FAST AND PRECISE AMPLITUDE SURROGATES WITH SYMMETRY EQUIVARIANT NETWORKS

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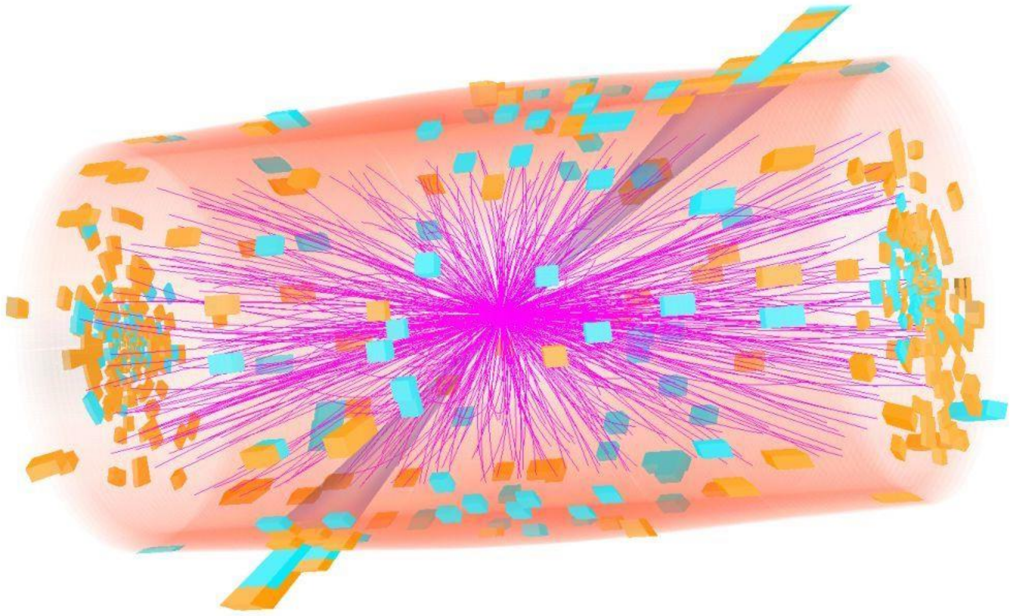
PROBLEMS IN COLLIDER PHYSICS



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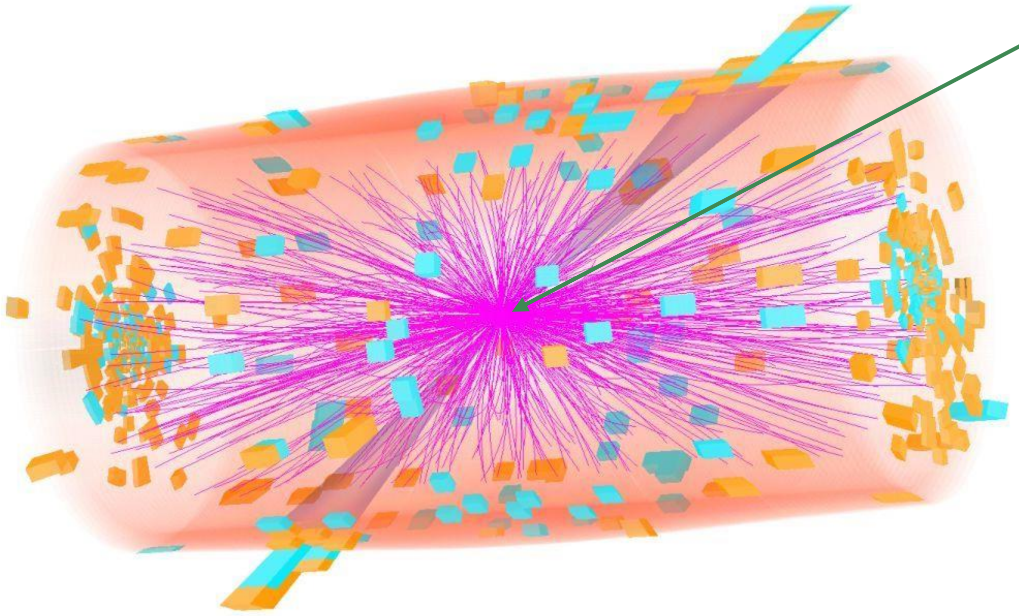

Predict the hard process interactions in a fast and precise way

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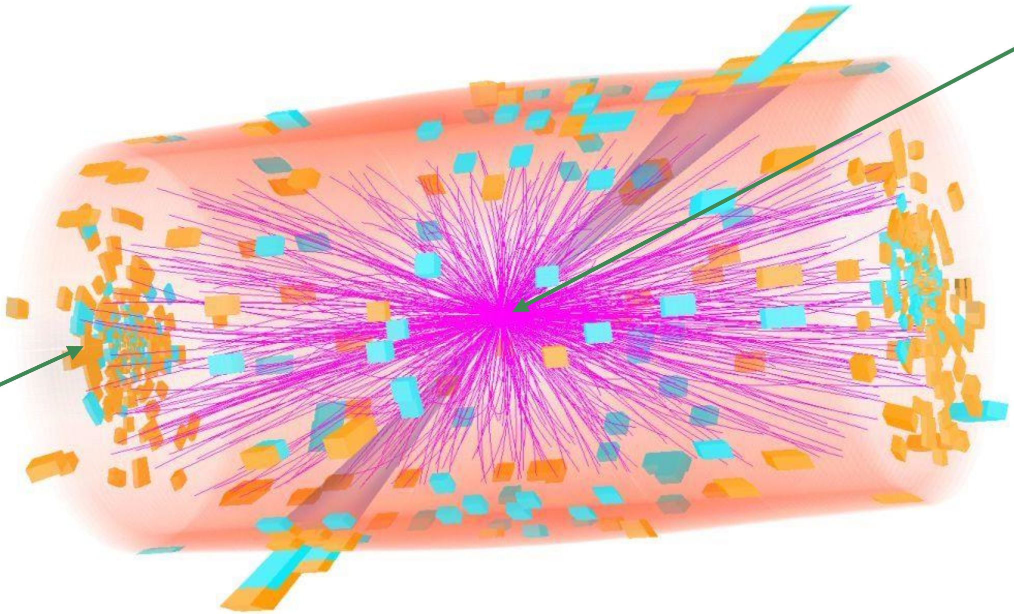

Identify particle structures at the detector level

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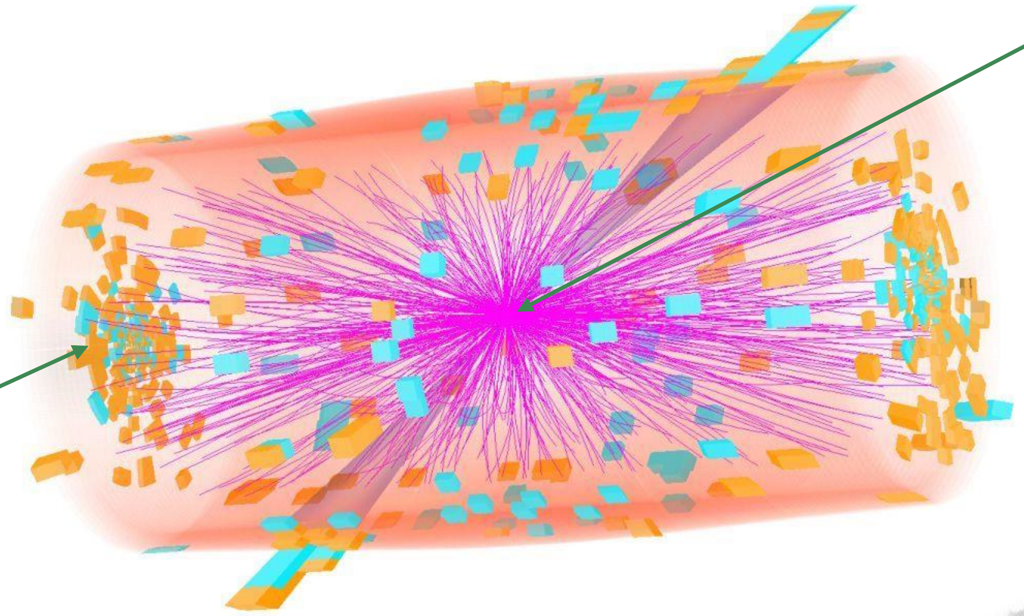
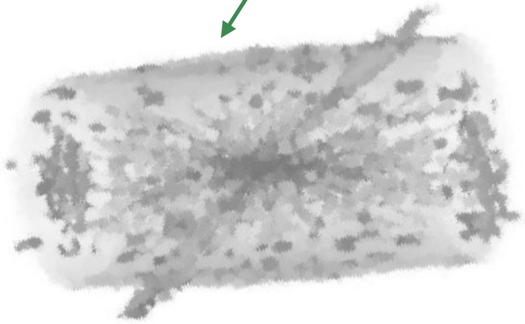


Simulate full events



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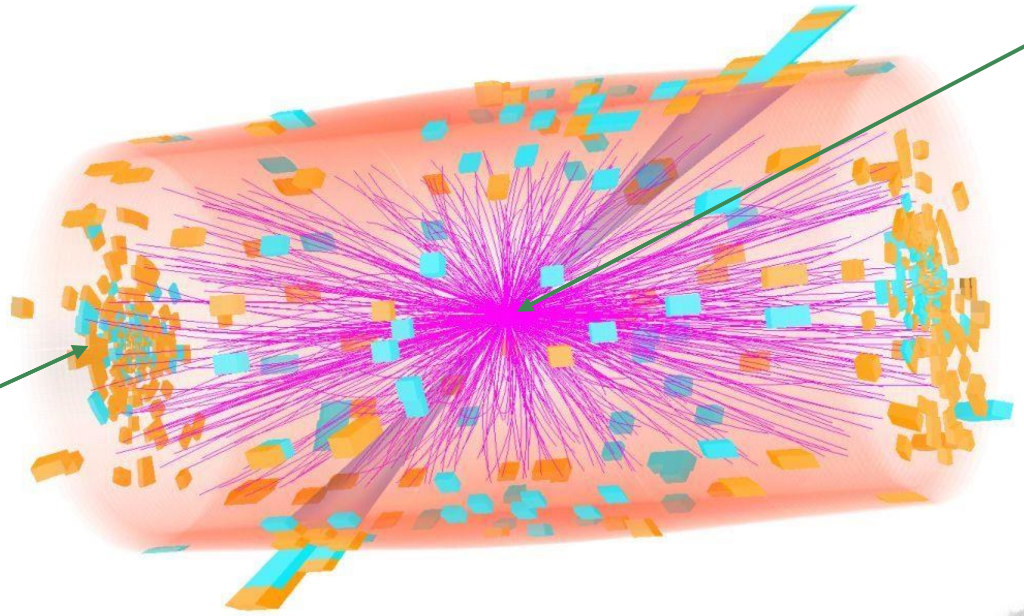

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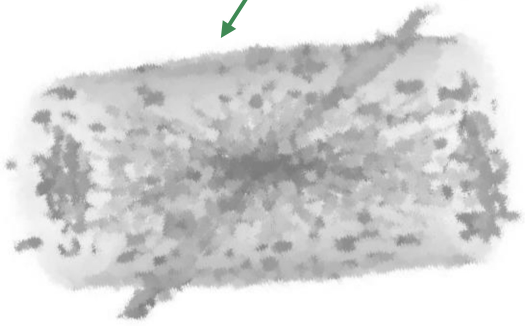

All collider processes are ruled by strict and concrete **symmetry laws**

GEOMETRIC ALGEBRA TRANSFORMER

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- Geometric inductive bias

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- Symmetry awareness

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- A general multivector in the Lorentz geometric algebra will be:

$$x = x_s + x_0e_0 + x_1e_1 + x_2e_2 + x_3e_3 + x_{01}e_0e_1 + x_{02}e_0e_2 + x_{03}e_0e_3 + x_{12}e_1e_2 + x_{13}e_1e_3 + x_{23}e_2e_3 \\ + x_{012}e_0e_1e_2 + x_{013}e_0e_1e_3 + x_{023}e_0e_2e_3 + x_{123}e_1e_2e_3 + x_{0123}e_0e_1e_2e_3$$

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Lorentz vectors

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Scalars — Lorentz vectors — Lorentz bilinears

$$x = \underbrace{x_s}_{\text{Scalars}} + \underbrace{x_0 e_0 + x_1 e_1 + x_2 e_2 + x_3 e_3}_{\text{Lorentz vectors}} + \underbrace{x_{01} e_0 e_1 + x_{02} e_0 e_2 + x_{03} e_0 e_3 + x_{12} e_1 e_2 + x_{13} e_1 e_3 + x_{23} e_2 e_3}_{\text{Lorentz bilinears}}$$
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- Auxiliary scalar representations (for non-geometric data)
- Positional embeddings
- Axial attention

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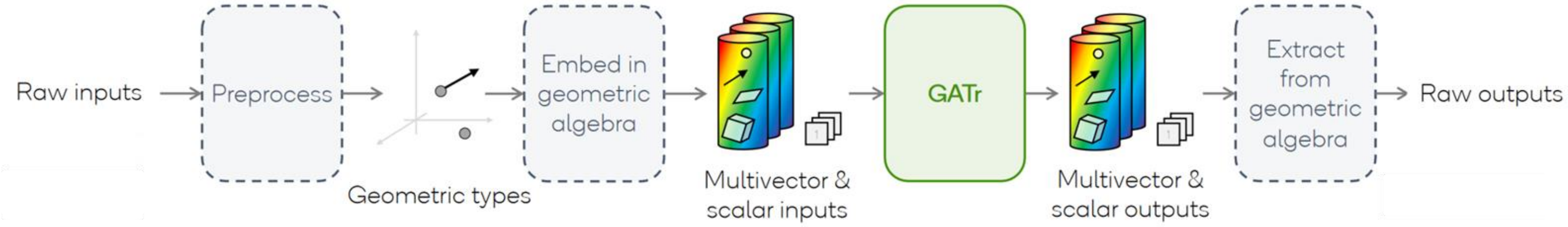

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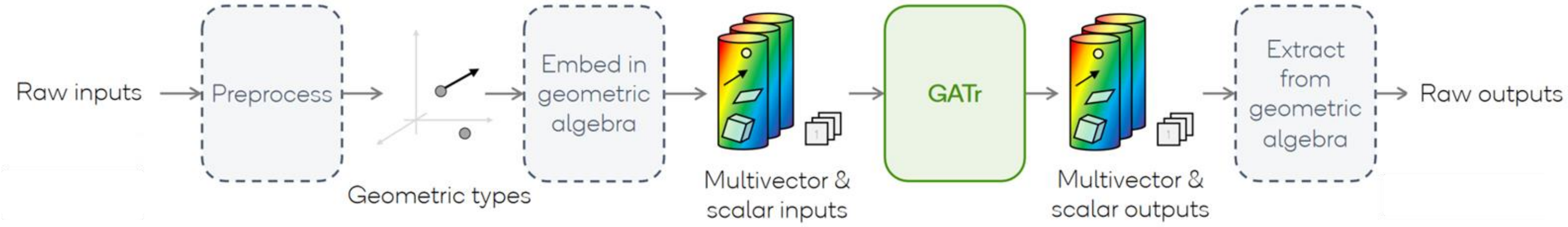

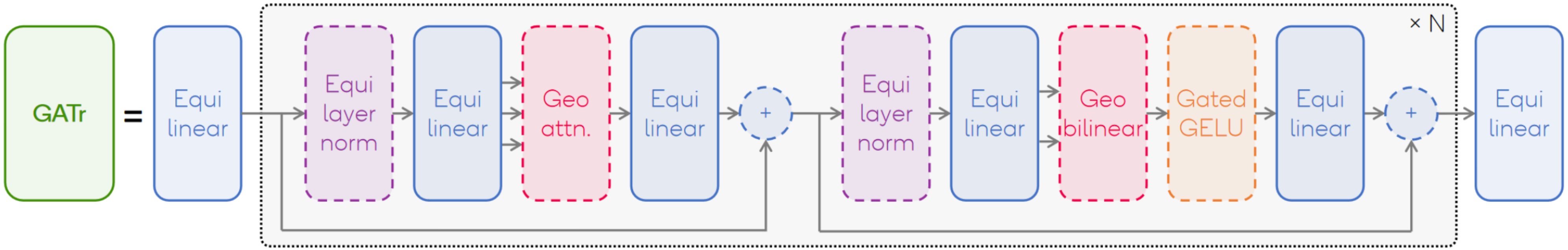

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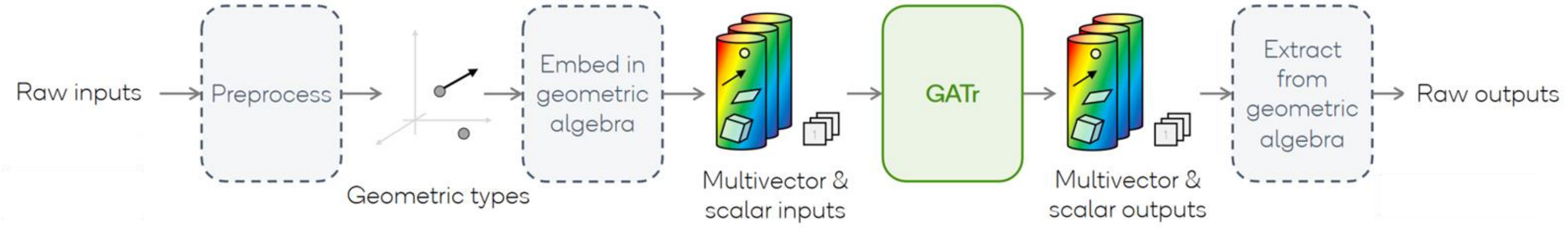

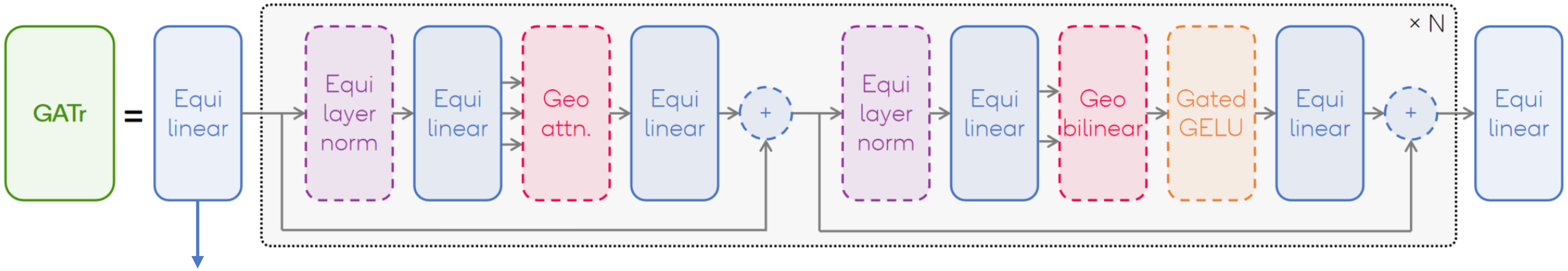

$$\phi(x) = \sum_{k=0}^{d} w_k \langle x \rangle_k$$

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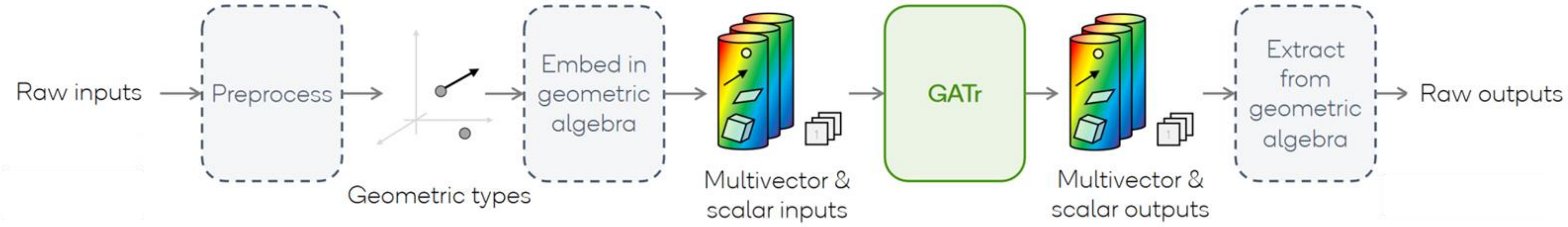
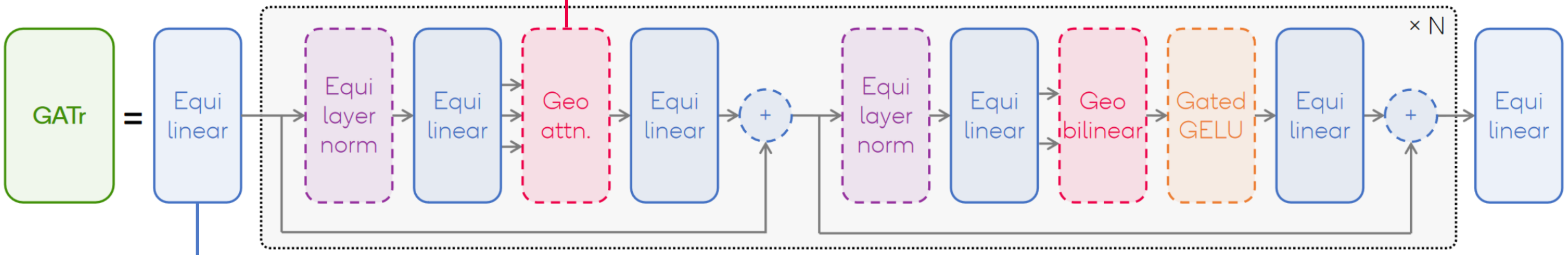


$$\text{Attention}(q,k,v)_{i'c'} = \sum_i \text{Softmax}_i \left(\frac{\sum_c \langle q_{i'c}, k_{ic} \rangle}{\sqrt{8 n_c}} \right) v_{ic'}$$



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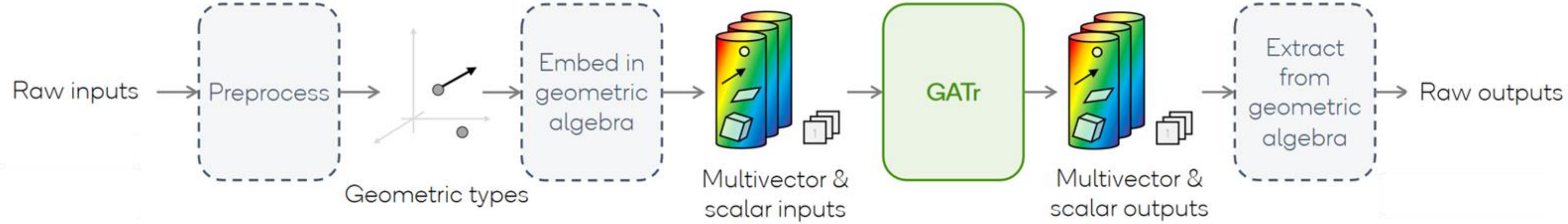

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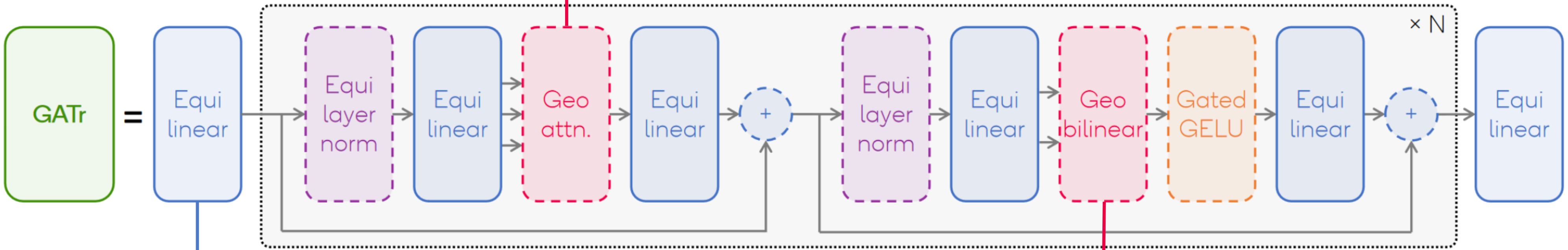

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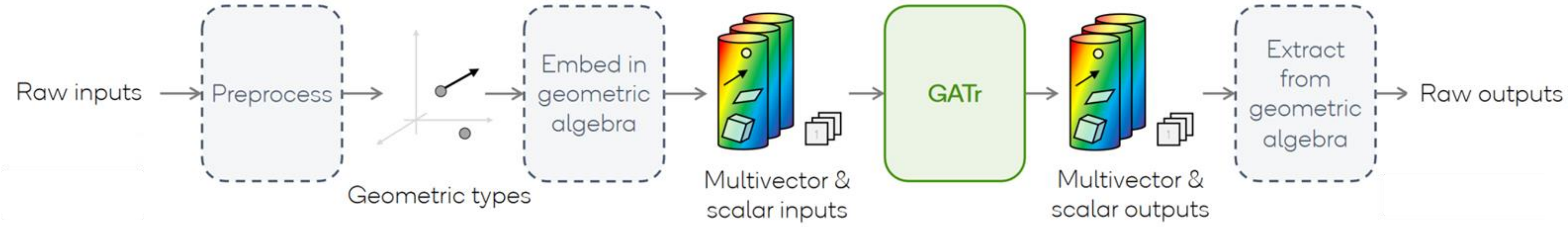

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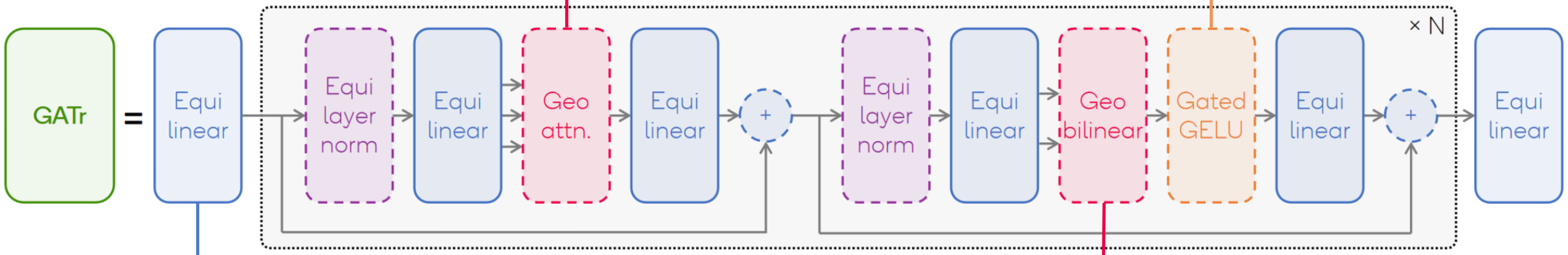
$$\text{GatedGELU}(x) = \text{GELU}(x_s)x$$



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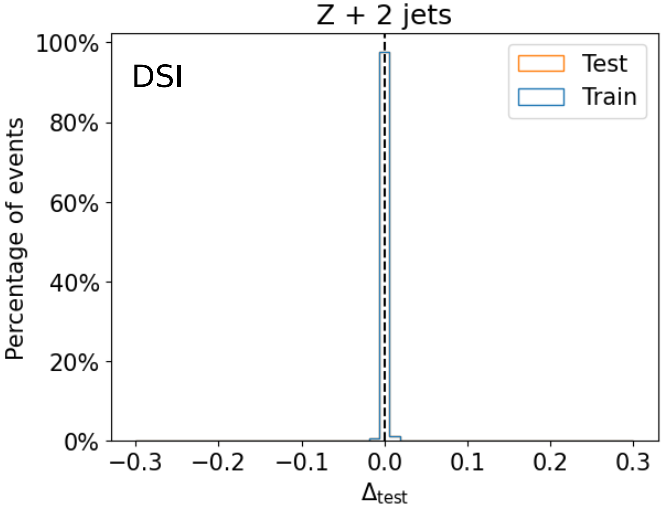

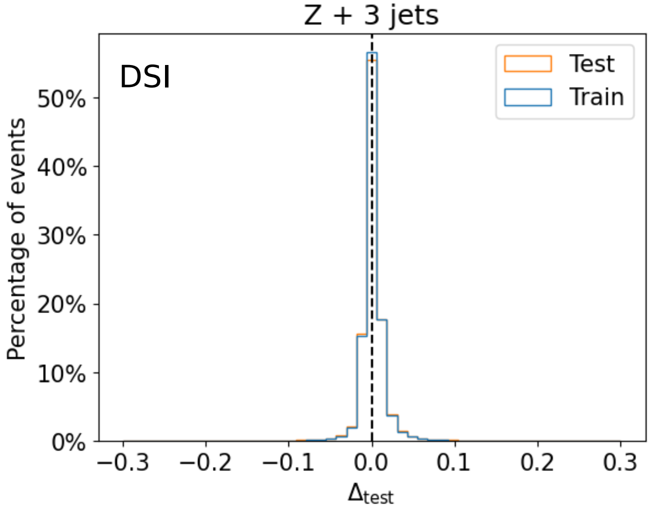

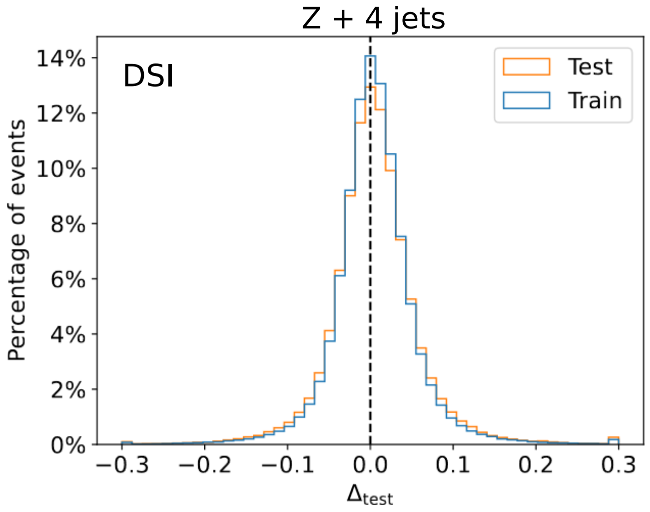

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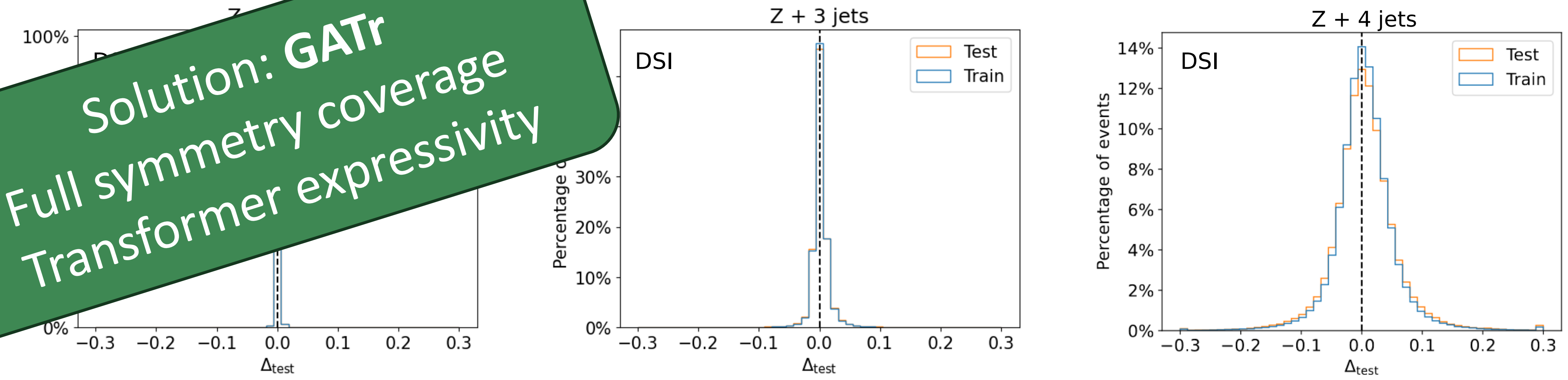
Solution: **GATr**

1. Full symmetry coverage
2. Transformer expressivity



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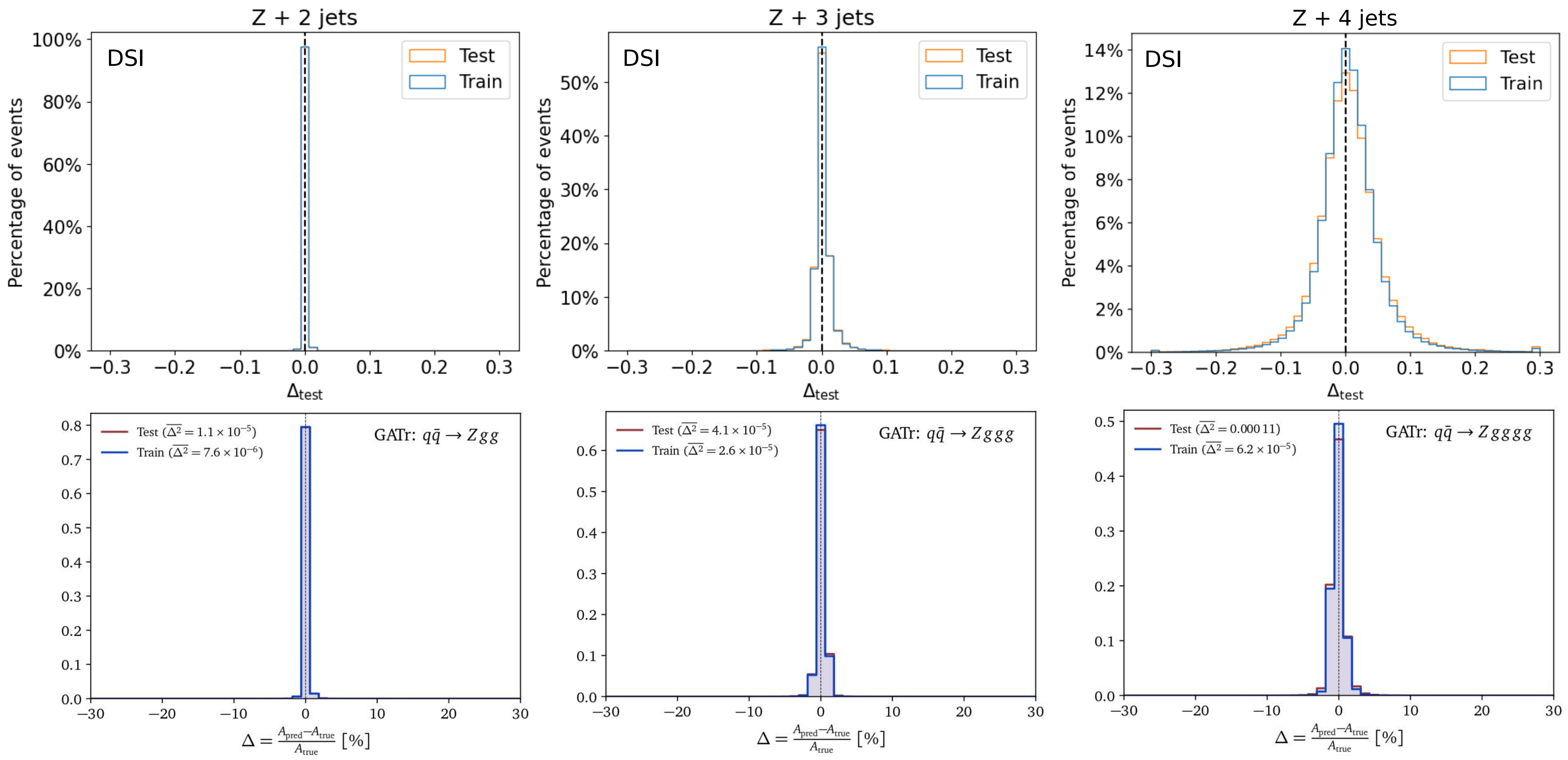
Preliminary results!



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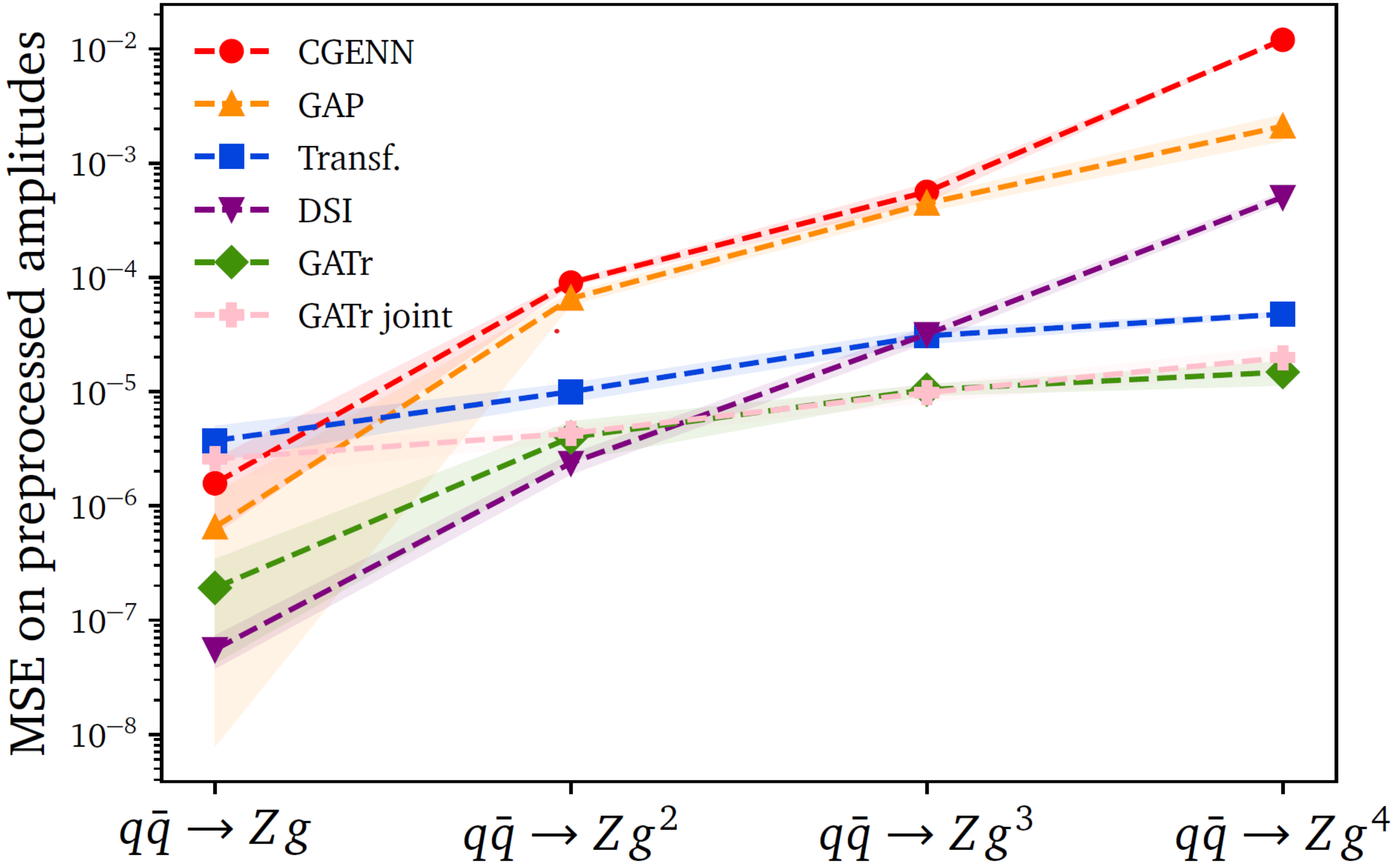

DSI: Deep Sets algorithm with momentum invariant inputs. Our main baseline.

GATr joint: GATr model trained with all data sets at the same time

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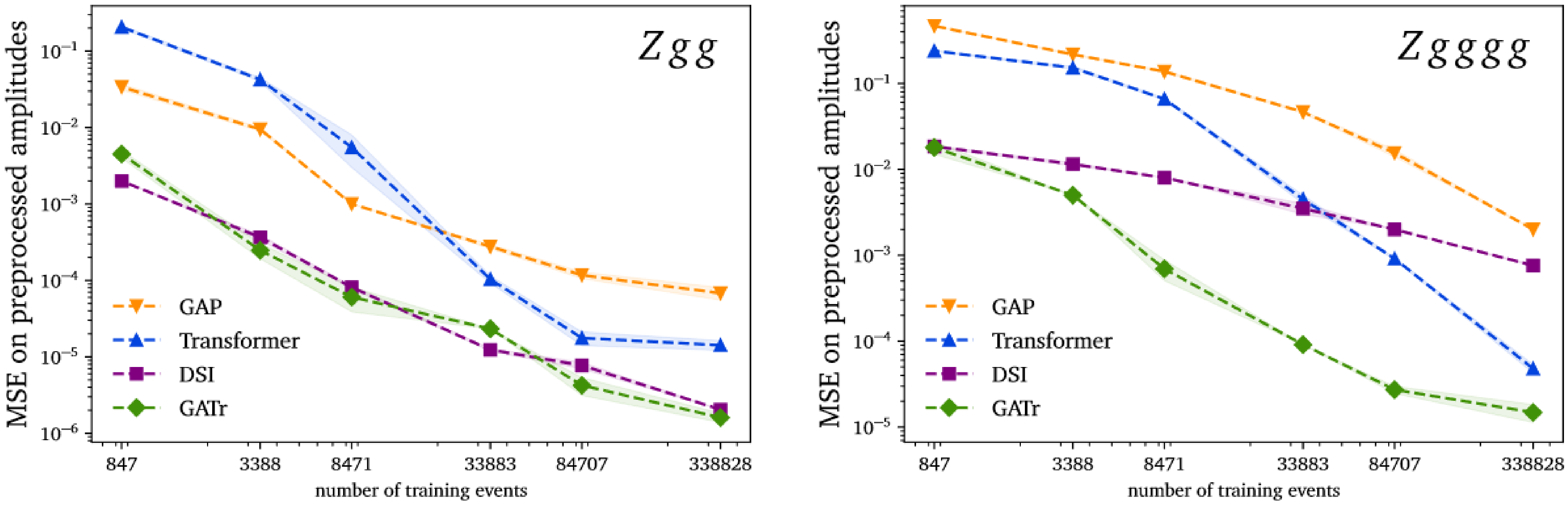

Excellent sample efficiency from **GATr**

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- Outlook:
  - NLO amplitude regression
  - Other collider physics tasks