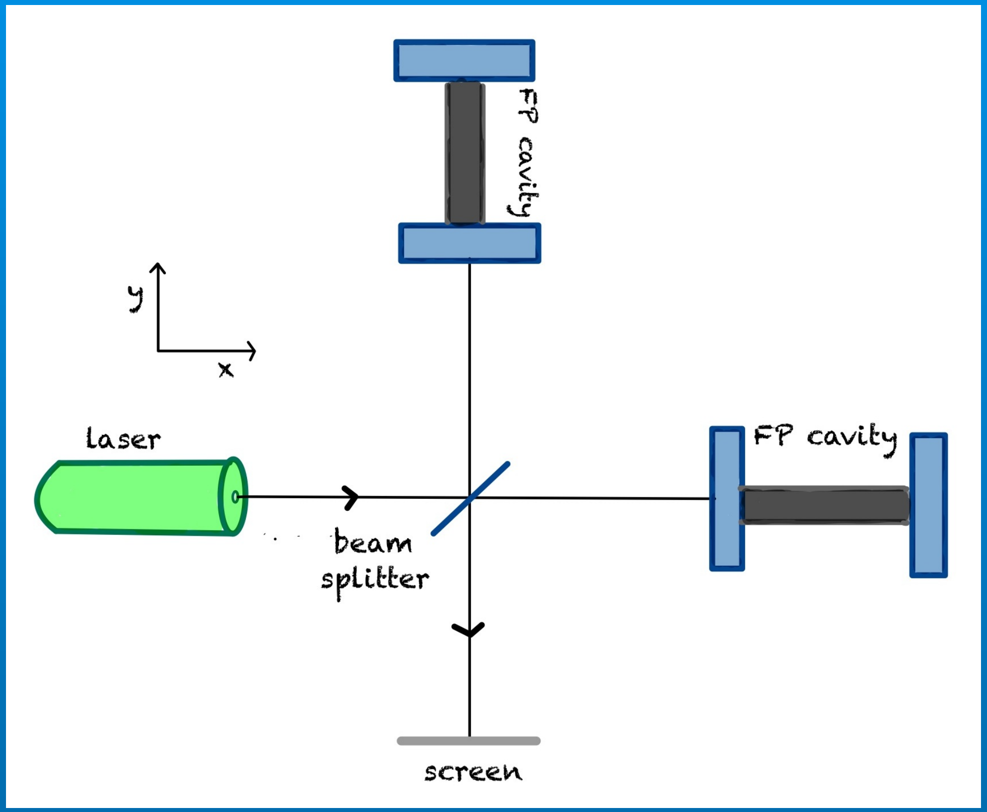
Image credits: WMAP,
Phys. Rev. Lett. 124, 041804

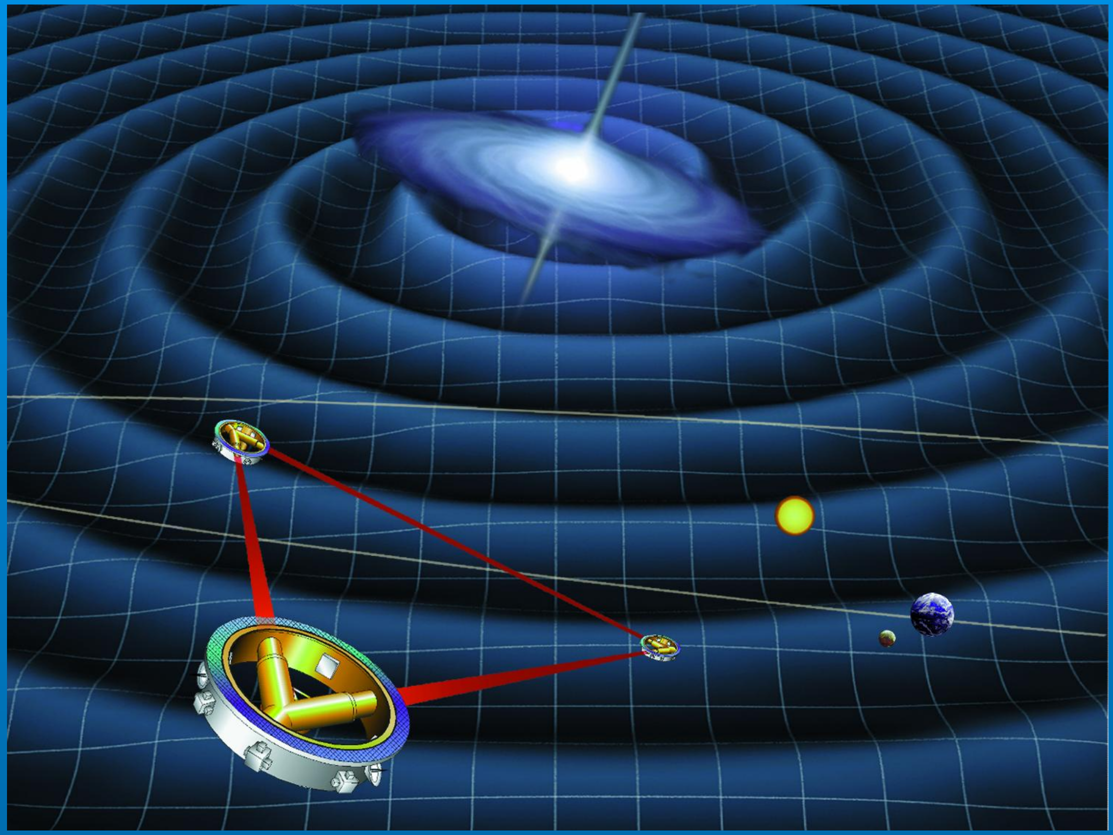
New physics is hard to test!

Reheating scale $10^{-3} - 10^{13}$ GeV
Leptogenesis scale: $10^{9} - 10^{14}$ GeV
Affleck Dine Scale: $\sim 10^{12} - 10^{15}$ GeV
Dark matter scale ~ ???

Lots of physics is difficult to test on Earth
Archaeology is not about certainty, it is using GW signals to figure out what is *likely*

Interferometer GW detectors

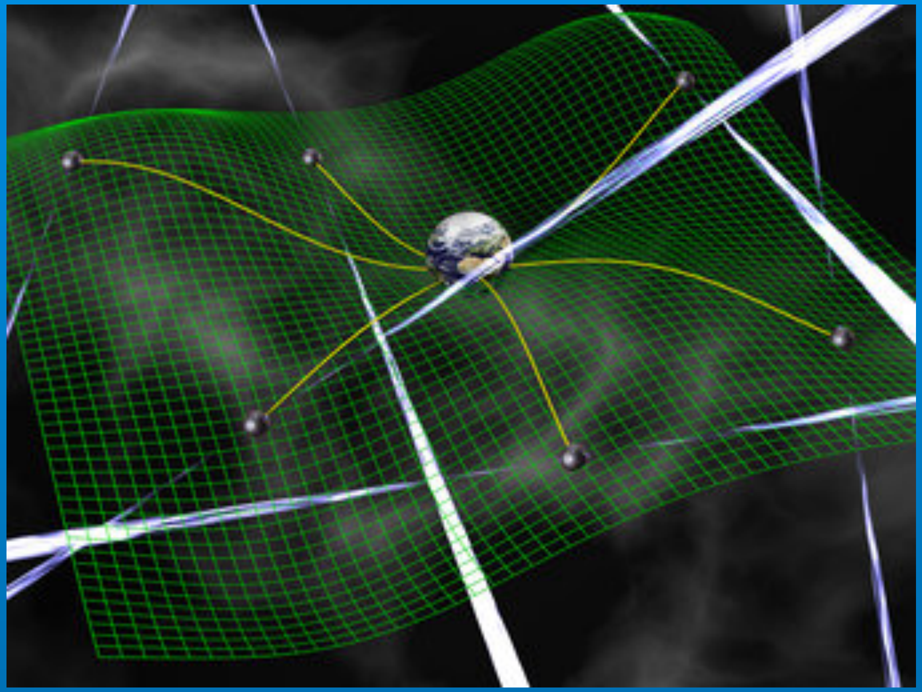




LISA, LIGO, Einstein Telescope, DeCigo…

Image credit ISA

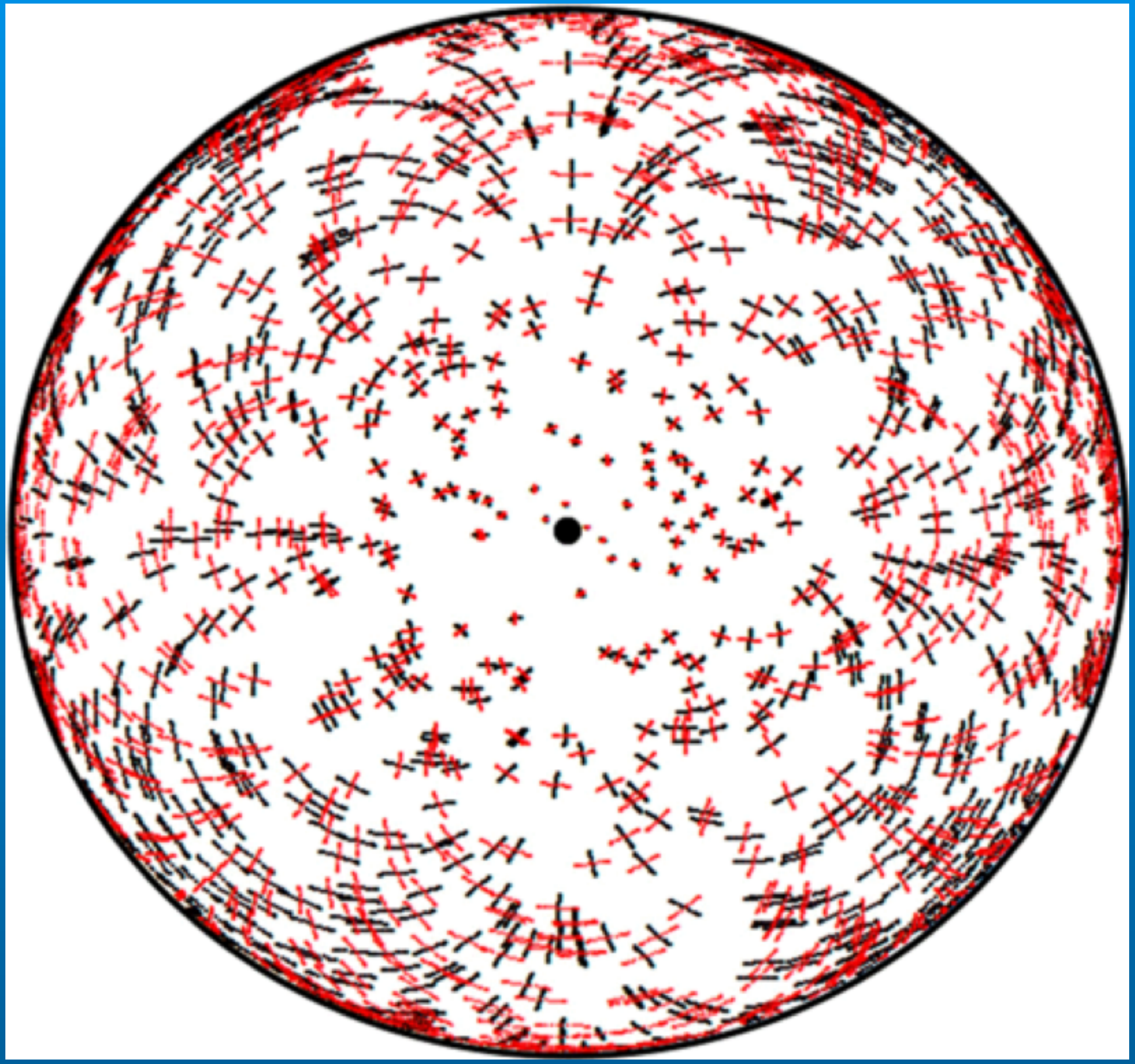
Pulsar Timing arrays

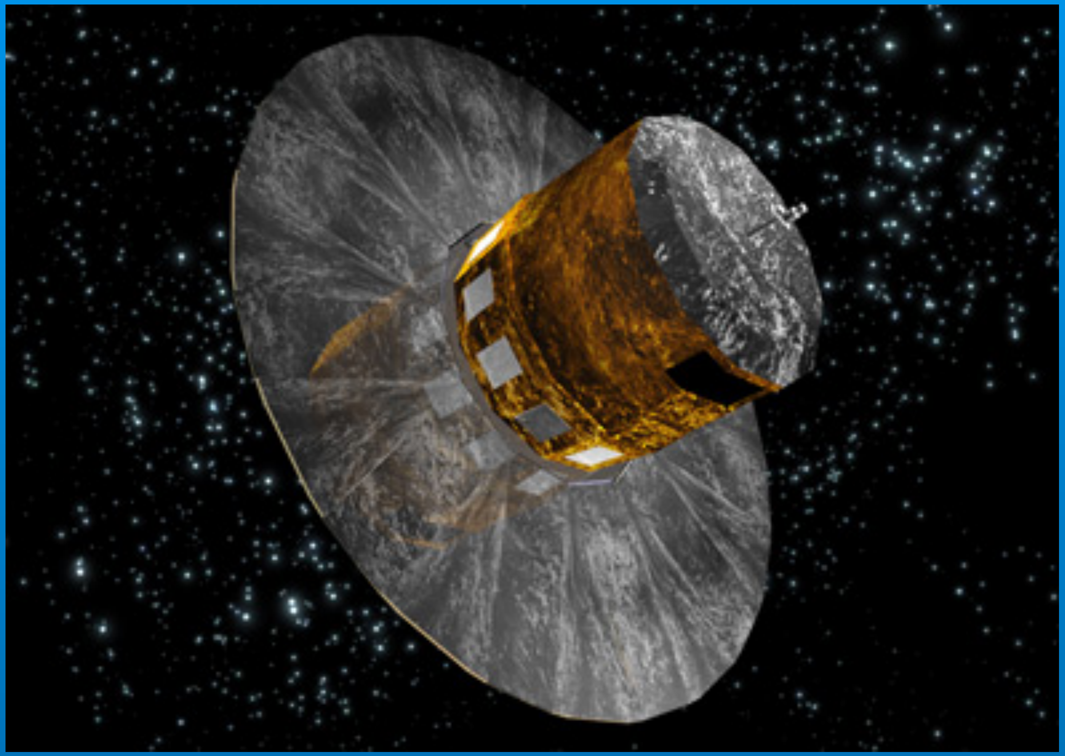


NANOGrav, EPTA, Square kilometer array..

Image credit: David Champion

Astrometry

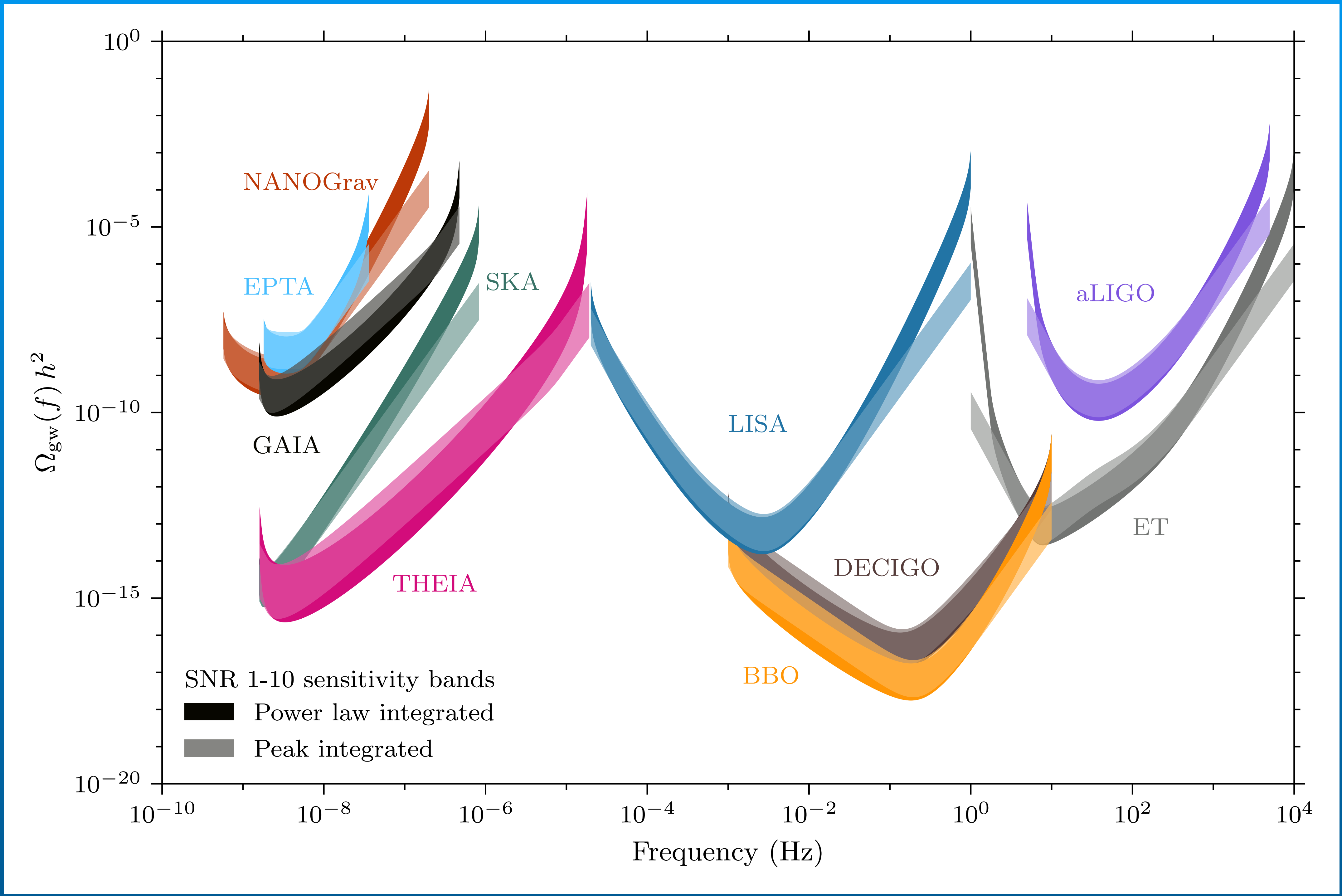




Gaia, Theia, asteroids…

Image credits: 1707.06239, C. Carreau

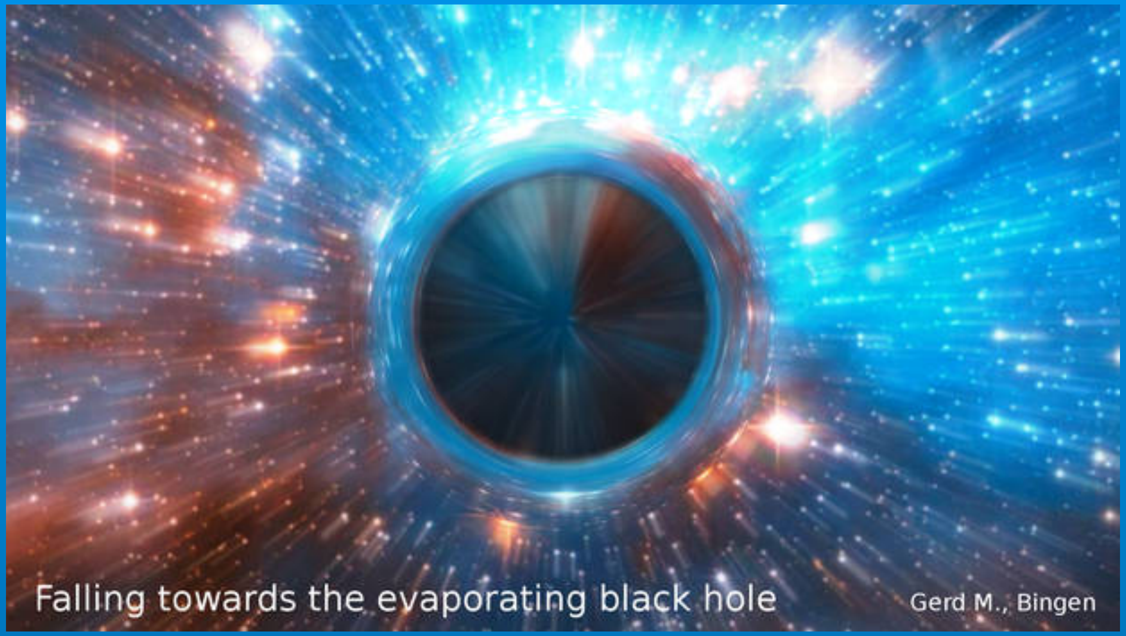
So many gravitational wave detectors!

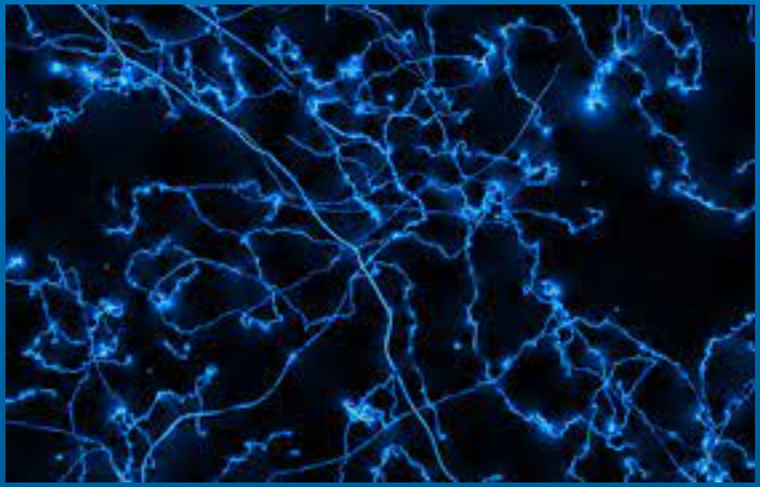


Juan Garcia-Bellido, Hitoshi Murayama, Graham White 2104.04778

(Mostly) Three types of post-inflation signals

Matter suddenly vanishing





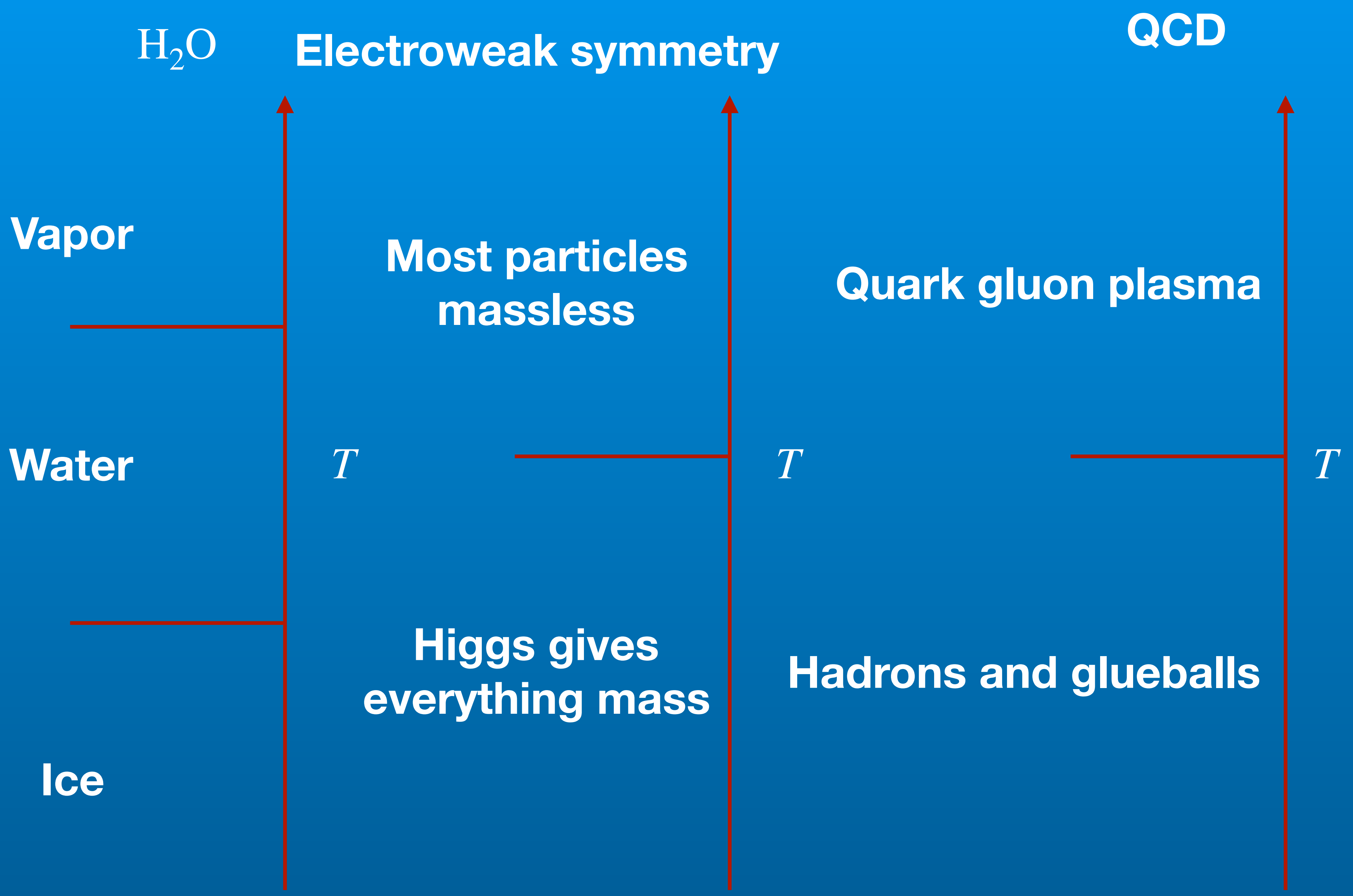
Cosmic defects

Boiling Universe

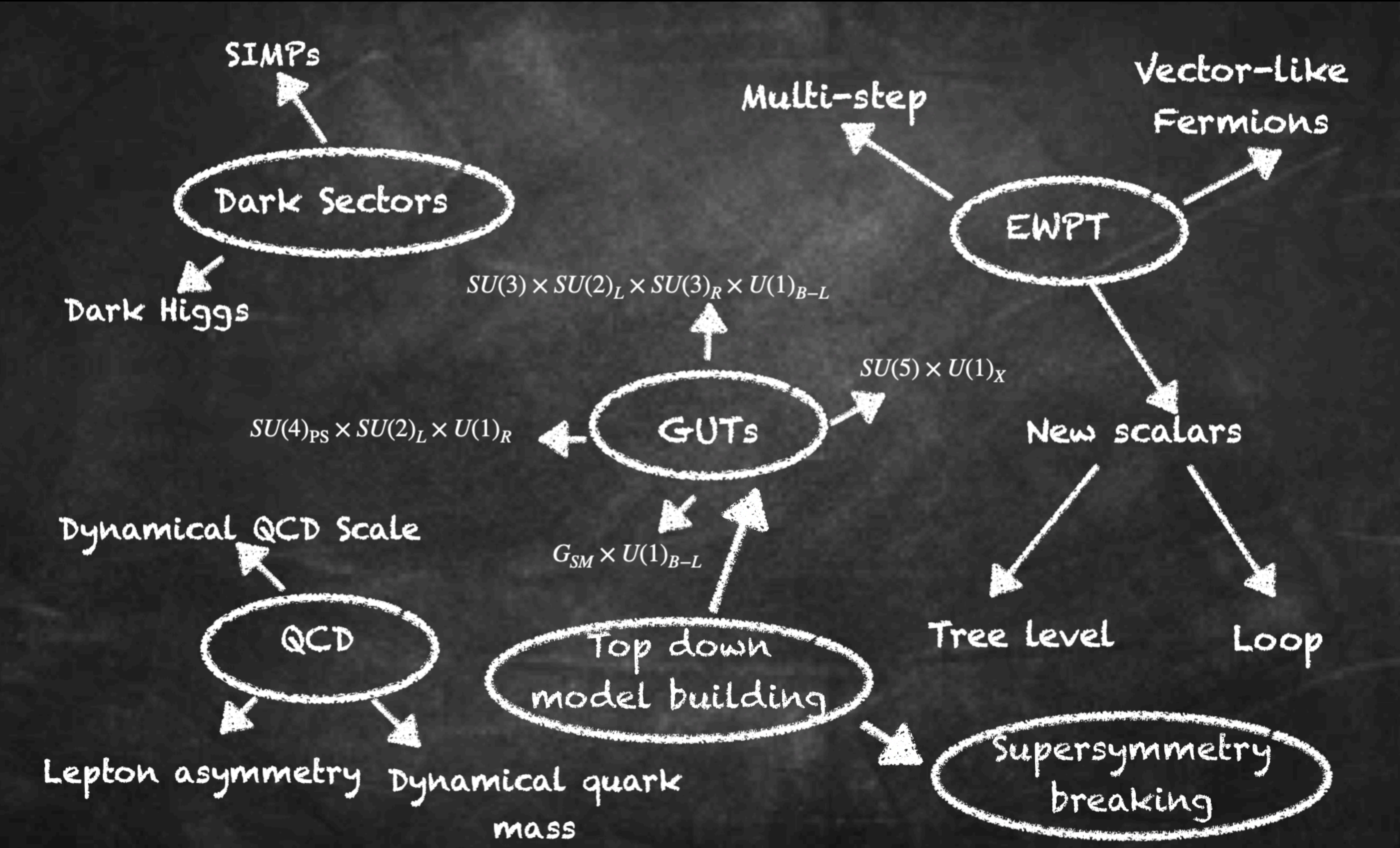
Image credits: icjlab, indi.ca

Part 1 Phase transitions

Phase transitions a change of ground state



Many reasons to hope for a cosmic phase transition



Why a potential will evolve with temperature

1) consider simple theory with one real scalar field

$$\mathscr{L} = \partial_\mu \Phi \partial^\mu \Phi - \frac{m^2}{2}\Phi^2 - \frac{\lambda}{4}\Phi^4$$

We can write the field as a classical field, v, with quantum field

$$\Phi = v + \phi$$

“v” is the average value of the field every where. It can have a different value everywhere v(x) which is why we call it a classical field

In practice though, the energy likes to be extremal. This occurs for

$$\partial_\mu \Phi = 0, \rightarrow \partial v = 0, \quad v = \text{constant}$$

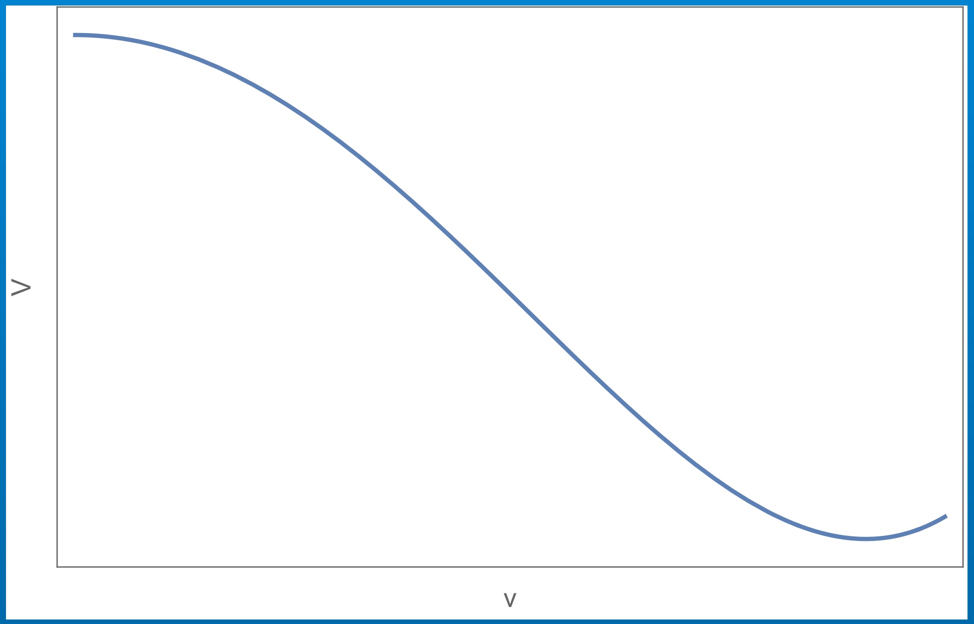
“v” is then a value chosen to minimize the function

$$V = \frac{m^2}{2}v^2 + \frac{\lambda}{4}v^4$$

Why a potential will evolve with temperature

$$V = \frac{m^2}{2} v^2 + \frac{\lambda}{4} v^4$$

This is called an "effective potential"
- if $m^2 < 0$ the minimum is for $\phi \neq 0$



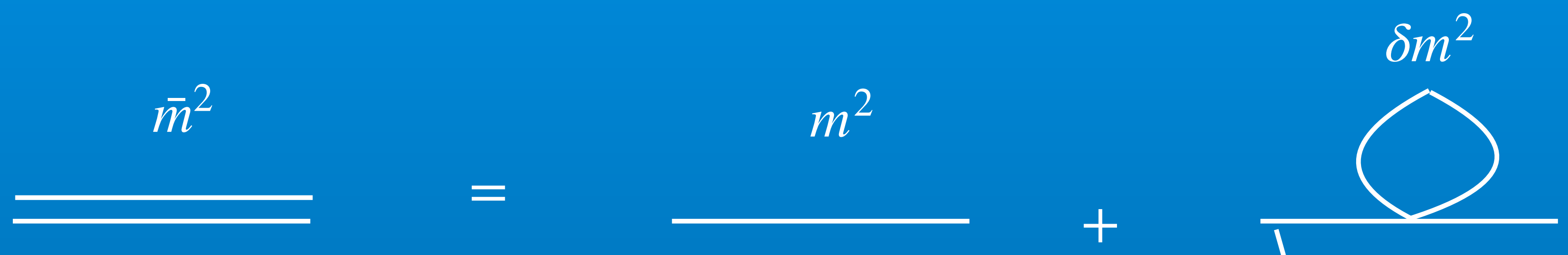
$$\frac{\partial V}{\partial v} = 0 \rightarrow v = \sqrt{\frac{-m^2}{\lambda}}$$

You might be familiar with the Higgs potential having $m^2 < 0$
- this is why the Higgs field is non zero everywhere
- the existence of a non-zero Higgs field is why lots of particles get a mass!

Why a potential will evolve with temperature

$$V = \frac{m^2}{2}v^2 + \frac{\lambda}{4}v^4$$

This is just the "tree level" potential as it only includes the "tree level" interactions. But we have quantum fields too!



Replace external legs with classical fields and this becomes a correction to the quadratic term

Also a change to the 4 point function

$\delta\lambda =$

Why a potential will evolve with temperature

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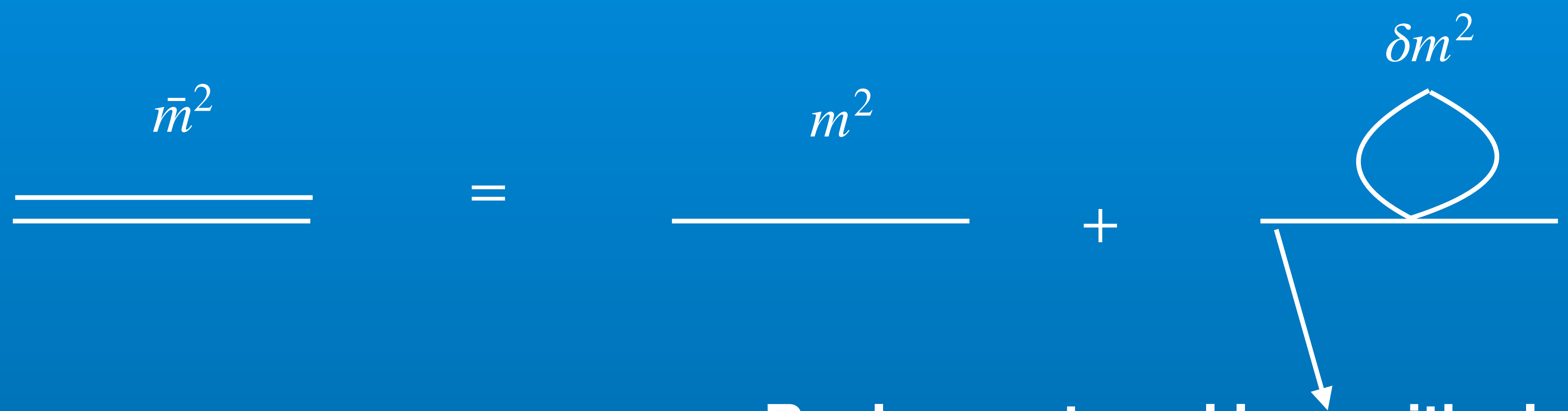
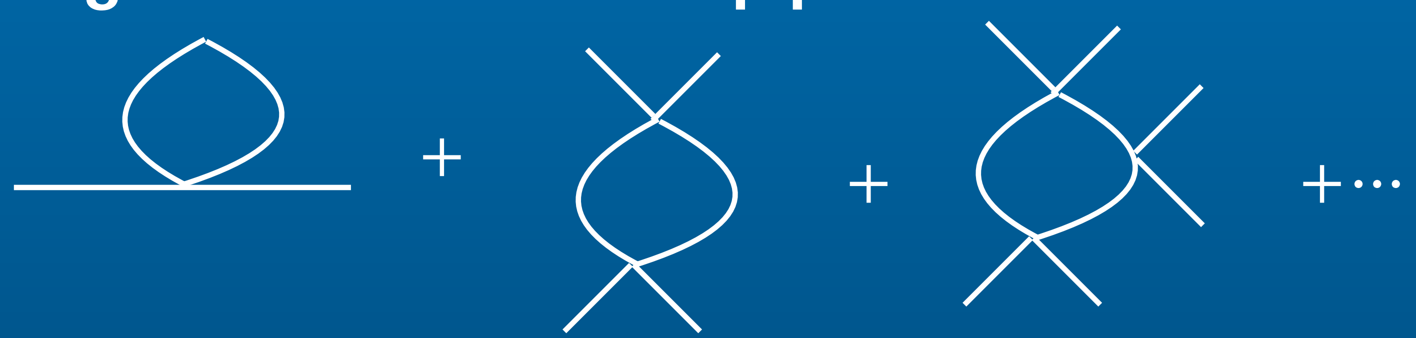


Replace external legs with classical fields and this becomes a correction to the quadratic term

Can keep going to have all the 1 loop pieces



Why a potential will evolve with temperature

$$V = \frac{m^2}{2}v^2 + \frac{\lambda}{4}v^4$$

Can keep going to have all the 1 loop pieces

This is an infinite sum! But it is straight forward to work out the answer if you have done some quantum field theory

$$V = \frac{m^2}{2}v^2 + \frac{\lambda}{4}v^4 + \frac{(m^2 + 3\lambda v^2)^2}{64\pi^2}\left[\log\frac{(m^2 + 3\lambda v^2)}{\mu^2} - \frac{3}{2}\right]$$

This μ^2 is the scale our theory of the effective potential is accurate at. It looks weird as it is our choice what μ^2 is! This dependence on this arbitrary scale is because we have calculated things at 1 loop rather than infinite!

Why a potential will evolve with temperature

$$V = \frac{m^2}{2}v^2 + \frac{\lambda}{4}v^4$$

More general 1 loop correction

$$V_{1-\text{loop}} = \sum_{i \in \text{bosons}} n_i \frac{M_i^4}{64\pi^2}\left[\log\frac{M^2}{\mu^2} - c_i\right] - \sum_{i \in \text{fermions}} n_i \frac{M_i^4}{64\pi^2}\left[\log\frac{M^2}{\mu^2} - \frac{3}{2}\right]$$

Why a potential will evolve with temperature

$$V = \frac{m^2}{2}v^2 + \frac{\lambda}{4}v^4$$

What about at finite temperature?

Finite temperature field theory is an advanced theory but I will sketch out some heuristics

When doing QFT diagrams we need to go right back to the beginning
A free scalar can be written in terms of momentum modes

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}(2\omega_p)^{1/2}}[a(p)e^{-ipx} + a^\dagger(p)e^{ipx}]$$

At finite temperature, in equilibrium

$$\langle a^\dagger(p)a(k)\rangle = n_B\delta^3(p-k), \quad n_B = \frac{1}{e^{\beta\omega} - 1}$$

So the mode occupation n_B goes inside your propagators and therefore your loops
All finite temperature effects are loop corrections, aka quantum corrections!

Why a potential will evolve with temperature

$$V = \frac{m^2}{2}v^2 + \frac{\lambda}{4}v^4$$

$$\langle a^\dagger(p)a(k)\rangle = n_B\delta^3(p-k), \quad n_B = \frac{1}{e^{\beta\omega}-1}$$

1 loop greens function at finite temperature is now temperature dependent

$$G(x-y) = \int \frac{d^4p}{(2\pi)^3}[\Theta(p^0)-\Theta(-p^0)]\delta(p^2-m^2)\left[\Theta(x^0-y^0)+n_B(p^0)\right]e^{-ip(x-y)}$$

So now our loop correction is

$$V_{1-\text{loop}} = \sum_{i\in\text{bosons}} n_i\frac{M_i^4}{64\pi^2}\left[\log\frac{M_i^2}{\mu^2}-c_i\right] + n_i\frac{T^4}{2\pi^2}J_B(M_i^2/T^2) - \sum_{i\in\text{fermions}} n_i\frac{M_i^4}{64\pi^2}\left[\log\frac{M_i^2}{\mu^2}-\frac{3}{2}\right] + n_i\frac{T^4}{2\pi^2}J_F(M_i^2/T^2)$$

Finite temperature loop function

Why a potential will evolve with temperature

$$V = \frac{m^2}{2}v^2 + \frac{\lambda}{4}v^4$$

$$V_{1-\mathrm{loop}} = \sum_{i \in \mathrm{bosons}} n_i \frac{M_i^4}{64\pi^2}\left[\log\frac{M_i^2}{\mu^2} - c_i\right] + n_i \frac{T^4}{2\pi^2} J_B(M_i^2/T^2) - \sum_{i \in \mathrm{fermions}} n_i \frac{M_i^4}{64\pi^2}\left[\log\frac{M_i^2}{\mu^2} - \frac{3}{2}\right] + n_i \frac{T^4}{2\pi^2} J_F(M_i^2/T^2)$$

So we need to know what this JB and JF looks like!

$$\frac{T^4}{2\pi^2}J_B(M^2/T^2) \sim \frac{M^2T^2}{24} - \frac{M^3T}{12\pi}, \quad \frac{T^4}{2\pi^2}J_F(M^2/T^2) \sim -\frac{M^2T^2}{48}$$

For simplicity imagine all our masses have some form $M \sim c\phi$

The largest pieces of the effective potential look like

$$V = DT^2v^2 - ETv^3 + \frac{\lambda}{4}v^4$$

Why a potential will evolve with temperature

$$V = DT^2v^2 - ETv^3 + \frac{\lambda}{4}v^4$$

Three powers means up to three turning points!

$$\frac{dV}{dv} = 0 \to v = \left(0, \frac{3ET \pm T\sqrt{9E - 8\lambda D}}{2\lambda}\right)$$

If E is large we have this

Why a potential will evolve with temperature

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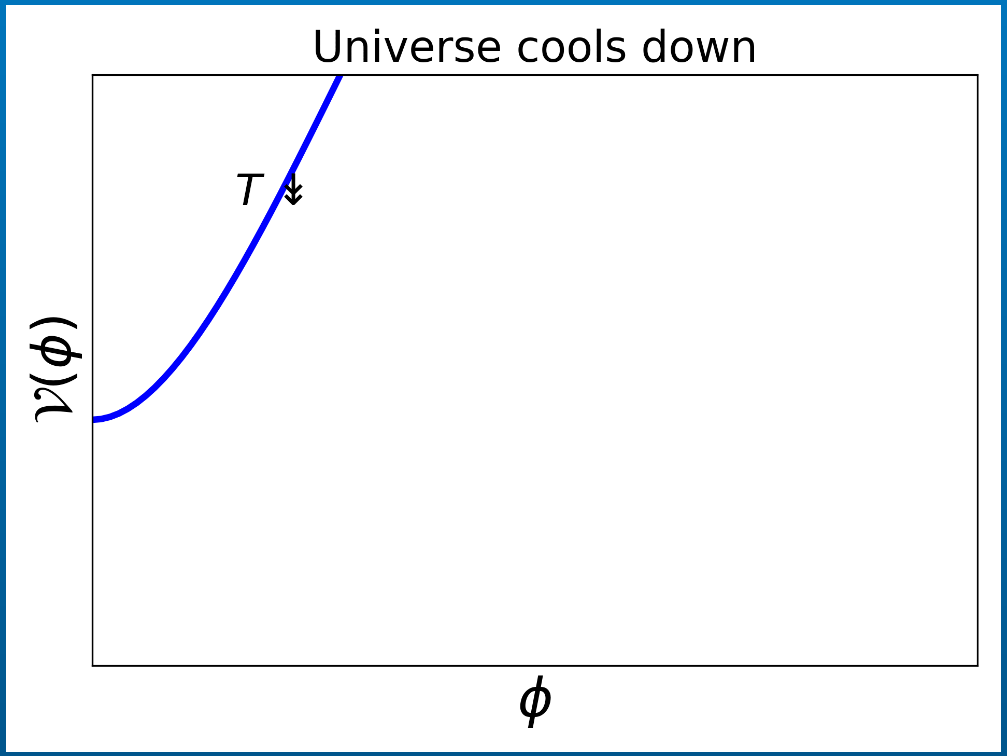

Why a potential will evolve with temperature

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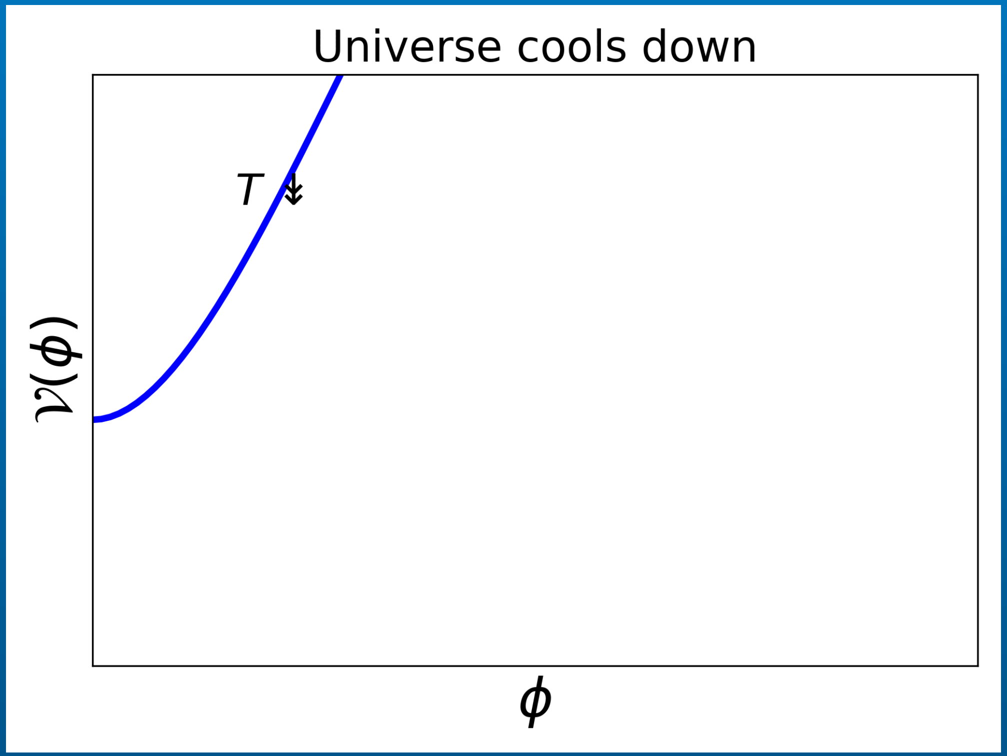


How to get bubbles - barrier between ground states

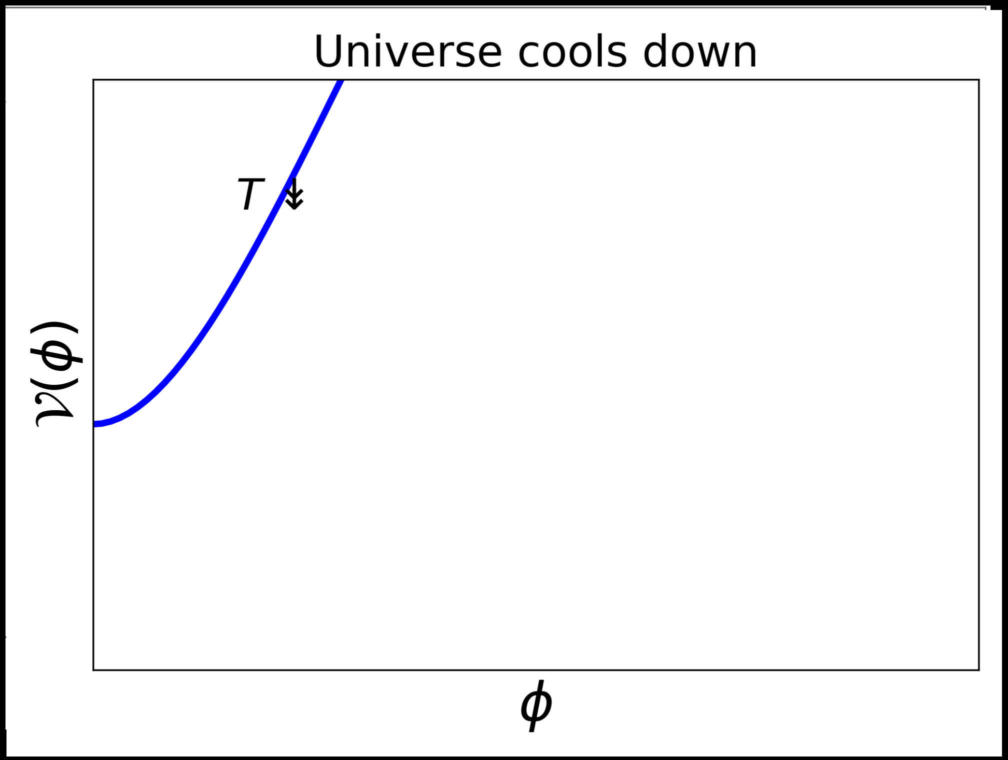


Image credit: Tim Dean

3 sources of energy - compare to a kettle

- bubble collision
- sound waves
- Turbulence

3 sources of energy - compare to a kettle

- bubble collision
- sound waves
- Turbulence

Gravitational wave source depends on 4 thermal parameters

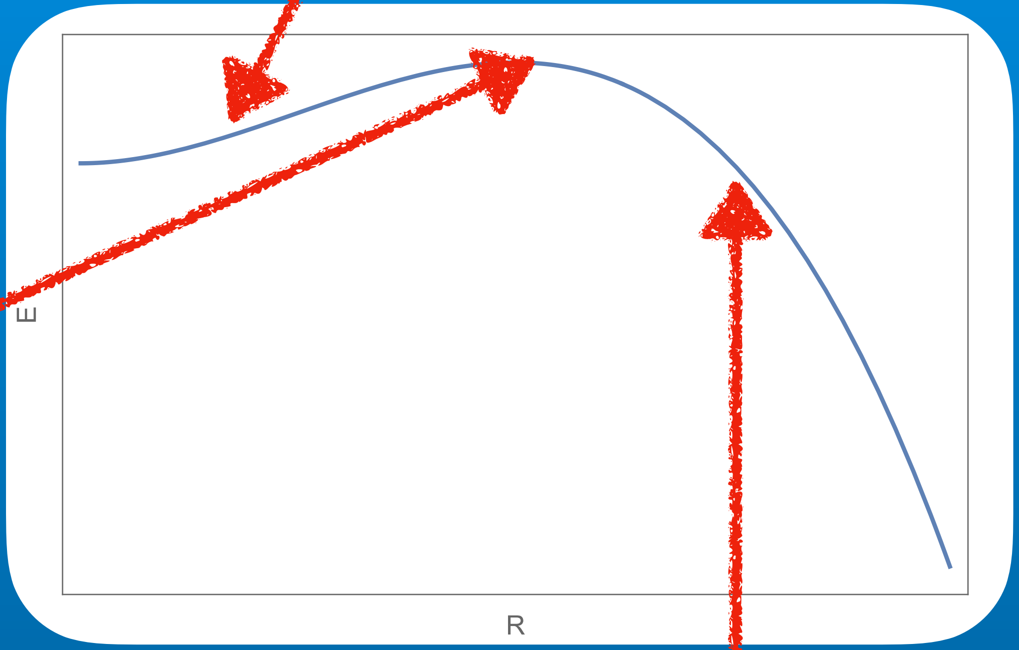
1) The velocity of the bubble walls
2) How much energy is dumped into the plasma
3) How far away bubbles are
4) The temperature this all occurs

How to understand the properties of a phase transition

Warm up: Classical nucleation theory

$$E = -\Delta p \left(\frac{4\pi}{3}R^3\right) + \left(4\pi R^2\right)\sigma$$

Bubble loses energy by shrinking



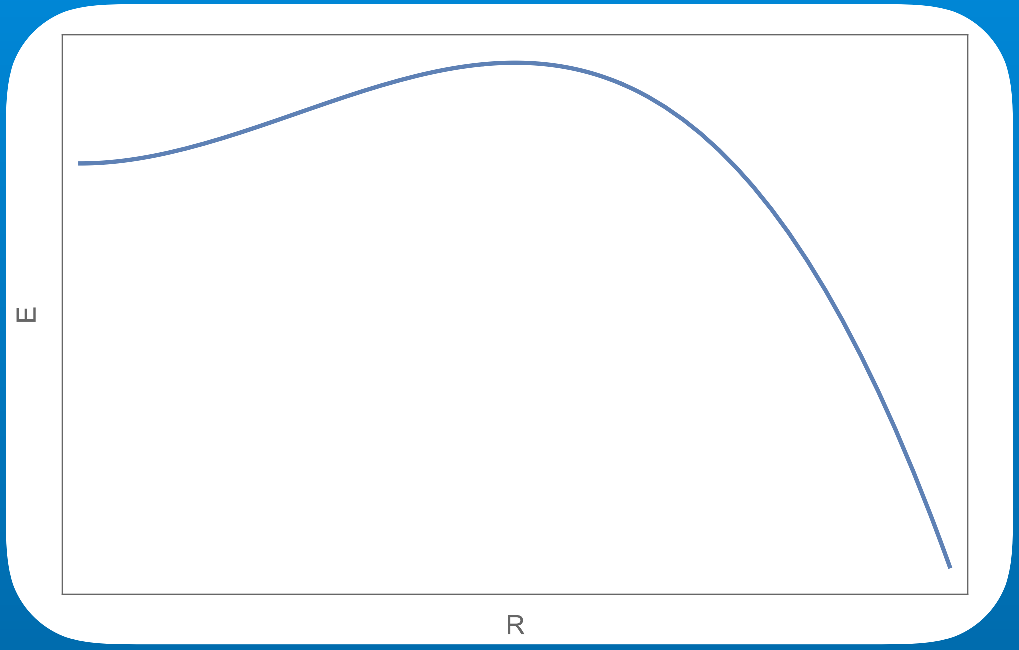
Smallest possible bubble that doesn't shrink

Bubble loses energy by expanding

How to understand the properties of a phase transition

Classical nucleation theory

$$E = -\Delta p \left(\frac{4\pi}{3} R^3\right) + \left(4\pi R^2\right)\sigma$$



$$\left.\frac{dE}{dR} = 0\right|_{R_c} \rightarrow R_c = \frac{\sigma}{2\Delta p}$$

$$E(R_c) = \frac{16\pi\sigma^3}{3\Delta p^2}$$

Nucleation rate $\Gamma \sim e^{-\frac{E(R_c)}{T}}$

How to understand the properties of a phase transition

Nucleation in an expanding background

Nucleation rate $\Gamma \sim e^{-\frac{E(R_c)}{T}}$ $E(R_c) = \frac{16\pi\sigma^3}{3\Delta p^2}$

If we have the phase transition in a *stationary* box we only need

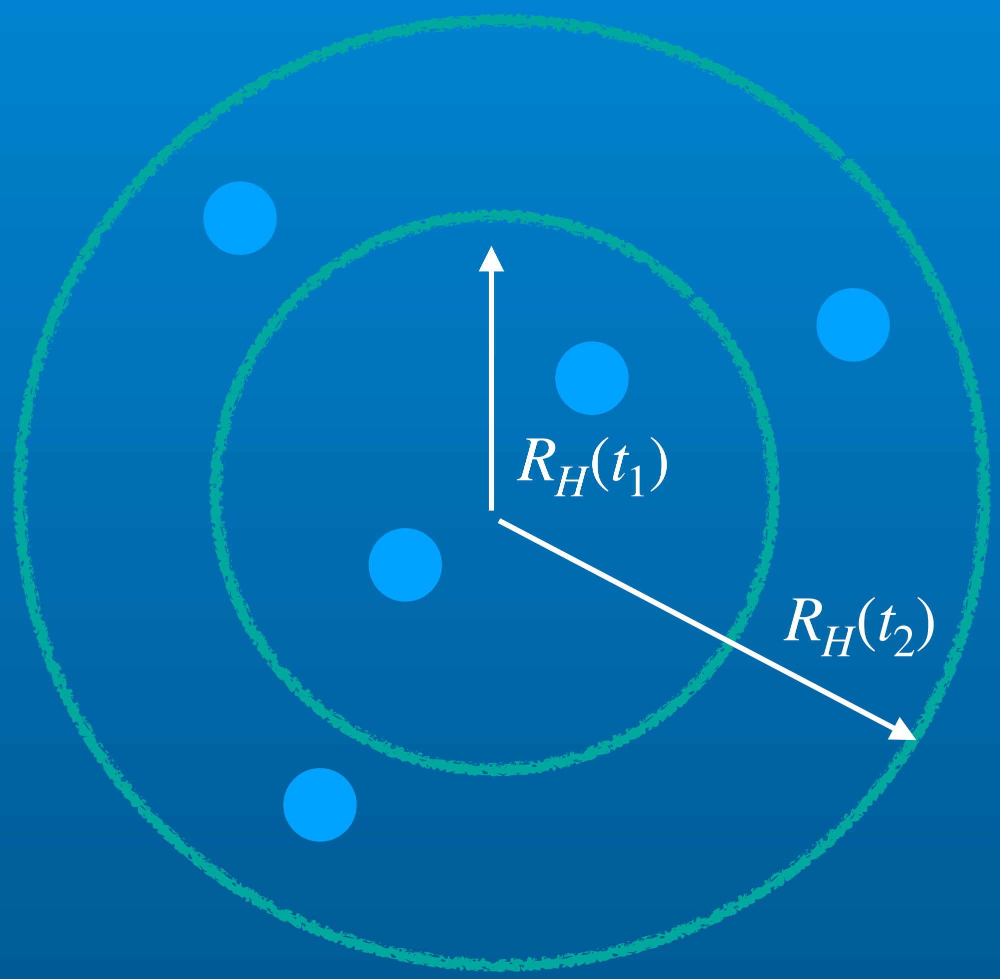
$\Gamma > \epsilon, \; \epsilon \to 0$ **For the phase transition to complete**

So we just need $\Delta p > 0$. The point where $\Delta p = 0$ is known as the *critical temperature*

How to understand the properties of a phase transition

Nucleation in an expanding background

But what about an expanding background?



The nucleation rate must be fast enough for the bubbles to grow and expand so that the volume fraction in the new phase *catches up* with the expanding volume

How to understand the properties of a phase transition

Nucleation in an expanding background

Key times:

$\Gamma = \epsilon \to T = T_C$ **Critical temperature when** $\Delta p = 0$

$\Gamma \sim e^{-140} \to T = T_n$ **Nucleation temperature when you have 1 bubble per Hubble volume**

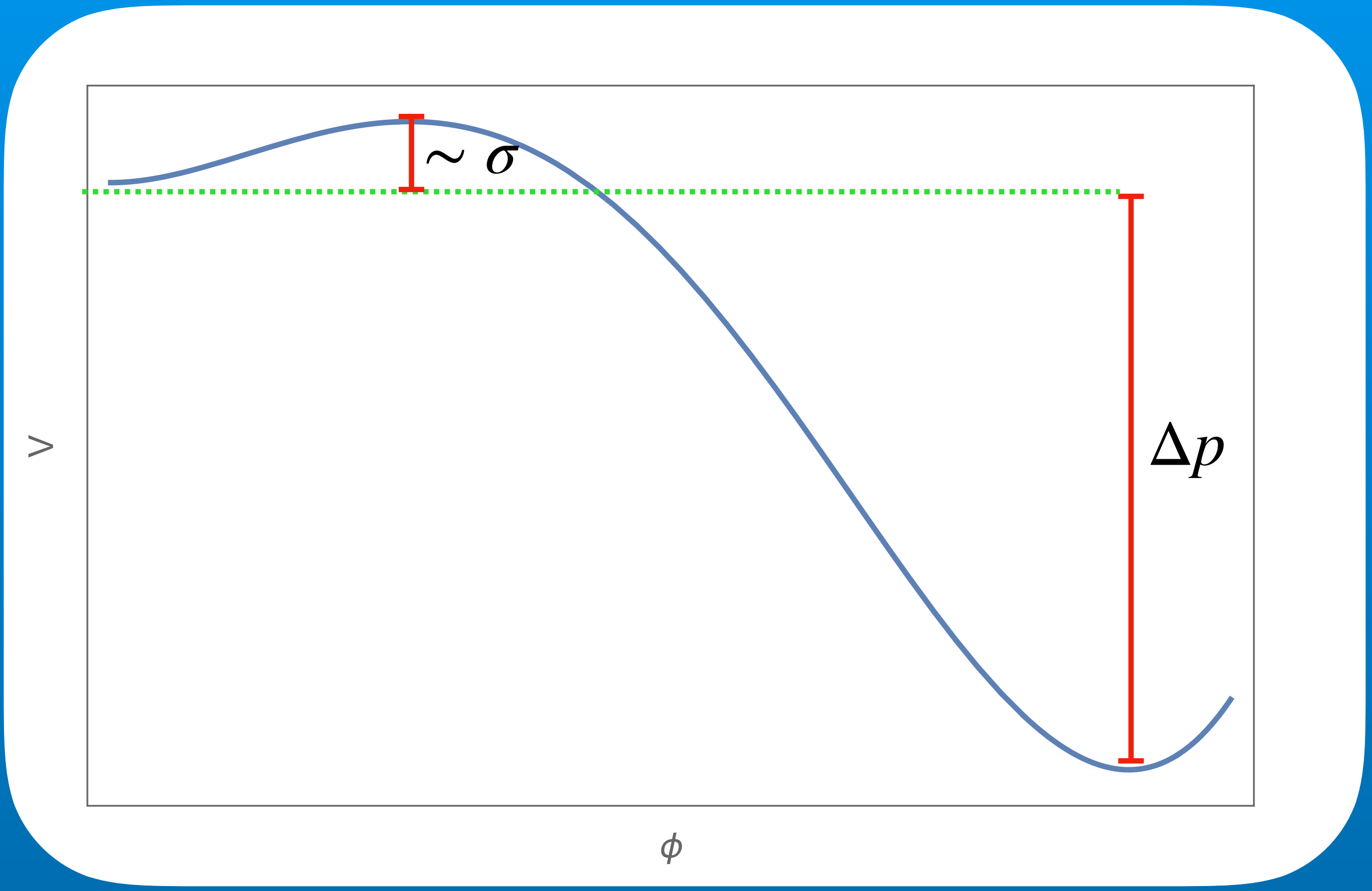
$T_p < T_n$ **Percolation temperature when bubbles are colliding**

$$\Gamma = e^{-S(t)} \sim e^{-\left[S_0 + \beta(t - t_n) + \frac{1}{2}\gamma(t - t_n)^2\right]}$$

In an expanding background it is *not* S_0 **but** $\beta = \frac{dS}{dt} = T\frac{d(E_c/T)}{dT}$

How to understand the properties of a phase transition

Translating to QFT



$$\sigma = \int_0^v d\phi \sqrt{2V},\ \Delta V = \Delta p$$

How to understand the properties of a phase transition

More precisely in the path integral formulation

$$\Gamma \sim \int [d\phi] e^{-S(\phi)} \sim e^{-S}$$

Where S is the action

$$S = \int d^4x \left(\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right)$$

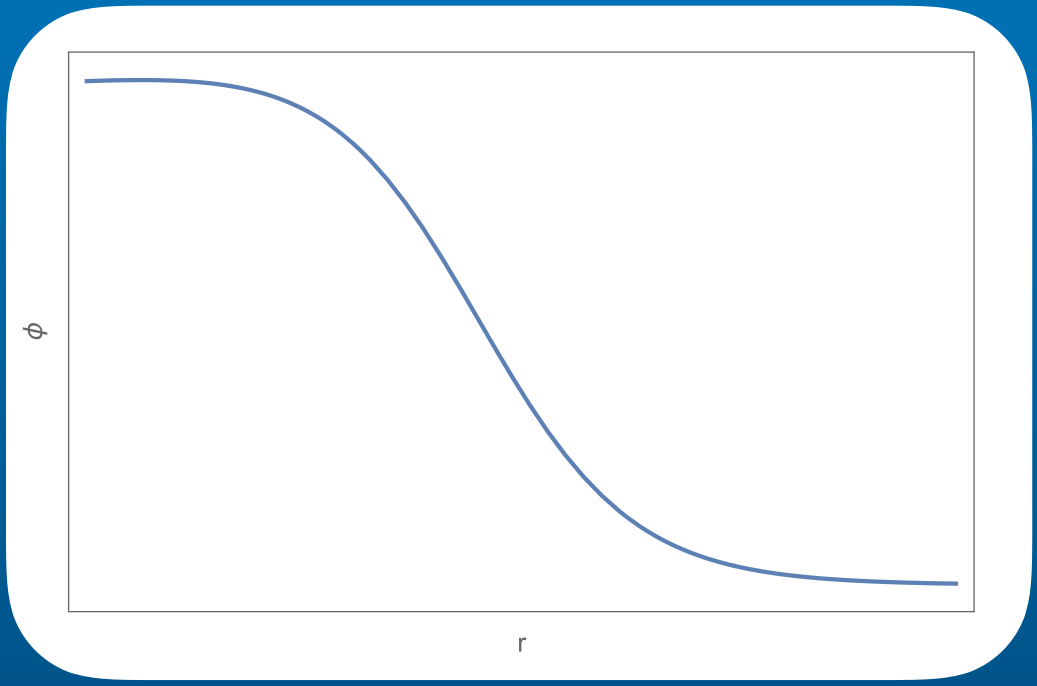
Δp

Equations of motion for a stationary (d/dt=0), spherically symmetric solution

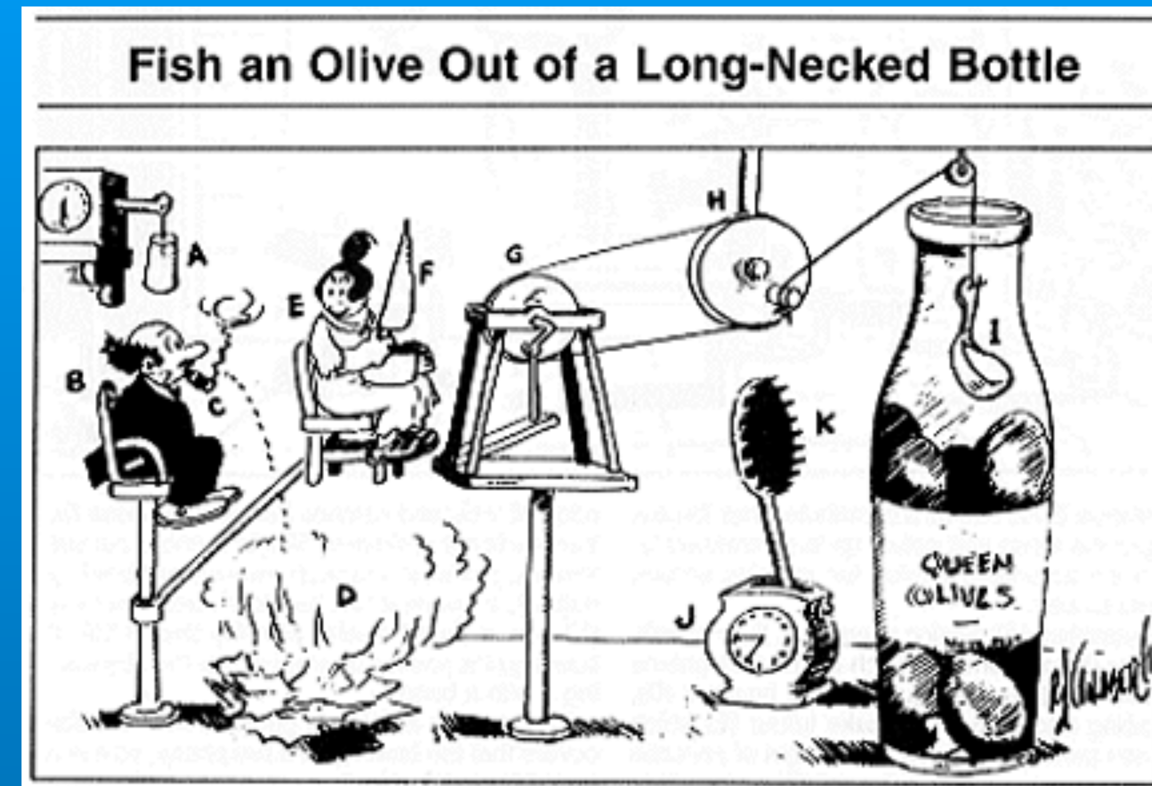
$$\frac{d^2\phi}{dr^2} + \frac{3}{r}\frac{d\phi}{dr} = \frac{\partial V}{d\phi}$$

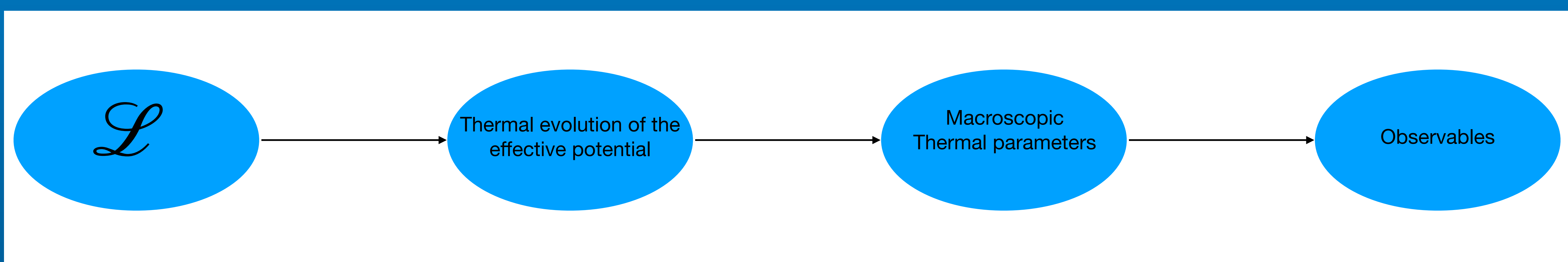
A bubble has boundary conditions

$$\phi(0) \sim v, \phi(\infty) = 0, \frac{d\phi}{dr}|_{r=0} = 0$$

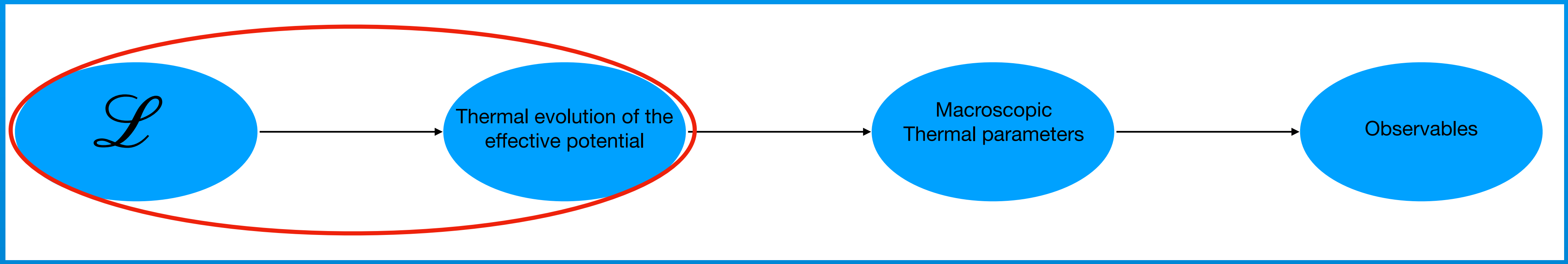


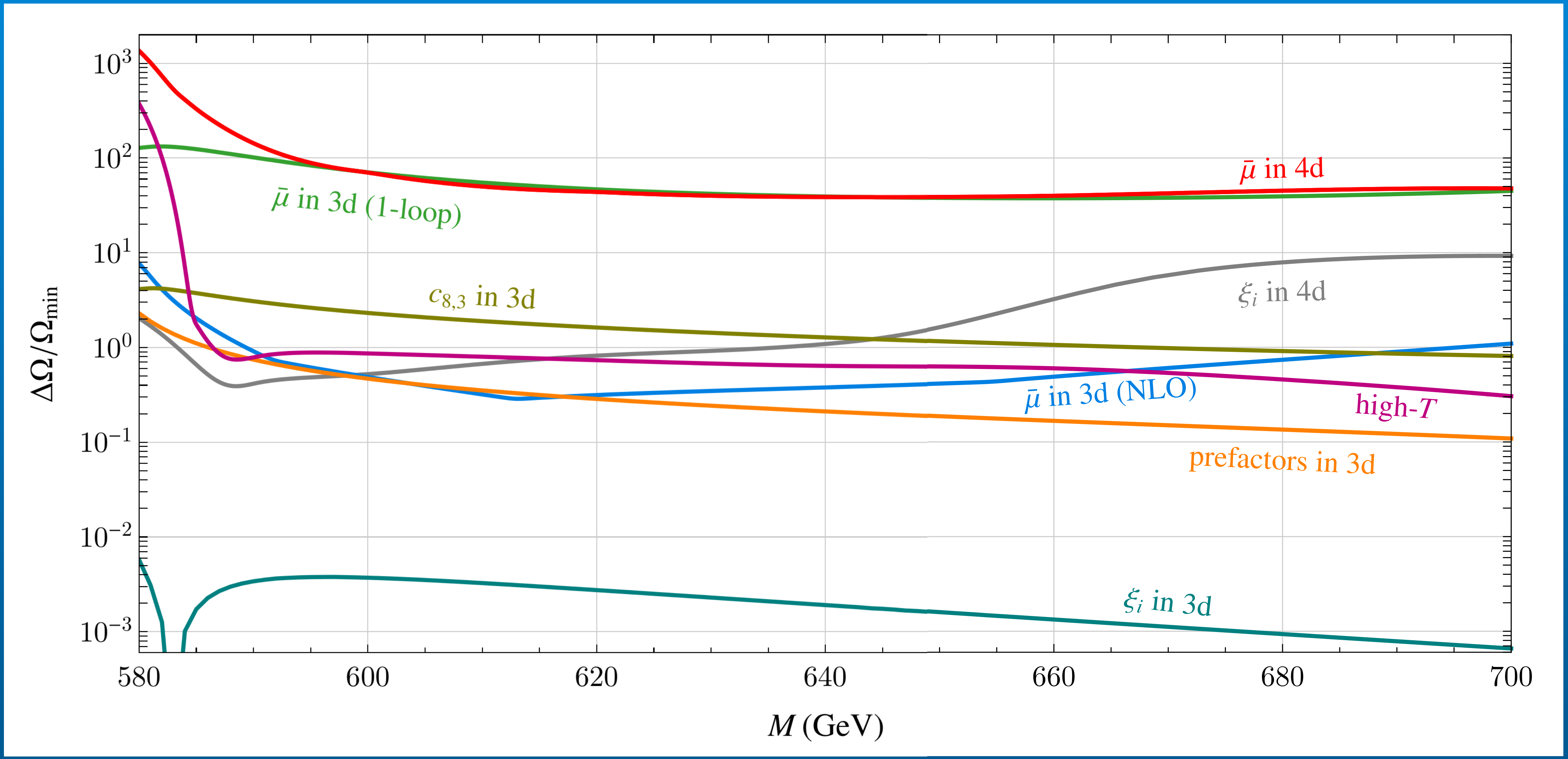
From a model to a GW spectrum, a Rube Goldberg machine





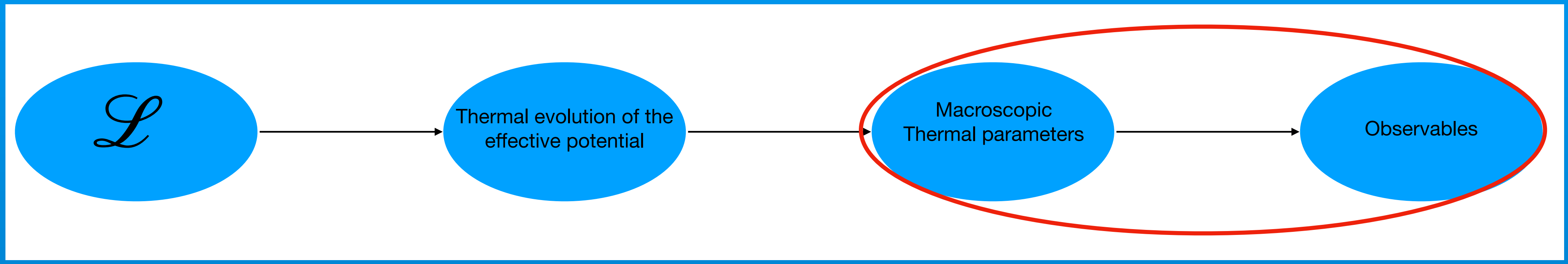
Theoretical uncertainties from thermal field theory

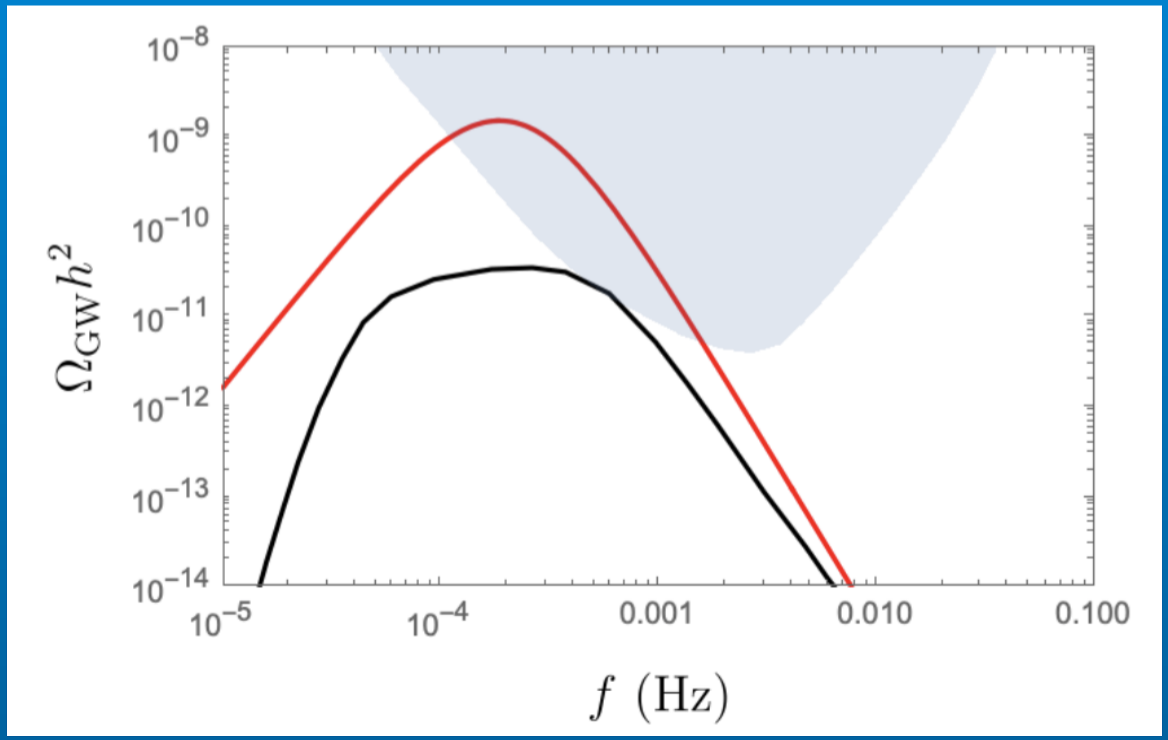




Croon, Gould, Schicho, Tenkanen and White: 2009.10080

Theoretical uncertainties (modelling)





Picture from my book, data from Gowling, Hindmarsh 2106.05984

Towards precision gravitational wave calculations

NLO impossible	LHC Run 1	LHC Run 2	N^3LO possible
2003	2008	2015	2022

$\frac{\Delta\Omega}{\Omega_{\rm min}}(\text{theory}) > 10^2$	SKA	LISA	μAres
2023	2027	2034	2050

That’s enough for now….

Next lecture:
-more on theoretical uncertainties in phase transitions

-scalar induced gravitational waves

Break

Gravitational wave archaeology

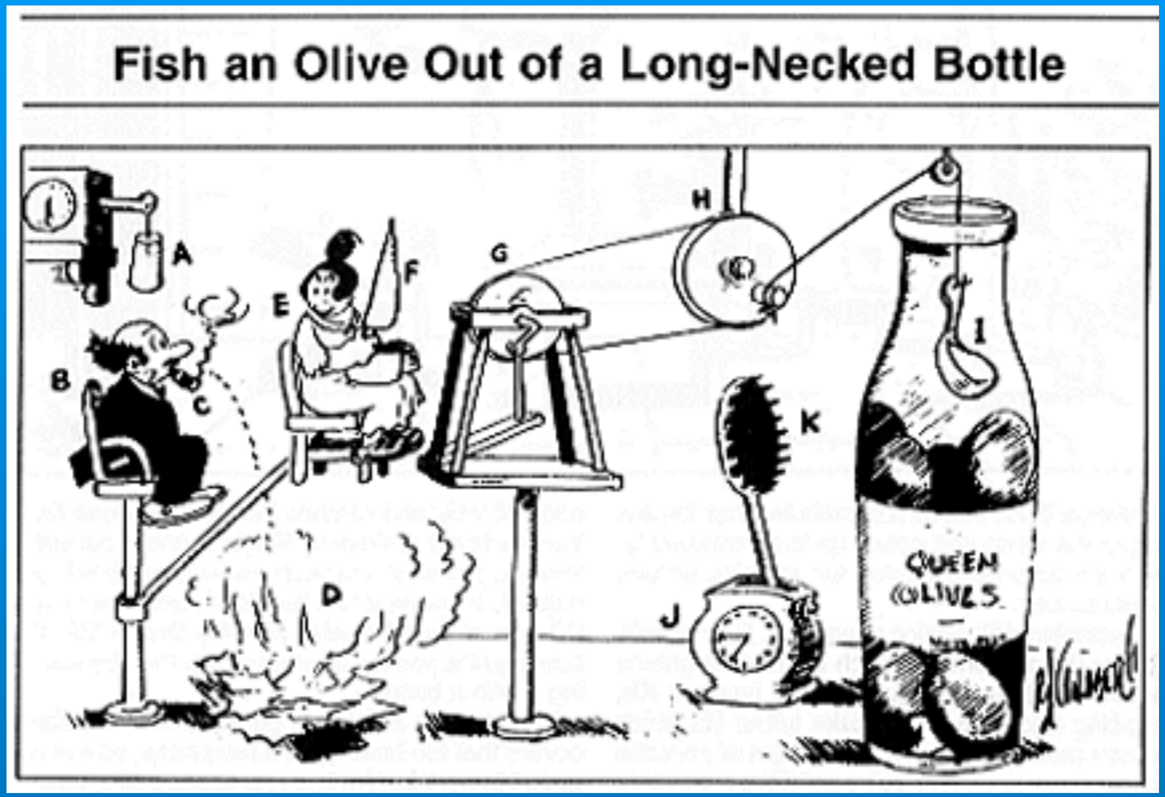
Lecture 2: scalar induced Gws andmore on phase transitions

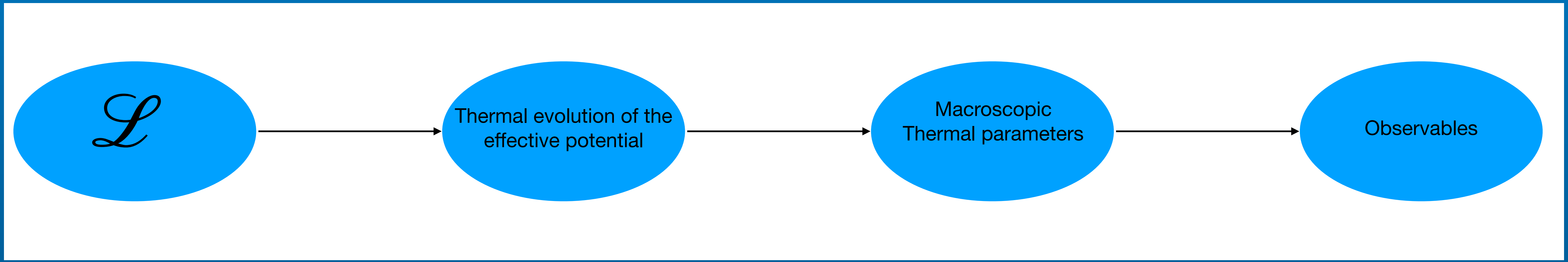
Graham White

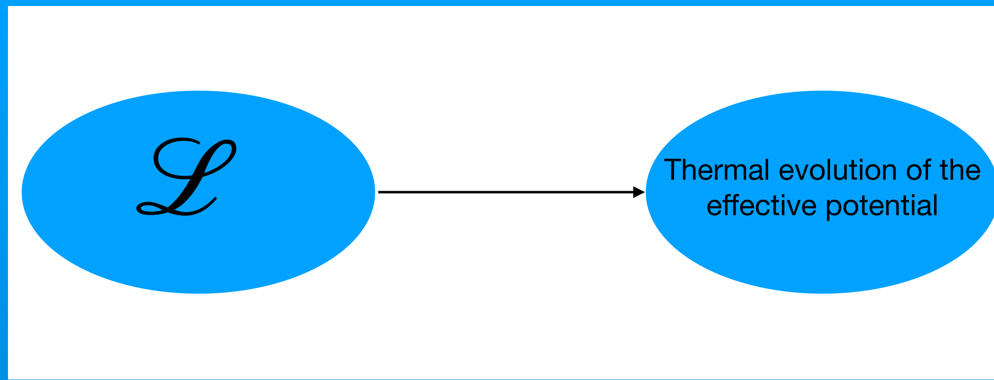
University of Southampton

SHEP Southampton High Energy Physics

From a model to a GW spectrum, a Rube Goldberg machine







Scale dependence as a probe of perturbation theory

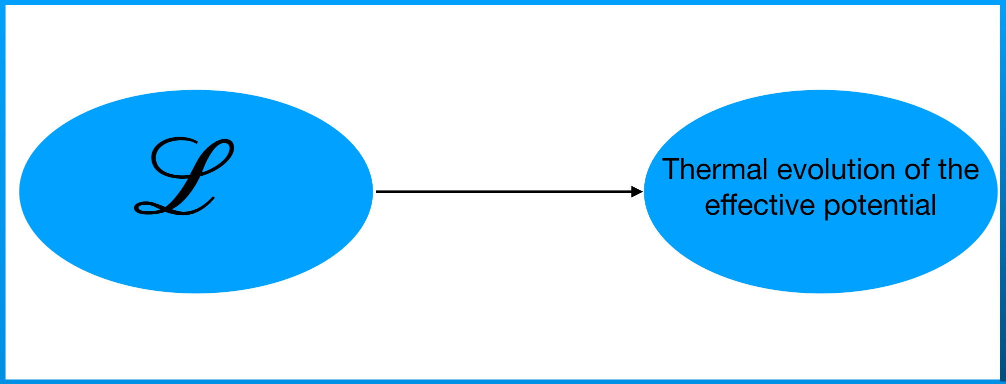
Start with zero temperature. Remember this weird scale dependence?

$$V(\phi, \mu) = V_{\text{tree}}(\phi, \mu) + V_1(\phi, \mu)$$

Implicit, $\lambda \equiv \lambda(\mu)$

Implicit + Explicit

$$\frac{\partial (V_{\text{tree}} + V_1)}{\partial \mu} \sim \mathcal{O}(V_2)$$



Scale dependence as a probe of perturbation theory

Now at finite T

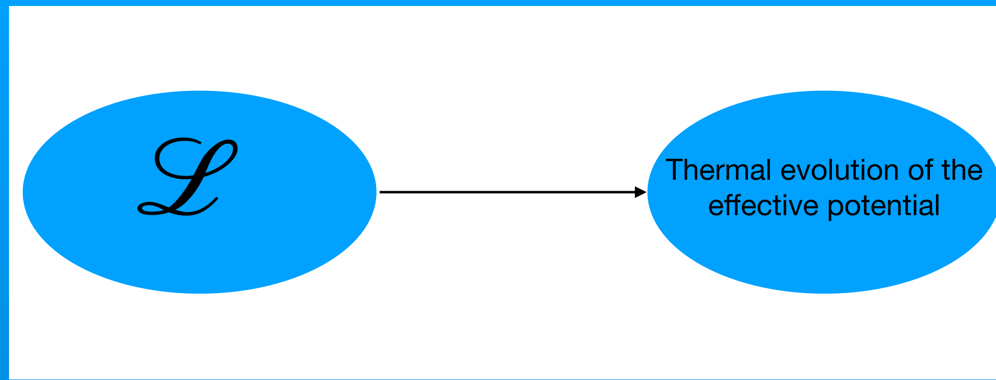
$$V(\phi, \mu) = V_{\text{tree}}(\phi, \mu) + V_1(\phi, T, \mu)$$

$$V_1 = V_{\text{CW}} + V_{1,T}$$

Implicit, $\lambda \equiv \lambda(\mu)$

Implicit + Explicit

$$\frac{\partial(V_{\text{tree}} + V_1)}{\partial\mu} \sim \mathcal{O}(V_2) + \frac{\partial V_{1,T}}{\partial\mu}$$



Scale dependence as a probe of perturbation theory

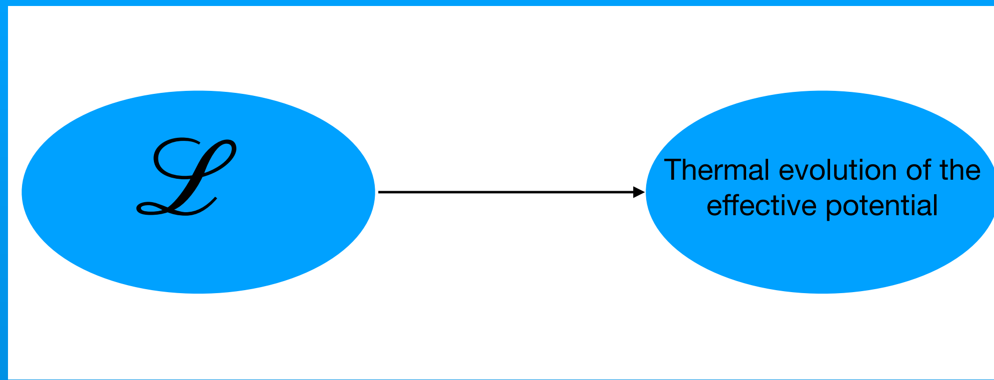
How big is $\frac{\partial V_{1,T}}{\partial \mu}$**?**

$$V_{1,T} \sim \frac{T^2\phi^2}{32}(8\lambda + 4y_t + 3g_2^2 + g_1^2) + \frac{3T\phi^3}{96\pi}\left(2g_2^3 + (g_1^2 + g_2^2)^{3/2}\right)$$

$$\mu\frac{\partial V_{1,T}}{\partial \mu} \supset \frac{T^2\phi^2}{32\pi^2}(-\frac{19}{16}g_2^4 + \frac{3}{8}y_t^3 - \frac{3}{4}g_3^2 y_t)$$

Compare to $$\mu\frac{\partial V_{\text{Tree}}}{\partial \mu} \supset -\frac{3}{32\pi^2}m^2\phi^2 - \frac{3}{32\pi}y_t^4\phi^4$$

Not small!

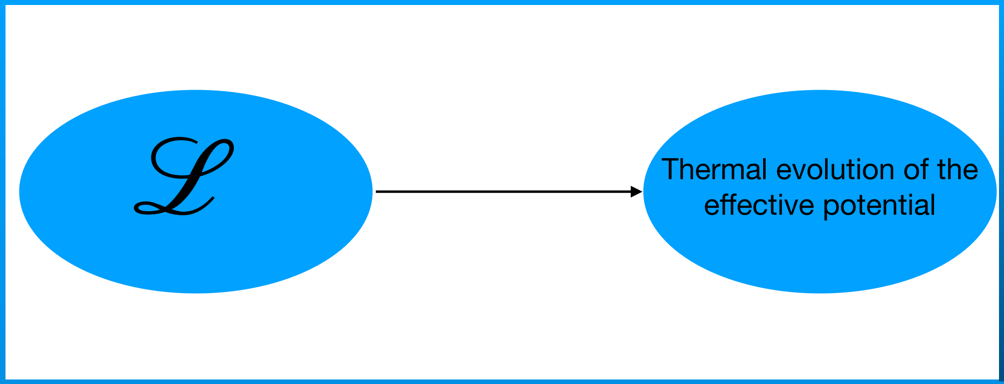


Dimensional reduction

Propagators at FT

$$\langle i\mathscr{M} \rangle \sim \int d^4p f\left(\frac{1}{p^2 - m^2}\right) \rightarrow \beta \sum_n \int d^3p f\left(\frac{1}{\omega_n^2 + \overrightarrow{p}^2 + m^2}\right) \quad \omega_n = 2\pi n T$$

In imaginary time, the theory looks like a 3d theory with a compactified dimension of size $1/T$



Dimensional reduction

Write a 3d effective theory integrating stuff out

$$\Lambda = \pi T \to \text{KK scale}$$

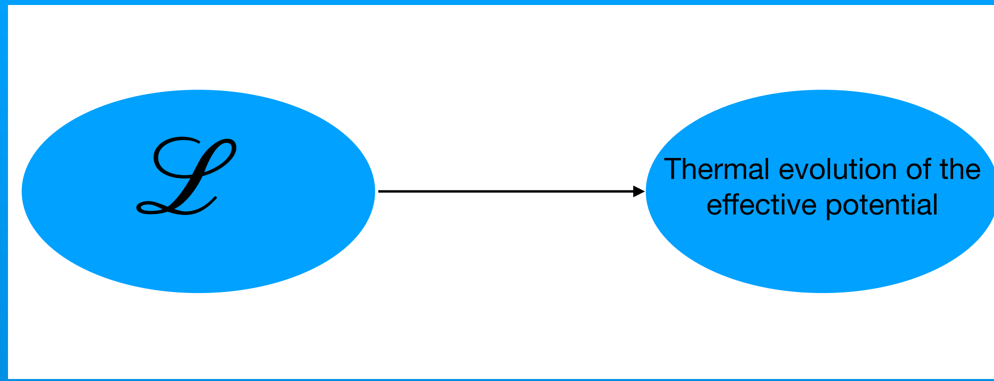
$$\Lambda = gT \to \text{thermal mass of gauge bosons (longtitudinal modes)}$$

$$\Lambda = g^2 T/\pi \to \text{roughly the scale of dynamical field}$$

Schematically

$$(\phi\phi)_{3\text{d}} = \frac{1}{T}(1 + \Pi'_{\phi\phi})(\phi\phi)_{4\text{d}} \quad \phi \in (h, A_0 \cdots)$$

Miracle: doing this automatically orders FT perturbation theory from largest to smallest terms, in contrast with a naive loop expansion



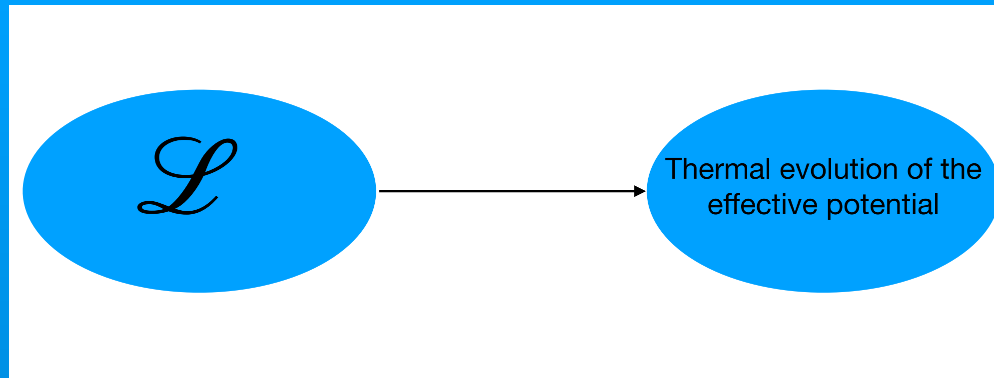
Dimensional reduction

NLO resummation $$\Pi_{\rm NLO} = \Pi_{\rm LO} + \Pi^{2-\rm loop} - \Pi_{\rm LO}\Pi'$$

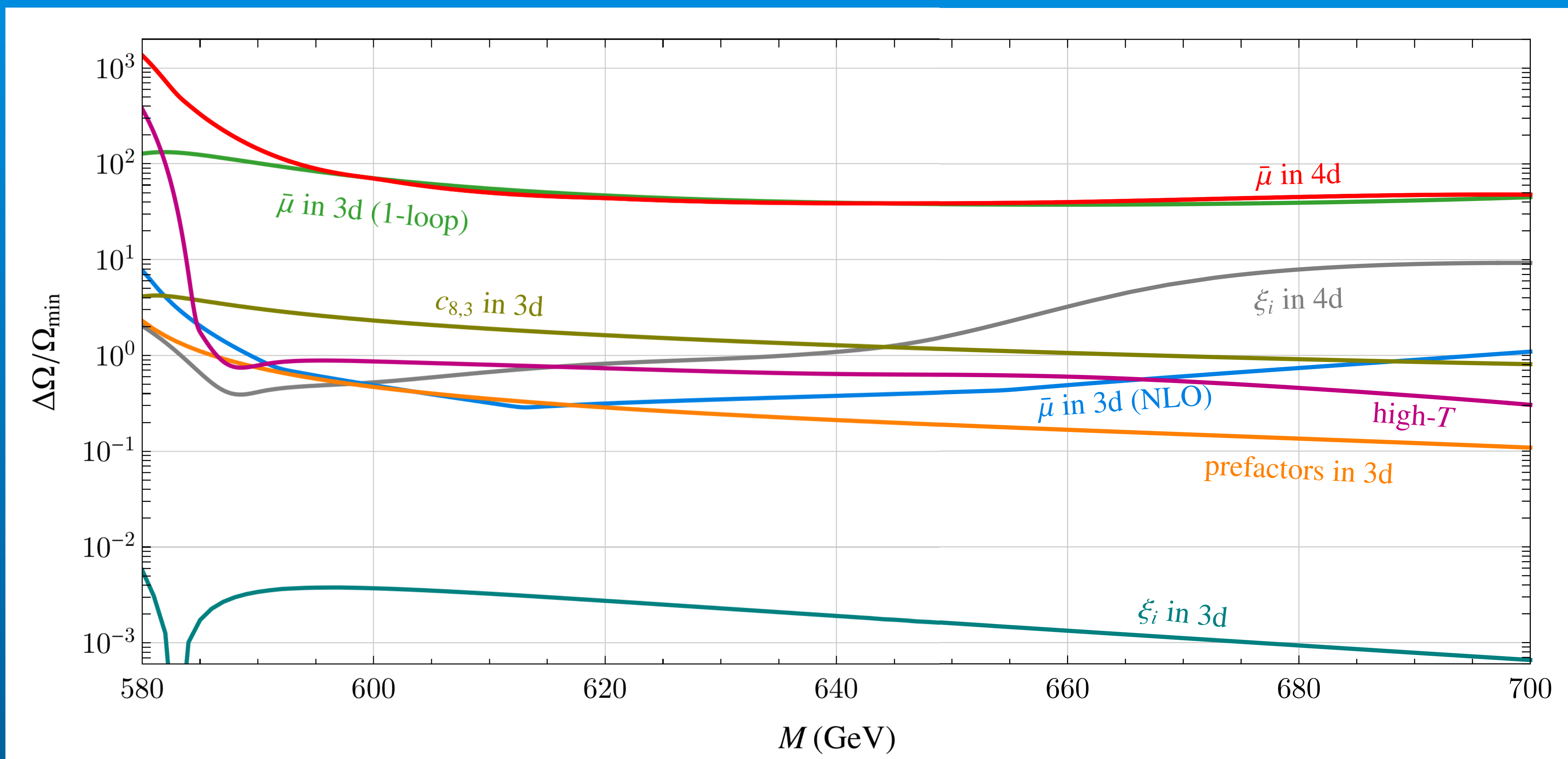
Explicit scale dependence

$$\frac{\partial (V_{\rm tree} + V_1(\phi, T, \Pi_{\rm NLO}))}{\partial \mu} \sim \mathcal{O}(V_2)$$

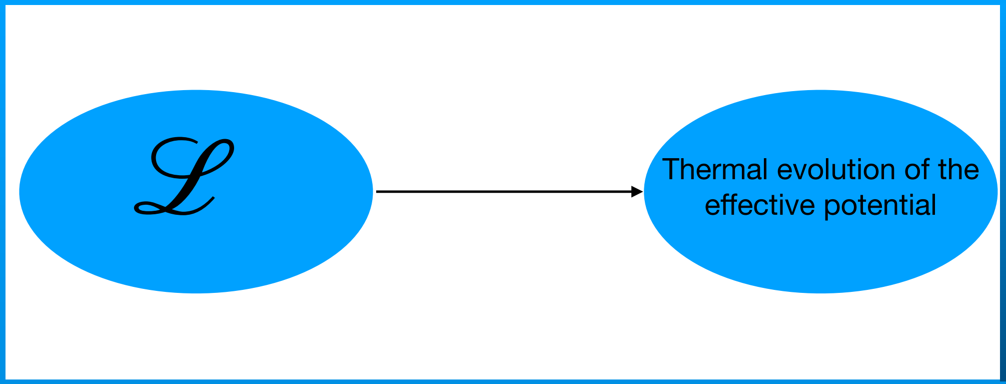
Free open source Mathematica notebook will do NLO dimensional reduction for you!
DRALGO



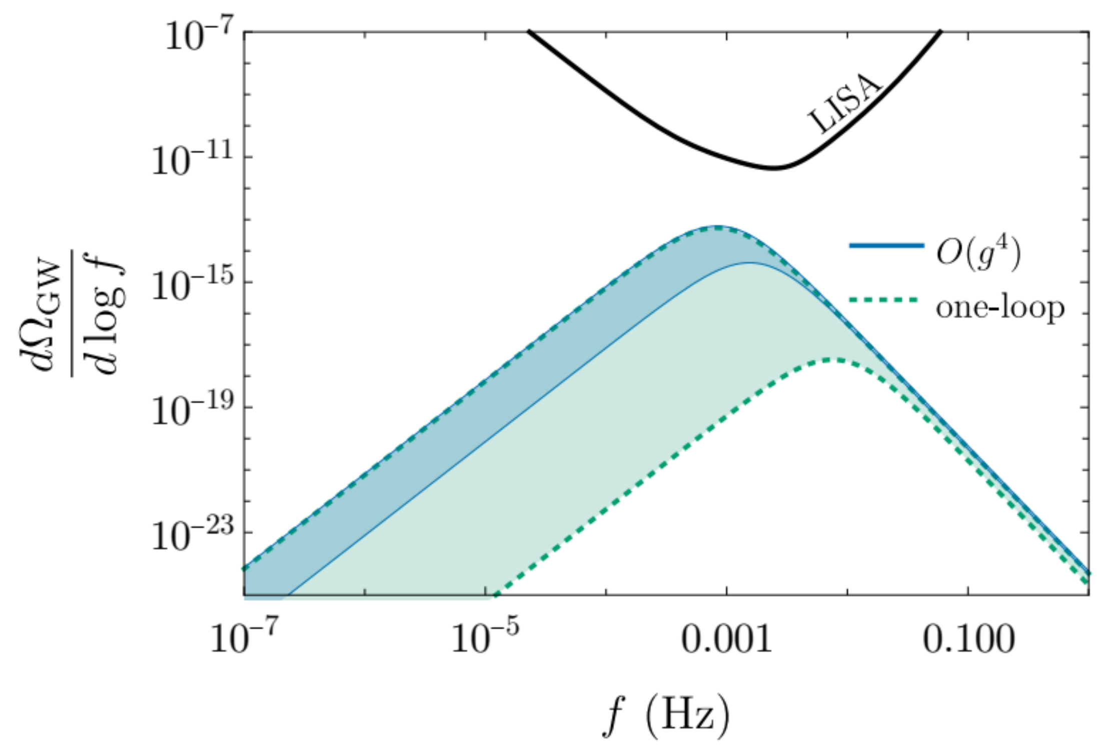
Scale dependence in SMEFT



Croon, Gould, Schicho, Tenkanen and White: 2009.10080

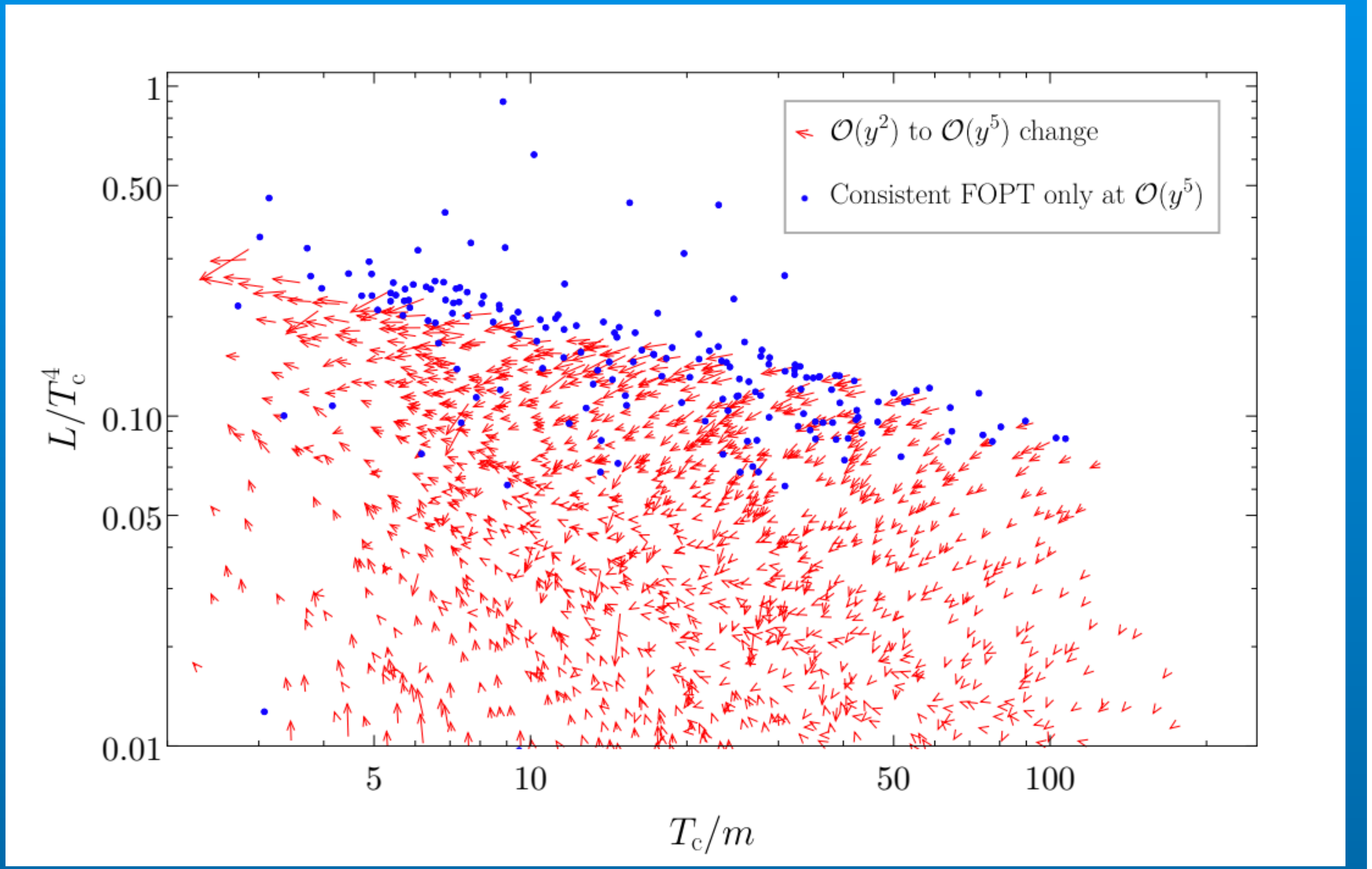


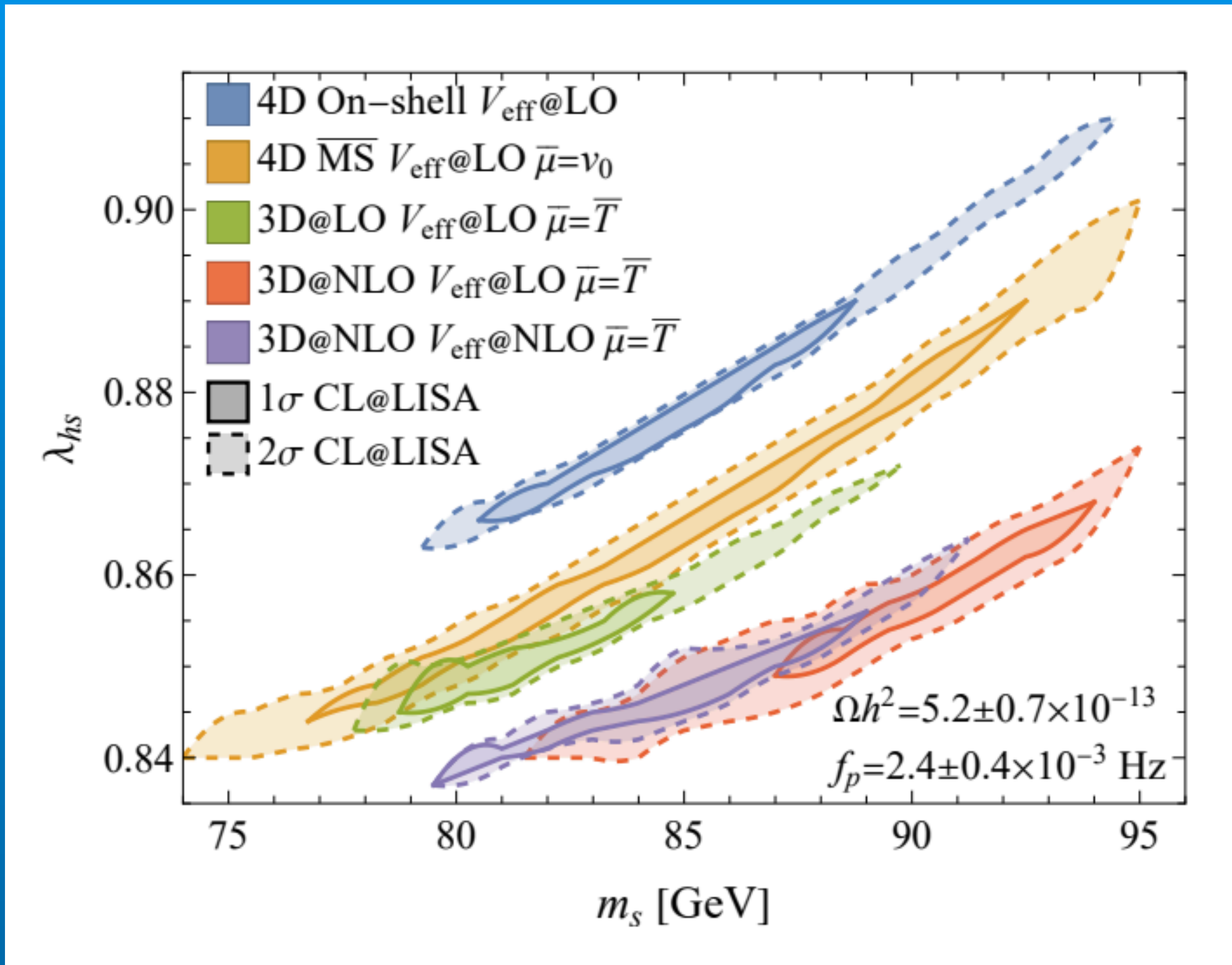
It’s not just SMEFT…..



Gould, Tankanen: 2104.04399

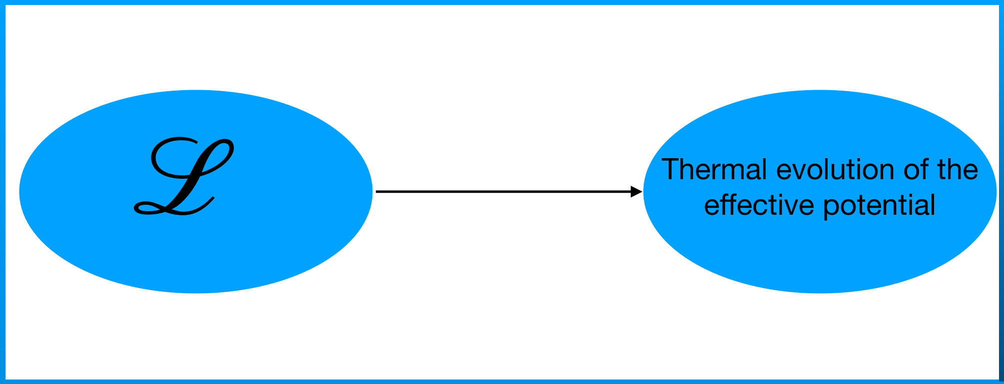
Does the blob move?





Gould Xie 2310.02308

Lewicki, Merchand, Sagunski, Schicho and Schmitt 2403.03769

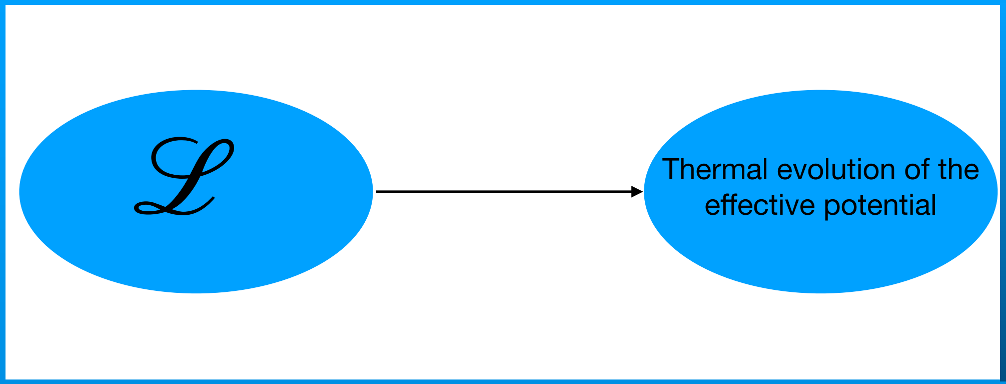


Limits of dimensional reduction: strong transitions

To integrate out KK modes we need

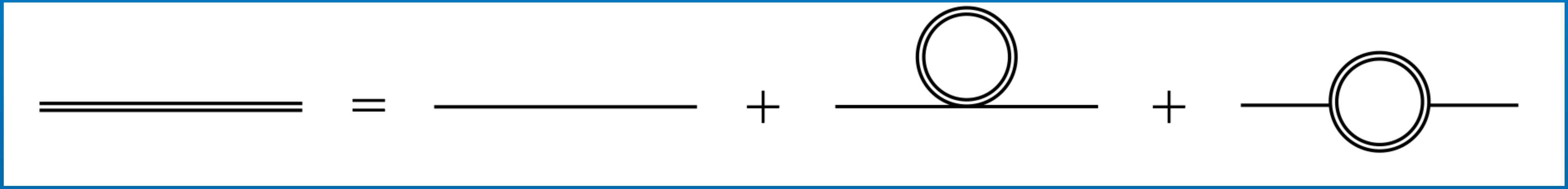
$$m \sim g\phi \ll \pi T$$

But for the strongest transitions, $\phi_C / T_C \gg 1,\ g \rightarrow \text{large}$
So this might break down!

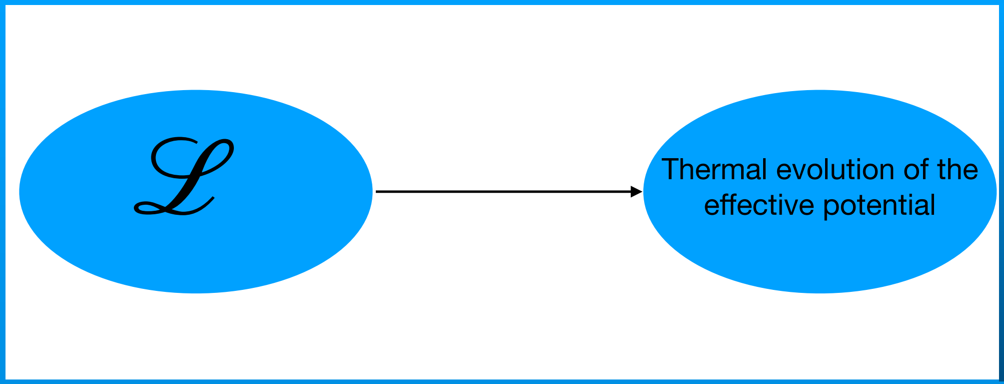


Limits of dimensional reduction: strong transitions

Possible solution are gap equations



Technology still being developed (2211.08218)

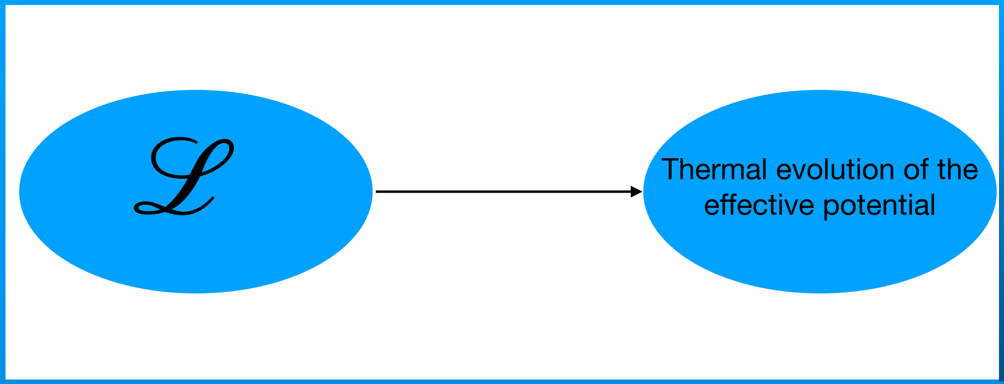


Limits of dimensional reduction: Weak transitions

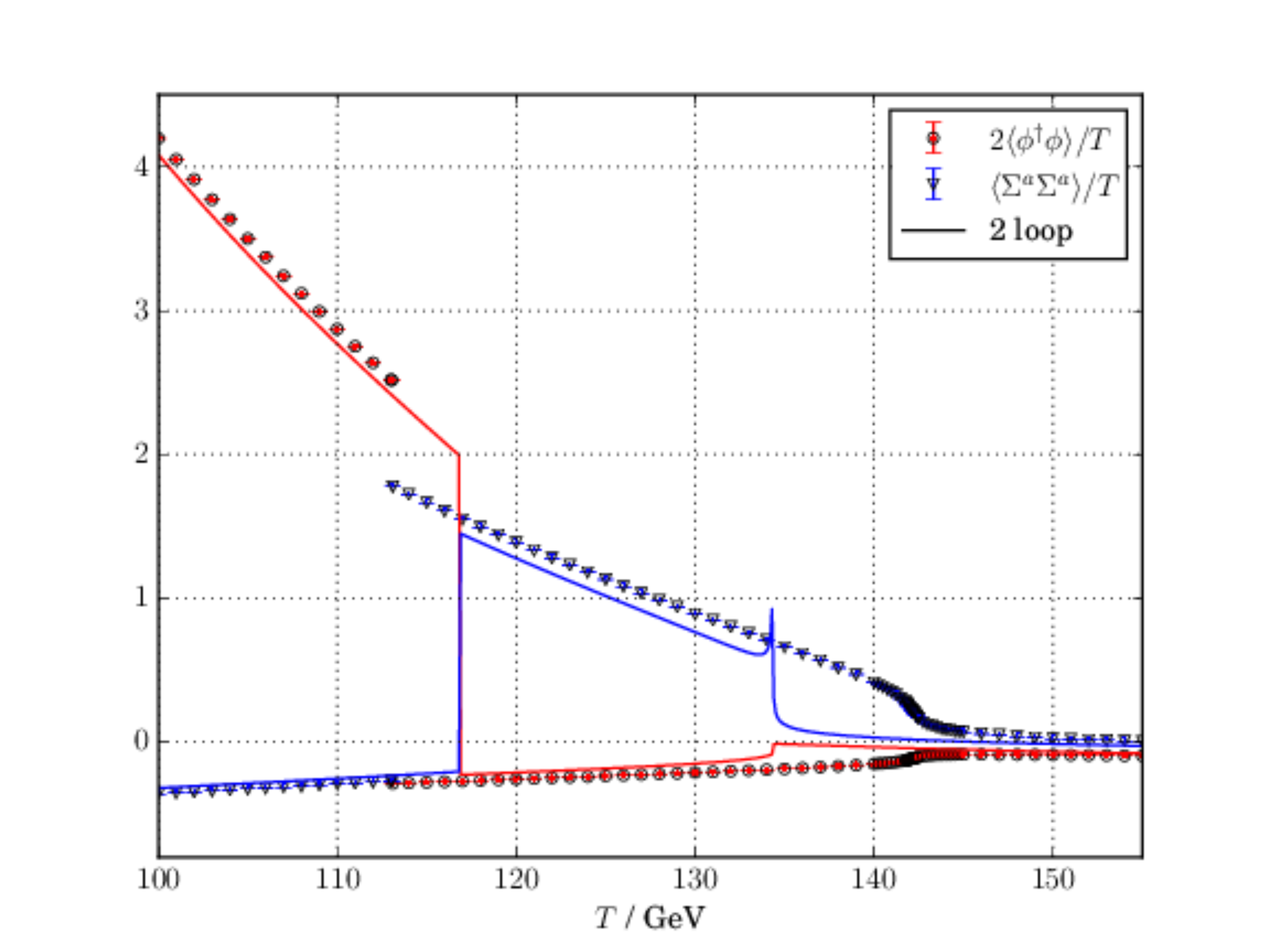
The infamous infrared problem $$g^2 \to g^2 n_b \to g^2 \frac{T}{m} \cdots \to g^2 \frac{T}{\sqrt{m^2 + \Pi}}$$

$$m_H = -\mu^2 + 3\lambda h^2 + \Pi$$

Diverges for weak transition!



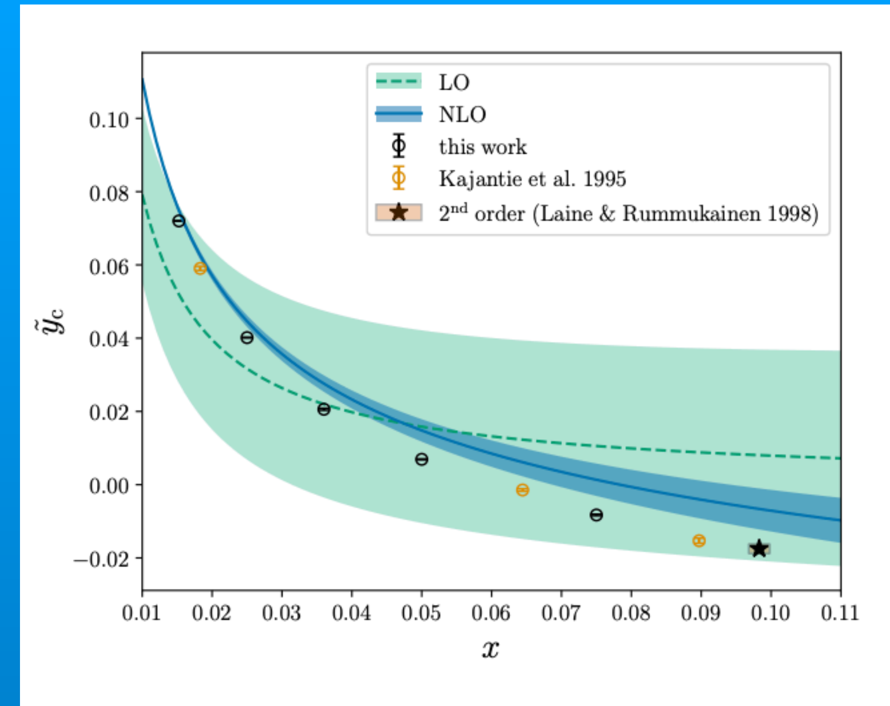
Weak transitions

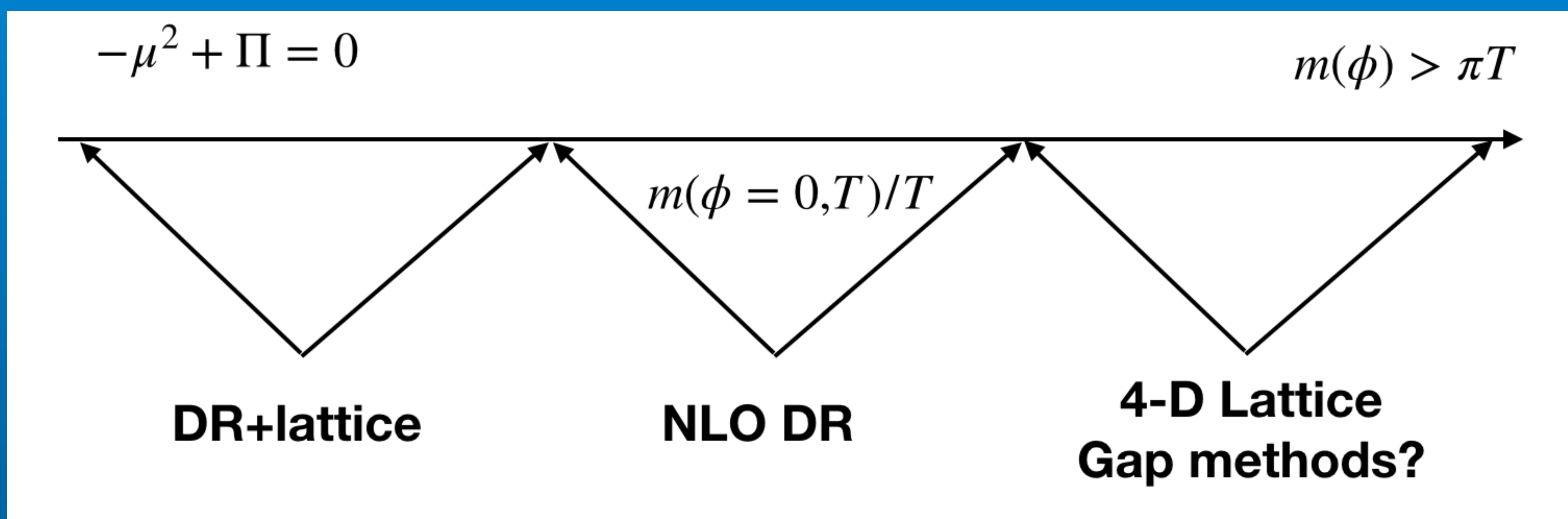


When (if ever) perturbation theory is valid is unknown

Nieme, Ramsey-Musolf, Tenkanen and Weir 2005.11332

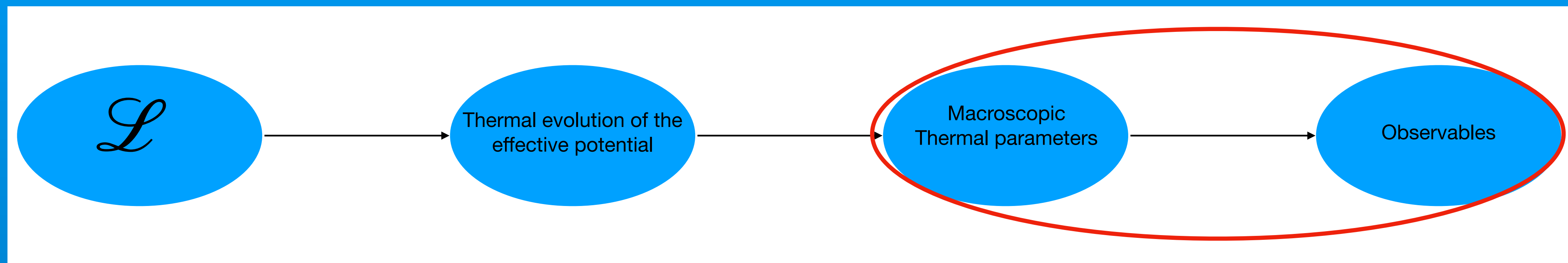
The Goldilocks zone?

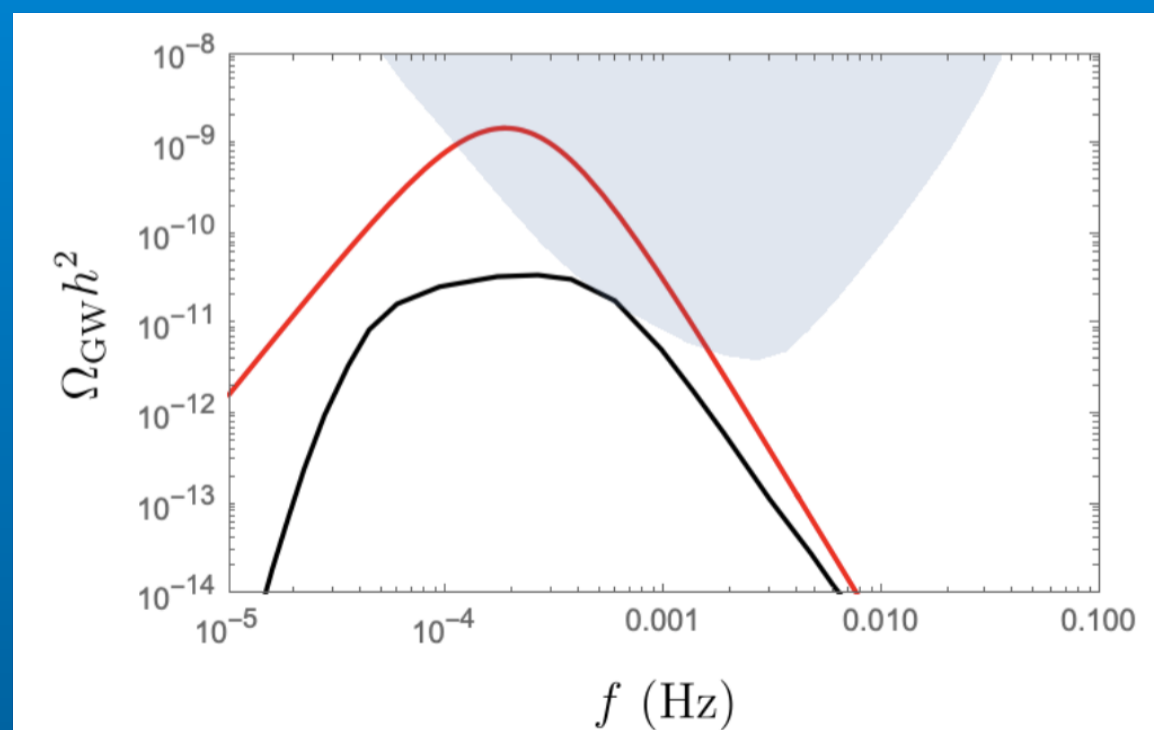




Gould, Guyer, Rummukainen: 2205.07238

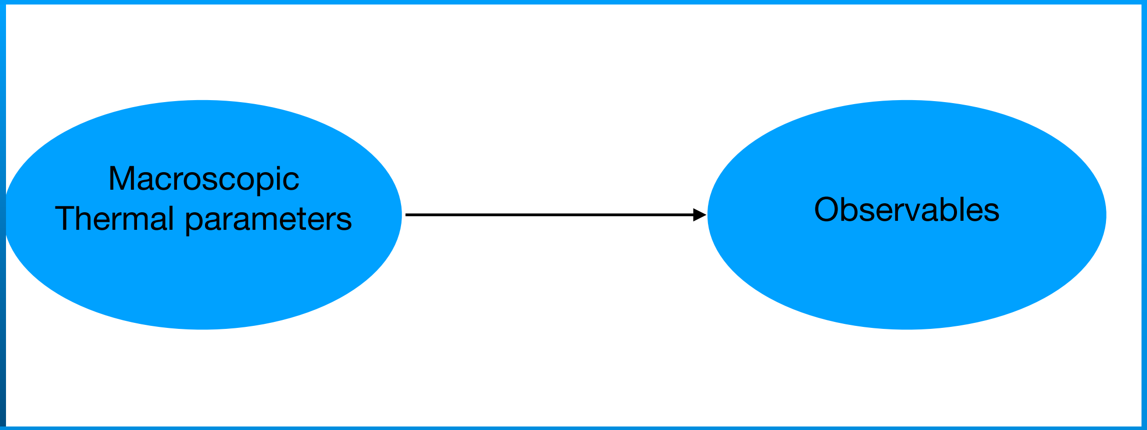
Theoretical uncertainties (modelling)





Picture from my book, data from Gowling, Hindmarsh 2106.05984

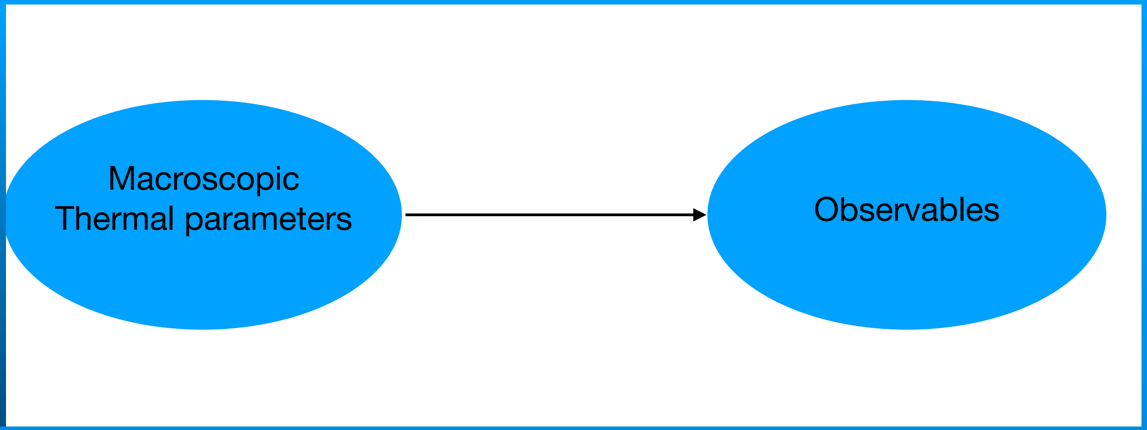
From thermal field theory to parameters



Guo, Sinha, Vagie White: 2103.06933

Effect	Range of error	Type
Tracking bubble distribution	$\mathcal{O}(10^{-4} - 10^{1})$	**Random**
Solve hydrodynamic eqns	$\mathcal{O}(10^{-2} - 10^{0})$	**Random**
Finite lifetime	$\mathcal{O}(10^{1} - 10^{3})$	**Overestimate**
Vorticity/reheating (1906.00480)	$\mathcal{O}(10^{-1} - 10^{0})$	**Random**
Full Sound shell	$\sim \mathcal{O}(10^{0} - 10^{2})$	Shape
Density fluctuations (2108.11947)	$\mathcal{O}(10^{1})?$	Underestimate + shape
Variable speed of sound (2004.06995)	$\mathcal{O}(10^{-1} - 10^{0})$	**Random**

From thermal field theory to parameters



Guo, Sinha, Vagie White: 2103.06933

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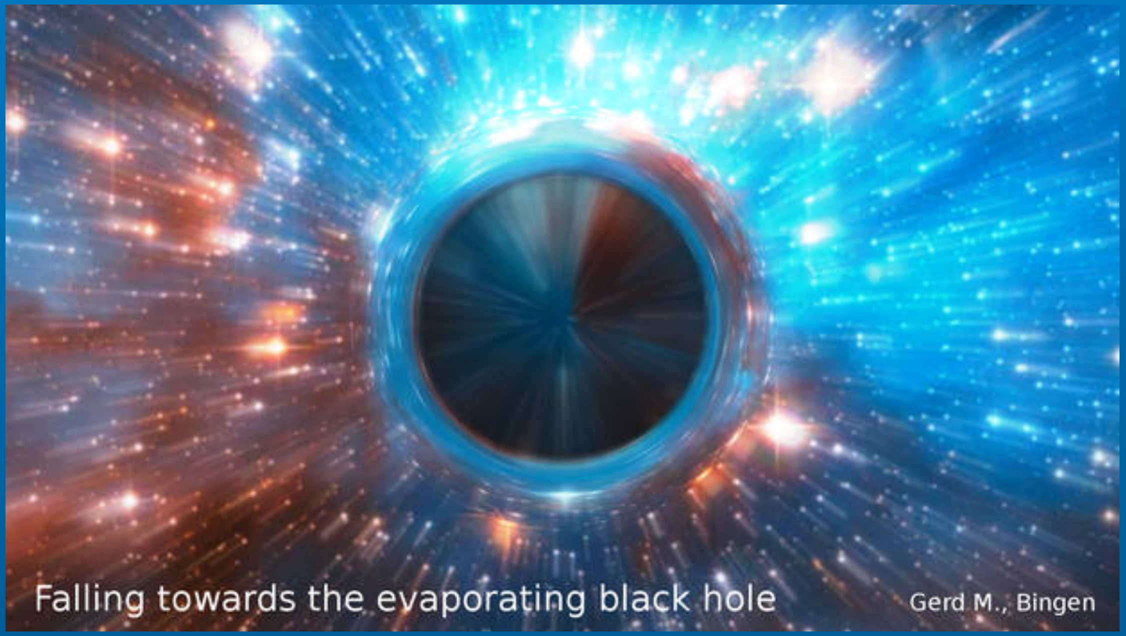
Gravitational waves from a sudden change in the equation of state



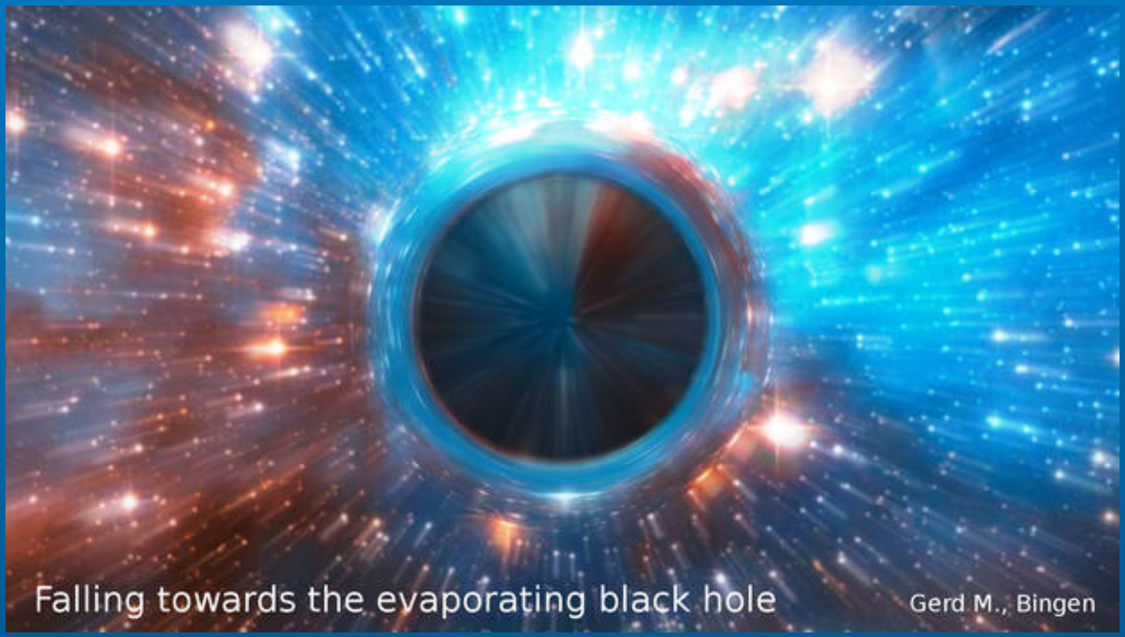
Scalar induced gravitational waves

- Scalar perturbations are a source for tensor perturbations at second order

$$\Omega_{\rm GW}^{\rm induced} h^2 \sim 10^{-1} \Omega_{r,0} h^2 \mathscr{P}_{\mathscr{R}}^2 \sim 10^{-6} \mathscr{P}_{\mathscr{R}}^2$$

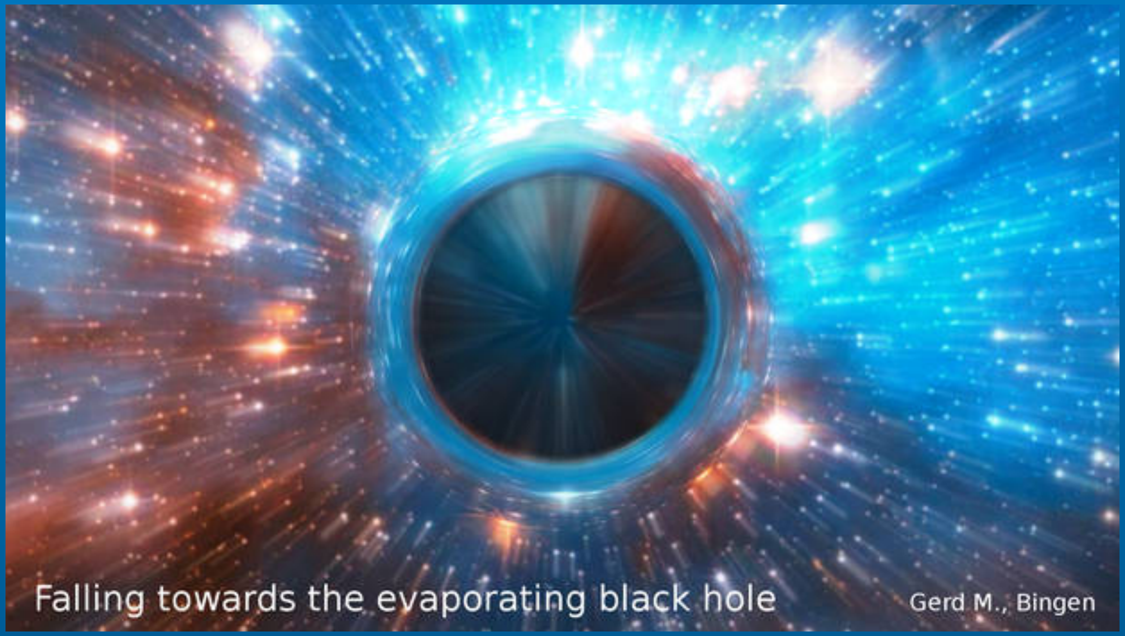
CMB extrapolation

$\mathscr{P}_{\mathscr{R}} \sim 10^{-9} \rightarrow \Omega_{\rm GW}^{\rm induced} h^2 \sim 10^{-24}$



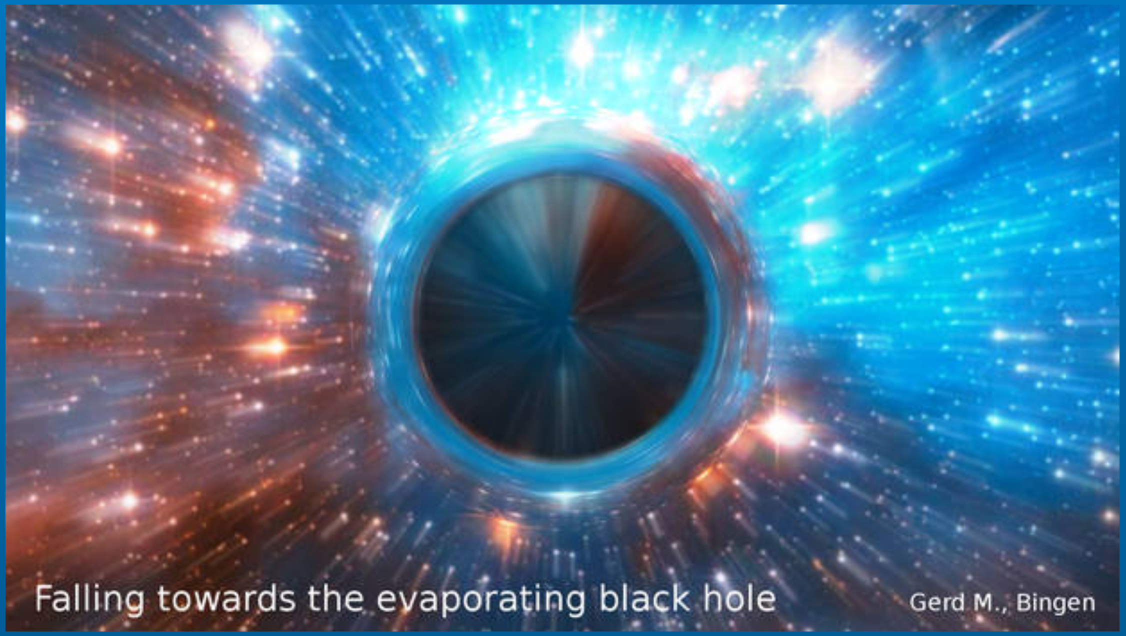
How to enhance $\mathscr{P}_{\mathscr{R}}$

1) Ultra slow roll
2) Sudden change in the equation of state



How to enhance $\mathscr{P}_{\mathscr{R}}$

1) Ultra slow roll
2) Sudden change in the equation of state
   - Matter domination to radiation domination



How to enhance $\mathscr{P}_{\mathscr{R}}$

Perturbations in conformal newtonian gauge

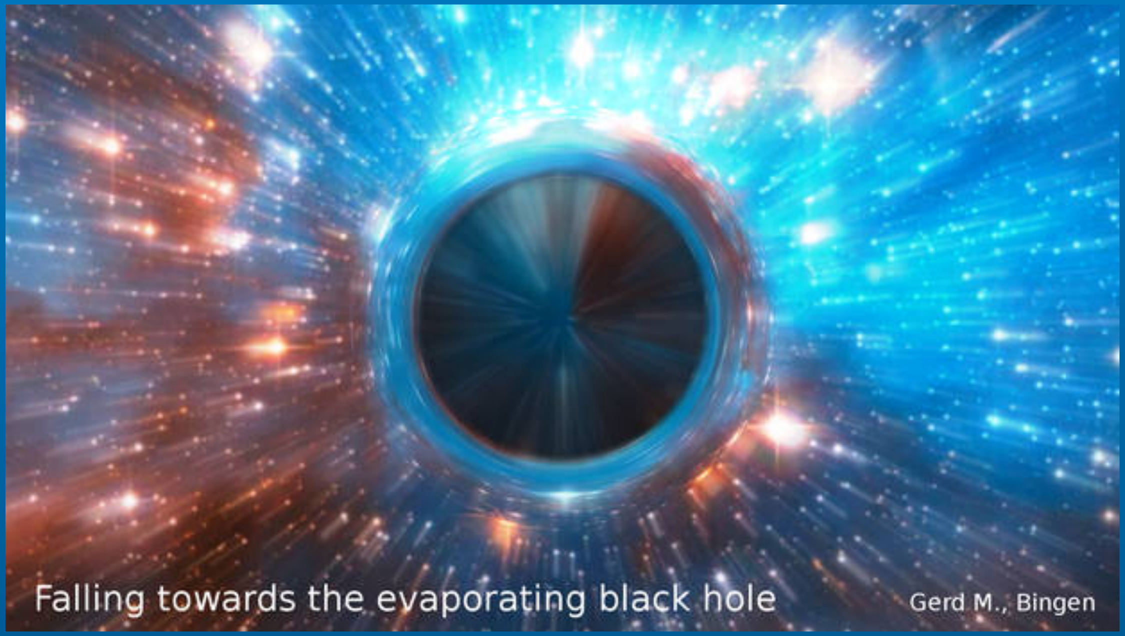
$$ds^2 = -a^2(1+2\Phi)d\eta^2 + a^2\left((1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\right)dx^i dx^j$$

Equations of motion

$$h'' + 2\mathscr{H}h' + k^2 h = 4S$$

$$S = \int \frac{d^3q}{(2\pi)^{3/2}} e_{ij} q_i q_j \left(2\Phi\Phi + \frac{4}{3(1+w)}(\mathscr{H}^{-1}\Phi' + \Phi)(\mathscr{H}^{-1}\Phi' + \Phi)\right)$$

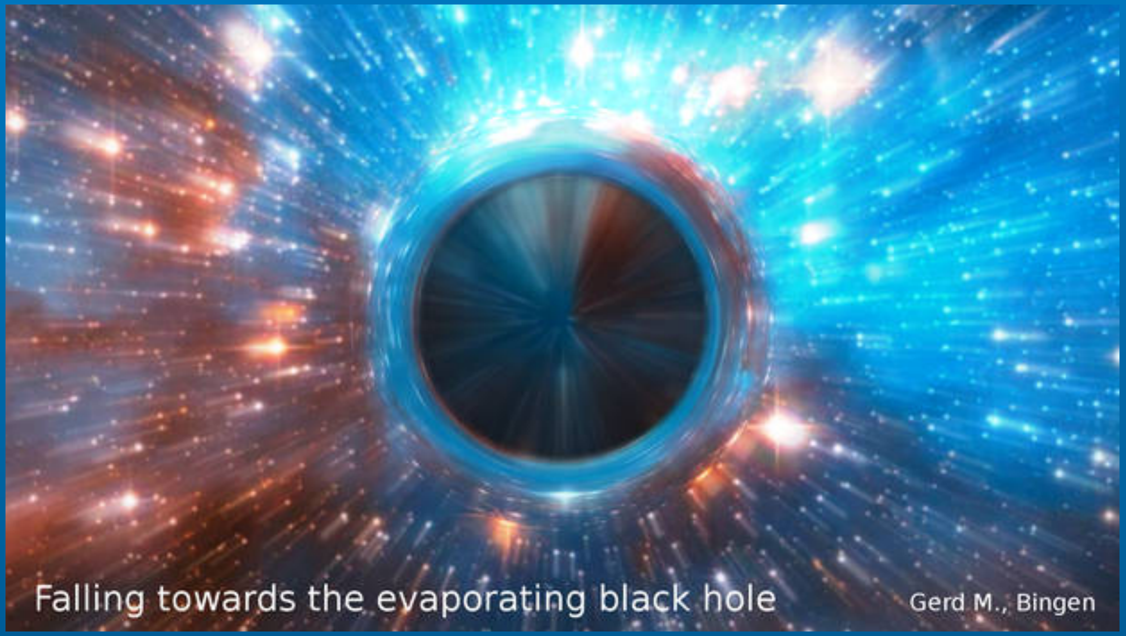
$$\langle hh \rangle = \frac{2\pi^2}{k^3}\delta(\Delta k) P_{\text{tensor}}$$



How tensor modes are sourced by scalar modes

$$\langle hh \rangle = \frac{2\pi^2}{k^3}\delta(\Delta k) P_{\text{tensor}}$$

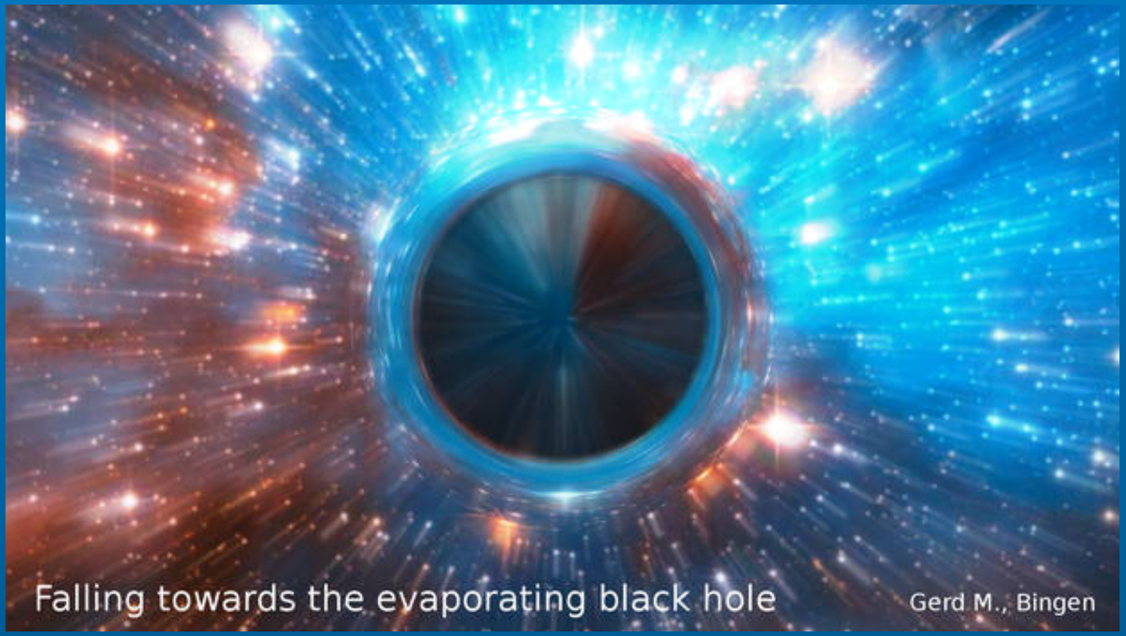
$$\bar{\mathscr{P}}_{\text{tensor}} = 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \bar{I}^2 \mathscr{P}_{\mathscr{R}}(uk)\mathscr{P}_{\mathscr{R}}(vk)$$



How tensor modes are sourced by scalar modes

$$\bar{\mathcal{P}}_{\text{tensor}} = 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2 - (1+v^2-u^2)^2}{4vu}\right)^2 \bar{I}^2 \mathcal{P}_{\mathcal{R}}(uk)\mathcal{P}_{\mathcal{R}}(vk)$$

$$I = \int_0^x d\bar{x} \frac{a(\bar{\eta})}{a(\eta)} kGf \qquad\qquad x = k\eta$$

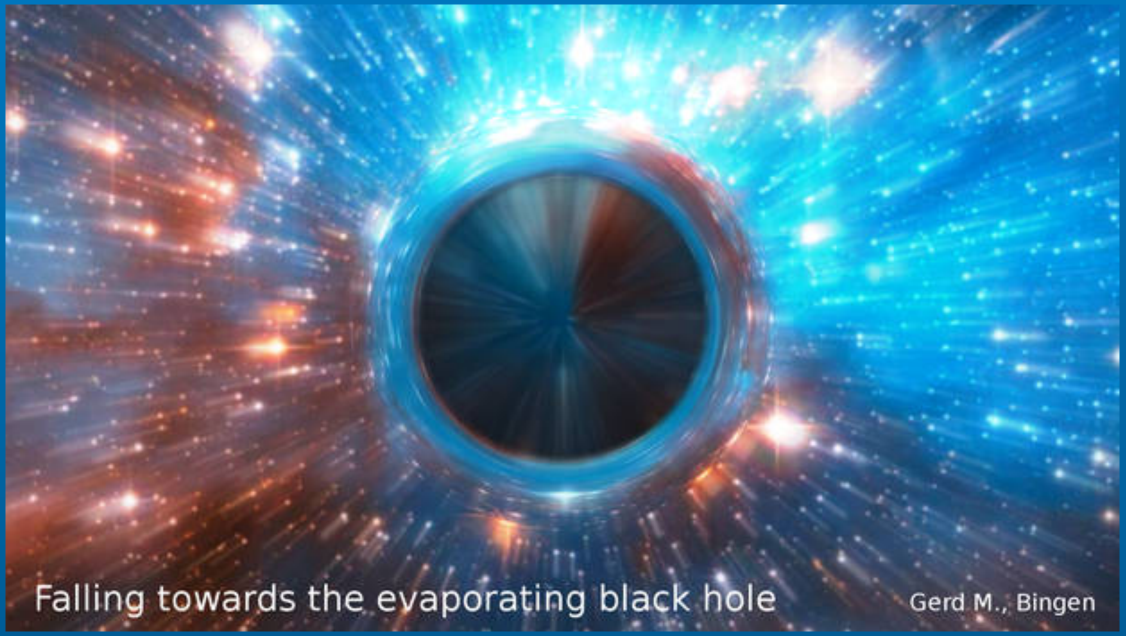


How tensor modes are sourced by scalar modes

$$\bar{\mathscr{P}}_{\rm tensor} = 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2 - (1+v^2-u^2)^2}{4vu}\right)^2 \bar{I}^2 \mathscr{P}_{\mathscr{R}}(uk)\mathscr{P}_{\mathscr{R}}(vk)$$

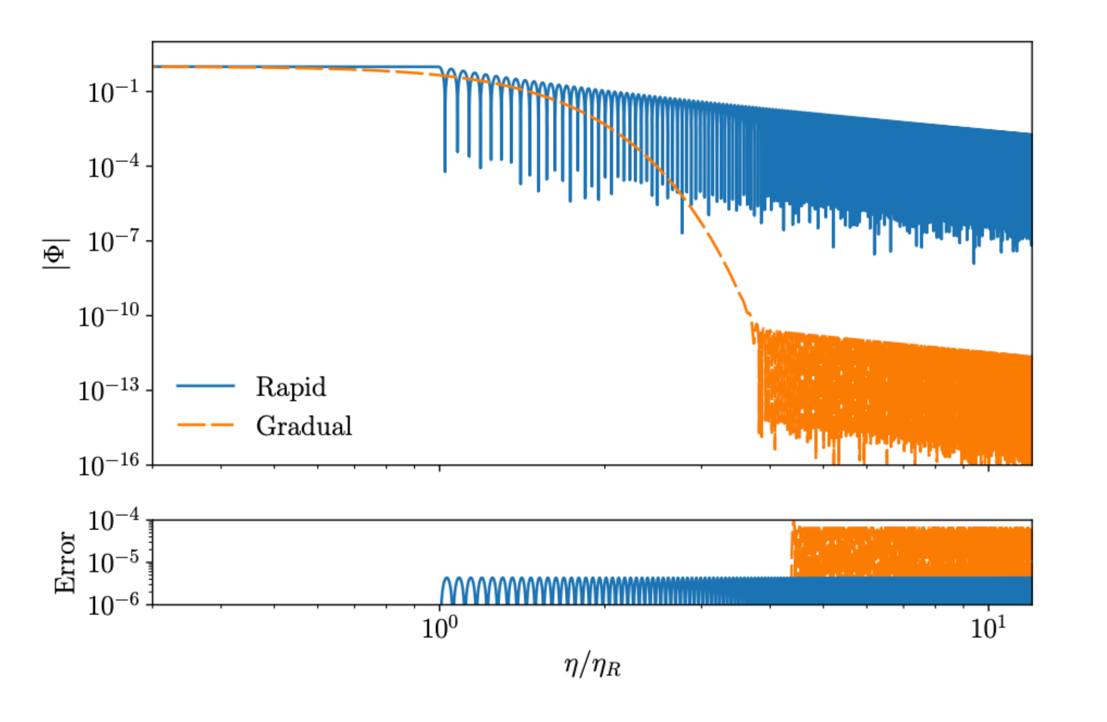
$$I = \int_0^x d\bar{x} \frac{a(\bar{\eta})}{a(\eta)} kGf \qquad x = k\eta$$

$$f = \frac{3(2(5+3w)\Phi\Phi + 4\mathscr{H}^{-1}(\Phi'\Phi + \Phi\Phi') + 4\mathscr{H}^{-2}\Phi'\Phi'}{25(1+w)}$$



Fast vs gradual transitions

$$I = \int kGf = \int kG^{\rm MD}f + \int kG^{\rm RD}f \to \Omega_{\rm GW} \sim I^2 = \Omega_{\rm MD} + \Omega_{\rm RD} + \Omega_{\rm cross}$$



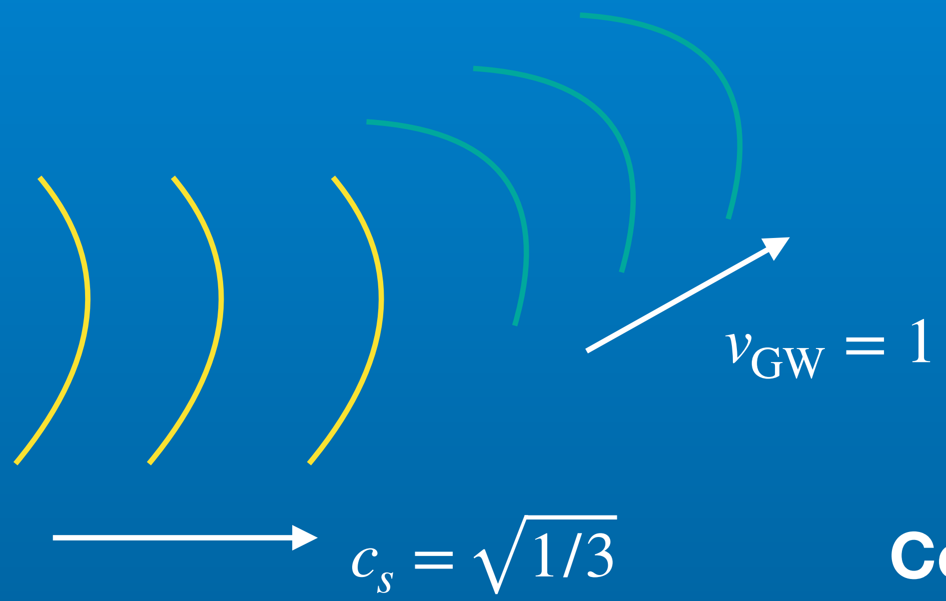
$$\Omega_{\rm cross} < 0$$

So this is reason why instantaneous transitions are stronger.
Mathematical explanation

1) Φ' does not vanish as quickly, therefore neither does f
2) Negative definite cross term vanishes

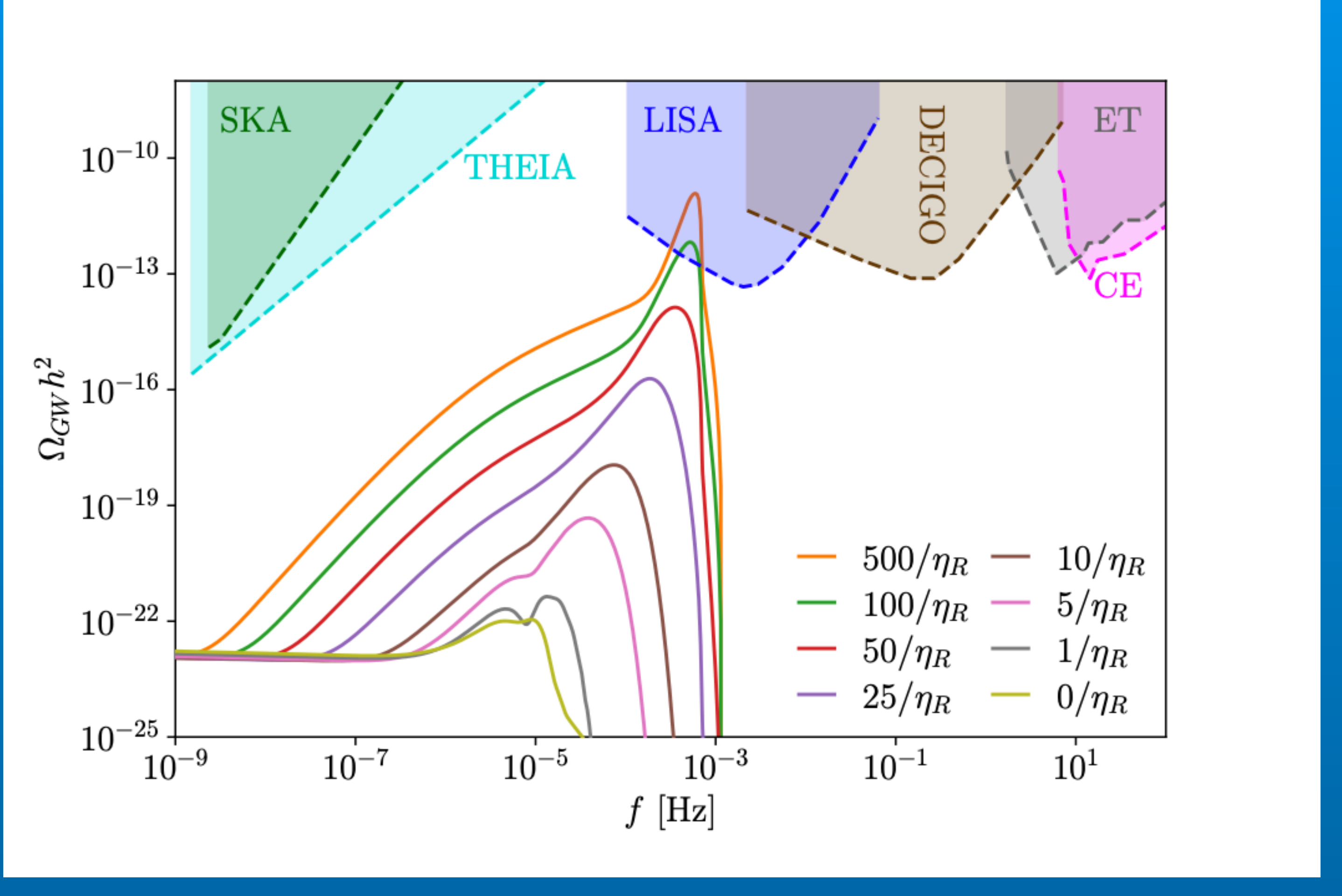
Physical explanation why the instantaneous transitions have larger GWs

- During matter domination perturbations grow
- Suppose the transition to radiation is very fast
- Perturbations cannot melt away in this case but produce sound waves



Constructive interference

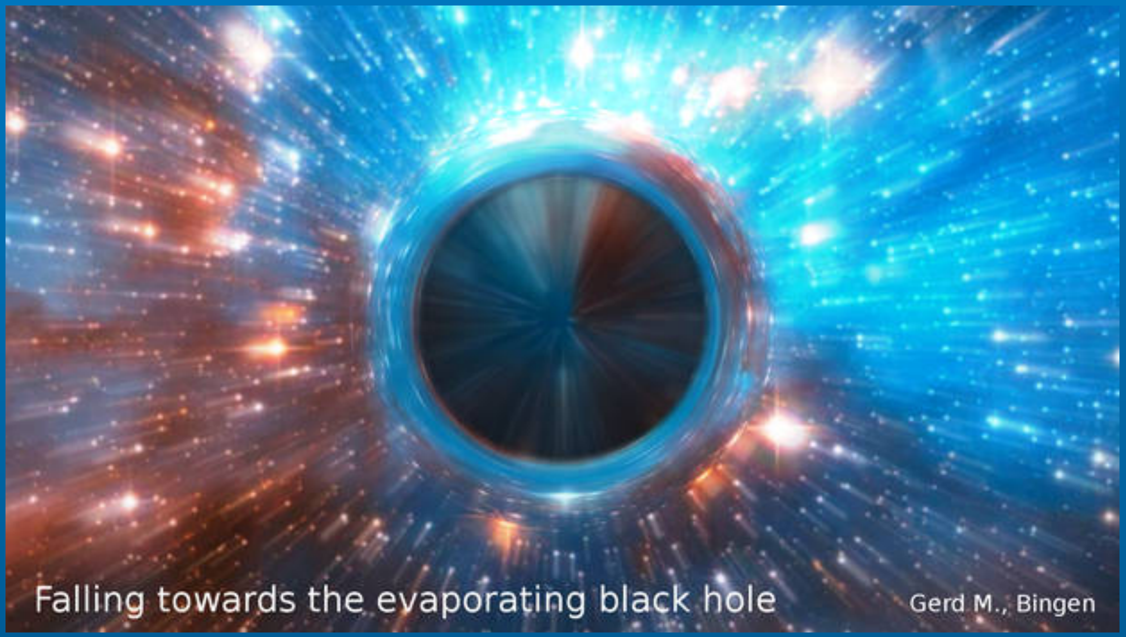
Spectrum



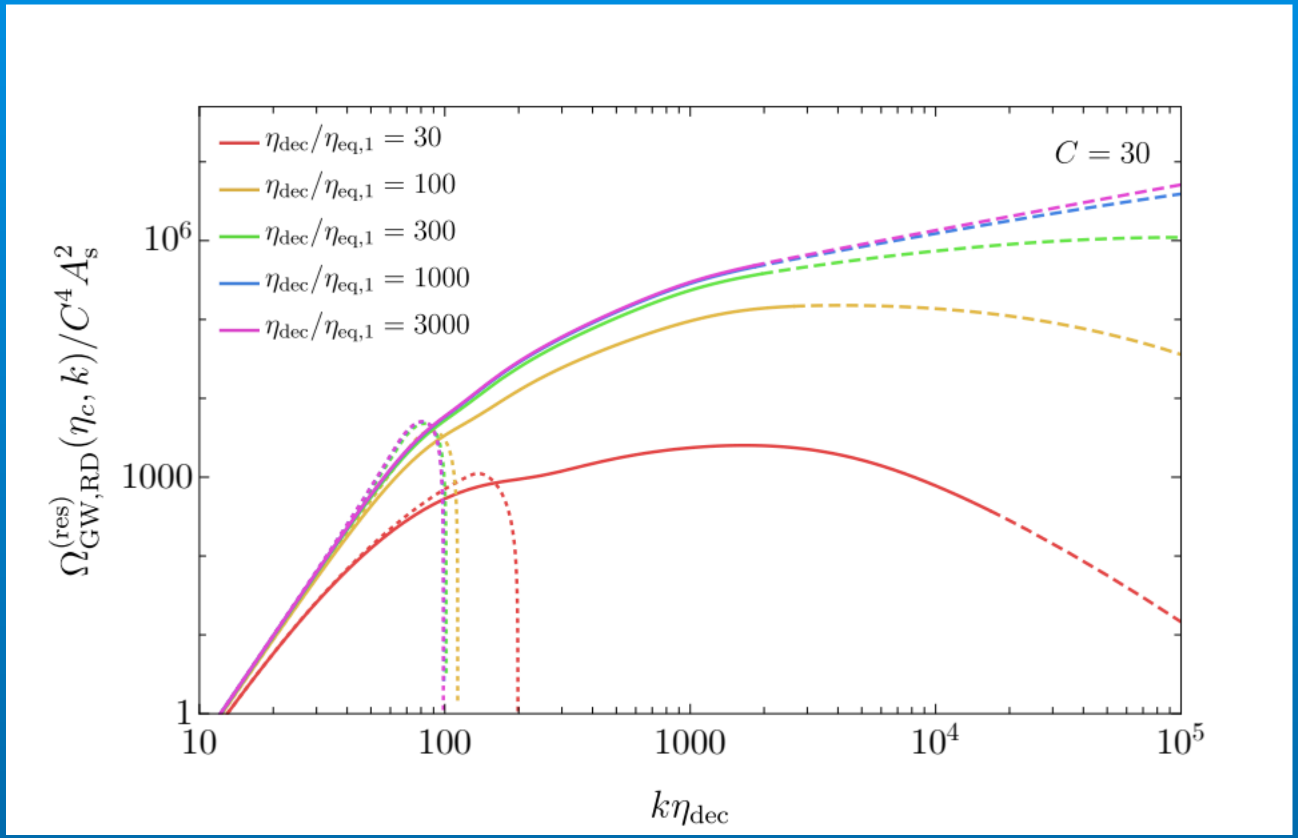
2311.12340

Possible reasons why there is a sudden change from matter to radiation

Very light primordial black holes
Clumps of field (Q-balls)
Particle whose decay is forbidden by a symmetry
Coherent scalar field



Beyond linear regime



Masahiro Kawasaki and Kai Murai.2308.13134

It wouldn't be a cosmological gravitational wave source if we knew what we were doing…

Gauge issues for scalar induced Gws

$$\rho_{\rm GW} \sim \langle h' h' \rangle$$

$$x^\mu \to x^\mu - \xi^\mu \quad \text{where } \xi^\mu = (T, \partial^i L)$$

$$\rho_{\rm GW} \to \rho_{\rm GW} + \langle \partial_i T \partial_j T \partial^i T \partial^j T \rangle$$

It wouldn't be a cosmological gravitational wave source if we knew what we were doing…

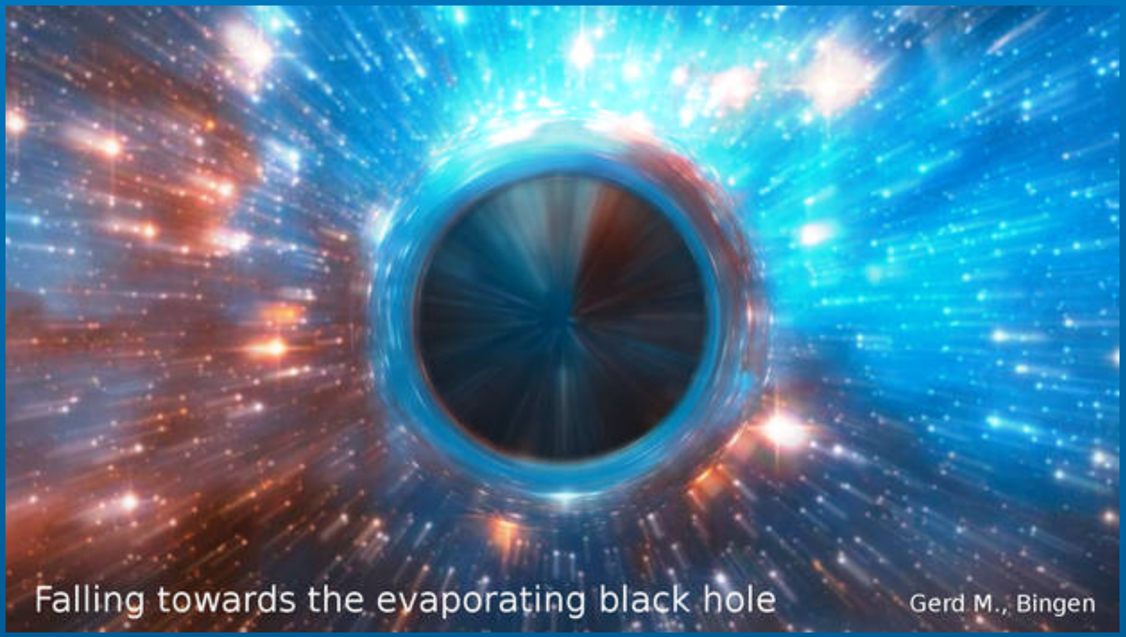
Gauge issues for scalar induced Gws

Least bad approach - gravitational waves aren't very numerically sensitive to gauge transformations as long as we choose gauges where

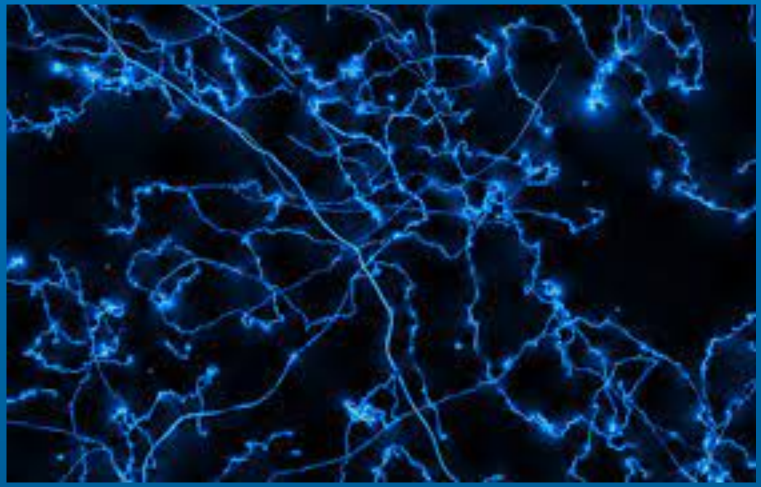
- We only consider sub-horizon scales
- We calculate the gravitational wave spectrum after the source has become negligible
- We use a gauge that is suitable for small distance calculations

Outlook

- Great potential for detecting any departure from early radiation domination
- Need work in understanding the strongest signals
- Need work in understanding intermediate transitions
- Huge potential for understanding high scale physics



Next lecture - topological defects



Gravitational wave archaeology

Lecture 3: topological defects

Graham White

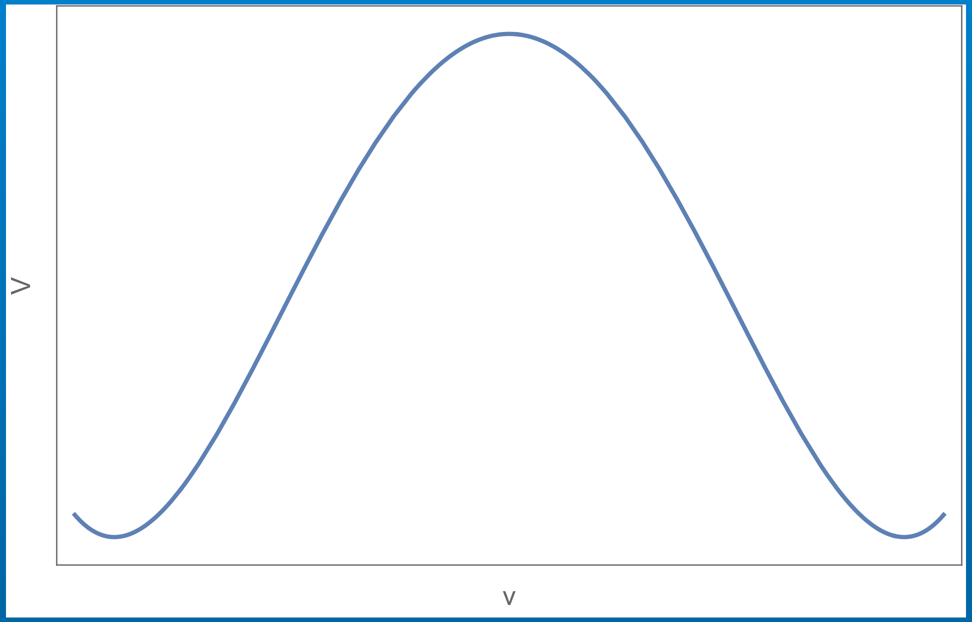
University of Southampton

SHEP Southampton High Energy Physics

Introduction to Topological defects

Consider the potential

$$V = -\mu^2 v^2 + \lambda v^4$$



There are two minima with the same value!

Introduction to Topological defects

Consider the potential

$$V = -\mu^2 v^2 + \lambda v^4$$

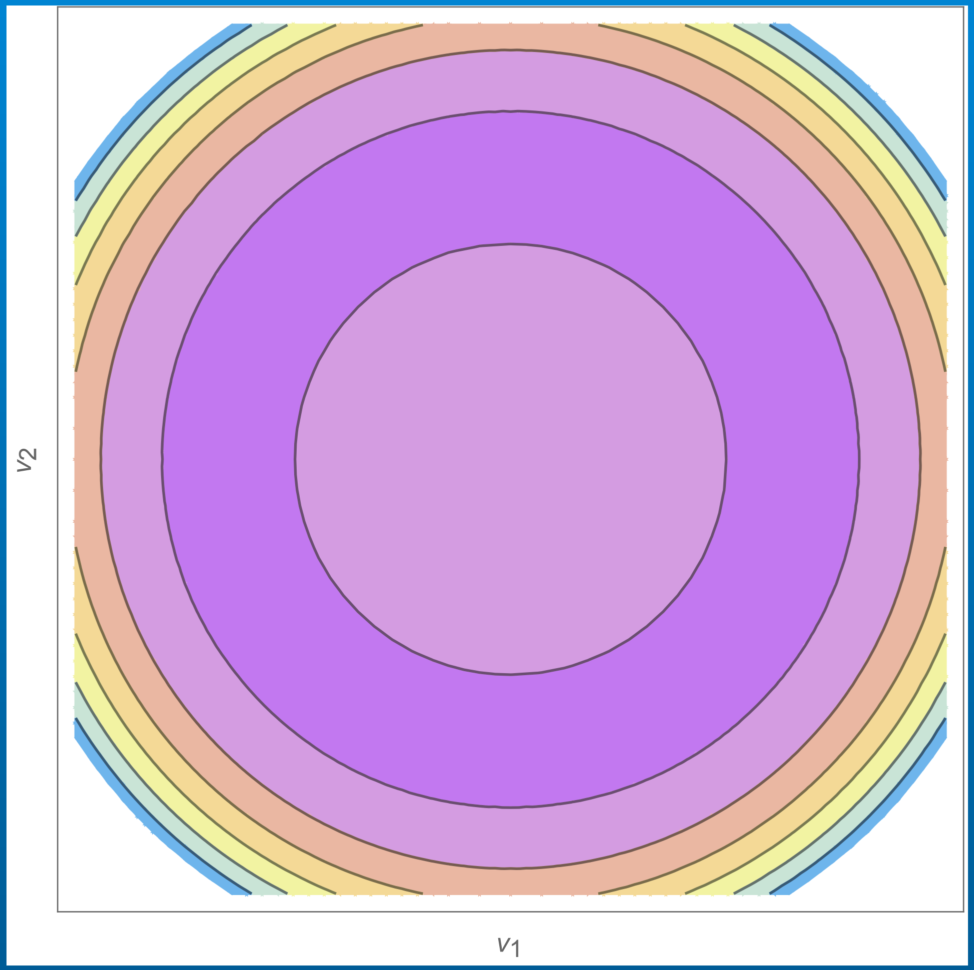
This probably feels like overkill for such a simple example, but we can depict the Vacuum manifold as two disconnected points

The key point is the vacuum manifold is *disconnected*

Introduction to Topological defects

But this isn't the only type on degeneracy we can have

$$V = -\mu^2(x^2+y^2)+\lambda(x^2+y^2)^2$$



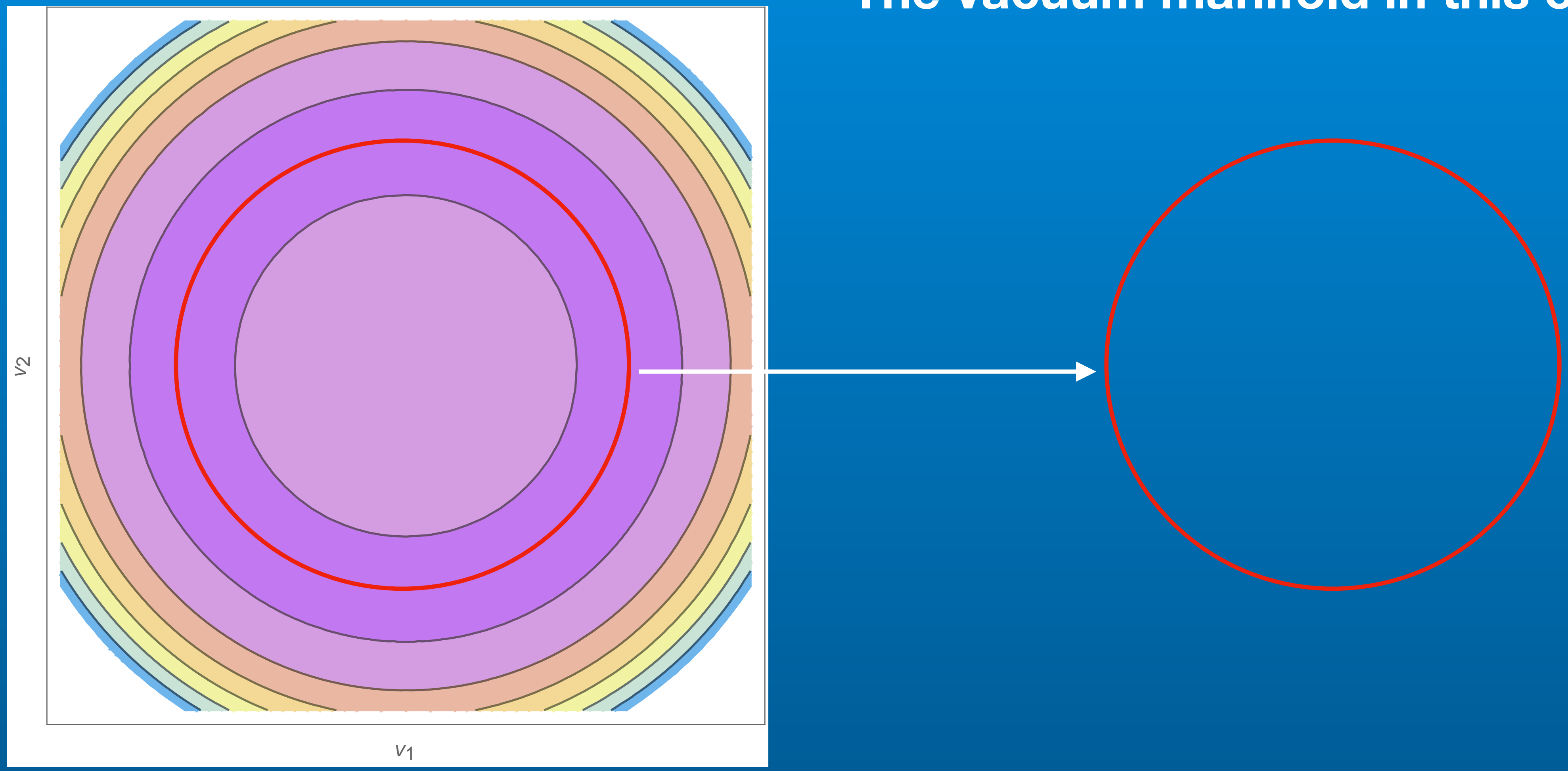
In this case the minima is degenerate for all $x^2+y^2=\text{constant}$

Introduction to Topological defects

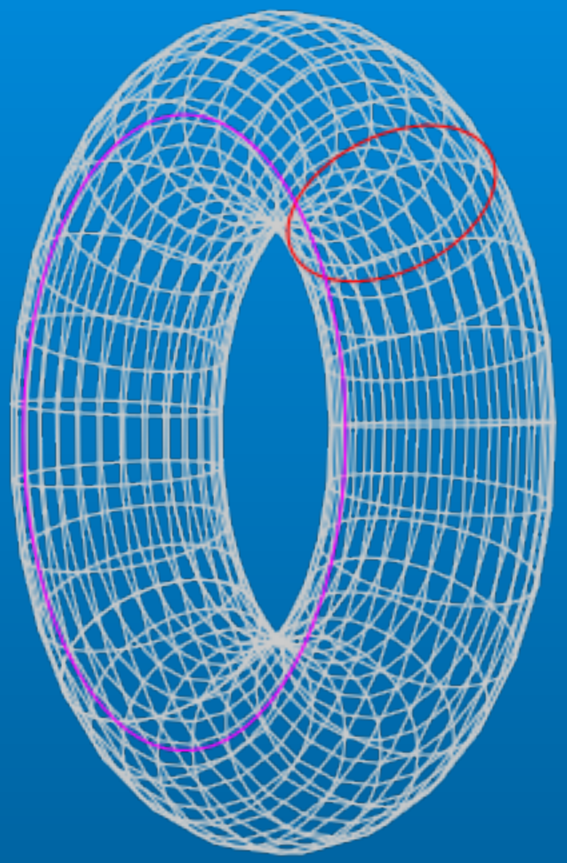
But this isn't the only type on degeneracy we can have

$$V = -\mu^2(x^2 + y^2) + \lambda(x^2 + y^2)^2$$

The vacuum manifold in this case is a circle



With more fields we can have more complicated vacuum manifolds



We can imagine a vacuum manifold that is not simply connected

e.g. if there are non contractable loops

A torus or a coffee mug are both not simply connected because they have a hole!

Introduction to Topological defects

A fancy way to categorize the features of a vacuum manifold is to use a mathematical tool called the homotopy group

We won't be going into the homotopy group much, but will simply use the terminology

$\pi_0(\mathscr{M}) \neq 1 \rightarrow$ Disconnected

$\pi_1(\mathscr{M}) \neq 1 \rightarrow$ Not simply connected

$\pi_2(\mathscr{M}) \neq 1 \rightarrow$ non contractable spheres etc

Introduction to Topological defects

So why care about any of this? Well you get weird solutions to the equations of motion

Consider a disconnected manifold $V = -\mu^2 v^2 + \lambda v^4$

A solution to the equation of motion is obviously when we solve the Euler Lagrange equations

$$\partial^\mu \partial_\mu v = \frac{dV}{dv}$$

We have the usual boring solutions $v = \pm \sqrt{\frac{\mu^2}{2\lambda}}$

Introduction to Topological defects

We have the usual boring solutions $v = \pm\sqrt{\frac{\mu^2}{2\lambda}}$

We also have a solution which continuously goes from one vacuum to another

$$v = \frac{\mu \tanh[z\mu]}{\sqrt{2\lambda}}$$

Three of the derivative terms in the Euler Langrange equations vanish

$$\frac{\partial^2 v}{dt^2} = \frac{\partial^2 v}{dx^2} = \frac{\partial^2 v}{dy^2} = 0$$

This leaves just $$\frac{\partial^2 v}{dz^2} = \frac{\partial V}{\partial v}$$

Subbing in our Tanh solution we find both sides equals $-\frac{\sqrt{2}\mu^3 \text{sech}[z\mu]^2 \tanh[z\mu]}{\sqrt{\lambda}}$

Introduction to Topological defects

So what is this weird tanh solution?

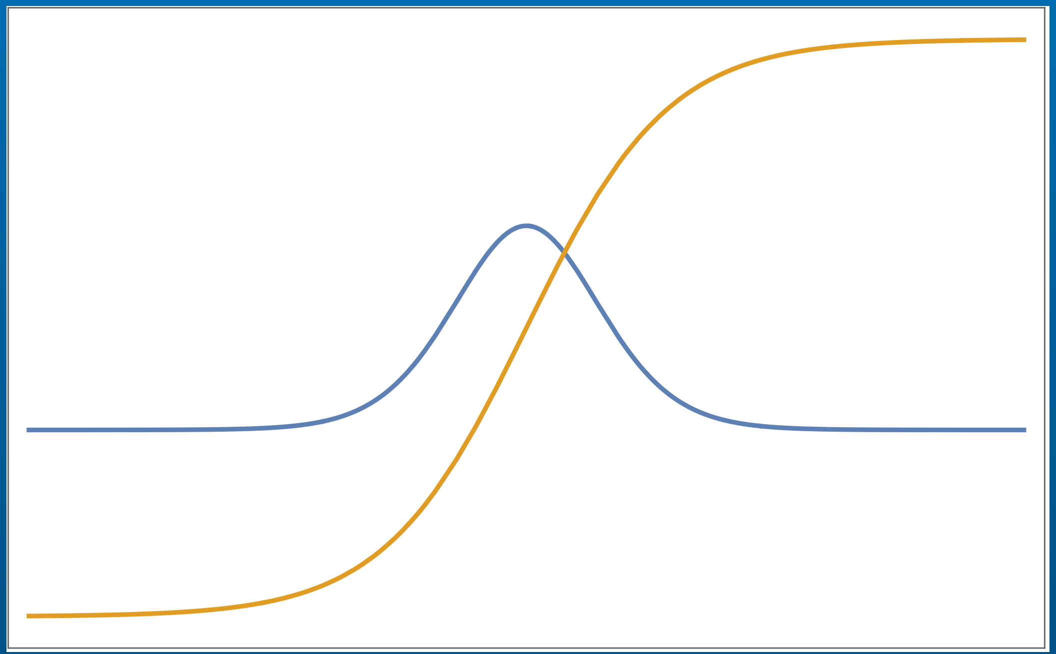
When we put it into the Lagrangian it gives a localized energy distribution

$$\int dzL = \int dz \left[\frac{1}{2}\left(\frac{dv}{dz}\right) + V\right]$$

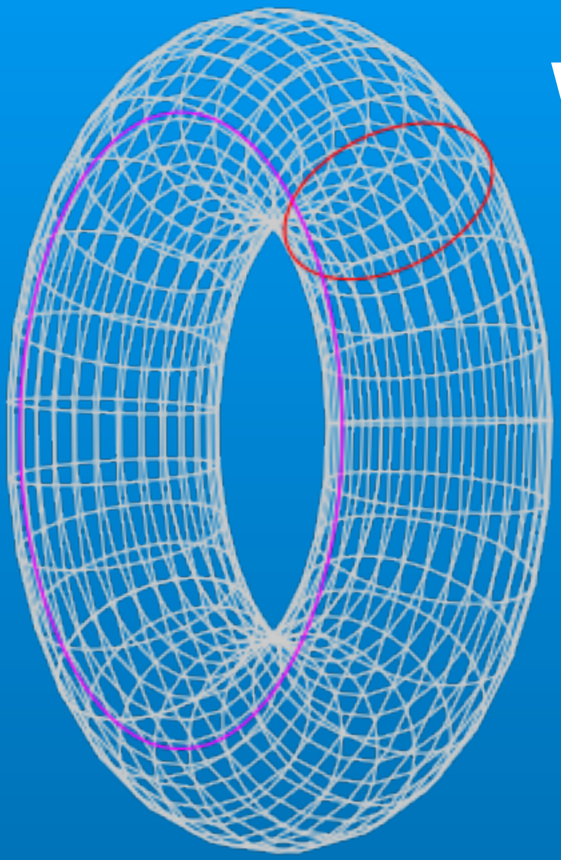
$$v = \frac{\mu \tanh[z\mu]}{\sqrt{2\lambda}} \rightarrow \int dzL = \int dz \frac{m^4(-1 + 2\mathrm{sech}[zu]^4)}{4\lambda}$$

The energy is clumpy! It looks like a wall of energy in space

This is why we call it a domain wall



Introduction to Topological defects



What about a more exotic vacuum manifold

$$L = |D\phi|^2 + V(\phi) + \frac{1}{4}F^2,\ D_\mu = \partial\mu - ieA_\mu$$

With eqns of motion

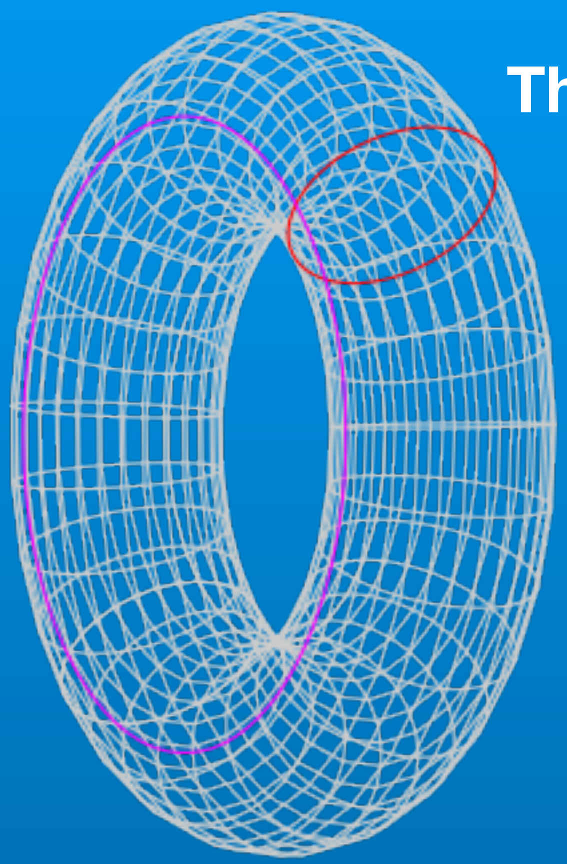
$$D^2\phi + \frac{\partial V}{\partial \phi} = 0,\ \partial^\mu F_{\mu\nu} - ie(\phi^* D_\nu \phi - D_\nu \phi^* \phi) = 0$$

We have a solution for *both* fields in ϕ, unlike the domain wall

$$\phi(r) = f(r) v e^{i\theta},\ A_i = \frac{1}{er} A_\theta(r) \hat{\theta}_i$$

$$A(\infty) = f(\infty) = 1,\ A(0) = f(0) = 0$$

Introduction to Topological defects



The wind around a non-contractable loop makes a string

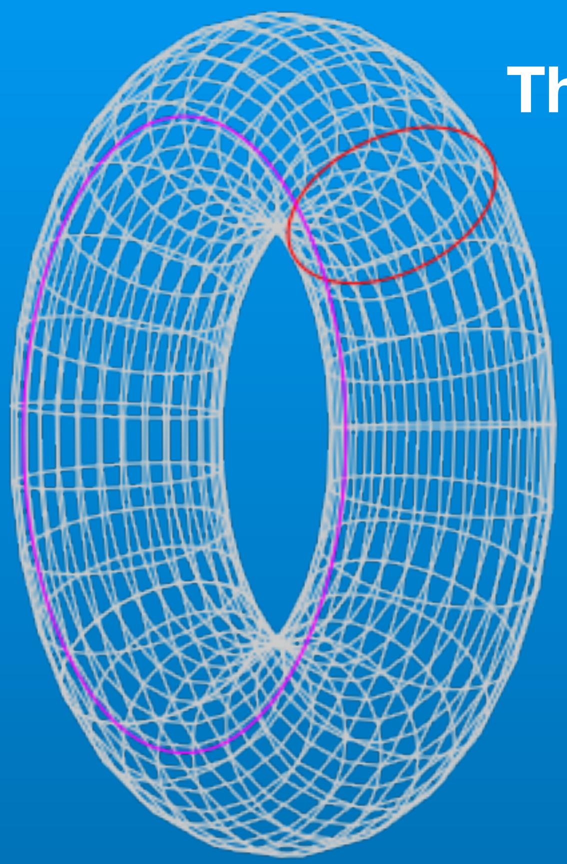
The equations that govern this string, once it is big enough, are the same ones you will see in string theory

$$S = -\mu \int d^2\xi \sqrt{\gamma}$$

Metric on the world sheet

$$\mu = \int r dr d\phi \left[\left| \frac{\partial \phi}{\partial r} \right|^2 + \frac{1}{4} \left| \frac{d\phi}{d\theta} - iqA'_\theta \phi \right|^2 + V(\phi) + \frac{(B')^2}{2} \right]$$

Introduction to Topological defects



The wind around a non-contractable loop makes a string

The equations that govern this string, once it is big enough, are the same ones you will see in string theory

$$S = -\mu \int d^2\xi \sqrt{\gamma}$$

Recall from string theory (if not just take this on faith) that fluctuations on the world sheet produce gravitons!

Similarly, domain walls will produce gravitons

$$S = -\sigma \int d^3\xi \sqrt{\gamma}$$

Introduction to Topological defects

Contribution to energy density from strings and domain walls

$$E_{\text{string}} = \mu R \to \rho_{\text{string}} = \mu R/R^3 = \mu/R^2 = \rho_{\text{string initial}} \frac{a_{\text{initial}}}{a^2}$$

$$E_{\text{DW}} = \sigma R^2 \to \rho_{\text{DW}} = \sigma R^2/R^3 = \sigma/R = \rho_{\text{DW initial}} \frac{a_{\text{initial}}}{a^1}$$

The energy density of radiation dilutes as a^{-4}, so the fraction of the total energy density of the Universe will *grow* as the Universe expands

With strings they will cut each other so their number density evolution as a function of size is more complicated, they tend to eventually maintain a *constant* fraction of the energy density.

However, if domain walls have no way of annihilating they will dominate the Universe!

Introduction to Topological defects

Making domain walls metastable

1) Local discrete symmetry - need to be eaten by strings
2) Global discrete symmetry - need to be annihilated

Introduction to Topological defects

Making domain walls metastable

1) Local discrete symmetry - need to be eaten by strings
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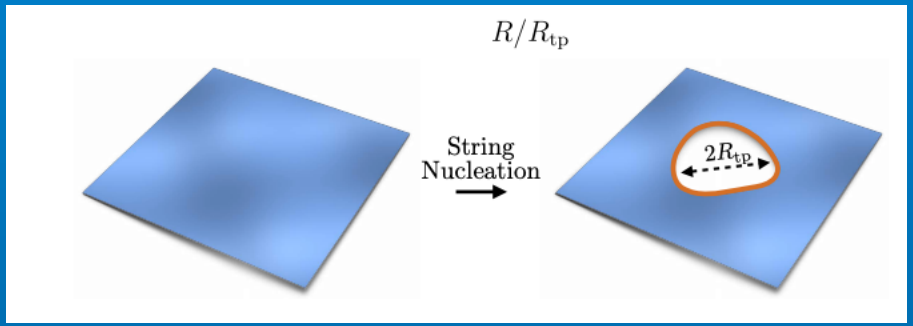
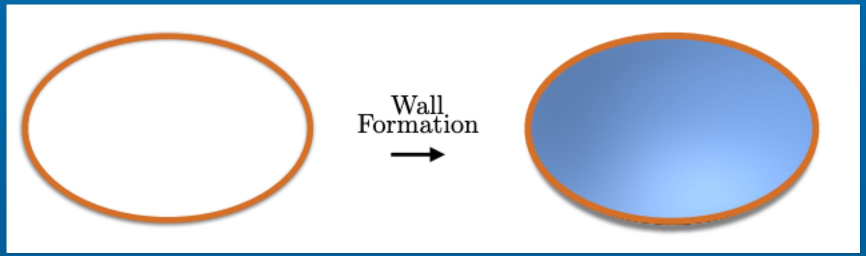


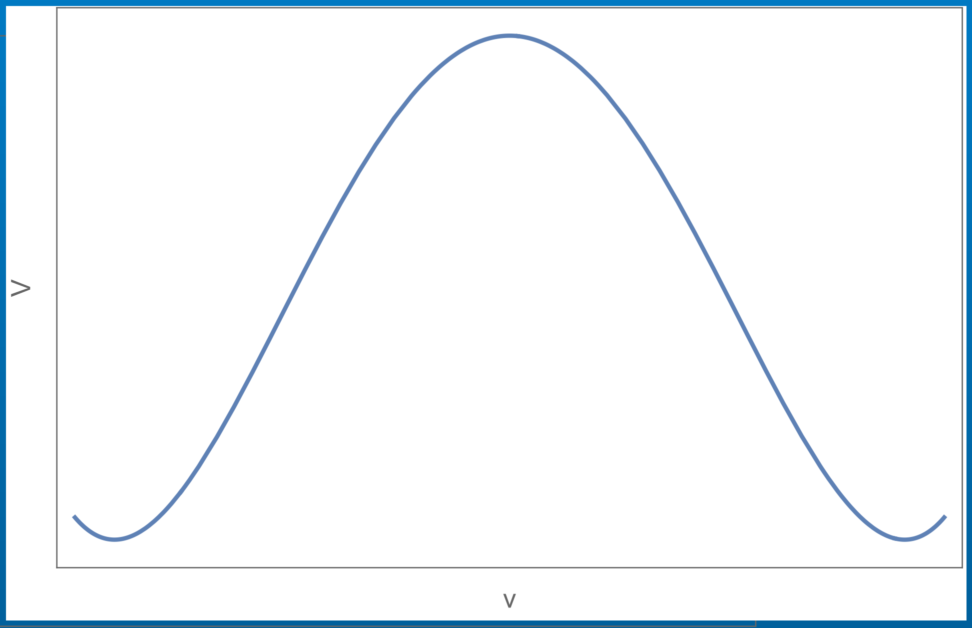


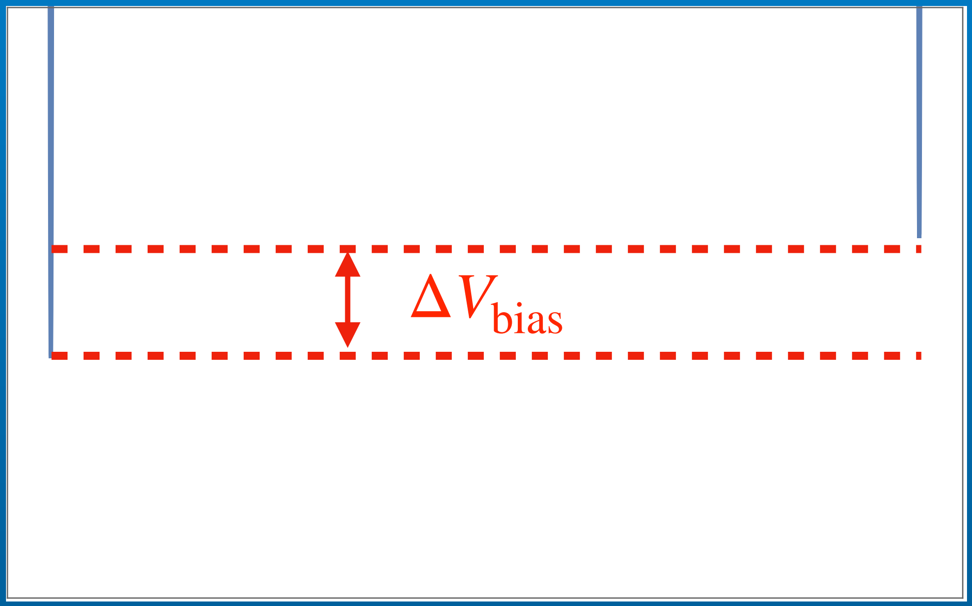
Introduction to Topological defects

Making domain walls metastable

1) Local discrete symmetry - need to be eaten by strings
2) Global discrete symmetry - need to be annihilated

$$V = -\mu^2 v^2 + \lambda v^4 + \frac{1}{\Lambda} v^5$$





Introduction to Topological defects

Gravitational waves

$$\frac{d\rho_{\rm GW}}{dt} = -\,n_{\rm defect}P_{\rm GW},\ \Omega_{\rm GW} = f\,\frac{d\rho_{\rm GW}/df}{\rho_c}$$

For domain walls this is relatively simple

$$P_{\rm GW,dw} \sim G\sigma M_{\rm DW} = G\sigma^2R^2,\ n_{\rm dw} = R^{-3},\ \frac{d\rho}{dt} \sim H(t) \to \frac{d\rho/dt}{\rho_{\rm rad}} \sim \frac{1}{H}$$

For radiation domination $a \sim t^{1/2}, f \sim a^{-1}, H \sim a^2 \to \dfrac{df}{dt} \sim t^{-3/2} \sim a^{-3}$

Using chain rule $$\frac{1}{\rho_{\rm rad}}\frac{d\rho_{\rm GW}}{df} = \frac{1}{\rho_{\rm rad}}\frac{d\rho_{\rm GW}}{dt}\left(\frac{df}{dt}\right)^{-1} \sim a^{-2}a^3 \sim a \sim f^{-1}$$

And this is exactly what we find in simulations

$$\Omega_{\rm GW}(f) = \Omega_{\rm max}\left(\Theta(f-f_{\rm peak})\left[\frac{f}{f_{\rm peak}}\right]^{-1} + \Theta(f_{\rm peak}-f)\left[\frac{f}{f_{\rm peak}}\right]^{3}\right)$$

Introduction to Topological defects

Gravitational waves

$$\frac{d\rho_{\rm GW}}{dt} = -\,n_{\rm defect} P_{\rm GW},\ \Omega_{\rm GW} = f\,\frac{d\rho_{\rm GW}/df}{\rho_c}$$

For strings it is tougher. All the cutting means you get a number distribution, also the emission of energy into gravitational waves shrinks the string

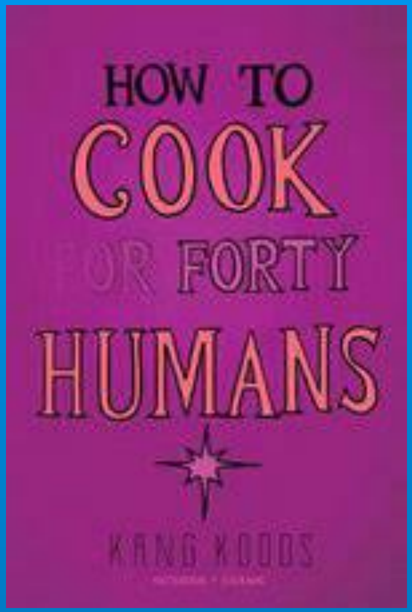
$$l = \alpha t_k - \Gamma G\mu(t - t_k)$$

$$\frac{\rho_{\rm GW}(t)}{df} = \int dt' \frac{a(t')^4}{a(t)^4} \int dl \frac{dn}{dl}\frac{dP}{df'}\frac{df'}{df}$$

This is a bit more complicated but here are two qualitative features

1) $\Omega_{\rm GW} \sim f^0$ **for radiation domination**
2) $\Omega_{\rm GW} \sim \mu^2 \sim v$

Controversy over Nambu-Goto strings



hep-ph/9708427

1903.05102

2103.16248

2302.03717

$$\mathscr{L} = \left(D_\mu \phi^\star D^\mu \phi + V + \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} \right)$$

Vs

$$S_{\rm NG} = -\mu \int d^2 \sqrt{-\gamma}$$

Gauge v global defects

A vacuum manifold can manifest as a result of breaking a gauge symmetry

$$\phi \rightarrow e^{i\alpha(x).\tau}\phi, \ \partial_\mu \rightarrow D_\mu = \partial_\mu - ia\tau^a A^a_\mu,$$

Or a global symmetry

$$\phi \rightarrow e^{i\alpha.\tau}\phi, \ \partial_\mu \rightarrow \partial_\mu = \partial_\mu$$

If the vacuum manifold that arises from breaking a *global* symmetry supports a cosmic defect, we call that a *global defect*
e.g. Global string, global monopole etc

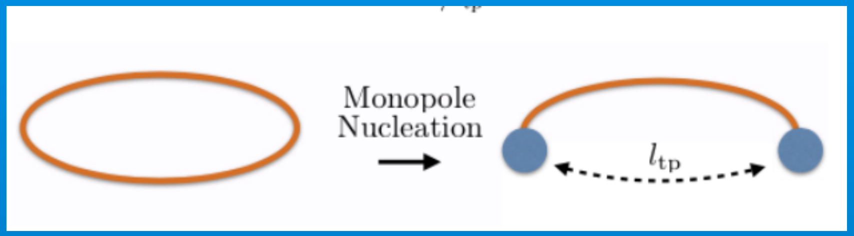
If the defect is supported from the breaking of a *gauge* symmetry it is called a local defect or a gauge defect

Gauge v global defects

	Gauge	Global
Strings	$\Omega_{\rm GW} \sim v$	$\Omega_{\rm GW} \sim v^4$
Domain walls	**Destroyed by strings**	**Destroyed by bias**
Textures	**Sharp peak**	$\Omega_{\rm GW} \sim v^4$

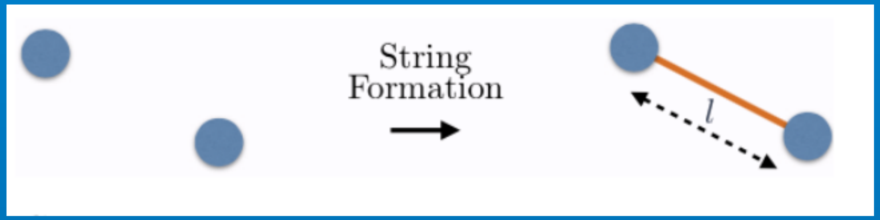
Hybrid defects

Arise when there are two symmetry breaking steps in a row and the defects are connected



$$\phi_1 \rightarrow v_1 \rightarrow \text{monopole}$$

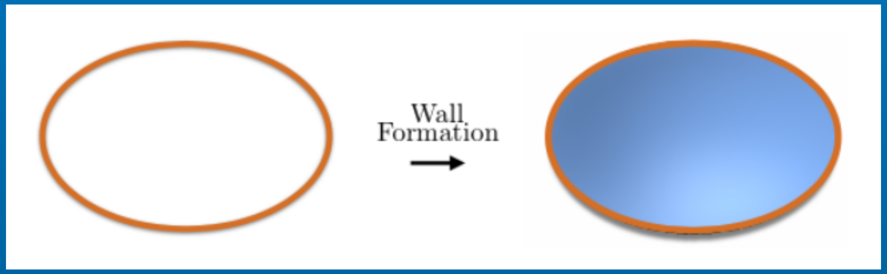
$$\phi_2 \rightarrow v_2 \rightarrow \text{string}$$

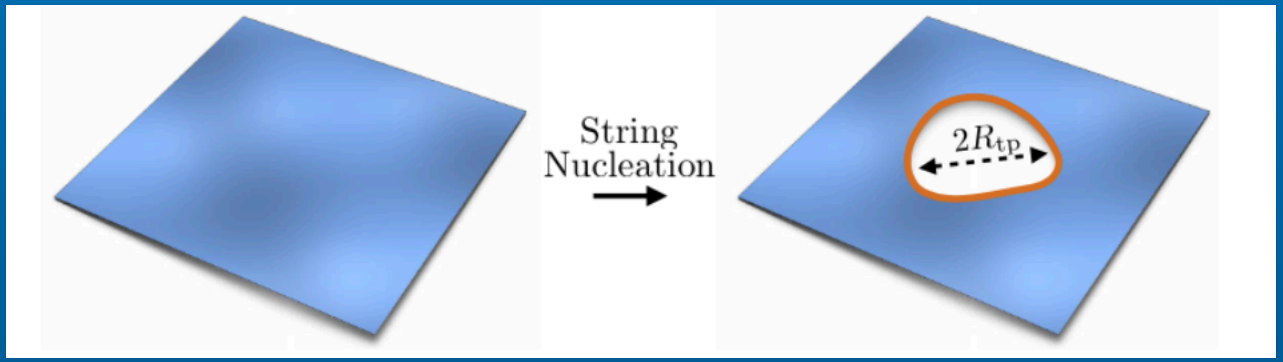


Or

$$\phi_1 \rightarrow v_1 \rightarrow \text{string}$$

$$\phi_2 \rightarrow v_2 \rightarrow \text{domain wall}$$

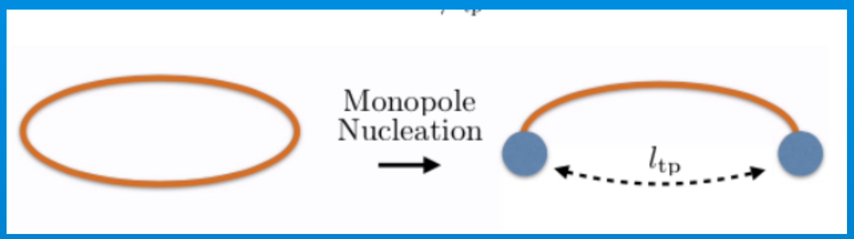




Dunsky, Ghoshal, Murayama and Sakakihara White 2111.08750

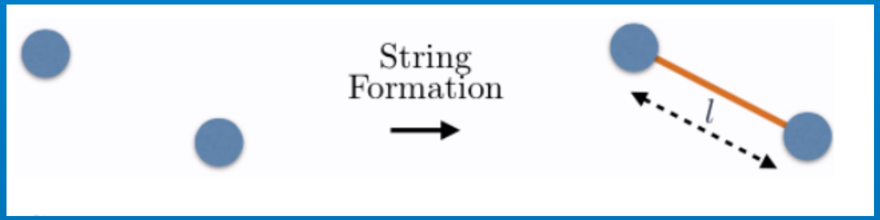
Hybrid defects

If v_1 is close to v_2 the hybrid defect appears through a tunneling process

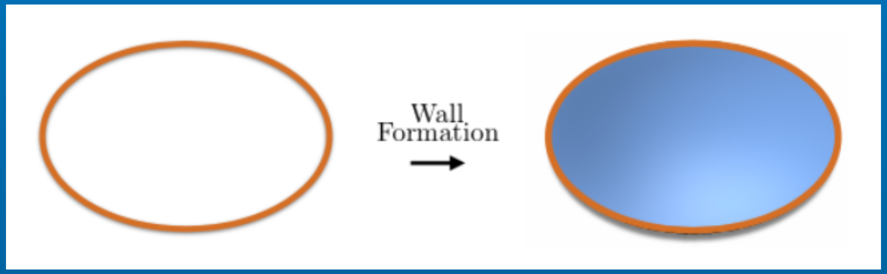


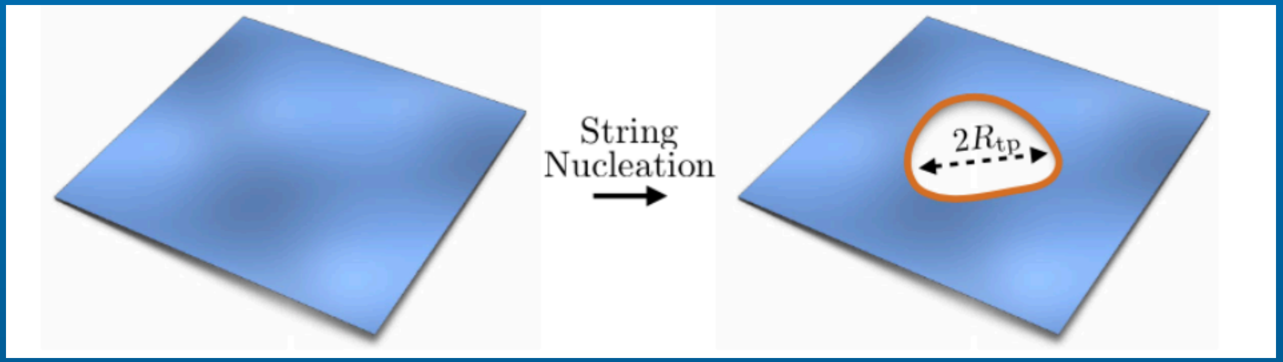
$$\Gamma_{\text{monopole}} \sim e^{-m^2/\mu} \sim e^{-v_1^2/v_2^2}$$

$$\Gamma_{\text{string}} \sim e^{-\mu^3/\sigma^2} \sim e^{-v_1^6/v_2^6}$$



Otherwise the strings/DWs appear as flux tubes/ sheets enclosed by monopoles/strings

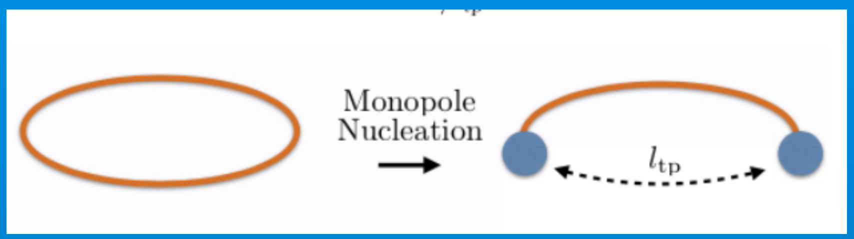


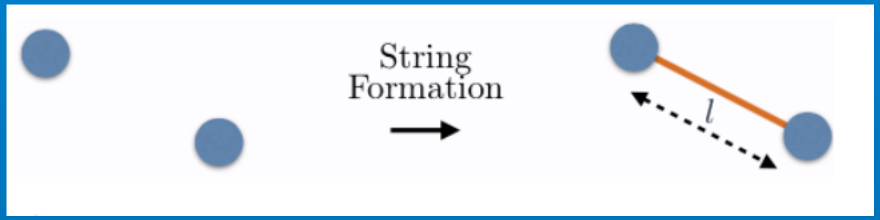


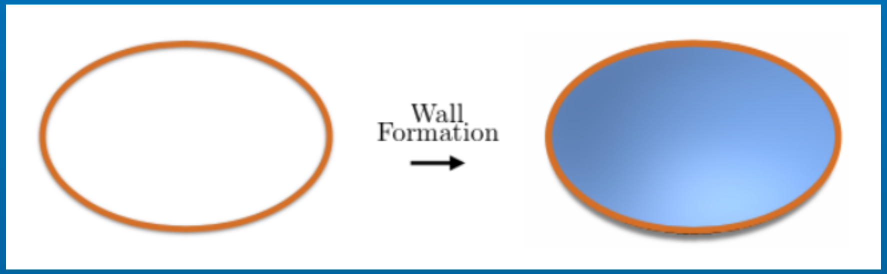
Dunsky, Ghoshal, Murayama and Sakakihara White 2111.08750

Hybrid defects

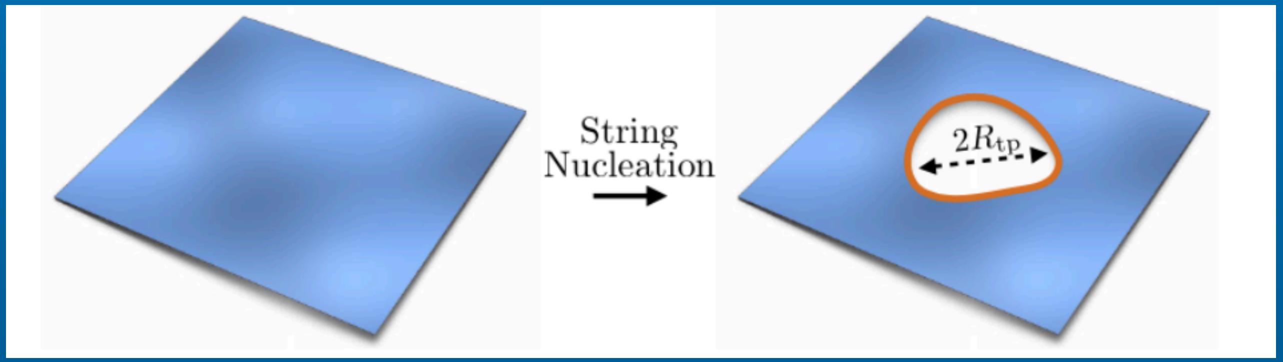
All four gravitational wave signals are distinguishable







Cosmic Course	IR	UV
Monopoles Eating String Network (Nucleation)	f^2	f^0
Strings Eating Domain Wall Network (Nucleation)	f^3	f^{-1}
Domain Walls Eating String Network (Collapse)*	f^3	f^0
Strings Eating Monopole Network (Collapse)*	f^3	$\ln f \to f^{-1}$

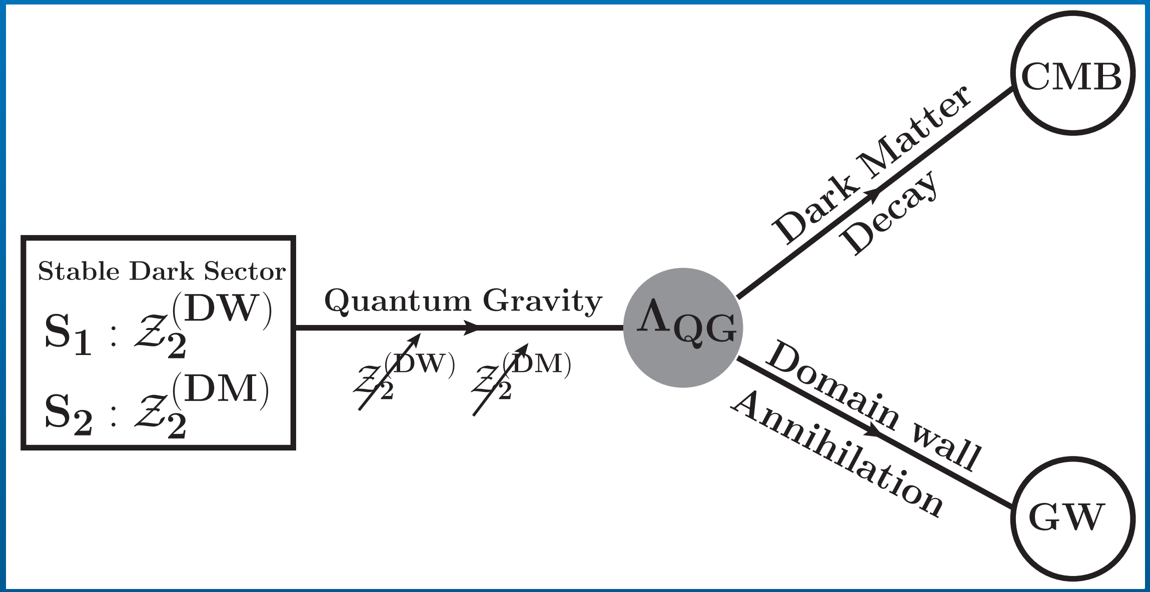


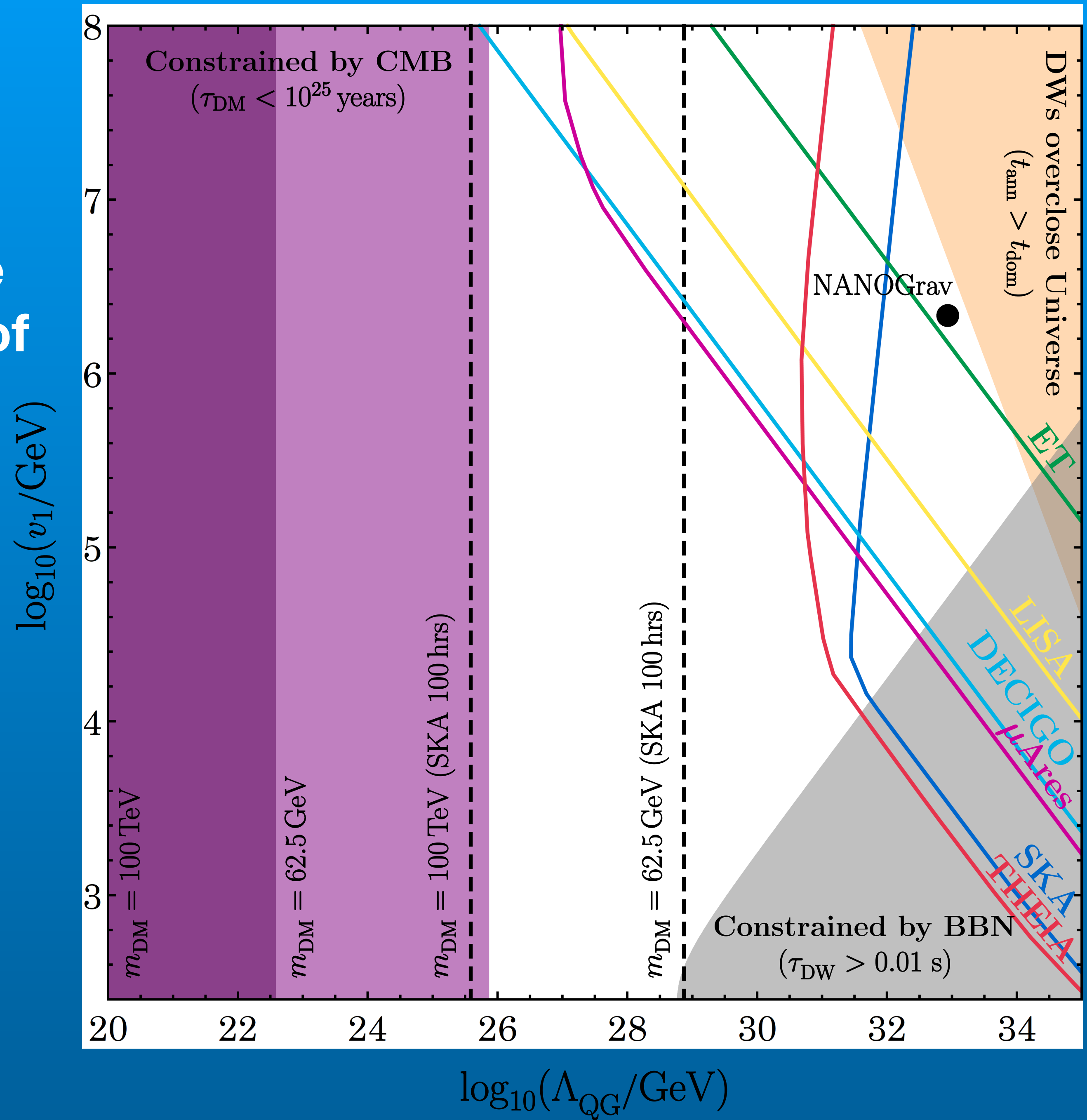
Dunsky, Ghoshal, Murayama and Sakakihara White 2111.08750

How to get a small bias term to kill domain walls

Quantum gravity is expected to kill all global symmetries
The scale of symmetry breaking is not the Planck scale, but above that as violation of global symmetries require gravitational instantons like wormholes

$$\mathcal{O}_{\text{sym,br}} \sim \frac{1}{\Lambda_{QG}} \sim \frac{e^{-S}}{M_p}$$





King, Roshan, Wang, White, Tamazaki 2308.03724

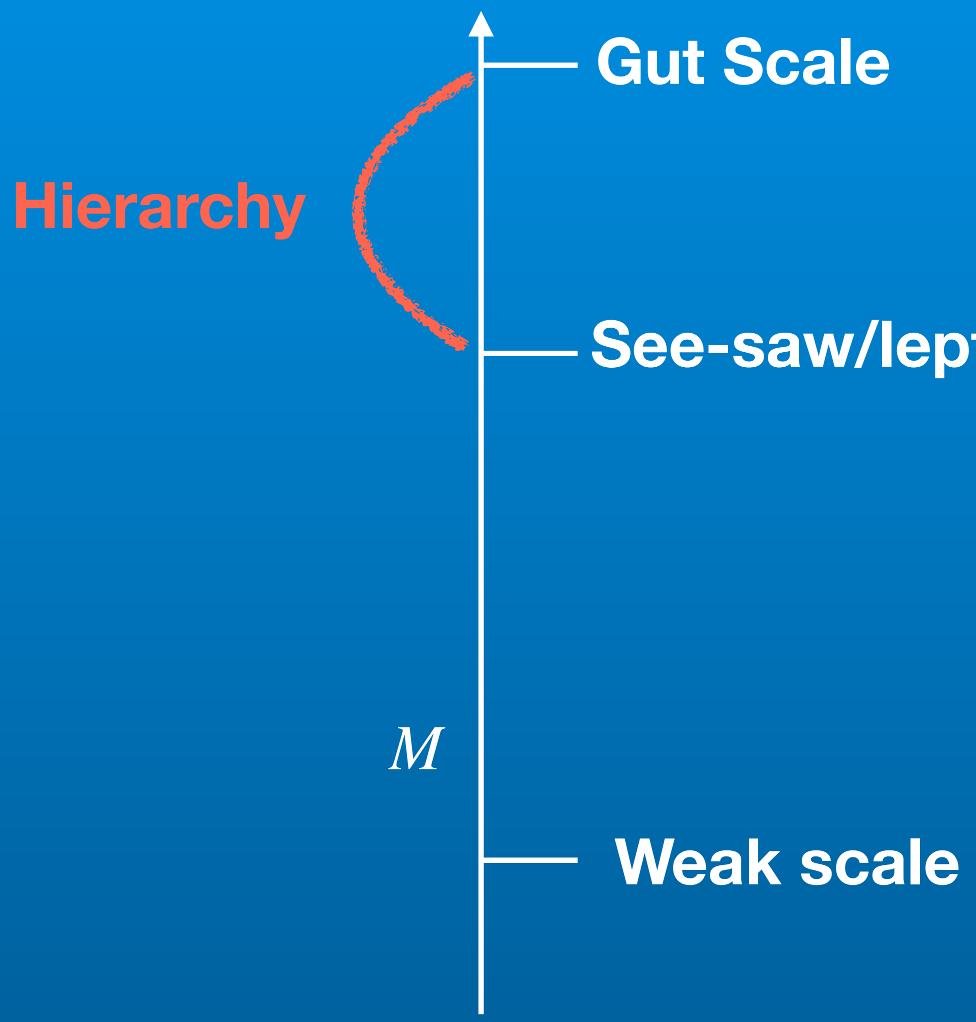
Application of strings to baryogenesis

$$\Delta\mathscr{L} = M\bar{N}^C N$$

- Explains light neutrinos
- Violates lepton number
- Their decay can explain why there is more matter than anti-matter

Dror, Hiramatsu, Kohri, Murayama White 1908.03227

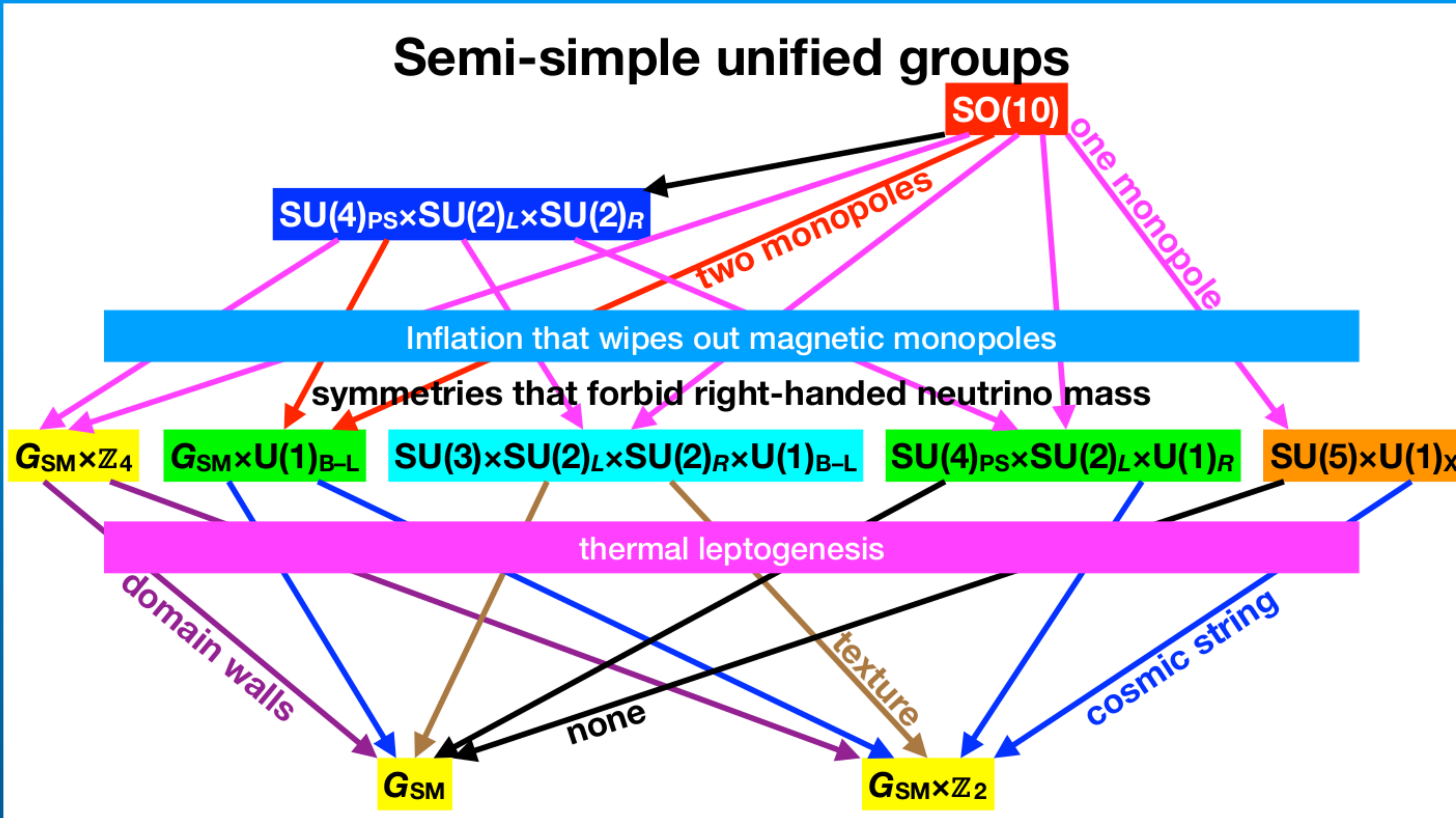
Application of strings to baryogenesis



Protect scale of seesaw-leptogenesis with a $U(1)_{B-L}$ gauge symmetry

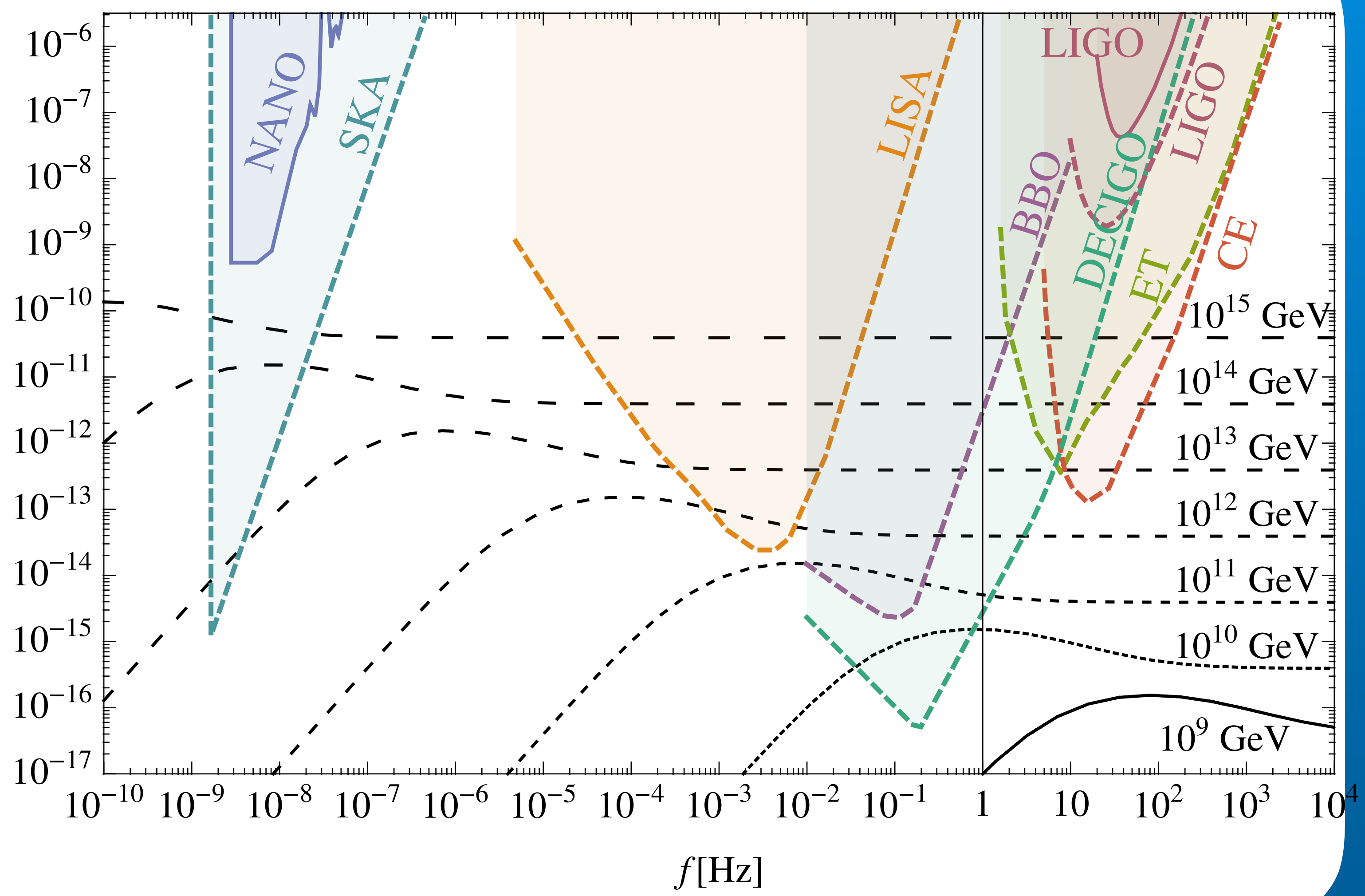
Dror, Hiramatsu, Kohri, Murayama White 1908.03227

Application of strings to baryogenesis



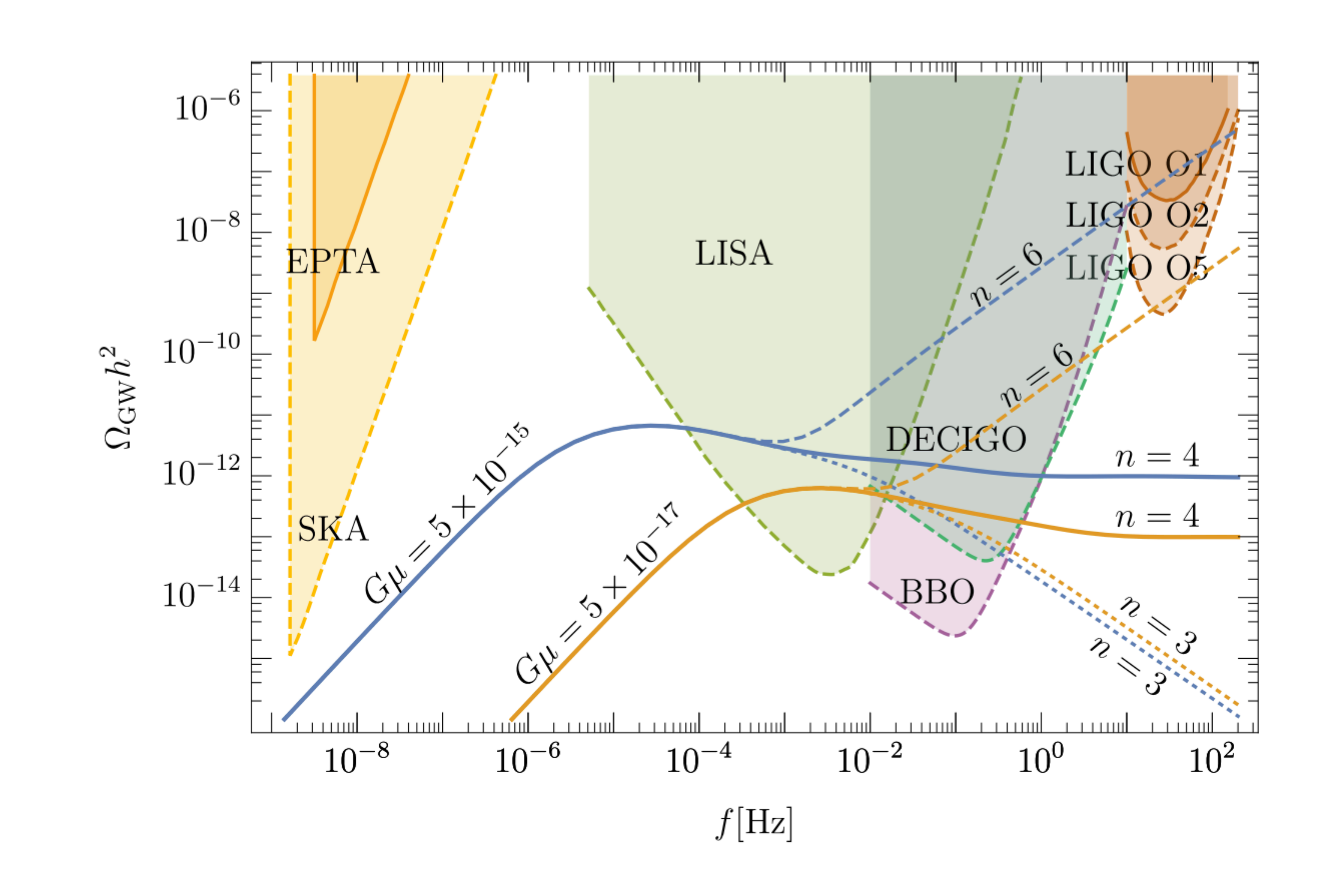
Dror, Hiramatsu, Kohri, Murayama White 1908.03227

Application of strings to baryogenesis



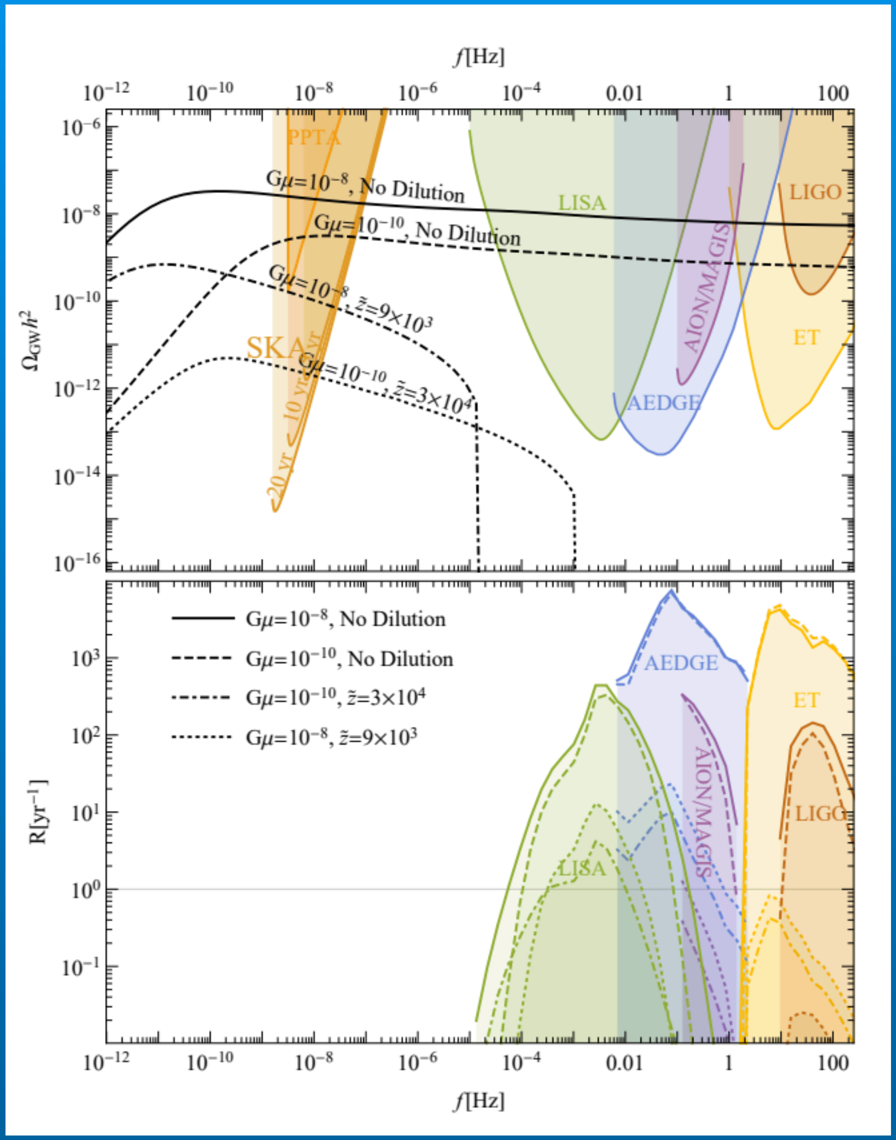
Dror, Hiramatsu, Kohri, Murayama White 1908.03227

Application of strings to cosmic history



Yanou Cui, Marek Lewicki, David E. Morrissey, and James D. Wells 1711.03104

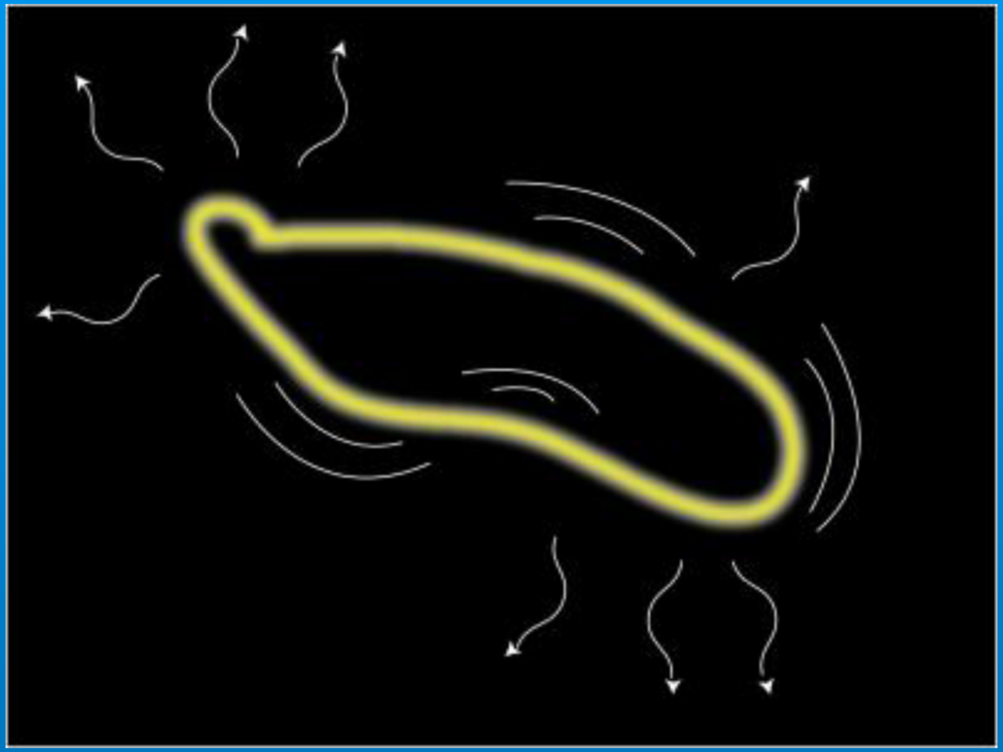
Application of strings to pre inflation

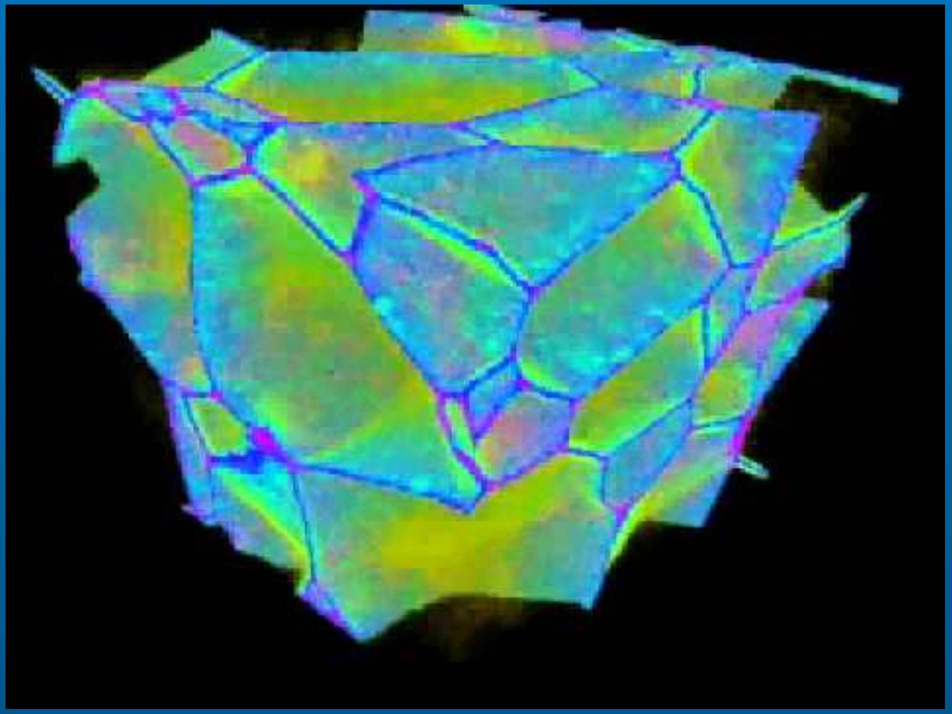


Yanou Cui, Marek Lewicki and David E. Morrissey 1912.08832

Outlook

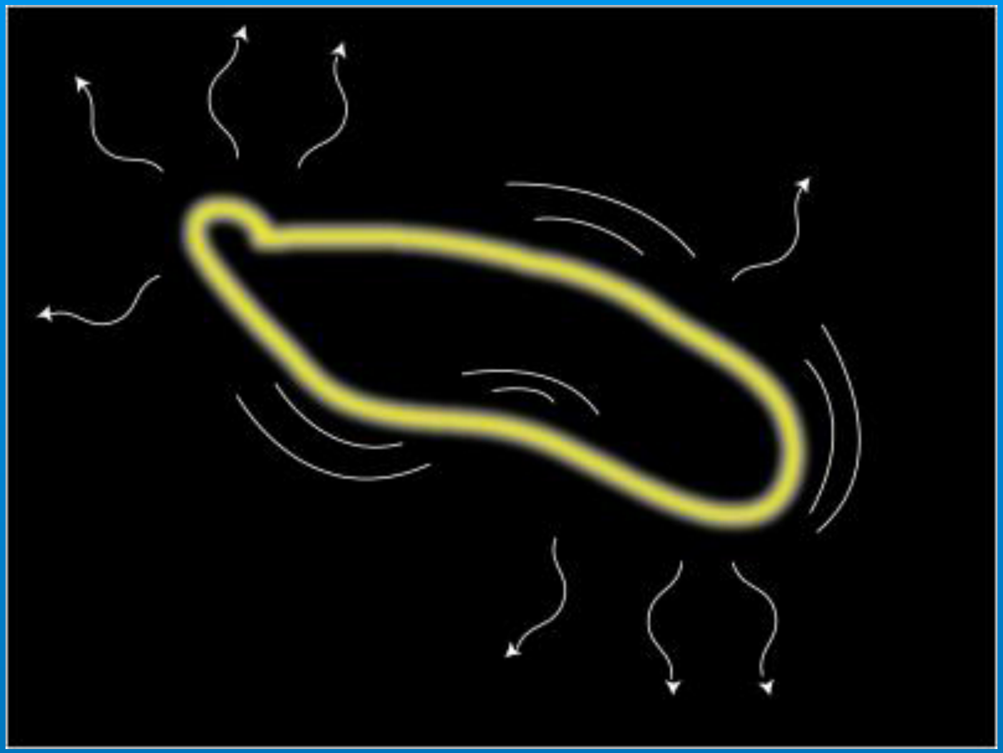
- Still exploring all applications
- Incredible potential to test high scale physics
- Large uncertainties for cosmic strings

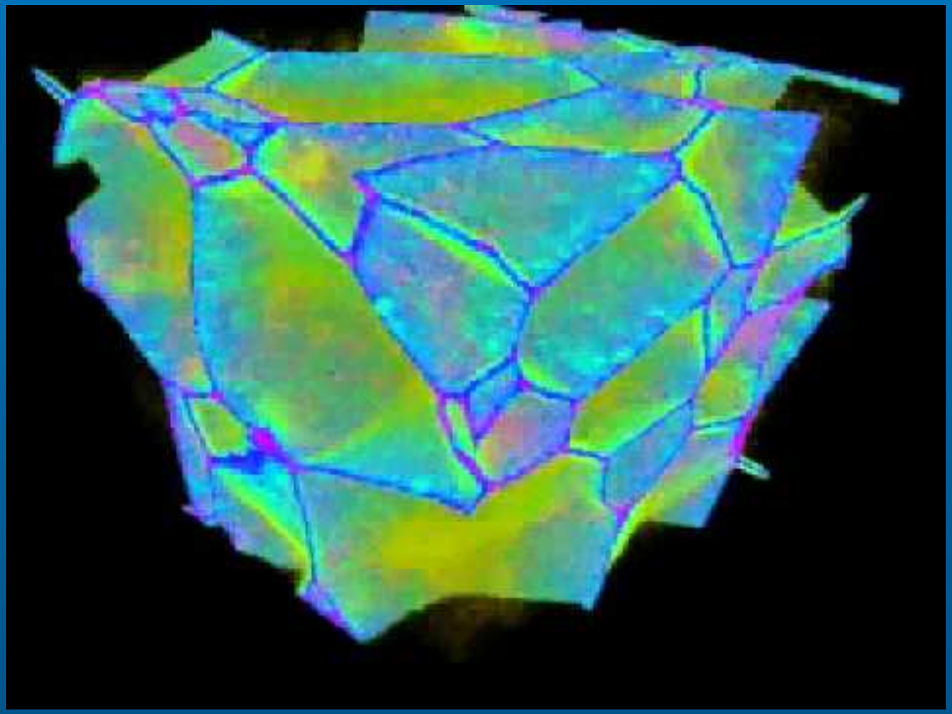




Summary cosmic defects

- Sometimes when a symmetry is spontaneously broken, the vacuum manifold is topologically interesting
- Strings, domain walls and textures (not covered) have strong gravitational wave signals
- Such defects are very common in grand unification
- There can be hybrid defects where strings connect to domain walls or monopoles. These have a unique GW signal
- Because the scale of new physics is not connected to a frequency peak in the signal, this is a great way to test high scale physics





Lecture series summary part 1

- The Universe is transparent to gravitational waves right to the beginning
- There are three *main* sources of gravitational waves the community focuses on
  - Phase transitions
  - Scalar induced Gws
  - Cosmic defects
- For all three sources we require theoretical and numerical work to improve our understanding

Lecture series summary part 2

- Source for phase transitions is peaked at the mean bubble separation - can probe up to around 10 TeV
- Source for early matter domination is peaked around Hubble size when the equation of state changes - can probe around 100 TeV
- Source for defects do not have a clear link between scale of physics and frequency window
  - Strings are flat and grow with the scale of new physics
  - Domain walls grow with how long they last, which is from the size of the bias term

Conclusion

- Gravitational wave archaeology is a unique probe of physics we cannot access on Earth
- All major sources of gravitational waves have major theoretical uncertainties
- Only just learning about the vast applications