

Is Quantum Computing energy efficient? An Investigation on a Quantum Annealer

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Abstract. The environmental impact of computing activities is starting to be acknowledged as relevant and several scientific initiatives and research lines are gathering momentum in the scientific community to identify and curb it. Governments, industries, and commercial businesses are now holding high expectations for quantum technologies as they have the potential to create greener and faster methods for information processing. The energy perspective of such technologies, however, has remained rather outside the scopes of current deployment strategies, which might limit future adoptions. In order to shed some light upon the interplay between classical/quantum computing and energy efficiency, we perform a benchmark study on the latest commercially available Quantum Annealing device by the company D-Wave Systems. In particular, we estimate the power consumption of this machine to run a quantum algorithm for Integer Factorization and analyze the outcome of such experiments.

1 Introduction

Awareness of the environmental impact of computing activities is gaining increasing recognition as a critical issue, prompting numerous scientific initiatives and research endeavors aimed at identifying and mitigating these effects. Governments, industries, and commercial enterprises are placing significant hopes on quantum technologies, which promise to offer greener and faster solutions for specific computational tasks. However, the energy consumption of these emerging technologies has not been adequately addressed in deployment strategies. Ignoring this crucial aspect could hinder the widespread adoption and long-term sustainability of quantum technologies. Therefore, it is important to integrate energy efficiency considerations into the development and implementation phases to ensure that quantum computing can fulfill its potential without environmental concerns. To empirically investigate the energy profile of the currently available quantum technology on a specific use-case, we pick a well-known NP-hard problem such as Integer Factorization. We implement an annealing-based quantum algorithm to solve the task and we estimate its energy consumption performing real experiments on the D-Wave Advantage quantum annealing machine.

2 Integer Factorization

Integer factorization is the task of decomposing a positive integer number N into two integer factors p and q . When the chosen number is big enough, no efficient classical factorization algorithm is known (the fastest being the “General Number Field Sieve Method” [1]), although it has not been formally proven it does not exist. The energy consumption of available algorithms, just like the solution time-scale, tends eventually to ∞ . Furthermore, if N happens to be a big semi-prime number (i.e. $N = p \cdot q$ where p and q are prime numbers) the factorization task is known to belong to the NP-hard complexity class of computational problems. This mathematical property is used, for instance, to develop secure key-encryption such as RSA.

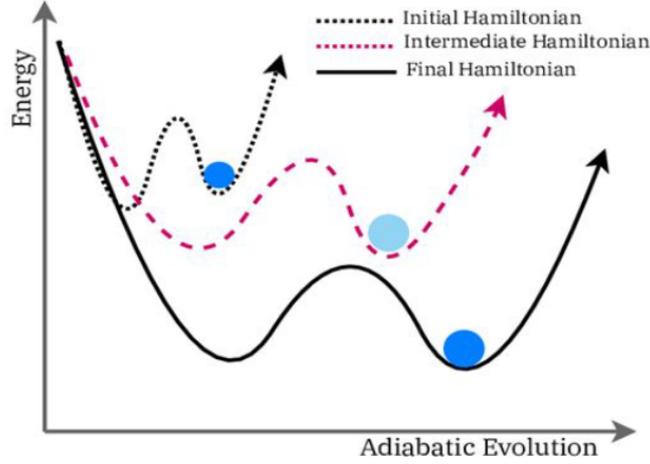


Figure 1: The adiabatic evolution allows the system to reach the ground state of the final Hamiltonian.

3 Quantum Annealing (QA)

Quantum annealing is a quantum-based computing technique employed to solve problems within the domain of Quadratic Unconstrained Binary Optimization (QUBO). The high-level idea of QA is based on the implementation of a quasi-adiabatic evolution of a quantum Hamiltonian system. The process begins by initializing the system in the ground state of a simple, easily solvable Hamiltonian. Over time, this Hamiltonian is gradually transformed into a more complex one, specifically designed to encode the optimal solution of the given QUBO problem into its ground state. According to the adiabatic theorem, if this transformation happens slowly enough, the system will stay in the ground state of the instantaneous Hamiltonian throughout the whole evolution (see Figure 1).

Consequently, at the end of the annealing process, the state of the system will be the final Hamiltonian ground state, corresponding to the optimal solution of the optimization problem of interest. More in detail, these are the fundamental steps of the QA protocol for optimization:

- The quantum system is prepared in the known ground state of an initial Ising Hamiltonian H_{init} .
- The given problem is formulated as a QUBO (formally equivalent to the Ising model) and the solution is encoded in the ground state of a final Hamiltonian H_{fin} .
- The quantum system evolution is controlled by the following time-dependent Hamiltonian:

$$H(t) = A(t)H_{\text{init}} + B(t)H_{\text{fin}} \quad (1)$$

where $A(t)$ and $B(t)$ define the annealing schedule. In particular, these functions will be chosen to have $A(t=0) = A_{\text{max}}, B(t=0) = 0$ and $A(t=T) = 0, B(t=T) = B_{\text{max}}$, with T being the annealing time.

- The solution of the QUBO problem is estimated by sampling the probability distribution corresponding to the ground state $|\psi\rangle$ of the final Hamiltonian ($H_{\text{fin}}|\psi\rangle = E^*|\psi\rangle$, where E^* is the minimum energy of the system).

4 Quantum Algorithm for Factorization

The quantum annealing approach for the Integer Factorization problem is presented and discussed by Jiang et al. and Mengoni et al.[2, 3]. It is a probabilistic algorithm based on the following steps:

- 1) Encode the integer numbers N, p, q into their binary representation as bitstring.
- 2) Compute the product $p \cdot q$ using the table method for bitstring multiplication (see Figure 2).
- 3) Write the objective function $O(p, q) = (N - p \cdot q)^2$ in the QUBO form $\sum_i \sum_j Q_{ij} x_i x_j = \vec{x}^T Q \vec{x}$, where Q is the QUBO matrix and \vec{x} is the vector of binary variables representing p, q , and n .

Columns	7	6	5	4	3	2	1	0
$\mathbf{p} \cdot \mathbf{q}$					1	p_2	p_1	1
				q_1	$p_2 q_1$	$p_1 q_1$	q_1	
			q_2	$p_2 q_2$	$p_1 q_2$	q_2		
		1	p_2	p_1	1			
Carries	c_{67}	c_{56}	c_{45}	c_{34}	c_{23}	c_{12}		
	c_{57}	c_{46}	c_{35}	c_{24}				
N	n_7	n_6	n_5	n_4	n_3	n_2	n_1	n_0

Figure 2: Multiplication table scheme used in the quantum algorithm implementation.

5 Methodology and Results

Once the problem is written in the QUBO form, we can run the algorithm accessing the Advantage quantum annealer, available through the D-Wave Leap cloud platform. To test the energy consumption of this quantum-based approach, we select 13 different Integer Factorization problem instances starting from 8-bits up to 32-bits numbers. For each problem instance, we run 100 experiments until success (i.e. correct identification of the p, q pair). At each experiment, the energy consumption E is estimated by:

$$E = k \cdot t_{\text{neal}} = k \cdot (\text{QPU}_{\text{time}} \cdot N_{\text{shots}})^1 \quad (2)$$

Following the official D-Wave System documentation, we set $k = 25 \text{ kW}$ as the maximal absorbed power and we estimated the total annealing time t_{neal} as the product of a single-shot access time QPU_{time} and the number of shots N_{shots} required to get the correct result. From the online monitoring dashboard of D-Wave Leap, it was possible to observe a $\text{QPU}_{\text{time}} = 16 \text{ ms}$ per shot.

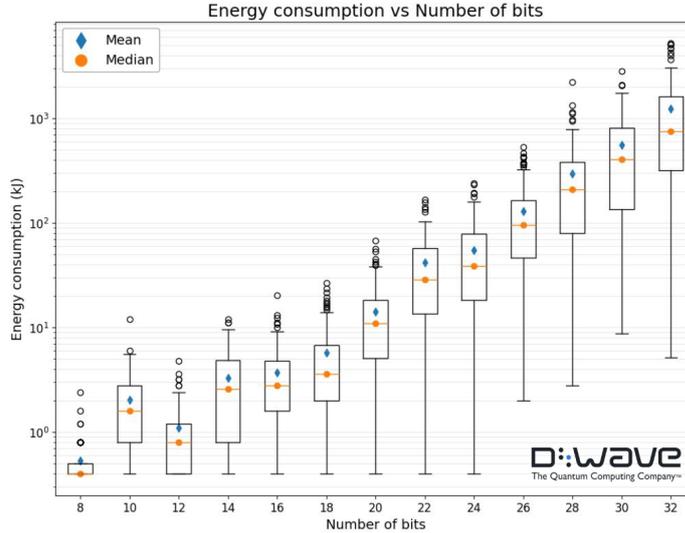


Figure 3: Distributions of the energy consumption as a function of the problem size. Each vertical box represents 100 successful experiments of each Integer Factorization problem instance.

In Figure 3 we show the distributions of energy consumption relative to the 100 quantum annealing successful runs executed for each Integer Factorization problem. As it can be seen, the mean and me-

¹A more comprehensive estimation would include also the energy required by the dilution refrigerator to cool the system down to its operational temperature ($T_{\text{ops}} \sim 15 \text{ mK}$). However, this information was not publicly available.

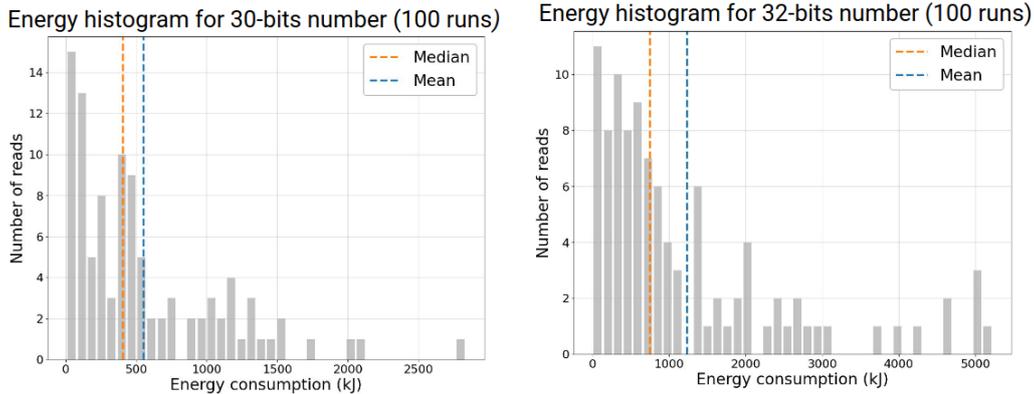


Figure 4: Energy consumption distribution over 100 experiments for the quantum-based factorization on a 30-bits integer number (left) and a 32-bits integer number (right).

dian energy consumption increase exponentially with the factorization problem size. Hence, the current quantum annealing approach does not indicate any advantage in terms of energy consumption scaling.

However, in Figure 4, we focus on the 30-bits and 32-bits factorization cases to notice how the distributions are actually skewed towards a low energy consumption level. This means that in most of the cases the quantum annealing-based algorithm is fairly efficient but also that the worst case scenario leads to a much higher level of energy consumption.

6 Conclusions

In this work we show that running the Integer Factorization algorithm on the D-Wave Advantage quantum annealer available nowadays does not offer any relevant advantage in terms of energy efficiency: the mean and median energy consumption scales exponentially with the problem size. However, it is possible to observe that the energy consumption distributions for each problem instance are skewed towards lower energy levels, suggesting that quantum algorithm optimizations should be deeper investigated in the analysis. Moreover, hardware improvements both in the quality of the qubits and their physical connectivity could lead to a better performance and overall efficiency in the near-term future.

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