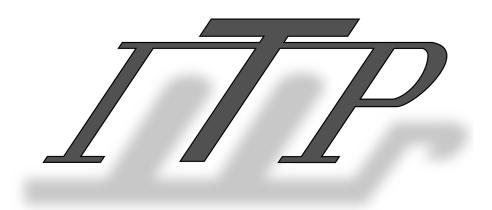
# Modern Machine Learning **Tools for Unfolding**

#### Javier Mariño Villadamigo





Institut für Theoretische Physik - University of Heidelberg



In collaboration with Nathan Huetsch, Anja Butter, Theo Heimel, Tilman Plehn

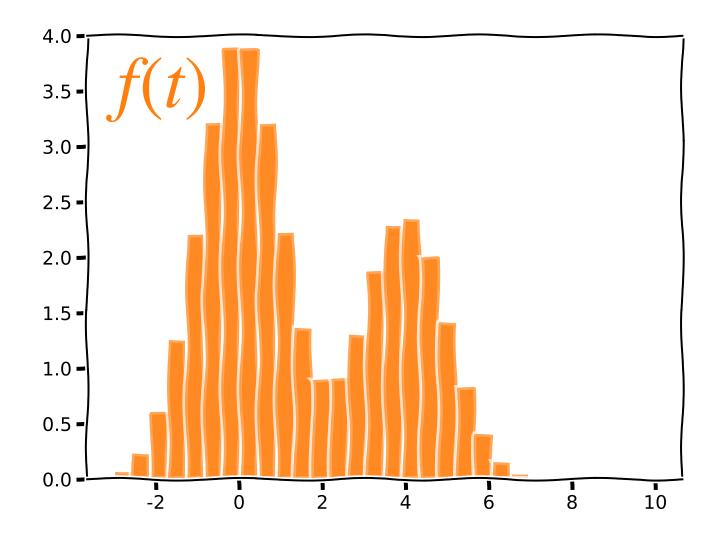
ACAT March 12th 2024



ZUKUNFT



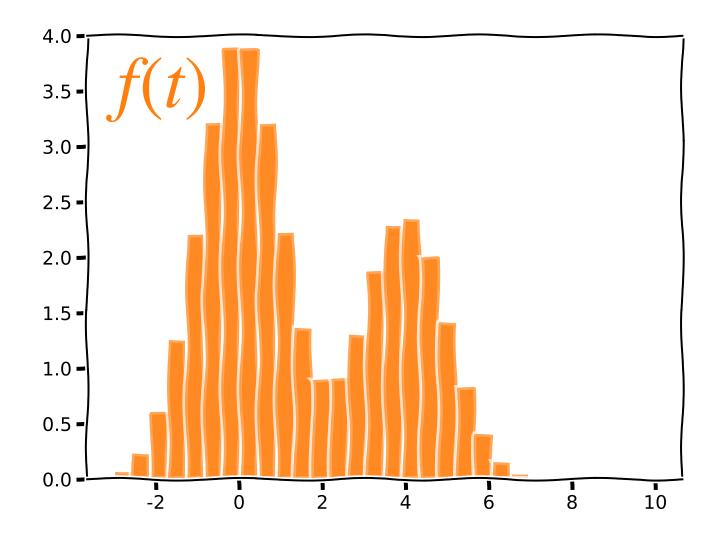
directly accessible.



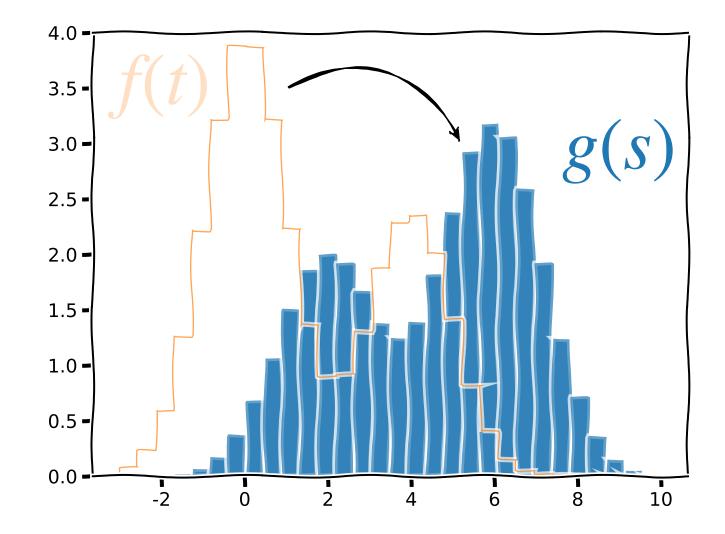
• Distributions f(t) of a physics variable t to be measured in particle physics experiments are often not



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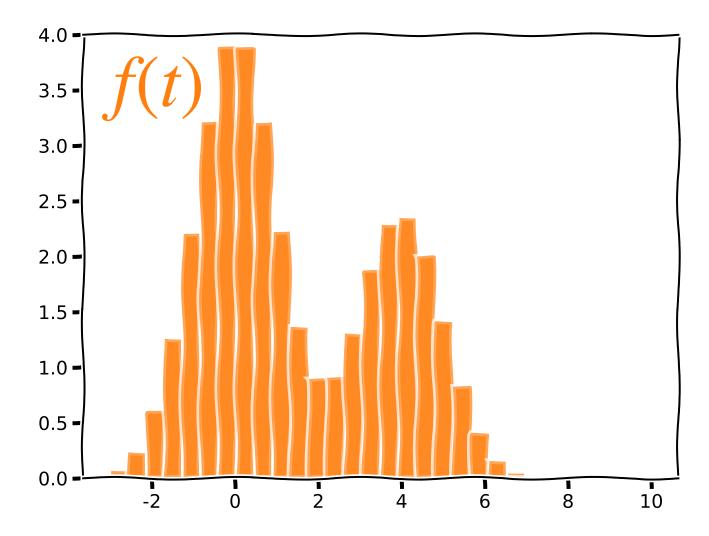


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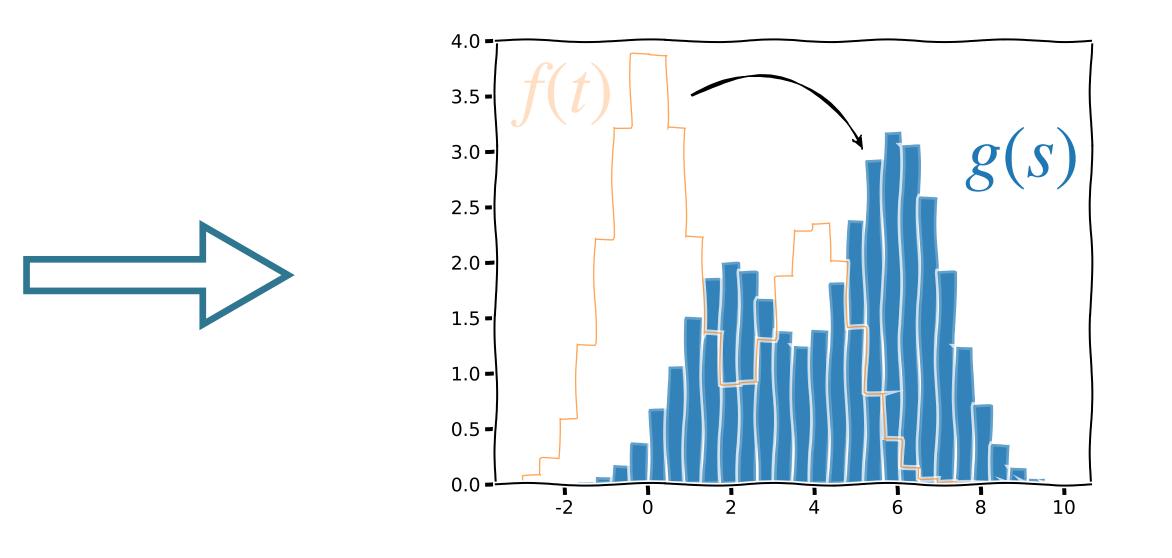


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measured distribution g(s) can be simulated.

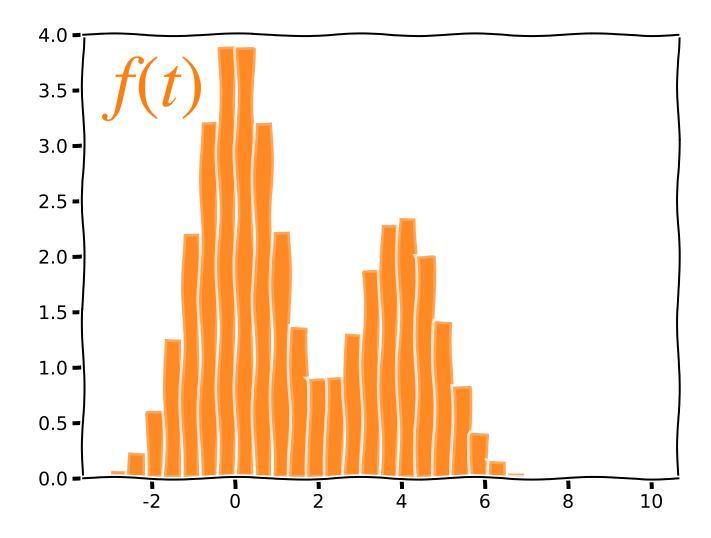
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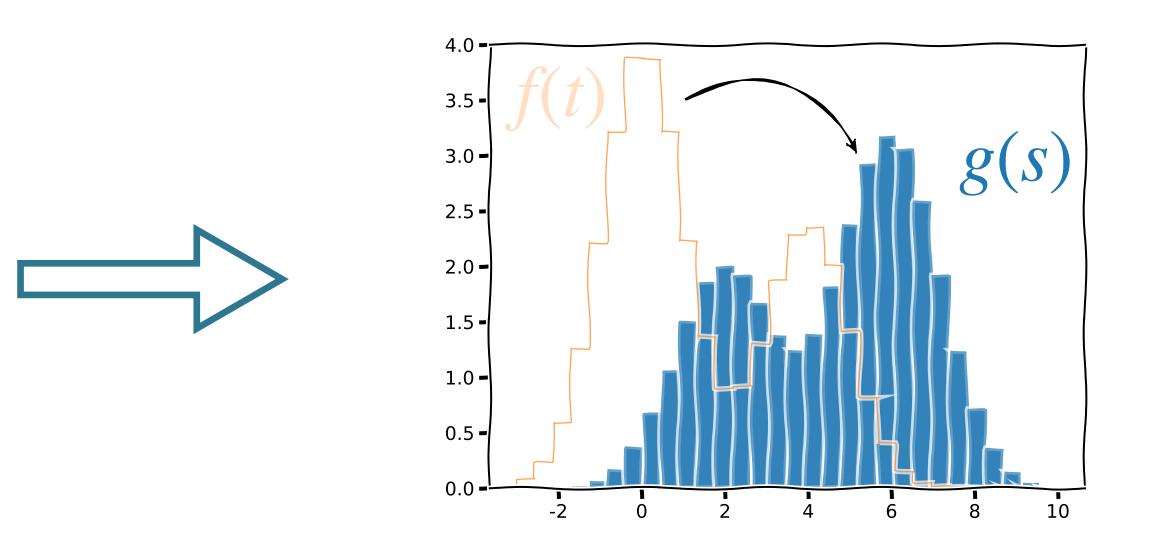


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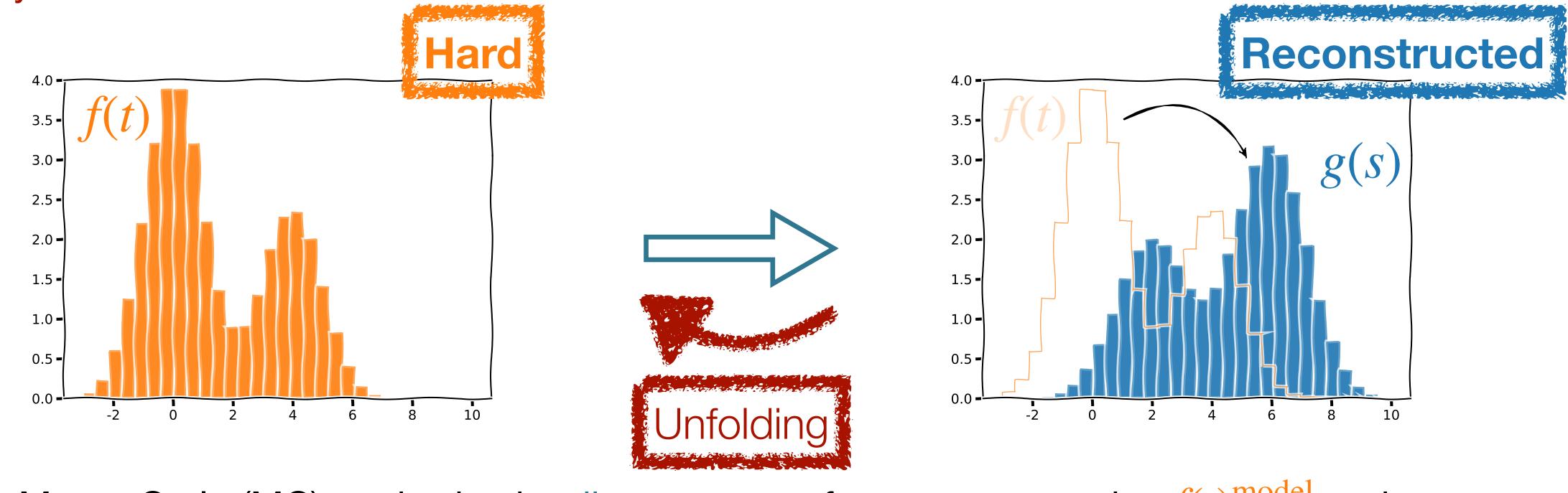
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Traditionally:

Matrix-based unfolding

$$g(s) = \int R(s \mid t) f(t) \, \mathrm{d}t$$



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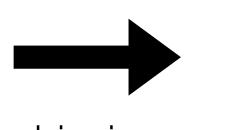


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 $r_i = \sum_j R_{ij} \cdot t_j$ 

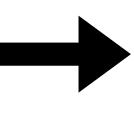


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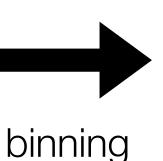
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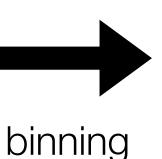
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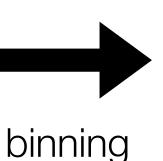
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Requires binning and can only unfold a few dimensions With neural networks:

- **ML-based** unfolding

  - Allows to unfold (and account for correlations in) many dimensions
  - Some methods allow for independent single-event unfolding



$$r_i = \sum_j R_{ij} \cdot t_j$$

General need for regularization: trade-off between bias and statistical uncertainty

Unbinned: advantageous if one wants to derive quantities from the unfolding observables





► (\*)

Omnifold [1911.09107]

#### **Distribution mapping**

- Direct Diffusion [2311.17175]
- Schrödinger Bridge [2308.12351]

(\*) These are not comprehensive lists. For a more extensive catalogue see for example the <u>HEP ML Living Review</u>

### Several approaches

#### **Conditional phase space sampling**

- GANs [1912.00477]
- Latent Diffusion [2305.10399]
- Conditional Flow Matching [2305.10475]
- cINN [<u>2212.08674</u>, <u>2006.06685</u>]
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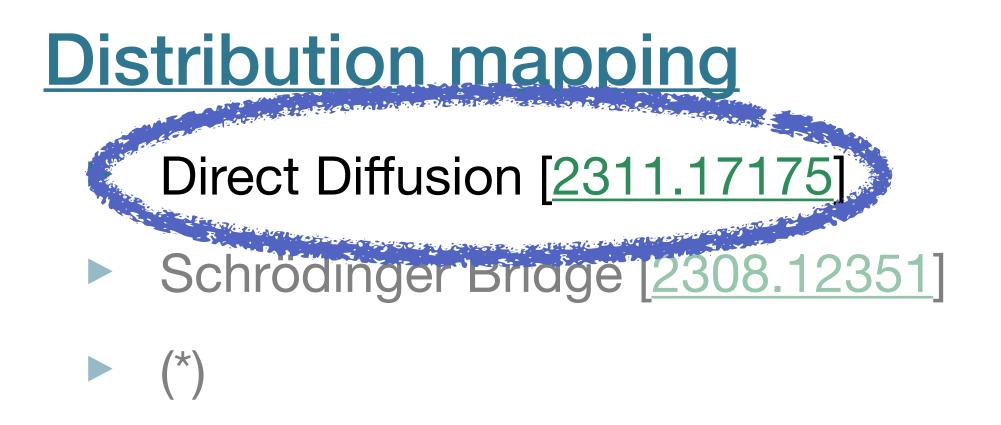






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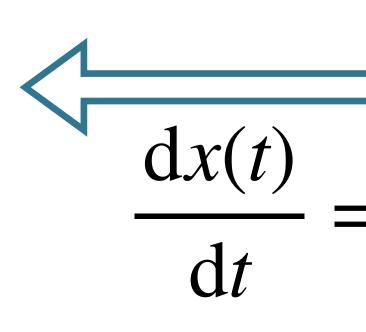
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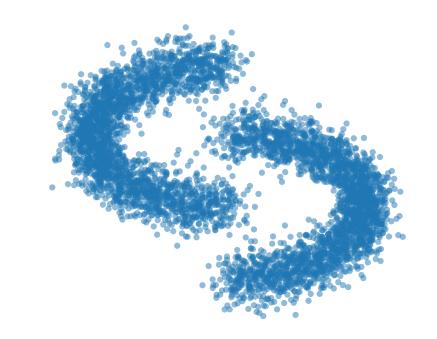




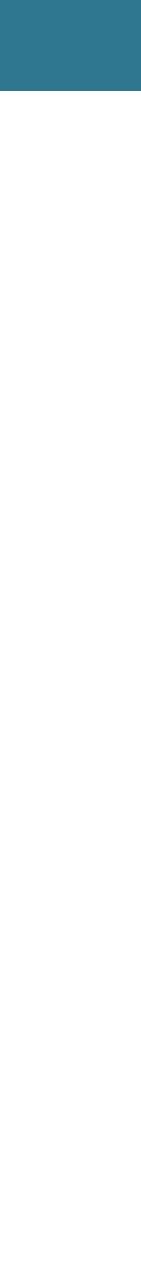


### $x_0 \sim p_{\text{model}}(x_{\text{hard}})$

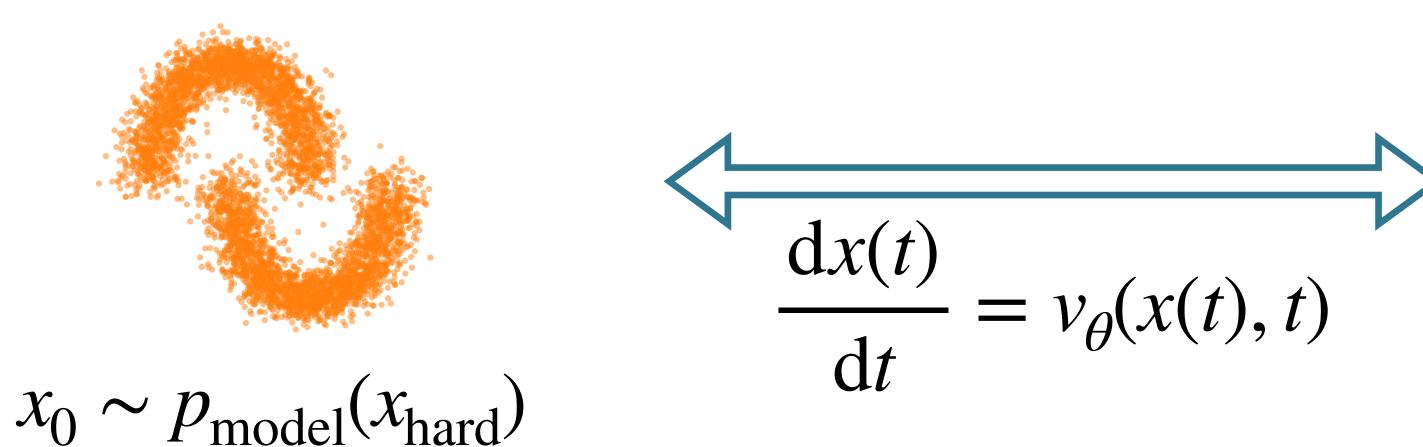
 $= v_{\theta}(x(t), t)$ 



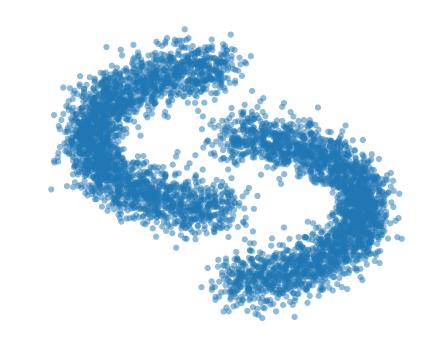
 $x_1 \sim p_{\text{reco}}(x_{\text{reco}})$ 





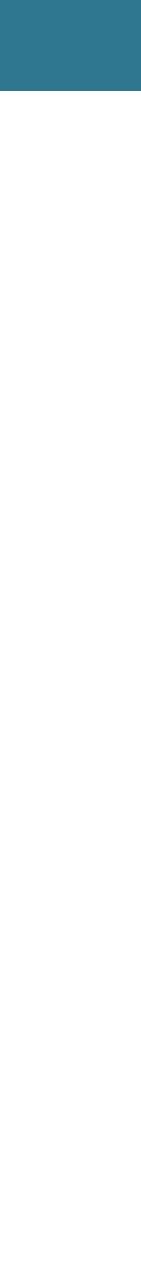


• Connect  $x_0$  and  $x_1$  with a linear trajectory:

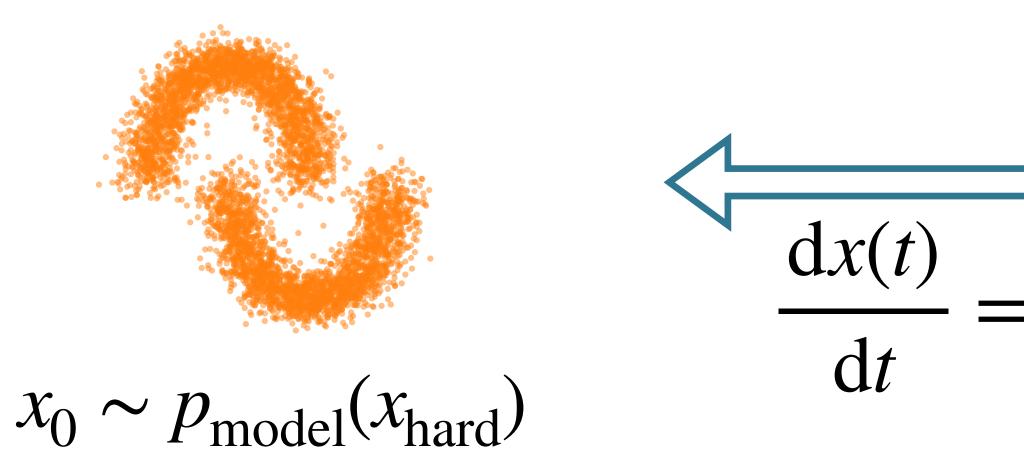


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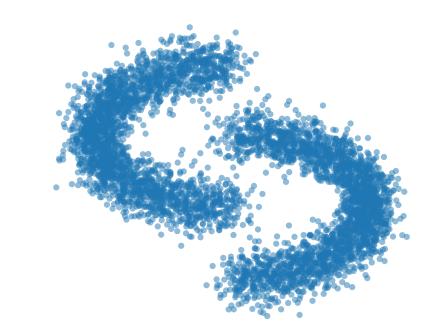
 $x(t) = (1 - t)x_0 + tx_1$ 







- Connect  $x_0$  and  $x_1$  with a linear trajectory:
- The NN is regressed to predict the velocity

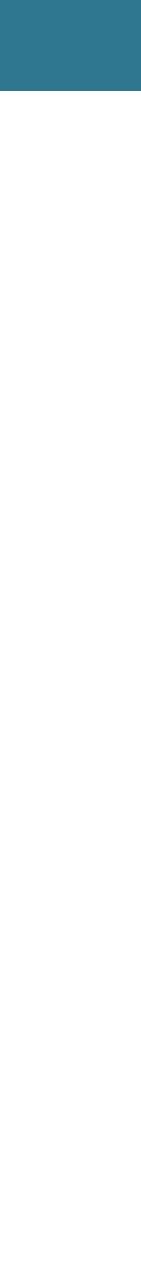


$$v_{\theta}(x(t), t)$$

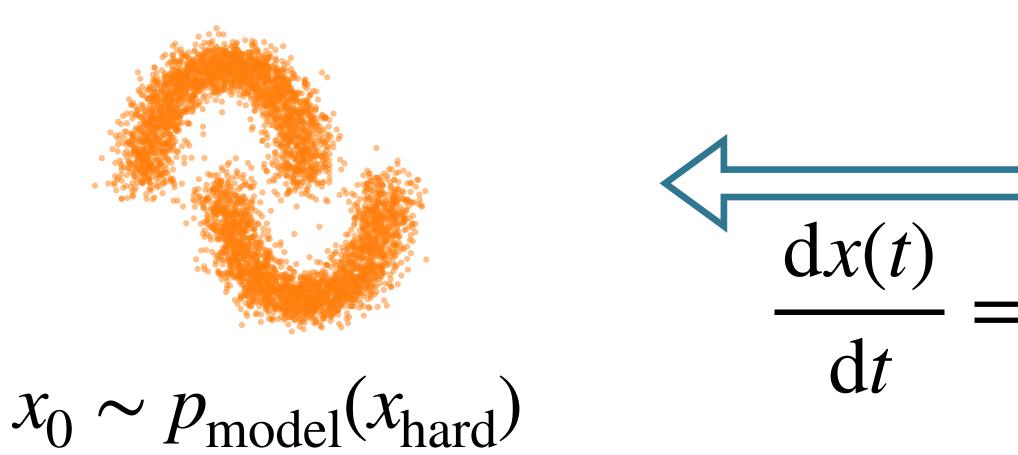
$$x_1 \sim p_{\text{reco}}(x_{\text{reco}})$$

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in the symmetry field:  

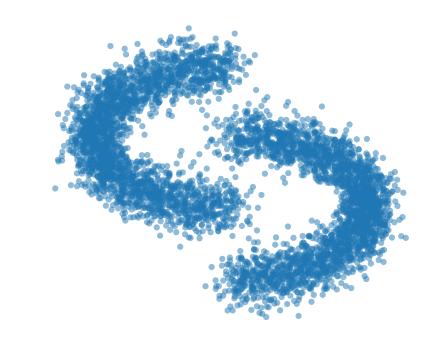
$$v_{\theta}(x(t), t) \approx \frac{\mathrm{d}x(t)}{\mathrm{d}t} = x_1 - x_0$$







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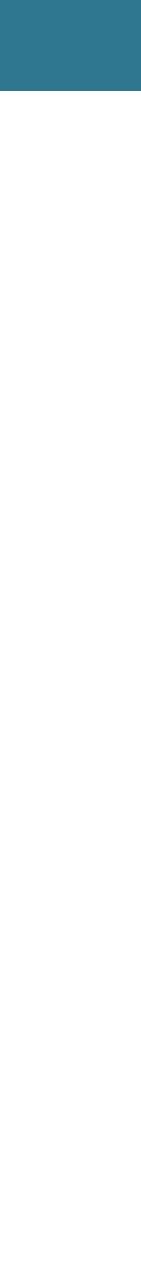
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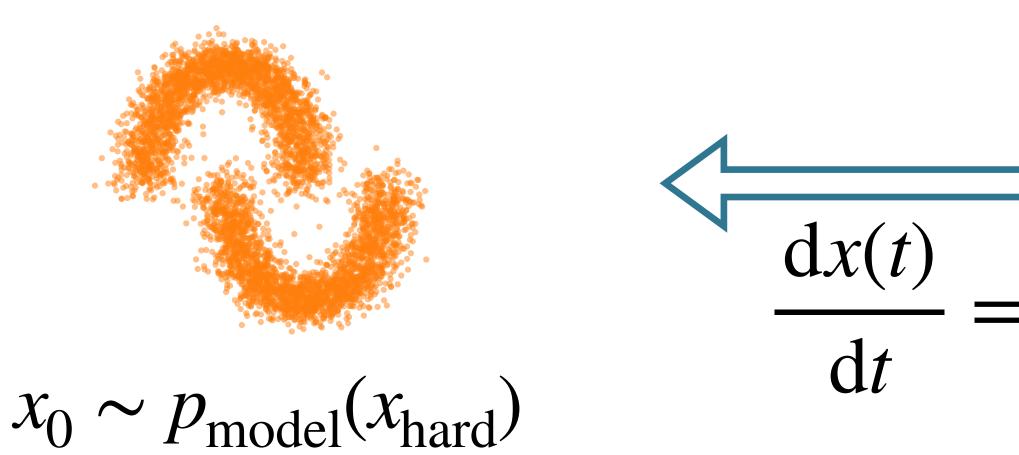
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$$v_{\theta}(x(t), t) \approx \frac{dx(t)}{dt} = x_1 - x_0$$

$$x_0 = x_1 + \int_1^0 v_{\theta}(x(t), t) dt$$

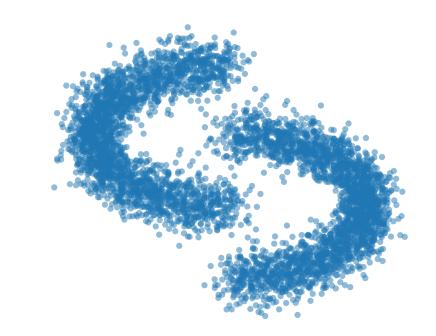






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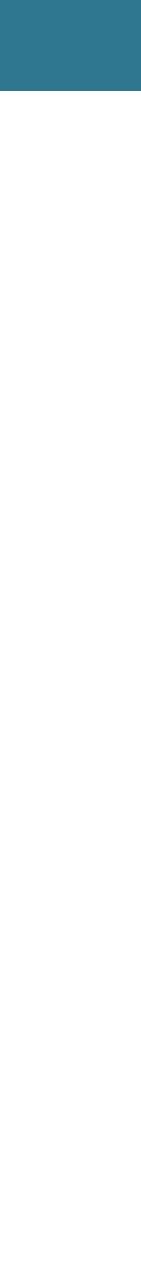
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 $-(x_1 - x_0)]^2 \rangle_{t \sim \mathcal{U}([0,1]), (x_0, x_1) \sim p(x_{\text{hard}}, x_{\text{reco}})}$ 







(\*)

Omnifold [1911.09107]

#### **Distribution mapping**

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### Several methods

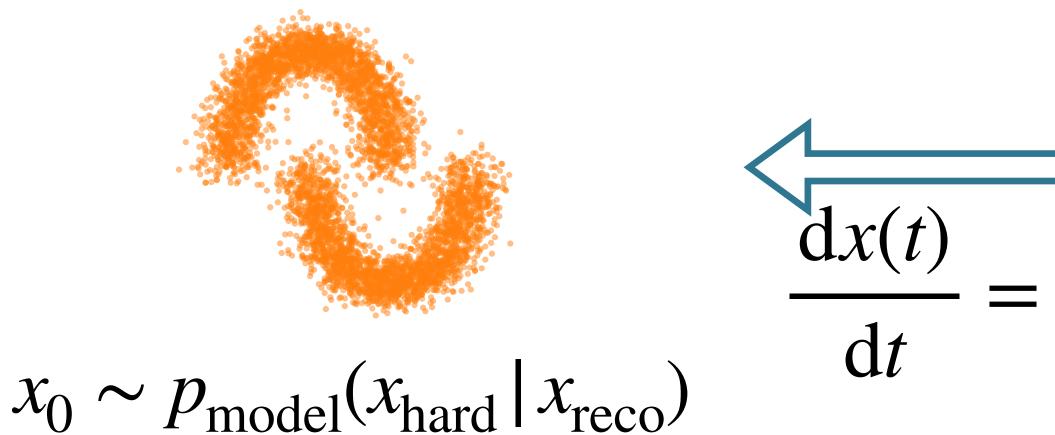
### **Conditional phase space sampling**

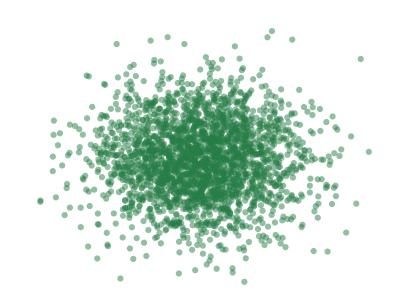
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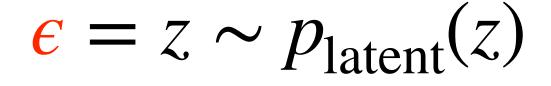


## Conditional Flow Matching (CFM)



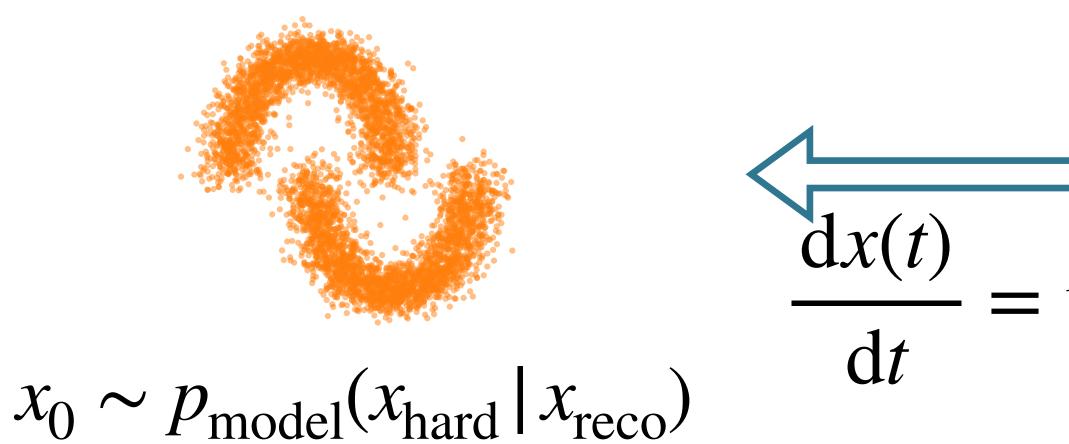


 $= v_{\theta}(x(t), t \,|\, x_{\text{reco}})$ 



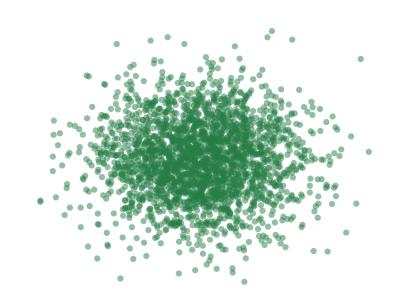


## Conditional Flow Matching (CFM)



- Connect  $x_0$  and  $\epsilon$  with a linear trajectory:
- The NN is regressed to predict the velocity field:
- For sampling, solve ODE starting from  $\epsilon$ :

• Loss: 
$$\mathscr{L}_{\text{CFM}} = \Big\langle [v_{\theta}((1-t)x_0 + t\epsilon, t, x_0)] + t\epsilon \Big\rangle \Big\rangle$$



$$v_{\theta}(x(t), t \mid x_{\text{reco}})$$

 $\epsilon = z \sim p_{\text{latent}}(z)$ 

$$x(t) = (1-t)x_0 + t\epsilon$$

ty field:  $v_{\theta}(x(t), t | x_{reco}) \approx \frac{dx(t)}{dt} = \epsilon - x_0$ 

$$x_0 = \epsilon + \int_1^0 v_{\theta}(x(t), t | x_{\text{reco}}) dt$$

 $x_{\text{reco}}) - (\epsilon - x_0)]^2 \rangle_{t \sim \mathcal{U}([0,1]), (x_0, x_{\text{reco}}) \sim p(x_{\text{hard}}, x_{\text{reco}}), \epsilon \sim \mathcal{N}(0,1)}$ 





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Omnifold [1911.09107]

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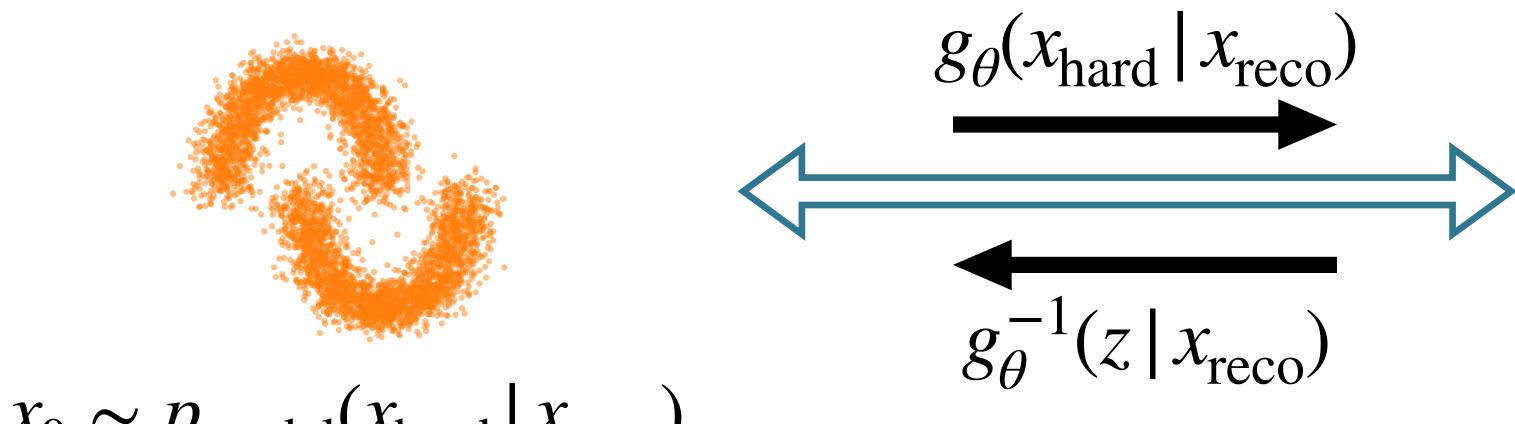
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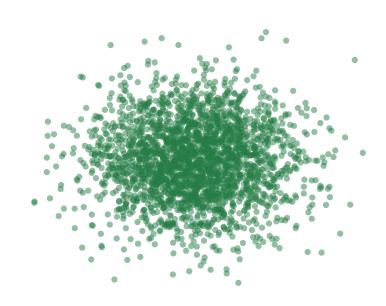




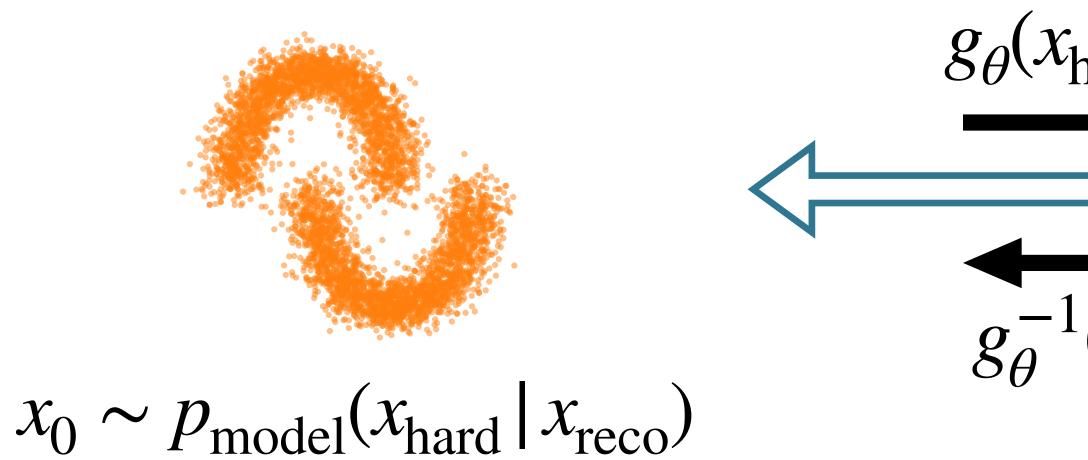
## Conditional INN (CINN)



### $x_0 \sim p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}})$



 $z \sim p_{\text{latent}}(z)$ 

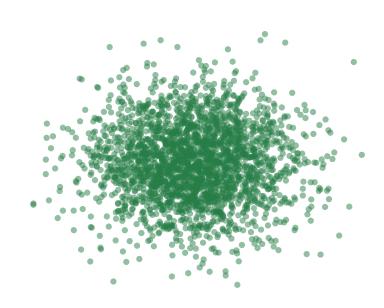


#### • Bijective function between $p_{\text{latent}}(z)$ and p

 $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) = p_{\text{latent}}(z)$ 

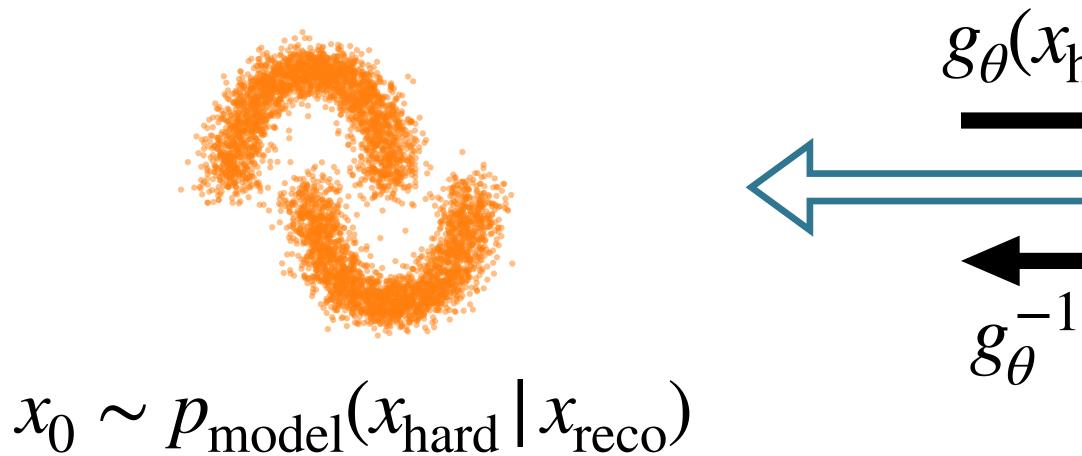


hard 
$$x_{reco}$$



$$z \sim p_{\text{latent}}(z)$$

$$p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}):$$
  
 $\left| \det \frac{\partial g_{\theta}(x_{\text{h}}, x_{\text{r}})}{\partial x_{\text{hard}}} \right| = p_{\text{lat.}}(z) \left| \det J_{g_{\theta}} \right|$ 



**Bijective function** between  $p_{\text{latent}}(z)$  and p

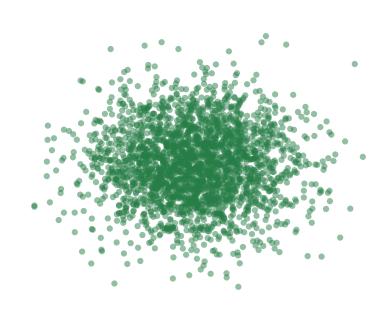
 $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) = p_{\text{latent}}(z)$ 

> Pairs  $(x_{hard}, x_{reco})$  are passed through the NN to the latent space:



$$S_{\theta}(x_{hard} | x_{reco})$$

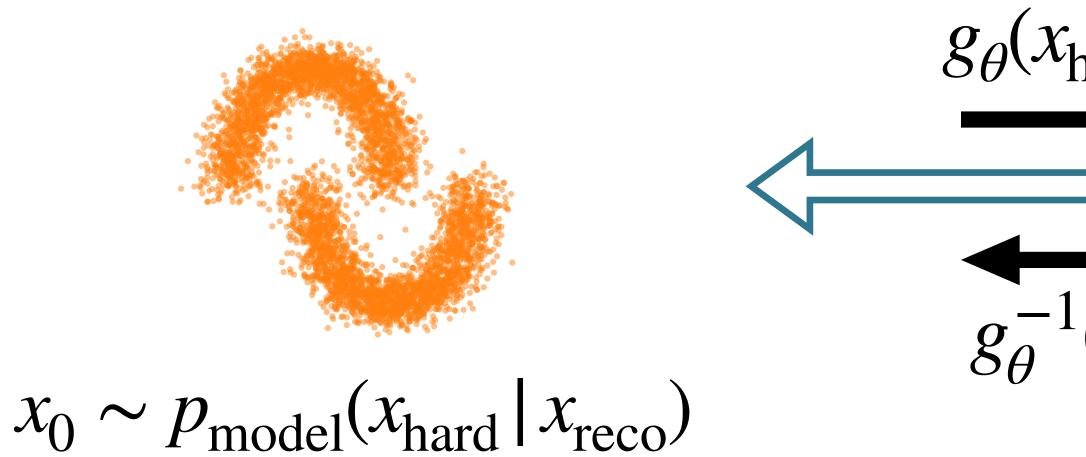
$$= g_{\theta}^{-1}(z | x_{reco})$$



 $z \sim p_{\text{latent}}(z)$ 

$$\mathcal{P}_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}):$$
  
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 $z = g_{\theta}(x_{\text{hard}} | x_{\text{reco}})$ 



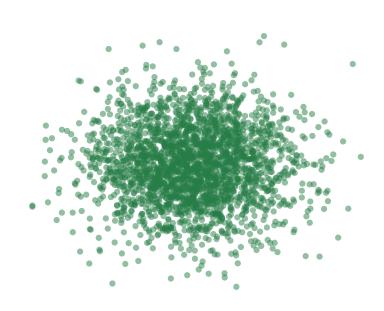
Bijective function

between 
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 and  $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}})$ :  
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• Pairs  $(x_{hard}, x_{reco})$  are passed through the NN to the latent space: • Once trained, one can sample -conditioned on reco- from the latent:  $p_{hard}(x) \approx p_{model}(x_{hard} | x_{reco})$ 



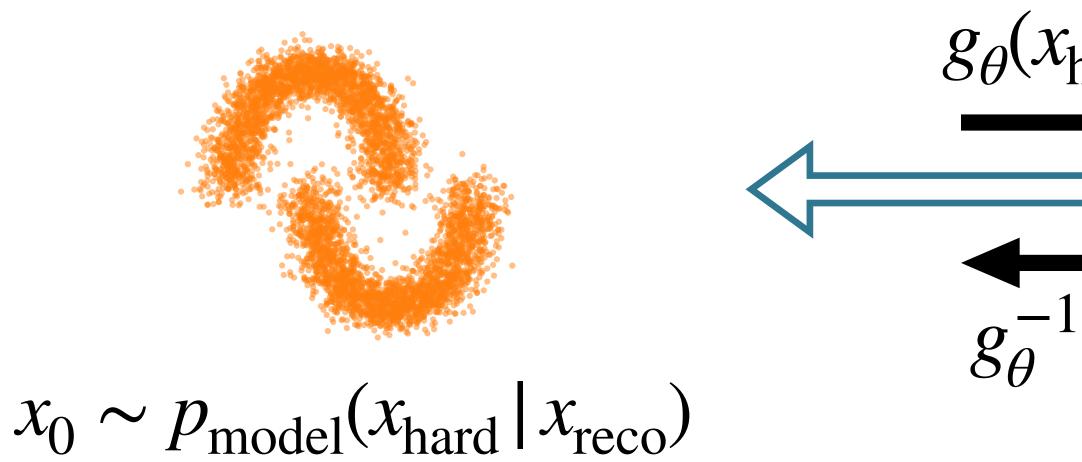
hard 
$$|x_{reco}\rangle$$
  
 $|(z|x_{reco})|$ 



 $z \sim p_{\text{latent}}(z)$ 

 $z = g_{\theta}(x_{\text{hard}} | x_{\text{reco}})$ 





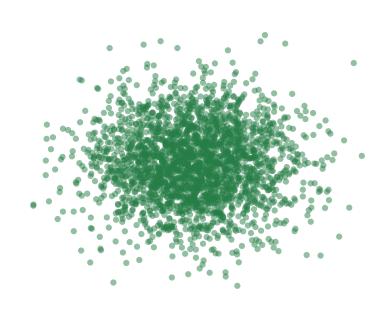
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> Pairs  $(x_{hard}, x_{reco})$  are passed through the NN to the latent space:  $\mathscr{L}_{\text{cINN}} = -\langle \log p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) \rangle_{(x_0, x_1) \sim p(x_{\text{hard}}, x_{\text{reco}})}$ Loss:



hard 
$$x_{reco}$$
  
 $(z | x_{reco})$ 



 $z \sim p_{\text{latent}}(z)$ 

 $z = g_{\theta}(x_{\text{hard}} | x_{\text{reco}})$ • Once trained, one can sample -conditioned on reco- from the latent:  $p_{hard}(x) \approx p_{model}(x_{hard} | x_{reco})$ 



## Z+jets events

# $Z(p_T > 200 \text{ GeV})$ + jets events generated at $\sqrt{s} = 14 \text{ TeV}$ with Pythia 8.244 and Delphes simulation 3.5.0 available on Zenodo. Slight modification from [1911.09107] dataset

## Z + jets events

# simulation 3.5.0 available on <u>Zenodo</u>. Slight modification from [<u>1911.09107</u>] dataset

Six widely-used jet substructure observables:

- Jet mass *m*
- Jet width *w*
- Jet constituents multiplicity N

 $Z(p_T > 200 \text{ GeV})$  + jets events generated at  $\sqrt{s} = 14 \text{ TeV}$  with Pythia 8.244 and Delphes

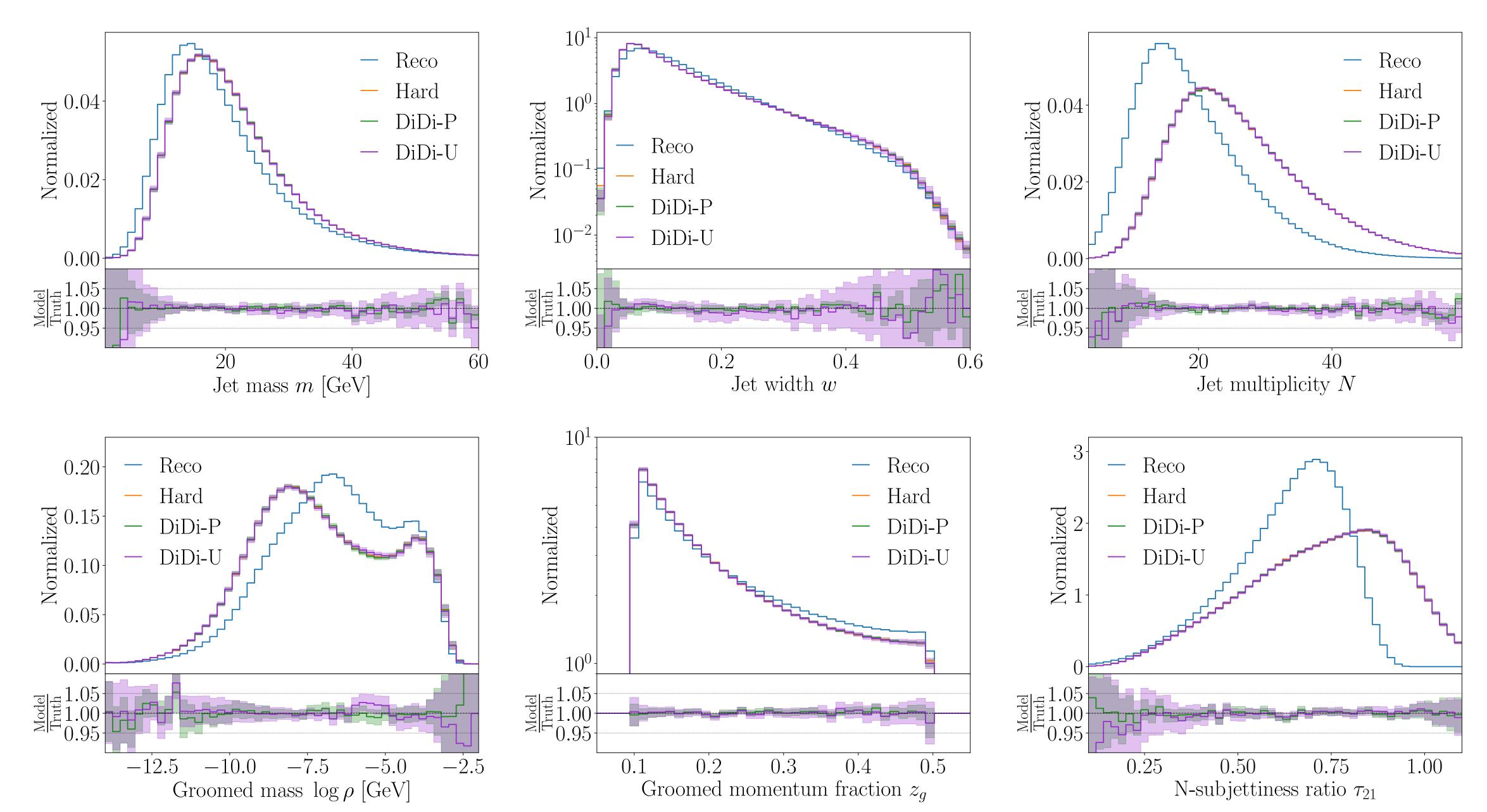
• Groomed mass  $\log \rho = 2 \log (m_{SD} / p_T)$ 

• Groomed momentum fraction  $z_g = \tau_1^{\beta=1}$ 

N-subjettiness ratio  $\tau_{21} = \tau_2^{\beta=1} / \tau_1^{\beta=1}$ 

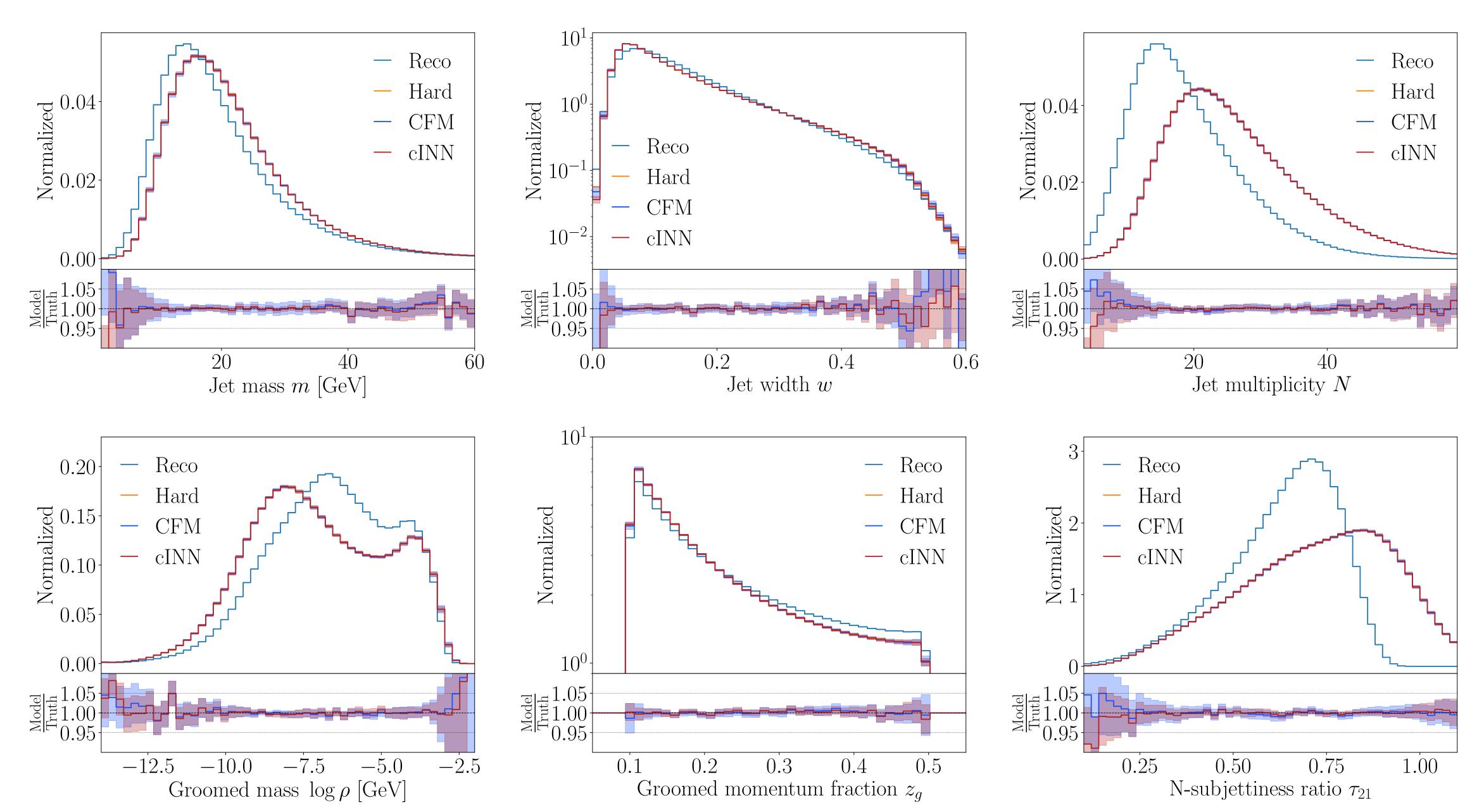


## Results (DiDi)

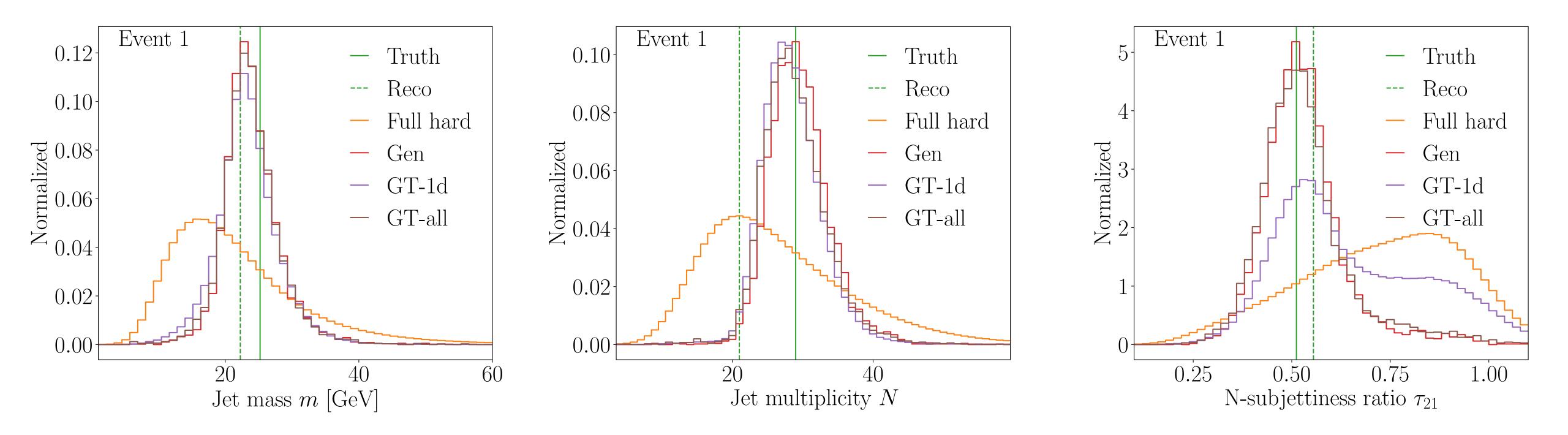




## Results (CFM & cINN)

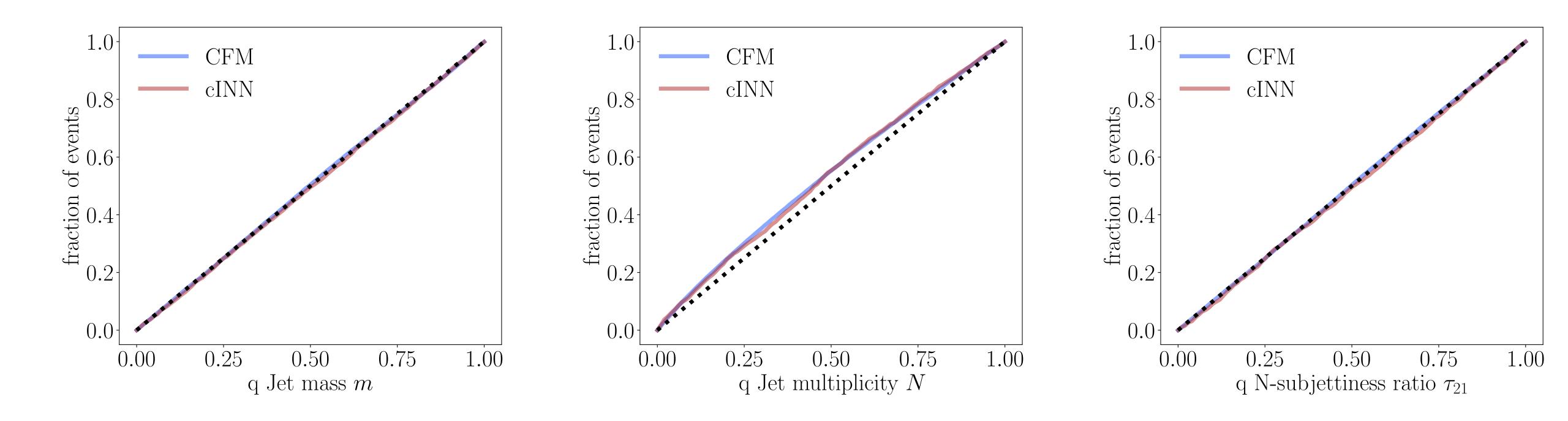


## Single event unfolding





## Calibration



Matrix elements are evaluated at  $\sqrt{s} = 13$  TeV using MadGraph\_aMC@NLO. Showering and hadronization are simulated with Pythia8, and detector response is simulated with Delphes with the standard CMS card. For a detailed description see [2305.10399].



Matrix elements are evaluated at  $\sqrt{s} = 13$  TeV using MadGraph\_aMC@NLO. Showering and the standard CMS card. For a detailed description see [2305.10399].

Unfolding from 6 final-state particles  $(bl\nu)(bqq)$ :

- 4 DoFs for the lepton
- > 3 DoFs for the missing  $p_T^{\nu}$
- 5 DoFs per jet (4-momentum + b-tag)

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**Total: 27 DoFs at reco-level** 

and 19 DoFs at parton-level



#### Much harder problem:

Unfolding to parton-level is not only inve forward simulation chain

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#### Much harder problem:

- forward simulation chain
- resonances
- Non-trivial combinatorics between physics objects at both levels

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#### Much harder problem:

- forward simulation chain
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Adding transformers:

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Faithful modeling of complex correlations at parton-level, i.e., W boson and top mass



#### Much harder problem:

- forward simulation chain
- resonances
- Non-trivial combinatorics between physics objects at both levels

#### Adding transformers:

employed to encode correlations at reco and parton-level.

Unfolding to parton-level is not only inverting detector effects, but rather inverting the entire

Faithful modeling of complex correlations at parton-level, i.e., W boson and top mass

Transfermer and Tra-CFM as an extension to the cINN and CFM [2310.07752]. A transformer is

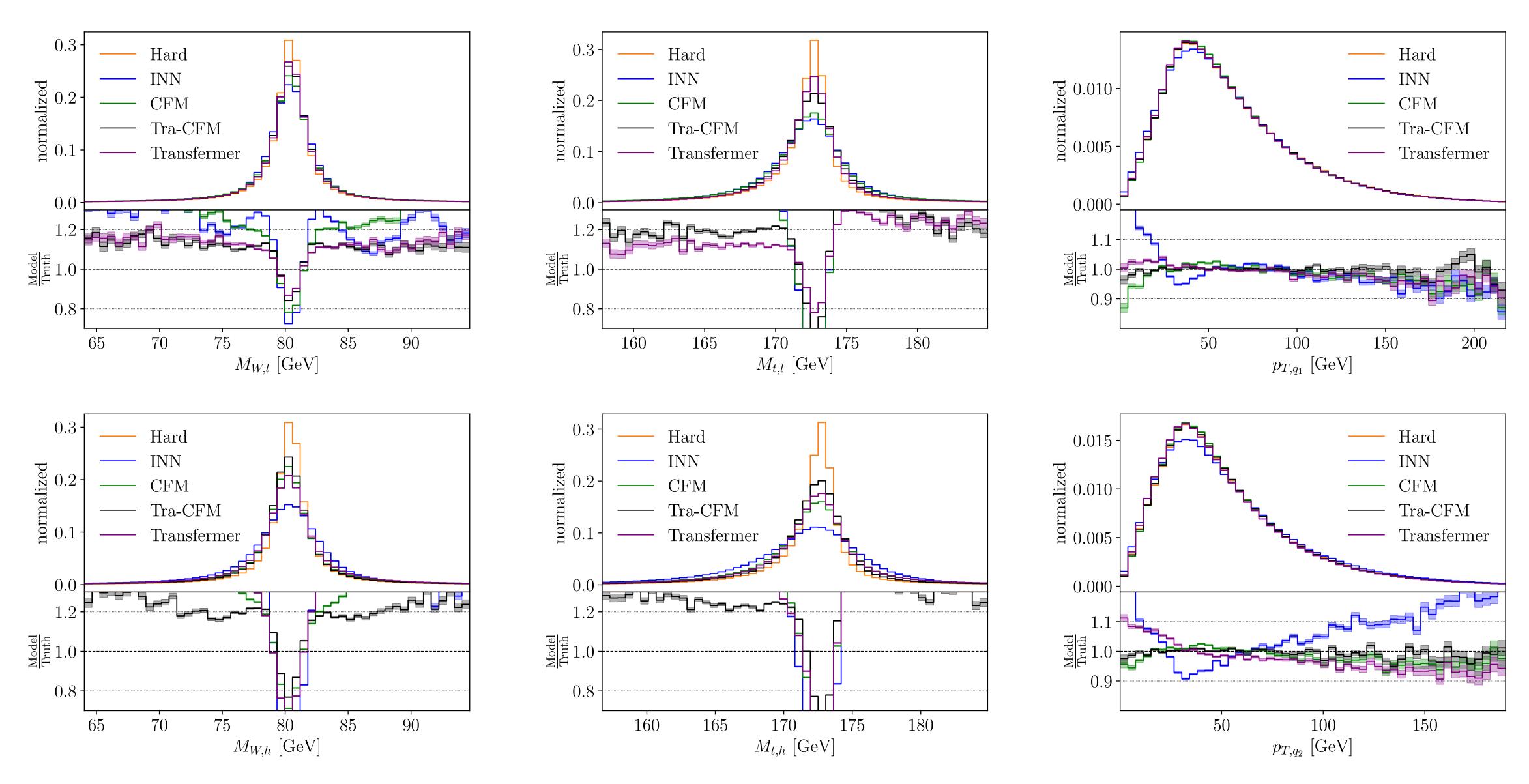








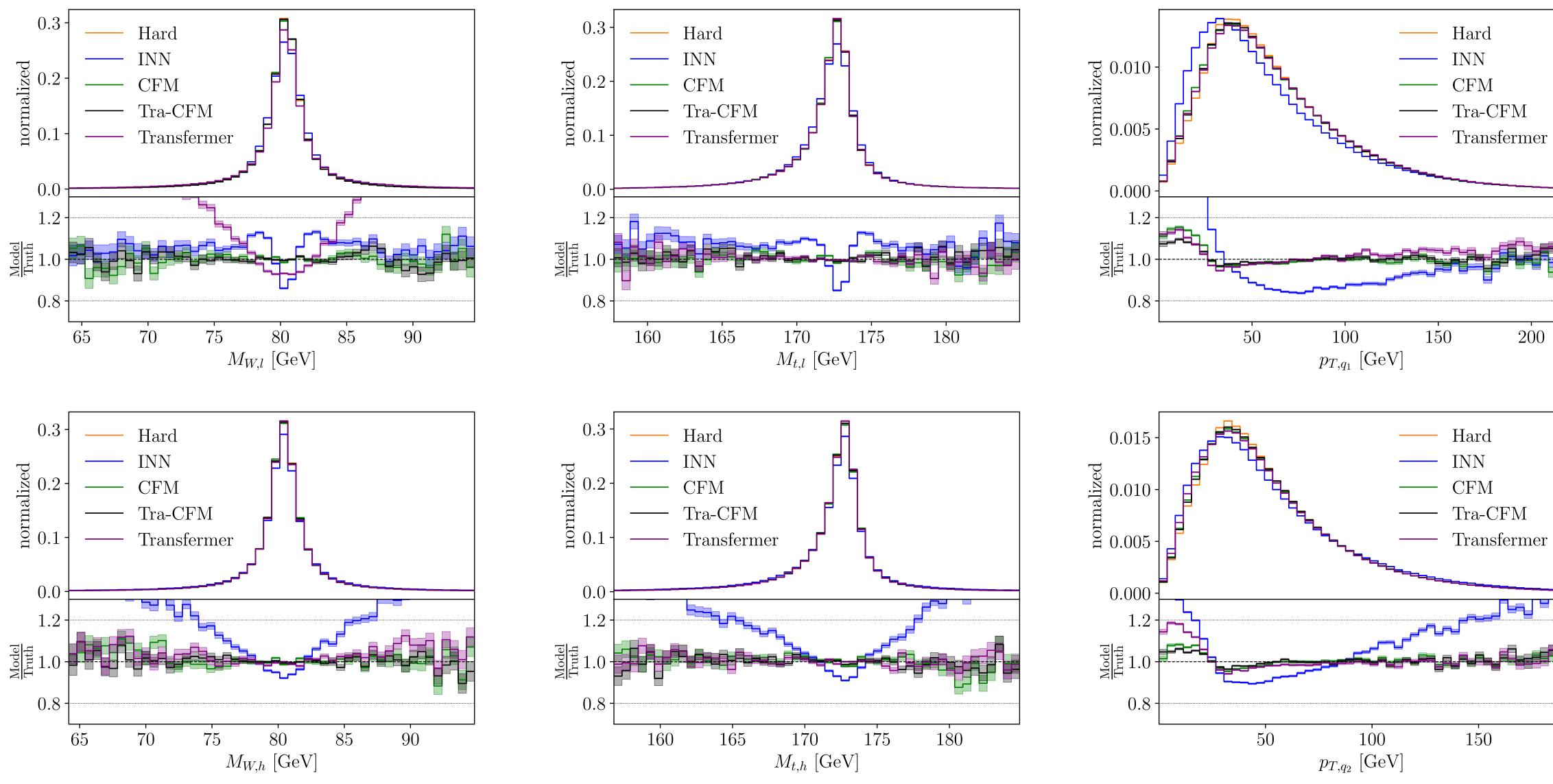
#### Unfold: $(p_{T,b_l},\eta_{b_l},\phi_{b_l},p_{T,l},\eta_l,\phi_l,p_{T,\nu},\eta_{\nu},\phi_{\nu},p_{T,b_h},\eta_{b_h},\phi_{b_h},m_{q_1},p_{T,q_1},\eta_{q_1},\phi_{q_1},p_{T,q_2},\eta_{q_2},\phi_{q_2})$



Results: naive parametrization

#### Results: mass parametrization

Unfold:



#### $(m_t, p_{T,t}^L, \eta_t^L, \phi_t^L, m_W, \eta_W^T, \phi_W^T, (m_{d_1}^W), \eta_{d_1}^W, \phi_{d_1}^W)$









- accross many dimensions.
- CFM and cINN both learn the phase space probabilities for each event, so it is best suited to describe complex detector effects, but their also more complex architectures to train.
- for correlations and resonances reconstruction
- The mass parametrization allows for more efficient unfolding without the need of very large networks
- Single-event unfolding, calibrated posteriors, compare to other models...

### Summary and outlook

ML-based unfolding is an unbinned transformative analysis tool capable of dealing with correlations

Distribution mapping can be trained on matched and unmatched data and is relatively fast to train

Parton-level unfolding is a reasonably complicated task, but transformers help greatly in accounting







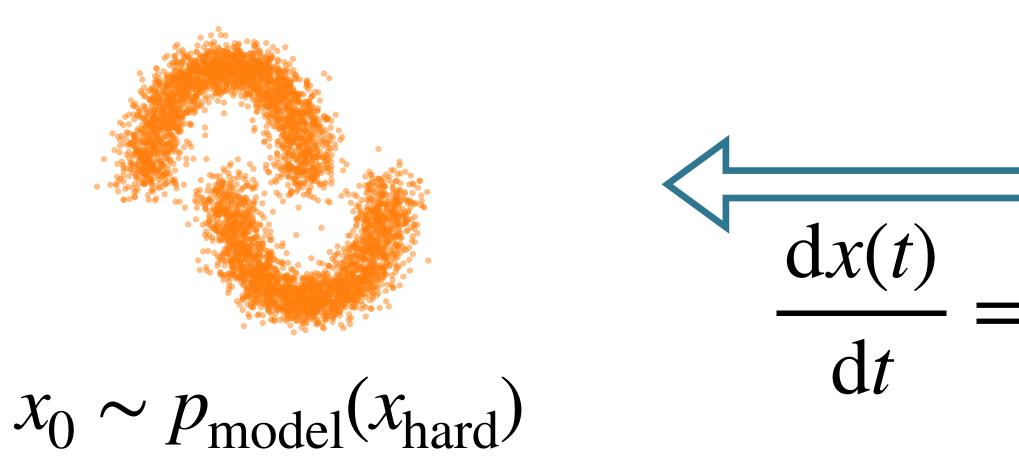




# Thanks for your attention!

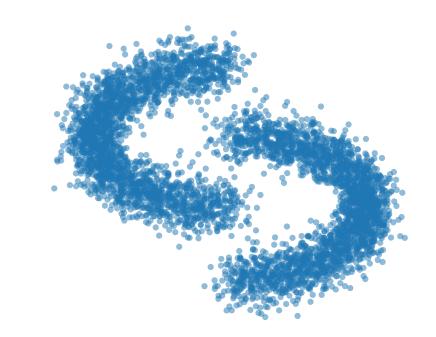


## **Direct Diffusion (DiDi)**



- Connect  $x_0$  and  $x_1$  with a linear trajectory:
- The NN is regressed to predict the velocity
- For

For sampling, solve ODE starting from 
$$x_1$$
:  $x_0 = x_1 + \int_1^0 v_\theta(x(t), t) dt$ 
Loss:
$$\mathscr{L}_{\text{DiDi-P}} = \left\langle \left[ v_\theta((1-t)x_0 + tx_1, t) - (x_1 - x_0) \right]^2 \right\rangle_{t \sim \mathscr{U}([0,1]), (x_0, x_1) \sim p(x_{\text{hard}}, x_{\text{reco}})} \\ \mathscr{L}_{\text{DiDi-U}} = \left\langle \left[ v_\theta((1-t)x_0 + tx_1, t) - (x_1 - x_0) \right]^2 \right\rangle_{t \sim \mathscr{U}([0,1]), x_0 \sim p(x_{\text{hard}}, x_1 \sim p(x_{\text{reco}}))} \right\rangle_{t \sim \mathscr{U}([0,1]), x_0 \sim p(x_{\text{hard}}, x_1 \sim p(x_{\text{reco}}))}$$



$$v_{\theta}(x(t), t)$$

$$x_1 \sim p_{\text{reco}}(x_{\text{reco}})$$

$$x(t) = (1 - t)x_0 + tx_1$$

ty field: 
$$v_{\theta}(x(t), t) \approx \frac{\mathrm{d}x(t)}{\mathrm{d}t} = x_1 - x_0$$



## Z + jets events

#### $Z(p_T > 200 \text{ GeV})$ + jets events generated at $\sqrt{s} = 14 \text{ TeV}$ with Pythia 8.244 and Delphes simulation 3.5.0 available on Zenodo

Six widely-used jet substructure observables:

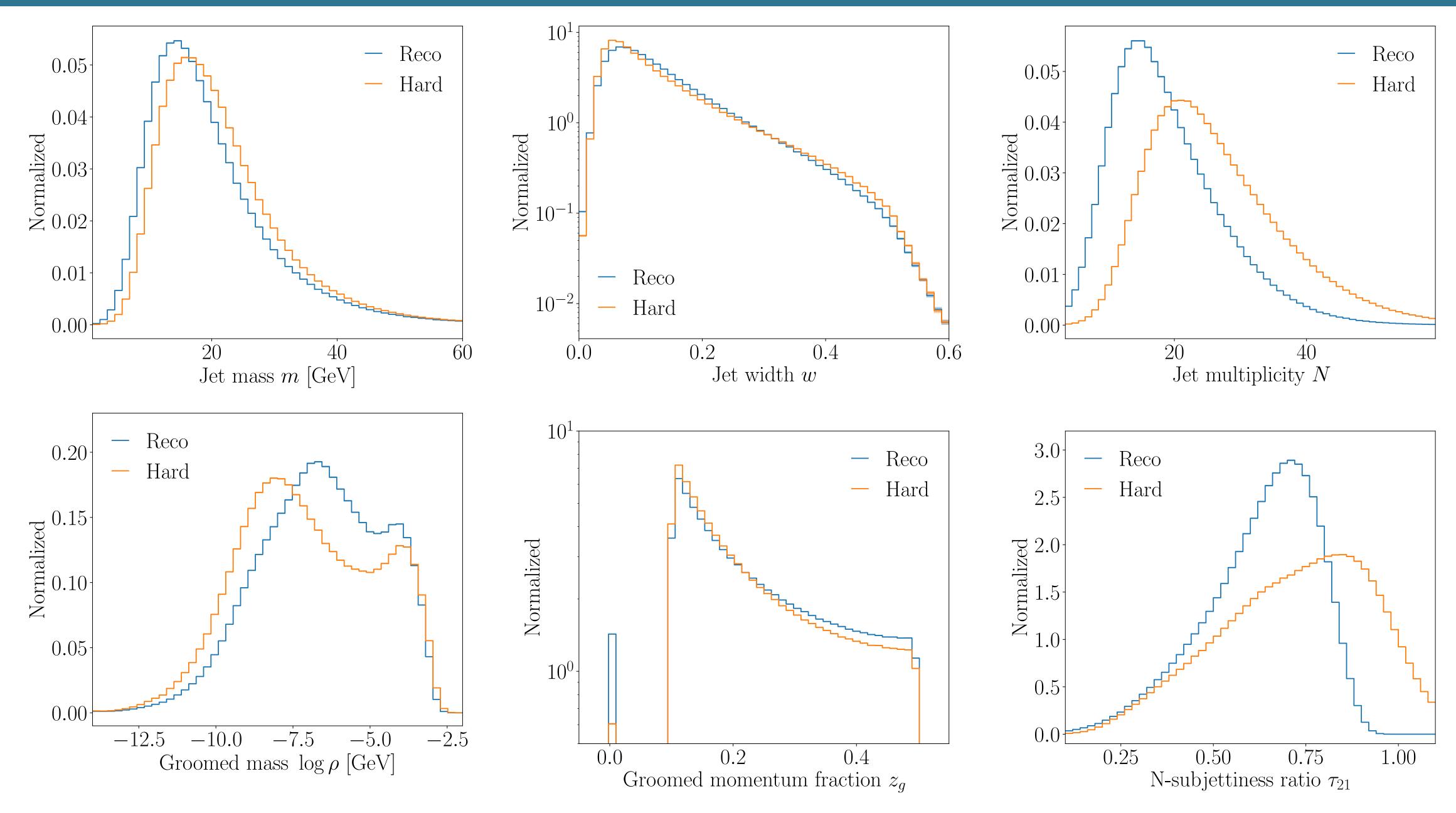
- Jet mass *m*
- ► Jet width *w*
- Jet constituents multiplicity N

Networks of ~3M parameters 19M training events and 1M validation events ~4M events for testing

- Groomed mass  $\log \rho = 2 \log (m_{SD} / p_T)$ • Groomed momentum fraction  $z_g = \tau_1^{\beta=1}$
- N-subjettiness ratio  $\tau_{21} = \tau_2^{\beta=1} / \tau_1^{\beta=1}$



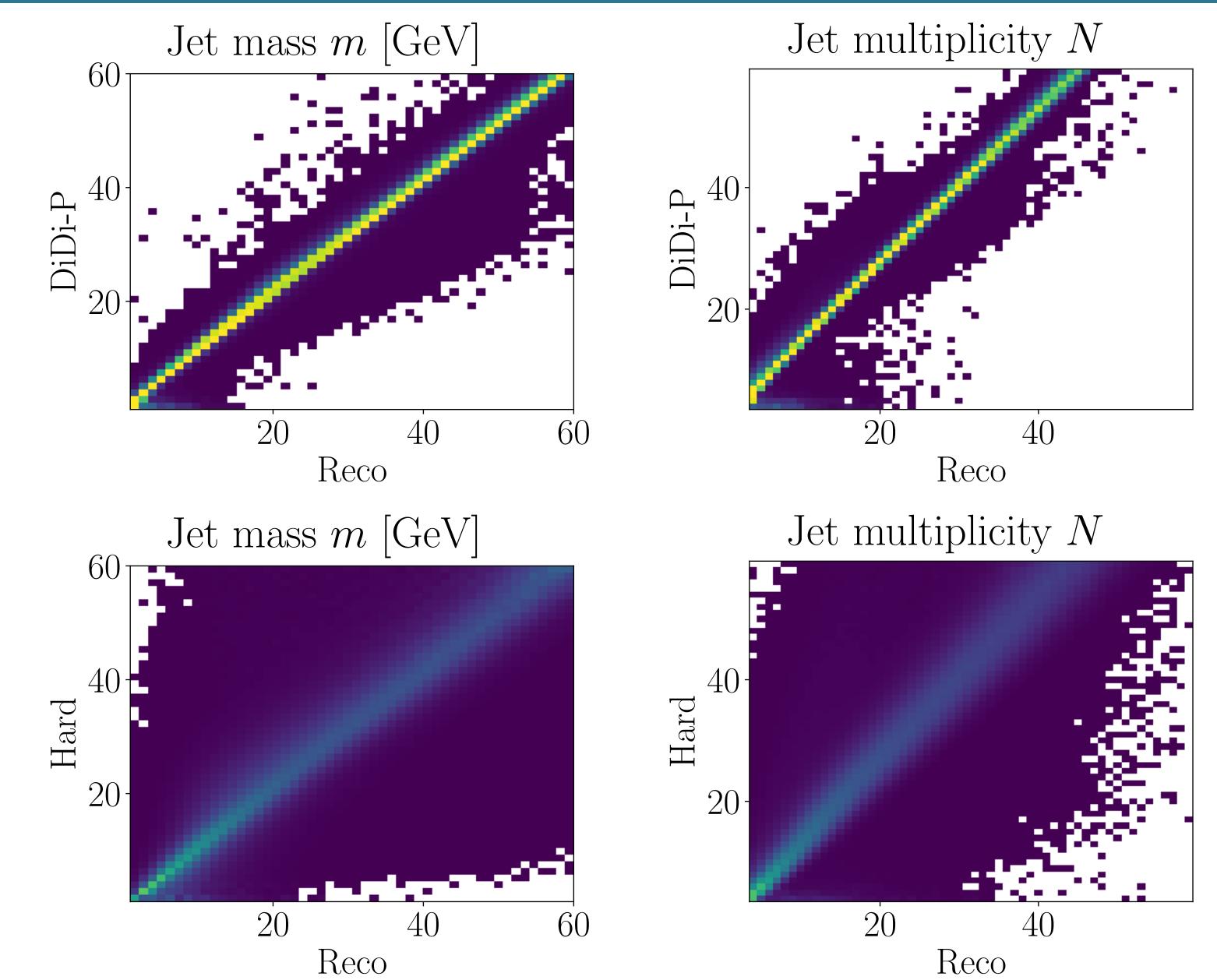




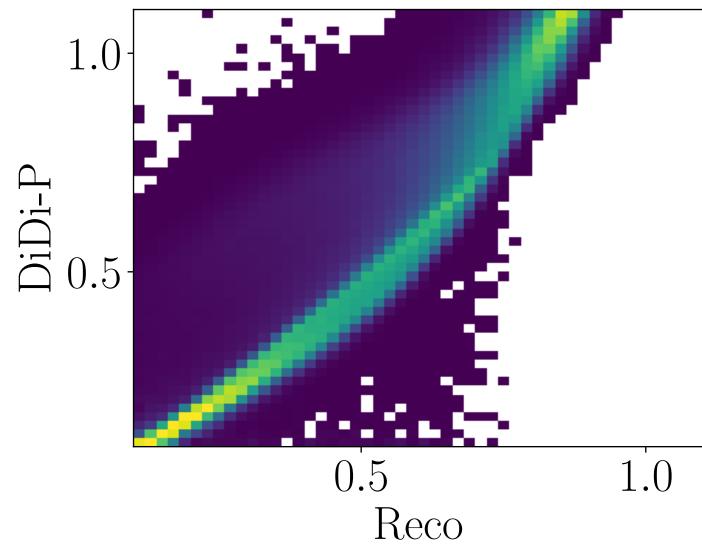
### The dataset

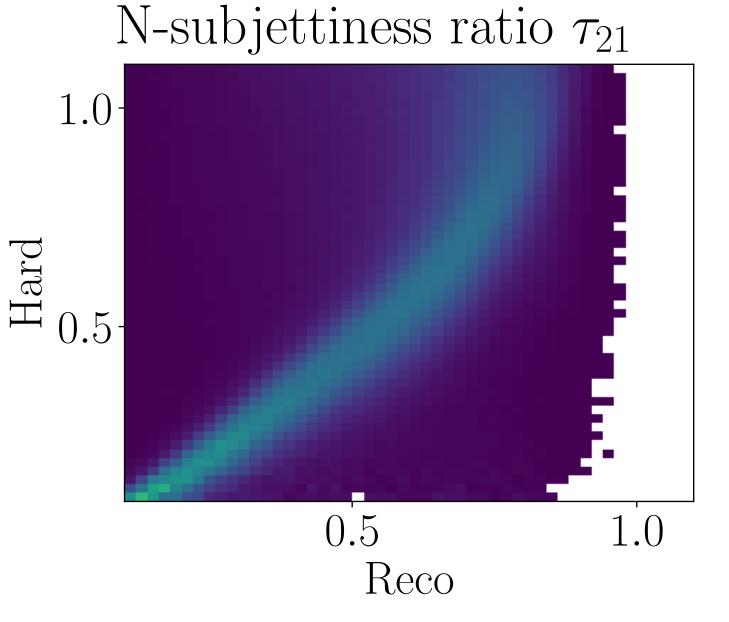


# Optimal transport (DiDi)



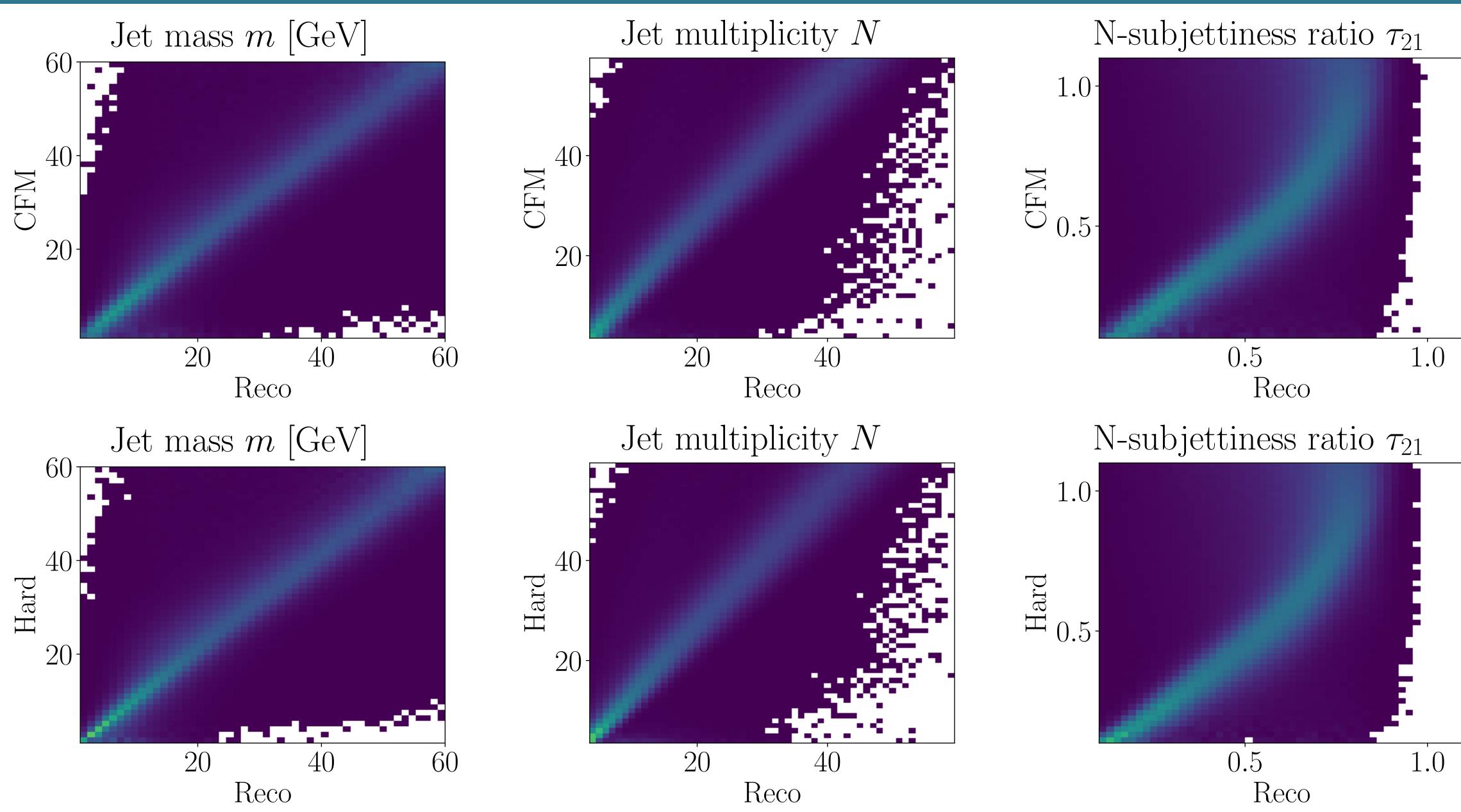
#### N-subjettiness ratio $\tau_{21}$





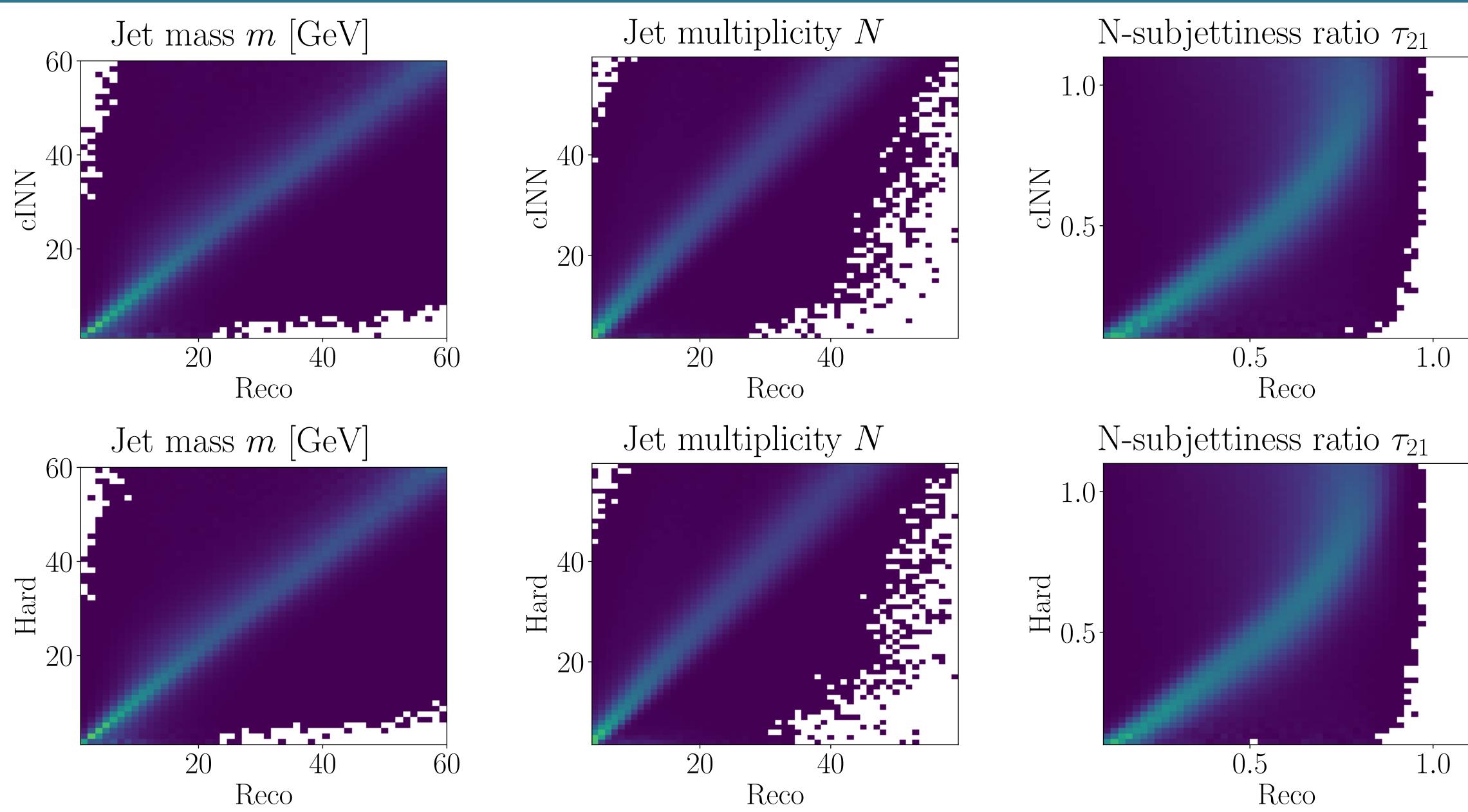


# Optimal transport (CFM)

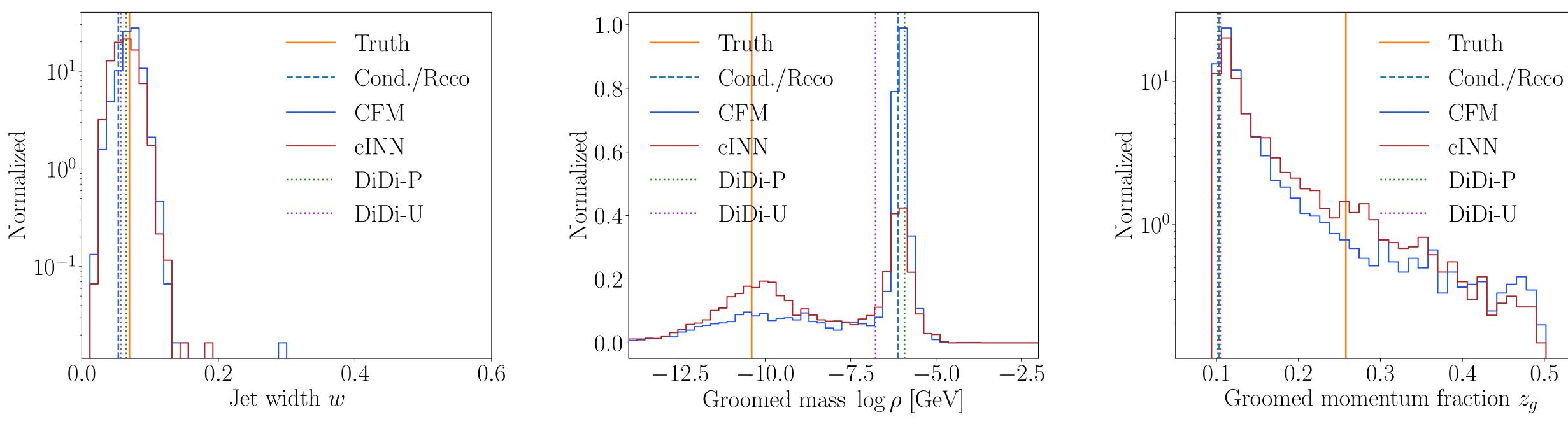




# Optimal transport (cINN)



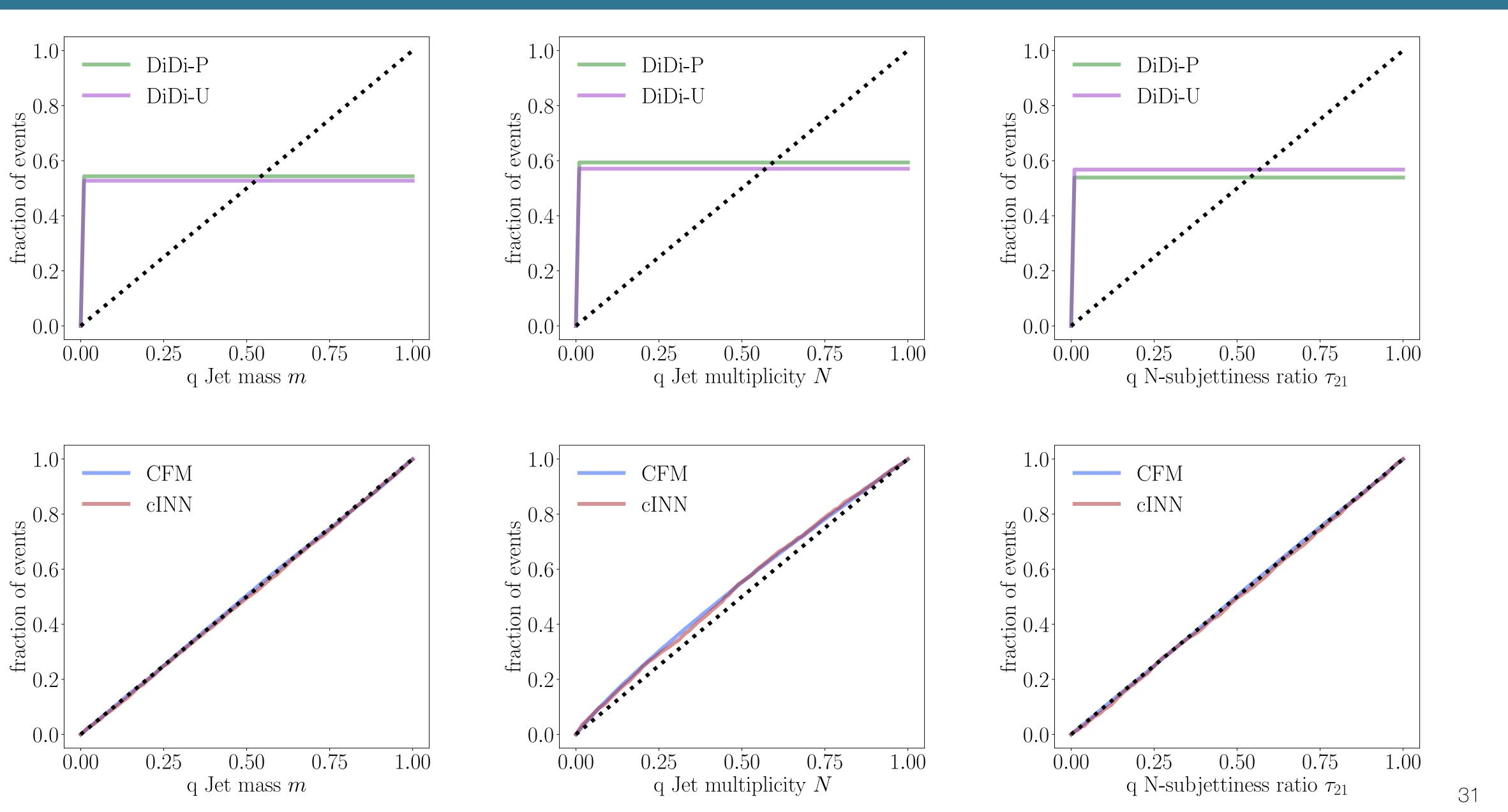








## Calibration



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Unfolding from 6 final-state particles  $(bl\nu)(bqq)$ :

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- 5 DoFs per jet (4-momentum + b-tag)

Non-bayesian networks

cINN ~ 8M parameters, CFM ~ 6M, Tra-CFM, Transfermer ~ 3M

10M training events and 1M testing events

hadronization are simulated with Pythia8, and detector response is simulated with Delphes with

**Total: 27 DoFs at reco-level** and 19 DoFs at parton-level



#### Adding transformers:

dimensions and reco-level event:

$$p_{\text{model}}(x_{\text{part}} | x_{\text{reco}}) = \prod_{i=1}^{n} p_{\text{model}}(x_{\text{part}}^{(i)} | c(x_{\text{part}}^{(0)}, \dots, x_{\text{part}}^{(i-1)}, x_{\text{reco}}))$$

each different dimension:

 $v(x_{\text{part}}(t), t \mid x_{\text{reco}}) = ($ 

For transfermer, likelihoods are factorized autoregressively on all previous parton-level

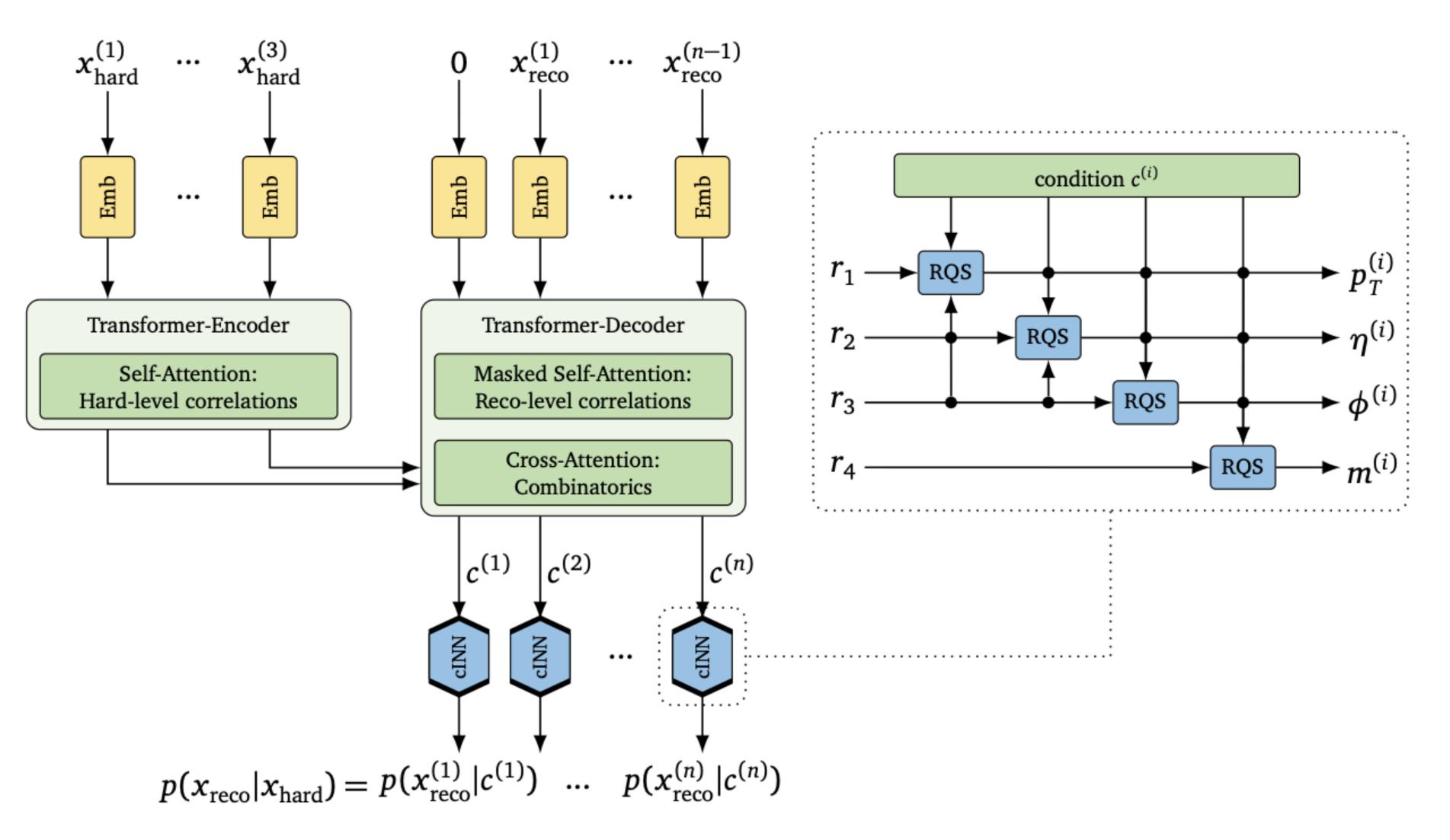
For Tra-CFM, the transformer is made time-dependent and a small CFM predicts velocities at

$$\left(v^{(1)}(c^{(1)},t), \dots, v^{(n)}(c^{(n)},t)\right)$$





#### Transfermer





#### Tra-CFM

