AXEL-2023

Introduction to Particle Accelerators

Transverse optics 1:

- ✓Relativity, Energy & Units
- ✓Accelerator co-ordinates
- ✓Magnets and their configurations
- ✓Hill's equation

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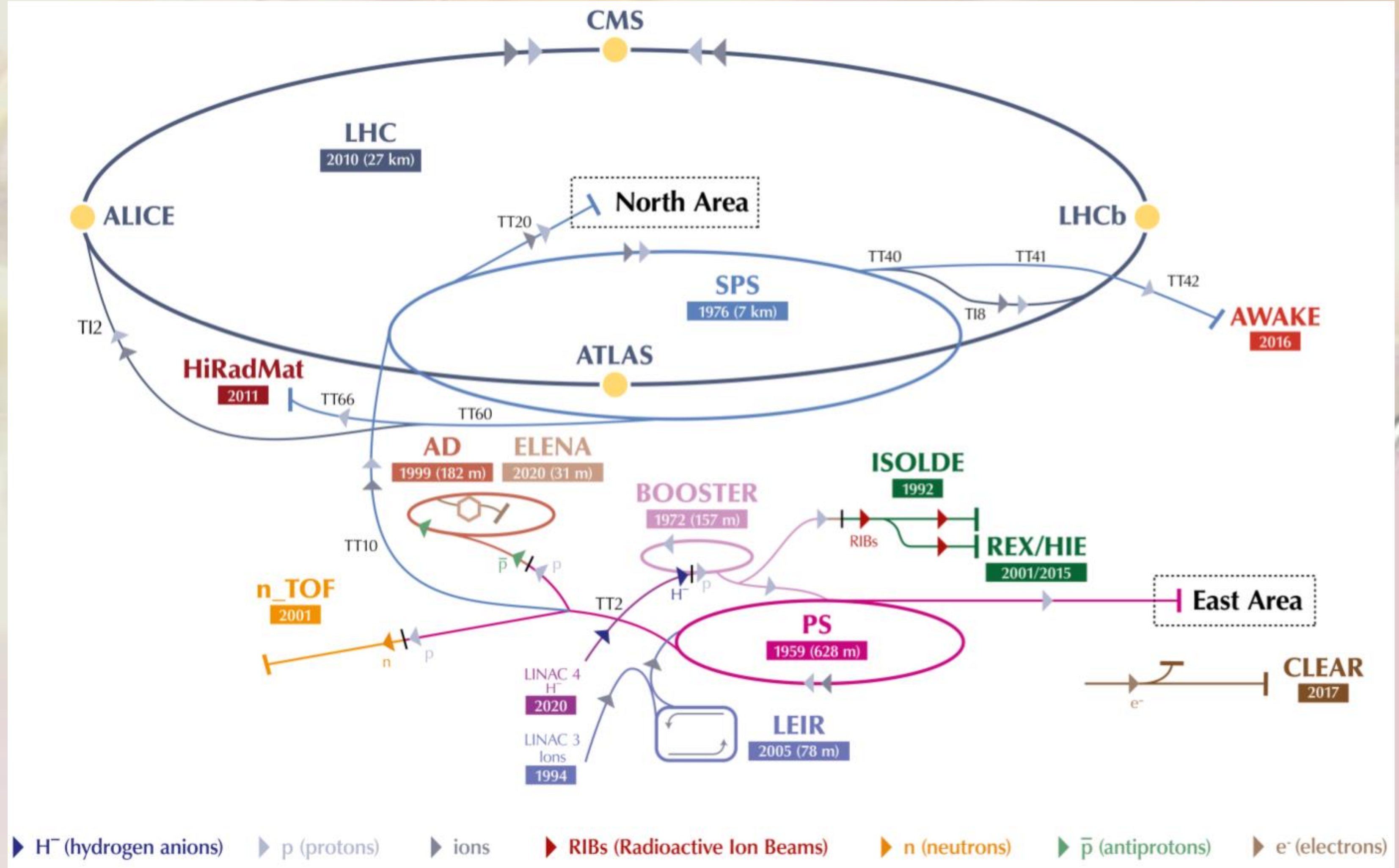
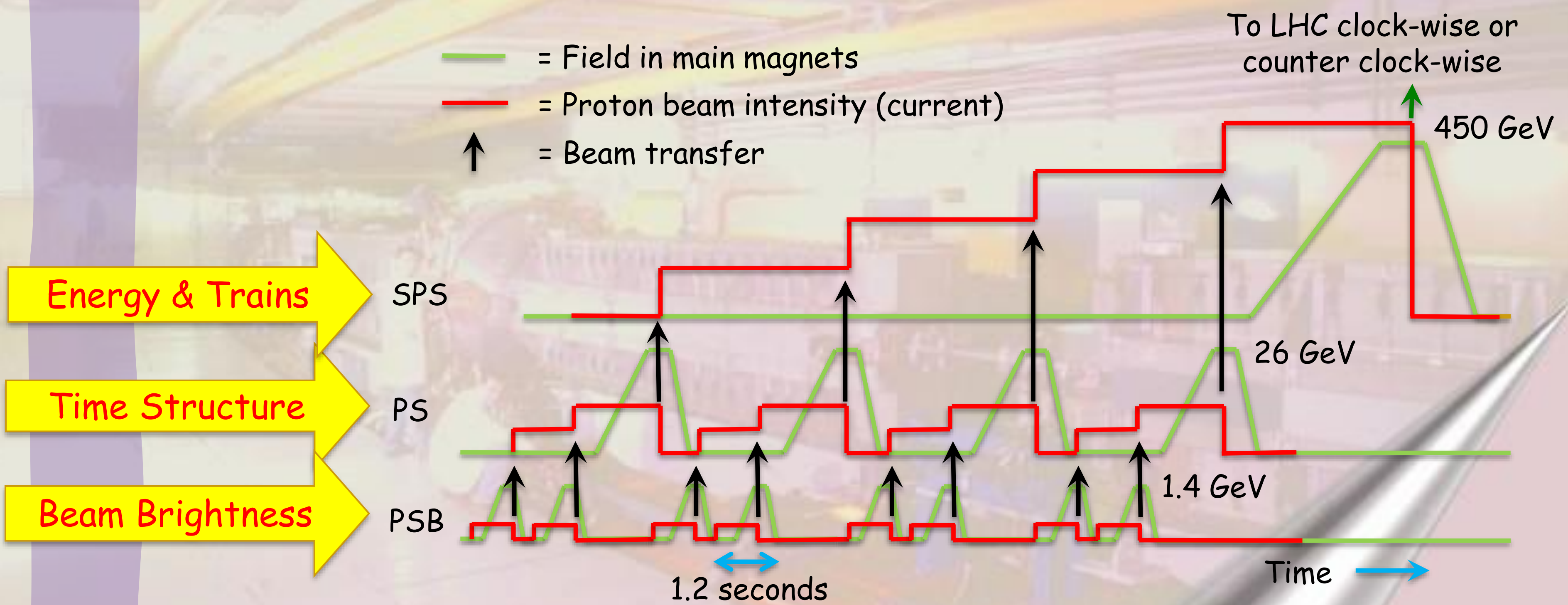
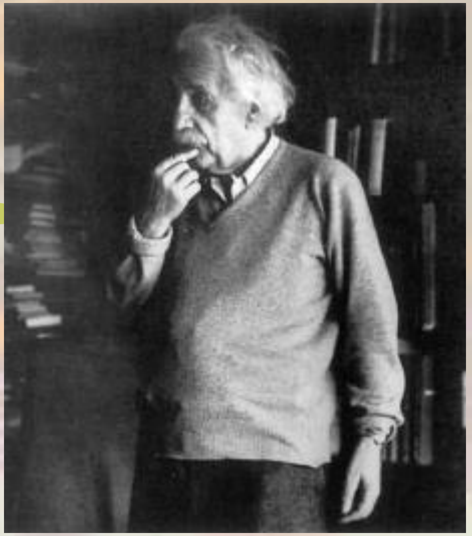
CERN Accelerators

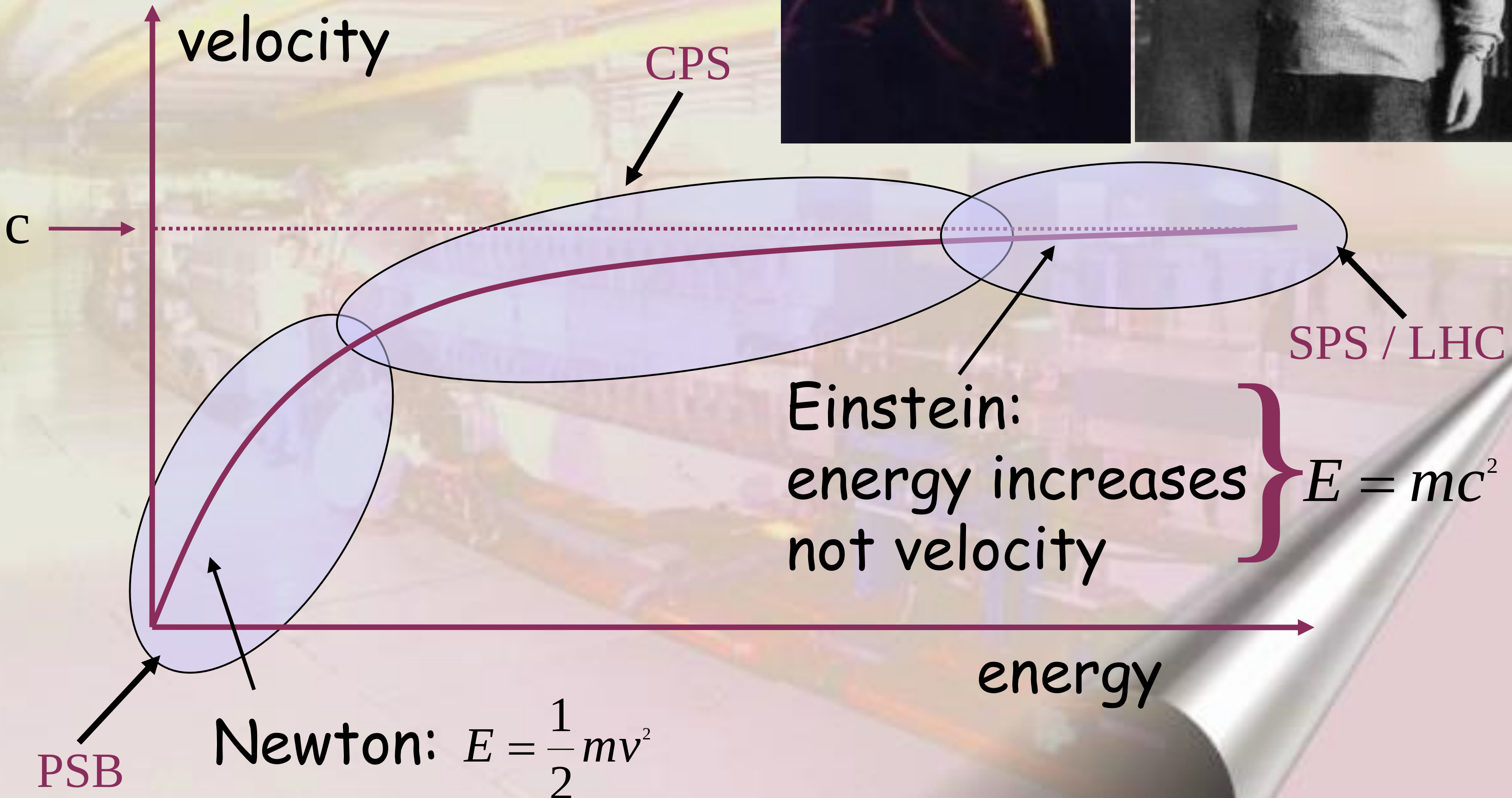


Classical Filling of the LHC with Protons



Relativity





velocity
c
CPS
SPS / LHC
Einstein:
energy increases
not velocity
$E = mc^2$
energy
PSB
Newton: $E = \frac{1}{2}mv^2$

Energy & Momentum

- Einstein's relativity formula: $E = mc^2$
- For a mass at rest this will be: $E_0 = m_0 c^2$

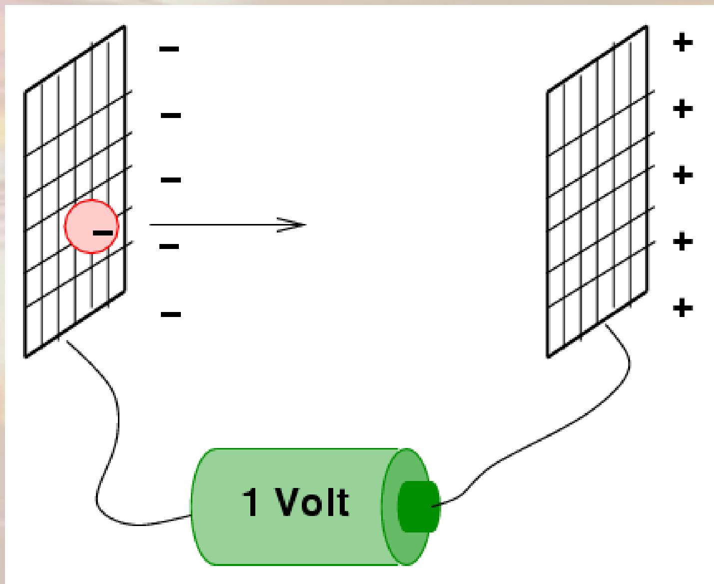
Rest mass

Rest energy

- Define: $\gamma = \frac{E}{E_0}$ As being the ratio between the total energy and the rest energy
- Then the mass of a moving particle is: $m = \gamma m_0$
- Define: $\beta = \frac{v}{c}$,then we can write: $\beta = \frac{mvc}{mc^2}$
- $p = mv$,which is always true and gives: $\beta = \frac{pc}{E}$ or $p = \frac{E\beta}{c}$

The Units we use for Energy

- The energy acquired by an electron in a potential of 1 Volts is defined as being 1 eV



- The unit eV is too small to be used today, we use:

  1 KeV = 10^3, MeV = 10^6, GeV = 10^9, TeV = 10^{12}

Energy: eV versus Joules

- The unit most commonly used for **Energy** is **Joules [J]**
- In accelerator and particle physics we talk about **eV**....!?
- The **energy** acquired by an **electron** in a potential of **1 Volt** is defined as being **1 eV**
- **1 eV** is **1 elementary charge** ‘pushed’ by **1 Volt**.

$$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{Joules}$$

Units: Energy & Momentum (2)

- However:

$$p = \frac{E\beta}{c}$$

Energy

Momentum

- Therefore the **units** for **momentum** are GeV/c…etc.

Attention:

when **β=1** **energy** and **momentum** are **equal**

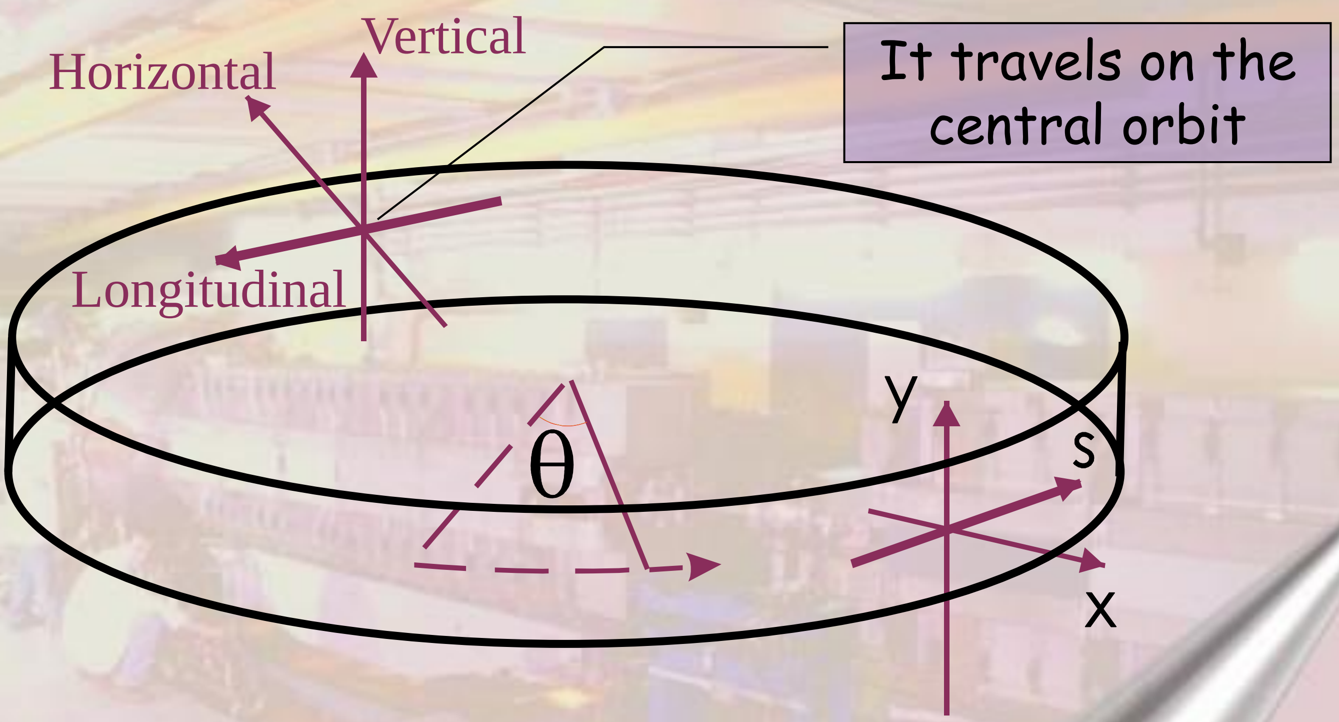
when **β<1** the **energy** and **momentum** are **not equal**

Units: Example PS injection

- ✓ Kinetic energy at injection $E_{kinetic}$ = 1.4 GeV
- ✓ Proton rest energy E_0=938.27 MeV
- ✓ The total energy is then: $E = E_{kinetic} + E_0$ =2.34 GeV
- ✓ We know that $\gamma = \frac{E}{E_0}$,which gives γ = 2.4921
- ✓ We can derive $\beta = \sqrt{1 - \frac{1}{\gamma^2}}$,which gives β = 0.91597
- ✓ Using $p = \frac{E\beta}{c}$ we get p = 2.14 GeV/c

✓ In this case: **Energy ≠ Momentum**

Accelerator co-ordinates



- ✓ We can speak about a:

<u>Rotating Cartesian Co-ordinate System</u>

Magnetic rigidity

- ✓ The force **evB** on a charged particle moving with velocity **v** in a dipole field of strength **B** is equal to its mass multiplied by its acceleration towards the centre of its circular path.

- ✓ As a formula this is:

$$F = evB = \frac{mv^2}{\rho}$$

Radius of curvature

Like for a stone attached to a rotating rope

- ✓ Which can be written as:

$$B\rho = \frac{mv}{e} = \frac{p}{e}$$

Momentum p=mv

- ✓ **Bρ** is called the **magnetic rigidity**, and if we put in all the correct units we get:

Bρ = 33.356·p [KG·m] = 3.3356·p [T·m] (if p is in [GeV/c])

Some LHC figures

- LHC circumference = 26658.883 m
  - Therefore the radius r = 4242.9 m
- There are 1232 main dipoles to make 360°
  - This means that each dipole deviates the beam by only 0.29 °
- The dipole length = 14.3 m
  - The total dipole length is thus 17617.6 m, which occupies 66.09 % of the total circumference
- The bending radius ρ is therefore
  - ρ = 0.6609 x 4242.9 m → ρ = 2804 m

Dipole magnet

- A dipole with a uniform dipolar field deviates a particle by an angle θ.
- The deviation angle θ depends on the length L and the magnetic field B.
- The angle θ can be calculated:

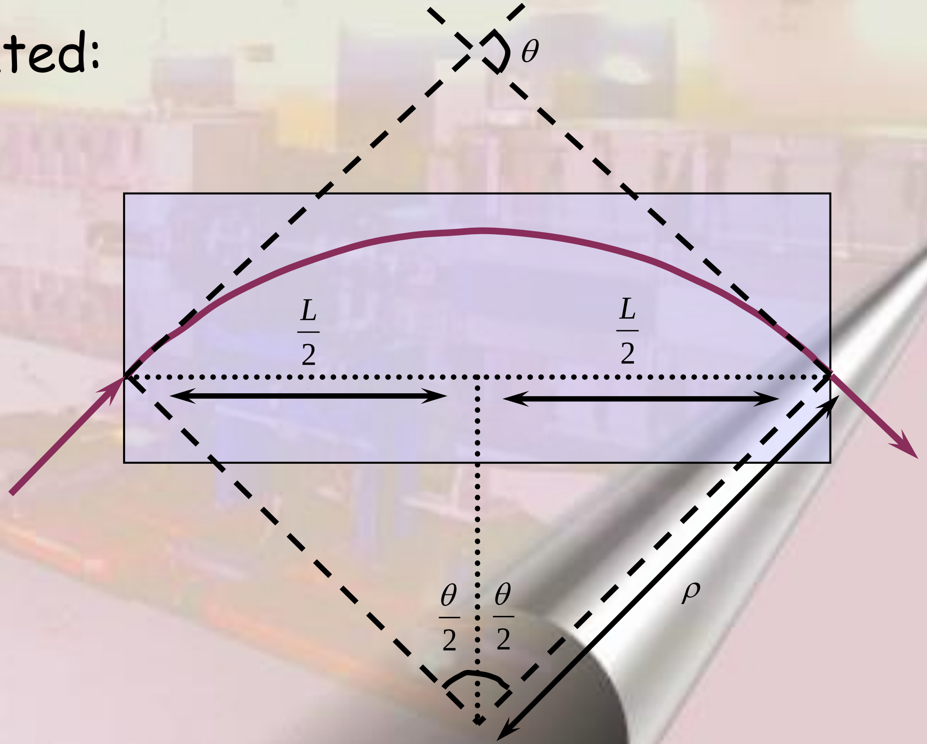
$$\sin\left(\frac{\theta}{2}\right) = \frac{L}{2\rho} = \frac{1}{2}\frac{LB}{(B\rho)}$$

- If θ is small:

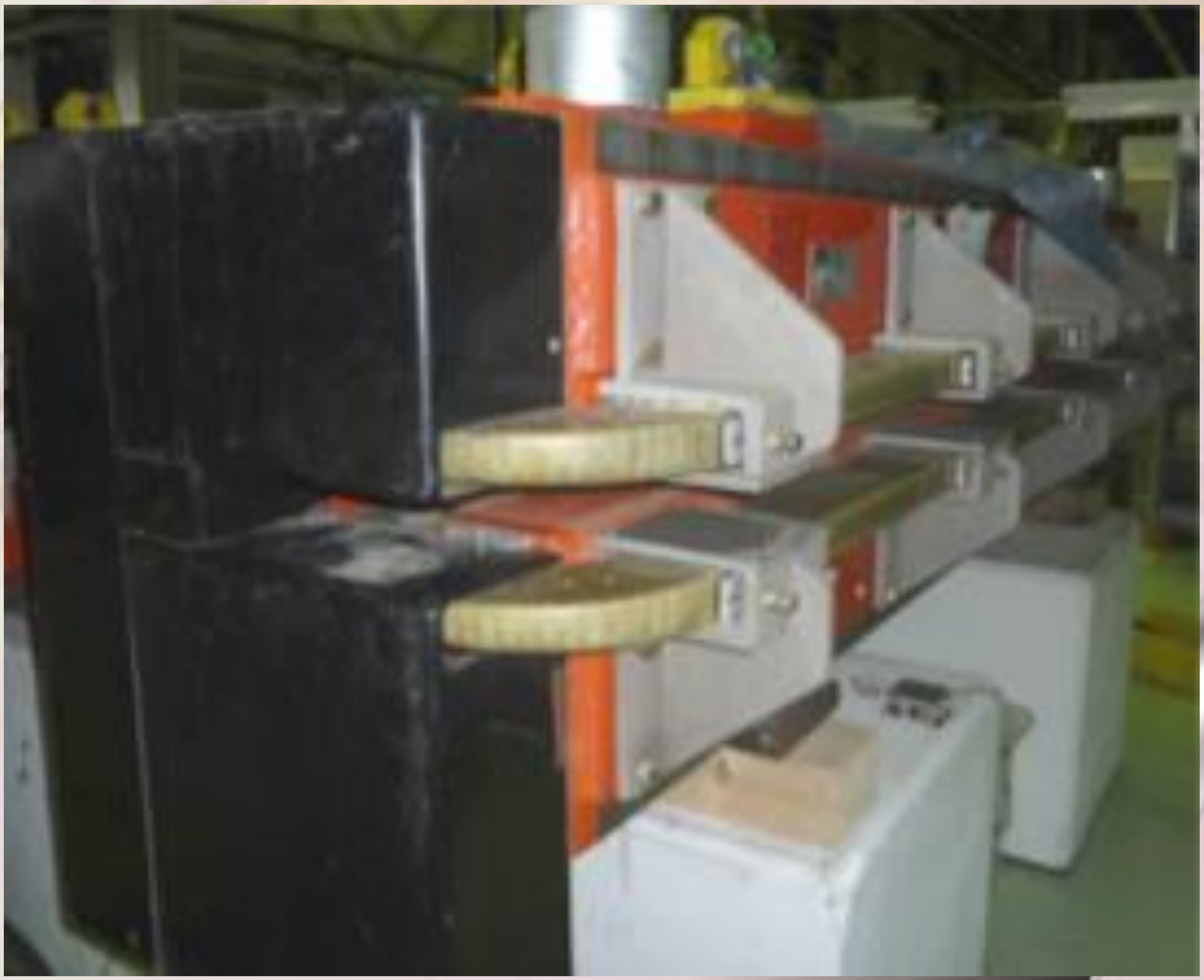
$$\sin\left(\frac{\theta}{2}\right) = \frac{\theta}{2}$$

- So we can write:

$$\theta = \frac{LB}{(B\rho)}$$

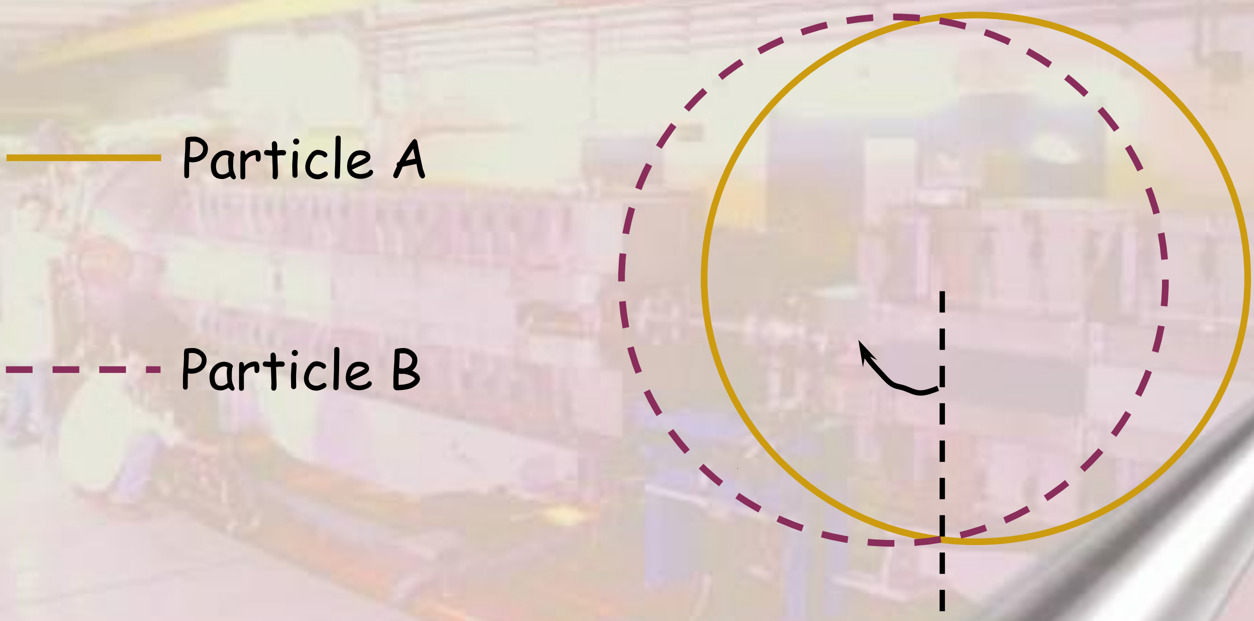


A Real Dipole Magent



Two particles in a dipole field

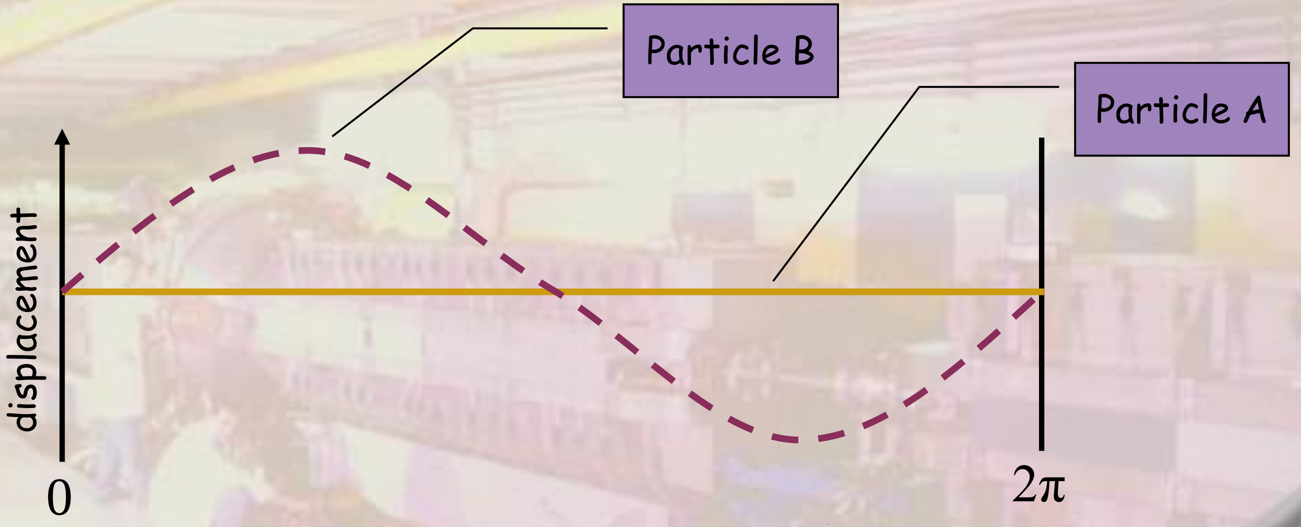
- What happens with two particles that travel in a dipole field with different initial angles, but with equal initial position and equal momentum ?



- Assume that $B\rho$ is the same for both particles.
- Lets unfold these circles……

The 2 trajectories unfolded

- ✓ The horizontal displacement of particle B with respect to particle A.



- ✓ Particle B oscillates around particle A.
- ✓ This type of oscillation forms the basis of all transverse motion in an accelerator.
- ✓ It is called **'Betatron Oscillation'**

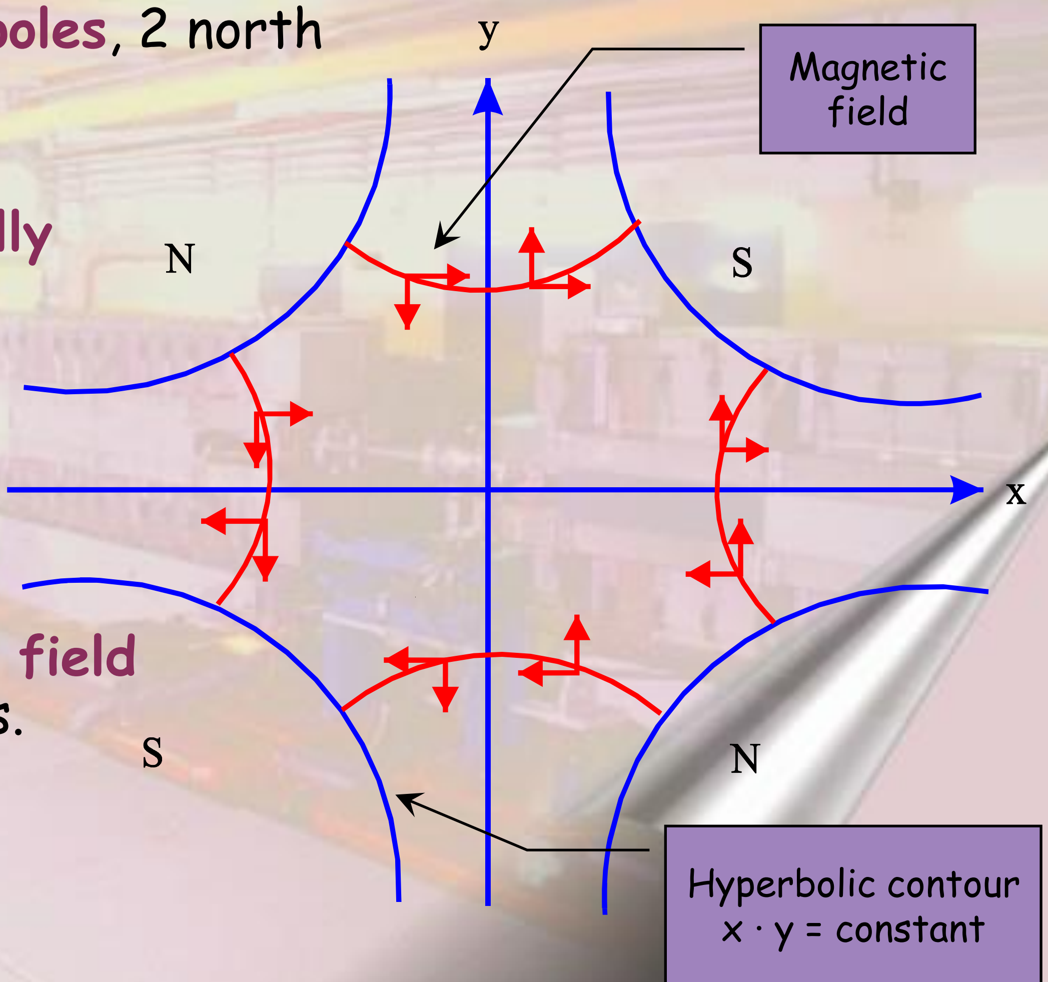
'Stable' or 'unstable' motion ?

- ✓ Since the horizontal trajectories close we can say that the horizontal motion in our simplified accelerator with only a horizontal dipole field is <u>**'stable'**</u>
- ✓ What can we say about the vertical motion in the same simplified accelerator ? Is it <u>**'stable'**</u> or <u>**'unstable'**</u> and why ?
- ✓ What can we do to make this motion stable ?
- ✓ We need some element that 'focuses' the particles back to the reference trajectory.
- ✓ This extra focusing can be done using:

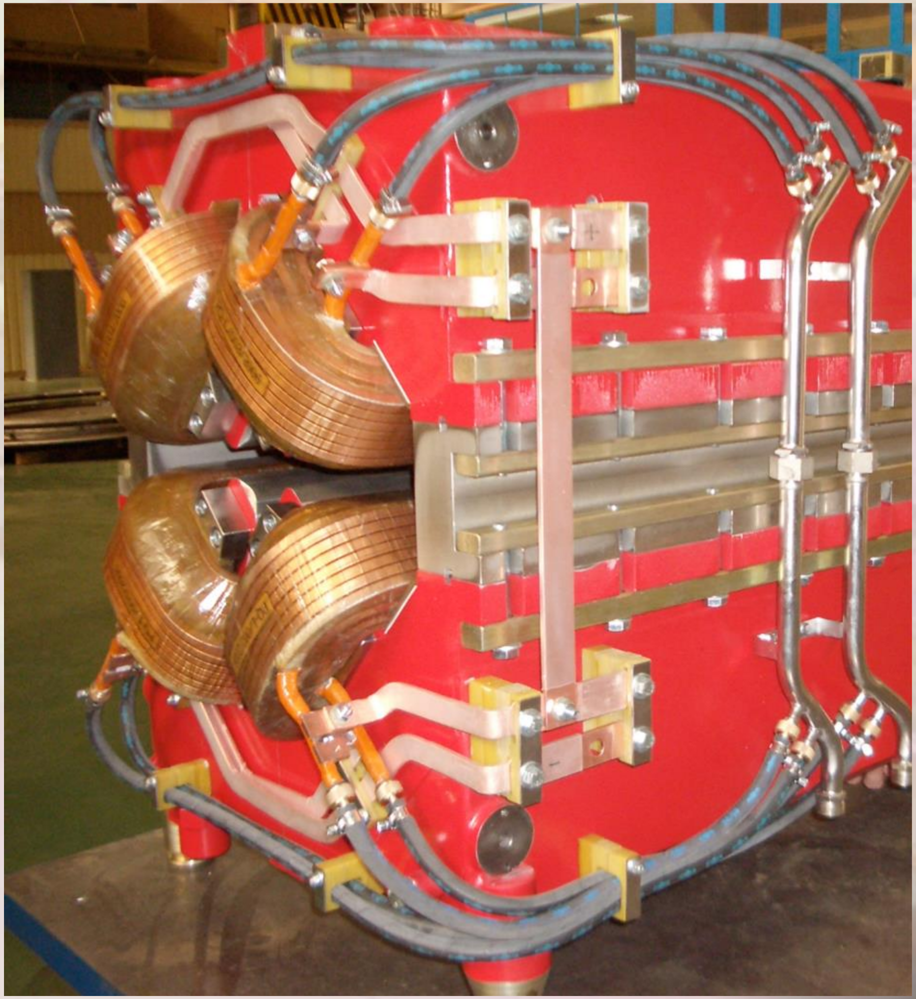
<u>**Quadrupole magnets**</u>

Quadrupole Magnet

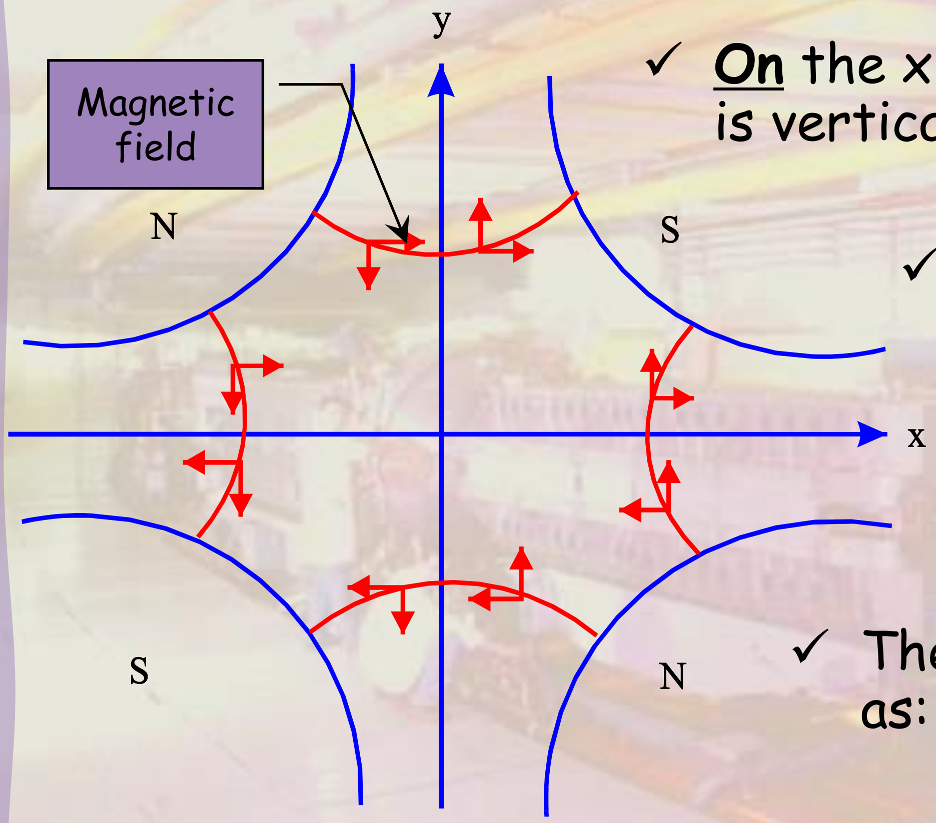
- ✓ A **Quadrupole** has **4 poles**, 2 north and 2 south
- ✓ They are **symmetrically arranged** around the centre of the magnet
- ✓ There is no **magnetic field** along the central axis.



A Real Quadrupole Magnet

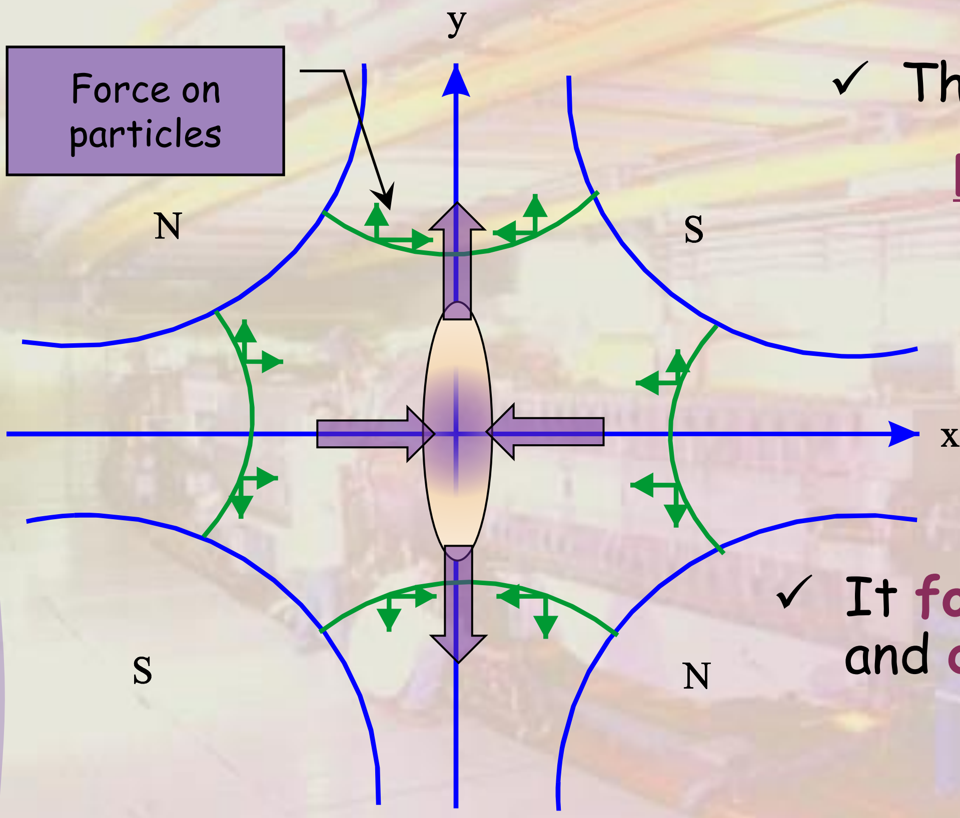


Quadrupole fields



- ✓ On the x-axis (horizontal) the field is vertical and given by: $B_y \propto x$
- ✓ On the y-axis (vertical) the field is horizontal and given by: $B_x \propto y$
- ✓ The field gradient, K is defined as: $\frac{d(By)}{dx} (Tm^{-1})$
- ✓ The 'normalised gradient', k is defined as: $\frac{K}{(B\rho)} (m^{-2})$

Types of quadrupoles



- This is a:

  Focusing Quadrupole (QF)

- It **focuses** the beam **horizontally** and **defocuses** the beam **vertically**.

- **Rotating** this magnet by **90°** will give a:

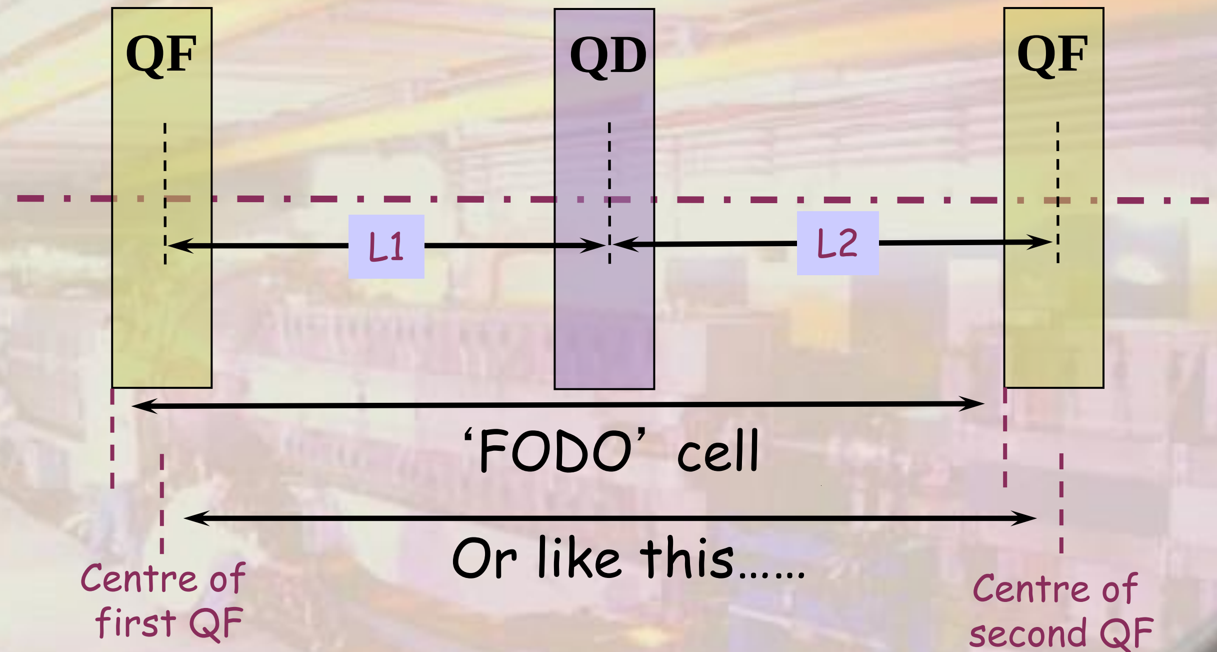
  Defocusing Quadrupole (QD)

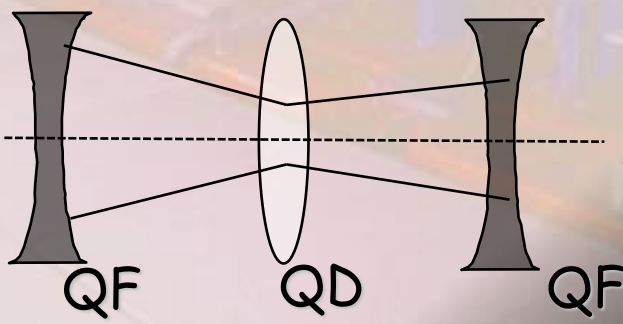
Focusing and Stable motion

- Using a combination of focusing (QF) and defocusing (QD) quadrupoles solves our problem of 'unstable' vertical motion.
- It will keep the beams focused in **both planes** when the position in the accelerator, type and strength of the quadrupoles are well chosen.
- By now our accelerator is composed of:
  - **Dipoles**, constrain the beam to some closed path (orbit).
  - **Focusing and Defocusing Quadrupoles**, provide horizontal and vertical focusing in order to constrain the beam in transverse directions.
- A combination of focusing and defocusing sections that is very often used is the so called: **FODO lattice**.
- This is a configuration of magnets where focusing and defocusing magnets alternate and are separated by non-focusing drift spaces.

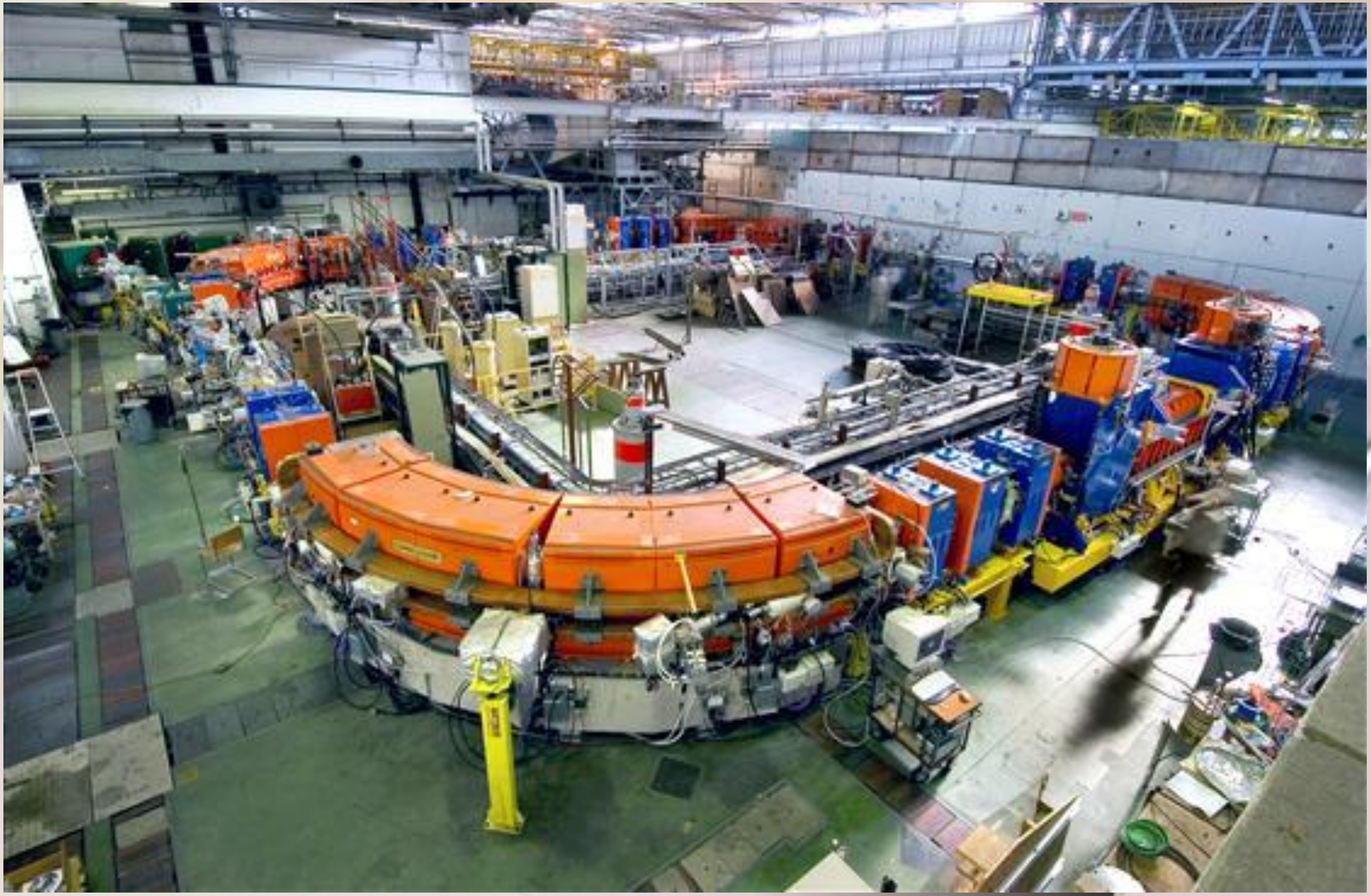
FODO cell

- ✓ The 'FODO' cell is defined as follows:



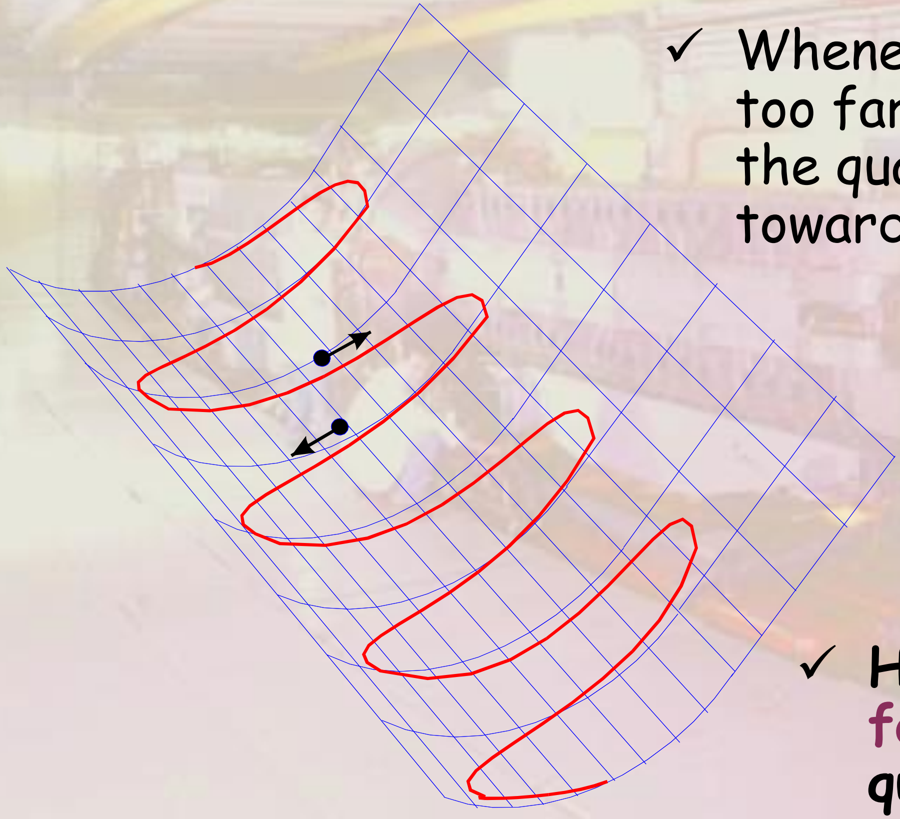


A Real Machine



The mechanical equivalent

- ✓ The gutter below illustrates how the particles in our accelerator behave due to the quadrupolar fields.

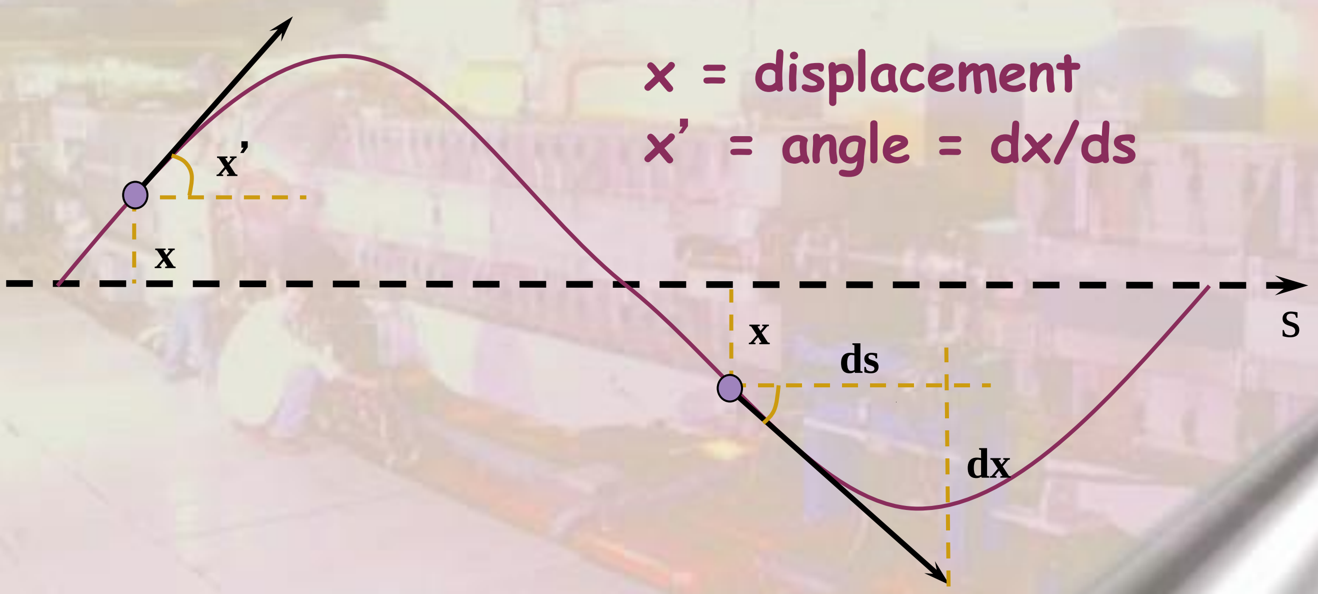


- ✓ Whenever a particle beam diverges too far away from the central orbit the quadrupoles focus them back towards the central orbit.

- ✓ **How can we represent the focusing gradient of a quadrupole in this mechanical equivalent ?**

The particle characterized

- A particle during its transverse motion in our accelerator is characterized by:
  - <u>Position</u> or displacement from the central orbit.
  - <u>Angle</u> with respect to the central orbit.



- This is a motion with a <u>constant restoring force</u>, like in the first lecture on differential equations, with the <u>pendulum</u>

Hill's equation

- These **betatron oscillations** exist in both **horizontal** and **vertical** planes.
- The **number of betatron oscillations per turn** is called the **betatron tune** and is defined as Qx and Qy.
- Hill's equation describes this motion mathematically

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

- If the restoring force, K is constant in 's' then this is just a **Simple Harmonic Motion**.
- 's' is the longitudinal displacement around the accelerator.

Hill's equation (2)

- In a real accelerator K varies strongly with 's'.
- Therefore we need to solve Hill's equation for K varying as a function of 's'

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

- What did we conclude on the mechanical equivalent concerning the shape of the gutter……?
- How is this related to Hill's equation……?

Questions….,Remarks…?