

Emergent Classicality from Information Bottleneck



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Zhelun Zhang, YZY. arXiv: 2306.14838

What is Classicality?

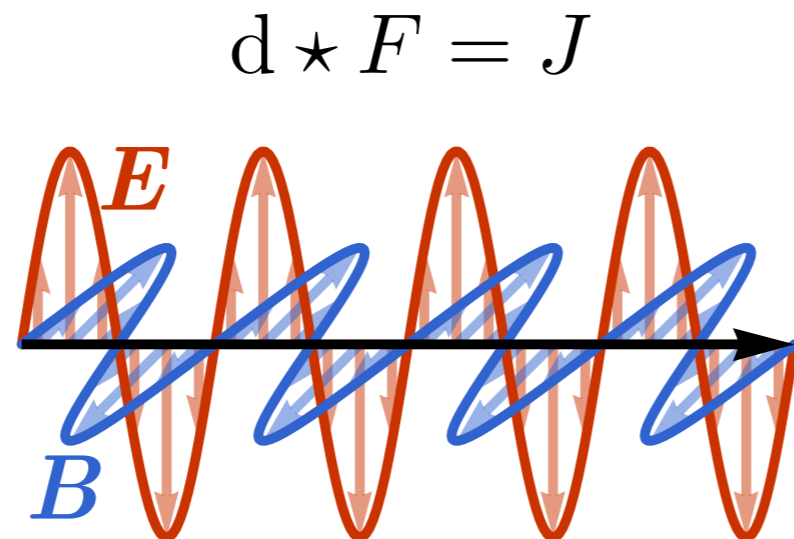
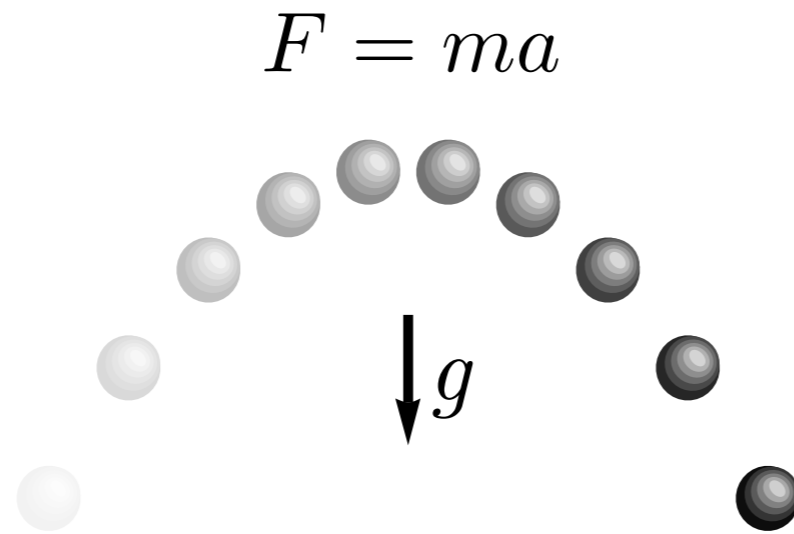
- In physics, the word “**classical**” is used in contrast to “**quantum**”: classical physics refers to physics before quantum mechanics.



Issac Newton



James Maxwell



- Classical physics is **deterministic**.
- It works pretty well in the **macroscopic** world.

How is Quantum Differed from Classical?

- In the early 20th century, it was realized that classical physics does not quite apply to the **microscopic** world.



Max Planck

Albert Eistein

Niels Bohr

E. Schrödinger

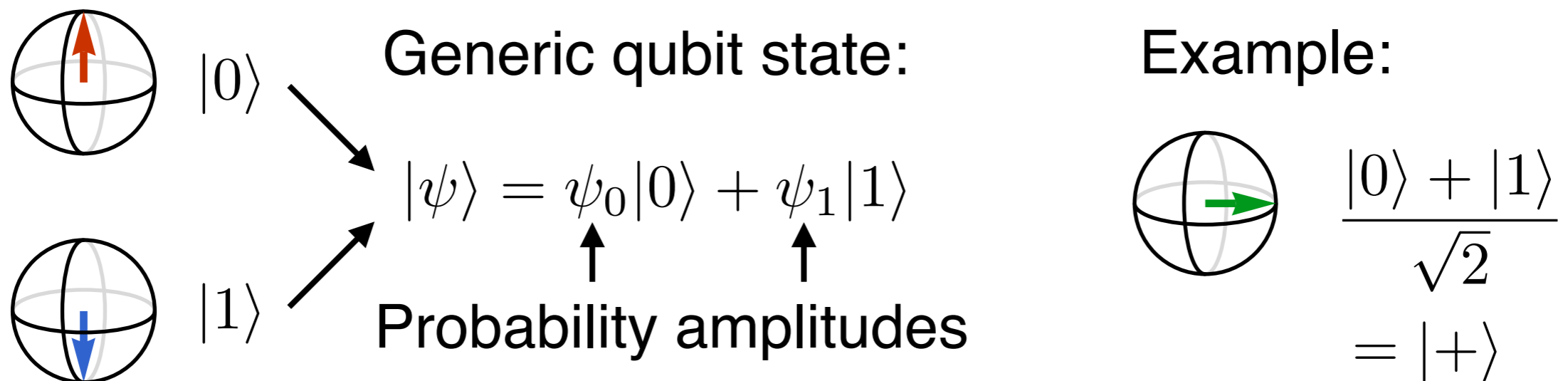
W. Heisenberg

- A new branch of physics — **quantum** mechanics — was established. It is intrinsically **probabilistic**.
- Quantum mechanics is more exotic: it describes the square root of probability — called **probability amplitude**.

$$\psi(x) \sim \pm \sqrt{p(x)}$$

Quantum Mechanics

- Quantum superposition
 - In quantum mechanics, (pure) **states** of a system are described by **vectors**, and they can be **linearly combined**.
 - Similar to word vectors in natural language processing.
 - A physical example: **qubit** — quantum bit.



When measured in 0/1 basis, the probability to observe 0/1 is given by: $p(0|\psi) = |\psi_0|^2$ or $p(1|\psi) = |\psi_1|^2$.

Quantum Mechanics

- In quantum mechanics, physical **observables** are represented by **matrices**.
- For a single qubit, there are three orthogonal observables

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

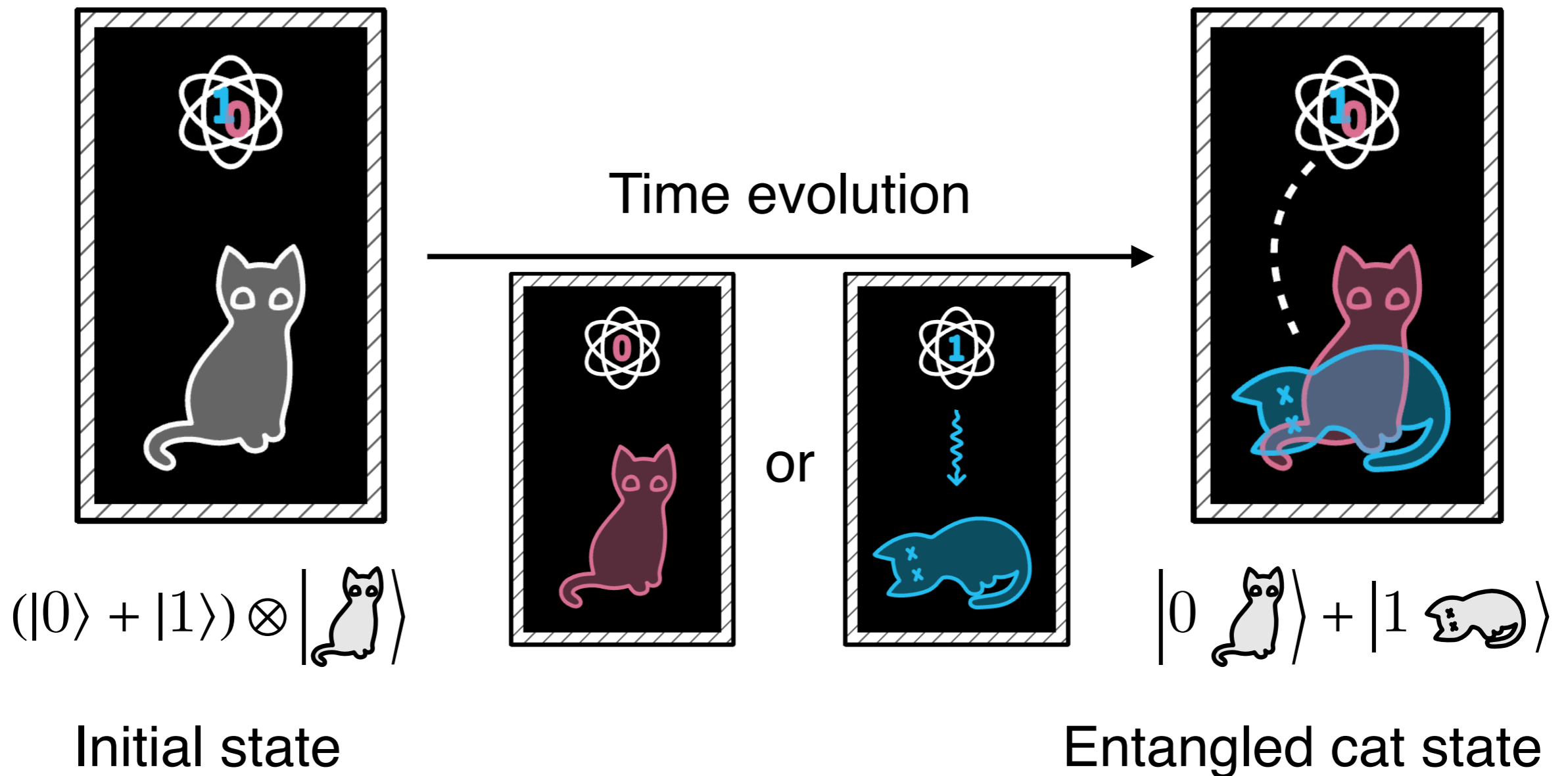
(Pauli observables)

They all have binary measurement outcomes ± 1 .

state	reality
$ 0\rangle$	$Z = +1$
$ 1\rangle$	$Z = -1$
$ +\rangle := \frac{ 0\rangle + 1\rangle}{\sqrt{2}}$	$X = +1$
$ -\rangle := \frac{ 0\rangle - 1\rangle}{\sqrt{2}}$	$X = -1$

Schrödinger's Cat

- Quantum superposition can become weirder when it comes to states of multiple qubits — a famous example is **Schrödinger's cat**.

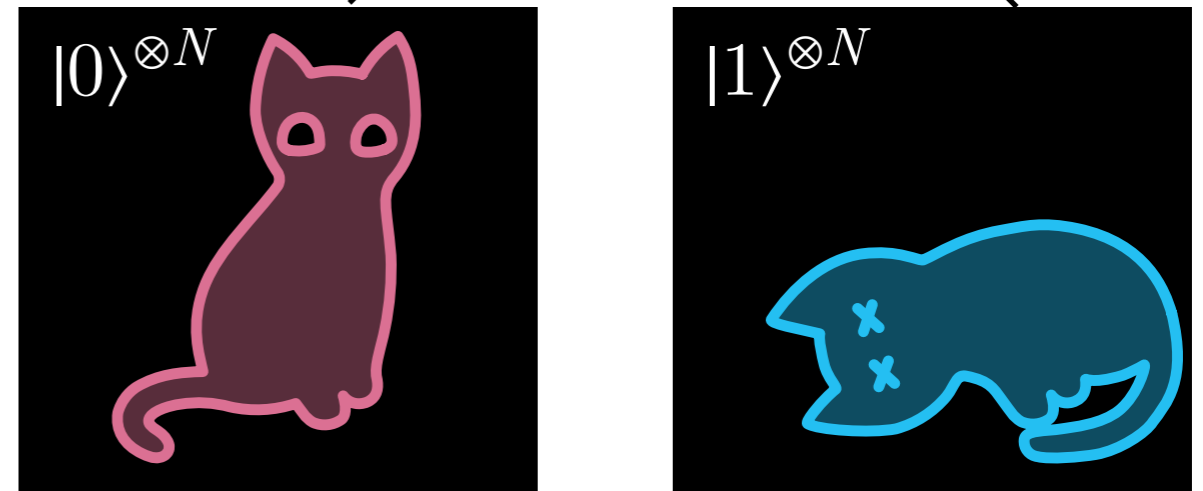


Schrödinger's Cat

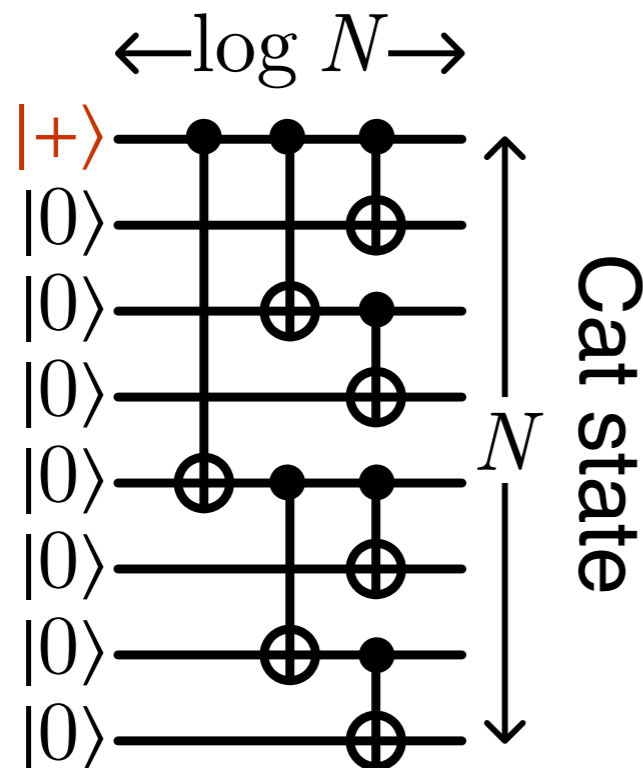
- The cat state can be modeled by a multi-qubit GHZ state,

$$|+\rangle := \frac{|0\rangle + |1\rangle}{\sqrt{2}} \xrightarrow[\text{(Quantum circuit)}]{\text{Time evolution}} \frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}$$

which can be prepared by a quantum circuit in $\log N$ depth (time).



Greenberger, Horne, Zeilinger 1989

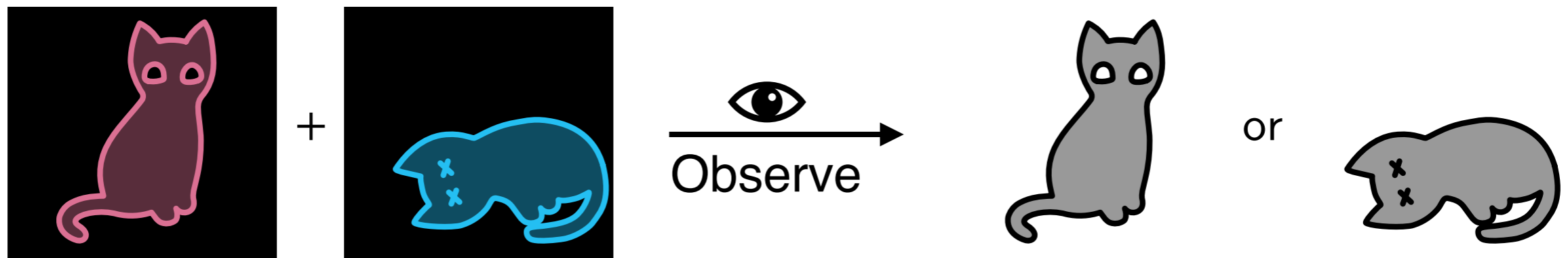


CNOT (controlled-NOT) gate

$$\begin{array}{c}
 a \\
 | \\
 b
 \end{array}
 \begin{array}{c}
 \bullet \\
 | \\
 \oplus
 \end{array}
 \begin{array}{c}
 a \\
 \\
 a \oplus b
 \end{array}
 \quad a, b \in \{0, 1\}
 \quad |a\rangle \otimes |b\rangle \rightarrow |a\rangle \otimes |a \oplus b\rangle$$

Quantum State Collapse

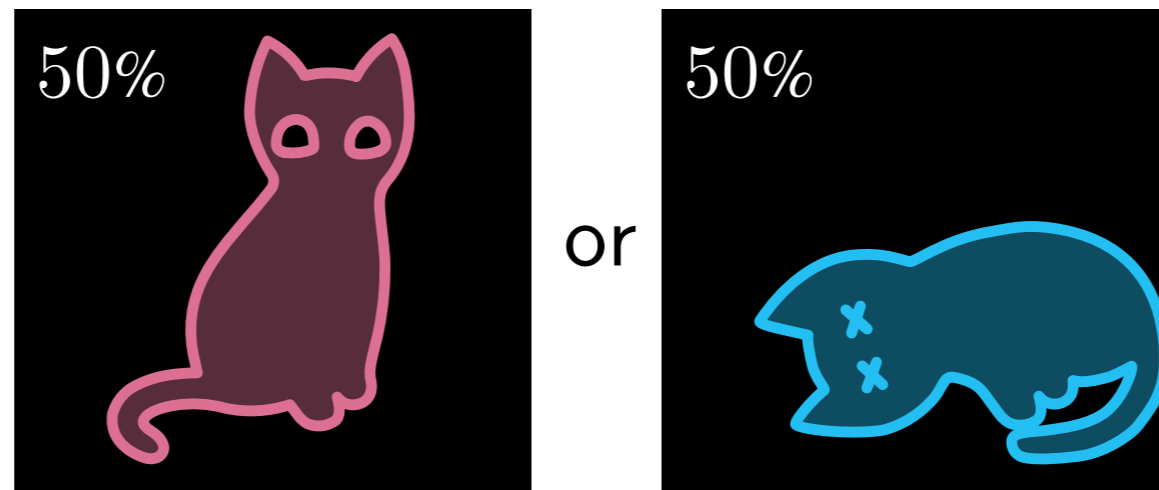
- But we never see a superposition cat in reality. Why?
- **Copenhagen Interpretation:** **Observing** the cat would cause the superposition to **collapse** into one of the two classical realities: cat alive or cat dead.



- What happens during the observation?
- Who qualifies as an observer?
- Should the observer be conscious/intelligent? ...

Quantum State Collapse

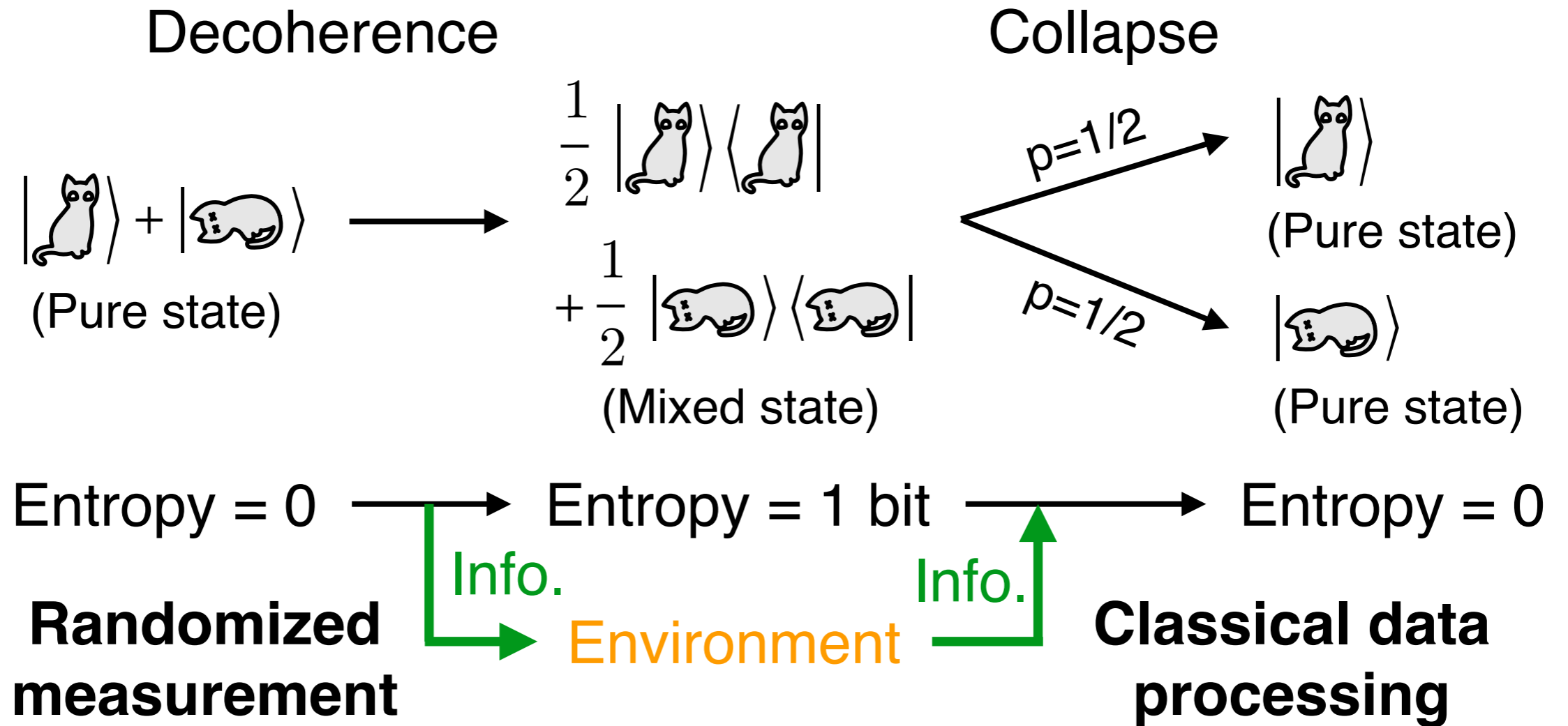
- **Modern understanding:** randomized measurement + classical data processing.
- Measurement: the system interacts with the environment.



- Interaction → entanglement (information sharing).
- Information loss = entropy increase:
pure cat state → mixed state ensemble of alive and dead.
- This process is called **quantum decoherence**. No intelligence is required at this step.

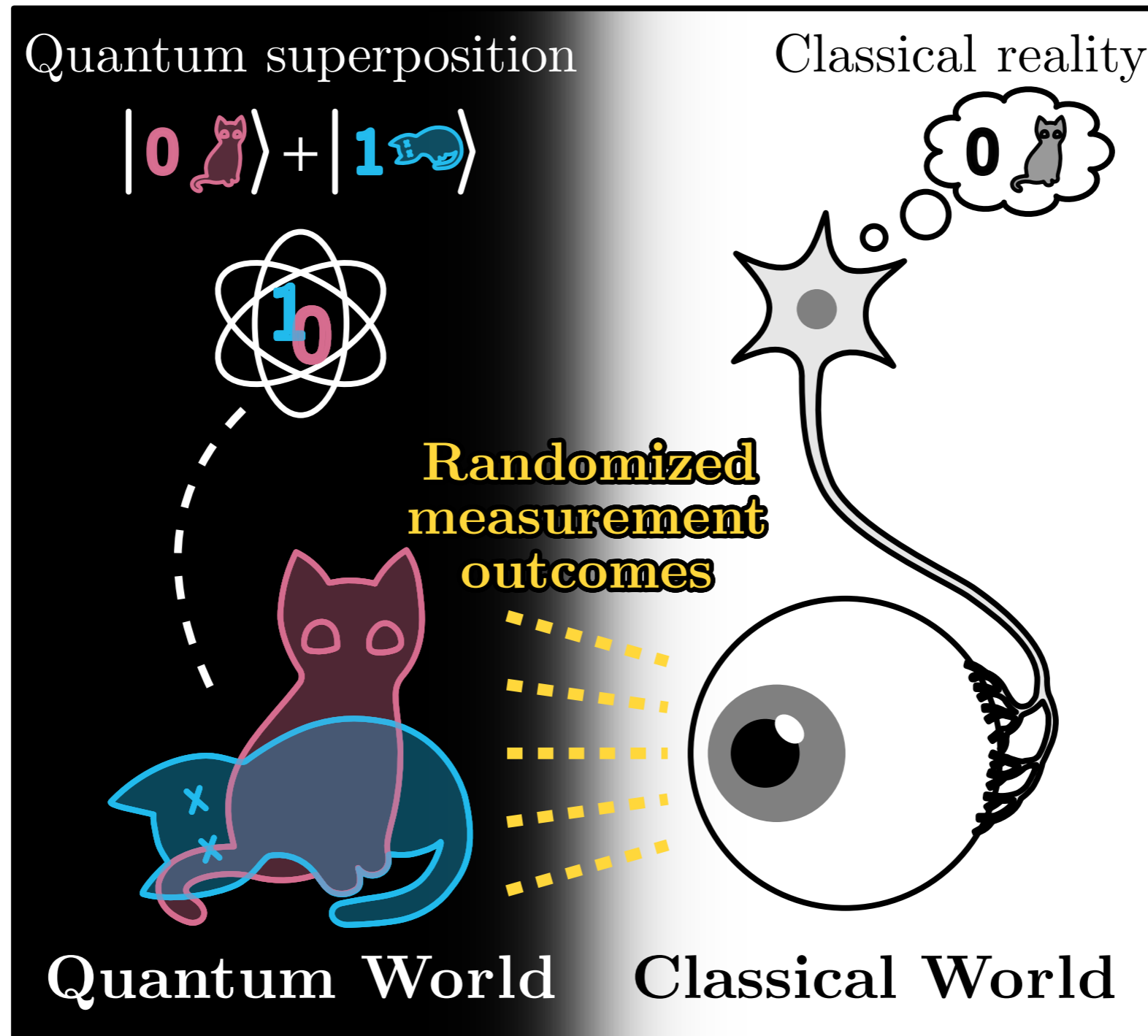
Quantum State Collapse

- **Modern understanding:** randomized measurement + classical data processing.
- **Emergent classical reality:** how to collapse from the mixed state back to one of the alive/dead pure states



General Idea

- **Idea:** use AI to process randomized measurement data.



Randomize Measurement

- **Randomized measurement** — estimate properties of an unknown quantum state by measuring random observables.
 - Philosophy: measure first, ask questions later.
- **Measurement scheme:**
 - Prepare an N -qubit GHZ state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N})$
 - Perform **random & local** measurements:
 - Draw a sequence of Pauli observables uniformly
$$\mathbf{x} = (x_1, x_2, \dots, x_N), \quad x_i \in \{X, Y, Z\}$$
 - Independently measure each qubit i by its corresponding observable x_i
 - Collect measurement outcomes as a sequence
$$\mathbf{y} = (y_1, y_2, \dots, y_N), \quad y_i \in \{\pm 1\}$$
 - Repeat ...

Randomize Measurement

- Randomized measurements collect a large amount of data.
- **Data structure:** a pair of sequences

$$(\mathbf{x}, \mathbf{y}) \quad \mathbf{x} \in \{X, Y, Z\}^{\times N}, \mathbf{y} \in \{\pm 1\}^{\times N}$$

- **Data distribution:** $p(\mathbf{x}, \mathbf{y}) = p(\mathbf{y}|\mathbf{x})p(\mathbf{x})$

$$p(\mathbf{x}) = 3^{-N} \quad (\text{Uniform, trivial})$$

$$p(\mathbf{y}|\mathbf{x}) = \langle \Psi | \bigotimes_i \frac{1 + y_i x_i}{2} | \Psi \rangle$$

↑
Non-trivial. Encodes all quantum information about the cat state $|\Psi\rangle$

Randomize Measurement

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- **Classical post-processing:** (\mathbf{x}, \mathbf{y}) are also called **classical shadows**, from which the quantum state can be recovered.

$$\rho := |\Psi\rangle\langle\Psi| = \mathbb{E}_{(\mathbf{x}, \mathbf{y})} \bigotimes_i \frac{1 + 3y_i x_i}{2}$$

Randomize Measurement

- Randomized measurements collect a large amount of data.

$$(\mathbf{x}, \mathbf{y}) \quad \mathbf{x} \in \{X, Y, Z\}^{\times N}, \mathbf{y} \in \{\pm 1\}^{\times N}$$

$$(\mathbf{x}, \mathbf{y}) \sim p(\mathbf{x}, \mathbf{y}) = p(\mathbf{y}|\mathbf{x})p(\mathbf{x})$$

- Examples ($N = 4$): classical shadows of Schrödinger's cat

x: XYYX	x: XYYZ	x: XZYG	x: XZXX	x: XYXY	x: XYZZ
y: -+--	y: -+--	y: --+-	y: -+-+	y: +-++	y: +-++
x: XZZZ	x: XZXY	x: YZZZ	x: YZXX	x: YXXY	x: YXZX
y: +----	y: ++--	y: ----	y: -++-	y: ++-+	y: ++++
x: YYXZ	x: YYYX	x: ZYXX	x: ZZYG	x: ZZZY	x: ZZZX
y: ++-+	y: +++-	y: -+--	y: ----+	y: ----+	y: ++++

Generative Modeling of Classical Shadows

- **Objective:** to model the conditional distribution $p(\mathbf{y}|\mathbf{x})$ of measurement outcomes given local observables.

\mathbf{x} : ZZXY Observables (question)

\mathbf{y} : ---+ Outcomes (answer)

- This maps to a **chat completion** task in natural language processing. — We can train a transformer-based **generative language model** to perform the task.

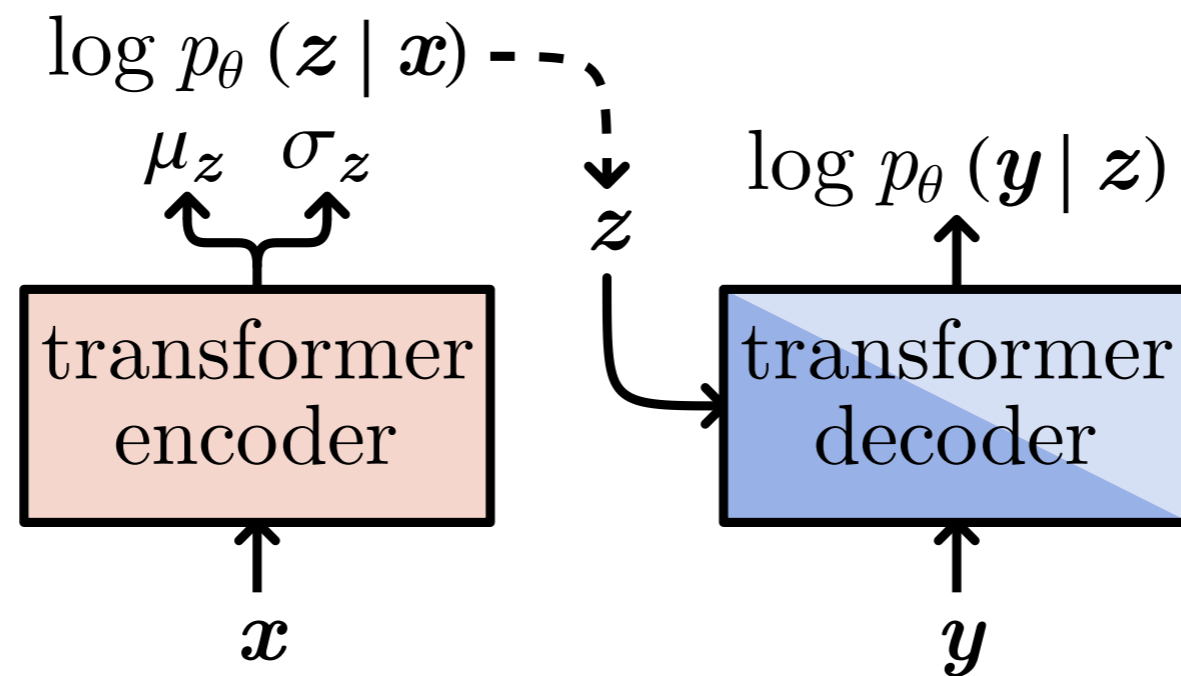
Vaswani et al. 2017; Devlin et al. 2019

- After training, the model can replace the quantum experiment to answer questions about the underlying quantum state (the cat state). — It can “speak” the quantum language.

Generative Modeling of Classical Shadows

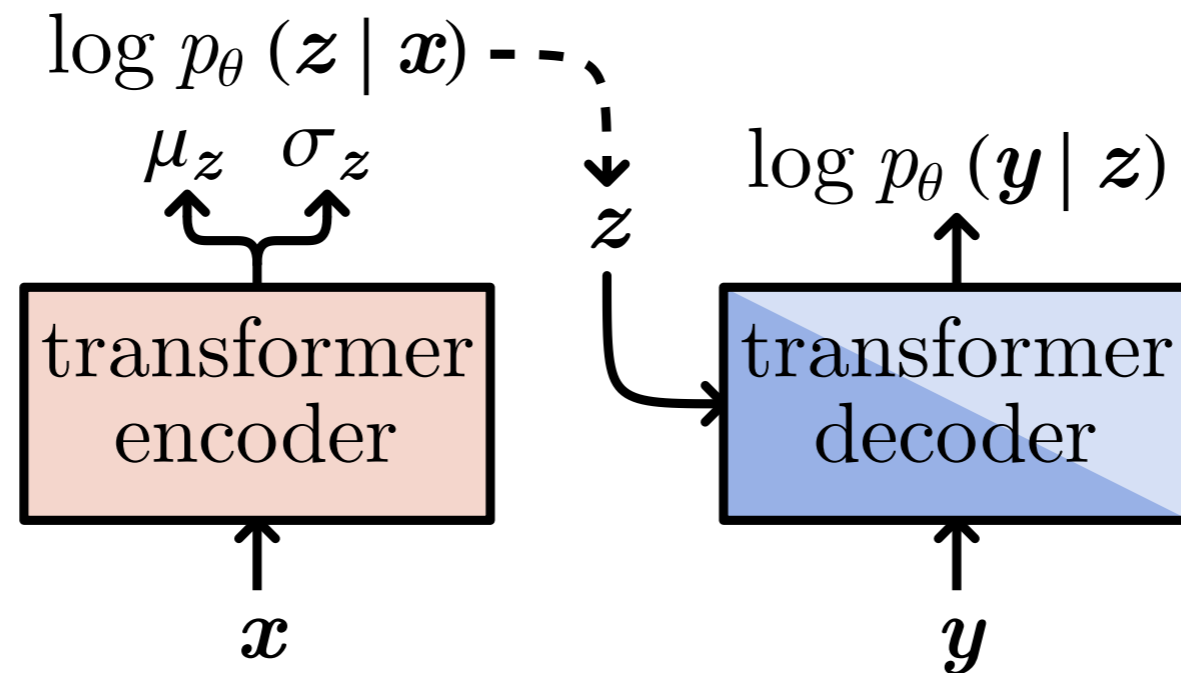
- **Objective:** to model the conditional distribution $p(\mathbf{y}|\mathbf{x})$ of measurement outcomes given local observables.
- **Architecture:** transformer-based β -VAE

$$p_{\theta}(\mathbf{y}|\mathbf{x}) = \int_{\mathbf{z}} p_{\theta}(\mathbf{y}|\mathbf{z})p_{\theta}(\mathbf{z}|\mathbf{x})$$



Generative Modeling of Classical Shadows

- **Loss function (ELBO):** $\mathcal{L} = \mathbb{E}_{(\mathbf{x}, \mathbf{y}) \sim p_{\text{dat}}} \mathcal{L}(\mathbf{x}, \mathbf{y})$



$$\mathcal{L}(\mathbf{x}, \mathbf{y}) = - \mathbb{E}_{\mathbf{z} \sim p_\theta(\mathbf{z} | \mathbf{x})} \log p_\theta(\mathbf{y} | \mathbf{z}) \quad \text{Negative log-likelihood}$$
$$+ \beta \text{KL}[p_\theta(\mathbf{z} | \mathbf{x}) || p_{\mathcal{N}}(\mathbf{z})] \quad \text{KL regularization}$$

- Hyperparameter β enables us to impose a variational **information bottleneck** on the transformer.

Model Evaluation

- **Evaluation metric: fidelity** — a measure of the closeness between quantum states.

- Original state ($|\Psi\rangle$ - the GHZ state):

$$|\Psi\rangle\langle\Psi| = \rho = \mathbb{E}_{(\mathbf{x}, \mathbf{y}) \sim p_{\text{dat}}} \bigotimes_i \frac{1 + 3y_i x_i}{2}$$

- Reconstructed state:

$$\tilde{\rho} = \mathbb{E}_{(\mathbf{x}, \tilde{\mathbf{y}}) \sim p_{\text{mdl}}} \bigotimes_i \frac{1 + 3\tilde{y}_i x_i}{2}$$

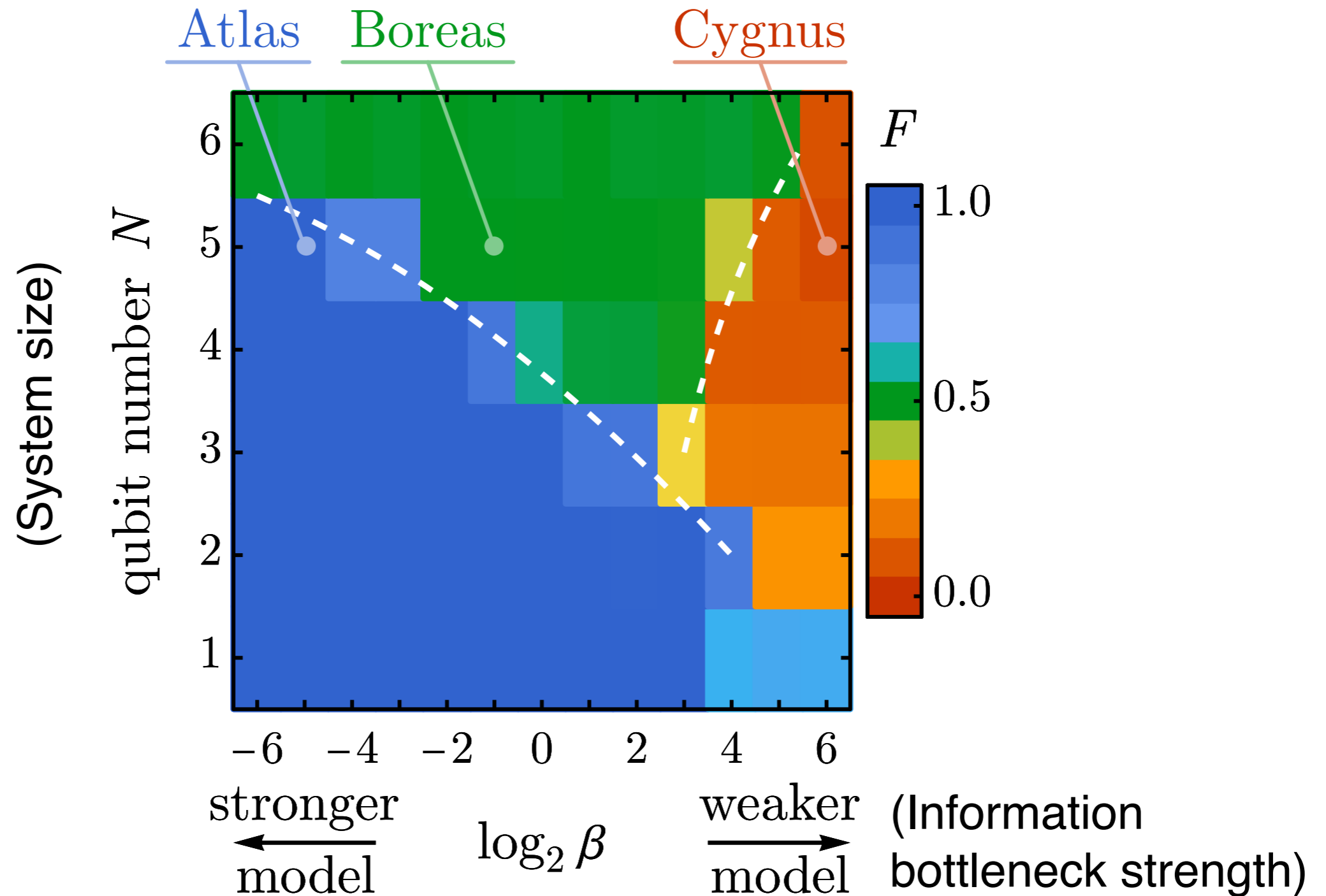
- Fidelity (the probability of observing $\tilde{\rho}$ given $|\Psi\rangle$)

$$F(\rho, \tilde{\rho}) := \left(\text{Tr} \sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}} \right)^2 = \langle \Psi | \tilde{\rho} | \Psi \rangle$$

In general, $0 \leq F(\rho, \tilde{\rho}) \leq 1$ (the larger the better).

Model Evaluation

- Fidelity of the model reconstructed quantum state



One-Shot Cat Classification

- To understand the difference between Atlas, Boreas and Cygnus, let us chat with them!
- We can ask them for the “one-shot cat classification”.

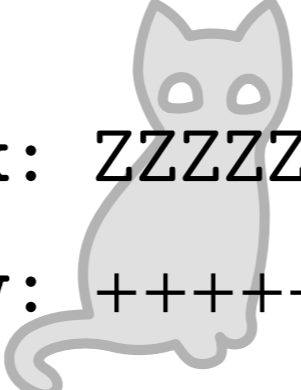

Task: given a one-shot observation of the cat, determine if it is alive or dead.



Prompt:

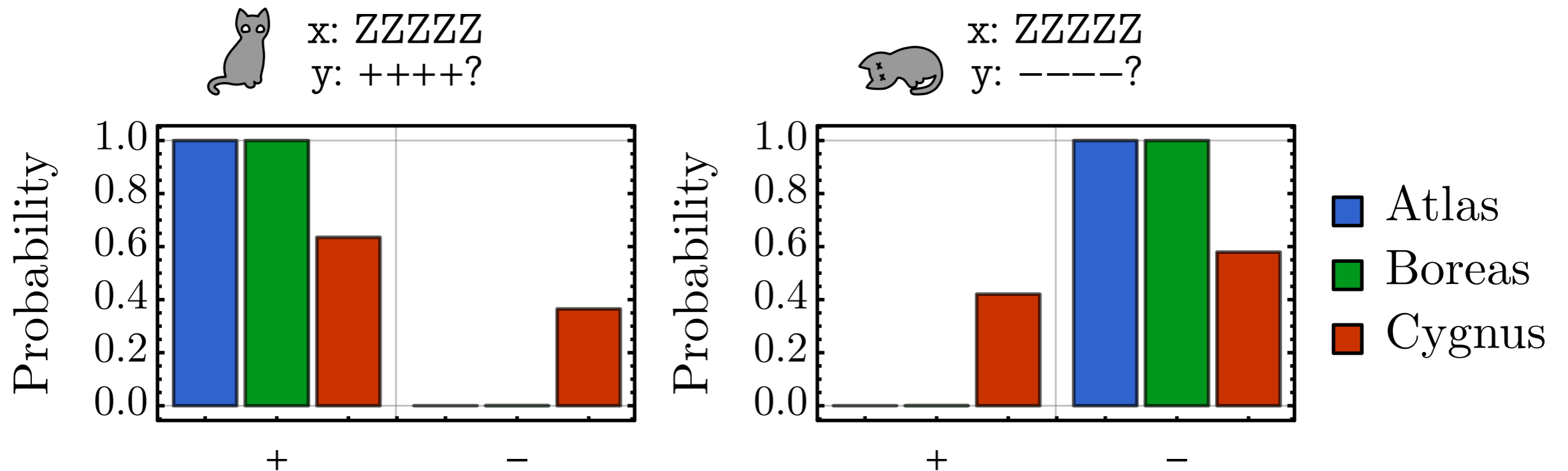
x: ZZZZZ	x: ZZZZZ	...
y: ++++?	y: ----?	...

Expectation:

x: ZZZZZ	x: ZZZZZ	...
y:  ++++	y:  -----	...

One-Shot Cat Classification

- In-distribution classification task



- **Atlas** and **Boreas** can perfectly determine the life and death of the cat.
- However, **Cygnus** is a weaker model and cannot make a clear judgment about the classical reality.

One-Shot Cat Classification

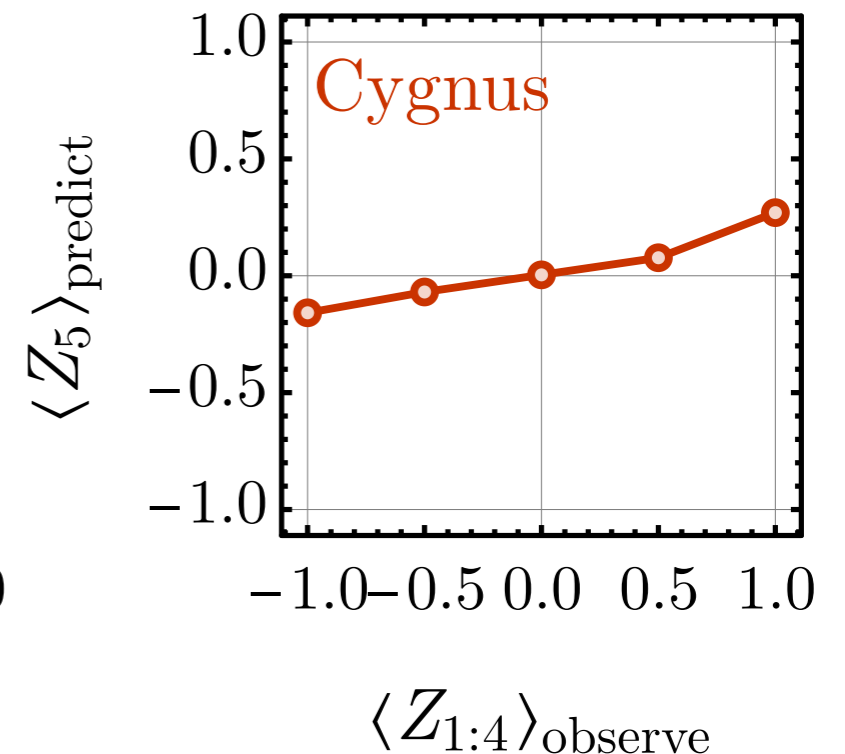
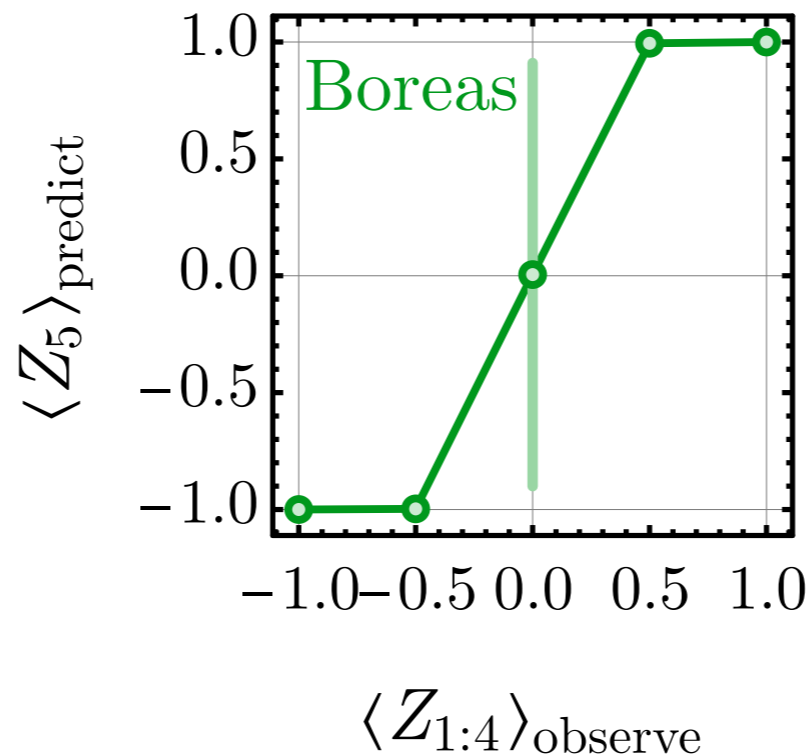
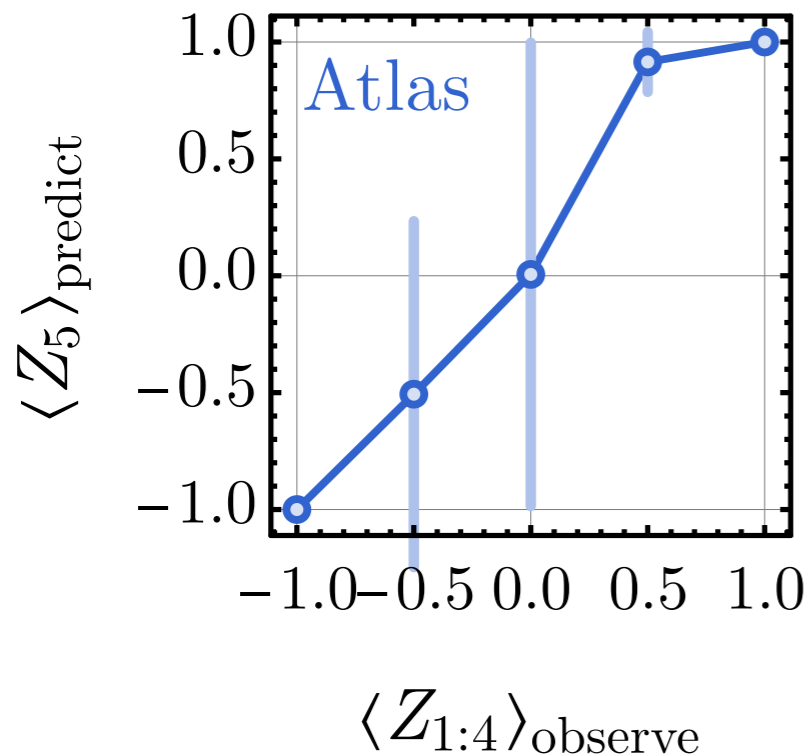
- Out-of-distribution classification task
 - What about the following prompt?

x: ZZZZZ

(This never appears in the classical shadow data of the GHZ state.)

y: -+-?

$\underbrace{\hspace{1.5cm}}_{Z_{1:4}} \quad \swarrow_{Z_5}$



One-Shot Cat Classification

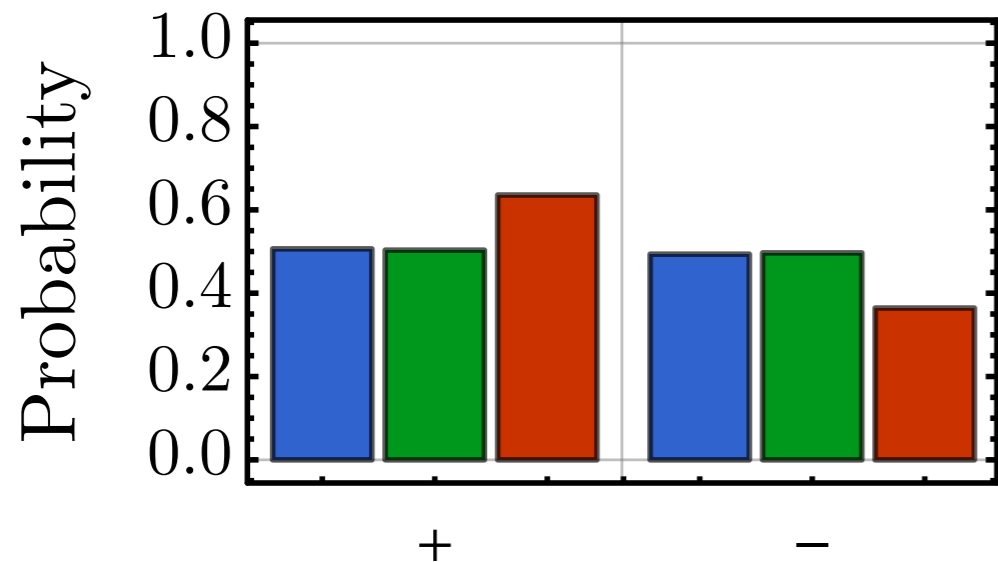
- Local Z -measurements destroy the quantum coherence of the cat state. Can we preserve the coherence?
- Consider local X -measurements:



x: XXXXZ
y: +++++?



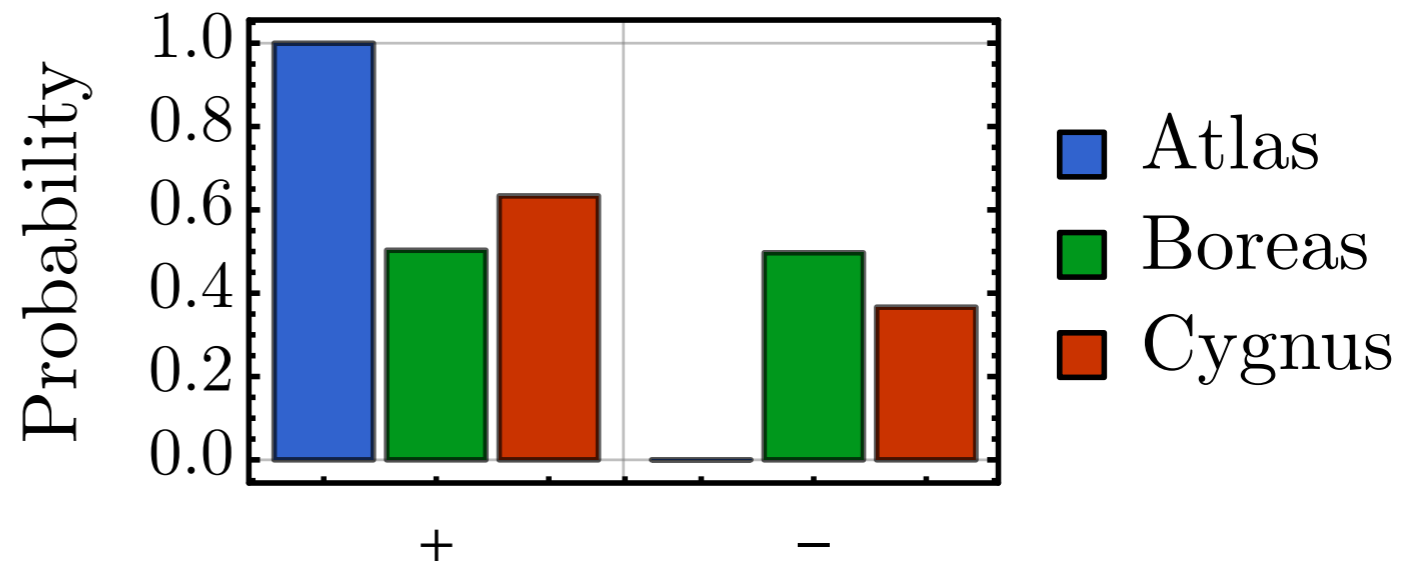
x: XXXXX
y: +++++?



Q: Is the Schrödinger cat alive or dead?

(+) Alive.

(-) Dead.



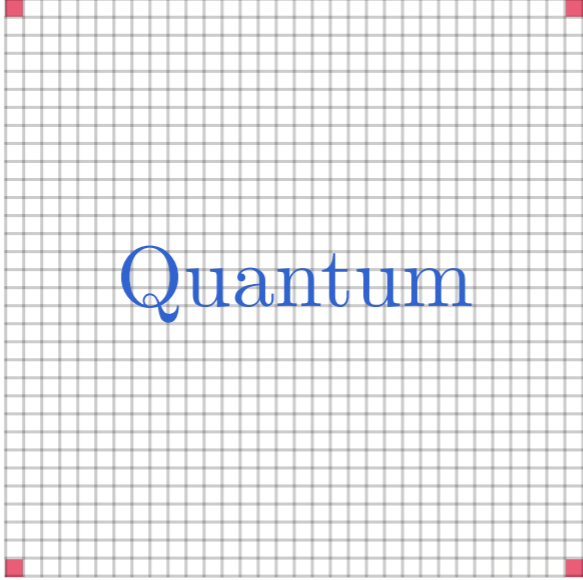
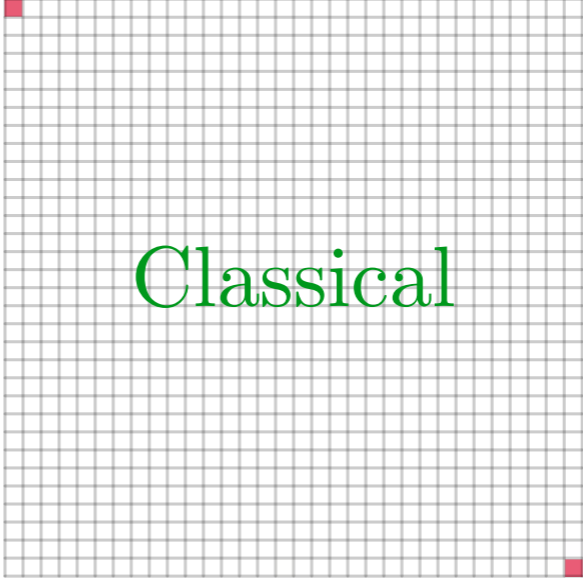
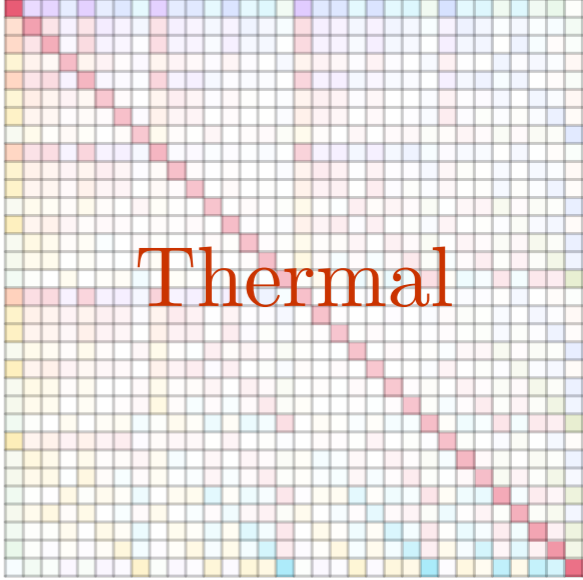
Q: What is the sign of quantum coherence?

(+) Positive.

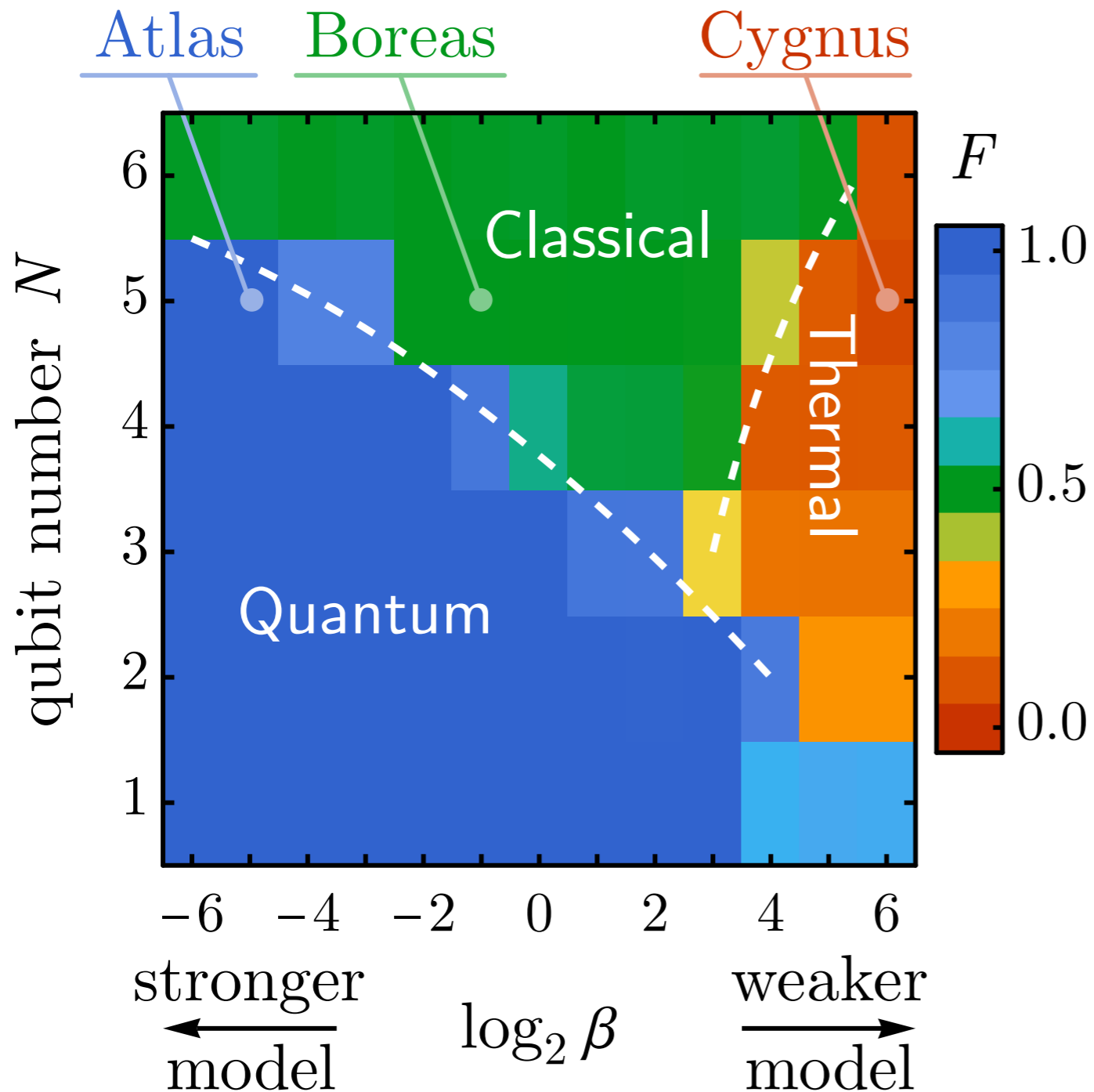
(-) Negative.

■ Atlas
■ Boreas
■ Cygnus

Characterize Representative Models

Model	Atlas	Boreas	Cygnus
$Z_{1:4} \rightarrow Z_5$ accuracy (\uparrow)	1.000	1.000	0.607
$X_{1:4} \rightarrow X_5$ accuracy (\uparrow)	1.000	0.503	0.634
$\tilde{\rho}$			
$F(\rho, \tilde{\rho})$ (\uparrow)	1.000	0.500	0.063
$S(\tilde{\rho})$ [bit] (\downarrow)	0.206	1.190	4.410

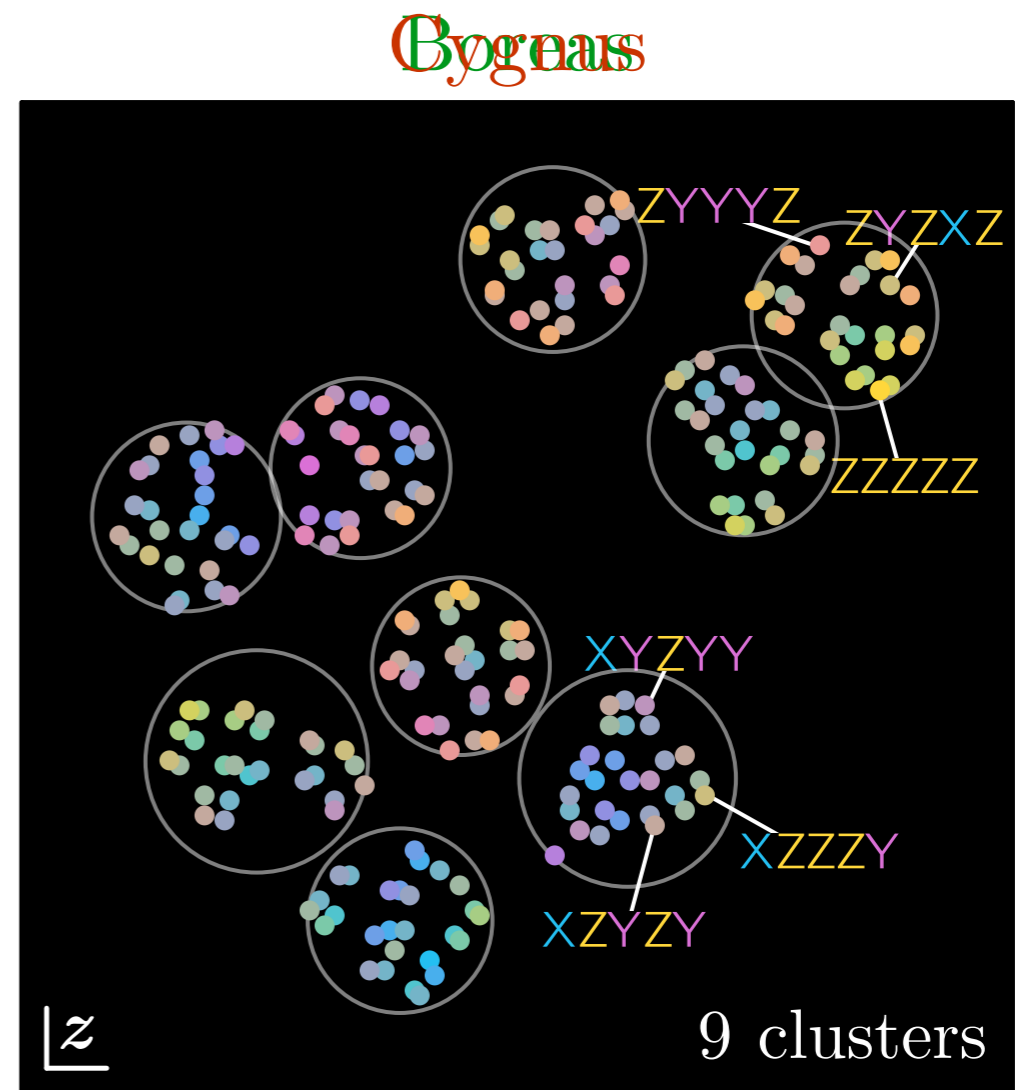
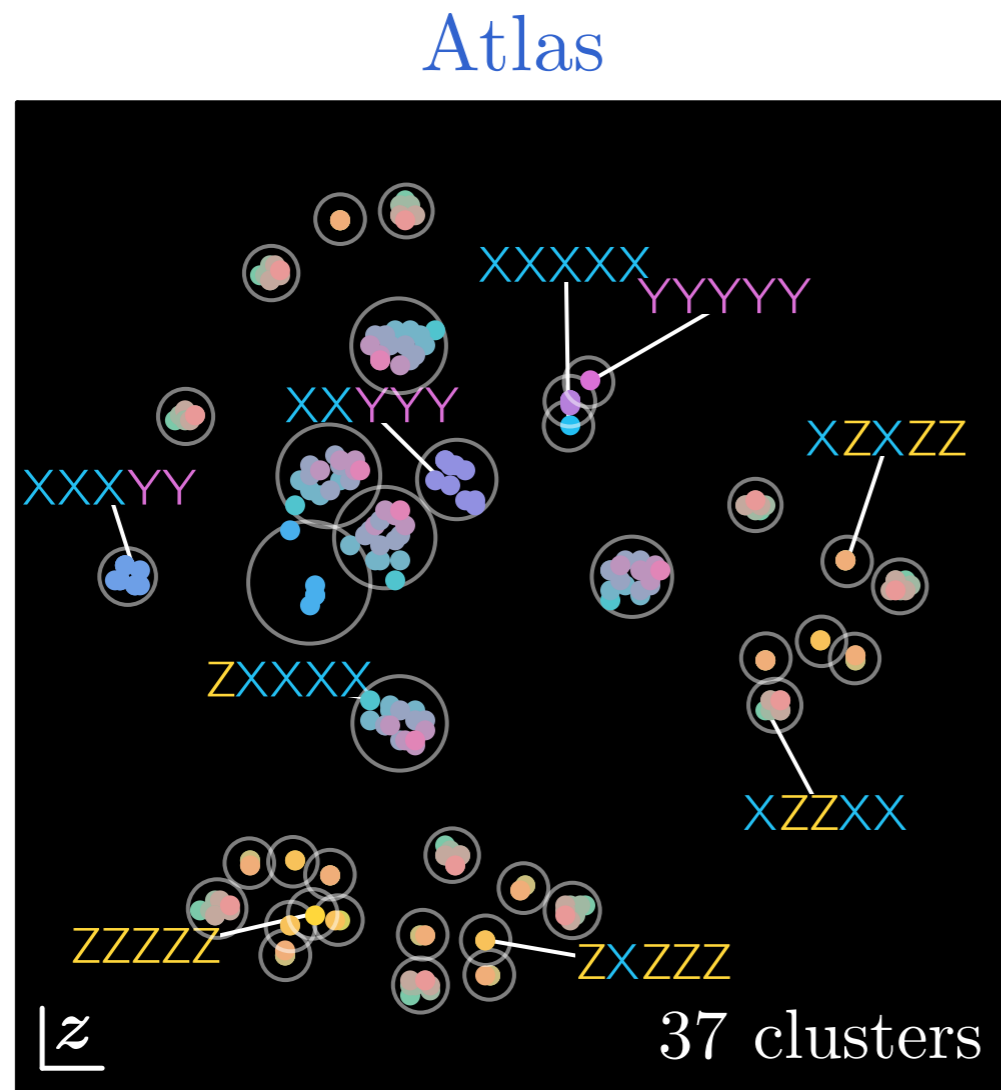
Emergent Classicality



- Classicality emerges with increasing —
- Qubit number (system size),
- Information bottleneck strength.
- Our world appears classical because —
- It involves too many qubits.
- We do not have enough classical data processing capability.

What Does the Latent Space Look Like?

- t-SNE visualization of operator embeddings.



- Each dot represents a sequence of observables.

$$\mathbf{x} \in \{X, Y, Z\}^N \xrightarrow[\text{Encoder}]{\text{Transformer}} \mathbf{z}$$

Summary

- We use a transformer-based **language model** to process **randomized measurement** data collected from Schrödinger's cat **quantum state**.
- **Classical reality** emerges in the language model due to the **information bottleneck**.
- Implying a fundamental **limitation** on our ability to understand the full quantum nature of the universe.
- A new avenue for using unlabeled **classical shadow** data to train generative models for **representation learning** of quantum operators
 - a step toward realizing AI quantum physicists.

Thanks!

Transformer-based β -VAE

