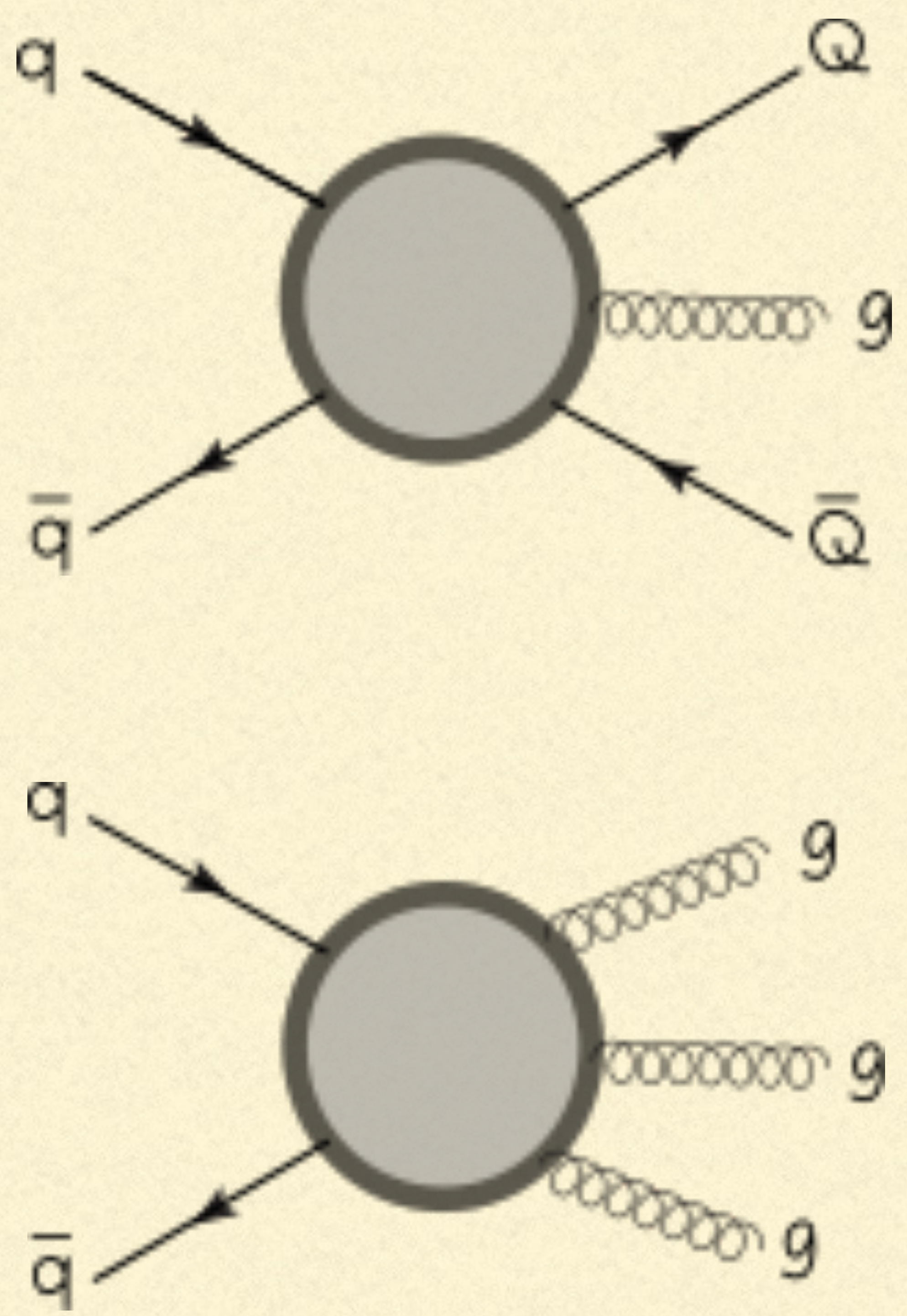
TWO-LOOP FIVE-POINT MASSLESS QCD AMPLITUDES IN FULL COLOR

In collaboration with: B.Agarwal, F.Buccioni, G.Gambuti, A.Von Manteuffel, L.Tancredi

Based on arXiv:2311.09870

In this talk:



- Relevance: phenomenology + "formal" aspects
- Outline of the calculation
- Summary and outlook

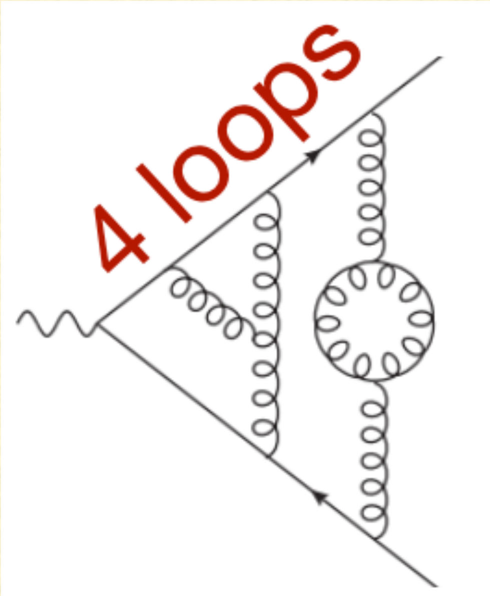
Computed in leading color approximation by Abreu et al. in [2102.13609]

This talk:
Full color

BRIEF HISTORY OF (MULTI-LOOP) QCD AMPLITUDES

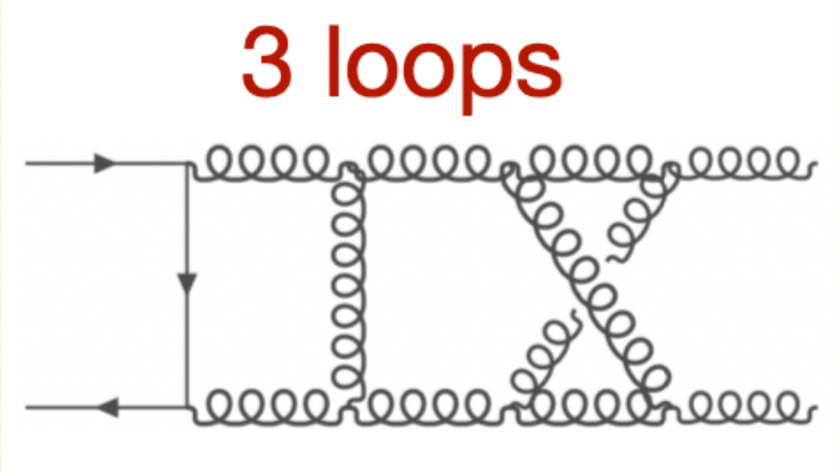
Reference list NOT exhaustive!

Form factor:



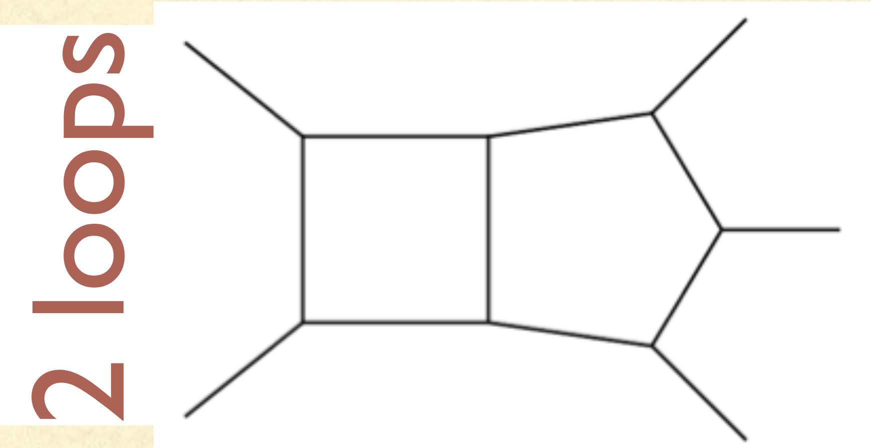
Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser: **2202.04660(PRL)**

$2 \to 2$ scattering:



Bargiela, Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi: **2011.13946(PRL), 2112.11097(PRL)**
Henn, Mistlberger: **1608.00850(PRL)**
Henn, Mistlberger, Smirnov, Wasser: **2002.09492(JHEP)**

$2 \to 3$ scattering:



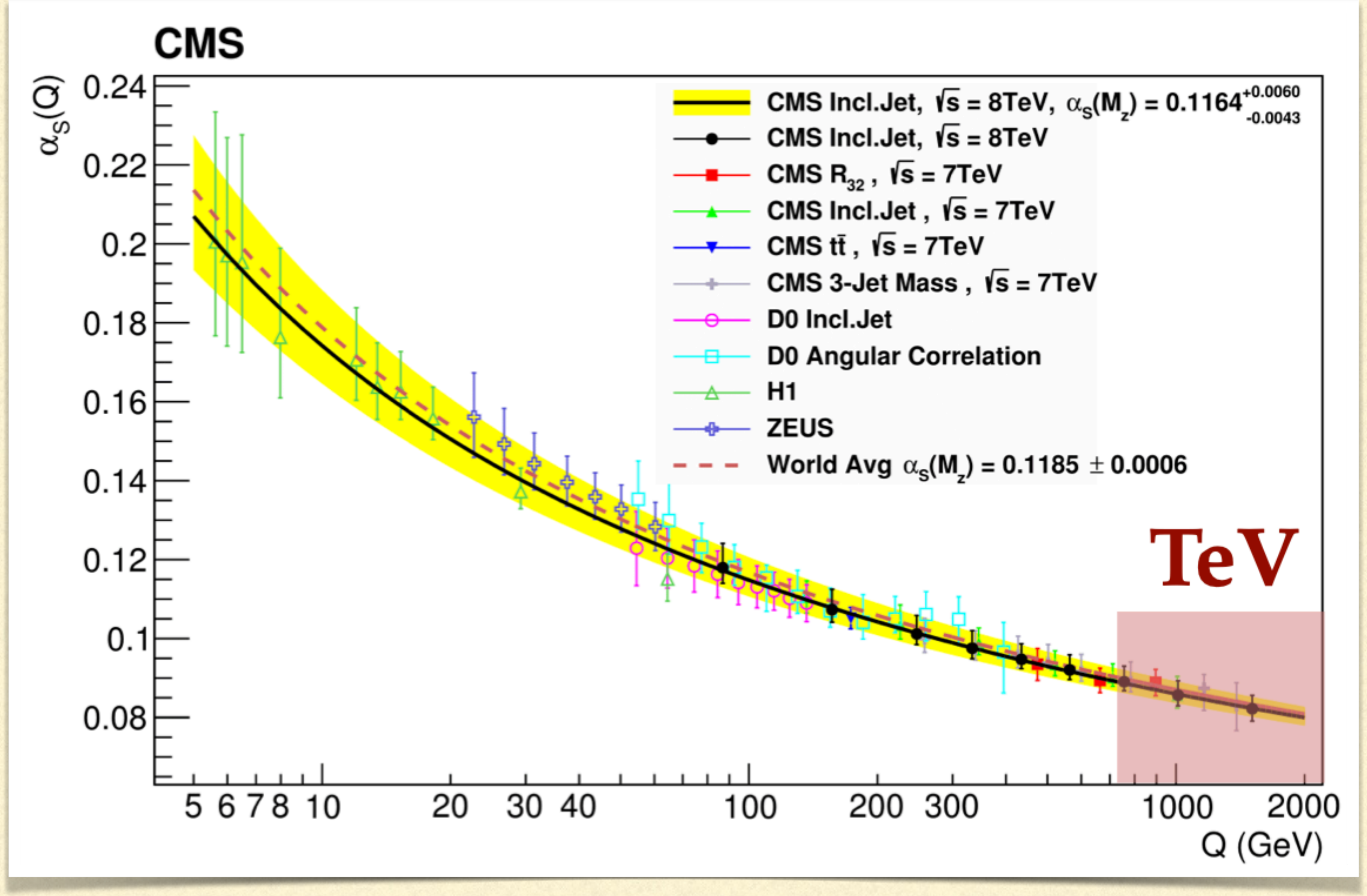
Badger, Gehrmann, Heinrich, Henn: **1905.03733(PRL)**
Abreu, Cordero, Ita, Page, Sotnikov: **2102.13609(JHEP)**
Agarwal, Buccioni, FD, Gambuti, von Manteuffel, Tancredi: **2311.09870**
De Laurentis, Ita, Klinkert, Sotnikov: **2311.10086,2311.18752**

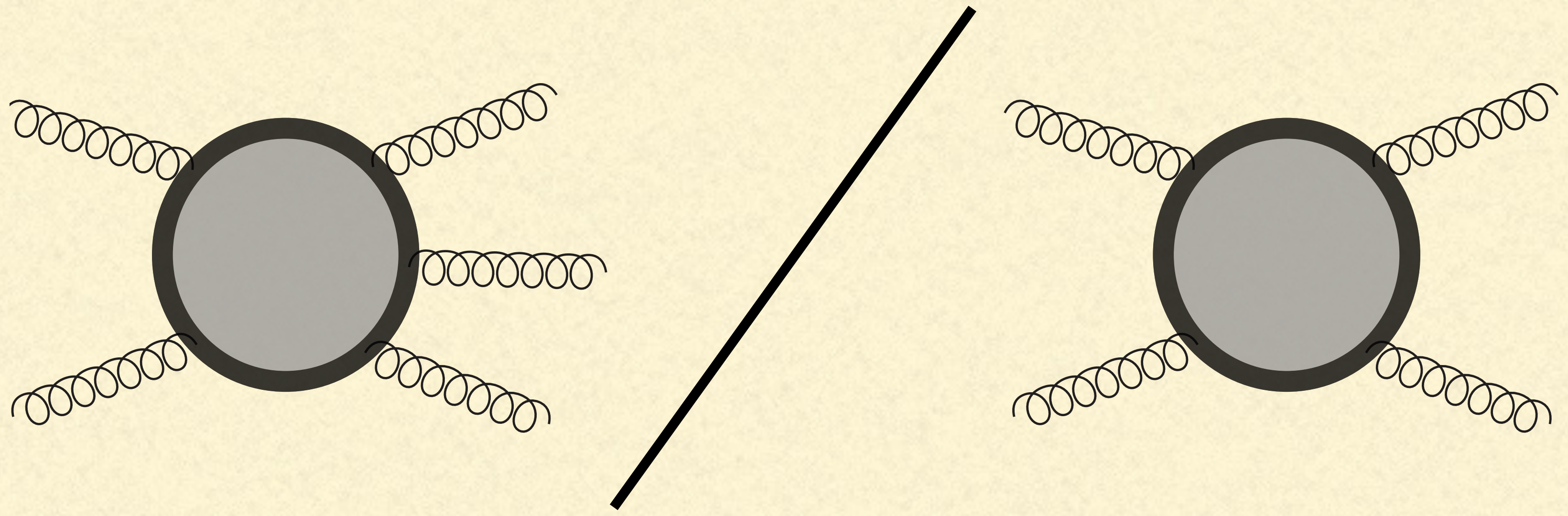
$2 \to n \geq 4$ scattering: 1 loop

On-shell methods/Unitarity (Bern, Dixon, Kosower), e.g. BlackHat, NJet
Off-shell/Tensor integral methods, e.g. OpenLoops, MadLoop

Three-jets @ LHC

- Three-to-two jet rates $R_{3/2}$ → **Extraction of α_s at LHC**



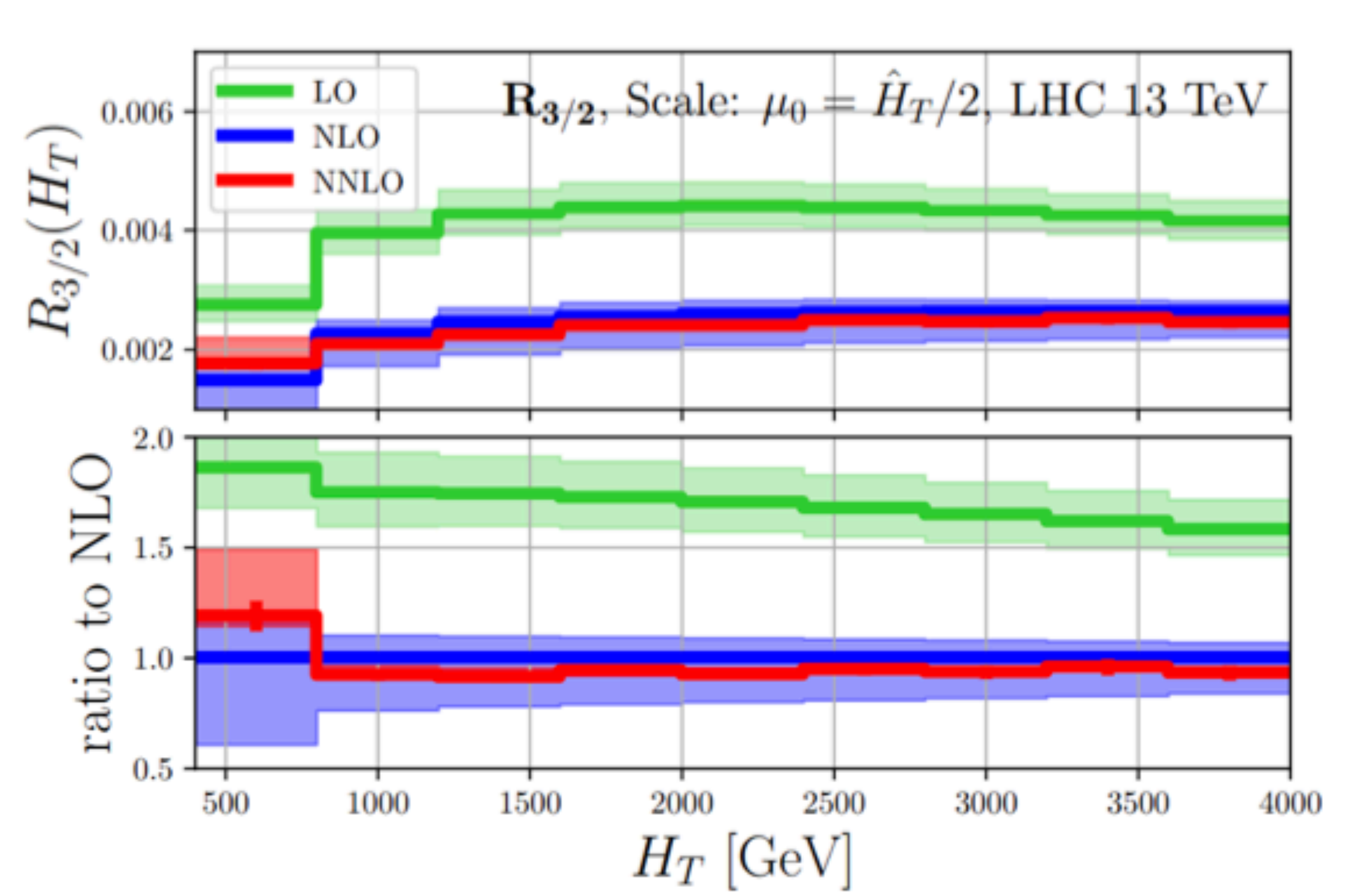


$\sim \alpha_s(Q)$

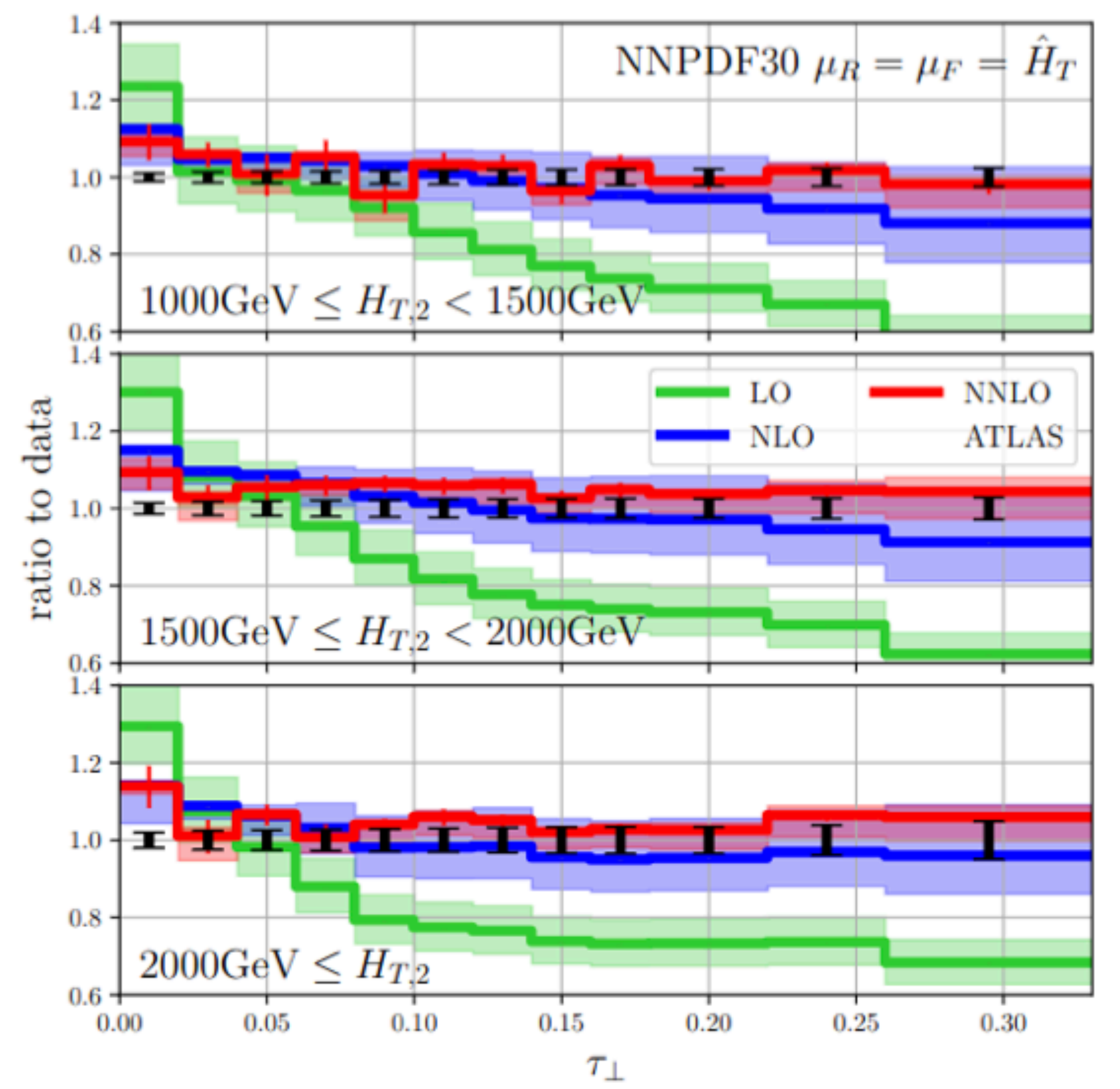
Running of strong coupling at TeV scale

Three-jets@NNLO QCD

First (impressive!) calculation of NNLO QCD corrections for 3 jets production at the LHC by Czakon, Mitov, Poncelet [2106.05331] & [2301.01086]



[Czakon, Mitov, Poncelet 2106.05331]



[Czakon, Mitov, Poncelet 2301.01086]

Caveat: double virtual contributions in leading color approximation

Study of QCD dynamics

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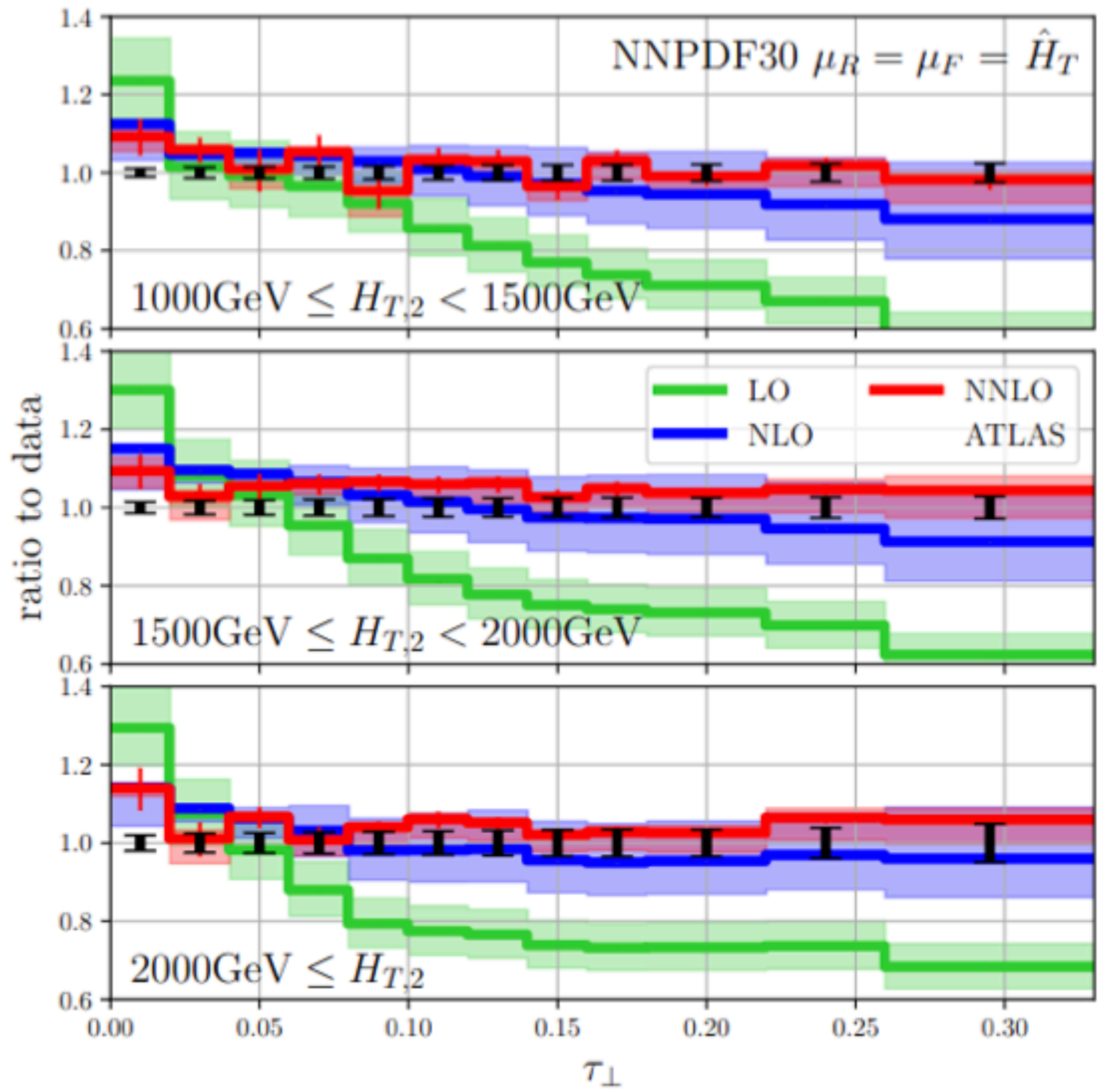

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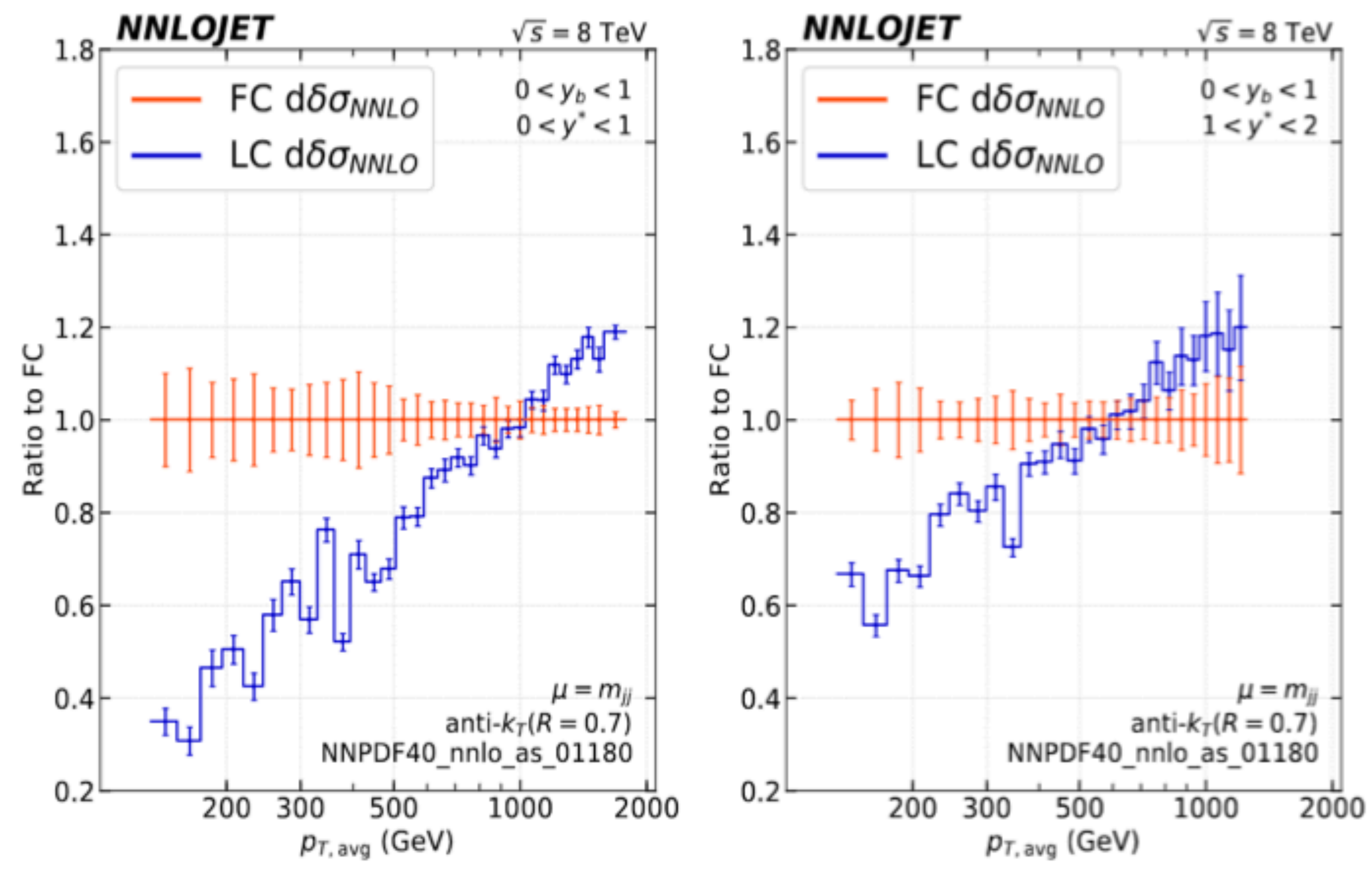

[Chen, Gehrmann, Glover, Huss, Mo 2204.10173]

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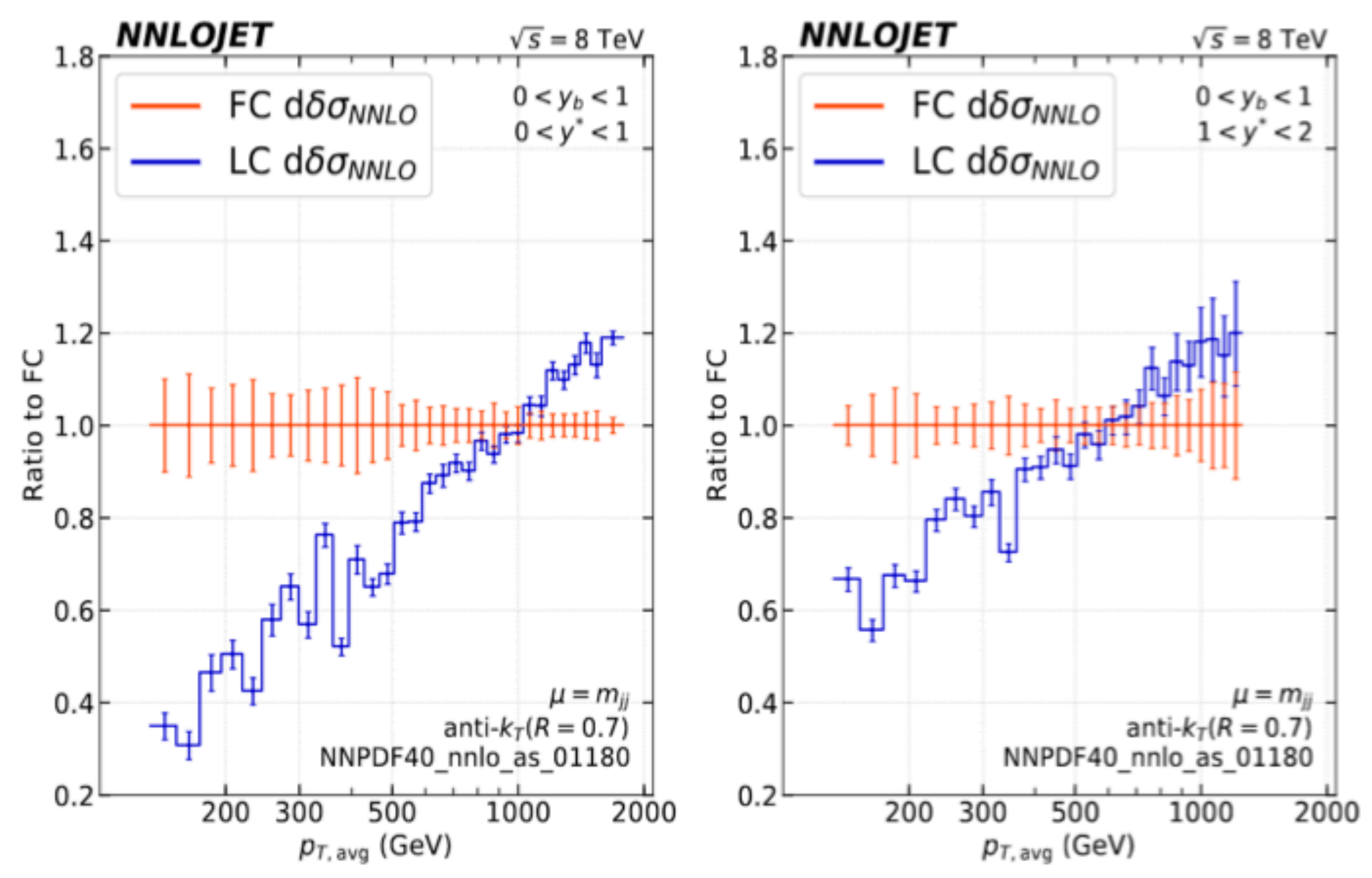

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Caveat: double virtual contributions in leading color approximation

Study of QCD dynamics

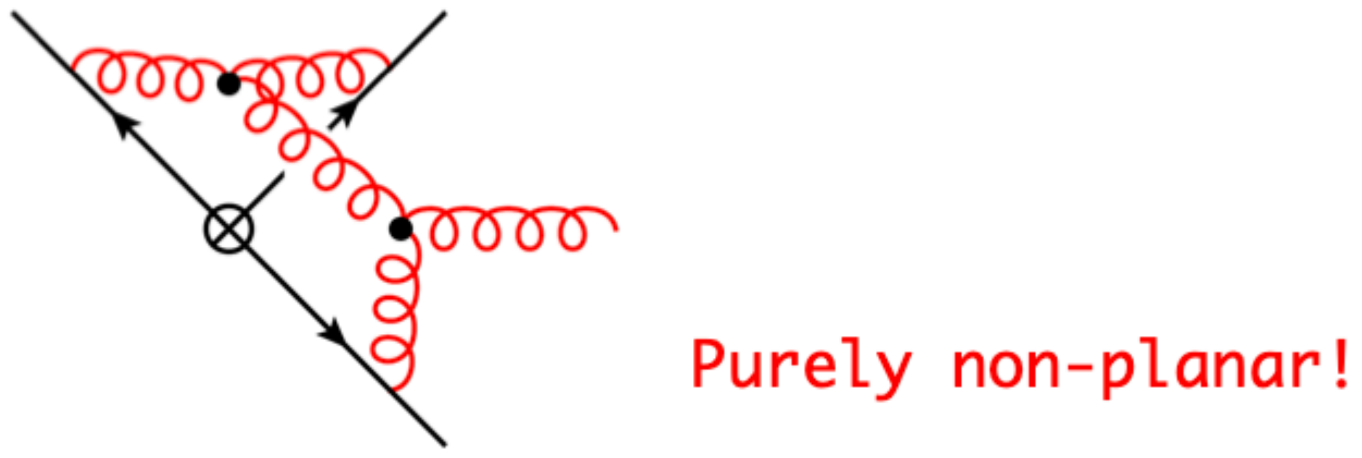
Triply differential dijet cross-section

Structure of QCD amplitudes

Explore QCD non-planar sector

IR limits: soft/collinear
Example: "tripole" soft gluon current

$$\boldsymbol{S}^{+,(2)}_{a,ikj} = \left(V^{q}_{ij}\right)^2 f^{a a_k b} f^{b a_i a_j} \boldsymbol{T}_i^{a_i} \boldsymbol{T}_j^{a_j} \boldsymbol{T}_k^{a_k} \left[\frac{\langle ik \rangle}{\langle iq \rangle \langle qk \rangle} F(z_k^{ij}, \epsilon) - \frac{\langle jk \rangle}{\langle jq \rangle \langle qk \rangle} F(z_k^{ji}, \epsilon) \right]$$



[Dixon, Herrmann, Yan, Zhu: 1912.09370]

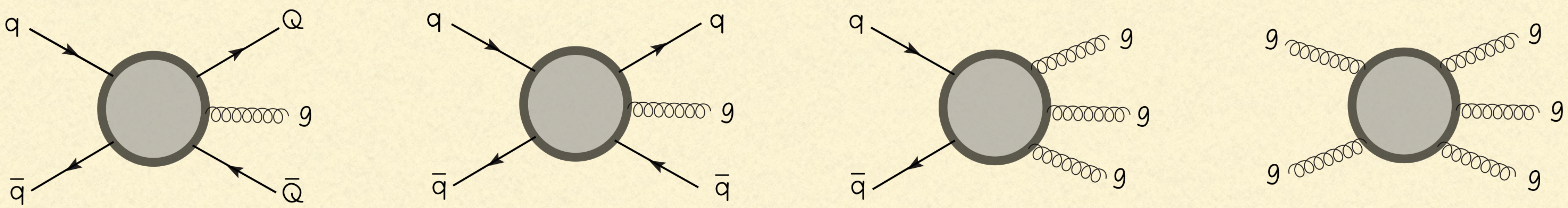
Image taken from G. Gambuti

High energy limits:
multi-Regge
kinematics

Most of these structures have been studied in more symmetric theories such as N=4 sYM

DETAILS OF THE CALCULATION

$\mathscr{A} = \sum$ Feynman diagrams



Feynman Diagrams	2522	4258	9136	28020
Helicities	8	8	16	32
Dimension colour space	4	4	11	22
Tot # colour structures	24	24	54	75

$$\mathscr{A} = \sum \text{Feynman diagrams}$$

Color decomposition

$$\mathscr{A}^{c_1 \dots c_5} = \sum_i^n A_i | c_1 \dots c_5 \rangle_i = \sum_i^n A_i | \mathscr{C}_i \rangle$$

Helicity decomposition

$$A_i \equiv A_i^{h_1 \dots h_5} = A_i^{\mu_1 \dots \mu_5} \epsilon_{\mu_1}^{h_1} \dots \epsilon_{\mu_5}^{h_5}$$

Projection to scalar form factors

$$A_i^{\mu_1 \dots \mu_5} = \sum_j F^j T_j^{\mu_1 \dots \mu_5}$$

IBP identities

$$F^j = \sum_k R^k I_k$$

Rational coefficients

Master integrals (MIs)

Algebraic manipulations + computation of MIs

Final result

$$\mathscr{A} = \sum \text{Feynman diagrams}$$

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Rational coefficients

Master integrals (MIs)

Algebraic

This is where the magic happens

Final result

Full color vs Leading color

Decompose amplitude in color space

$$\mathscr{A}^{c_1 \ldots c_5} = \sum_i^n A_i | c_1 \ldots c_5 \rangle_i = \sum_i^n A_i | \mathscr{C}_i \rangle$$

Polynomials in Nc, nf

Color basis

Example @ 2-loops:

Subleading color

$$A_i = b_i^{(2,0)} N_c^2 + b_i^{(1,0)} N_c + b_i^{(0,0)} 1 + b_i^{(-1,0)} N_c^{-1} + b_i^{(-2,0)} N_c^{-2} + b_i^{(1,1)} N_c n_f + \ldots$$

Leading color

Color decomposition

$$\mathscr{A}^{c_1\ldots c_5} = \sum_i^n A_i \,|\, c_1 \ldots c_5\rangle_i = \sum_i^n A_i \,|\, \mathscr{C}_i\rangle$$

Full color: span the whole color space

A_i → "Partial amplitudes"

$\vert\mathscr{C}_i\rangle$	$ggggg$	$q\bar{q}ggg$	$q\bar{q}Q\bar{Q}g$
Tree level	$\mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}T^{a_5}) - \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}T^{a_2}T^{a_1})$ + permutations	$(T^{a_1}T^{a_2}T^{a_3})_{ji}$ + permutations	$T^a_{ij}\delta_{kl}$ $T^a_{ik}\delta_{jl}$
Beyond tree	$\mathrm{Tr}(T^{a_1}T^{a_2}) \times (\mathrm{Tr}(T^{a_3}T^{a_4}T^{a_5}) - \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}))$ + permutations	$\mathrm{Tr}(T^{a_1}T^{a_2})T^{a_3}_{ij}$ $(\mathrm{Tr}(T^{a_3}T^{a_4}T^{a_5}) - \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}))\delta_{ij}$ $(\mathrm{Tr}(T^{a_3}T^{a_4}T^{a_5}) + \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}))\delta_{ij}$	Same as tree

Helicity decomposition

tHV scheme: external particles in d=4

$$A_c \equiv A_c^{h_1 \ldots h_5} = A_c^{\mu_1 \ldots \mu_5} \epsilon_{\mu_1}^{h_1} \ldots \epsilon_{\mu_5}^{h_5} = \sum_j F_c^j T_j^{\mu_1 \ldots \mu_5} \epsilon_{\mu_1}^{h_1} \ldots \epsilon_{\mu_5}^{h_5}$$

5 gluons: 1724 structures in d dimensions $\xrightarrow[\epsilon_i \cdot p_{i+1} = 0]{\epsilon \cdot p = 0}$ 142 structures… still a lot!

Many of these do not contribute to the physical helicity amplitudes!

qqQQg: Even worse! Dirac algebra does not close in d dimensions!

Key idea: rotate away the -2ϵ subspace

[Peraro, Tancredi 1906.03298, 2012.00820]

# of independent structures = # helicity configurations !

Reduction to master integrals

$$A_i^{\mu_1\ldots\mu_5} = \sum_{j=1}^{32} F^j T_j^{\mu_1\ldots\mu_5} \quad \longrightarrow \quad F^j = \sum_k R^k I_k$$

Scalar objects

IBP reduction

FinRed

[A.v. Manteuffel]

Master Integrals

Wise choice of MIs?

How FinRed overcomes the complexity:

- Finite-fields arithmetics [von Manteuffel, Schabinger 1406.4513; Peraro 1905.08019]
- Syzygy techniques [Gluza, Kadja, Kosower 1009.0472, Ita 1510.05626; Larsen, Zhang, 1511.01071, Agarwal, Jones, von Manteuffel 2011.15113]
- Denominator guessing [Abreu, Dormans, Febres Cordero, Ita, Page 1812.04586; Heller, von Manteuffel 2101.08283]

Reduction to master integrals

$$A_i^{\mu_1 \ldots \mu_5} = \sum_{j=1}^{32} F^j \, T_j^{\mu_1 \ldots \mu_5}$$

Scalar objects

Major bottleneck

eduction

FinRed

[A.v. Manteuffel]

$$F^j = \sum_k R^k I_k$$

Master Integrals

Wise choice of MIs?

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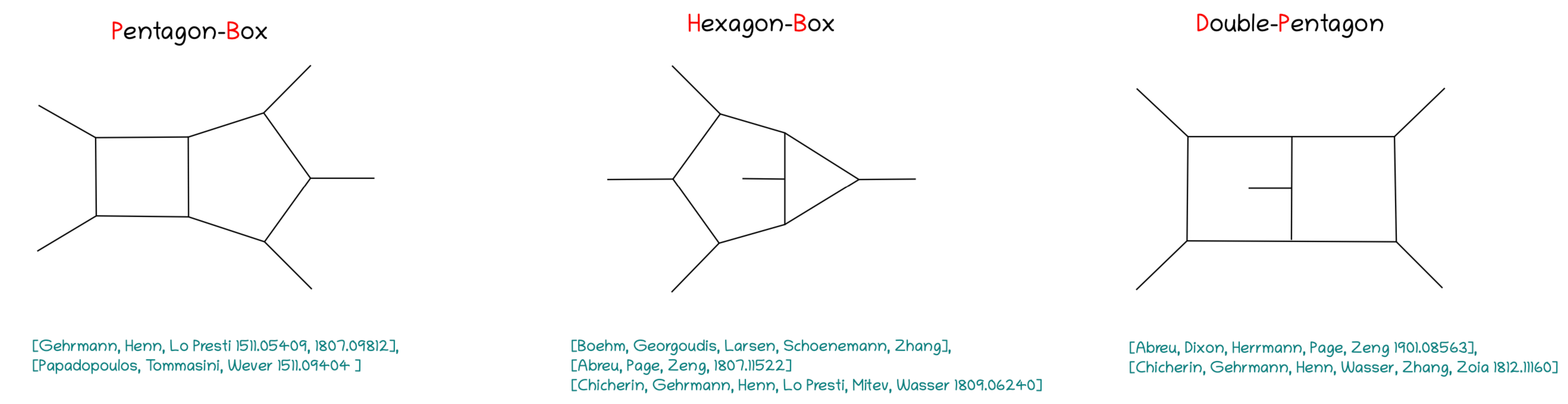
MIs: Pentagon Functions



- Canonical basis
- UT weight integrals

Expressed as Chen iterated integrals, full set available [Chicherin, Sotnikov 2009.07803]

$$f^{(\omega)}(\mathbf{x}) = \int_\gamma \mathrm{d}\log W_{i_1} \ldots \mathrm{d}\log W_{i_n}$$

Evaluation time: $\sim 1s$

Reduction to master integrals - II

$$I(s_{ij}, d) = \sum_{k=1}^{M} a_k(s_{ij}, d) \mathcal{I}(s_{ij}, d)$$

Common denominator

$$a_k(s_{ij}, d) = \frac{N(s_{ij}, d)}{Q(d) D(s_{ij})}$$

Partial-fractioned form

$$a_k(s_{ij}, d) = \sum_{l} g_l(d) R_l(s_{ij})$$

Crucial step: drastic reduction in size!

We employ MultivariateApart for multivariate partial fractioning:
[Heller, von Manteuffel 2101.08283]

- Avoids spurious denominators
- Produces unique results when applied to terms of a sum separately

Simplifying the rational coefficients

$$a_k(s_{ij}, d) = \frac{N(s_{ij}, d)}{Q(d)D(s_{ij})} \longrightarrow a_k(s_{ij}, d) = \sum_l g_l(d)\, R_l(s_{ij})$$

MultivariateApart + Singular

[Heller, von Manteuffel 2101.08283][Decker, Greuel, Pfister, Schoenemann]

Drastic simplifications occur:

	CD		PF
PB: INT[TA,8,255,8,5,{1,1,1,1,1,1,1,1,-5,0,0}]	162 MB	→	3.9 MB
HB: INT[TB,8,255,8,5,{1,1,1,1,1,1,1,1,-4,0,-1}]	513 MB	→	9.9 MB
DP: INT[TB,8,510,8,5,{0,1,1,1,1,1,1,1,1,-5,0}]	2.9 GB	→	24 MB

$\sim \mathcal{O}(100)$!!

Final result: infrared structure

$$A_i(\mathbf{h}) = \sum_l r_l(s_{ij}, \epsilon_5) f_l(s_{ij}, \epsilon_5)$$

UV and IR subtraction

$$\mathbf{Z}^{-1} = 1 - \frac{\alpha_s}{2\pi}\mathbf{I}_1 - \left(\frac{\alpha_s}{2\pi}\right)^2 \mathbf{I}_2$$

$$A_{ren}(\epsilon, p_i) = \mathbf{Z}(\epsilon, p_i, \mu) A_{fin}(p_i, \mu)$$

I_1 and I_2 diagonal in LC approximation, but not in full color

$$\mathbf{I}_1(\epsilon) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \sum_i \left(\frac{1}{\epsilon^2} - \frac{\gamma_0^i}{2\epsilon}\frac{1}{\mathbf{T}_i^2}\right) \sum_{j\neq i} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \left(\frac{\mu^2}{-s_{ij}}\right)^\epsilon$$

$$\mathbf{I}_2(\epsilon) = \frac{e^{-\epsilon\gamma_E}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \sum_i \left(\frac{\gamma_1^{cusp}}{8} + \frac{\beta_0}{2\epsilon}\right) \mathbf{I}_1(2\epsilon) - \frac{1}{2}\mathbf{I}_1(\epsilon)\left(\mathbf{I}_1(\epsilon) + \frac{\beta_0}{\epsilon}\right) + \mathbf{H}_2(\epsilon)$$

Contains non-trivial triple color correlated contributions beyond LC

Final result: checks

Results available at: https://zenodo.org/records/10227683

1-loop:

- Match prediction for IR poles @ NLO
- For 5 gluon amplitudes: check color trace identities [Bern, Kosower Nucl.Phys.B 362 (1991)]
- Reproduce available results and checked vs OpenLoops

2-loop:

- ggggg $\to 0, q\bar{q}ggg \to 0, q\bar{q}Q\bar{Q}g \to 0$ completed
- Match prediction for IR poles @ NNLO
- For all channels: agreement with LC result + Full color results from [2311.10086] & [2311.18752] [Abreu, Febres Cordero, Ita, Page, Sotnikov: 2102.13609][De Laurentis, Ita, Sotnikov, Klinkert]
- For ggggg: agreement with all-plus full color Yang Mills result [Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia 1905.03733]

Outlook and conclusions

- Calculation of 5-points 2-loops QCD massless amplitudes in full color
- In principle everything is there to study numerical impact and analyse structure
- Unique opportunity to study non-planar sectors of QCD and various kinematic limit (study the impact on collinear factorization, high-energy factorization etc.)

For the future…

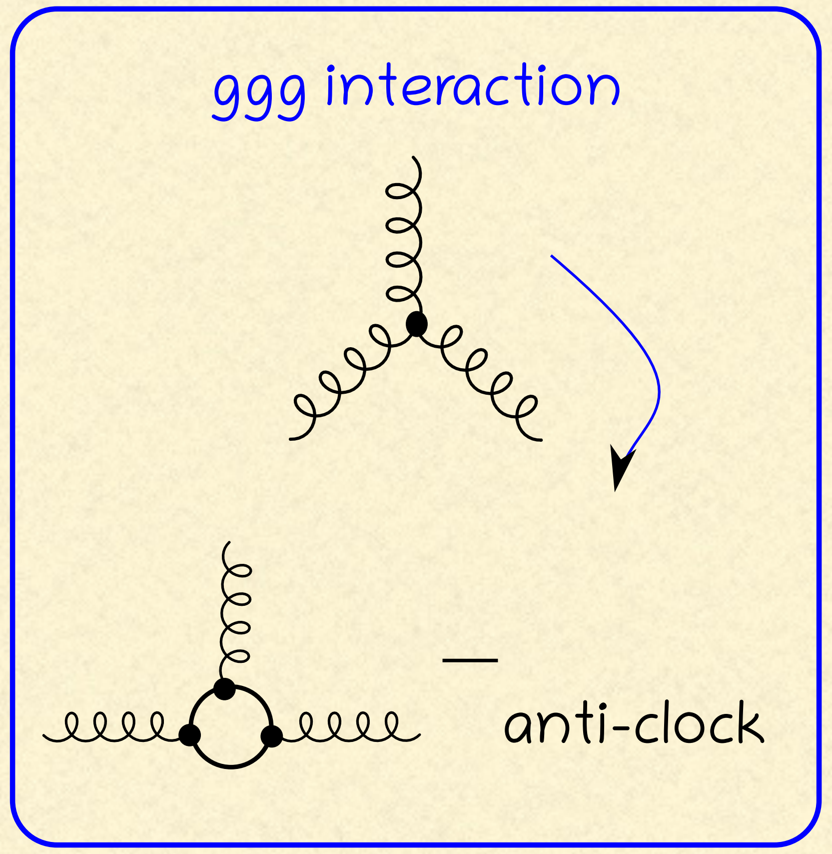
- Study impact of subleading color in phenomenological studies
- Multi-Regge kinematics of QCD amplitudes + IR limits (soft-gluon current etc.) WIP

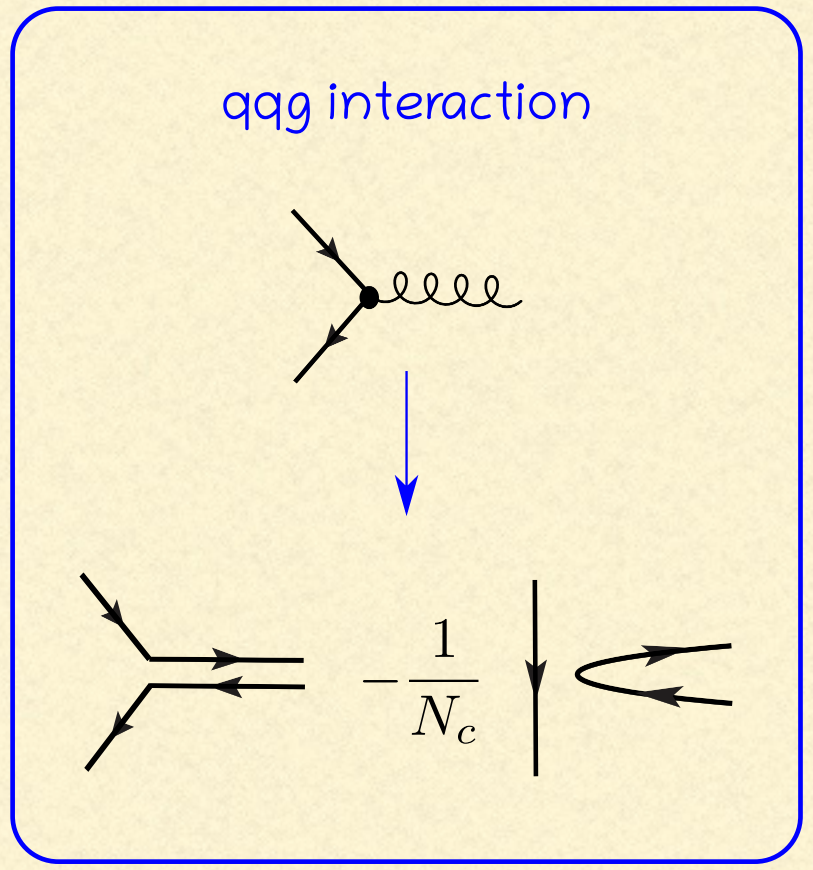
THANK YOU!

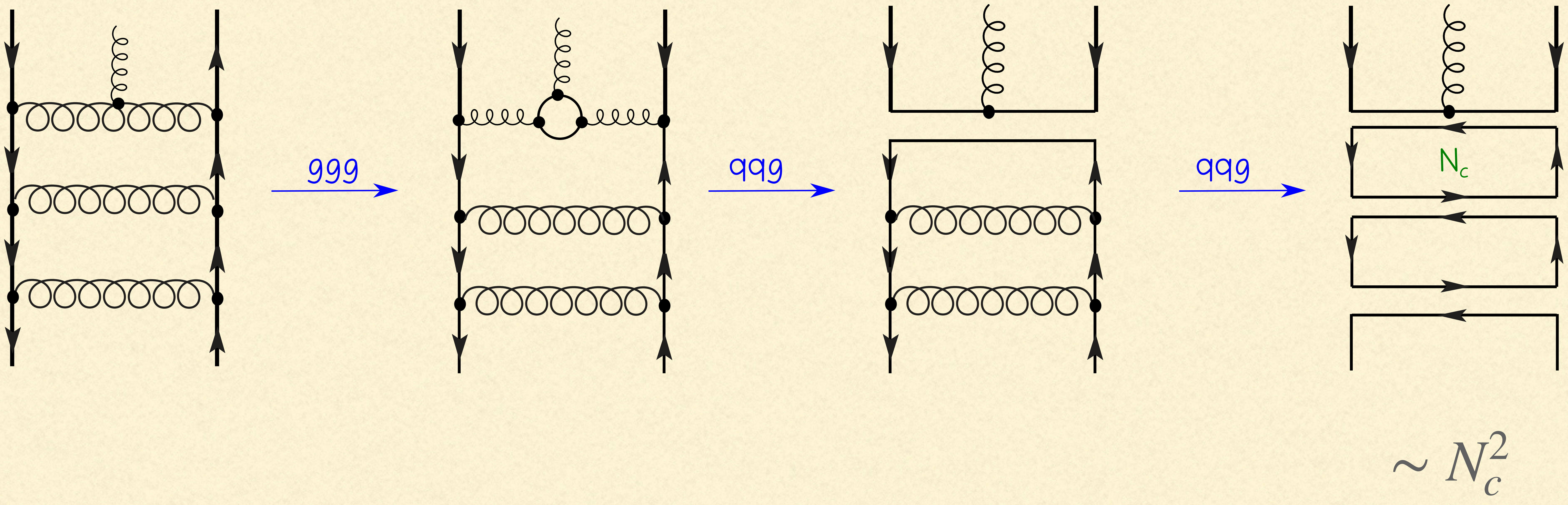
BACKUP

Full color vs Leading color - II

In QCD: **Leading color** → **Planar topologies** ['t Hooft]







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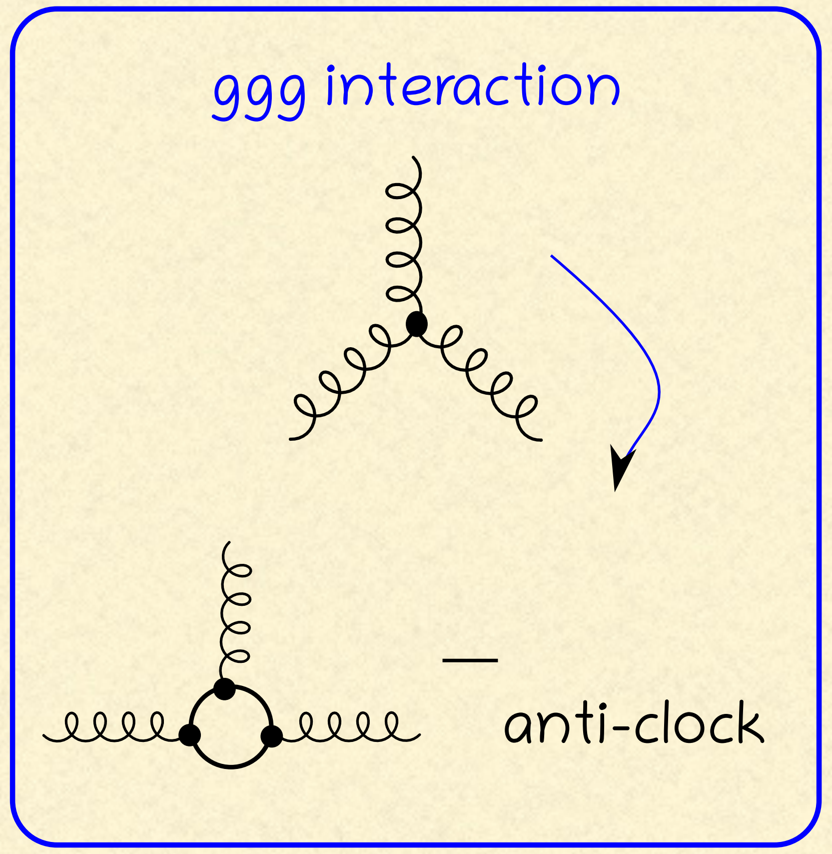
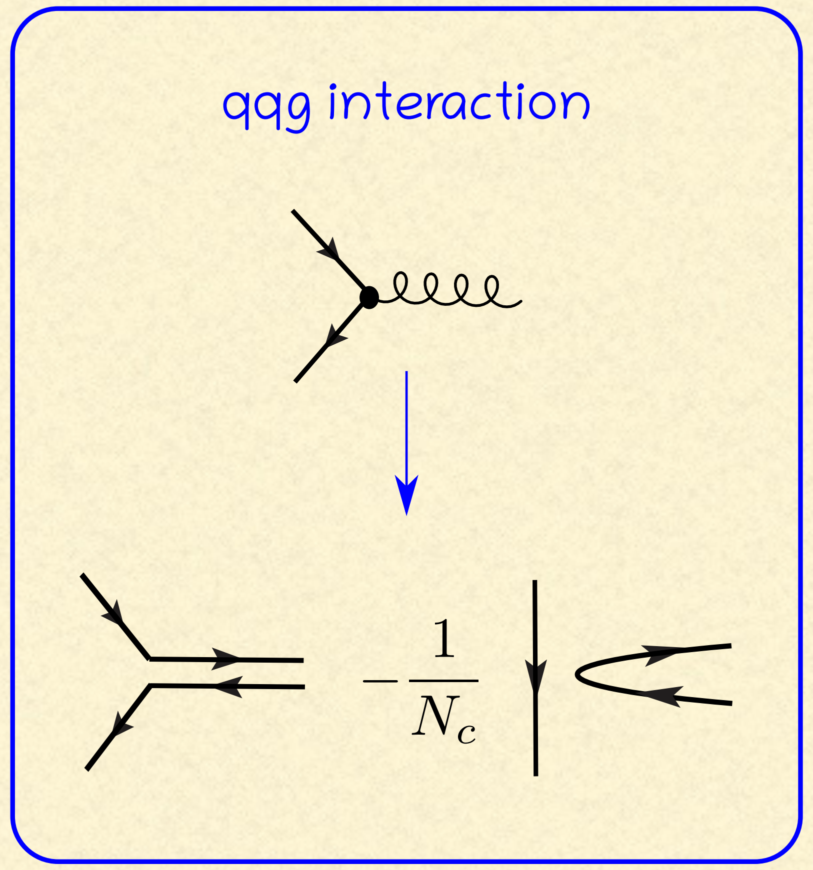
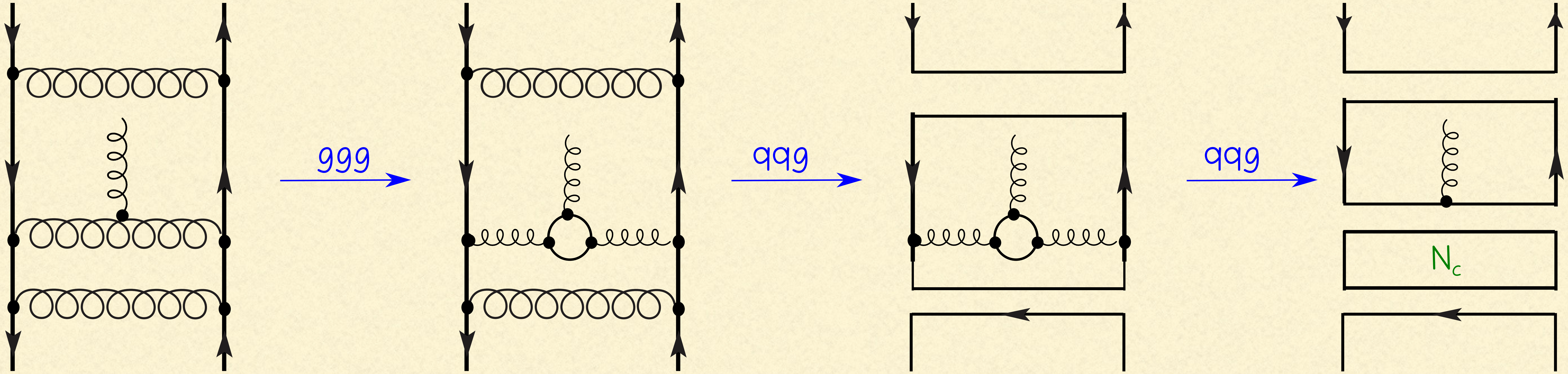






$\sim N_c^{-2}$

Helicity decomposition

$$A_i \equiv A_i^{h_1 \dots h_5} = A_i^{\mu_1 \dots \mu_5} \epsilon_{\mu_1}^{h_1} \dots \epsilon_{\mu_5}^{h_5}$$

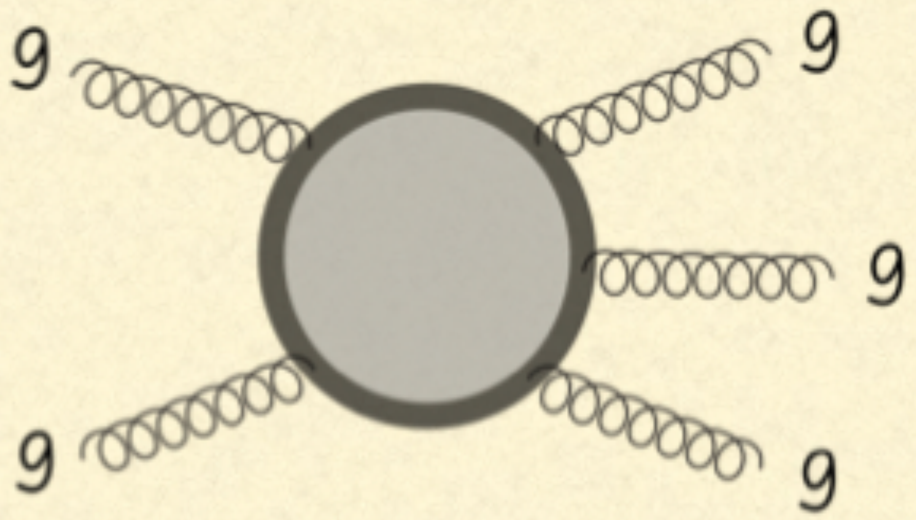
For n>4 can be constructed in terms of external momenta!

Independent in d=4

[Peraro, Tancredi 1906.03298, 2012.00820]

Transversality + choice reference vector

# of independent structures = # helicity configurations !



tHV scheme: external particles in d=4

$$A_i^{\mu_1 \dots \mu_5} = \sum_{j=1}^{32} F^j \, T_j^{\mu_1 \dots \mu_5}$$

Tensors

Form Factors

$$F^j = \sum_{pol} \mathscr{P}^j(\mathbf{h}) A(\mathbf{h})$$

Projectors

Helicity decomposition

qqQQg: Even worse! Dirac algebra does not close in d dimensions!

Example: 4 quarks amplitude

$$T_1 = \bar{u}(p_2)\gamma_{\mu_1} u(p_1)\,\bar{u}(p_4)\gamma^{\mu_1} u(p_3)$$

$$T_2 = \bar{u}(p_2)\not{p}_5\, u(p_1)\,\bar{u}(p_4)\not{p}_5\, u(p_3)$$

$$T_3 = \bar{u}(p_2)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} u(p_1)\,\bar{u}(p_4)\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} u(p_3)$$

$$T_4 = \bar{u}(p_2)\gamma_{\mu_1}\not{p}_5\gamma_{\mu_3} u(p_1)\,\bar{u}(p_4)\gamma^{\mu_1}\not{p}_5\gamma^{\mu_3} u(p_3)$$

: grows w/ # loops !

Key idea: rotate away the -2ϵ subspace

[Peraro, Tancredi 1906.03298, 2012.00820]

Only T_1, T_2 are left in d=4!

$$T_1 = \bar{u}(p_2)\gamma_{\mu_1} u(p_1)\,\bar{u}(p_4)\gamma^{\mu_1} u(p_3)$$

$$T_2 = \bar{u}(p_2)\not{p}_5\, u(p_1)\,\bar{u}(p_4)\not{p}_5\, u(p_3)$$

$$T_3 = \cancel{T_3^{4}} + (d-4)\,T_3^{-2\epsilon}$$

$$T_4 = \cancel{T_4^{4}} + (d-4)\,T_4^{-2\epsilon}$$

\# of independent structures = # helicity configurations !