Joint Universities Accelerator School

JUAS 2024

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Normal-conducting accelerator magnets
Lecture 5: Analytical design

Thomas Zickler

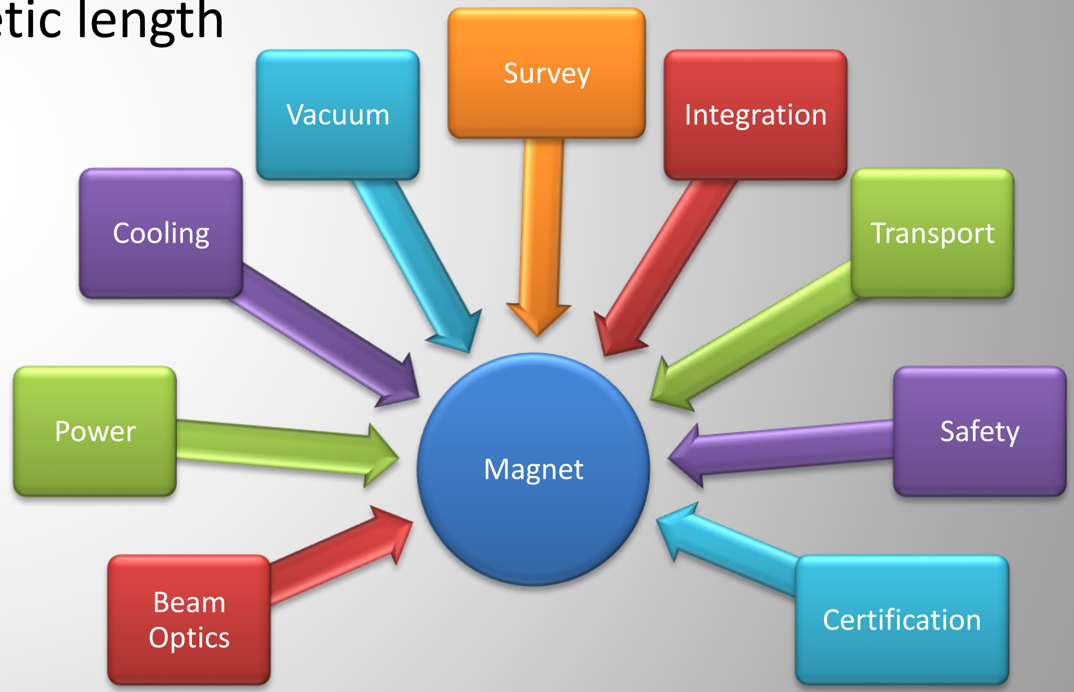
CERN

Design process

Electro-magnetic design is an iterative process:

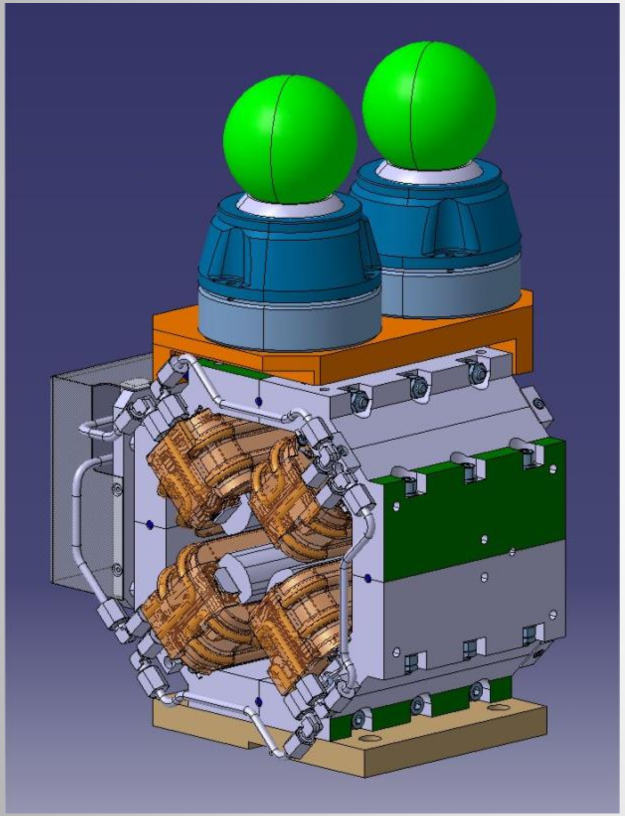
Collect input data → Analytical design → Numerical 2D/3D simulations → Mechanical design → Drawings & specifications

- Field strength (gradient) and magnetic length
- Integrated field strength (gradient)
- Aperture and ‚good field region‘
- Field quality:
  - field homogeneity
  - maximum allowed multi-pole errors
  - settling time (time constant)
- Operation mode: continous, cycled
- Electrical parameters
- Mechanical dimensions
- Cooling requirements



A magnet is not a stand-alone device!

Magnet Components



Alignment targets

Yoke

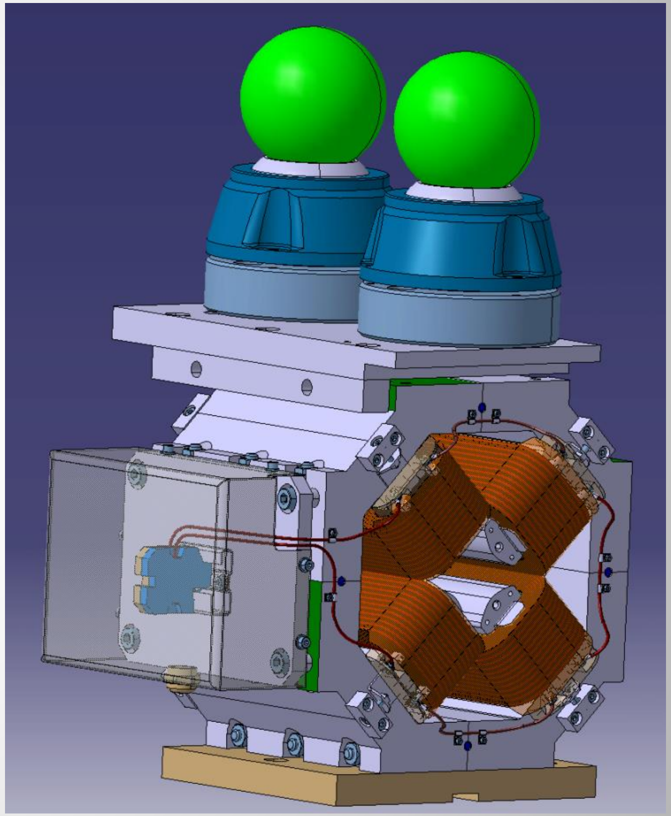
Coils

Sensors

Cooling circuit

Connections

Support



Beam rigidity

Beam rigidity $(B\rho)$ [Tm]: $$(B\rho) = \frac{p}{q} = \frac{1}{qc}\sqrt{{E_k}^2 + 2E_k E_0}$$

- p : particle momentum [kg m s^{-1}]
- q : particle charge [C or A s]
- c : speed of light [m s^{-1}]
- E_k : kinetic beam energy [eV]
- E_0 : particle rest mass energy [eV] (0.51 MeV for electrons; 938 MeV for protons)

" …resistance of the particle beam against a change of direction when applying a bending force…"

Magnetic induction

Dipole bending field B [T]:

$$B = \frac{(B\rho)}{r_M}$$

B: Flux density or magnetic induction (vector) [T]

r_M: magnet bending radius [m]

Quadrupole field gradient B' [T/m]:

$$B' = (B\rho)k$$

k: quadrupole strength [m^{-2}]

Sextupole differential gradient B'' [T/m^2]:

$$B'' = (B\rho)j$$

j: sextupole strength [m^{-3}]

Excitation current in a dipole

Ampere's law $\oint \vec{H} \cdot d\vec{s} = NI$ and $\vec{B} = \mu \vec{H}$ with $\mu = \mu_0 \mu_r$

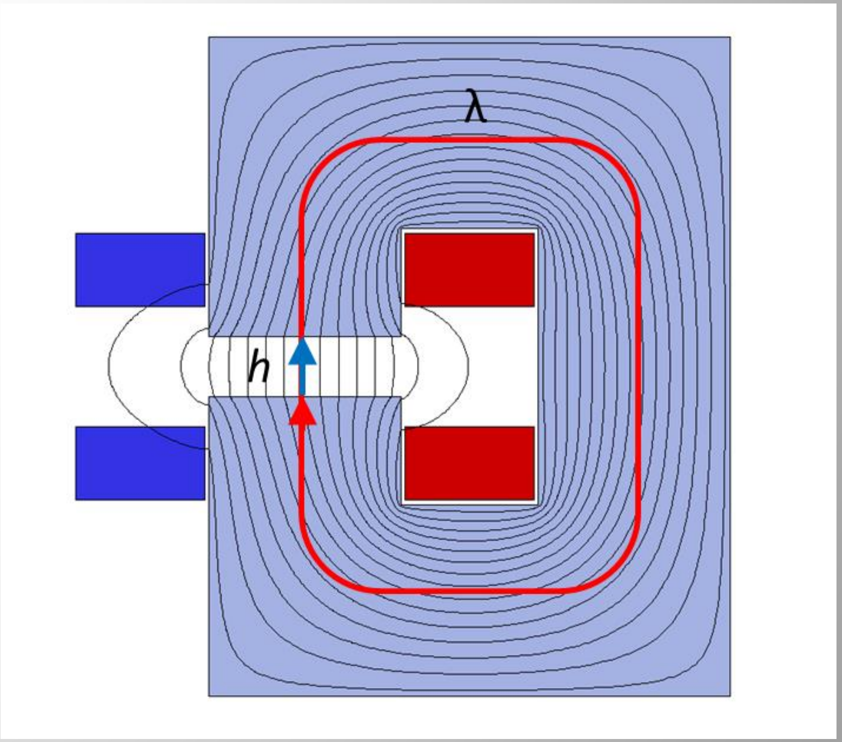
leads to $$NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{s} = \int_{air} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{s} + \int_{iron} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{s} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$$

assuming, that *B* is constant along the path

If the iron is not saturated: $$\frac{h}{\mu_{air}} \gg \frac{\lambda}{\mu_{iron}}$$

then: $$NI_{(perpole)} \approx \frac{Bh}{2\mu_0}$$

h: gap height [m]



Excitation current in a Quadrupole

Choosing the shown integration path gives: $NI = \oint \vec{H} \cdot d\vec{s} = \int_{s1} \overrightarrow{H_1} \cdot d\vec{s} + \int_{s2} \overrightarrow{H_2} \cdot d\vec{s} + \int_{s3} \overrightarrow{H_3} \cdot d\vec{s}$

For a quadrupole, the gradient $B' = \frac{dB}{dr}$ is constant

and $B_y = B'x$ $B_x = B'y$

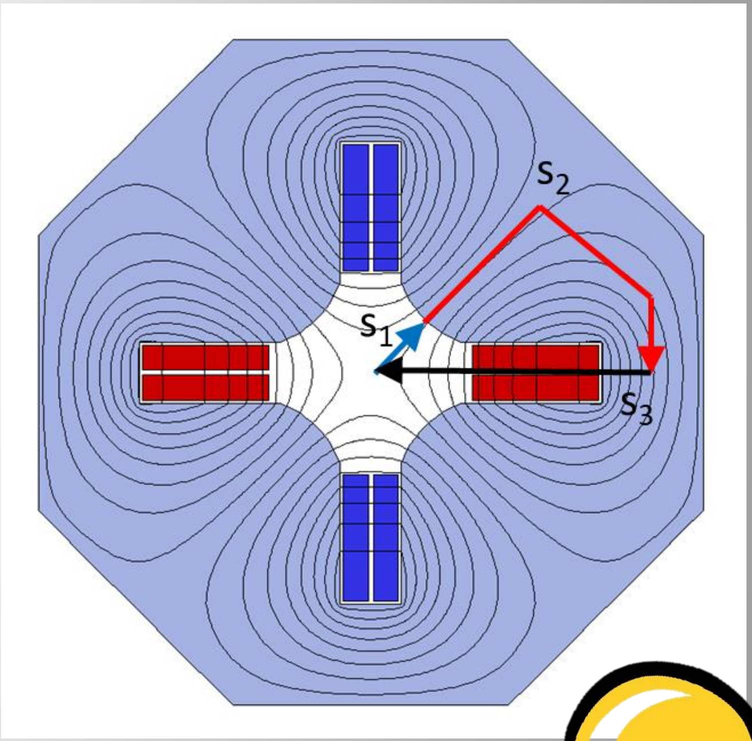
Field <u>modulus</u> along s_1: $|H(r)| = \sqrt{H_x^2 + H_y^2} = \frac{B'}{\mu_0}\sqrt{x^2 + y^2} = \frac{B'}{\mu_0} r$

Neglecting H in s_2 because: $R_{M,s2} = \frac{s_2}{\mu_{iron}} << \frac{s_1}{\mu_{air}}$

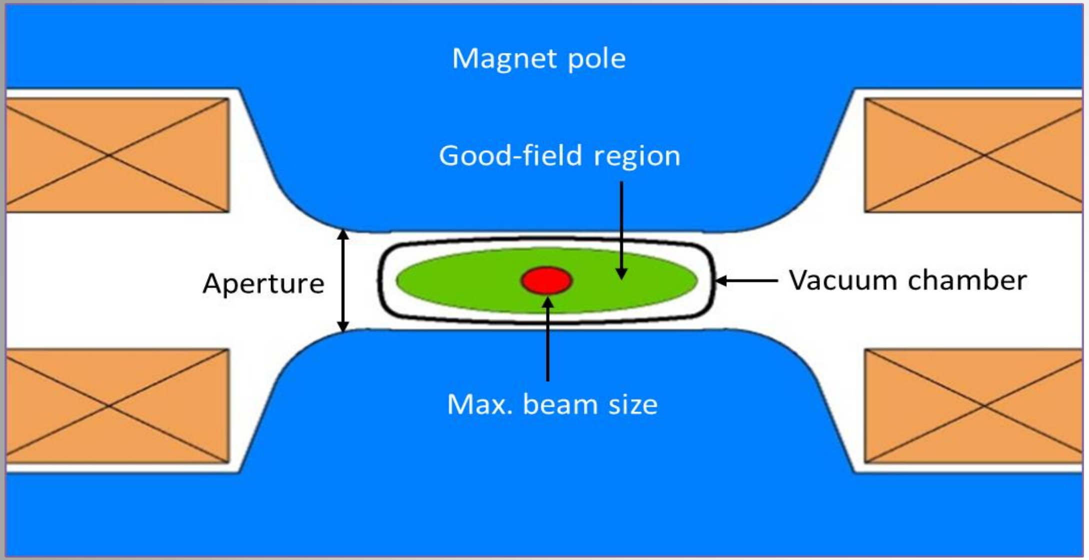
and along s_3: $\int_{s3} \overrightarrow{H_3} \cdot d\vec{s} = 0$

Leads to: $NI \approx \int_0^R H(r)dr = \frac{B'}{\mu_0}\int_0^R r \cdot dr$

$$NI_{(per\ pole)} = \frac{B' r^2}{2\mu_0}$$



Aperture size



"…good-field region: central region around the theoretical beam trajectory where the field quality has to be within certain tolerances…"

- Aperture
  - Good-field region
    - Max. beam size envelope (typical 3-sigma)
      - Lattice functions: beta functions and dispersion
      - Geometrical transverse emittances (energy depended)
      - Momentum spread
    - Closed orbit distortions (few mm)
  - Vacuum chamber thickness (0.5 – 5 mm)
  - Installation and alignment margin (0 – 10 mm)

Pole design

It is easy to derive an ideal mathematical pole configuration for a specific field configuration

In practice, poles are not ideal: finite width and end effects result in (allowed) multi-pole errors disturbing the fundamental field

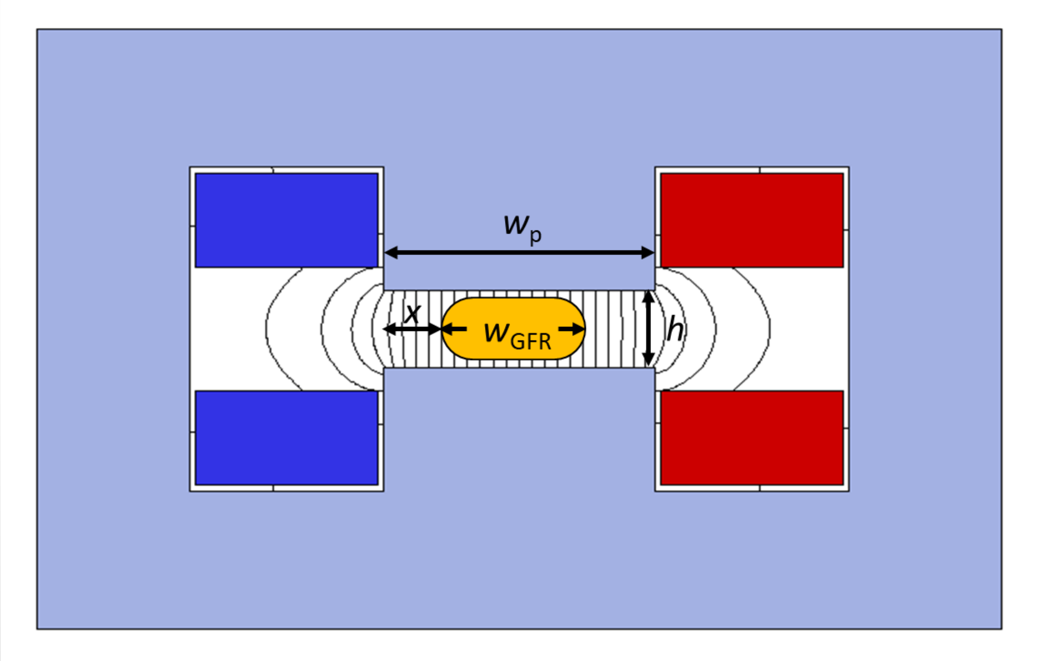
The uniform field region is limited to a small fraction of the pole width:

- Estimate the size of the poles and calculate the resulting fields numerically
- Better approach: calculate the necessary pole overhang

$$x_p = \frac{h}{2} f\left(\frac{\Delta B}{B_0}\right)$$

$$f\left(\frac{\Delta B}{B_0}\right)_{unopt.} = -0.36 \ln \frac{\Delta B}{B_0} - 0.90$$

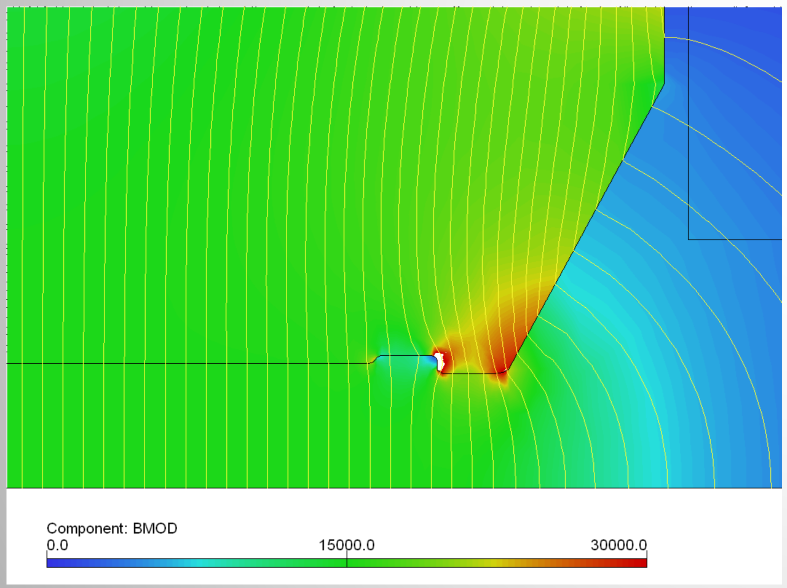
x_p : pole overhang: excess pole beyond the edge of the GFR



Pole optimization

To improve the field quality in the GFR we can either increase the pole width or optimize the pole profile by ‚shimming‘:

- Add or remove material on the pole profile (shims)
- Taper or round-off pole edges to reduce saturation
- Often done by trial-and-error



For an optimized pole:

$$f\left(\frac{\Delta B}{B_0}\right)_{opt.} = -0.14 \ln \frac{\Delta B}{B_0} - 0.25$$

Please note: the final field quality will depend on the longitudinal pole profile, coil ends, mechanical tolerances, assembly errors, possible inhomogeneities in the magnetic properties, and saturation

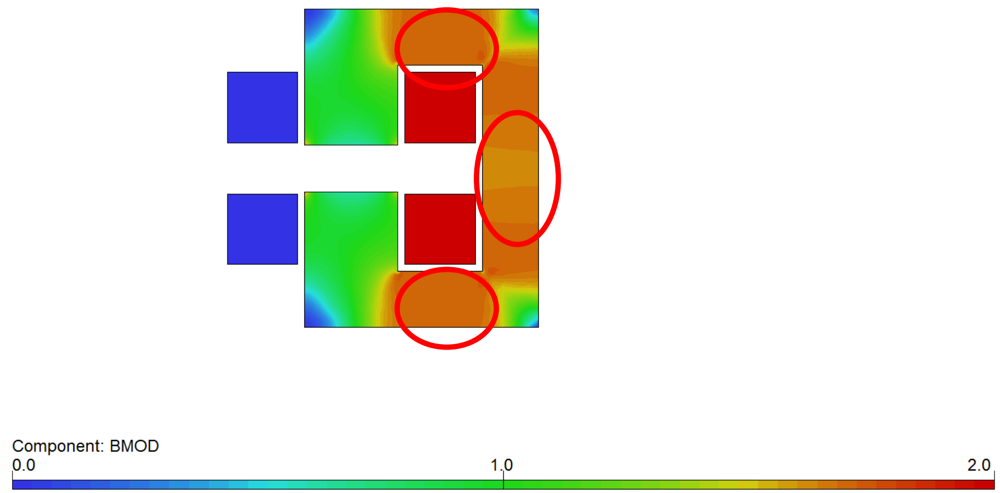
Yoke dimensioning

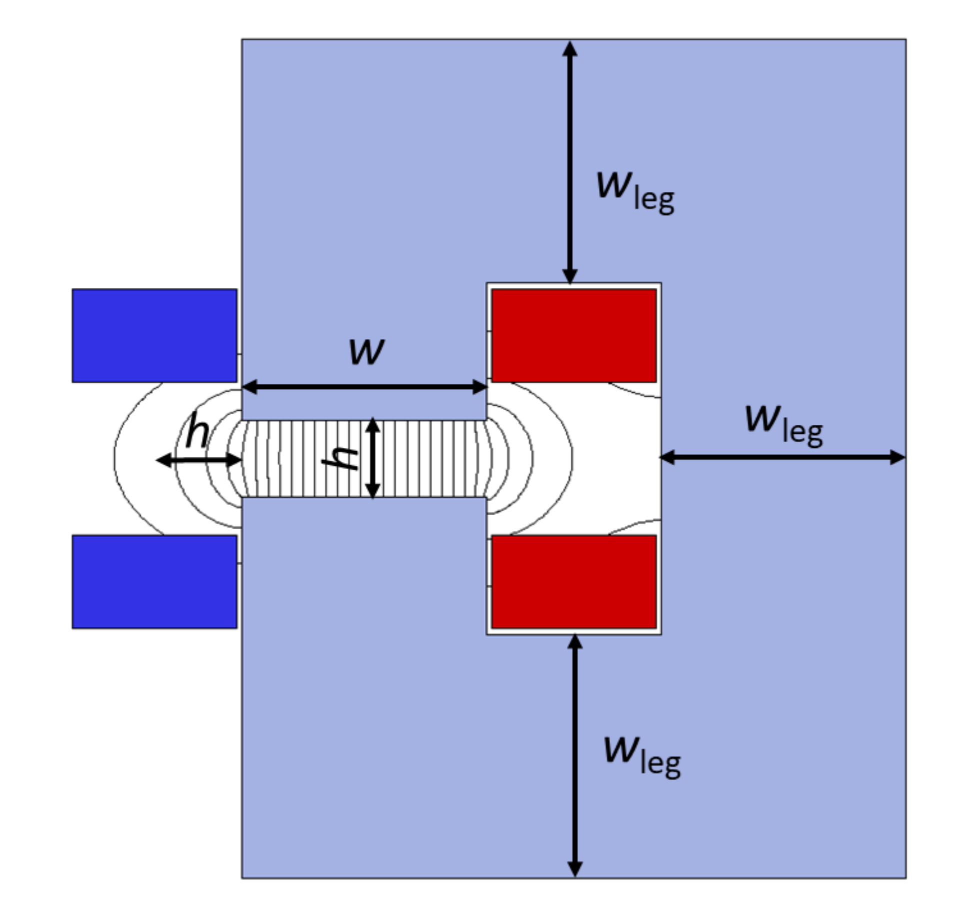
Total flux in the return yoke includes the flux from the aperture and the stray flux outside the gap

$$\Phi = \int_a B \cdot da \approx B_{gap}(w + 2h)\, l_{eff}$$

$$B_{leg} \cong B_{gap} \frac{w + 2h}{w_{leg}}$$ (ignoring the 3rd dimension)

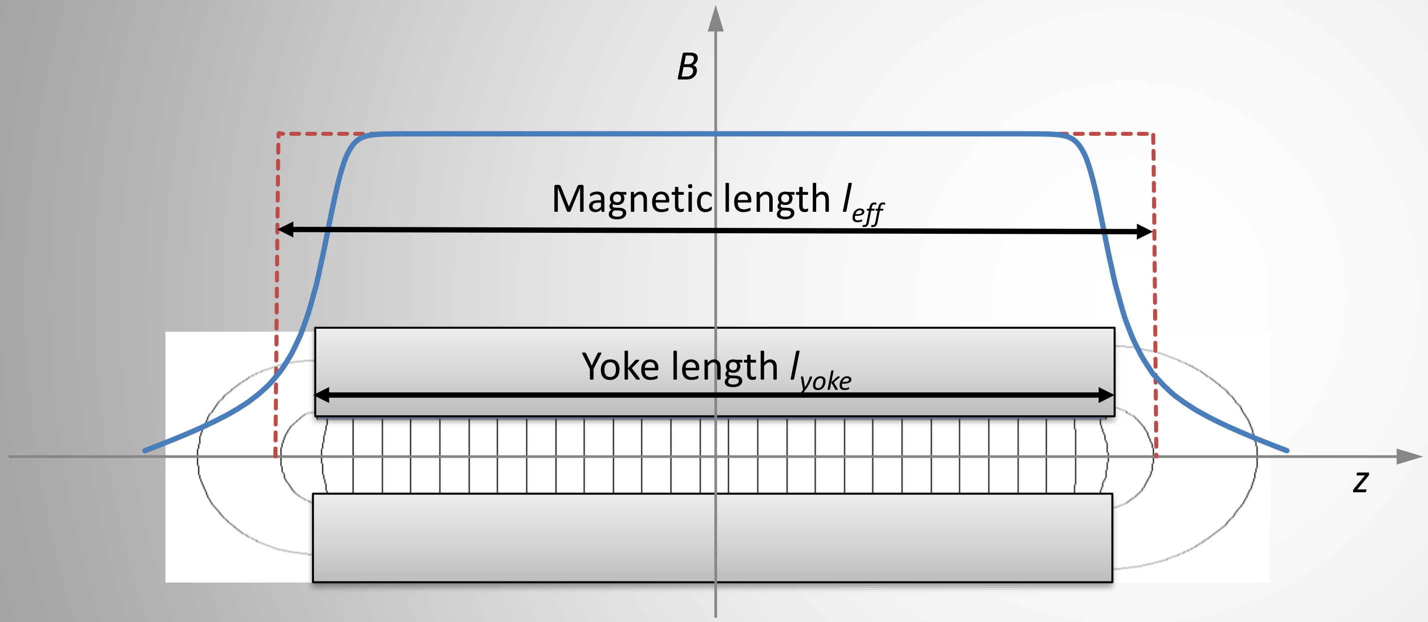
Avoid saturation in the yoke





Effective length

Coming from ∞, B increases towards the magnet center:



Effective length > yoke length

Approximation for a dipole: $l_{eff} \approx l_{yoke} + h$

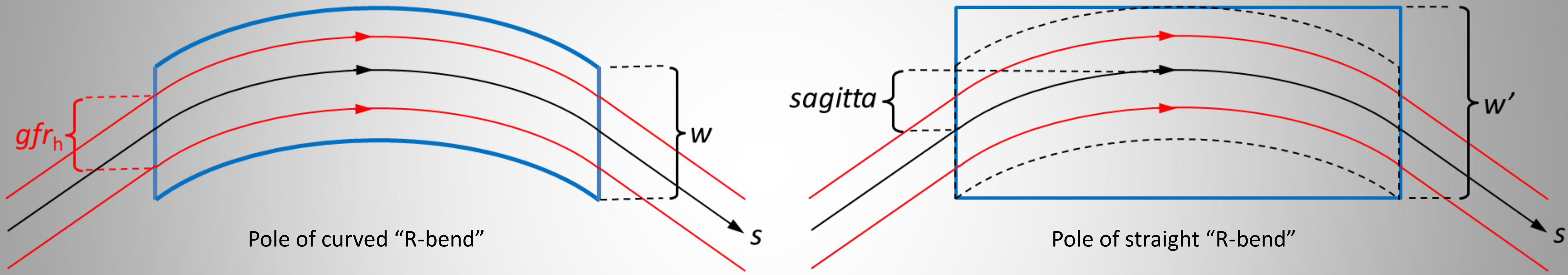
Approximation works only if:

- pole length >> gap height
- saturation is negligible

Effective or magnetic length: $l_{eff} = \dfrac{\int_{-\infty}^{\infty} B(z)\, dz}{B_0}$

Sagitta

View on the pole from the top



For straight magnets, the horizontal pole width has to be enlarged by the sagitta:

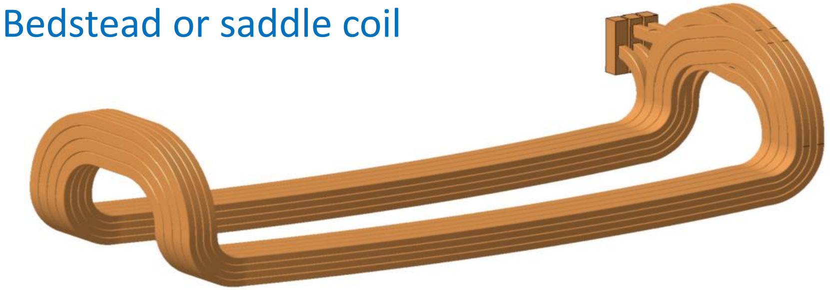
$$s = r_M(1 - \cos(\frac{\alpha}{2}))$$

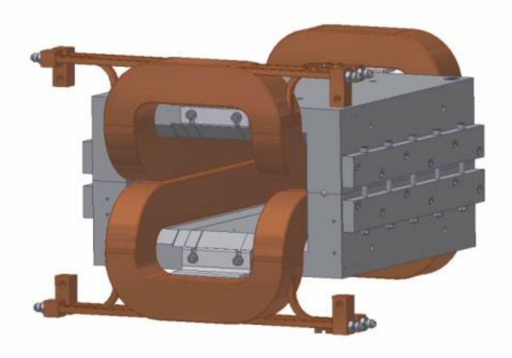
Coil design

Ampere-turns *NI* are determined, but for the coil design, the number of turns *N* and the current density *J* need to be found

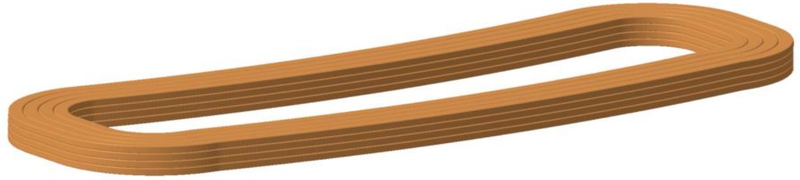
- Coil type selection
- Conductor selection
- Power requirements
- Cooling circuit computation
- Optimization

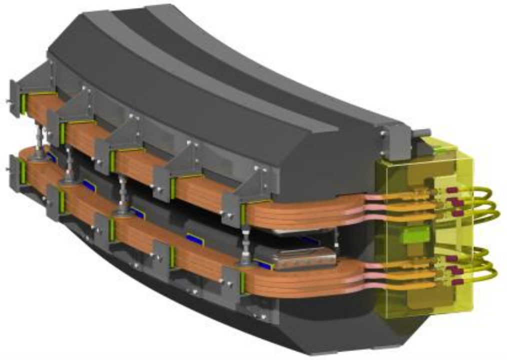
Bedstead or saddle coil





Racetrack coil





Number of turns

The determined ampere-turns *NI* have to be divided into *N* and current *I*

Large N = low current = high voltage	Small N = high current = low voltage
• Small terminals	• Large terminals
• Small conductor cross-section	• Large conductor cross-section
• Thick insulation for coils and cables	• Thin insulation in coils and cables
• Less good filling factor in the coils	• Good filling factor in the coils
• Low power transmission loss	• High power transmission loss

The number of turns *N* are chosen to match the impedances of the power converter and connections

Current density

A sensible choice of the current density J is crucial for a robust and economical magnet design:

$$J = \frac{I}{a_c}$$

J: current density [A m^{-2}]

a_c: net conductor cross section [m^2]

Once magnet cross-section and the yoke length are fix, the ohmic power loss P_Ω depends mainly on the current density J

$$P_{\Omega,dip} = \rho \frac{Bh}{\mu_0} J\, l_{avg}$$

$$P_{\Omega,quad} = 2\rho \frac{B' r^2}{\mu_0} J\, l_{avg}$$

The current density J has a direct impact on coil size, coil cooling, power converter choice, operation costs and investment costs

Electrical parameters

Basic relations:

$R \propto N^2 J$ $U \propto NJ$ $P_\Omega \propto J$

Coil resistance:

$$R = \frac{N\, l_{avg}}{a_c\, \sigma}$$

Resistance R [Ω]
El. conductivity σ [S m^{-1}]
Number of turns per coil N []
Avg. turn length l_{avg} [m]
Conductor cross-section a_{con} [m^2]

Ohm's law:

$$U = R\, I$$

Voltage drop per magnet U [V]
Coil resistance R [Ω]
Current I [A]

Ohmic losses:

$$P_\Omega = U\, I = R\, I^2$$

Power losses (ohmic) P[W]
Voltage drop U [V]
Current I [A]
Resistance R [Ω]

Attention: Electrical resistance is temperature depending

$$R(T) = R(T_0)\left(1 + \alpha\left(T_{avg} - T_0\right)\right)$$

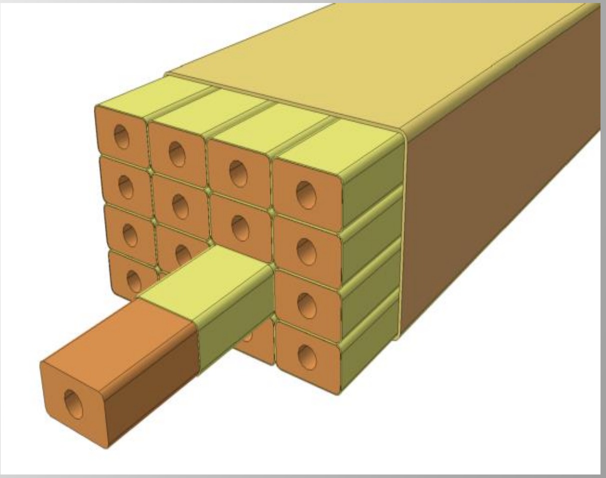
Coil cooling

Air cooling by natural convection:

- Current density
  $J < 2$ A mm^{-2} for small, thin coils
- Cooling enhancement
  Heat sink with enlarged radiation surface
  Forced air flow (cooling fan)
- Only for magnets with limited strength (e.g. correctors)

Direct water cooling:

- Typical current density $J \leq 10$ A mm^{-2}
- Requires demineralized water (low conductivity) and hollow conductor profiles



Indirect water cooling:

- Current density $J \leq 3$ A mm^{-2}
- Tap water can be used

Direct water cooling

Practical recommendations and canonical values:

- Water cooling: 2 A mm^{-2} ≤ J ≤ 10 A mm^{-2}
- Pressure drop: 1 ≤ ΔP ≤ 10 bar (possible up to 20 bar)
- Low pressure drop might lead to more complex and expensive coil design
- Flow velocity should be high enough, so flow is turbulent (Reynolds number Re > 4000)
- Flow velocity v_{avg} ≤ 3 m s^{-1} to avoid erosion and vibrations
- Acceptable temperature rise: ΔT ≤ 30°C but for advanced stability: ΔT ≤ 15°C

Cooling water properties:

- For the cooling of hollow conductor coils demineralised water is used (exception: indirect cooled coils)
- Water quality is essential for performance and reliability of the coil (corrosion, erosion, short circuits)
- Resistivity > 0.1 x 10^{6} Ωm
- pH between 6 and 6.5 (= neutral)
- Dissolved oxygen below 0.1 ppm
- Filters to remove particles and loose deposits to avoid cooling duct obstruction

Direct water cooling

Useful simplified formulas using water as cooling fluid:

Water flow Q [litre/min] necessary to remove power P_Ω : $Q = 14.5 \frac{P_\Omega}{\Delta T}$

P_Ω : dissipated power [kW]

ΔT : temperature increase [°C]

Average water velocity v_{avg} [m s^{-1}] in a round tube: $v_{avg} = 16.67 \frac{Q}{a_h} = 66.67 \frac{Q}{d_h^2 \pi}$

$a_h = \frac{\pi d^2}{4}$: bore cross-section [mm^2]

d_h : hydraulic diameter [mm]

Pressure drop ΔP [bar] : $\Delta P = 53.32 \frac{Q^{1.75}}{d_h^{4.75}} l_h$ (from Blasius' law)

l_h : cooling circuit length [m]

Reynolds number Re []: $Re = \frac{v_{avg}\, d_h}{\nu}$

Re: dimensionless quantity used to help predict similar flow patterns in different fluid flow situations

ν : kinematic viscosity of coolant is temperature depending, for simplification it is assumed to be constant ($6.58 \cdot 10^{-7}$ m^2 s^{-1} for water at 40°C)

Note: for convenience practical (non-SI) units are used in the formulae of this slide

Design recipe for cooling circuits

Already determined: current density *J*, current *I*, and number of turns *N*

1. Calculate the conductor net cross-section from the current density and the current.
2. From the number of turns and the average turn length compute the coil resistance.
3. Define the allowed temperature rise and correct the coil resistance for the average conductor temperature before computing the ohmic losses in the coil.
4. Calculate the required flow rate to evacuate the total ohmic losses from the coil by keeping the temperature increase within the defined limit.
5. For a given cooling duct diameter and the flow rate, determine the required pressure drop. Alternatively, calculate the diameter of the cooling duct from the flow rate and a give pressure drop.
6. If necessary, change the pressure drop, the hydraulic diameter or the number of cooling circuits per coil and go back to point 5.
7. Check that the coolant velocity remains below the limit where erosion phenomena will become apparent.
8. Finally, verify that the Reynolds number is in the turbulent flow regime for which Blasius equations holds. The Reynold number should be between 4000 and 100 000.

Cost estimate

Production specific tooling:

10 to 20 k€/tooling

Material:

Steel sheets: 1.0 - 1.5 € /kg

Copper conductor: 10 to 20 € /kg

Yoke manufacturing:

Dipoles: 6 to 10 € /kg (> 1000 kg)

Quads/Sextupoles: 50 to 80 € /kg (> 200 kg)

Small magnets: up to 300 € /kg

Coil manufacturing:

Dipoles: 30 to 50 € /kg (> 200 kg)

Quads/Sextupoles: 65 to 80 € /kg (> 30 kg)

Small magnets: up to 300 € /kg

Contingency:

10 to 20 %

Magnet	***Magnet type***	***Dipole***	
	Number of magnets (incl. spares)	18	
	Total mass/magnet	8330	kg
Fixed costs	Design	14	kEuros
	Punching die	12	kEuros
	Stacking tool	15	kEuros
	Winding/molding tool	30	kEuros
Yoke	Yoke mass/magnet	7600	kg
	Used steel (incl. blends)/magnet	10000	kg
	Yoke manufacturing costs	8	Euros/kg
	Steel costs	1.5	Euros/kg
Coil	Coil mass/magnet	730	kg
	Coil manufacturing costs	50	Euros/kg
	Cooper costs (incl. insulation)	12	Euros/kg
Total costs	Total order mass	150	Tonnes
	Total fixed costs	71	kEuros
	Total Material costs	428	kEuros
	Total manufacturing costs	1751	kEuros
	Total magnet costs	2250	kEuros
	Contingency	20	%
	Total overall costs	**2700**	**kEuros**

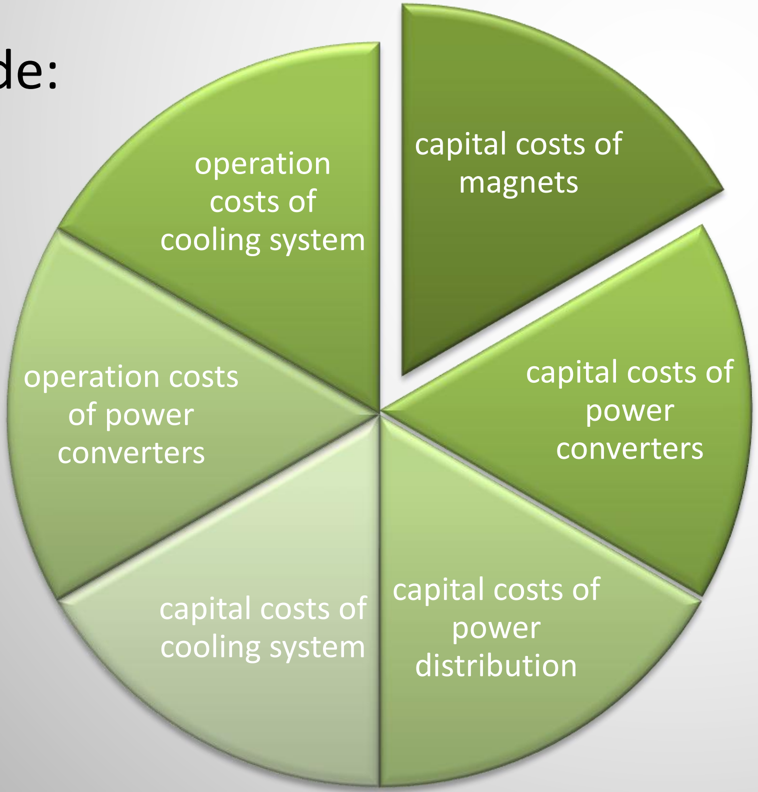
NOT included: magnetic design, supports, cables, water connections, alignment equipment, magnetic measurements, transport, installation
Prices for **2012**

Costs and optimization

Focus on economic design!

Design goal: Minimum total costs over projected magnet lifetime by optimization of capital (investment) costs against running costs (power consumption)

Total costs include:



Attention: $Power \propto current\ desity$

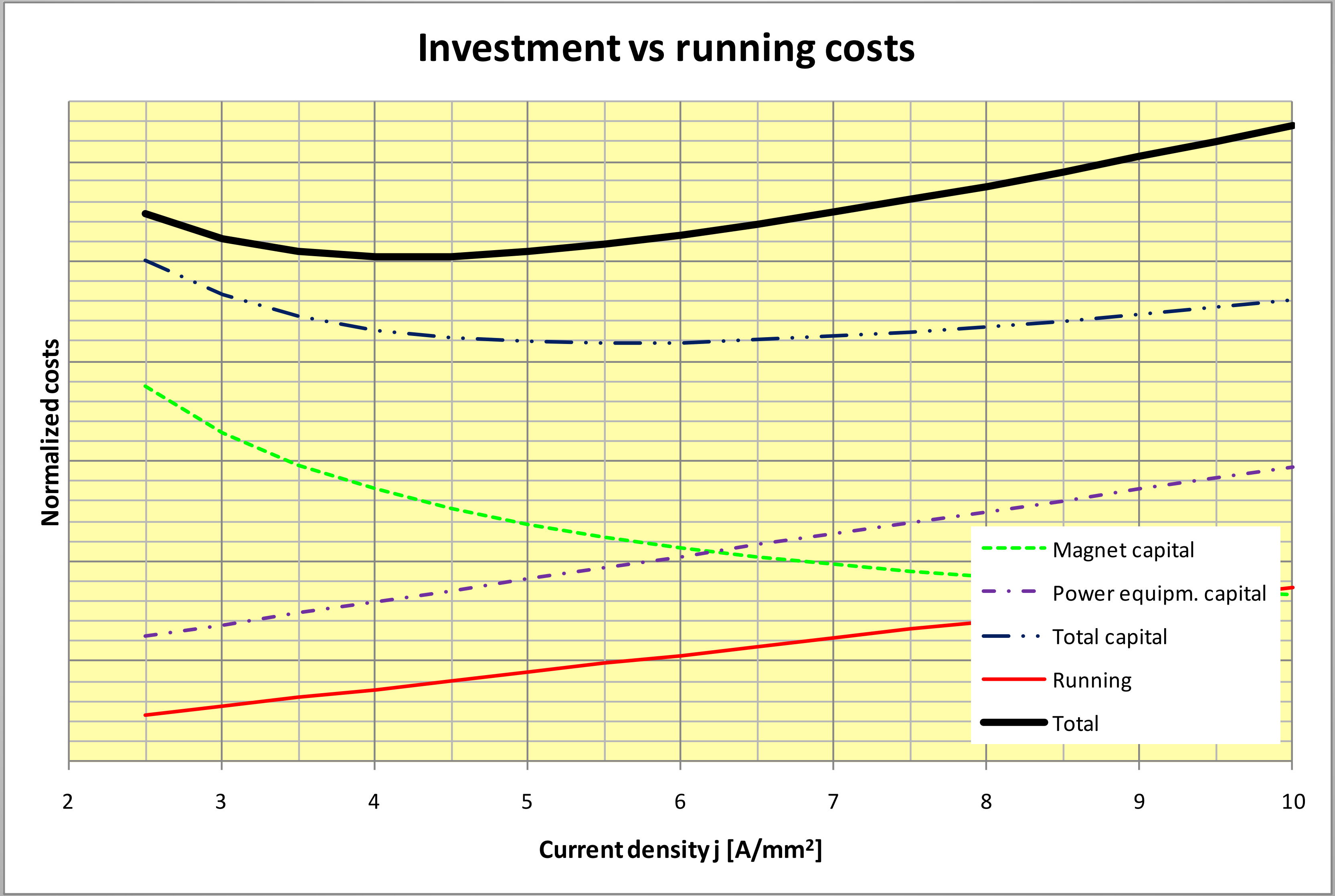
Decreasing current density means:

- ➢ increasing coil cross section
- ➢ increasing material (coil & yoke) cost
- ➢ increasing manufacturing cost

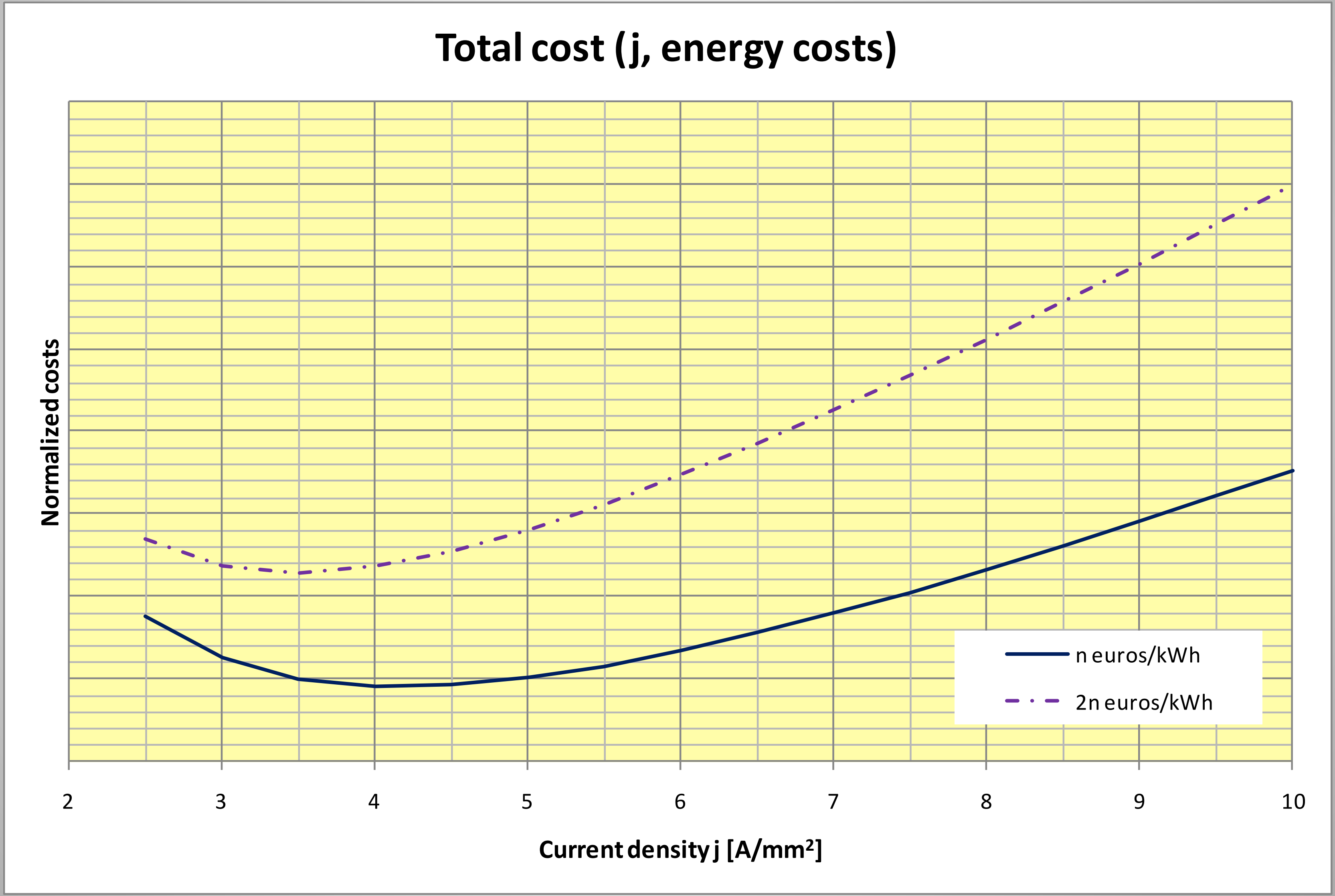
But:

- ➢ decreasing capital costs for power converter and cooling system
- ➢ decreasing operation costs

Cost optimization



Cost optimization



Thanks for your attention…