A fresh look at the Nested Soft-Collinear subtraction scheme: NNLO QCD corrections to N-gluon final state $q\bar{q}$ annihilation

NEW YORK - ACAT 2024

Davide Maria Tagliabue

In collaboration with:
[F. Devoto, K. Melnikov, R. Röntsch, C. Signorile-Signorile,
arXiv 2310.17598, *JHEP* 02 (2024) 016]

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DEGLI STUDI
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WHAT IS THIS TALK ABOUT?

✓ For any collider process:
i) compute the differential cross-section
ii) use **fixed-order perturbation theory**

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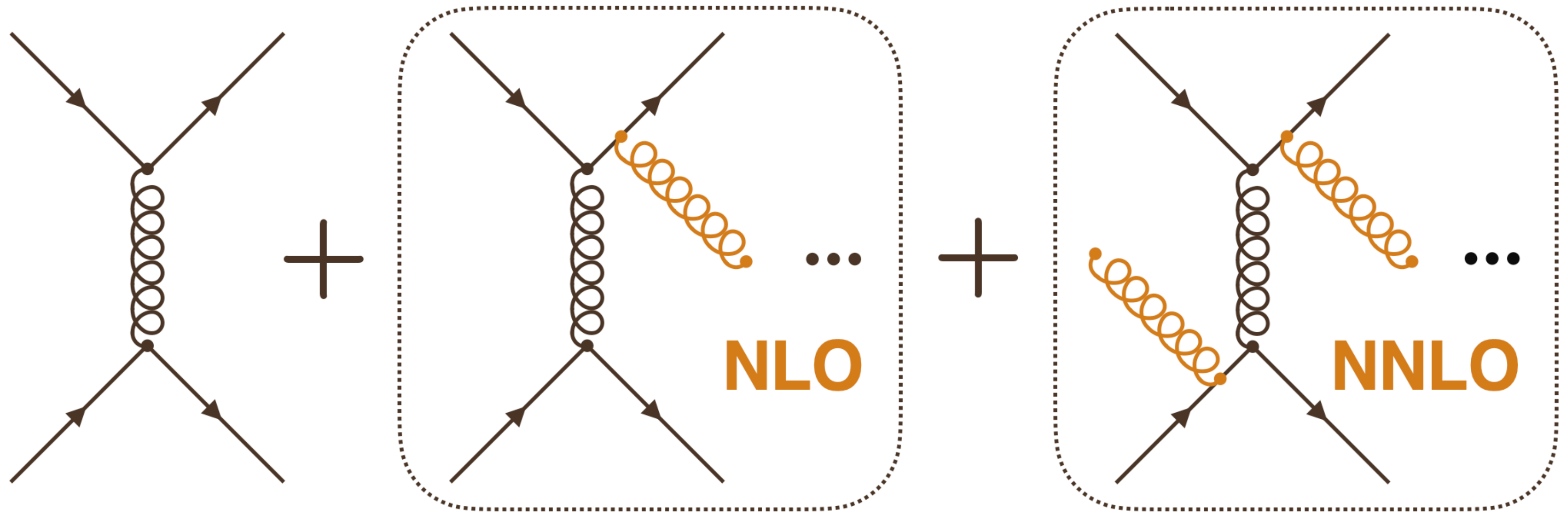

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NLO: solved in full generality two decades ago
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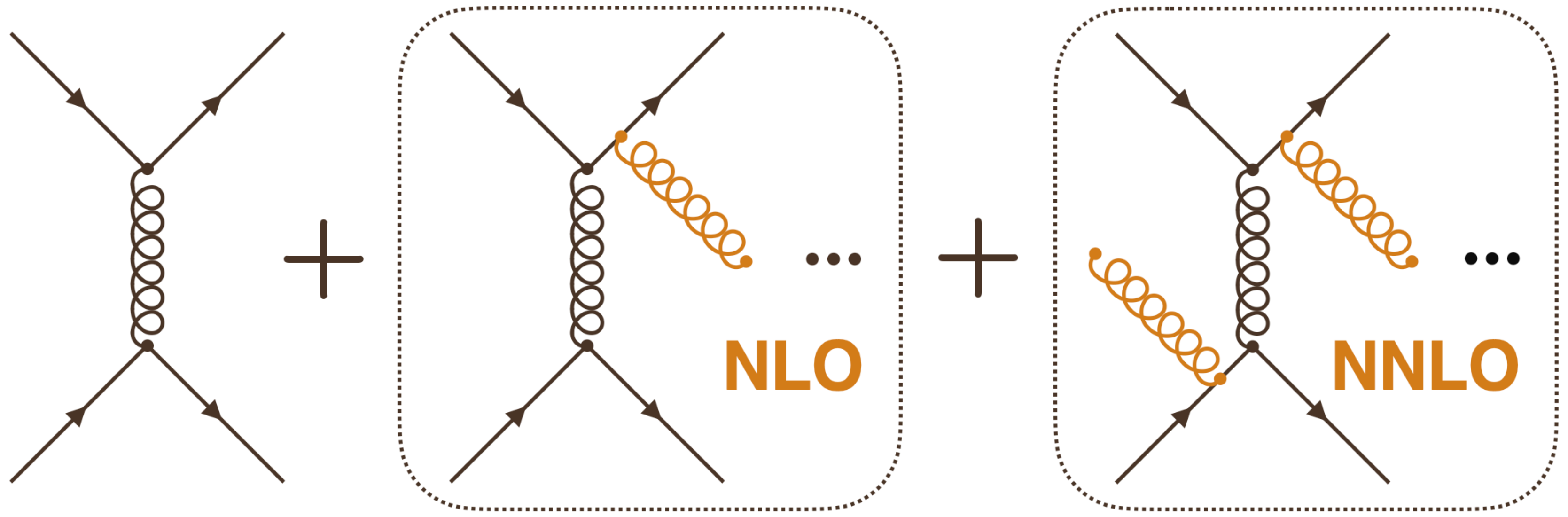

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✓ Two main difficulties: **IR singularities**, arising from real and virtual radiation, and **multi-loop amplitude** calculations

Recommended talks:

i) *Herschel Chawdhry*: this afternoon, at 3:50 pm
ii) *Federica Devoto*: tomorrow at 3:50 pm

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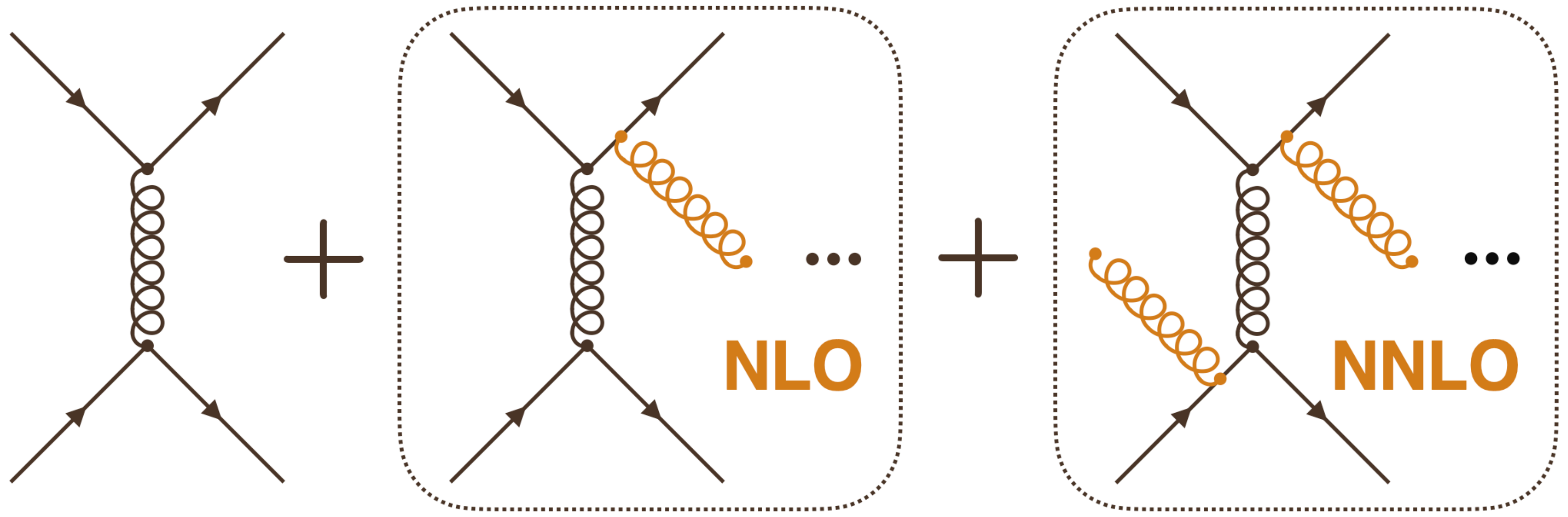

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- ✓ Two main difficulties: **IR singularities**, arising from real and virtual radiation, and **multi-loop amplitude** calculations

This talk! *IR singularities*:
- i) they are unphysical: require **SUBTRACTION SCHEMES**
- ii) we use **NESTED-SOFT COLLINEAR** [Caola, Melnikov, Röntsch, ‘17]

NESTED SOFT COLLINEAR

$$\int |\mathscr{M}|^2 \, \mathrm{d}^{(d)}\phi \overset{?}{=}$$

NESTED SOFT COLLINEAR

$$\int |\mathscr{M}|^2 \, \mathrm{d}^{(d)}\phi = \int \left[|\mathscr{M}|^2 - K \right] \mathrm{d}^{(4)}\phi \; + \int K \, \mathrm{d}^{(d)}\phi$$

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$$\int |\mathscr{M}|^2 \, \mathrm{d}^{(d)}\phi = \underbrace{\boxed{\int \left[|\mathscr{M}|^2 - K\right] \mathrm{d}^{(4)}\phi}}_{\text{Finite}} + \int K \, \mathrm{d}^{(d)}\phi$$

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Laurent Series

$$c_0 + \frac{c_1}{\epsilon} + \frac{c_2}{\epsilon^2} + \frac{c_3}{\epsilon^3} + \frac{c_4}{\epsilon^4} + \dots$$

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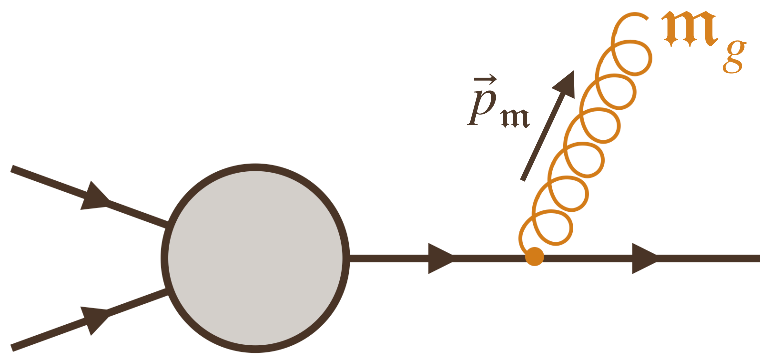
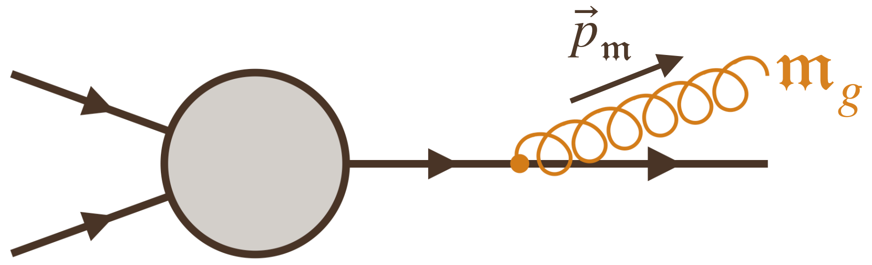
☠ Problem of **OVERLAPPING SOFT** and **COLLINEAR** real emissions



Soft emission

$S_{\mathfrak{m}}: |\vec{p}_{\mathfrak{m}}| \to 0$



Collinear emission

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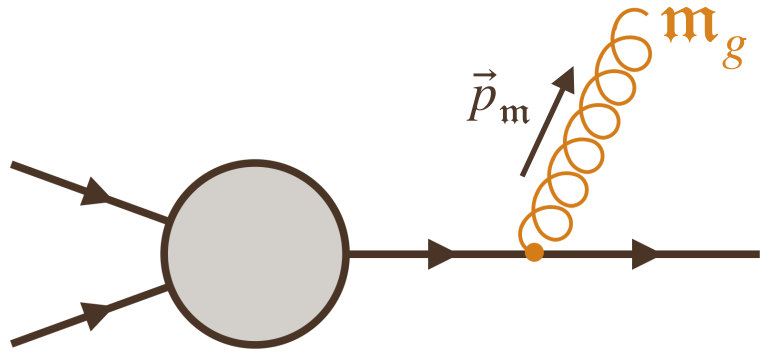

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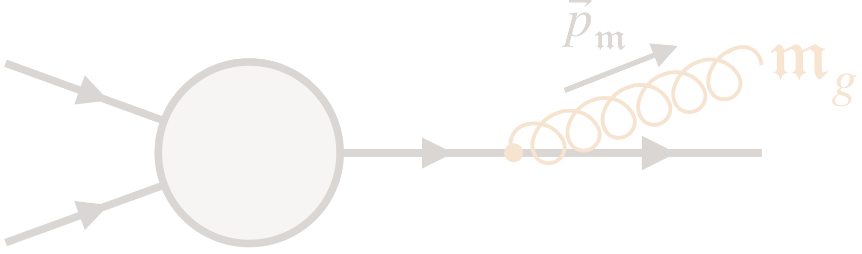

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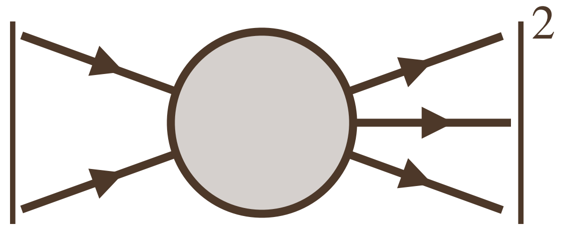

$$\overset{?}{=}$$

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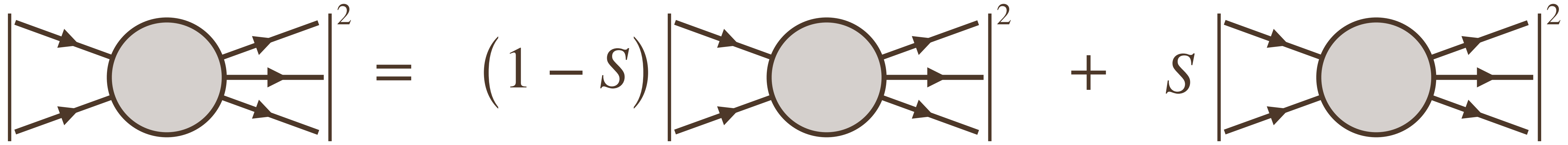

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Divergent

Laurent Series

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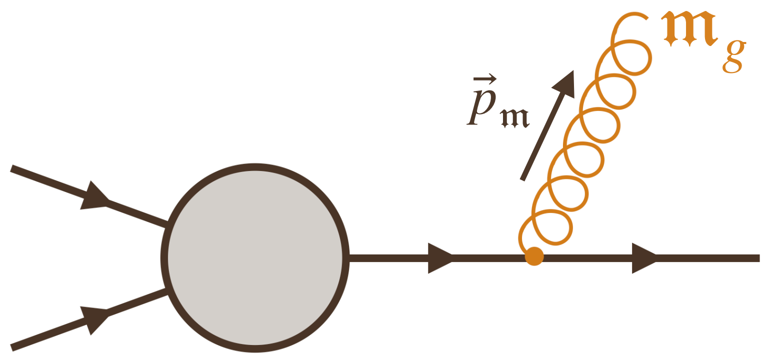

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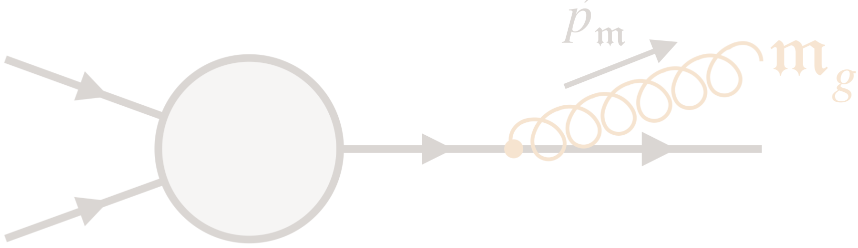
S_{m}: $|\vec{p}_{\mathrm{m}}| \to 0$



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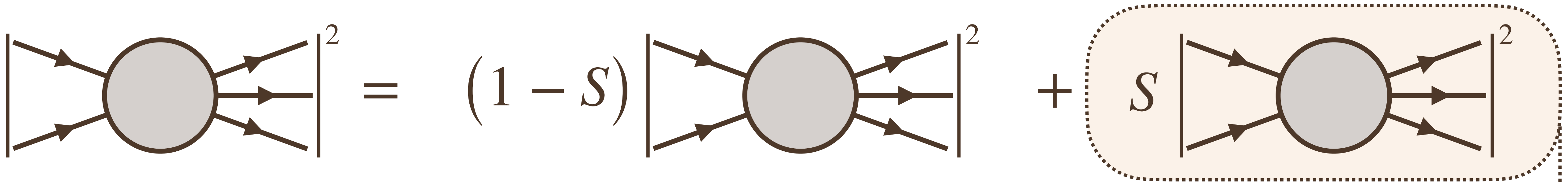

Soft-counterterm provides the formula for the soft poles

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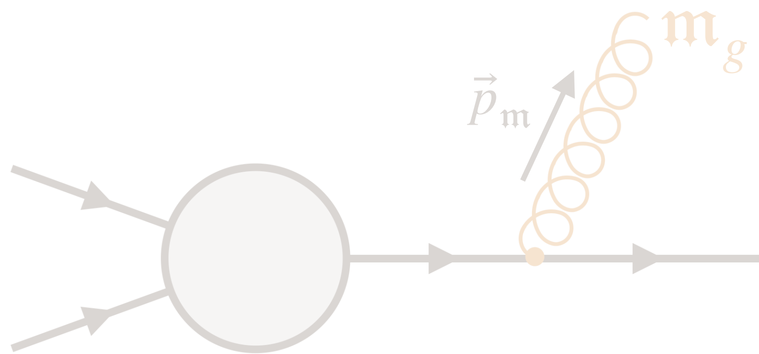

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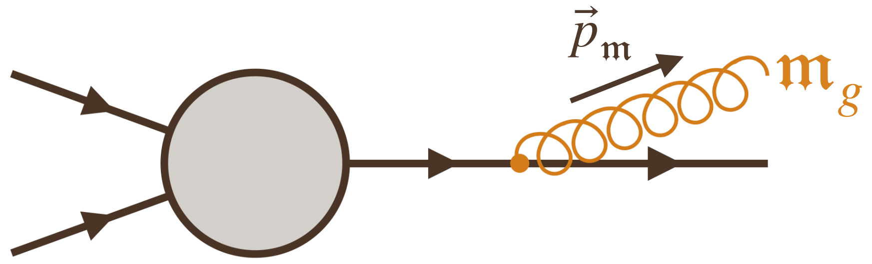

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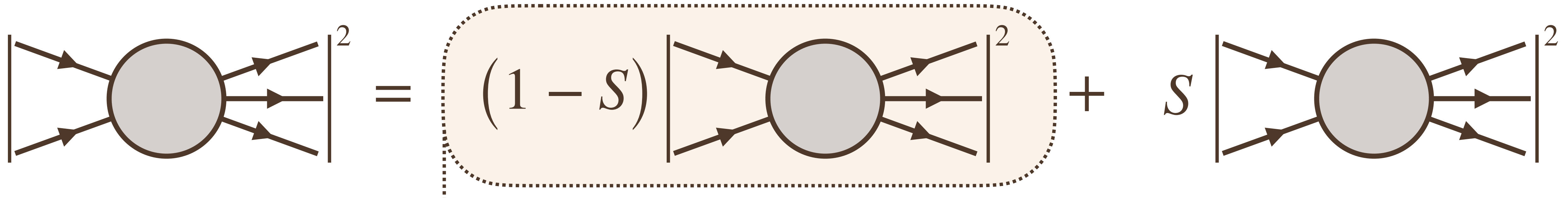

Soft-regulated term
still contains collinear
divergences

The **soft-regulated** term needs a similar treatment for **COLLINEAR** divergences: all the singular configurations can be separated out

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Finite

Divergent

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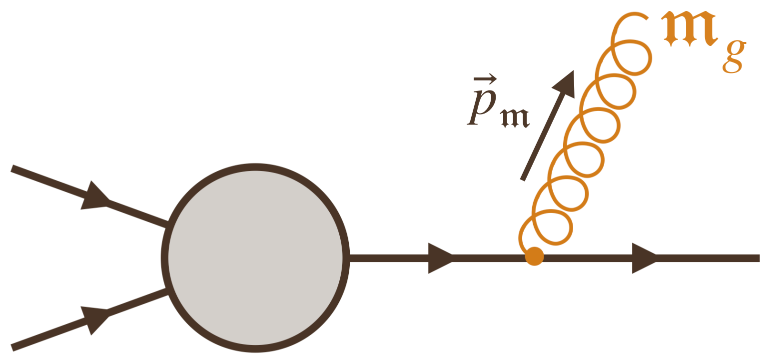

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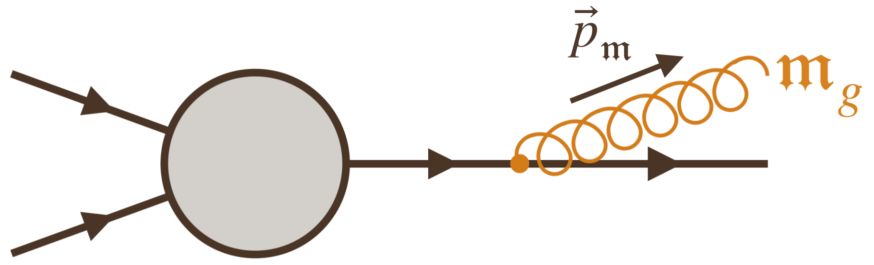

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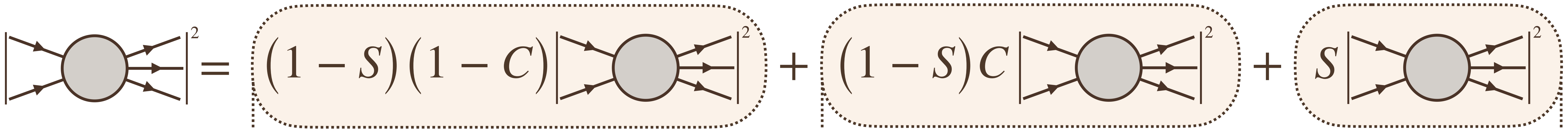

fully-regulated term
can be implemented
in a numerical code

Finite

Collinear-counterterm
provides the formula
for the collinear poles

Divergent

Soft-counterterm
provides the formula
for the soft poles

Divergent

ANALYTIC VERSUS NUMERICAL

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Sticking to an **ANALYTIC** approach to compute counterterms is complicated. Why don't we go for a **NUMERICAL** strategy?

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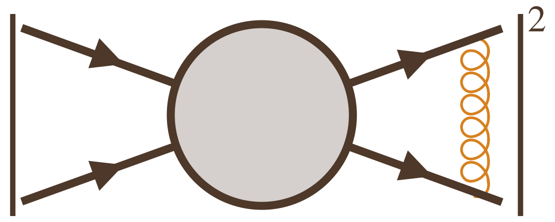
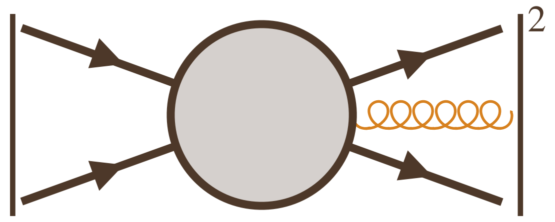


$$= + \frac{c_2}{\epsilon^2} + \frac{c_1}{\epsilon} + c_0$$

$$\int \mathrm{d}^{(d)}\phi$$



$$= - \frac{c_2}{\epsilon^2} - \frac{c_1}{\epsilon} + \tilde{c}_0$$

ANALYTIC
VERSUS
NUMERICAL

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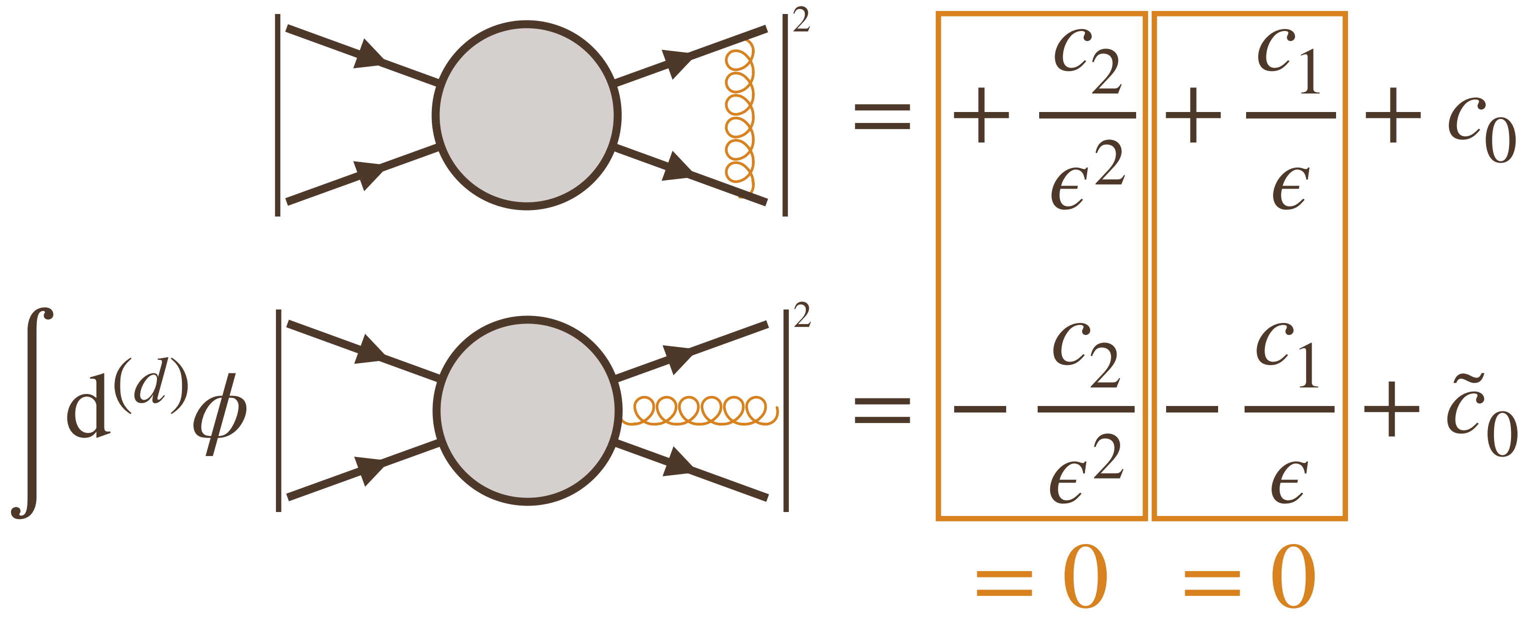

$$= \boxed{+\frac{c_2}{\epsilon^2}} \boxed{+\frac{c_1}{\epsilon}} + c_0$$

$$\int \mathrm{d}^{(d)}\phi \qquad = \boxed{-\frac{c_2}{\epsilon^2}} \boxed{-\frac{c_1}{\epsilon}} + \tilde{c}_0$$

$$= 0 \qquad = 0$$

The cancellation is **ALWAYS EXACT**

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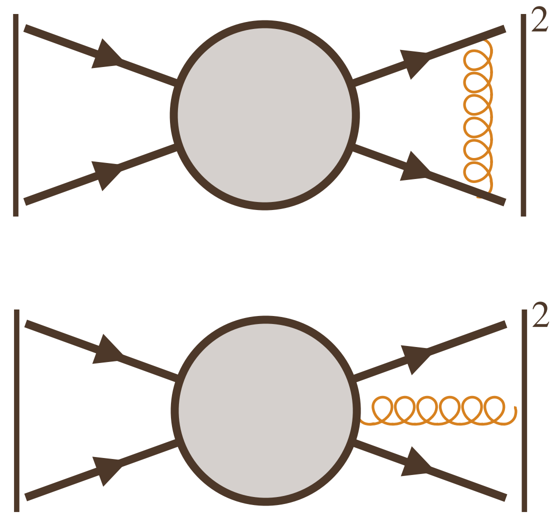

$$\begin{aligned} &= \boxed{+\frac{c_2}{\epsilon^2}} \boxed{+\frac{c_1}{\epsilon}} + c_0 \\ \int d^{(d)}\phi &= \boxed{-\frac{c_2}{\epsilon^2}} \boxed{-\frac{c_1}{\epsilon}} + \tilde{c}_0 \end{aligned}$$

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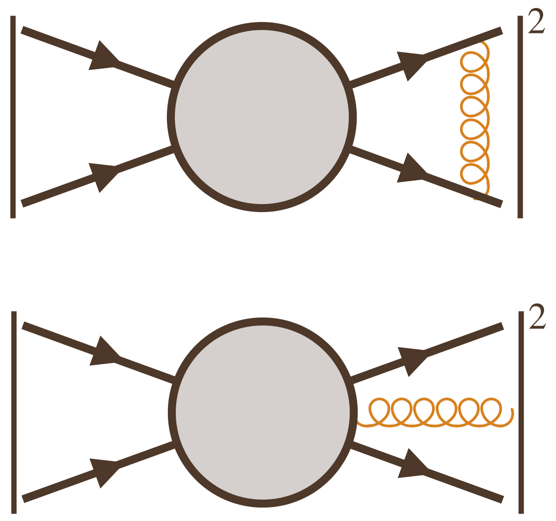

$$\begin{aligned} &= +1.1235813213455 + c_0 \\ \int d^{(4)}\phi &= -1.1235813213454 + \tilde{c}_0 \end{aligned}$$

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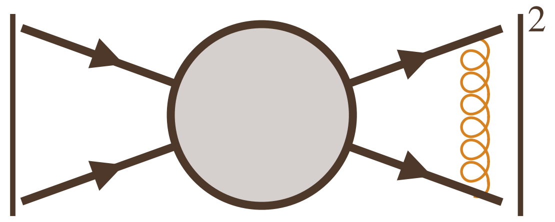
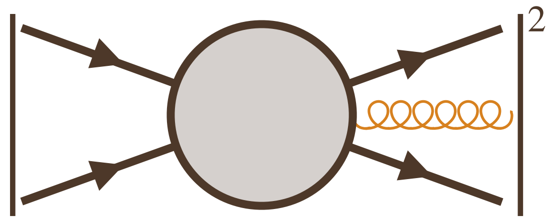


$$= \boxed{+\frac{c_2}{\epsilon^2}}\boxed{+\frac{c_1}{\epsilon}} + c_0$$

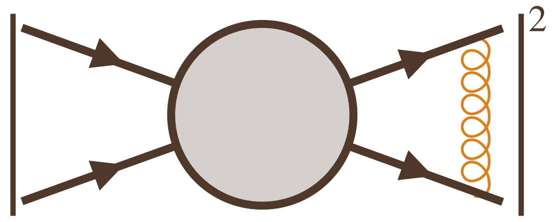
$\int d^{(d)}\phi$



$$= \boxed{-\frac{c_2}{\epsilon^2}}\boxed{-\frac{c_1}{\epsilon}} + \tilde{c}_0$$

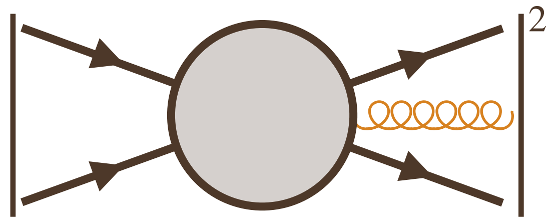
$= 0$ $= 0$

The cancellation is **ALWAYS EXACT**



$$= +1.1235813213455 + c_0$$

$\int d^{(4)}\phi$



$$= -1.1235813213454 + \tilde{c}_0$$

$= 10^{-13}$

The cancellation is **ALMOST EXACT**
We are happy!

ANALYTIC VERSUS NUMERICAL

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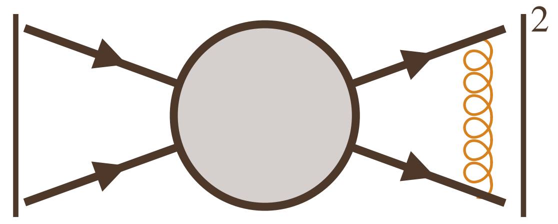

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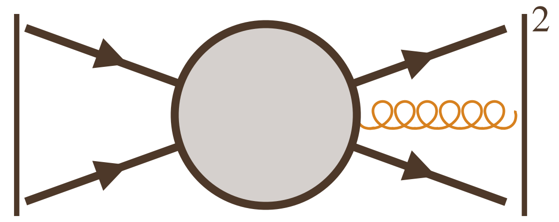
$$\int d^{(d)}\phi$$



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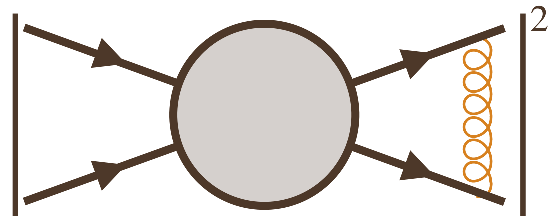

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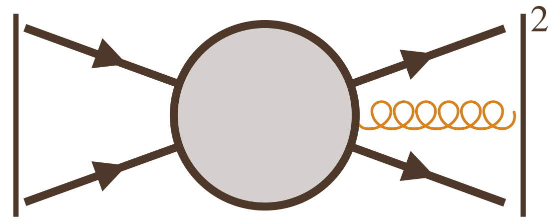
$$\int d^{(4)}\phi$$

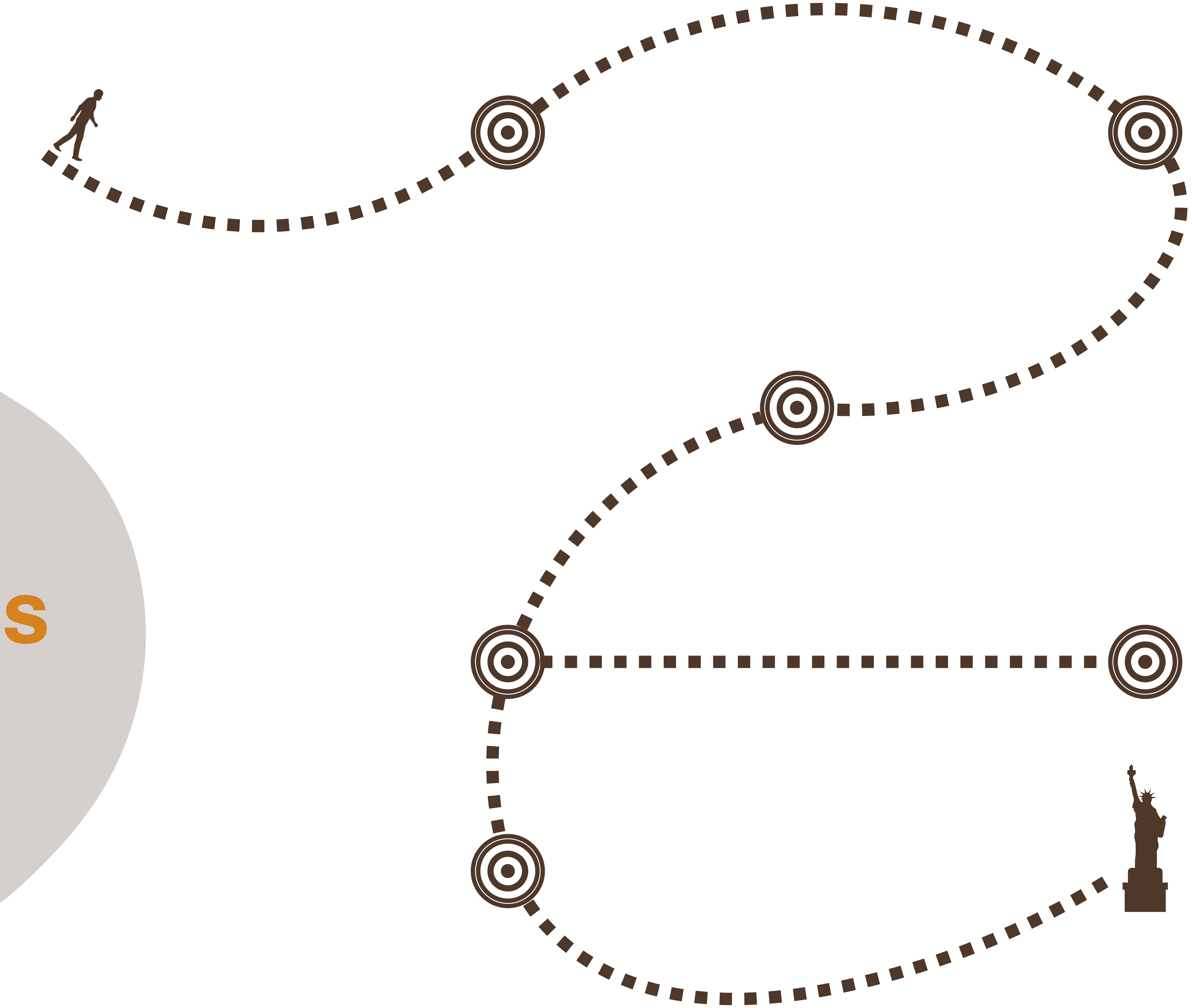


$$= -1.1235812000000 + \tilde{c}_0$$

$$= 10^{-7}$$

The cancellation is
IMPERFECT
How do we feel?

WHY WE STUDY $P + P \rightarrow X + N$ gluons AT NNLO



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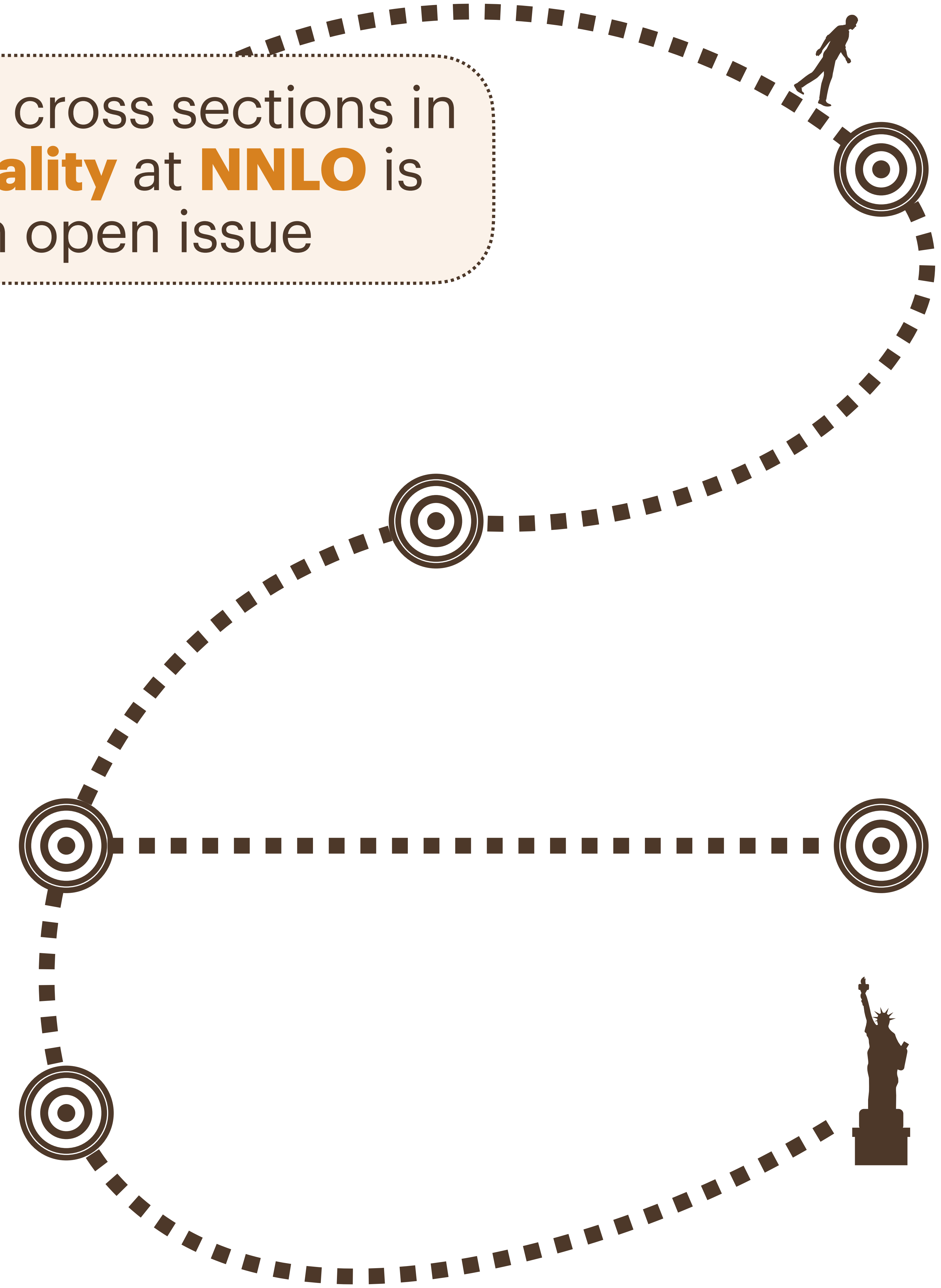

Computing cross sections in **full generality** at **NNLO** is still an open issue

WHY WE STUDY
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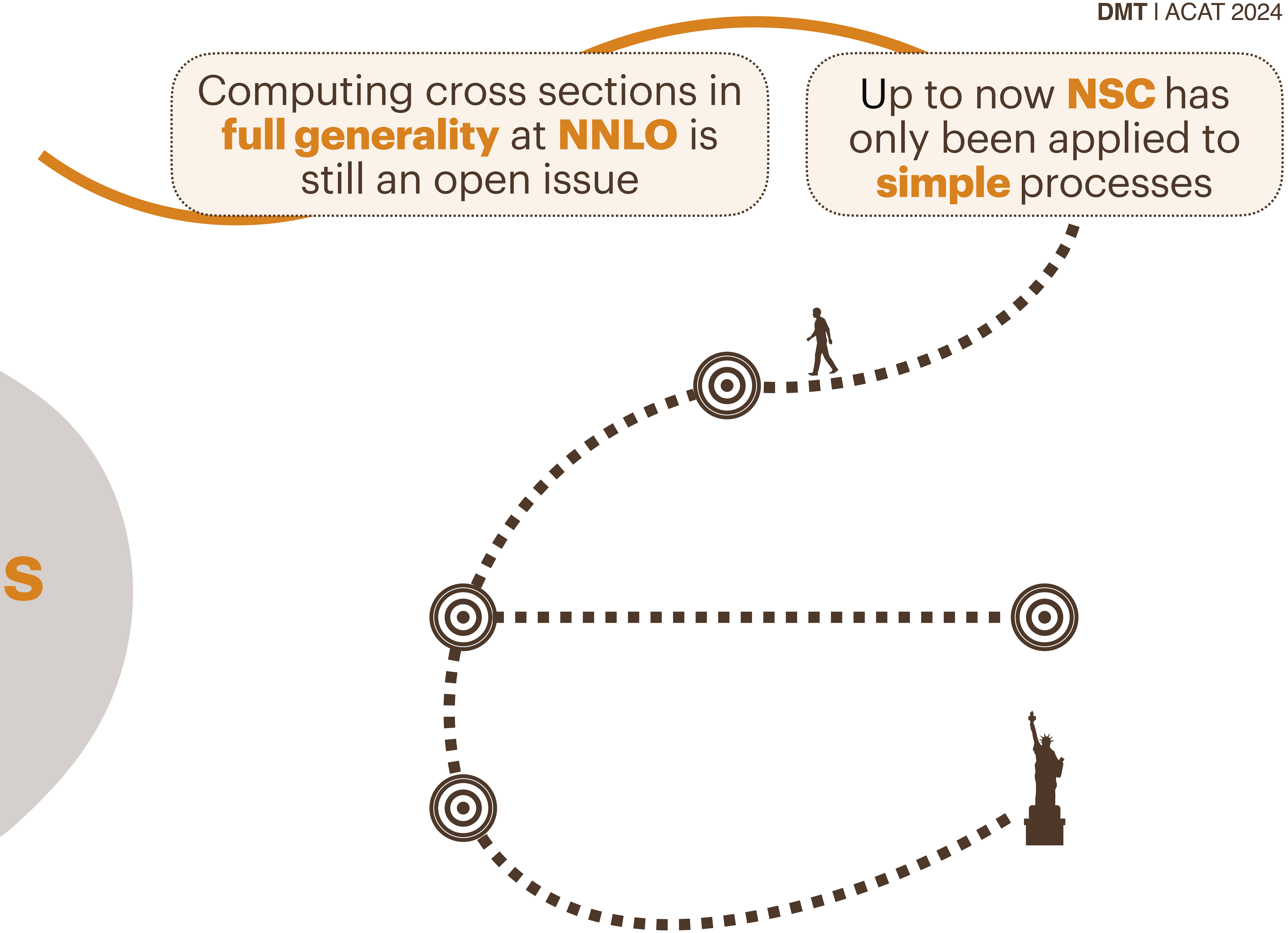
AT NNLO



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Up to now **NSC** has only been applied to **simple** processes

WHY WE STUDY P + P → X + N gluons AT NNLO

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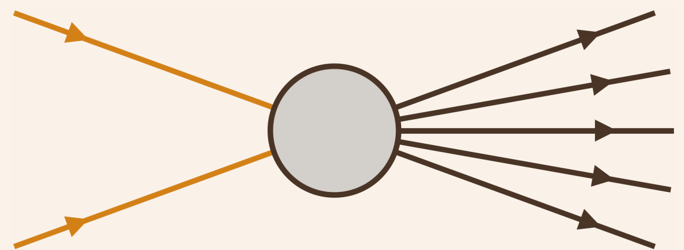
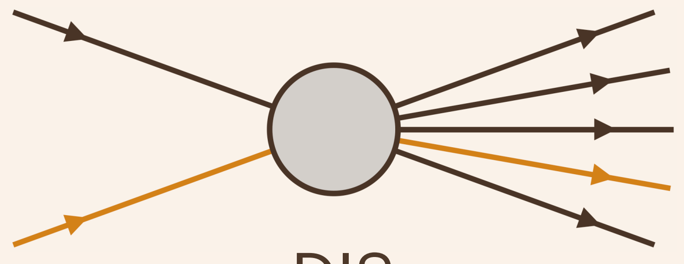
Up to now **NSC** has only been applied to **simple** processes

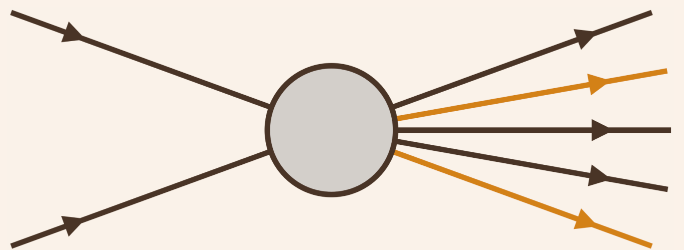
Simple = limited number of hard partons



Drell-Yan
[Caola, Melnikov, Röntsch '19]



DIS
[Asteriadis, Caola, Melnikov, Röntsch '19]



Higgs decay
[Caola, Melnikov, Röntsch '19]

WHY WE STUDY
P + P → X + N gluons
AT NNLO

Computing cross sections in **full generality** at **NNLO** is still an open issue

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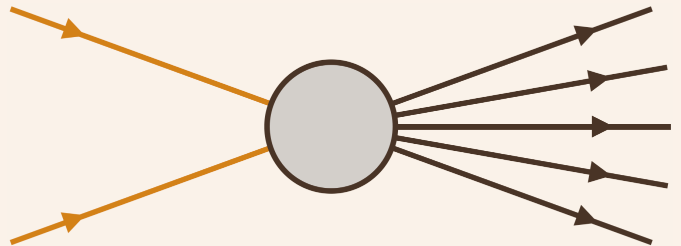

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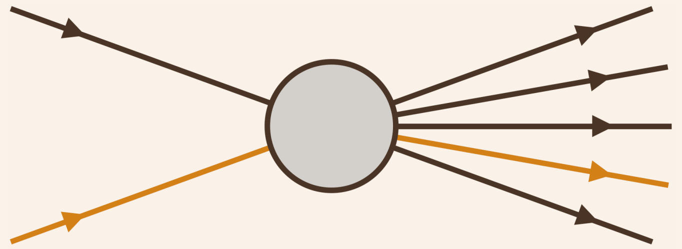

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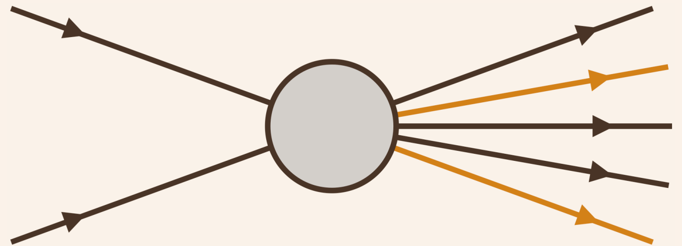

Higgs decay
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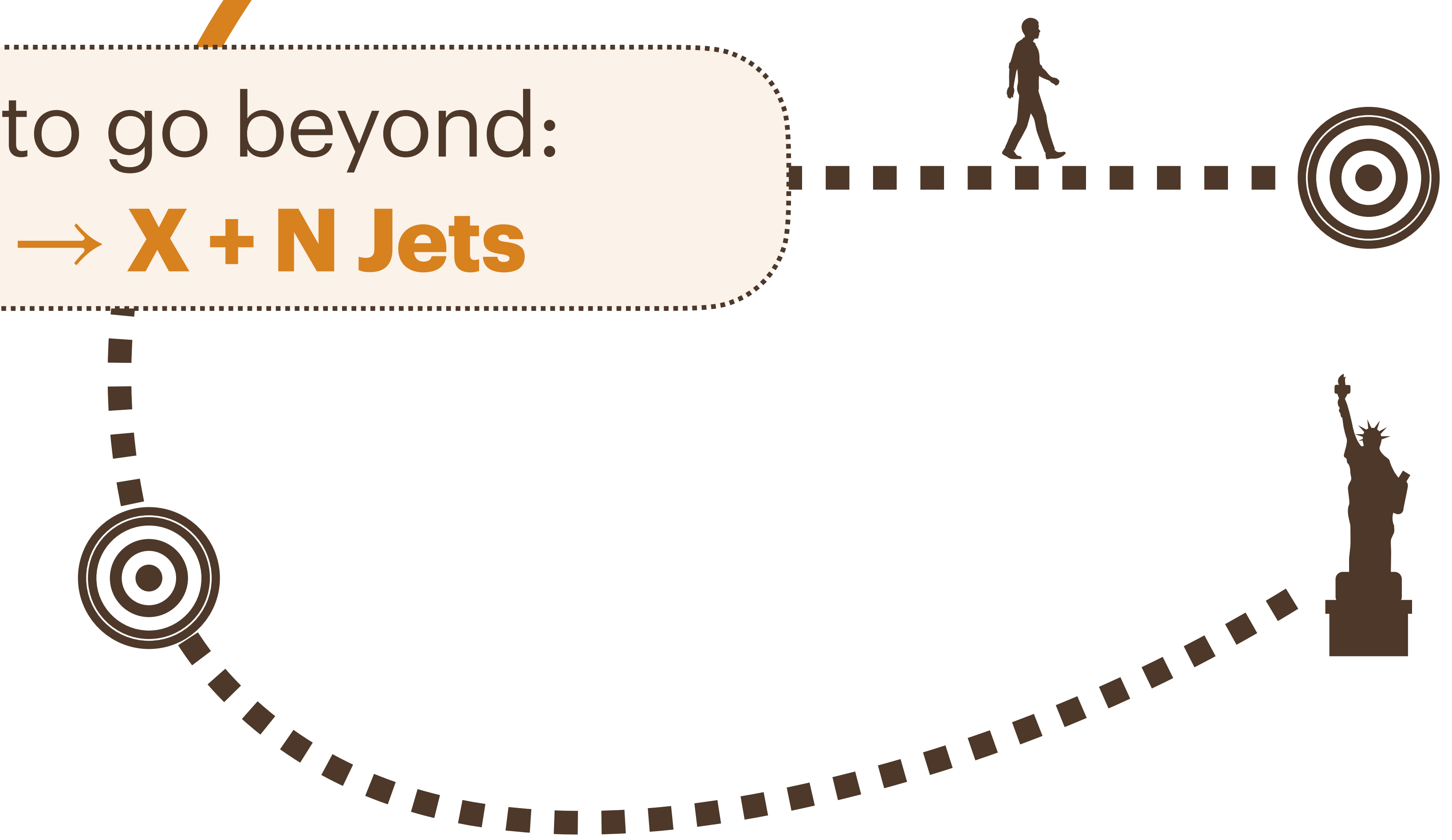
Need to go beyond:
P + P → X + N Jets



WHY WE STUDY
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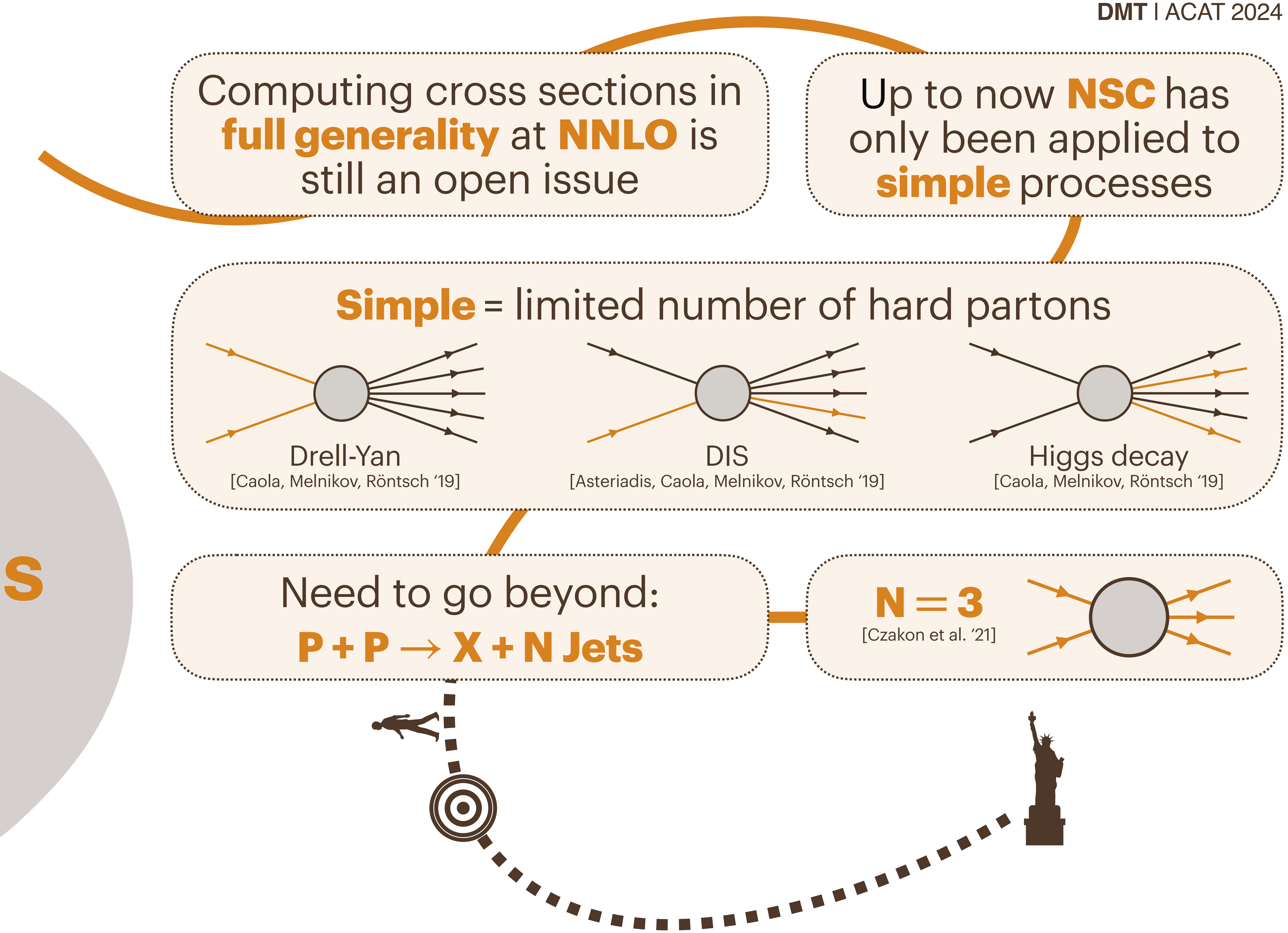
AT NNLO



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$N = 3$
[Czakon et al. '21]

WHY WE STUDY P + P → X + N gluons AT NNLO

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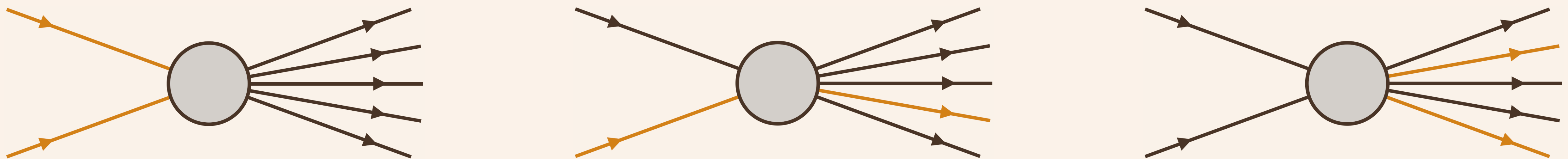

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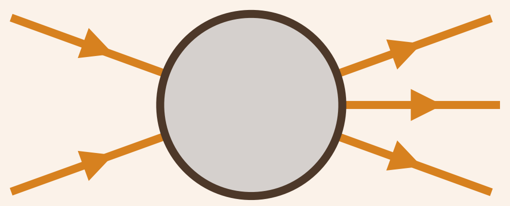
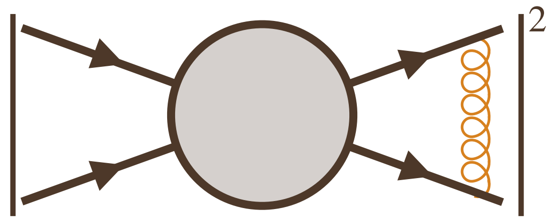


What is a good prototype of the problem?
P + P → X + N gluons

This talk!

RECURRING OPERATORS AT NLO

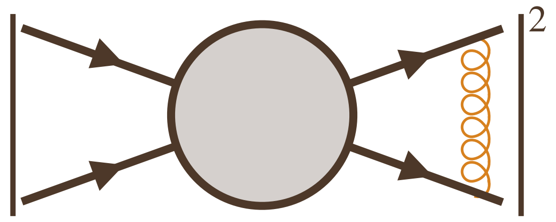
✓ **Virtual corrections $\mathrm{d}\hat{\sigma}^{\mathrm{V}}$**: the IR content of virtual amplitudes is well-known [Catani '98]



$$= + \frac{c_2}{\epsilon^2} + \frac{c_1}{\epsilon} + c_0$$

RECURRING OPERATORS AT NLO

Virtual corrections $\mathrm{d}\hat{\sigma}^{\mathrm{V}}$: the IR content of virtual amplitudes is well-known [Catani '98]



$$\left|\ \cdot\ \right|^2 = \boxed{+\frac{c_2}{\epsilon^2}+\frac{c_1}{\epsilon}} + c_0$$

$$\boldsymbol{I}_{\mathrm{V}}(\boldsymbol{\epsilon}) = \bar{I}_1(\epsilon) + \bar{I}_1^{\dagger}(\epsilon)$$

$$\bar{I}_1(\epsilon) = \frac{1}{2}\sum_{i\neq j}^{\boxed{Np}}\left(\frac{1}{\epsilon^2}+\frac{\gamma_i/\boldsymbol{T}_i^2}{\epsilon}\right)\left(-\frac{\mu^2}{s_{ij}}\right)^{\epsilon}\boxed{(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)}$$

$$N_p = N+2$$

Color Correlations

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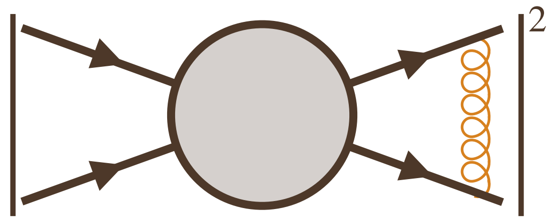

$$= \boxed{+\frac{c_2}{\epsilon^2}+\frac{c_1}{\epsilon}} + c_0$$

$$I_{\mathrm{V}}(\epsilon) = \bar{I}_1(\epsilon) + \bar{I}_1^{\dagger}(\epsilon)$$

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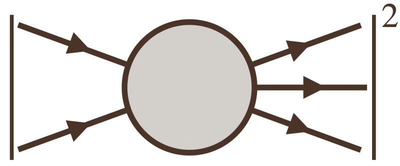
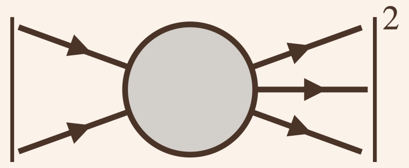
Color Correlations

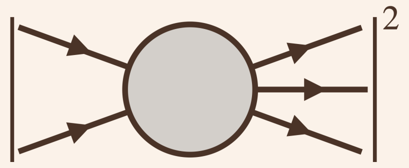
✓ **Real corrections $\mathrm{d}\hat{\sigma}^{\mathrm{R}}$**: we would like something similar



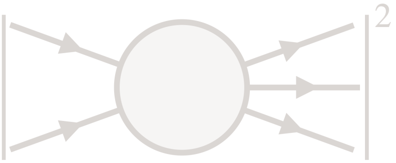
$= S$



$+ (1-S)C$



$+ (1-S)(1-C)$



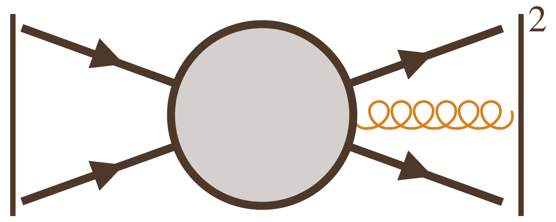
Making use of **NSC** formalism to regularize this divergences we obtain [Caola, Melnikov, Röntsch '17]

$$\mathrm{d}\hat{\sigma}^{\mathrm{R}} = \underbrace{\left\langle S_{\mathfrak{m}}F_{\mathrm{LM}}(\mathfrak{m})\right\rangle}_{\substack{\text{Soft term}\\ [S_{\mathfrak{m}}:\, E_{\mathfrak{m}}\to 0]}} + \underbrace{\sum_{i=1}^{N_p}\left\langle(1-S_{\mathfrak{m}})C_{i\mathfrak{m}}\Delta^{(\mathfrak{m})}F_{\mathrm{LM}}(\mathfrak{m})\right\rangle}_{\substack{\text{Hard-Collinear term}\\ [C_{i\mathfrak{m}}:\, \theta_{i\mathfrak{m}}\to 0]}} + \left\langle \mathcal{O}_{\mathrm{NLO}}\,\Delta^{(\mathfrak{m})}F_{\mathrm{LM}}(\mathfrak{m})\right\rangle$$

RECURRING OPERATORS AT NLO

✓ It turns out that the **SOFT TERM** can be written by means of an **operator** that, at least in principle, is very **close to $I_V(\epsilon)$**:

$$\int \mathrm{d}^{(d)}\phi \; S \left| \quad \right|^2$$



$$I_S(\epsilon) = -\frac{(2E_{\max}/\mu)^{-2\epsilon}}{\epsilon^2}\sum_{i\neq j}^{\boxed{N_p}} \eta_{ij}^{-\epsilon} K_{ij} \boxed{(\boldsymbol{T}_i \cdot \boldsymbol{T}_j)}$$

Color Correlations

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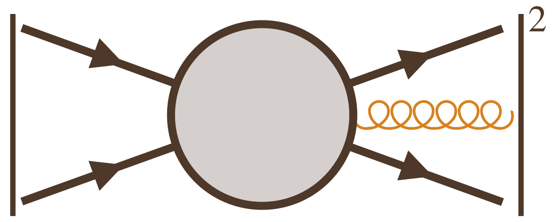

$$\int \mathrm{d}^{(d)}\phi \; S \left| \quad \right|^2 \quad \Rightarrow \quad I_{\mathrm{S}}(\epsilon) = -\frac{(2E_{\mathrm{max}}/\mu)^{-2\epsilon}}{\epsilon^2} \sum_{i\neq j}^{N_p} \eta_{ij}^{-\epsilon} K_{ij} \left(\boldsymbol{T}_i \cdot \boldsymbol{T}_j\right)$$

✓ Combination of $I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon)$:

$$I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon) = -\sum_{i=1}^{N_p} \frac{1}{\epsilon}\left(2\boldsymbol{T}_i^2\, L_i + \gamma_i\right) + \mathcal{O}(\epsilon^0)$$

$L_i = \log\left(E_{\mathrm{max}}/E_i\right)$
$\gamma_q = 3/2\, C_F$
$\gamma_g = \beta_0$

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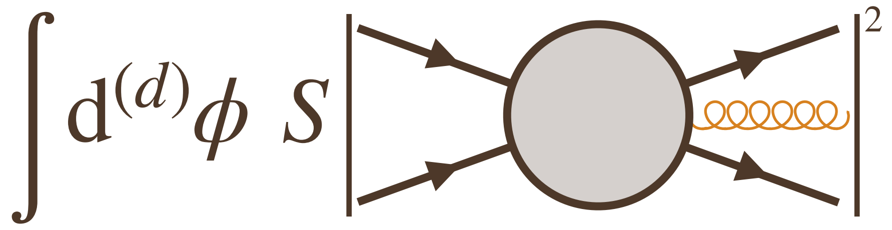

$$\Rightarrow \qquad I_{\mathrm{S}}(\epsilon) = -\frac{(2E_{\mathrm{max}}/\mu)^{-2\epsilon}}{\epsilon^2}\sum_{i\neq j}^{N_p} \eta_{ij}^{-\epsilon} K_{ij}\,(\boldsymbol{T}_i \cdot \boldsymbol{T}_j)$$

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- the pole of $\mathcal{O}(\epsilon^{-2})$ **vanishes**

RECURRING OPERATORS AT **NLO**

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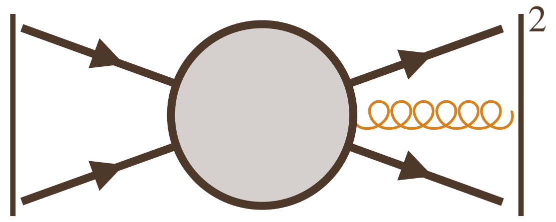
$\int \mathrm{d}^{(d)}\phi\ S$



⇨

$$I_{\mathrm{S}}(\epsilon) = -\frac{(2E_{\max}/\mu)^{-2\epsilon}}{\epsilon^2}\sum_{i\neq j}^{N_p}\eta_{ij}^{-\epsilon}K_{ij}\boxed{(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)}$$

- ✓ Combination of $I_{\mathrm{V}}(\epsilon)+I_{\mathrm{S}}(\epsilon)$:

NO Color Correlations

$$I_{\mathrm{V}}(\epsilon)+I_{\mathrm{S}}(\epsilon) = -\sum_{i=1}^{N_p}\boxed{\frac{1}{\epsilon}\left(2\boldsymbol{T}_i^2\,L_i+\gamma_i\right)}+\mathcal{O}(\epsilon^0)$$

$L_i = \log\left(E_{\max}/E_i\right)$
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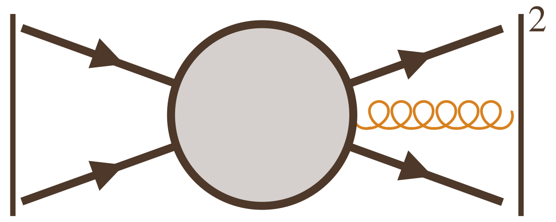
$$\int \mathrm{d}^{(d)}\phi \; S \left| \quad \right|^2$$



$$\Rightarrow \quad I_{\mathrm{S}}(\epsilon) = -\frac{(2E_{\max}/\mu)^{-2\epsilon}}{\epsilon^2} \sum_{i \neq j}^{N_p} \eta_{ij}^{-\epsilon} K_{ij} \left(\boldsymbol{T}_i \cdot \boldsymbol{T}_j\right)$$

✓ Combination of $I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon)$:

$$I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon) = -\sum_{i=1}^{\boxed{N_p}} \frac{1}{\epsilon}\left(2\boldsymbol{T}_i^2 \, L_i + \gamma_i\right) + \mathcal{O}(\epsilon^0)$$

$L_i = \log\left(E_{\max}/E_i\right)$
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- the pole of $\mathcal{O}(\epsilon^{-2})$ **vanishes**
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- **trivially dependent on** the number of hard partons N_p

RECURRING OPERATORS AT NLO

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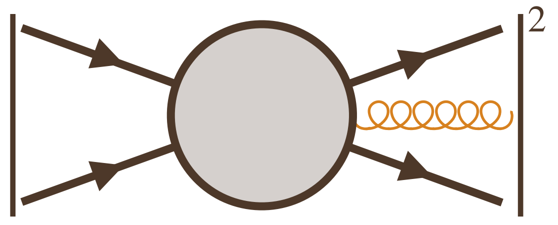
$\int \mathrm{d}^{(d)}\phi \; S$



$\Rightarrow$

$$I_{\mathrm{S}}(\epsilon) = -\frac{(2E_{\max}/\mu)^{-2\epsilon}}{\epsilon^2} \sum_{i\neq j}^{N_p} \eta_{ij}^{-\epsilon} K_{ij} \left(\boldsymbol{T}_i \cdot \boldsymbol{T}_j\right)$$

✓ Combination of $I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon)$:

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- **trivially dependent on** the number of hard partons N_p

THERE STILL IS A MISSING INGREDIENT

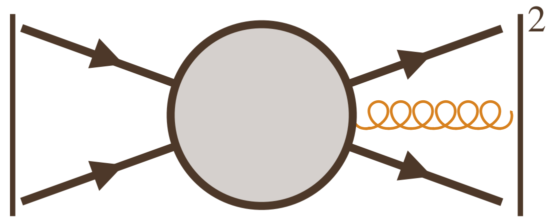
RECURRING OPERATORS AT NLO

$$I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon) = -\sum_{i=1}^{N_p} \frac{1}{\epsilon}\left(2\boldsymbol{T}_i^2\, L_i + \gamma_i\right) + \mathcal{O}(\epsilon^0)$$

$L_i = \log\left(E_{\max}/E_i\right)$
$\gamma_q = 3/2\, C_F$
$\gamma_g = \beta_0$

✓ *Last ingredient*: **hard-collinear term**. Some parts vanish against the DGLAP contribution, the remaining one **can be collected** within the **COLLINEAR OPERATOR**

$$\int \mathrm{d}^{(d)}\phi\ (1-S)\ C \left|\quad\right|^2$$



⇨

$$I_{\mathrm{C}}(\epsilon) = \sum_{i=1}^{N_p} \frac{\Gamma_{i,f_i}}{\epsilon}$$

RECURRING OPERATORS AT NLO

$$I_{\mathrm{V}}(\epsilon)+I_{\mathrm{S}}(\epsilon) = \boxed{-\sum_{i=1}^{N_p}\frac{1}{\epsilon}\left(2\boldsymbol{T}_i^2\,L_i+\gamma_i\right)} + \mathcal{O}(\epsilon^0)$$

$L_i = \log\left(E_{\max}/E_i\right)$
$\gamma_q = 3/2\,C_F$
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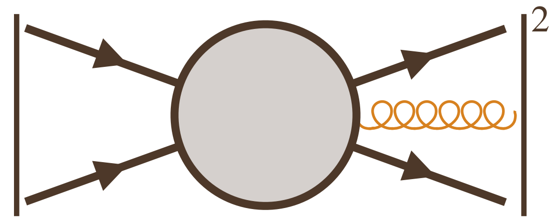
✓ *Last ingredient*: **hard-collinear term**. Some parts vanish against the DGLAP contribution, the remaining one **can be collected** within the **COLLINEAR OPERATOR**

$$\int \mathrm{d}^{(d)}\phi\ (1-S)\ C$$



$$I_{\mathrm{C}}(\epsilon) = \boxed{\sum_{i=1}^{N_p}\frac{\Gamma_{i,f_i}}{\epsilon}}$$

Expand in Series!

$$I_{\mathrm{C}}(\epsilon) = \boxed{+\sum_{i=1}^{N_p}\frac{1}{\epsilon}\left(2\boldsymbol{T}_i^2\,L_i+\gamma_i\right)} + \mathcal{O}(\epsilon^0)$$

$I_{\mathrm{C}}(\epsilon)$ cancels perfectly the pole of $\mathcal{O}(\epsilon^{-1})$ left by $I_{\mathrm{V}}(\epsilon)+I_{\mathrm{S}}(\epsilon)$

RECURRING OPERATORS AT NLO

$$I_{\mathrm{V}}(\epsilon)+I_{\mathrm{S}}(\epsilon) = -\sum_{i=1}^{N_p}\frac{1}{\epsilon}\left(2\boldsymbol{T}_i^2\,L_i+\gamma_i\right)+\mathcal{O}(\epsilon^0)$$

$L_i = \log\left(E_{\max}/E_i\right)$
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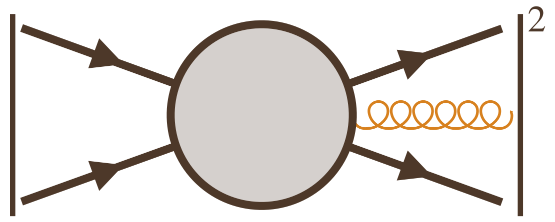
✓ *Last ingredient*: **hard-collinear term**. Some parts vanish against the DGLAP contribution, the remaining one **can be collected** within the **COLLINEAR OPERATOR**

$$\int d^{(d)}\phi\;(1-S)\,C\left|\quad\right|^2$$



$$\Rightarrow\qquad I_{\mathrm{C}}(\epsilon)=\sum_{i=1}^{N_p}\frac{\Gamma_{i,f_i}}{\epsilon}$$

$$I_{\mathrm{C}}(\epsilon) = +\sum_{i=1}^{N_p}\frac{1}{\epsilon}\left(2\boldsymbol{T}_i^2\,L_i+\gamma_i\right)+\mathcal{O}(\epsilon^0)$$

$I_{\mathrm{C}}(\epsilon)$ cancels perfectly the pole of $\mathcal{O}(\epsilon^{-1})$ left by $I_{\mathrm{V}}(\epsilon)+I_{\mathrm{S}}(\epsilon)$

TOTAL OPERATOR

$$I_{\mathrm{T}}(\epsilon) = I_{\mathrm{V}}(\epsilon)+I_{\mathrm{S}}(\epsilon)+I_{\mathrm{C}}(\epsilon)$$

- pole free
- fully general w.r.t. N_p

WHAT HAPPENS AT NNLO?

$$\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}} = \mathrm{d}\hat{\sigma}^{\mathrm{VV}} + \mathrm{d}\hat{\sigma}^{\mathrm{RV}} + \mathrm{d}\hat{\sigma}^{\mathrm{RR}} + \mathrm{d}\hat{\sigma}^{\mathrm{pdf}}$$

WHAT HAPPENS AT NNLO?

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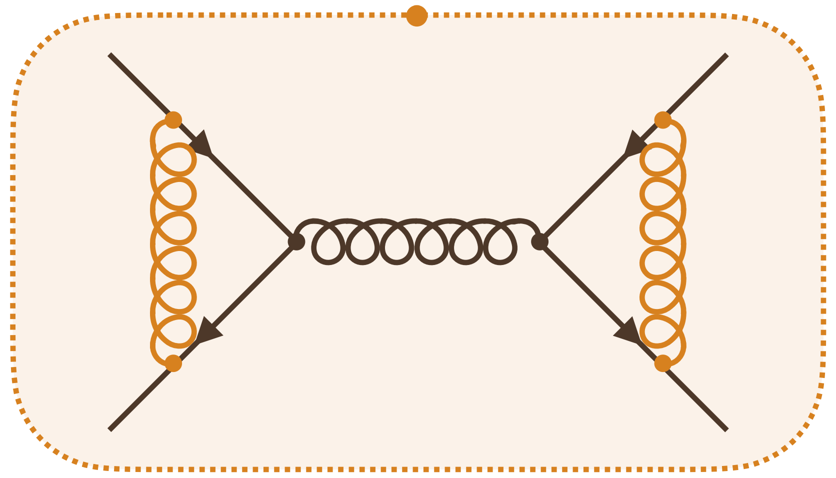
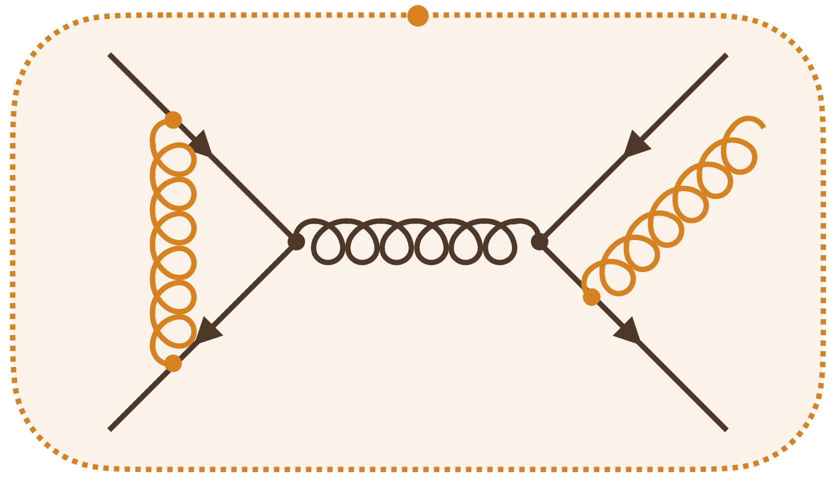


DOUBLE VIRTUAL

WHAT HAPPENS AT NNLO?

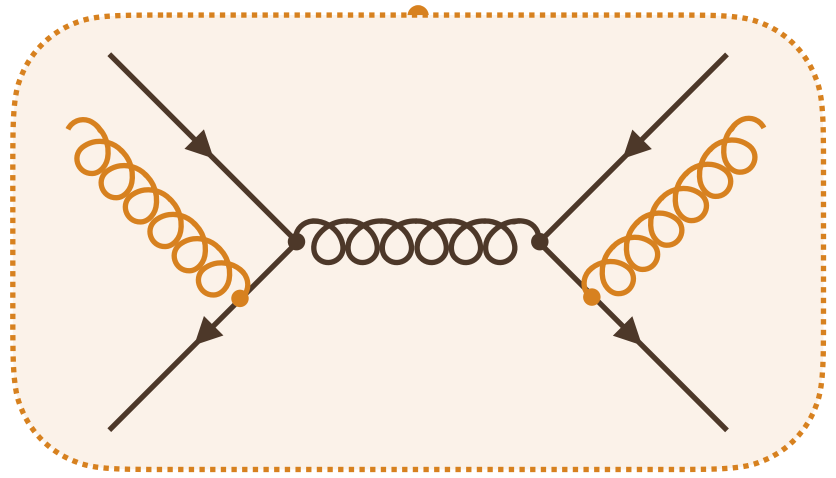
$$d\hat{\sigma}^{\mathrm{NNLO}} = d\hat{\sigma}^{\mathrm{VV}} + \boxed{d\hat{\sigma}^{\mathrm{RV}}} + d\hat{\sigma}^{\mathrm{RR}} + d\hat{\sigma}^{\mathrm{pdf}}$$



REAL VIRTUAL

WHAT HAPPENS AT NNLO?

$$d\hat{\sigma}^{\mathrm{NNLO}} = d\hat{\sigma}^{\mathrm{VV}} + d\hat{\sigma}^{\mathrm{RV}} + \boxed{d\hat{\sigma}^{\mathrm{RR}}} + d\hat{\sigma}^{\mathrm{pdf}}$$



DOUBLE REAL

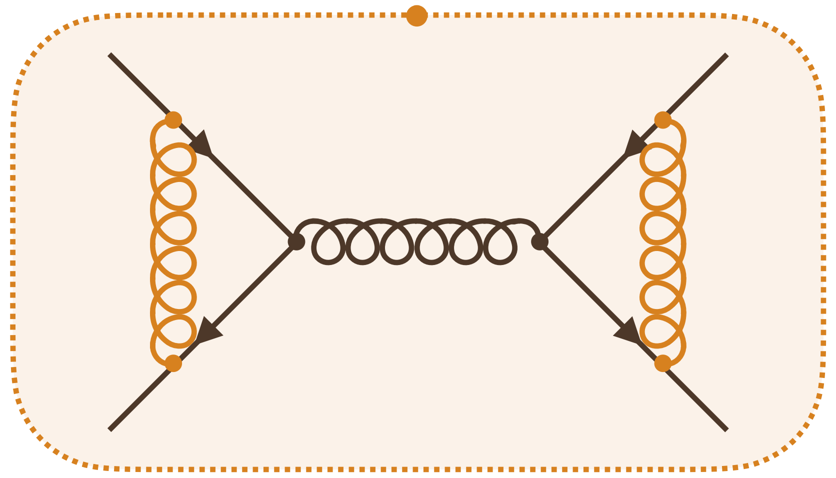
WHAT HAPPENS AT NNLO?

$$\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}} = \mathrm{d}\hat{\sigma}^{\mathrm{VV}} + \mathrm{d}\hat{\sigma}^{\mathrm{RV}} + \mathrm{d}\hat{\sigma}^{\mathrm{RR}} + \boxed{\mathrm{d}\hat{\sigma}^{\mathrm{pdf}}}$$

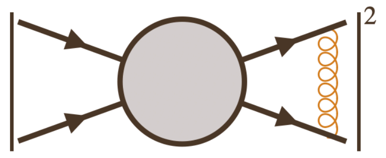
PDFs RENORM.

WHAT HAPPENS AT **NNLO**?

$$\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}} = \boxed{\mathrm{d}\hat{\sigma}^{\mathrm{VV}}} + \mathrm{d}\hat{\sigma}^{\mathrm{RV}} + \mathrm{d}\hat{\sigma}^{\mathrm{RR}} + \mathrm{d}\hat{\sigma}^{\mathrm{pdf}}$$



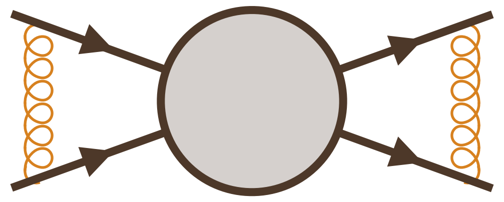
DOUBLE VIRTUAL



$$= \boxed{+ \frac{c_2}{\epsilon^2} + \frac{c_1}{\epsilon}} + c_0$$

$$\boldsymbol{I_{\mathrm{V}}(\epsilon)} = \bar{I}_1(\epsilon) + \bar{I}_1^{\dagger}(\epsilon)$$

We expect the **same** to happen for $\mathbf{d\hat{\sigma}^{VV}}$

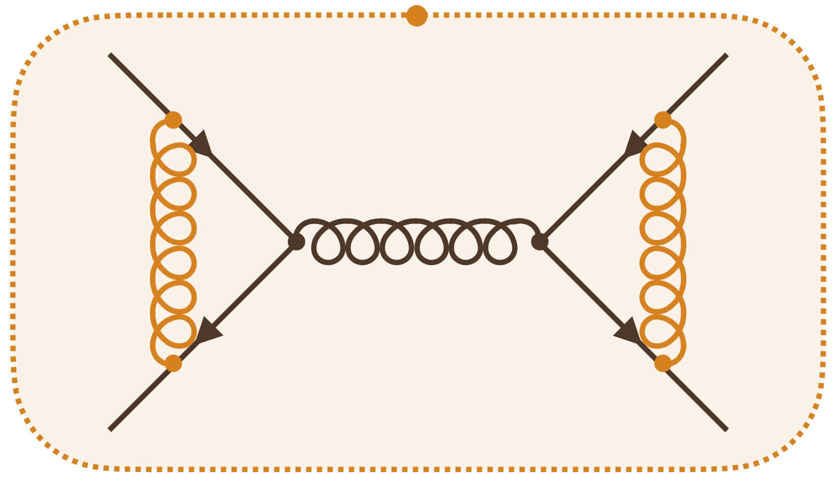


$$= \boxed{+ \frac{c_4}{\epsilon^4} + \frac{c_3}{\epsilon^3} + \frac{c_2}{\epsilon^2} + \frac{c_1}{\epsilon}} + c_0$$

$$\sim \boldsymbol{I_{\mathrm{V}}^2(\epsilon)}$$

WHAT HAPPENS AT **NNLO**?

$$d\hat{\sigma}^{\mathrm{NNLO}} = \boxed{d\hat{\sigma}^{\mathrm{VV}}} + d\hat{\sigma}^{\mathrm{RV}} + d\hat{\sigma}^{\mathrm{RR}} + d\hat{\sigma}^{\mathrm{pdf}}$$



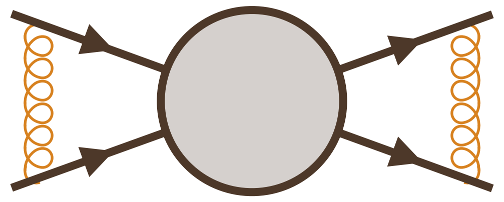
DOUBLE VIRTUAL

$$\bar{I}_1(\epsilon) = \frac{1}{2}\sum_{i\neq j}^{Np}\left(\frac{1}{\epsilon^2}+\frac{\gamma_i/\boldsymbol{T}_i^2}{\epsilon}\right)\left(-\frac{\mu^2}{s_{ij}}\right)^{\epsilon}\boxed{(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)}$$

$$N_p = N+2$$

Color Correlations

We expect the **same** to happen for $d\hat{\sigma}^{\mathrm{VV}}$



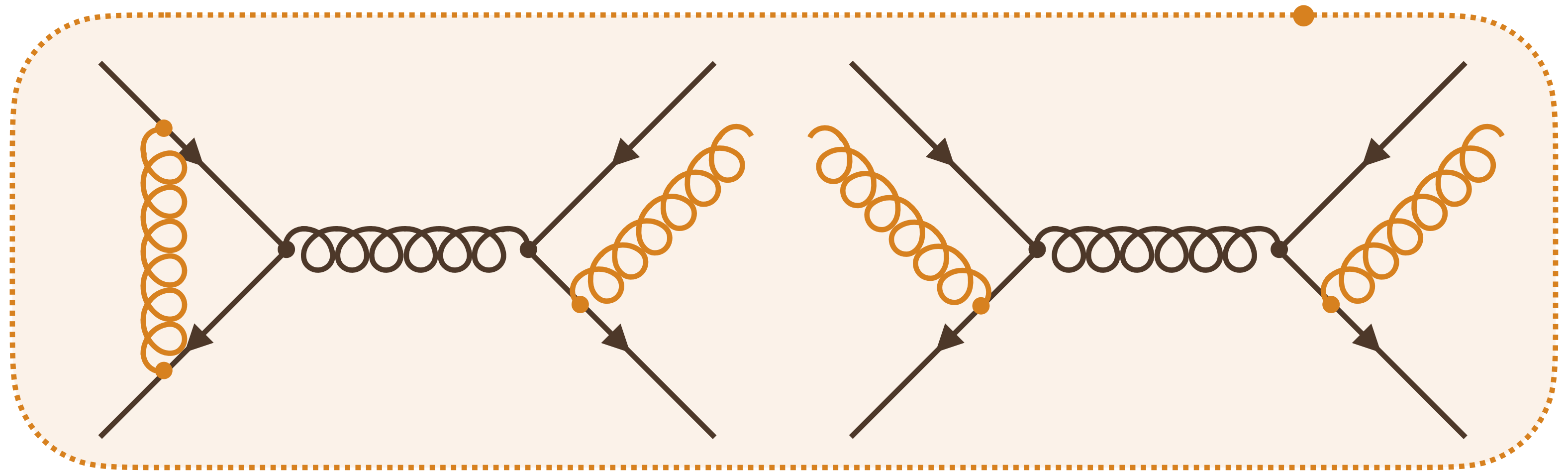
$$= \boxed{+\frac{c_4}{\epsilon^4}+\frac{c_3}{\epsilon^3}+\frac{c_2}{\epsilon^2}+\frac{c_1}{\epsilon}} + c_0$$

$$\sim \boldsymbol{I}_{\mathrm{V}}^2(\epsilon) \propto (\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,(\boldsymbol{T}_k\cdot\boldsymbol{T}_l)$$

QUARTIC Color Correlations

WHAT HAPPENS AT **NNLO**?

$$\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}} = \mathrm{d}\hat{\sigma}^{\mathrm{VV}} + \boxed{\mathrm{d}\hat{\sigma}^{\mathrm{RV}} + \mathrm{d}\hat{\sigma}^{\mathrm{RR}}} + \mathrm{d}\hat{\sigma}^{\mathrm{pdf}}$$

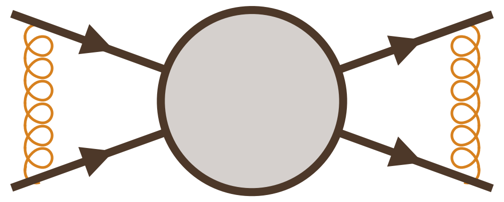


REAL VIRTUAL

DOUBLE REAL

Mhh... I must expect $(\boldsymbol{T}_i \cdot \boldsymbol{T}_j)(\boldsymbol{T}_k \cdot \boldsymbol{T}_l)$ also in $\mathbf{d}\hat{\boldsymbol{\sigma}}^{\mathbf{RV}}$ and $\mathbf{d}\hat{\boldsymbol{\sigma}}^{\mathbf{RR}}$

We expect the **same** to happen for $\mathbf{d}\hat{\boldsymbol{\sigma}}^{\mathbf{VV}}$



$$= \boxed{+\frac{c_4}{\epsilon^4} + \frac{c_3}{\epsilon^3} + \frac{c_2}{\epsilon^2} + \frac{c_1}{\epsilon}} + c_0$$

$$\sim \boldsymbol{I}_{\mathrm{V}}^2(\epsilon) \propto (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)(\boldsymbol{T}_k \cdot \boldsymbol{T}_l)$$

QUARTIC Color Correlations

And now... **HOW** can I **CANCEL** the poles of all these objects???

QUARTIC
COLOR
CORRELAT.

Here it is what we find [Devoto, Melnikov, Röntsch, Signorile-Signorile, **D.M.T.**, 2310.17598]

$$Y_{\mathrm{VV}} = \frac{[\alpha_s]^2}{2} \left\langle M_0 \middle| I_{\mathrm{V}}^2 \middle| M_0 \right\rangle + \dots$$

$$Y_{\mathrm{RR}}^{(\mathrm{ss})} = \frac{[\alpha_s]^2}{2} \langle M_0 | I_{\mathrm{S}}^2 | M_0 \rangle + \dots$$

$$Y_{\mathrm{RR}}^{(\mathrm{shc})} = [\alpha_s]^2 \langle M_0 | I_{\mathrm{S}} \, I_{\mathrm{C}} | M_0 \rangle + \dots$$

$$Y_{\mathrm{RR}}^{(\mathrm{cc})} = \frac{[\alpha_s]^2}{2} \langle M_0 | I_{\mathrm{C}}^2 | M_0 \rangle + \dots$$

$$Y_{\mathrm{RV}}^{(\mathrm{s})} = \frac{[\alpha_s]^2}{2} \left\langle M_0 \middle| I_{\mathrm{S}} \, I_{\mathrm{V}} + I_{\mathrm{V}} \, I_{\mathrm{S}} \middle| M_0 \right\rangle + \dots$$

$$Y_{\mathrm{RV}}^{(\mathrm{shc})} = [\alpha_s]^2 \left\langle M_0 \middle| I_{\mathrm{V}} \, I_{\mathrm{C}} \middle| M_0 \right\rangle + \dots$$

QUARTIC COLOR CORRELAT.

Here it is what we find [Devoto, Melnikov, Röntsch, Signorile-Signorile, **D.M.T.**, 2310.17598]

$$Y_{\rm VV} = \frac{[\alpha_s]^2}{2} \left\langle M_0 \middle| I_{\rm V}^2 \middle| M_0 \right\rangle + \ldots$$

$$\boldsymbol{Y_{\rm RR}^{(\rm ss)} = \frac{[\alpha_s]^2}{2} \langle M_0 \,|\, I_{\rm S}^2 \,|\, M_0 \rangle + \ldots}$$

$$Y_{\rm RR}^{(\rm shc)} = [\alpha_s]^2 \langle M_0 \,|\, I_{\rm S}\, I_{\rm C} \,|\, M_0 \rangle + \ldots$$

$$Y_{\rm RR}^{(\rm cc)} = \frac{[\alpha_s]^2}{2} \langle M_0 \,|\, I_{\rm C}^2 \,|\, M_0 \rangle + \ldots$$

$$Y_{\rm RV}^{(\rm s)} = \frac{[\alpha_s]^2}{2} \left\langle M_0 \middle| I_{\rm S}\, I_{\rm V} + I_{\rm V}\, I_{\rm S} \middle| M_0 \right\rangle + \ldots$$

$$Y_{\rm RV}^{(\rm shc)} = [\alpha_s]^2 \left\langle M_0 \middle| I_{\rm V}\, I_{\rm C} \middle| M_0 \right\rangle + \ldots$$

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$$Y_{\mathrm{RV}}^{(\mathrm{shc})} = [\alpha_s]^2 \left\langle M_0 \middle| I_{\mathrm{V}}\, I_{\mathrm{C}} \middle| M_0 \right\rangle + \ldots$$

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$$Y_{\rm RV}^{\rm (shc)} = [\alpha_s]^2 \Big\langle M_0 \Big| I_{\rm V}\, I_{\rm C} \Big| M_0 \Big\rangle + \ldots$$

Once combined, these objects return

NB square of NLO

$$Y = \frac{[\alpha_s]^2}{2} \Big\langle M_0 \Big| \big[I_{\rm V} + I_{\rm S} + I_{\rm C}\big]^2 \Big| M_0 \Big\rangle + \ldots \equiv \boxed{\frac{[\alpha_s]^2}{2} \langle \boldsymbol{M_0} \,|\, \boldsymbol{I_{\rm T}^2} \,|\, \boldsymbol{M_0} \rangle} + \ldots$$

QUARTIC COLOR CORRELAT.

The benefits of introducing these Catani-like operators:

Problem of **QUARTIC COLOR-CORRELATED** poles disappear, since everything is written in terms of $I_{\mathrm{T}}^2(\epsilon) \sim \mathcal{O}(\epsilon^0)$

NB square of NLO

$$Y = \frac{[\alpha_s]^2}{2}\left\langle M_0 \middle| \left[I_{\mathrm{V}} + I_{\mathrm{S}} + I_{\mathrm{C}}\right]^2 \middle| M_0 \right\rangle + \ldots \equiv \boxed{\frac{[\alpha_s]^2}{2}\langle M_0 | I_{\mathrm{T}}^2 | M_0 \rangle} + \ldots$$

QUARTIC COLOR CORRELAT.

The benefits of introducing these Catani-like operators:

- Problem of **QUARTIC COLOR-CORRELATED** poles disappear, since everything is written in terms of $\boldsymbol{I_{\mathrm{T}}^2(\epsilon) \sim \mathcal{O}(\epsilon^0)}$

- $\boldsymbol{I_{\mathrm{T}}(\epsilon)}$ depends trivially on $\boldsymbol{N_p}$, so the result we got is **FULLY GENERAL** w.r.t. the number of final state gluons

NB square of NLO

$$Y = \frac{[\alpha_s]^2}{2}\left\langle M_0 \middle| \left[I_{\mathrm{V}} + I_{\mathrm{S}} + I_{\mathrm{C}}\right]^2 \middle| M_0 \right\rangle + \ldots \equiv \boxed{\frac{[\alpha_s]^2}{2}\langle \boldsymbol{M_0} | \boldsymbol{I_{\mathrm{T}}^2} | \boldsymbol{M_0} \rangle} + \ldots$$

QUARTIC COLOR CORRELAT.

The benefits of introducing these Catani-like operators:

- Problem of **QUARTIC COLOR-CORRELATED** poles disappear, since everything is written in terms of $I_{\mathrm{T}}^{2}(\epsilon) \sim \mathcal{O}(\epsilon^{0})$
- $I_{\mathrm{T}}(\epsilon)$ depends trivially on N_p, so the result we got is **FULLY GENERAL** w.r.t. the number of final state gluons
- We **DO NOT EXPLICITLY CALCULATE** all the **SUB-BLOCKS** of the process. Instead, we write each of these in terms of $I_{\mathrm{V}}(\epsilon)$, $I_{\mathrm{S}}(\epsilon)$ and $I_{\mathrm{C}}(\epsilon)$, then recombine them to get $I_{\mathrm{T}}(\epsilon)$. The **CANCELLATION OF THE POLES** takes place **AUTOMATICALLY**

NB square of NLO

$$Y = \frac{[\alpha_s]^2}{2}\left\langle M_0 \middle| \left[I_{\mathrm{V}} + I_{\mathrm{S}} + I_{\mathrm{C}}\right]^2 \middle| M_0 \right\rangle + \ldots \equiv \boxed{\frac{[\alpha_s]^2}{2}\langle M_0 | I_{\mathrm{T}}^2 | M_0 \rangle} + \ldots$$

CONCLUSIONS AND OUTLOOK

1 We find **recurring building blocks**, i.e. $I_{\mathrm{V}}(\epsilon)$, $I_{\mathrm{S}}(\epsilon)$, $I_{\mathrm{C}}(\epsilon)$ and $I_{\mathrm{T}}(\epsilon)$, which let us solve the problem of color-correlated poles

2 The **procedure** is (almost) entirely **process independent**

3 *Work in progress*: next step is a generalization to **asymmetric initial state** and **arbitrary final state**

4 *Work in progress*: implementation of the results in a **numerical code**

5 *Outlook*: application of the method to **pheno-studies**