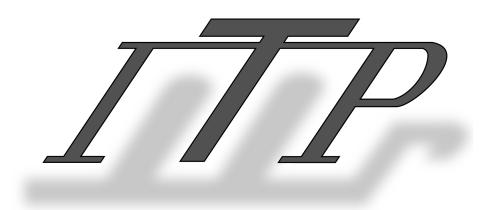
Modern Machine Learning **Tools for Unfolding**

Javier Mariño Villadamigo





Institut für Theoretische Physik - University of Heidelberg



In collaboration with Nathan Huetsch, Anja Butter, Theo Heimel, Tilman Plehn

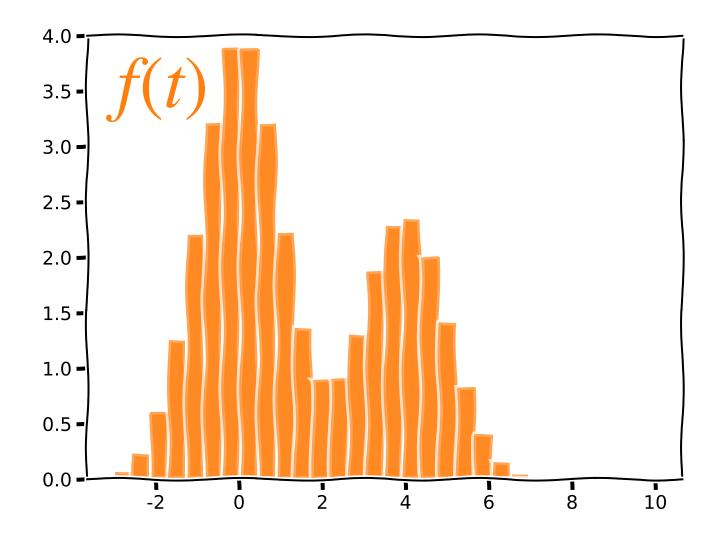
ACAT March 12th 2024



ZUKUNFT



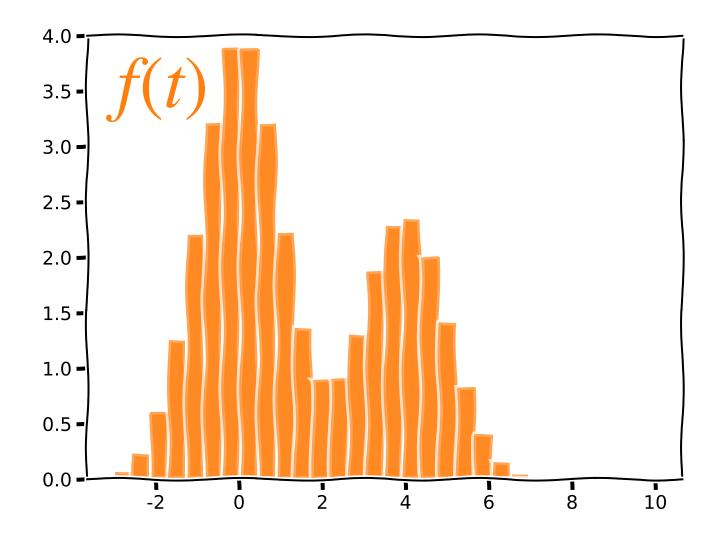
directly accessible.



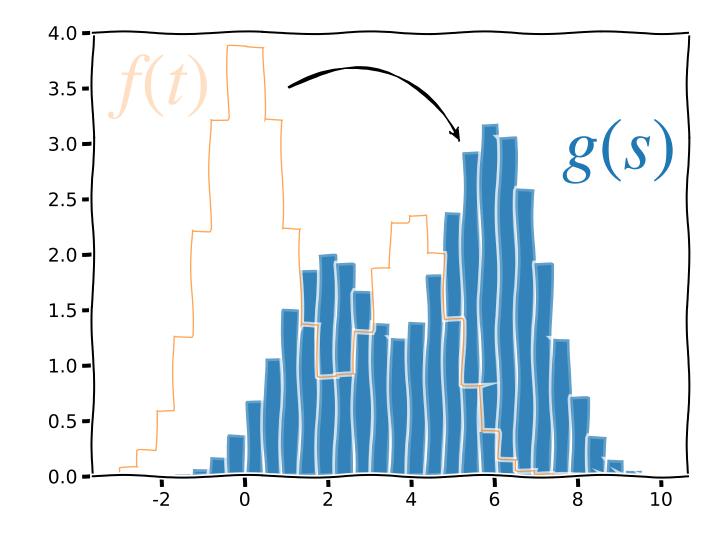
• Distributions f(t) of a physics variable t to be measured in particle physics experiments are often not



directly accessible.

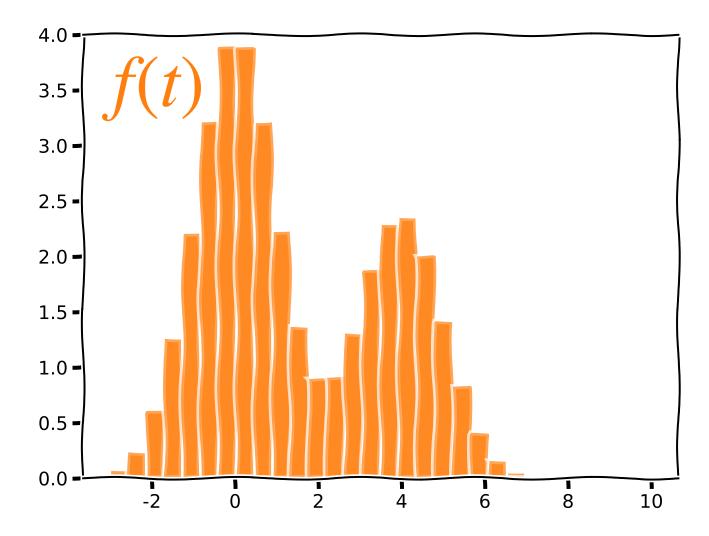


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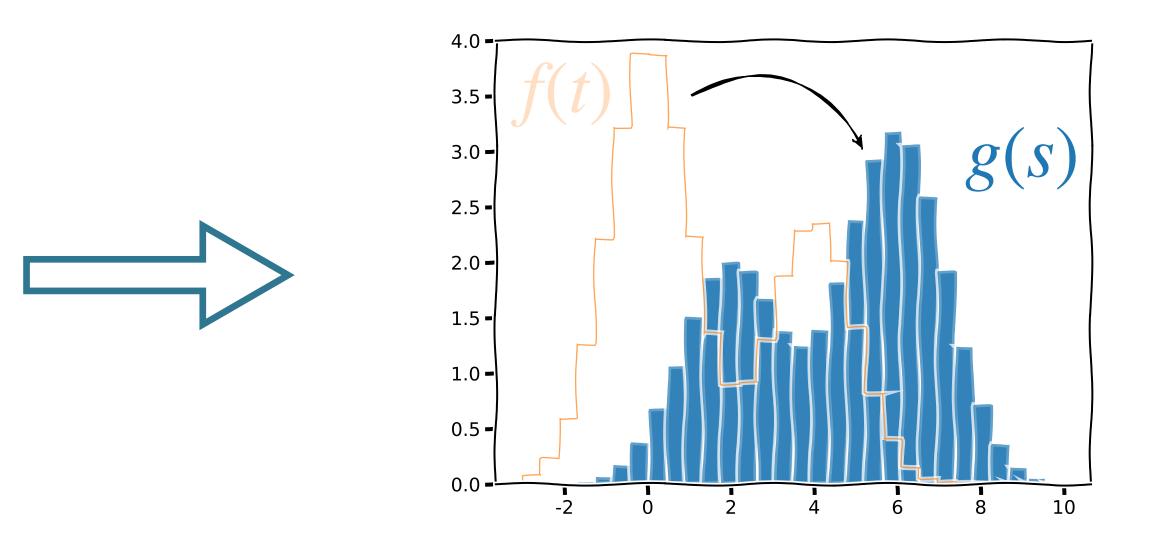


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measured distribution g(s) can be simulated.

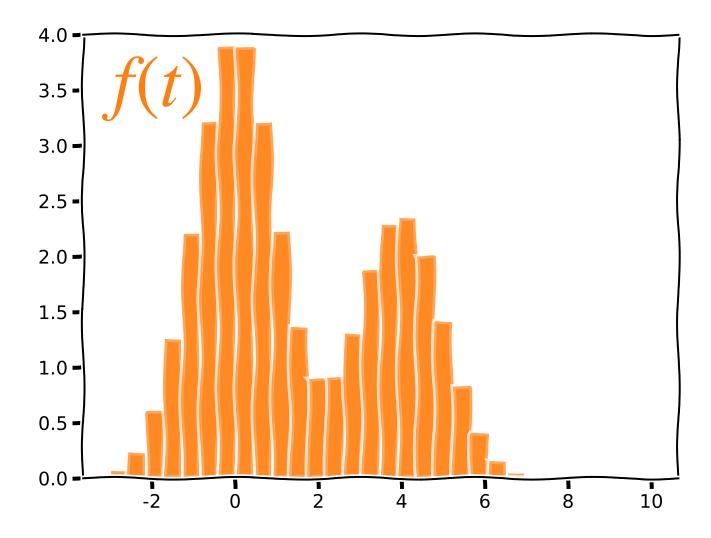
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Using Monte Carlo (MC) methods, the direct process from an assumption $f(t)^{\text{model}}$ to the expected

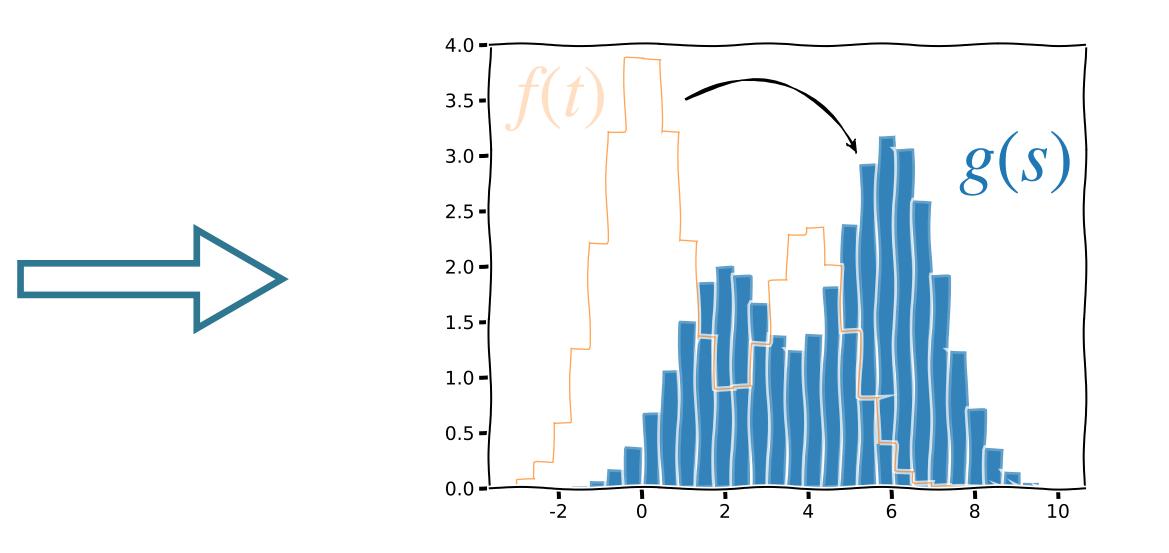


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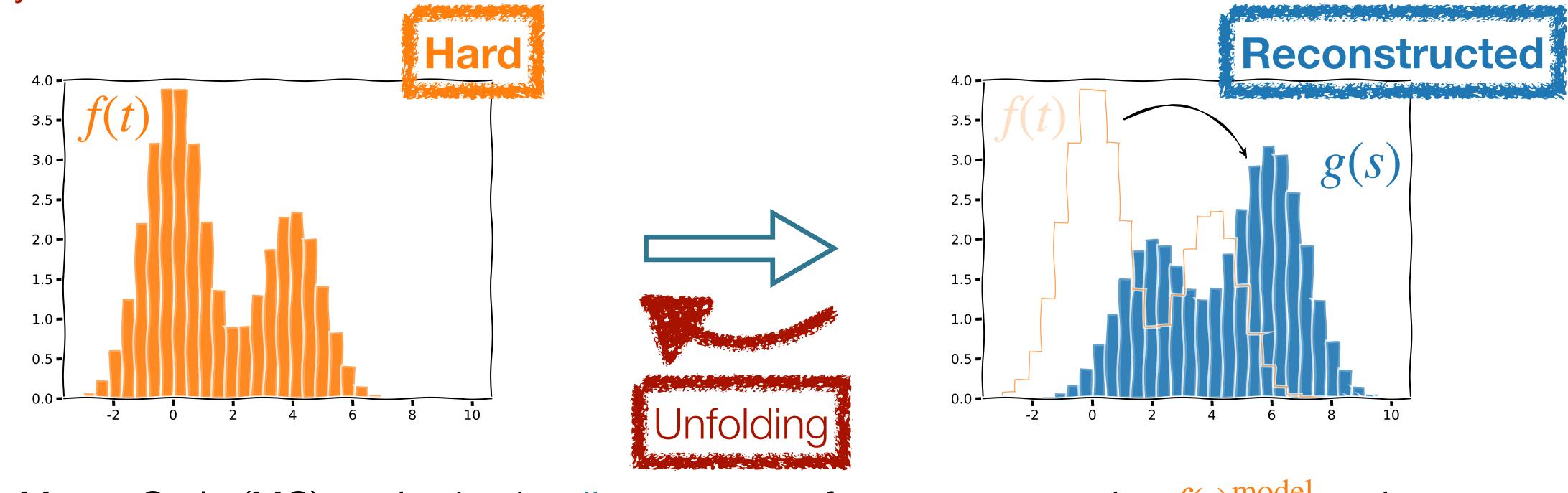
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Traditionally:

Matrix-based unfolding

$$g(s) = \int R(s \mid t) f(t) \, \mathrm{d}t$$



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detector response matrix

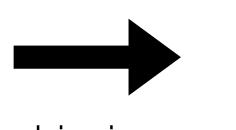


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 $r_i = \sum_j R_{ij} \cdot t_j$



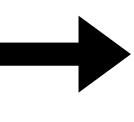
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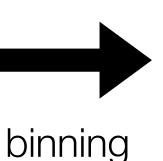
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Requires binning and can only unfold a few dimensions With neural networks:

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 - Unbinned: advantageous if one wants to derive quantities from the unfolding observables



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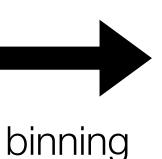
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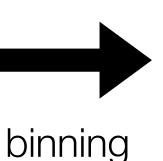
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- Various ways to invert the detector response matrix: SVD, IBU, IDS, etc.

Requires binning and can only unfold a few dimensions With neural networks:

- **ML-based** unfolding

 - Allows to unfold (and account for correlations in) many dimensions
 - Some methods allow for independent single-event unfolding



$$r_i = \sum_j R_{ij} \cdot t_j$$

General need for regularization: trade-off between bias and statistical uncertainty

Unbinned: advantageous if one wants to derive quantities from the unfolding observables





► (*)

Omnifold [1911.09107]

Distribution mapping

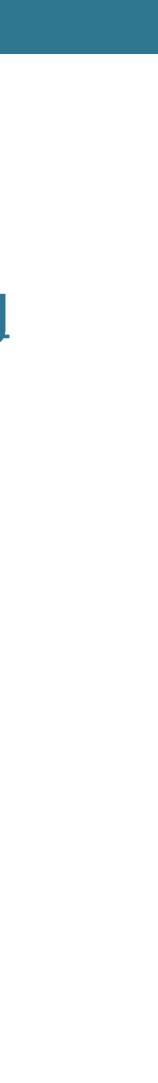
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(*) These are not comprehensive lists. For a more extensive catalogue see for example the <u>HEP ML Living Review</u>

Several approaches

Conditional phase space sampling

- GANs [1912.00477]
- Latent Diffusion [2305.10399]
- Conditional Flow Matching [2305.10475]
- cINN [<u>2212.08674</u>, <u>2006.06685</u>]
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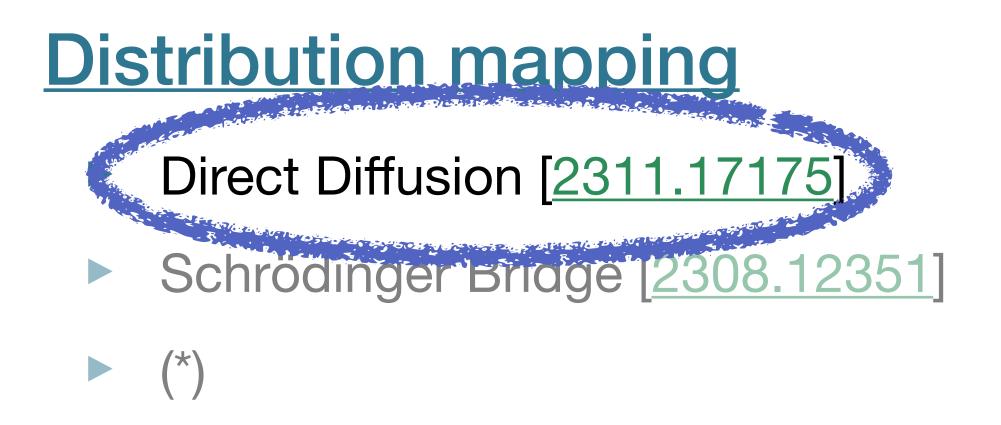






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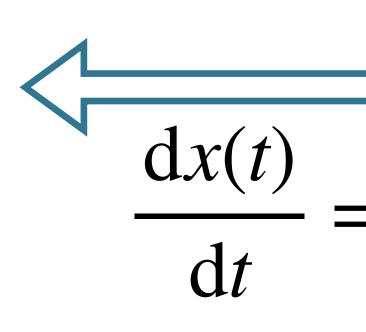
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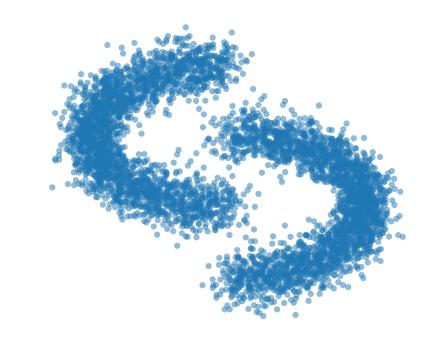






$x_0 \sim p_{\text{model}}(x_{\text{hard}})$

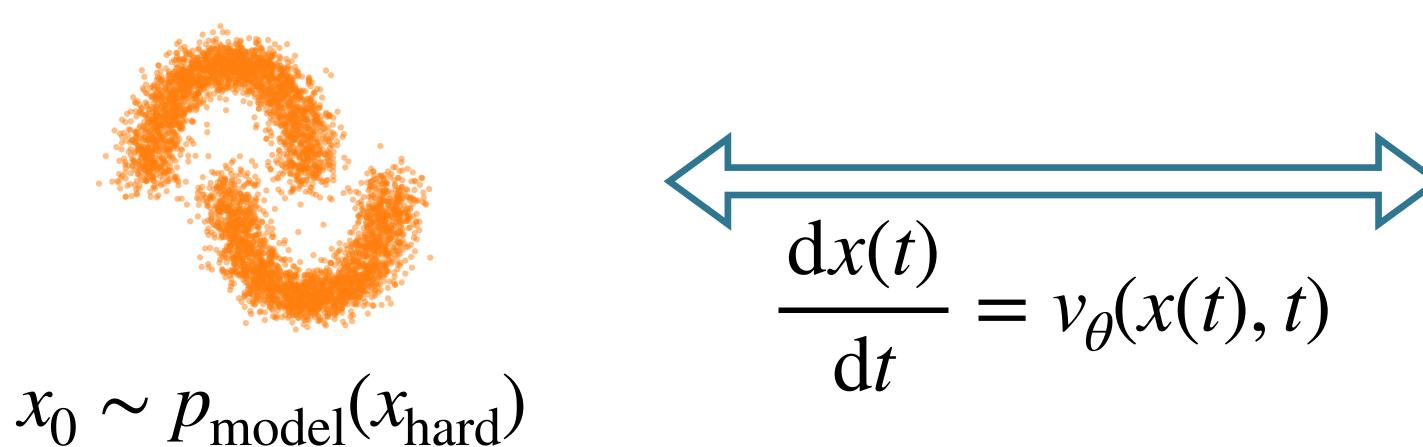
 $= v_{\theta}(x(t), t)$



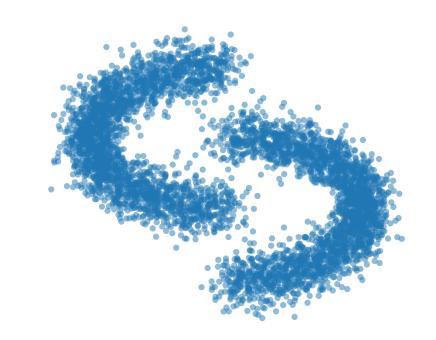
 $x_1 \sim p_{\text{reco}}(x_{\text{reco}})$







• Connect x_0 and x_1 with a linear trajectory:

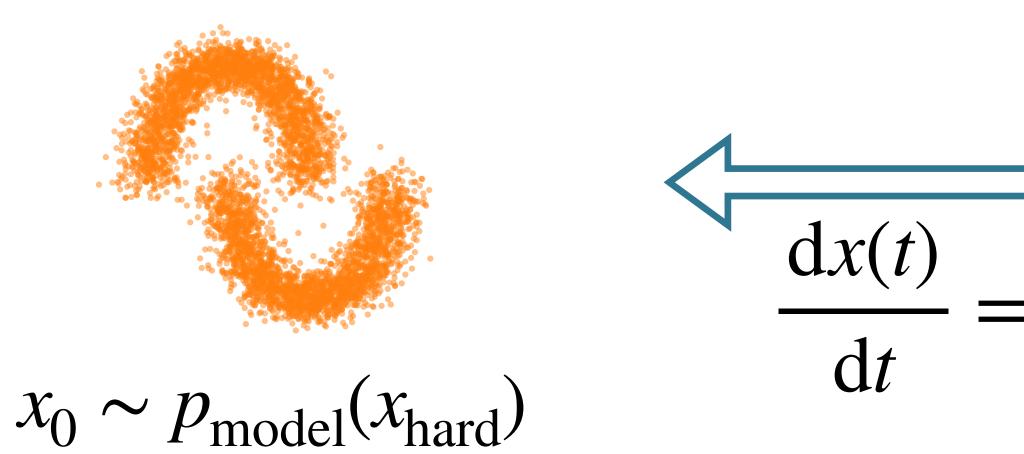


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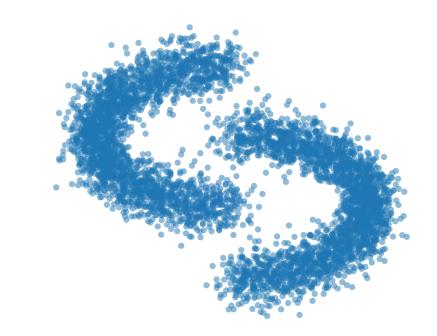
 $x(t) = (1 - t)x_0 + tx_1$







- Connect x_0 and x_1 with a linear trajectory:
- The NN is regressed to predict the velocity



$$v_{\theta}(x(t), t)$$

$$x_1 \sim p_{\text{reco}}(x_{\text{reco}})$$

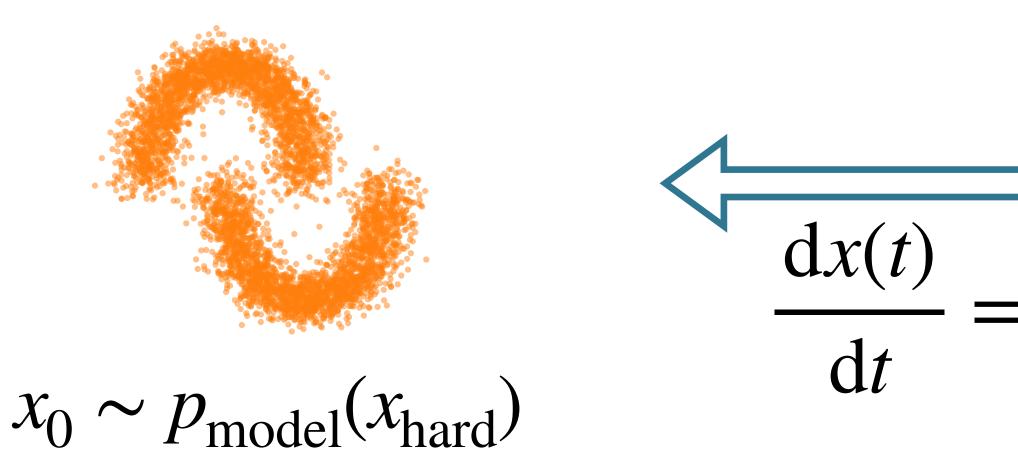
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in the symmetry field:

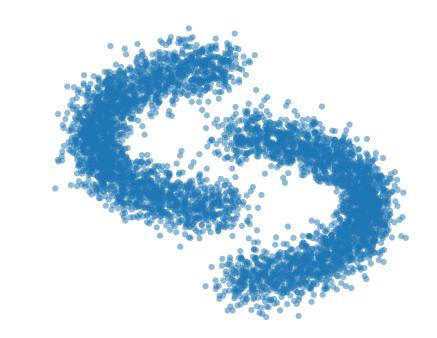
$$v_{\theta}(x(t), t) \approx \frac{\mathrm{d}x(t)}{\mathrm{d}t} = x_1 - x_0$$







- Connect x_0 and x_1 with a linear trajectory:
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- For sampling, solve ODE starting from x_1 :



$$v_{\theta}(x(t), t)$$

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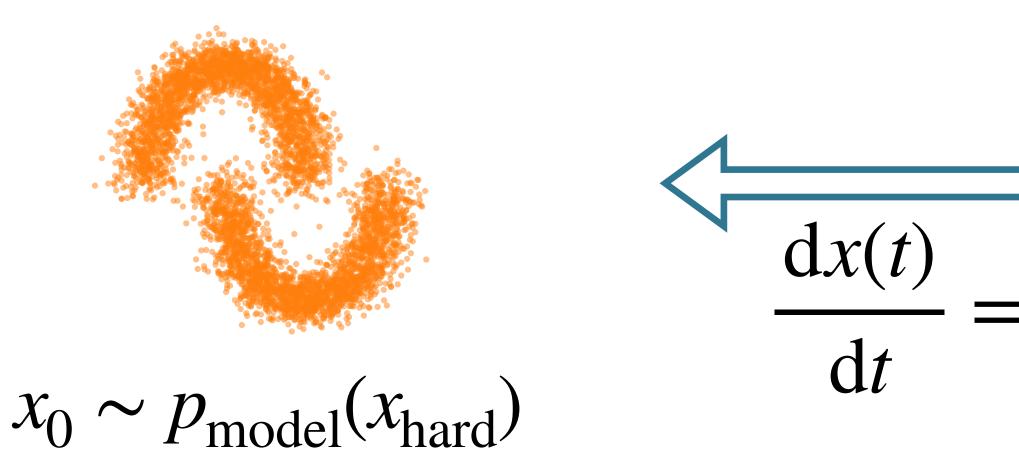
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$$v_{\theta}(x(t), t) \approx \frac{dx(t)}{dt} = x_1 - x_0$$

$$x_0 = x_1 + \int_1^0 v_{\theta}(x(t), t) dt$$

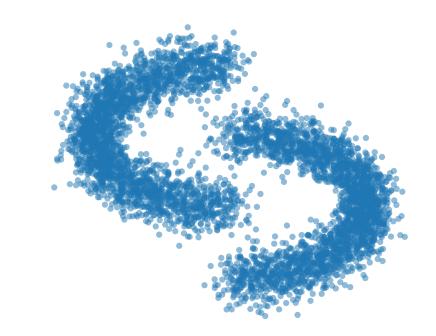






- Connect x_0 and x_1 with a linear trajectory:
- The NN is regressed to predict the velocity
- For sampling, solve ODE starting from x_1 :

• Loss:
$$\mathscr{L}_{\text{DiDi}} = \left\langle [v_{\theta}((1-t)x_0 + tx_1, t) \right\rangle$$



$$v_{\theta}(x(t), t)$$

$$x_1 \sim p_{\text{reco}}(x_{\text{reco}})$$

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$$x_0 = x_1 + \int_1^0 v_{\theta}(x(t), t) dt$$

 $-(x_1 - x_0)]^2 \rangle_{t \sim \mathcal{U}([0,1]), (x_0, x_1) \sim p(x_{\text{hard}}, x_{\text{reco}})}$







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Omnifold [1911.09107]

Distribution mapping

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Several methods

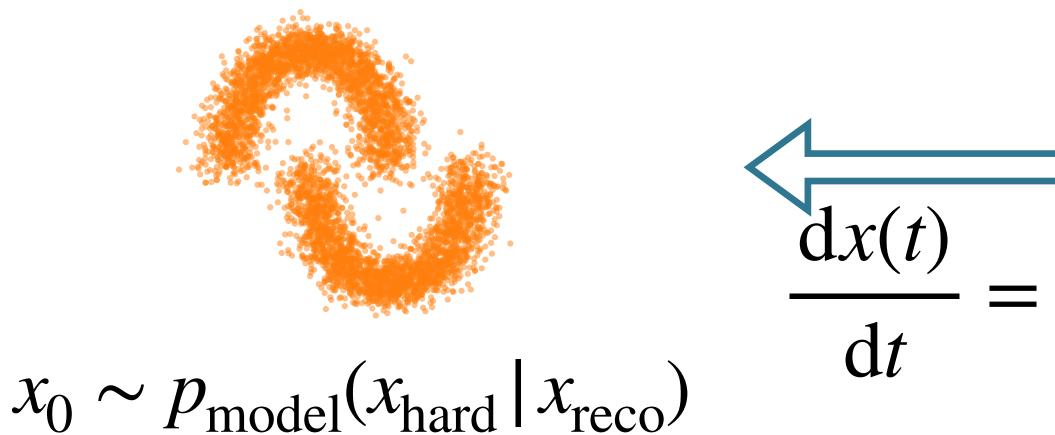
Conditional phase space sampling

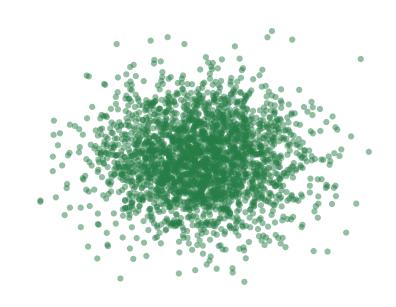
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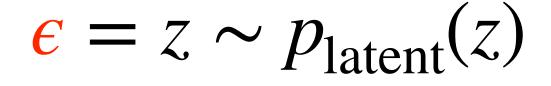


Conditional Flow Matching (CFM)



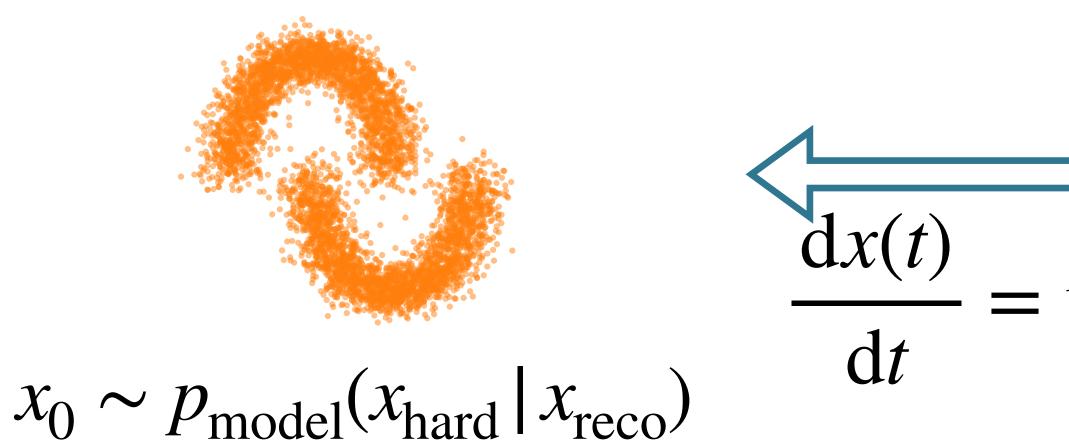


 $= v_{\theta}(x(t), t \,|\, x_{\text{reco}})$



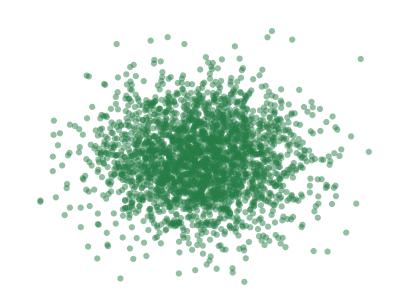


Conditional Flow Matching (CFM)



- Connect x_0 and ϵ with a linear trajectory:
- The NN is regressed to predict the velocity field:
- For sampling, solve ODE starting from ϵ :

• Loss:
$$\mathscr{L}_{\text{CFM}} = \Big\langle [v_{\theta}((1-t)x_0 + t\epsilon, t, x_0)] + t\epsilon \Big\rangle \Big\rangle$$



$$v_{\theta}(x(t), t \mid x_{\text{reco}})$$

 $\epsilon = z \sim p_{\text{latent}}(z)$

$$x(t) = (1-t)x_0 + t\epsilon$$

ty field: $v_{\theta}(x(t), t | x_{reco}) \approx \frac{dx(t)}{dt} = \epsilon - x_0$

$$x_0 = \epsilon + \int_1^0 v_{\theta}(x(t), t | x_{\text{reco}}) dt$$

 $x_{\text{reco}}) - (\epsilon - x_0)]^2 \rangle_{t \sim \mathcal{U}([0,1]), (x_0, x_{\text{reco}}) \sim p(x_{\text{hard}}, x_{\text{reco}}), \epsilon \sim \mathcal{N}(0,1)}$





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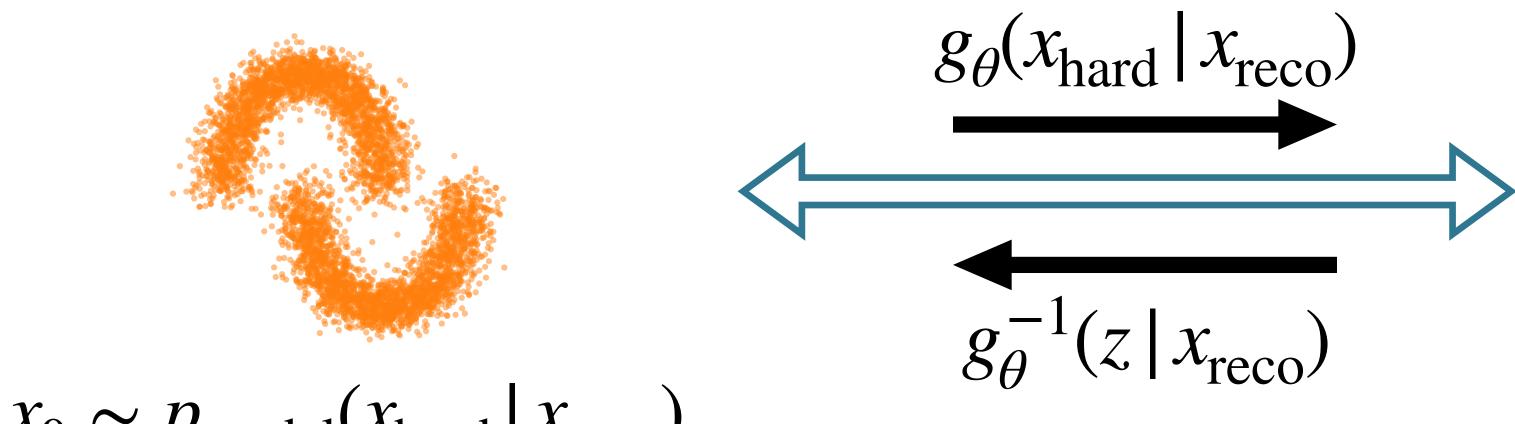
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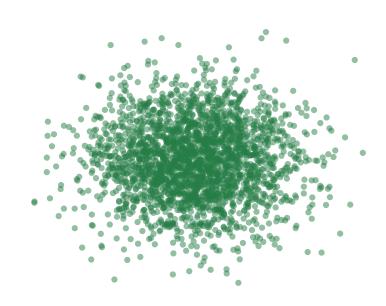




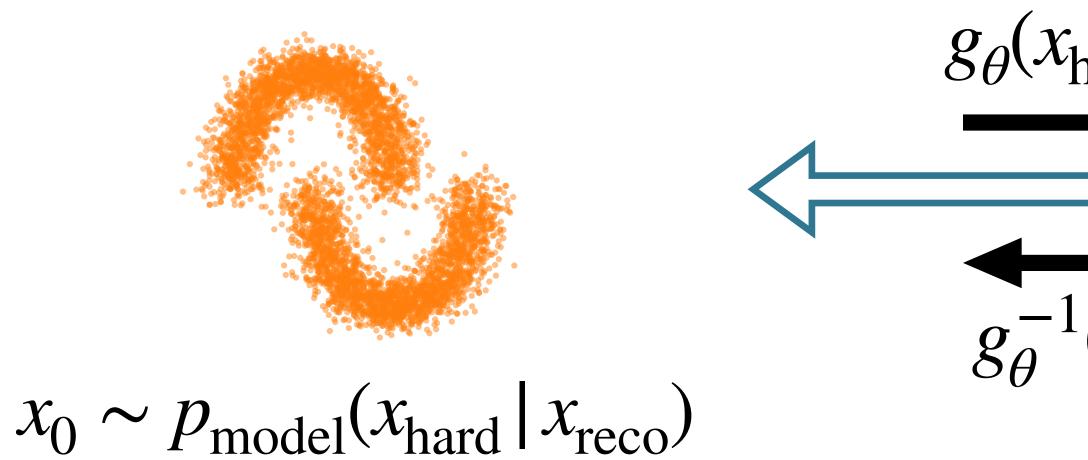
Conditional INN (CINN)



$x_0 \sim p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}})$



 $z \sim p_{\text{latent}}(z)$

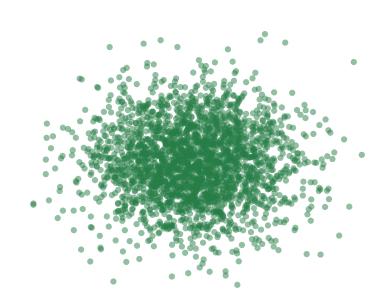


• Bijective function between $p_{\text{latent}}(z)$ and p

 $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) = p_{\text{latent}}(z)$



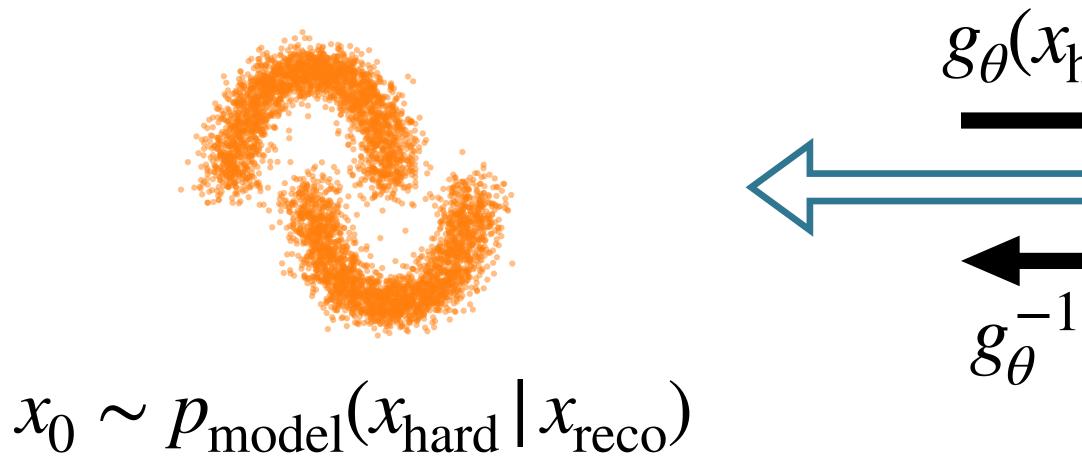
hard
$$x_{reco}$$



$$z \sim p_{\text{latent}}(z)$$

$$p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}):$$

 $\left| \det \frac{\partial g_{\theta}(x_{\text{h}}, x_{\text{r}})}{\partial x_{\text{hard}}} \right| = p_{\text{lat.}}(z) \left| \det J_{g_{\theta}} \right|$



Bijective function between $p_{\text{latent}}(z)$ and p

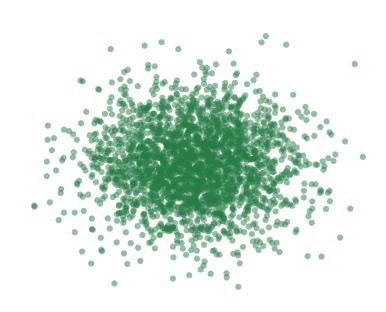
 $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) = p_{\text{latent}}(z)$

> Pairs (x_{hard}, x_{reco}) are passed through the NN to the latent space:



$$S_{\theta}(x_{hard} | x_{reco})$$

$$= g_{\theta}^{-1}(z | x_{reco})$$

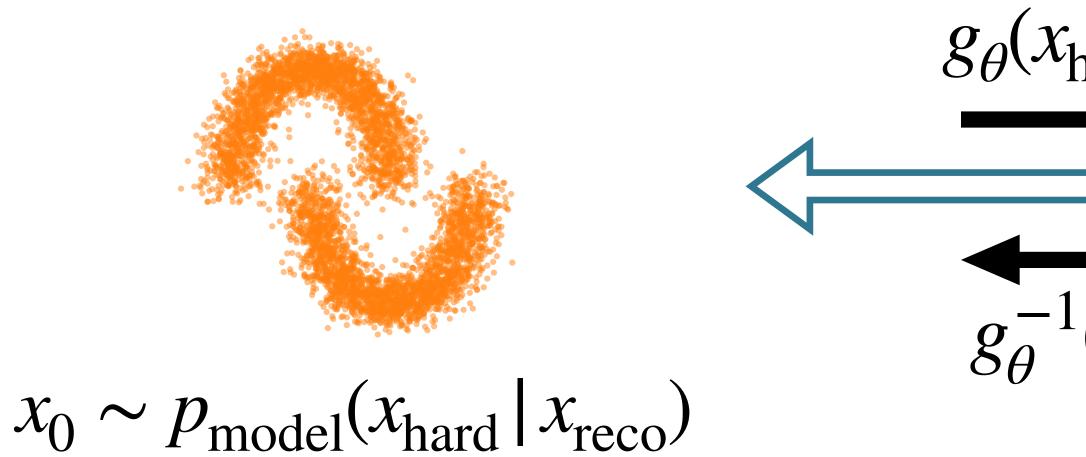


 $z \sim p_{\text{latent}}(z)$

$$\mathcal{P}_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}):$$

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 $z = g_{\theta}(x_{\text{hard}} | x_{\text{reco}})$



Bijective function

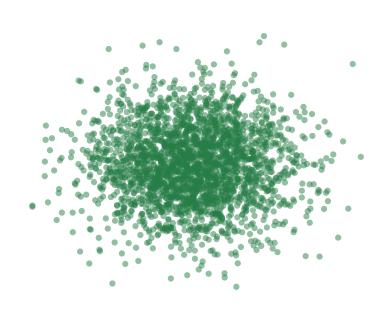
between
$$p_{\text{latent}}(z)$$
 and $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}})$:
 $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) = p_{\text{latent}}(z) \left| \det \frac{\partial g_{\theta}(x_{\text{h}}, x_{\text{r}})}{\partial x_{\text{hard}}} \right| = p_{\text{lat.}}(z) \left| \det J_{g_{\theta}} \right|$

• Pairs (x_{hard}, x_{reco}) are passed through the NN to the latent space: • Once trained, one can sample -conditioned on reco- from the latent: $p_{hard}(x) \approx p_{model}(x_{hard} | x_{reco})$



hard
$$|x_{reco}\rangle$$

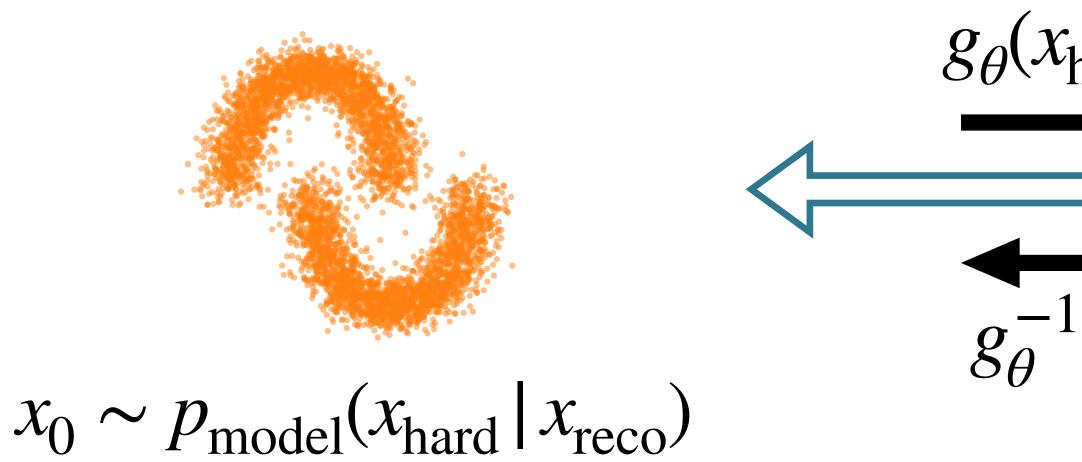
 $|(z|x_{reco})|$



 $z \sim p_{\text{latent}}(z)$

 $z = g_{\theta}(x_{\text{hard}} | x_{\text{reco}})$





Bijective function

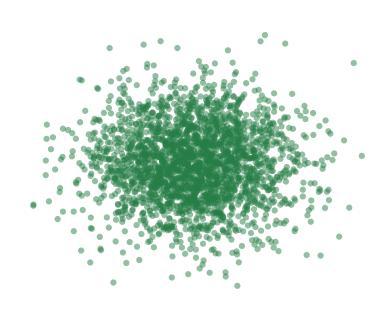
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 $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) = p_{\text{latent}}(z) \left| \det \frac{\partial g_{\theta}(x_{\text{h}}, x_{\text{r}})}{\partial x_{\text{hard}}} \right| = p_{\text{lat.}}(z) \left| \det J_{g_{\theta}} \right|$

> Pairs (x_{hard}, x_{reco}) are passed through the NN to the latent space: $\mathscr{L}_{\text{cINN}} = -\langle \log p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) \rangle_{(x_0, x_1) \sim p(x_{\text{hard}}, x_{\text{reco}})}$ Loss:



hard
$$x_{reco}$$

 $(z | x_{reco})$



 $z \sim p_{\text{latent}}(z)$

 $z = g_{\theta}(x_{\text{hard}} | x_{\text{reco}})$ • Once trained, one can sample -conditioned on reco- from the latent: $p_{hard}(x) \approx p_{model}(x_{hard} | x_{reco})$



Z+jets events

$Z(p_T > 200 \text{ GeV})$ + jets events generated at $\sqrt{s} = 14 \text{ TeV}$ with Pythia 8.244 and Delphes simulation 3.5.0 available on Zenodo. Slight modification from [1911.09107] dataset

Z + jets events

simulation 3.5.0 available on <u>Zenodo</u>. Slight modification from [<u>1911.09107</u>] dataset

Six widely-used jet substructure observables:

- Jet mass *m*
- Jet width *w*
- Jet constituents multiplicity N

 $Z(p_T > 200 \text{ GeV})$ + jets events generated at $\sqrt{s} = 14 \text{ TeV}$ with Pythia 8.244 and Delphes

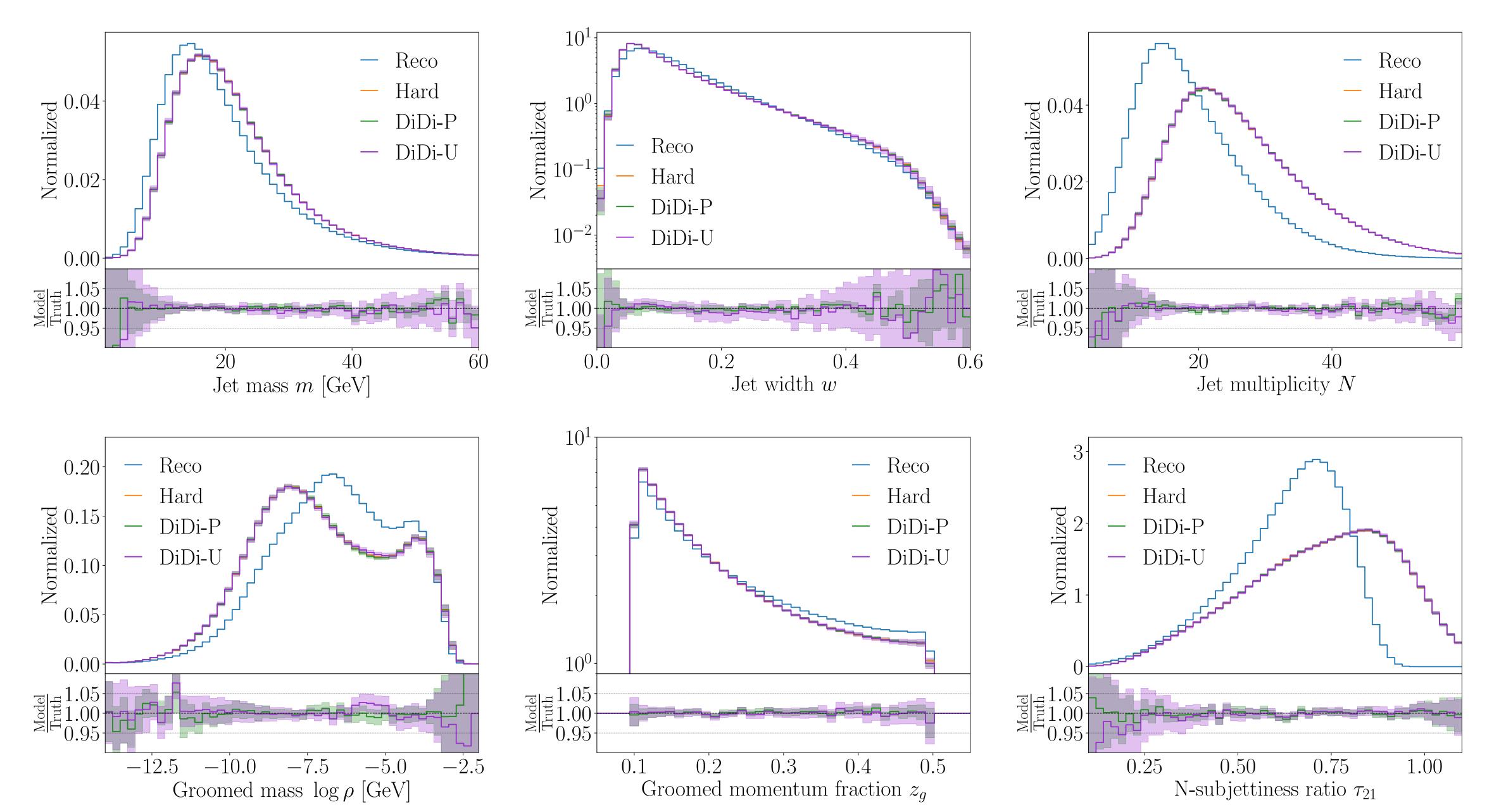
• Groomed mass $\log \rho = 2 \log (m_{SD} / p_T)$

• Groomed momentum fraction $z_g = \tau_1^{\beta=1}$

N-subjettiness ratio $\tau_{21} = \tau_2^{\beta=1} / \tau_1^{\beta=1}$

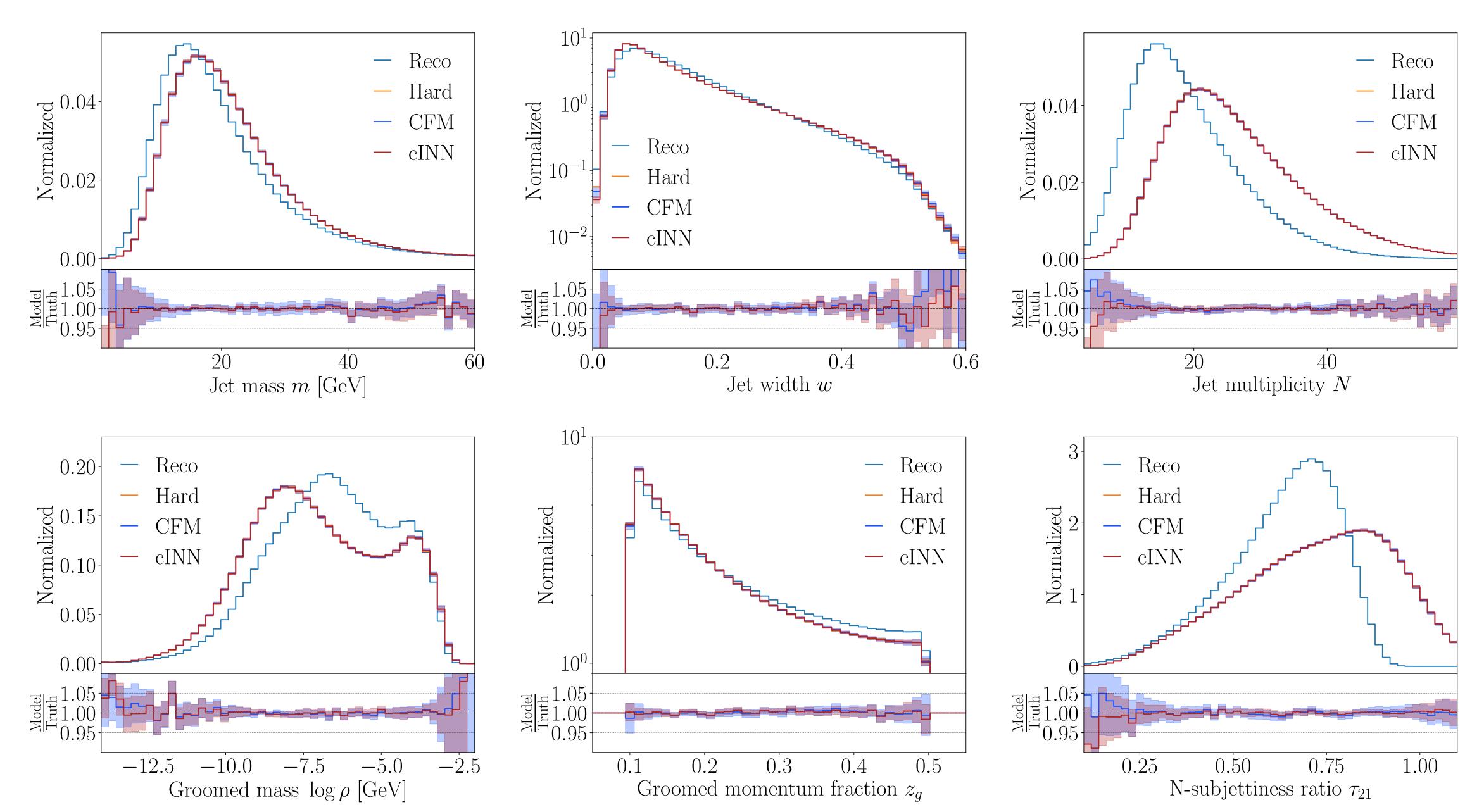


Results (DiDi)

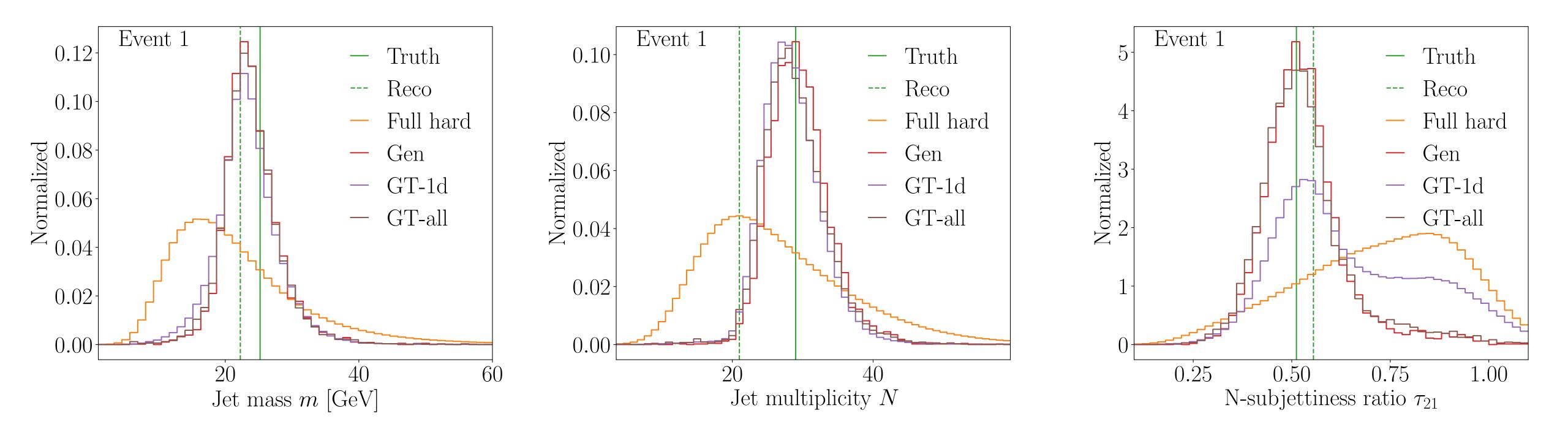




Results (CFM & cINN)

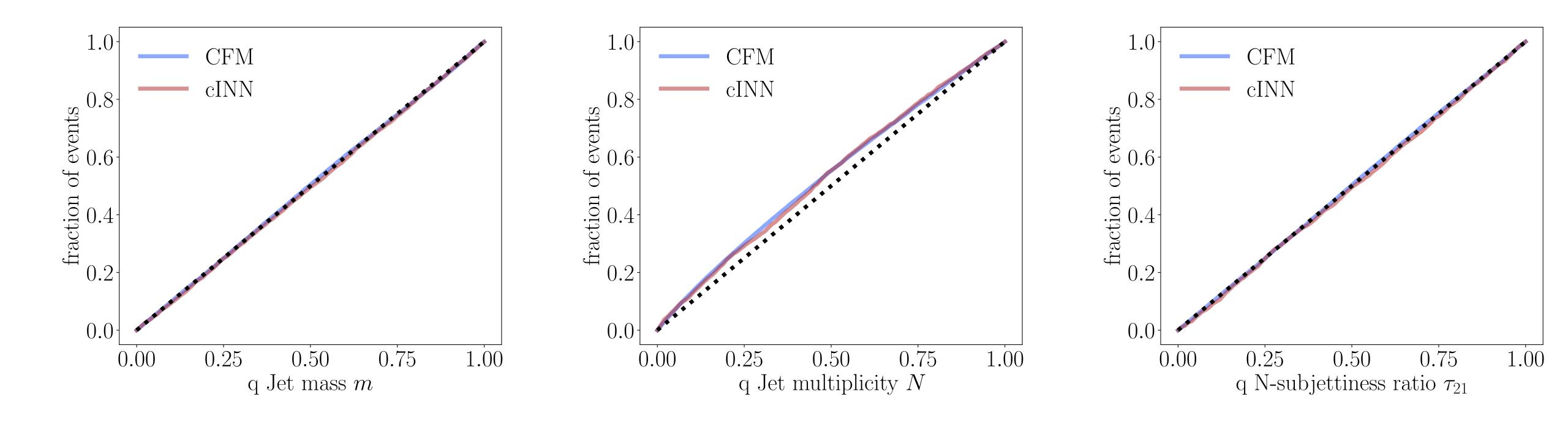


Single event unfolding





Calibration



Matrix elements are evaluated at $\sqrt{s} = 13$ TeV using MadGraph_aMC@NLO. Showering and hadronization are simulated with Pythia8, and detector response is simulated with Delphes with the standard CMS card. For a detailed description see [2305.10399].



Matrix elements are evaluated at $\sqrt{s} = 13$ TeV using MadGraph_aMC@NLO. Showering and the standard CMS card. For a detailed description see [2305.10399].

Unfolding from 6 final-state particles $(bl\nu)(bqq)$:

- 4 DoFs for the lepton
- > 3 DoFs for the missing p_T^{ν}
- 5 DoFs per jet (4-momentum + b-tag)

hadronization are simulated with Pythia8, and detector response is simulated with Delphes with



Matrix elements are evaluated at $\sqrt{s} = 13$ TeV using MadGraph_aMC@NLO. Showering and the standard CMS card. For a detailed description see [2305.10399].

Unfolding from 6 final-state particles $(bl\nu)(bqq)$:

- 4 DoFs for the lepton
- ► 3 DoFs for the missing p_T^{ν}
- 5 DoFs per jet (4-momentum + b-tag)

hadronization are simulated with Pythia8, and detector response is simulated with Delphes with

Total: 27 DoFs at reco-level

and 19 DoFs at parton-level



Much harder problem:

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Much harder problem:

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- resonances
- Non-trivial combinatorics between physics objects at both levels

Adding transformers:

employed to encode correlations at reco and parton-level.

Unfolding to parton-level is not only inverting detector effects, but rather inverting the entire

Faithful modeling of complex correlations at parton-level, i.e., W boson and top mass

Transfermer and Tra-CFM as an extension to the cINN and CFM [2310.07752]. A transformer is



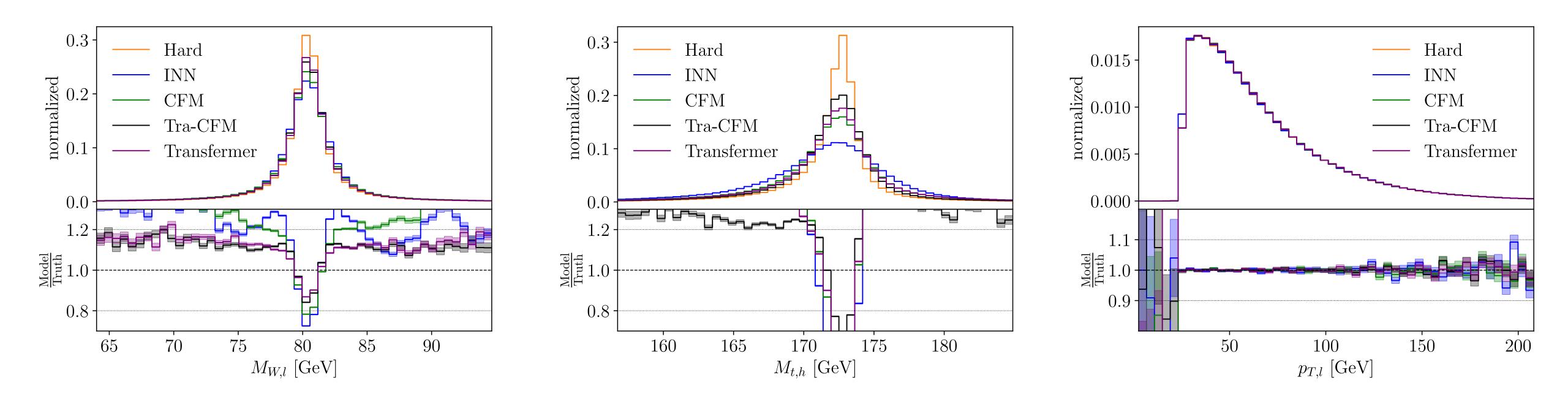






Results: naive parametrization

Unfold:



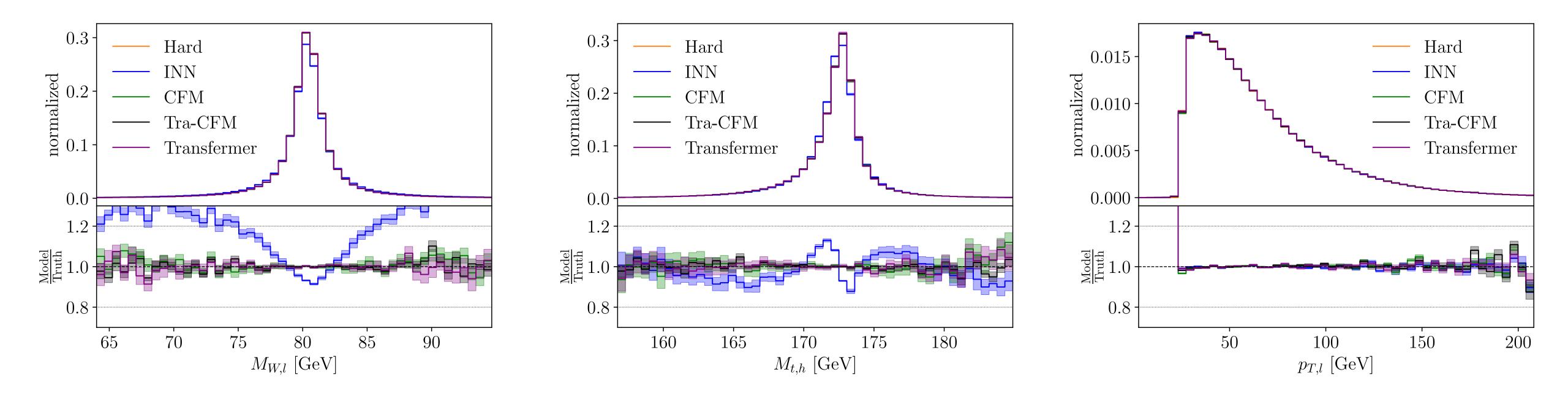
$(p_{T,b_l},\eta_{b_l},\phi_{b_l},p_{T,l},\eta_l,\phi_l,p_{T,v},\eta_v,\phi_v,p_{T,b_h},\eta_{b_h},\phi_{b_h},m_{q_1},p_{T,q_1},\eta_{q_1},\phi_{q_1},p_{T,q_2},\eta_{q_2},\phi_{q_2})$



Results: mass parametrization



 $(m_t, p_{T,t}^L, \eta_t^L, \phi_t^L, m_W, \eta_W^T, \phi_W^T, (m_{d_1}^W), \eta_{d_1}^W, \phi_{d_1}^W)$







- accross many dimensions.
- CFM and cINN both learn the phase space probabilities for each event, so it is best suited to describe complex detector effects, but their also more complex architectures to train.
- for correlations and resonances reconstruction
- The mass parametrization allows for more efficient unfolding without the need of very large networks
- Single-event unfolding, calibrated posteriors, compare to other models...

Summary and outlook

ML-based unfolding is an unbinned transformative analysis tool capable of dealing with correlations

Distribution mapping can be trained on matched and unmatched data and is relatively fast to train

Parton-level unfolding is a reasonably complicated task, but transformers help greatly in accounting







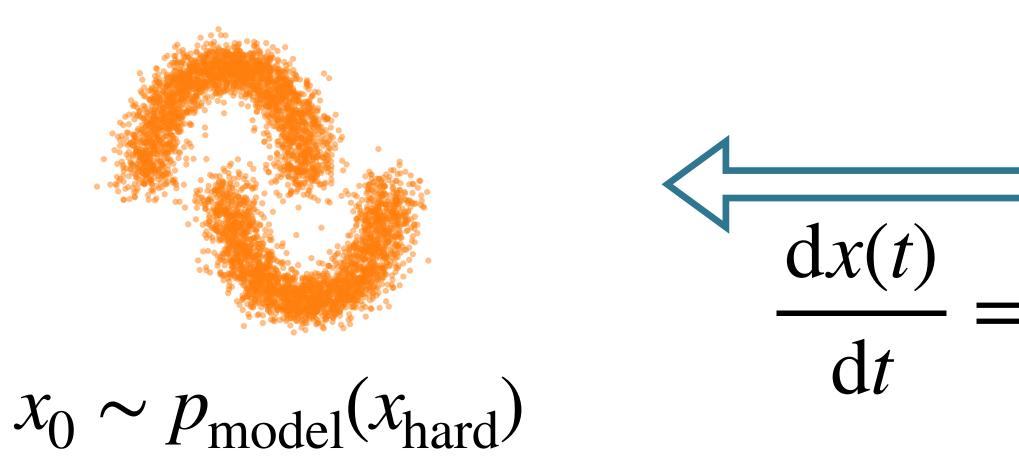




Thanks for your attention!

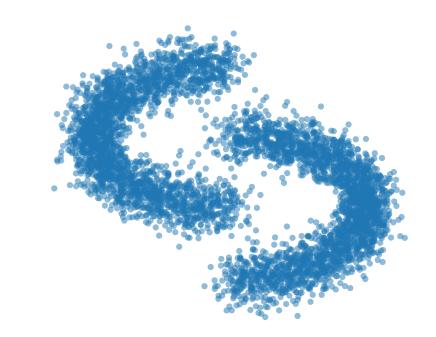


Direct Diffusion (DiDi)



- Connect x_0 and x_1 with a linear trajectory:
- The NN is regressed to predict the velocity
- For

For sampling, solve ODE starting from
$$x_1$$
: $x_0 = x_1 + \int_1^0 v_\theta(x(t), t) dt$
Loss:
$$\mathscr{L}_{\text{DiDi-P}} = \left\langle \left[v_\theta((1-t)x_0 + tx_1, t) - (x_1 - x_0) \right]^2 \right\rangle_{t \sim \mathscr{U}([0,1]), (x_0, x_1) \sim p(x_{\text{hard}}, x_{\text{reco}})} \\ \mathscr{L}_{\text{DiDi-U}} = \left\langle \left[v_\theta((1-t)x_0 + tx_1, t) - (x_1 - x_0) \right]^2 \right\rangle_{t \sim \mathscr{U}([0,1]), x_0 \sim p(x_{\text{hard}}, x_1 \sim p(x_{\text{reco}}))} \right\rangle_{t \sim \mathscr{U}([0,1]), x_0 \sim p(x_{\text{hard}}, x_1 \sim p(x_{\text{reco}}))}$$



$$v_{\theta}(x(t), t)$$

$$x_1 \sim p_{\text{reco}}(x_{\text{reco}})$$

$$x(t) = (1 - t)x_0 + tx_1$$

ty field:
$$v_{\theta}(x(t), t) \approx \frac{\mathrm{d}x(t)}{\mathrm{d}t} = x_1 - x_0$$



Z + jets events

$Z(p_T > 200 \text{ GeV})$ + jets events generated at $\sqrt{s} = 14 \text{ TeV}$ with Pythia 8.244 and Delphes simulation 3.5.0 available on Zenodo

Six widely-used jet substructure observables:

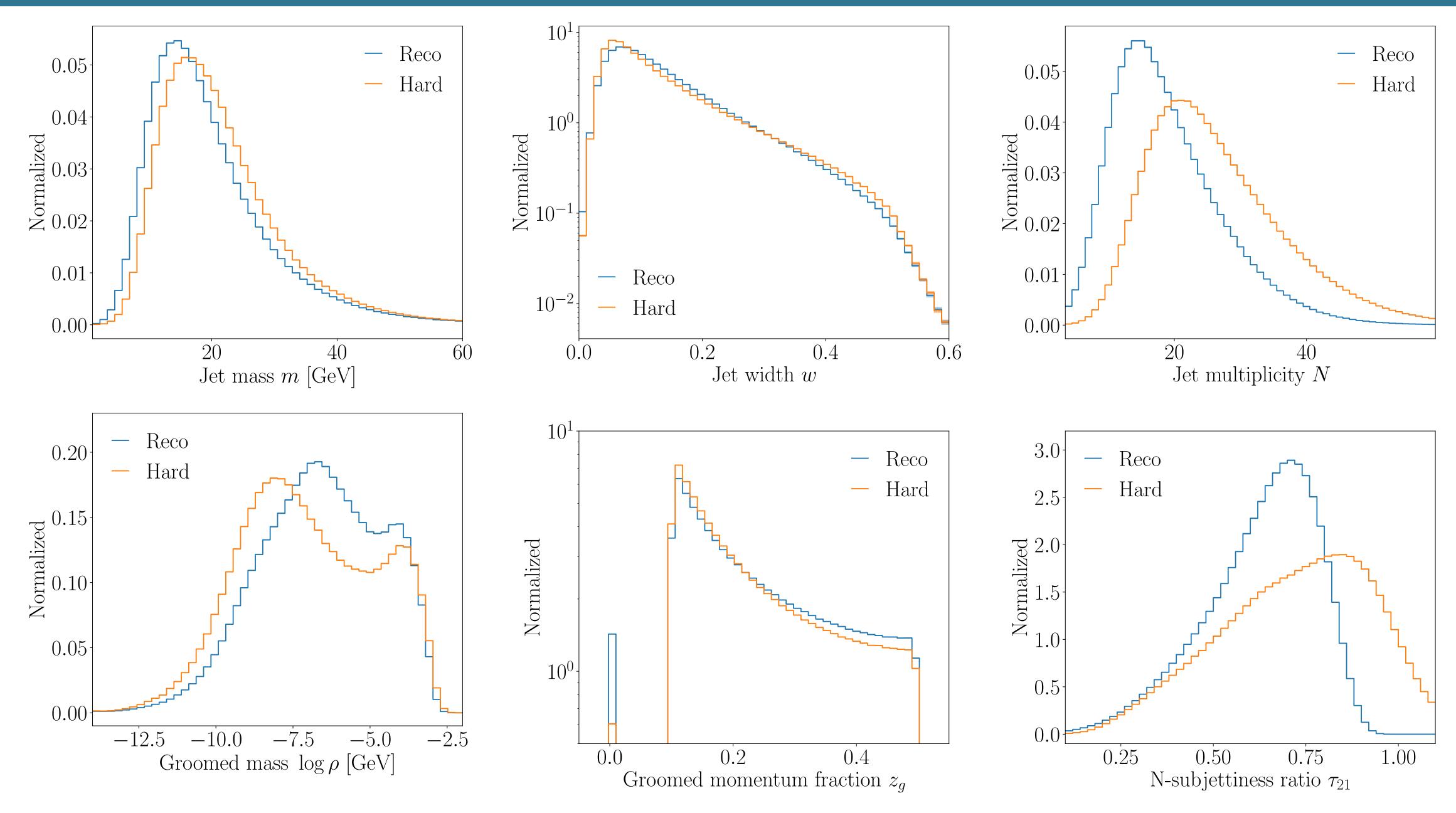
- Jet mass *m*
- ► Jet width *w*
- Jet constituents multiplicity N

Networks of ~3M parameters 19M training events and 1M validation events ~4M events for testing

- Groomed mass $\log \rho = 2 \log (m_{SD} / p_T)$ • Groomed momentum fraction $z_g = \tau_1^{\beta=1}$
- N-subjettiness ratio $\tau_{21} = \tau_2^{\beta=1} / \tau_1^{\beta=1}$



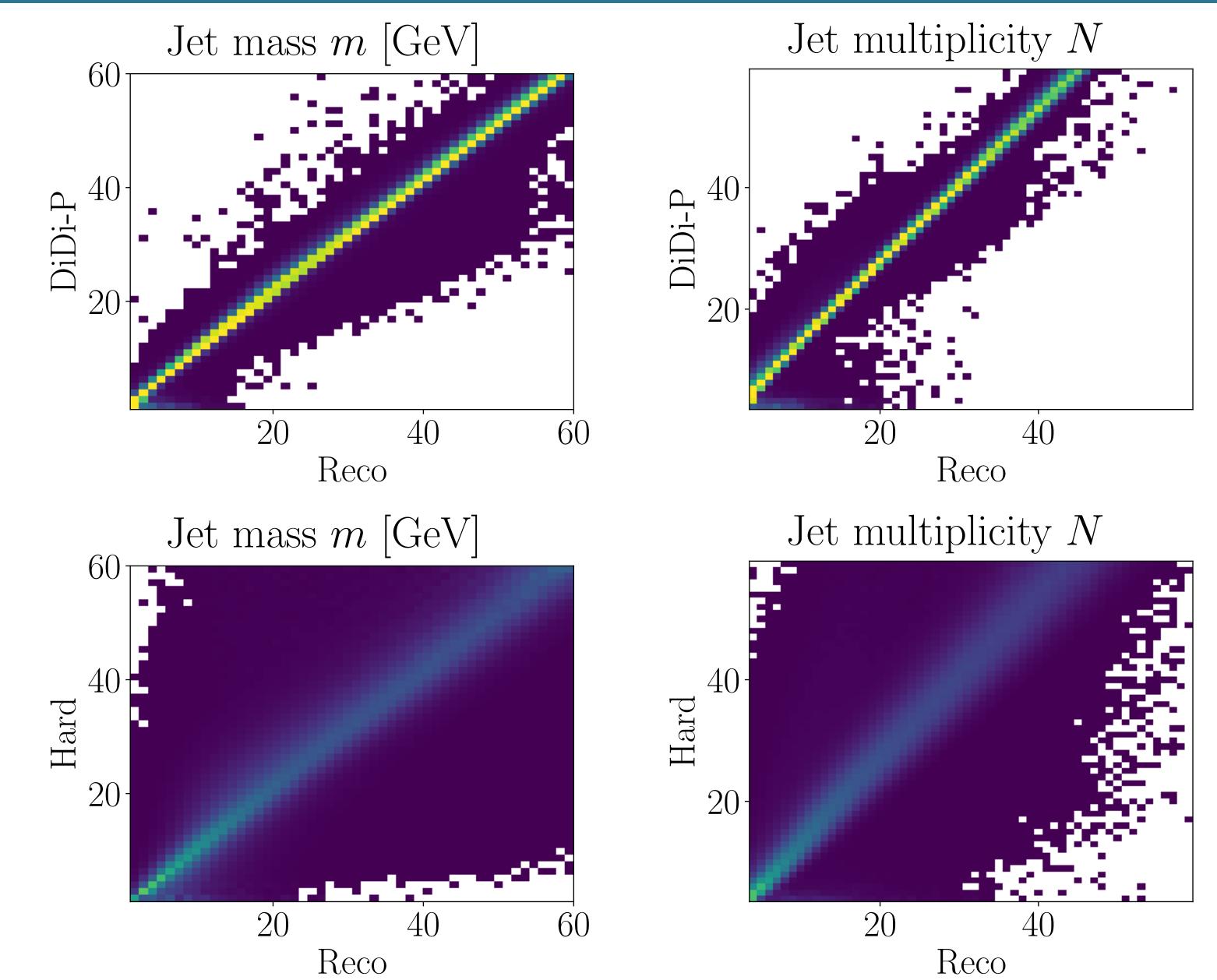




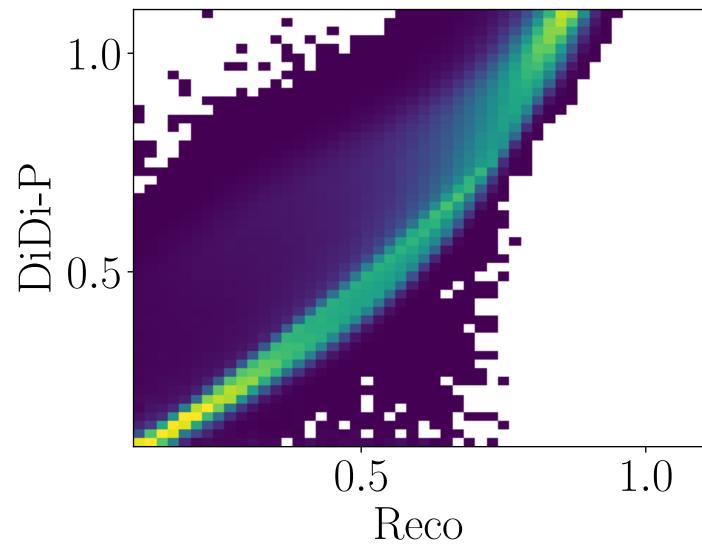
The dataset

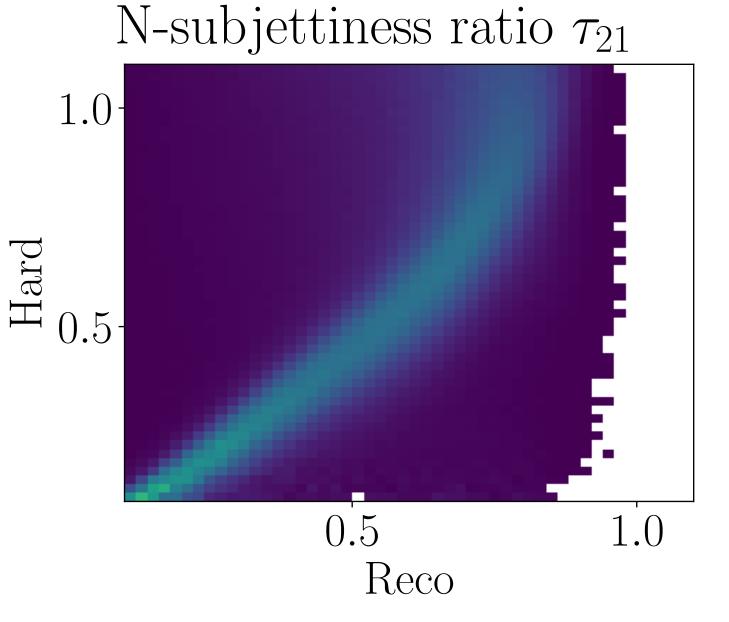


Optimal transport (DiDi)



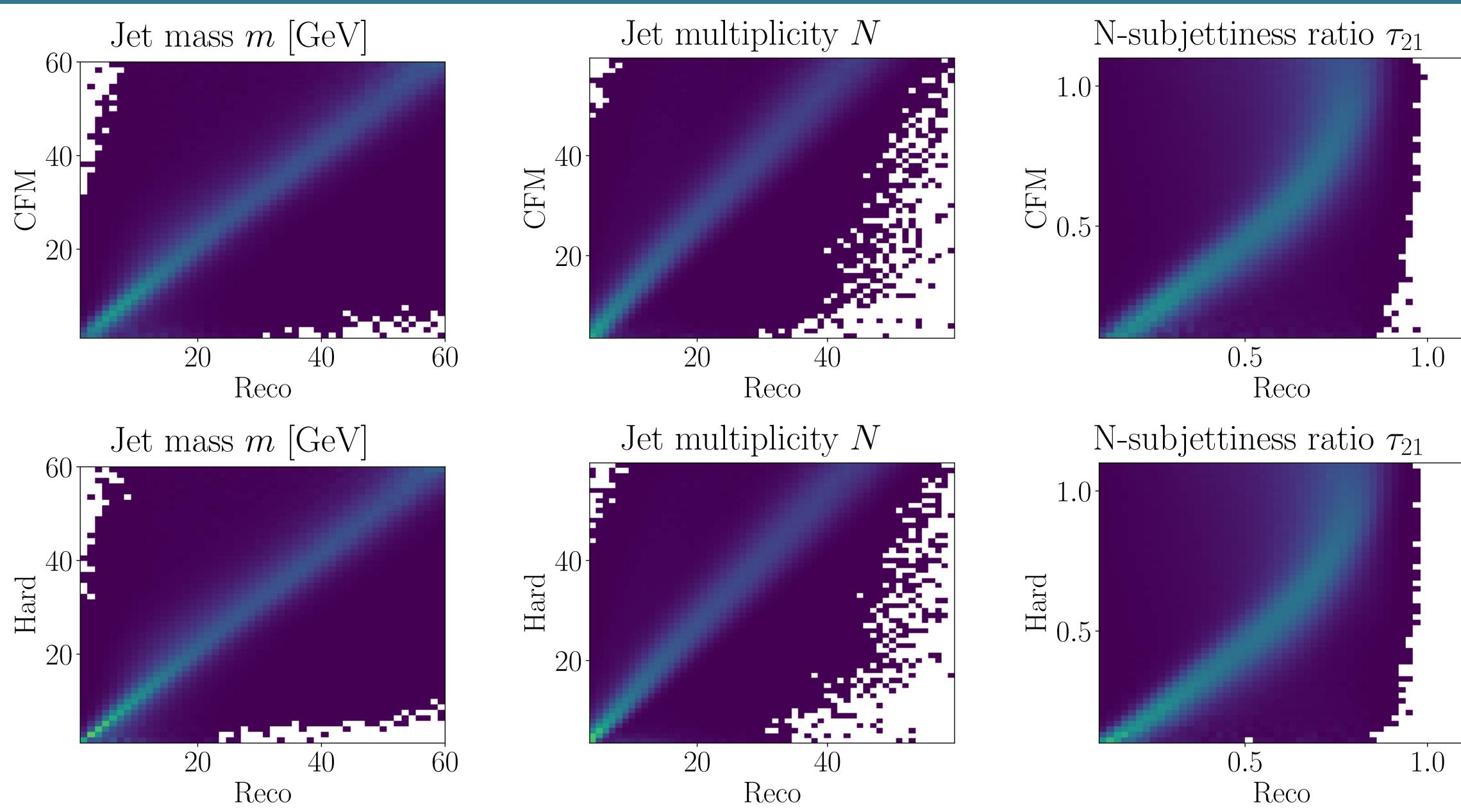
N-subjettiness ratio τ_{21}





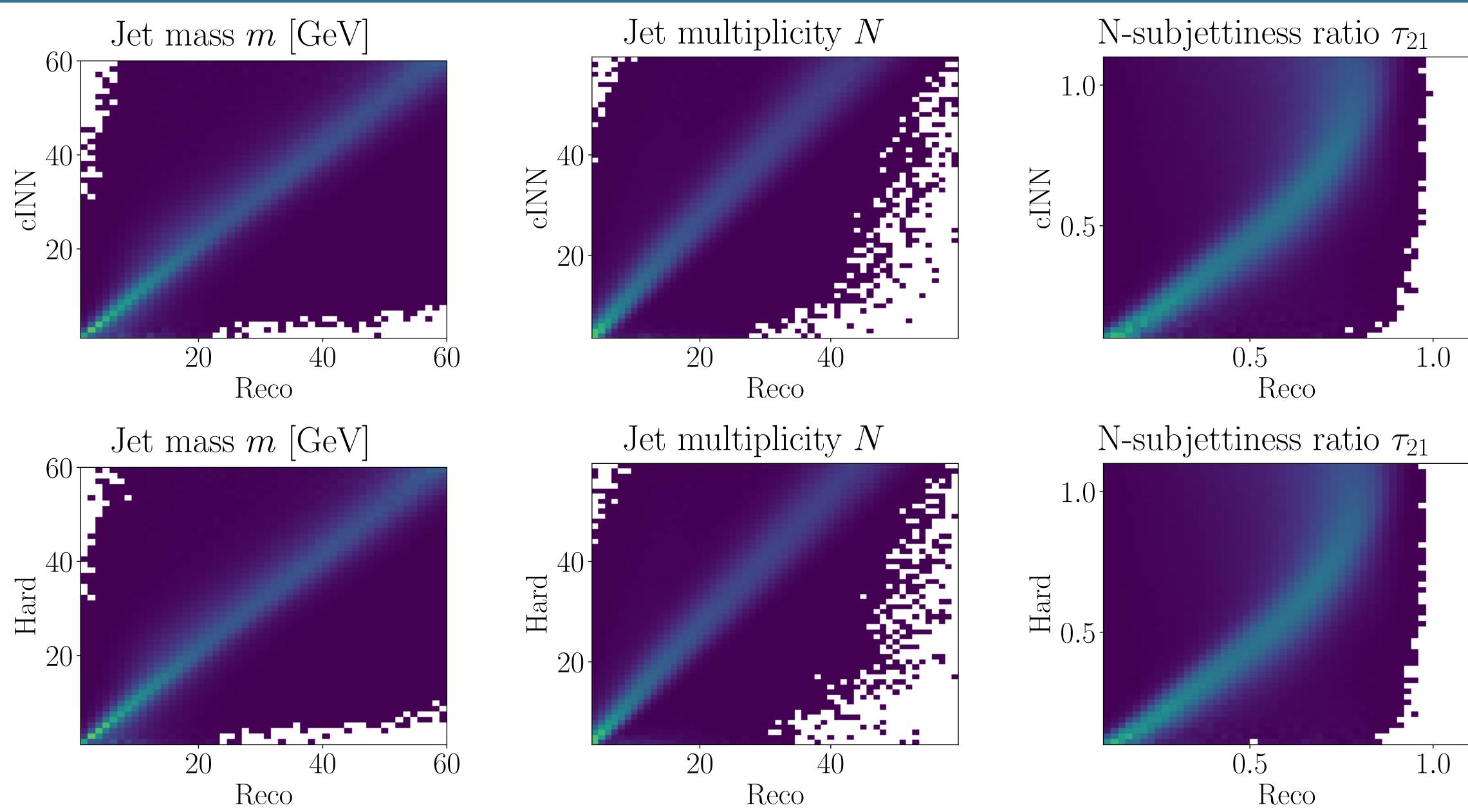


Optimal transport (CFM)

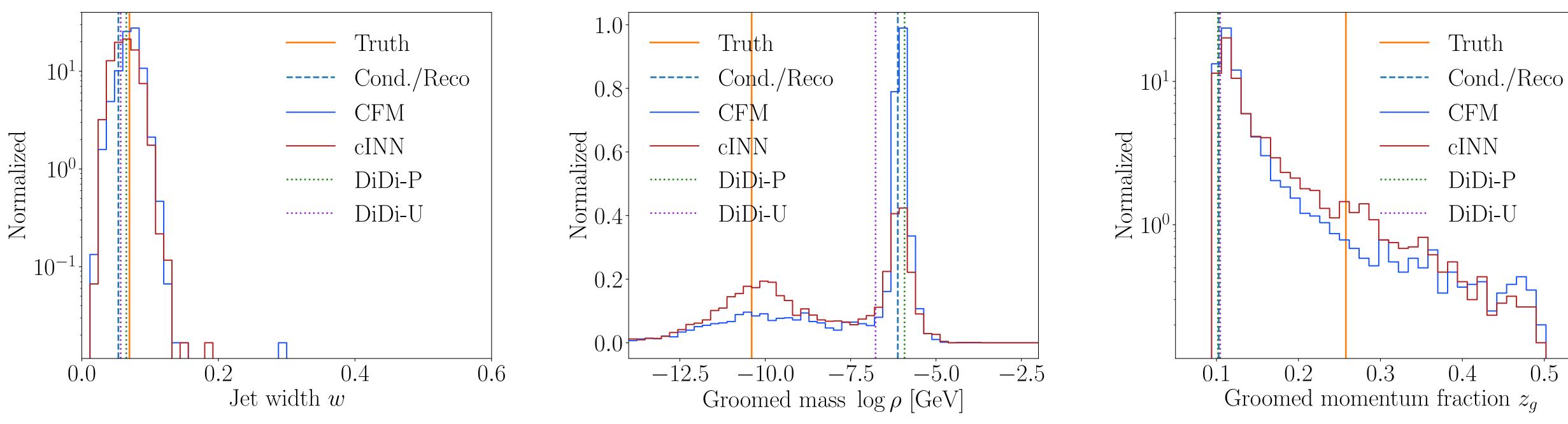




Optimal transport (cINN)



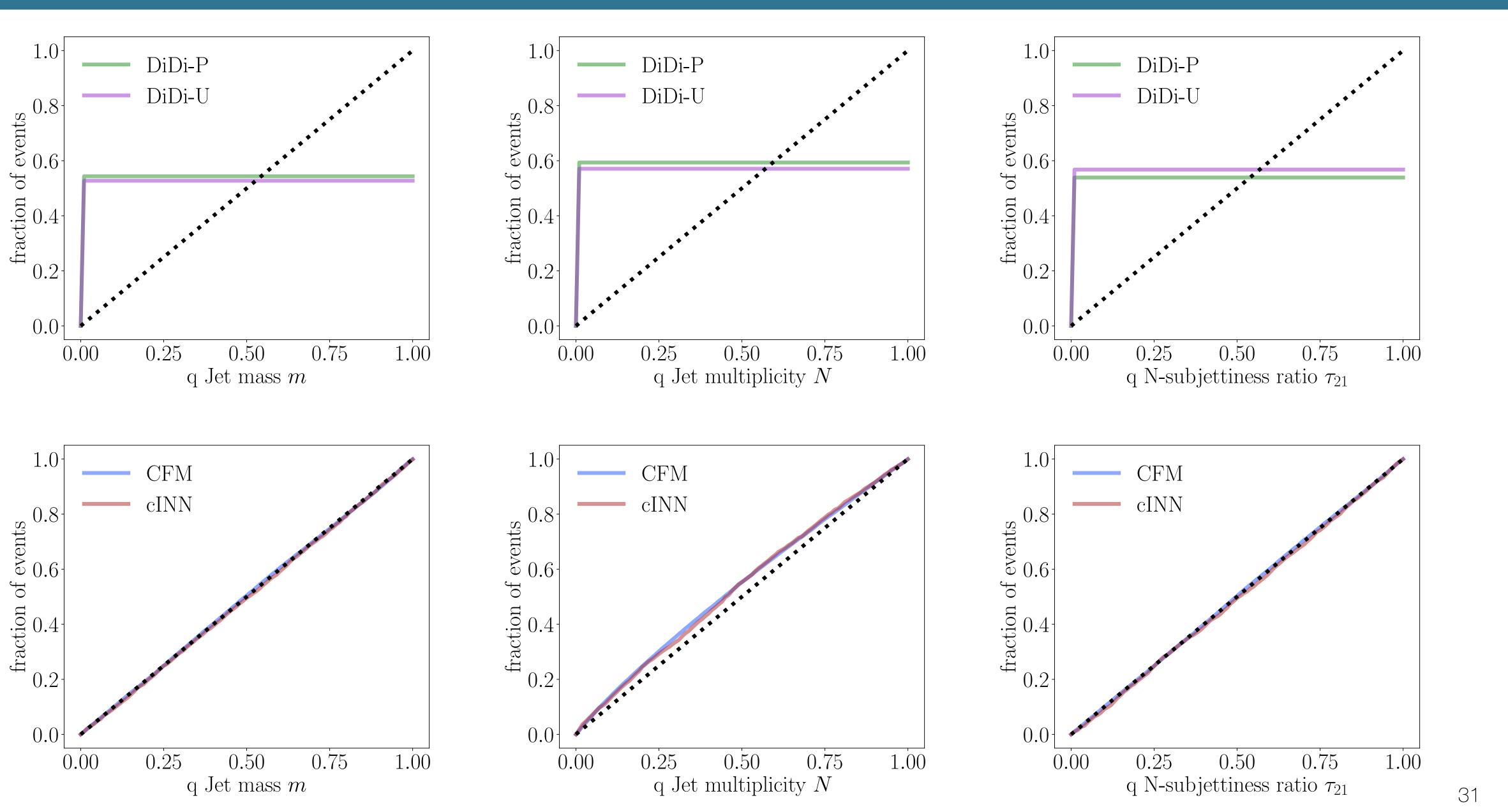








Calibration



Matrix elements are evaluated at $\sqrt{s} = 13$ TeV using MadGraph_aMC@NLO. Showering and the standard CMS card. For a detailed description see [2305.10399].

Unfolding from 6 final-state particles $(bl\nu)(bqq)$:

- 4 DoFs for the lepton
- > 3 DoFs for the missing p_T^{ν}
- 5 DoFs per jet (4-momentum + b-tag)

Non-bayesian networks

cINN ~ 8M parameters, CFM ~ 6M, Tra-CFM, Transfermer ~ 3M

10M training events and 1M testing events

hadronization are simulated with Pythia8, and detector response is simulated with Delphes with

Total: 27 DoFs at reco-level and 19 DoFs at parton-level



Adding transformers:

dimensions and reco-level event:

$$p_{\text{model}}(x_{\text{part}} | x_{\text{reco}}) = \prod_{i=1}^{n} p_{\text{model}}(x_{\text{part}}^{(i)} | c(x_{\text{part}}^{(0)}, \dots, x_{\text{part}}^{(i-1)}, x_{\text{reco}}))$$

each different dimension:

 $v(x_{\text{part}}(t), t \mid x_{\text{reco}}) = ($

For transfermer, likelihoods are factorized autoregressively on all previous parton-level

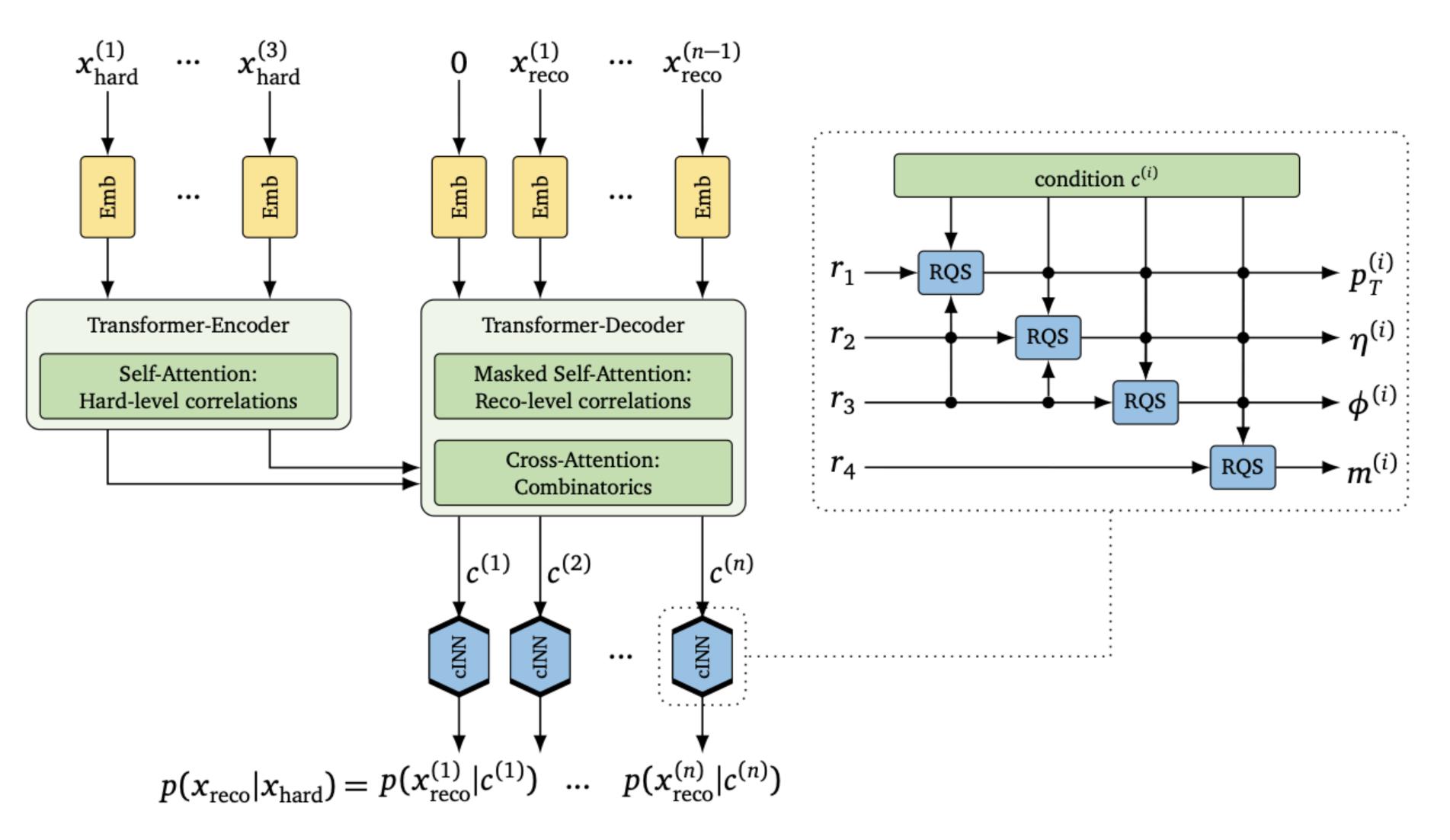
For Tra-CFM, the transformer is made time-dependent and a small CFM predicts velocities at

$$\left(v^{(1)}(c^{(1)},t), \dots, v^{(n)}(c^{(n)},t)\right)$$





Transfermer





Tra-CFM

