# Modern Machine Learning Tools for Unfolding 

Javier Mariño Villadamigo<br>In collaboration with Nathan Huetsch, Anja Butter, Theo Heimel, Tilman Plehn



ACAT March $12^{\text {th }} 2024$


## Fundamentals of unfolding

- Distributions $f(t)$ of a physics variable $t$ to be measured in particle physics experiments are often not directly accessible.



## Fundamentals of unfolding

- Distributions $f(t)$ of a physics variable $t$ to be measured in particle physics experiments are often not directly accessible.




## Fundamentals of unfolding

- Distributions $f(t)$ of a physics variable $t$ to be measured in particle physics experiments are often not directly accessible.


- Using Monte Carlo (MC) methods, the direct process from an assumption $f(t)^{\text {model }}$ to the expected measured distribution $g(s)$ can be simulated.


## Fundamentals of unfolding

- Distributions $f(t)$ of a physics variable $t$ to be measured in particle physics experiments are often not directly accessible.


- Using Monte Carlo (MC) methods, the direct process from an assumption $f(t)^{\text {model }}$ to the expected measured distribution $g(s)$ can be simulated.
- The inverse process from the actual measured distribution $g(s)$ to the true distribution $f(t)$ is difficult and ill-posed: small changes in $g(s)$ can cause large modifications in the reconstructed $\tilde{f}(t)$.


## Fundamentals of unfolding

Distributions $f(t)$ of a physics variable $t$ to be measured in particle physics experiments are often not directly accessible.



- Using Monte Carlo (MC) methods, the direct process from an assumption $f(t)^{\text {model }}$ to the expected measured distribution $g(s)$ can be simulated.
- The inverse process from the actual measured distribution $g(s)$ to the true distribution $f(t)$ is difficult and ill-posed: small changes in $g(s)$ can cause large modifications in the reconstructed $\tilde{f}(t)$.


## Why neural networks?

Traditionally:

- Matrix-based unfolding

$$
g(s)=\int R(s \mid t) f(t) \mathrm{d} t
$$

## Why neural networks?

Traditionally:

- Matrix-based unfolding

$$
g(s)=\iint_{\substack{\text { detector } \\ \text { response } \\ \text { matrix }}} R(s \mid t) f(t) \mathrm{d} t
$$

## Why neural networks?

Traditionally:

- Matrix-based unfolding

$$
g(s)=\iint_{\substack{\text { detector } \\ \text { response } \\ \text { matrix }}}^{R(s \mid t)} f(t) \mathrm{d} t \quad r_{i}=\sum_{j} R_{i j} \cdot t_{j}
$$

## Why neural networks?

## Traditionally:

- Matrix-based unfolding

$$
g(s)=\iint_{\substack{\text { detector } \\ \text { response }}}^{R(s \mid t)} f(t) \mathrm{d} t \quad r_{i}=\sum_{j} R_{i j} \cdot t_{j}
$$

- Various ways to invert the detector response matrix: SVD, IBU, IDS, etc.


## Why neural networks?

## Traditionally:

- Matrix-based unfolding

$$
g(s)=\iint_{\substack{\text { detector } \\ \text { response }}}^{R(s \mid t)} f(t) \mathrm{d} t \quad r_{i}=\sum_{j} R_{i j} \cdot t_{j}
$$

- Various ways to invert the detector response matrix: SVD, IBU, IDS, etc.
- General need for regularization: trade-off between bias and statistical uncertainty


## Why neural networks?

## Traditionally:

- Matrix-based unfolding

$$
g(s)=\iint_{\substack{\text { detector } \\ \text { response }}}^{R(s \mid t)} f(t) \mathrm{d} t \quad r_{i}=\sum_{j} R_{i j} \cdot t_{j}
$$

- Various ways to invert the detector response matrix: SVD, IBU, IDS, etc.
- General need for regularization: trade-off between bias and statistical uncertainty
- Requires binning and can only unfold a few dimensions


## Why neural networks?

Traditionally:

- Matrix-based unfolding

$$
g(s)=\iint_{\substack{\text { detector } \\ \text { response }}}^{R(s \mid t)} f(t) \mathrm{d} t \quad r_{i}=\sum_{j} R_{i j} \cdot t_{j}
$$

- Various ways to invert the detector response matrix: SVD, IBU, IDS, etc.
- General need for regularization: trade-off between bias and statistical uncertainty
- Requires binning and can only unfold a few dimensions

With neural networks:

- ML-based unfolding


## Why neural networks?

Traditionally:

- Matrix-based unfolding

$$
g(s)=\iint_{\substack{\text { detector } \\ \text { response }}}^{R(s \mid t)} f(t) \mathrm{d} t \quad r_{i}=\sum_{j} R_{i j} \cdot t_{j}
$$

- Various ways to invert the detector response matrix: SVD, IBU, IDS, etc.
- General need for regularization: trade-off between bias and statistical uncertainty
- Requires binning and can only unfold a few dimensions

With neural networks:

- ML-based unfolding
- Unbinned: advantageous if one wants to derive quantities from the unfolding observables


## Why neural networks?

Traditionally:

- Matrix-based unfolding

$$
g(s)=\iint_{\substack{\text { detector } \\ \text { response }}}^{R(s \mid t)} f(t) \mathrm{d} t \quad r_{i}=\sum_{j} R_{i j} \cdot t_{j}
$$

- Various ways to invert the detector response matrix: SVD, IBU, IDS, etc.
- General need for regularization: trade-off between bias and statistical uncertainty
- Requires binning and can only unfold a few dimensions

With neural networks:

- ML-based unfolding
- Unbinned: advantageous if one wants to derive quantities from the unfolding observables
- Allows to unfold (and account for correlations in) many dimensions


## Why neural networks?

Traditionally:

- Matrix-based unfolding

$$
g(s)=\iint_{\substack{\text { detector } \\ \text { response }}}^{R(s \mid t)} f(t) \mathrm{d} t \quad r_{i}=\sum_{j} R_{i j} \cdot t_{j}
$$

- Various ways to invert the detector response matrix: SVD, IBU, IDS, etc.
- General need for regularization: trade-off between bias and statistical uncertainty
- Requires binning and can only unfold a few dimensions

With neural networks:

- ML-based unfolding
- Unbinned: advantageous if one wants to derive quantities from the unfolding observables
- Allows to unfold (and account for correlations in) many dimensions
- Some methods allow for independent single-event unfolding


## Several approaches

## Event reweighting

- Omnifold [1911.09107]
- (*)


## Distribution mapping

- Direct Diffusion [2311.17175]
- Schrödinger Bridge [2308.12351]
- (*)


## Conditional phase space sampling

- GANs [1912.00477]
- Latent Diffusion [2305.10399]
- Conditional Flow Matching [2305.10475]
- cINN [2212.08674, 2006.06685]
- (*)


## Several approaches

## Event reweighting

- Omnifold [1911.09107]
- (*)


## Distribution mapping

- Direct Diffusion [2311.17175]
- Schrödinger Bridge [2308.12351]
- (*)


## Conditional phase space sampling

- GANs [1912.00477]
- Latent Diffusion [2305.10399]
- Conditional Flow Matching [2305.10475]
- cINN [2212.08674, 2006.06685]
- (*)


## Several approaches

## Event reweighting

- Omnifold [1911.09107]
- (*)


## Distribution mapping

Direct Diffusion [2311.17175]

## Conditional phase space sampling

- GANs [1912.00477]
- Latent Diffusion [2305.10399]
- Conditional Flow Matching [2305.10475]
- cINN [2212.08674, 2006.06685]
- (*)
- Schroainger ifage [2308.12351]
- (*)


## Direct Diffusion (DiDi)



$$
x_{1} \sim p_{\text {reco }}\left(x_{\text {reco }}\right)
$$

## Direct Diffusion (DiDi)



$$
x_{0} \sim p_{\text {model }}\left(x_{\text {hard }}\right)
$$



$$
x_{1} \sim p_{\text {reco }}\left(x_{\text {reco }}\right)
$$

- Connect $x_{0}$ and $x_{1}$ with a linear trajectory: $\quad x(t)=(1-t) x_{0}+t x_{1}$


## Direct Diffusion (DiDi)



$$
x_{1} \sim p_{\text {reco }}\left(x_{\text {reco }}\right)
$$

- Connect $x_{0}$ and $x_{1}$ with a linear trajectory: $x(t)=(1-t) x_{0}+t x_{1}$
- The NN is regressed to predict the velocity field: $\quad v_{\theta}(x(t), t) \approx \frac{\mathrm{dx}(\mathrm{t})}{\mathrm{dt}}=x_{1}-x_{0}$


## Direct Diffusion (DiDi)



$$
x_{0} \sim p_{\text {model }}\left(x_{\text {hard }}\right)
$$



$$
x_{1} \sim p_{\text {reco }}\left(x_{\text {reco }}\right)
$$

- Connect $x_{0}$ and $x_{1}$ with a linear trajectory: $\quad x(t)=(1-t) x_{0}+t x_{1}$
- The NN is regressed to predict the velocity field: $\quad v_{\theta}(x(t), t) \approx \frac{\mathrm{dx}(\mathrm{t})}{\mathrm{dt}}=x_{1}-x_{0}$
- For sampling, solve ODE starting from $x_{1}: \quad x_{0}=x_{1}+\int_{1}^{0} v_{\theta}(x(t), t) \mathrm{dt}$


## Direct Diffusion (DiDi)



$$
x_{1} \sim p_{\text {reco }}\left(x_{\text {reco }}\right)
$$

- Connect $x_{0}$ and $x_{1}$ with a linear trajectory: $x(t)=(1-t) x_{0}+t x_{1}$
- The NN is regressed to predict the velocity field: $\quad v_{\theta}(x(t), t) \approx \frac{\mathrm{dx}(\mathrm{t})}{\mathrm{dt}}=x_{1}-x_{0}$
- For sampling, solve ODE starting from $x_{1}: \quad x_{0}=x_{1}+\int_{1}^{0} v_{\theta}(x(t), t) \mathrm{dt}$
- Loss: $\quad \mathscr{L}_{\text {DiDi }}=\left\langle\left[v_{\theta}\left((1-t) x_{0}+t x_{1}, t\right)-\left(x_{1}-x_{0}\right)\right]^{2}\right\rangle_{t \sim \mathcal{U}([0,1]),\left(x_{0}, x_{1}\right) \sim p\left(x_{\text {nard }}, x_{\text {reco }}\right)}$


## Several methods

## Event reweighting

- Omnifold [1911.09107]
$\left.{ }^{\bullet}{ }^{*}\right)$


## Distribution mapping

- Direct Diffusion [2311.17175]
- Schrödinger Bridge [2308.12351]
- (*)


## Conditional phase space sampling

- GANs [1912.00477]
- Latent Diffusion [2325103991

Conditional Flow Matching [2305.10475]

- cINN [2212.08674, 2000.06685$]$
- (*)


## Conditional Flow Matching (CFM)

$$
x_{0} \sim p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)
$$



$$
\epsilon=z \sim p_{\text {latent }}(z)
$$

## Conditional Flow Matching (CFM)

$$
x_{0} \sim p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right) \quad \mathrm{d} t \quad \epsilon=z \sim p_{\text {latent }}(z)
$$

- Connect $x_{0}$ and $\epsilon$ with a linear trajectory: $\quad x(t)=(1-t) x_{0}+t \epsilon$
- The NN is regressed to predict the velocity field: $\quad v_{\theta}\left(x(t), t \mid x_{\mathrm{reco}}\right) \approx \frac{\mathrm{dx}(\mathrm{t})}{\mathrm{dt}}=\epsilon-x_{0}$
- For sampling, solve ODE starting from $\epsilon: \quad x_{0}=\epsilon+\int_{1}^{0} v_{\theta}\left(x(t), t \mid x_{\text {reco }}\right) \mathrm{dt}$
- Loss: $\quad \mathscr{L}_{\mathrm{CFM}}=\left\langle\left[v_{\theta}\left((1-t) x_{0}+t \epsilon, t, x_{\mathrm{reco}}\right)-\left(\epsilon-x_{0}\right)\right]^{2}\right\rangle_{t \sim \mathcal{U}([0,1]),\left(x_{0}, x_{\text {reco }}\right) \sim p\left(x_{\text {nard }}, x_{\text {reco }}\right), \epsilon \sim \mathcal{N}(0,1)}$


## Several methods

## Event reweighting

- Omnifold [1911.09107]
$\left.{ }^{\bullet}{ }^{*}\right)$


## Distribution mapping

- Direct Diffusion [2311.17175]
- Schrödinger Bridge [2308.12351]
- (*)


## Conditional phase space sampling

- GANs [1912.00477]
- Latent Diffusion [2305.10399]
- Conditional Flow Matching [2305. 10475] cINN [2212.08674, 2006.06685]
- (*)


## Conditional INN (cINN)

$$
x_{0} \sim p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)
$$



$$
z \sim p_{\text {latent }}(z)
$$

## Conditional INN (cINN)



$$
x_{0} \sim p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)
$$



$$
z \sim p_{\text {latent }}(z)
$$

- Bijective function between $p_{\text {latent }}(z)$ and $p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)$ :

$$
p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)=p_{\text {latent }}(z)\left|\operatorname{det} \frac{\partial g_{\theta}\left(x_{\mathrm{h}}, x_{\mathrm{r}}\right)}{\partial x_{\text {hard }}}\right|=p_{\text {lat. }}(z)\left|\operatorname{det} J_{g_{\theta}}\right|
$$

## Conditional INN (cINN)



$$
x_{0} \sim p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)
$$



$$
z \sim p_{\text {latent }}(z)
$$

- Bijective function between $p_{\text {latent }}(z)$ and $p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)$ :

$$
p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)=p_{\text {latent }}(z)\left|\operatorname{det} \frac{\partial g_{\theta}\left(x_{\mathrm{h}}, x_{\mathrm{r}}\right)}{\partial x_{\text {hard }}}\right|=p_{\text {lat. }}(z)\left|\operatorname{det} J_{g_{\theta}}\right|
$$

- Pairs $\left(x_{\text {hard }}, x_{\text {reco }}\right)$ are passed through the NN to the latent space: $z=g_{\theta}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)$


## Conditional INN (cINN)



$$
z \sim p_{\text {latent }}(z)
$$

- Bijective function between $p_{\text {latent }}(z)$ and $p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)$ :

$$
p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)=p_{\text {latent }}(z)\left|\operatorname{det} \frac{\partial g_{\theta}\left(x_{\mathrm{h}}, x_{\mathrm{r}}\right)}{\partial x_{\text {hard }}}\right|=p_{\text {lat. }}(z)\left|\operatorname{det} J_{g_{\theta}}\right|
$$

- Pairs $\left(x_{\text {hard }}, x_{\text {reco }}\right)$ are passed through the NN to the latent space: $z=g_{\theta}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)$
- Once trained, one can sample -conditioned on reco- from the latent: $\quad p_{\text {hard }}(x) \approx p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)$


## Conditional INN (cINN)



$$
z \sim p_{\text {latent }}(z)
$$

- Bijective function between $p_{\text {latent }}(z)$ and $p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)$ :

$$
p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)=p_{\text {latent }}(z)\left|\operatorname{det} \frac{\partial g_{\theta}\left(x_{\mathrm{h}}, x_{\mathrm{r}}\right)}{\partial x_{\text {hard }}}\right|=p_{\text {lat. }}(z)\left|\operatorname{det} J_{g_{\theta}}\right|
$$

- Pairs $\left(x_{\text {hard }}, x_{\text {reco }}\right)$ are passed through the NN to the latent space: $z=g_{\theta}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)$
- Once trained, one can sample -conditioned on reco- from the latent: $\quad p_{\text {hard }}(x) \approx p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)$
- Loss: $\mathscr{L}_{\text {cINN }}=-\left\langle\log p_{\text {model }}\left(x_{\text {hard }} \mid x_{\text {reco }}\right)\right\rangle_{\left(x_{0}, x_{1}\right) \sim p\left(x_{\text {nard }}, x_{\text {reco }}\right)}$


## $Z+$ jets events

$Z\left(p_{T}>200 \mathrm{GeV}\right)+$ jets events generated at $\sqrt{s}=14 \mathrm{TeV}$ with Pythia 8.244 and Delphes simulation 3.5.0 available on Zenodo. Slight modification from [1911.09107] dataset

## $Z+$ jets events

$Z\left(p_{T}>200 \mathrm{GeV}\right)+$ jets events generated at $\sqrt{s}=14 \mathrm{TeV}$ with Pythia 8.244 and Delphes simulation 3.5.0 available on Zenodo. Slight modification from [1911.09107] dataset

Six widely-used jet substructure observables:

- Jet mass $m$
- Jet width $w$
- Jet constituents multiplicity $N$
- Groomed mass $\log \rho=2 \log \left(m_{\mathrm{SD}} / p_{T}\right)$
- Groomed momentum fraction $z_{g}=\tau_{1}^{\beta=1}$
- N -subjettiness ratio $\tau_{21}=\tau_{2}^{\beta=1} / \tau_{1}^{\beta=1}$


## Results (DiDi)








## Results (CFM \& cINN)








## Single event unfolding





## Calibration





## Top-pair events: unfolding to parton-level

Matrix elements are evaluated at $\sqrt{s}=13 \mathrm{TeV}$ using MadGraph_aMC@NLO. Showering and hadronization are simulated with Pythia8, and detector response is simulated with Delphes with the standard CMS card. For a detailed description see [2305.10399].

## Top-pair events: unfolding to parton-level

Matrix elements are evaluated at $\sqrt{s}=13 \mathrm{TeV}$ using MadGraph_aMC@NLO. Showering and hadronization are simulated with Pythia8, and detector response is simulated with Delphes with the standard CMS card. For a detailed description see [2305.10399].

Unfolding from 6 final-state particles (blv)(bqq):

- 4 DoFs for the lepton
- 3 DoFs for the missing $p_{T}^{\nu}$
- 5 DoFs per jet (4-momentum + b-tag)


## Top-pair events: unfolding to parton-level

Matrix elements are evaluated at $\sqrt{s}=13 \mathrm{TeV}$ using MadGraph_aMC@NLO. Showering and hadronization are simulated with Pythia8, and detector response is simulated with Delphes with the standard CMS card. For a detailed description see [2305.10399].

Unfolding from 6 final-state particles (blv)(bqq):

- 4 DoFs for the lepton
- 3 DoFs for the missing $p_{T}^{\nu}$
- 5 DoFs per jet (4-momentum + b-tag)
Total: 27 DoFs at reco-level
and 19 DoFs at parton-level


## Top-pair events: unfolding to parton-level

Much harder problem:

- Unfolding to parton-level is not only inverting detector effects, but rather inverting the entire forward simulation chain


## Top-pair events: unfolding to parton-level

## Much harder problem:

- Unfolding to parton-level is not only inverting detector effects, but rather inverting the entire forward simulation chain
- Faithful modeling of complex correlations at parton-level, i.e., $W$ boson and top mass resonances


## Top-pair events: unfolding to parton-level

## Much harder problem:

- Unfolding to parton-level is not only inverting detector effects, but rather inverting the entire forward simulation chain
- Faithful modeling of complex correlations at parton-level, i.e., $W$ boson and top mass resonances
- Non-trivial combinatorics between physics objects at both levels


## Top-pair events: unfolding to parton-level

## Much harder problem:

- Unfolding to parton-level is not only inverting detector effects, but rather inverting the entire forward simulation chain
- Faithful modeling of complex correlations at parton-level, i.e., $W$ boson and top mass resonances
- Non-trivial combinatorics between physics objects at both levels


## Adding transformers:

## Top-pair events: unfolding to parton-level

## Much harder problem:

- Unfolding to parton-level is not only inverting detector effects, but rather inverting the entire forward simulation chain
- Faithful modeling of complex correlations at parton-level, i.e., $W$ boson and top mass resonances
- Non-trivial combinatorics between physics objects at both levels


## Adding transformers:

- Transfermer and Tra-CFM as an extension to the cINN and CFM [2310.07752]. A transformer is employed to encode correlations at reco and parton-level.


## Results: naive parametrization

- Unfold: $\quad\left(p_{T, b_{l}}, \eta_{b_{l}}, \phi_{b_{l}}, p_{T, l}, \eta_{l}, \phi_{l}, p_{T, v}, \eta_{v}, \phi_{v}, p_{T, b_{h}}, \eta_{b_{h}}, \phi_{b_{h}}, m_{q_{1}}, p_{T, q_{1}}, \eta_{q_{1}}, \phi_{q_{1}}, p_{T, q_{2}}, \eta_{q_{2}}, \phi_{q_{2}}\right)$





## Results: mass parametrization

- Unfold:
$\left(m_{t}, p_{T, t}^{L}, \eta_{t}^{L}, \phi_{t}^{L}, m_{W}, \eta_{W}^{T}, \phi_{W}^{T},\left(m_{d_{1}}^{W}\right), \eta_{d_{1}}^{W}, \phi_{d_{1}}^{W}\right)$





## Summary and outlook

- ML-based unfolding is an unbinned transformative analysis tool capable of dealing with correlations accross many dimensions.
- Distribution mapping can be trained on matched and unmatched data and is relatively fast to train
- CFM and cINN both learn the phase space probabilities for each event, so it is best suited to describe complex detector effects, but their also more complex architectures to train.
- Parton-level unfolding is a reasonably complicated task, but transformers help greatly in accounting for correlations and resonances reconstruction
- The mass parametrization allows for more efficient unfolding without the need of very large networks
- Single-event unfolding, calibrated posteriors, compare to other models...

Thanks for your attention!

## Backup

## Direct Diffusion (DiDi)



- Connect $x_{0}$ and $x_{1}$ with a linear trajectory: $\quad x(t)=(1-t) x_{0}+t x_{1}$
- The NN is regressed to predict the velocity field: $\quad v_{\theta}(x(t), t) \approx \frac{\mathrm{dx}(\mathrm{t})}{\mathrm{dt}}=x_{1}-x_{0}$
- For sampling, solve ODE starting from $x_{1}$ : $\quad x_{0}=x_{1}+\int_{1}^{0} v_{\theta}(x(t), t) \mathrm{dt}$
- Loss:

$$
\begin{aligned}
& \mathscr{L}_{\text {DiDi-P }}=\left\langle\left[v_{\theta}\left((1-t) x_{0}+t x_{1}, t\right)-\left(x_{1}-x_{0}\right)\right]^{2}\right\rangle_{t \sim U([0,1]),\left(x_{0}, x_{1}\right) \sim p\left(x_{\text {hard }}, x_{\text {reco }}\right)} \\
& \mathscr{L}_{\text {DiDi-U }}=\left\langle\left[v_{\theta}\left((1-t) x_{0}+t x_{1}, t\right)-\left(x_{1}-x_{0}\right)\right]^{2}\right\rangle_{t \sim U([0,1]), x_{0} \sim p\left(x_{\text {hard }}\right), x_{1} \sim p\left(x_{\text {reco }}\right)}
\end{aligned}
$$

## $Z+$ jets events

$Z\left(p_{T}>200 \mathrm{GeV}\right)+$ jets events generated at $\sqrt{s}=14 \mathrm{TeV}$ with Pythia 8.244 and Delphes simulation 3.5.0 available on Zenodo

## Six widely-used jet substructure observables:

- Jet mass $m$
- Jet width $w$
- Jet constituents multiplicity $N$

Networks of $\sim 3 \mathrm{M}$ parameters
19M training events and 1M validation events
$\sim 4 \mathrm{M}$ events for testing

- Groomed mass $\log \rho=2 \log \left(m_{\mathrm{SD}} / p_{T}\right)$
- Groomed momentum fraction $z_{g}=\tau_{1}^{\beta=1}$
- N -subjettiness ratio $\tau_{21}=\tau_{2}^{\beta=1} / \tau_{1}^{\beta=1}$


## The dataset








## Optimal transport (DiDi)








## Optimal transport (CFM)



## Optimal transport (cINN)







## Single events



## Calibration








## Top-pair events: unfolding to parton-level

Matrix elements are evaluated at $\sqrt{s}=13 \mathrm{TeV}$ using MadGraph_aMC@NLO. Showering and hadronization are simulated with Pythia8, and detector response is simulated with Delphes with the standard CMS card. For a detailed description see [2305.10399].

Unfolding from 6 final-state particles (blu)(bqq):

- 4 DoFs for the lepton
- 3 DoFs for the missing $p_{T}^{\nu}$
- 5 DoFs per jet (4-momentum + b-tag)


## Total: 27 DoFs at reco-level <br> and 19 DoFs at parton-level

Non-bayesian networks
cINN ~ 8M parameters, CFM ~ 6M, Tra-CFM, Transfermer ~ 3M
10M training events and 1 M testing events

## Top-pair events: unfolding to parton-level

## Adding transformers:

- For transfermer, likelihoods are factorized autoregressively on all previous parton-level dimensions and reco-level event:

$$
p_{\text {model }}\left(x_{\text {part }} \mid x_{\text {reco }}\right)=\prod_{i=1}^{n} p_{\text {model }}\left(x_{\text {part }}^{(i)} \mid c\left(x_{\text {part }}^{(0)}, \ldots, x_{\text {part }}^{(i-1)}, x_{\text {reco }}\right)\right)
$$

- For Tra-CFM, the transformer is made time-dependent and a small CFM predicts velocities at each different dimension:

$$
v\left(x_{\text {part }}(t), t \mid x_{\text {reco }}\right)=\left(v^{(1)}\left(c^{(1)}, t\right), \ldots, v^{(n)}\left(c^{(n)}, t\right)\right)
$$

## Top-pair events: unfolding to parton-level

## Transfermer



## Top-pair events: unfolding to parton-level

## Tra-CFM



