Is Quantum Computing energy efficient? An Investigation on a Quantum Annealer

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Introduction

The environmental impact of computing activities is starting to be acknowledged as relevant and several scientific initiatives and research lines are gathering momentum to identify and curb it. Governments, industries, and commercial businesses are now holding **high expectations on quantum technologies** as they have the **potential to create greener and faster methods** to solve specific computational tasks. The energy perspective of such technologies, however, has remained rather outside the scopes of deployment strategies, which might limit future adoptions.

To empirically investigate the energy perspective of the current quantum technology on a specific use-case, we pick a well-known NP-hard problem such as **Integer Factorization**. We implement an **annealing-based quantum algorithm** to solve the task and we estimate its energy consumption performing real experiments on the **D-Wave Advantage quantum annealing** machine.

Integer Factorization: problem definition

Given an integer number N, find the primes p and q such that

$$N = p \cdot q$$

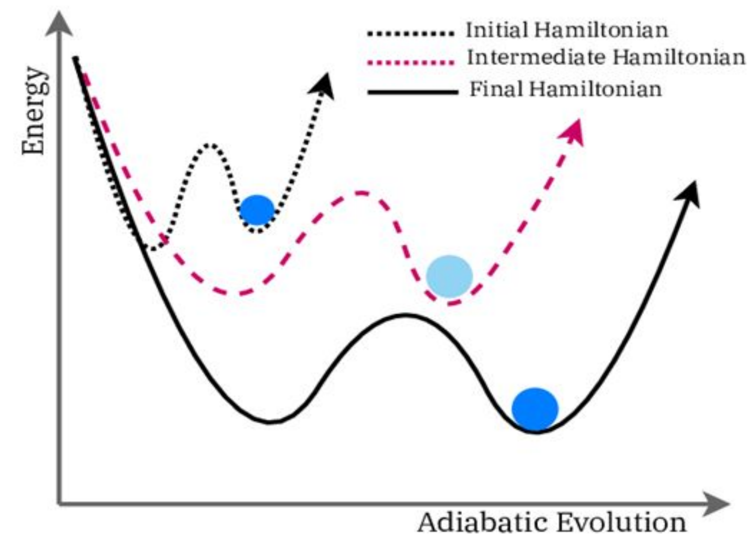
What is quantum annealing?

- The quantum system is prepared in the known ground-state of an <u>initial Hamiltonian</u>
- The target solution is encoded in the ground-state of a <u>final Hamiltonian</u>, written as the energy/cost function of a **Quadratic Unconstrained Binary Optimization** problem (QUBO problem)
- The system evolution is controlled by the following time-dependent Hamiltonian:

$$H(t) = A(t)H_{init} + B(t)H_{fin}$$

$A(t)$↓ $B(t)$↑

- **Quantum Adiabatic theorem**: "if the evolution is slow enough, the quantum-mechanical system stays close the ground-state of the instantaneous Hamiltonian"

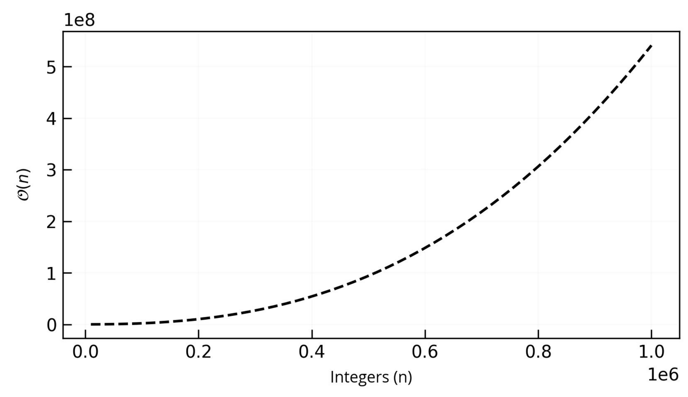


Classical algorithm

All known classical factoring algorithms which are deterministic and don't have unproved hypotheses **require exponential time in** $\log(N)$.

Thus, the Integer Factorization problem is used as the basic hardness assumption for many encryption methods including the **RSA cryptographic system**.

The fastest known classical algorithm is the **"General Number Field Sieve Method"** [1].



Quantum algorithm

The quantum annealing approach for the Integer Factorization problem is presented and discussed by Jiang et al. in [2] and Mengoni et al. in [3]. It is a **probabilistic algorithm** based on the following steps:

1. Encode the integer numbers N, p, and q into their bitstring representation
2. Compute the bitstrings product $p \cdot q$ using the "multiplication table method"
3. Write the objective $O(p,q) = (N - p \cdot q)^2$
4. Convert the objective into a **QUBO problem**

$$O(p,q) = (p_1 + q_1 - n_1 - 2c_{12})^2 + (p_2 + p_1q_1 + q_2 + c_{12} - n_2 - 2c_{23} - 4c_{24})^2 + \ \dots + (1 + c_{56} + c_{46} - n_6 - 2c_{67}) + (c_{57} + c_{67} - n_7)^2$$

Columns	7	6	5	4	3	2	1	0
p · q					1	p_2	p_1	1
				q_1	p_2q_1	p_1q_1	q_1	
			q_2	p_2q_2	p_1q_2	q_2		
		1	p_2	p_1	1			
Carries	c_{67} c_{57}	c_{56} c_{46}	c_{45} c_{35}	c_{34} c_{24}	c_{23}	c_{12}		
N	n_7	n_6	n_5	n_4	n_3	n_2	n_1	n_0

Methodology and Results

Once the problem is written in the QUBO form, we can run the **quantum annealing solver** on the **D-Wave Advantage device**.

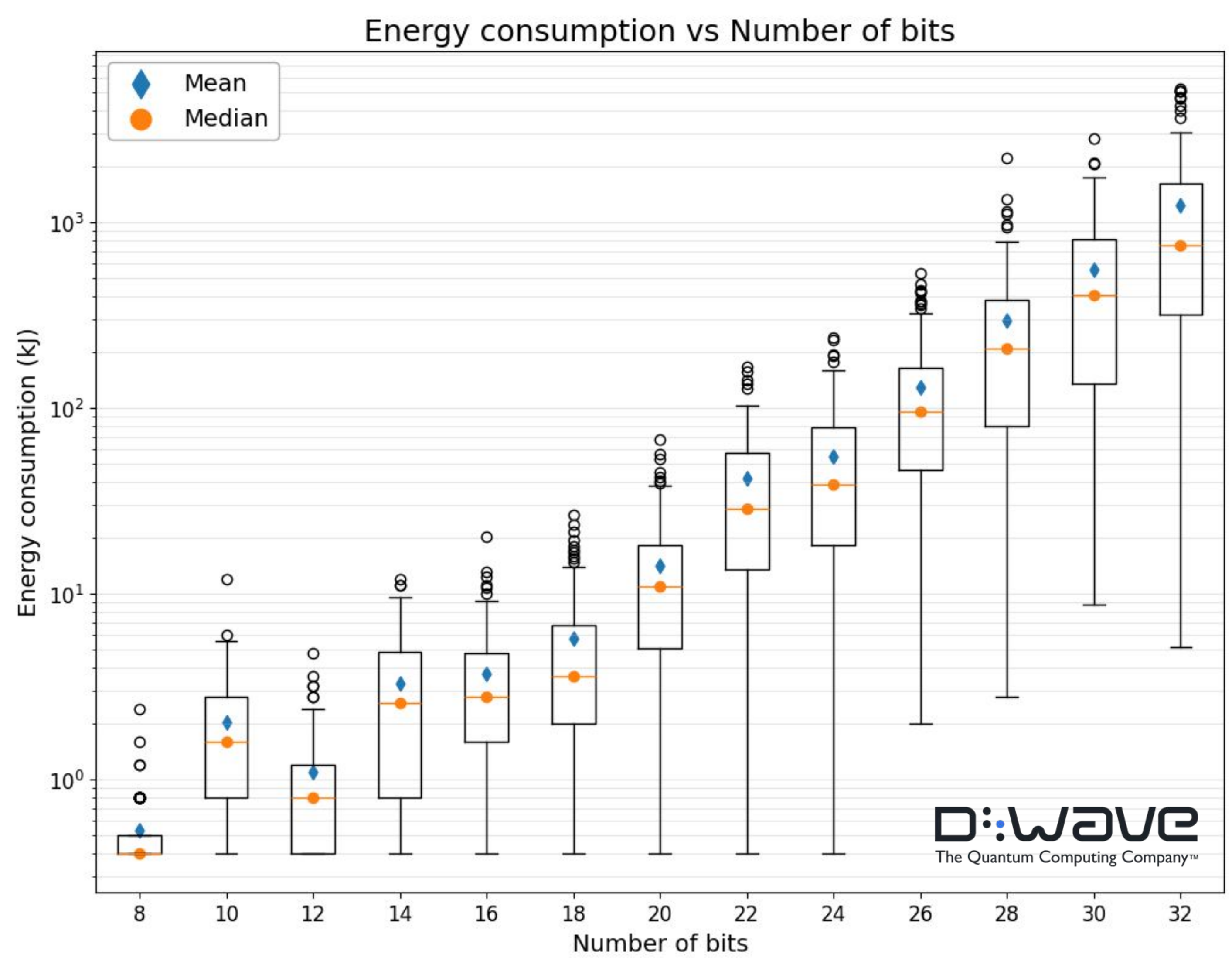
We select 13 different Integer Factorization problem instances starting from 143 (8-bits) to 4˙151˙998˙087 (32-bits). For each instance we run 100 experiments until success.

For each experiment the energy consumption E is estimated using the following relation:

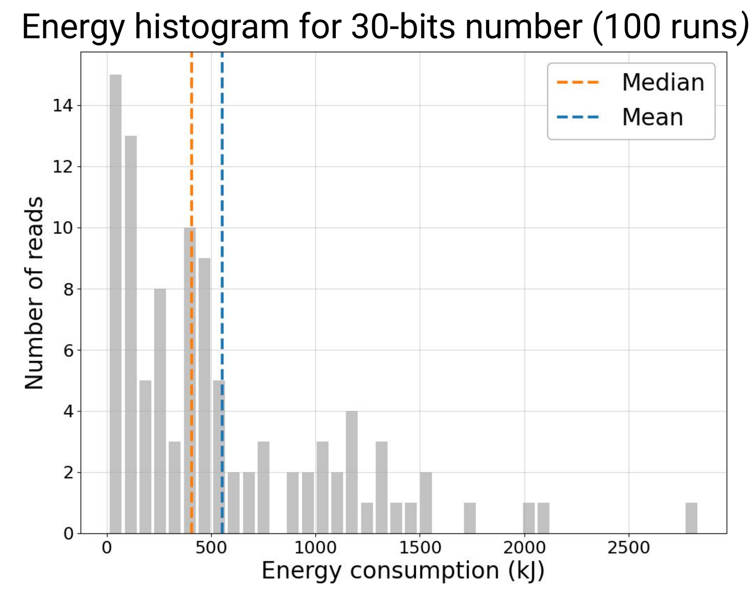
$$E = k \cdot t = k \cdot (QPU_{access} \cdot N_{shots})$$

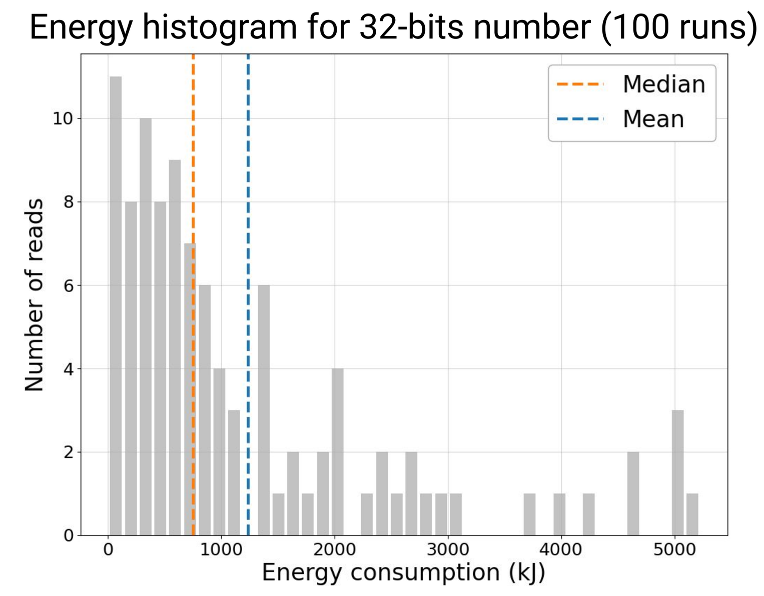
$k = 25\ kW$ is the maximal absorbed power (D-Wave docs)
t is the total QPU time required to get the correct solution
$QPU_{access} = 16\ ms$ is the QPU time per shot
N_{shots} is the number of shots required to get the correct solution

Remark: the estimate ignores the energy used by the dilution refrigerator to cool down the QPU to the operational temperature $\mathbf{T \sim 15\ mK}$



Distributions of energy consumptions as a function of the problem size. Each box represents 100 experiments for the same problem instance.





Conclusion

- Running the Integer Factorization algorithm on the D-Wave Advantage quantum annealer doesn't offer any advantage in terms of energy efficiency: <u>the mean/median energy consumption scales exponentially with the problem size</u>
- <u>The energy consumption distributions for all the problem instances are skewed towards lower energies</u>, suggesting that further **algorithm optimizations** or **hardware improvements** could lead to a better performance and efficiency

References

[1] Lenstra et al., "The Number Field Sieve", 2001
[2] Jiang et al., "Quantum Annealing for Prime Factorization", 2018
[3] Mengoni et al., "Breaking RSA Security With A Low Noise D-Wave 2000Q Quantum Annealer: Computational Times, Limitations And Prospects", 2020

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ACAT 2024 - Stony Brook University, Long Island NY, USA (11th-15th March)