

AXEL-2023

Introduction to Particle Accelerators

Longitudinal instabilities:

- ✓ *Single bunch longitudinal instabilities*
- ✓ *Multi bunch longitudinal instabilities*
- ✓ *Different modes*
- ✓ *Bunch lengthening*

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Instabilities (1)

- # Until now we have only considered independent particle motion.
- # We call this incoherent motion.
 - single particle synchrotron/betatron oscillations
 - each particle moves independently of all the others
- # Now we have to consider what happens if all particles move in phase, coherently, in response to some excitations

Synchrotron & betatron
oscillations

Instabilities (2)

- # We cannot ignore interactions between the charged particles
- # They interact with each other in two ways:

- Direct Coulomb interaction between particles

Space charge effects, intra beam scattering

- Via the vacuum chamber

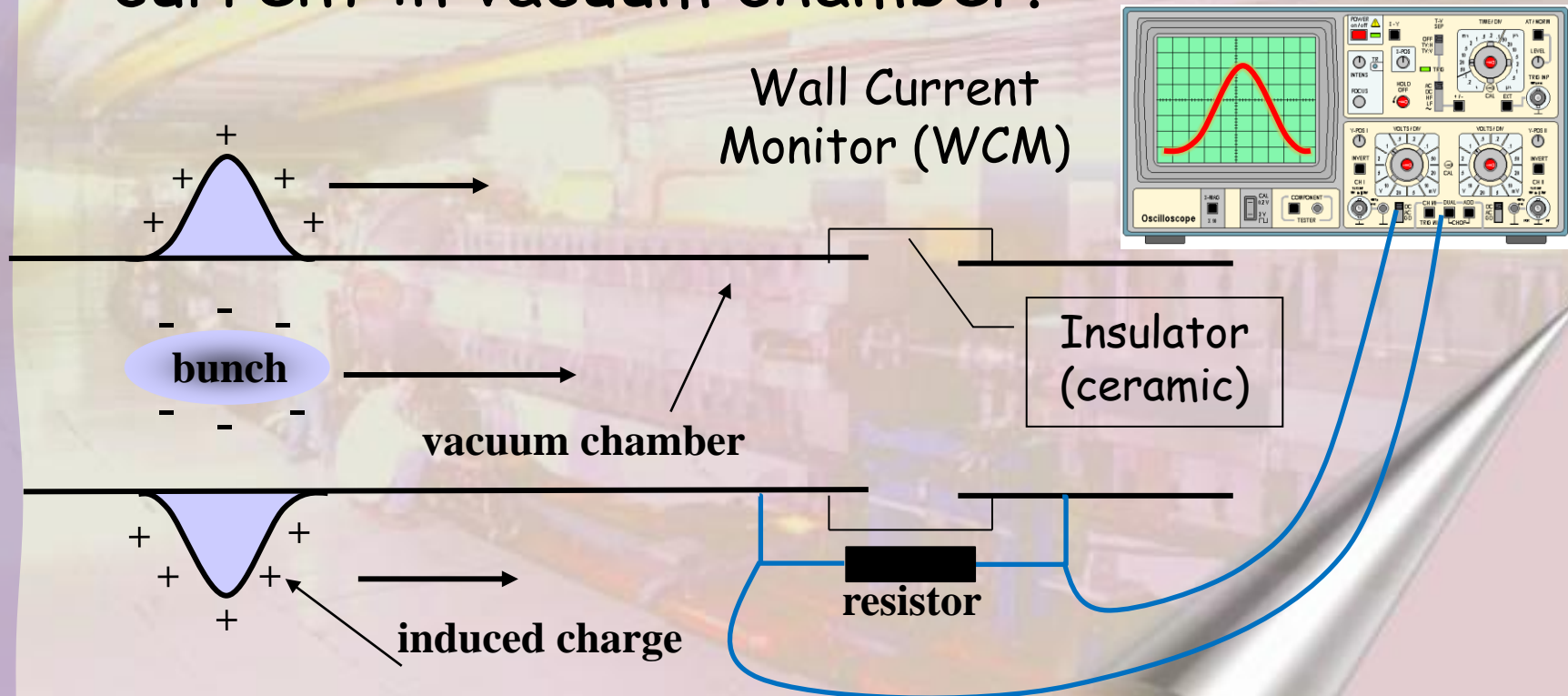
Longitudinal and transverse beam instabilities

Why do Instabilities arise?

- # A circulating bunch induces electro magnetic fields in the vacuum chamber
- # These fields act back on the particles in the bunch
- # Small perturbations to the bunch motion, change the induced EM fields
- # If this change amplifies the perturbation then we have an instability

Measuring Longitudinal Instabilities

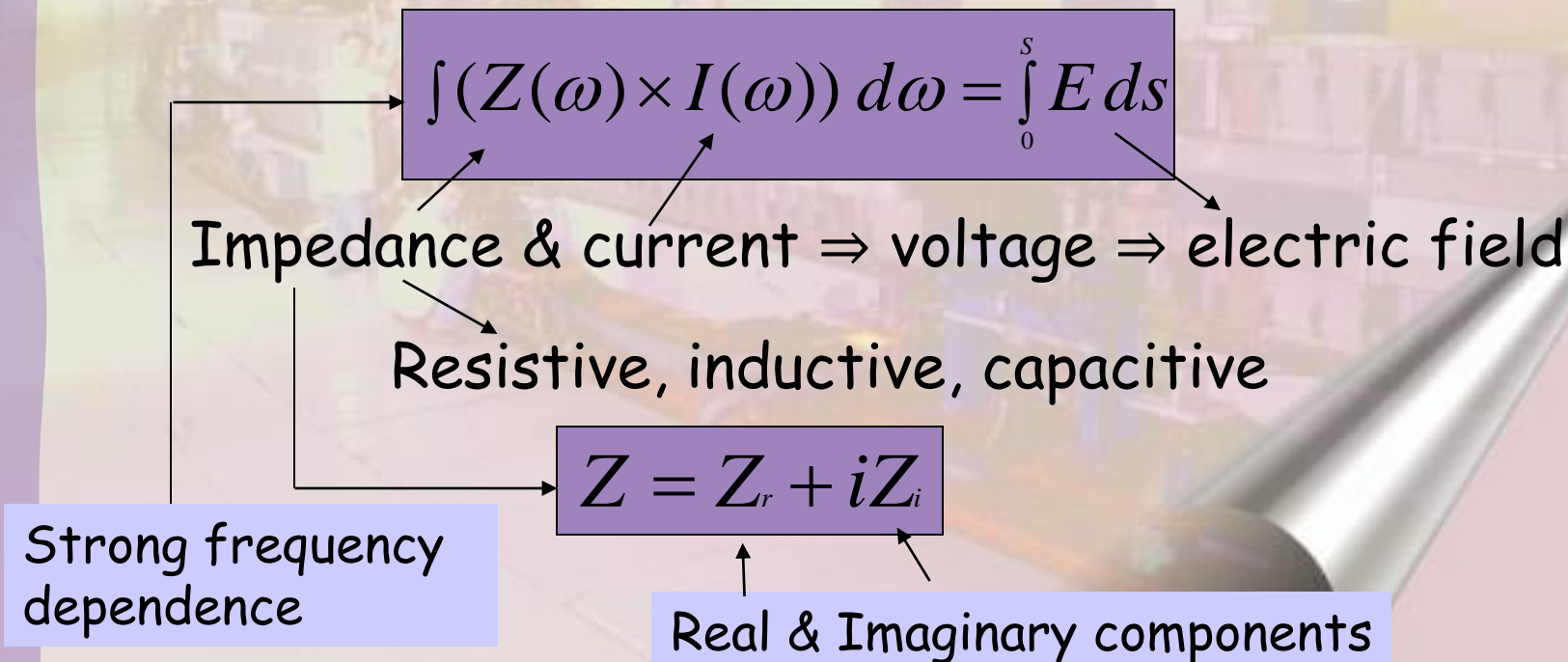
- # A circulating bunch creates an image current in vacuum chamber.



- # The induced image current is the same size but has the opposite sign to the bunch current.

Impedance and Wall current (1)

- # The vacuum chamber presents an impedance to this induced wall current (changes of shape, material etc.)
- # The image current combined with this impedance induces a voltage, which in turn affects the charged particles in the bunch



Impedance and Wall current (2)

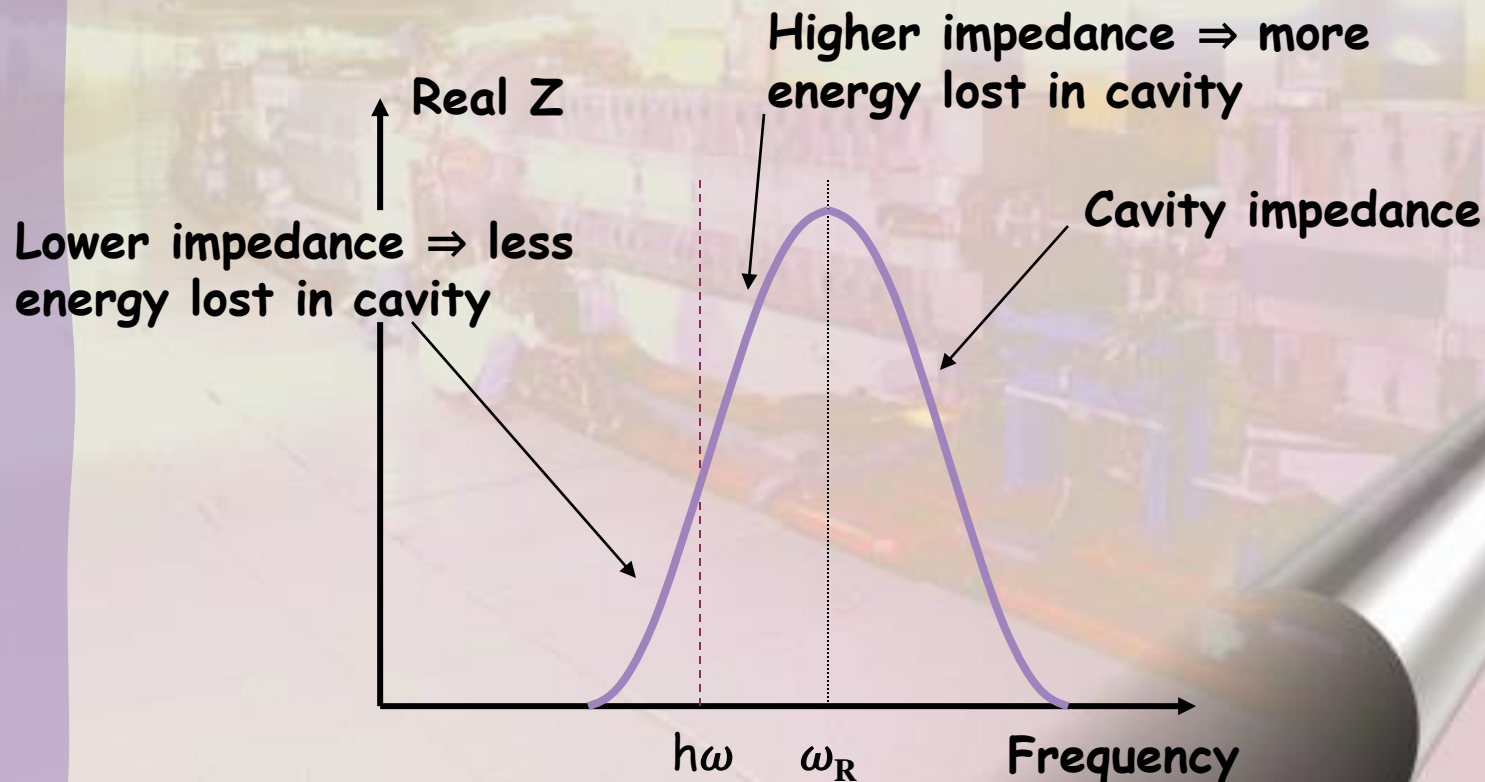
- # Any change of cross section or material leads to a finite impedance
- # We can describe the vacuum chamber as a series of cavities
 - **Narrow band** - High Q resonators - RF Cavities tuned to some harmonic of the revolution frequency
 - **Broad band** - Low Q resonators - rest of the machine
- # For any cavity two frequencies are important:
 - ω = Excitation frequency (bunch frequency)
 - ω_R = Resonant frequency of the cavity
- # If $h\omega \approx \omega_R$ then the induced voltage will be large and will build up with repeated passages of the bunch

h is an
integer

Single bunch Longitudinal Instabilities (1)

Lets consider:

- A single bunch with a revolution frequency = ω
- That this bunch is not centered in the long. Phase Space
- A single high-Q cavity which resonates at ω_R ($\omega_R \approx h\omega$)

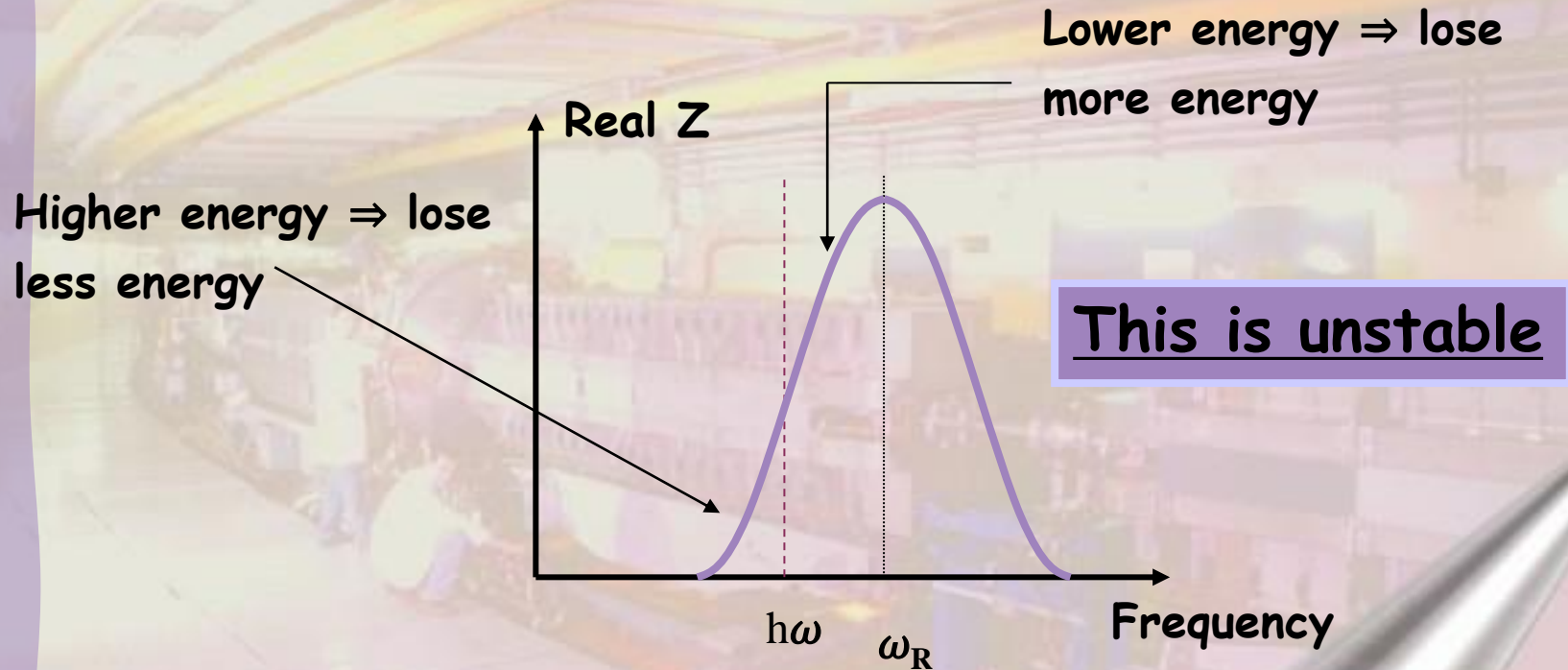


Single bunch Longitudinal Instabilities (2)

- # Lets start a coherent synchrotron oscillation (above transition)
- # The bunch will gain and lose energy/momentum
- # There will be a decrease and increase in revolution frequency
- # Therefore the bunch will see changing cavity impedance
- # Lets consider two cases:
 - First case, consider $\omega_R > h\omega$
 - Second case, consider $\omega_R < h\omega$

Single bunch Longitudinal Instabilities (3)

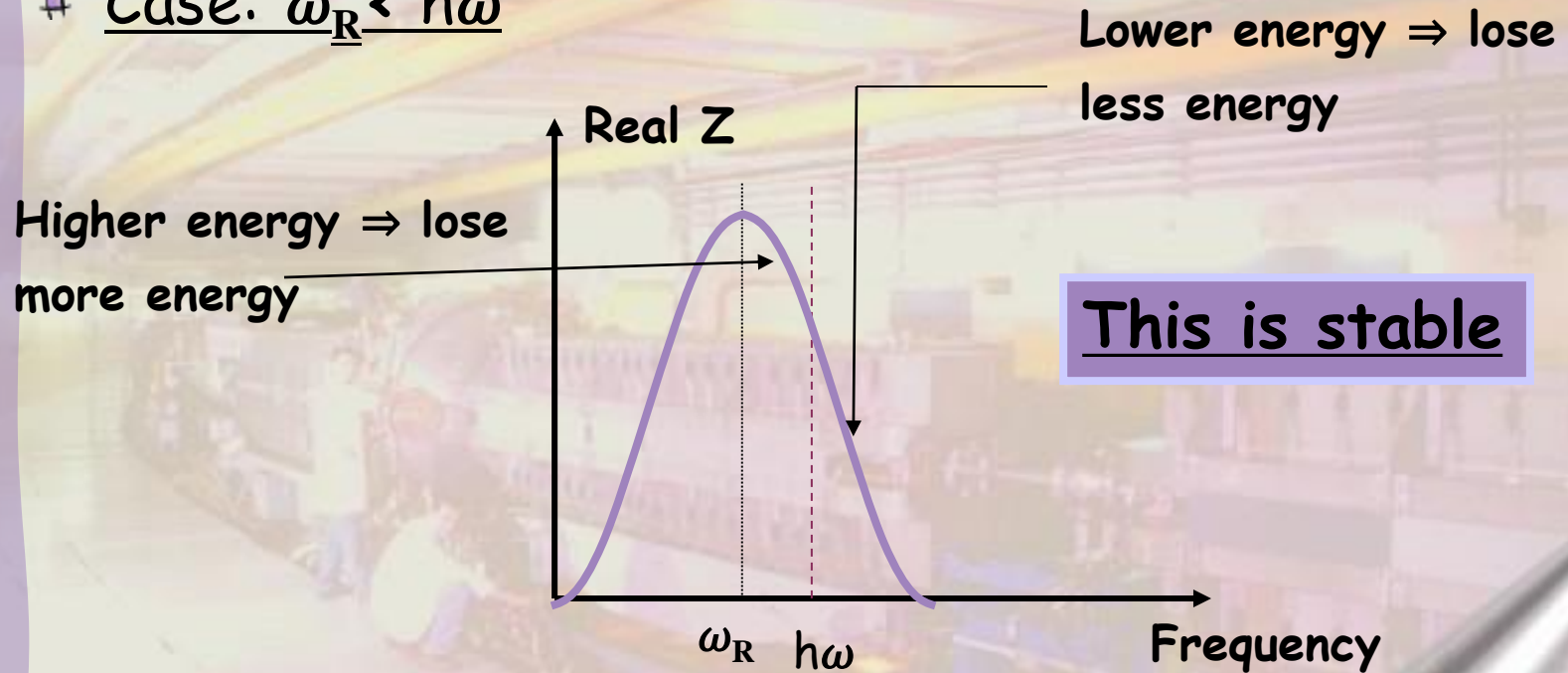
Case: $\omega_R > h\omega$



- # The cavity tends to increase the energy oscillations
- # Now retune cavity so that $\omega_R < h\omega$

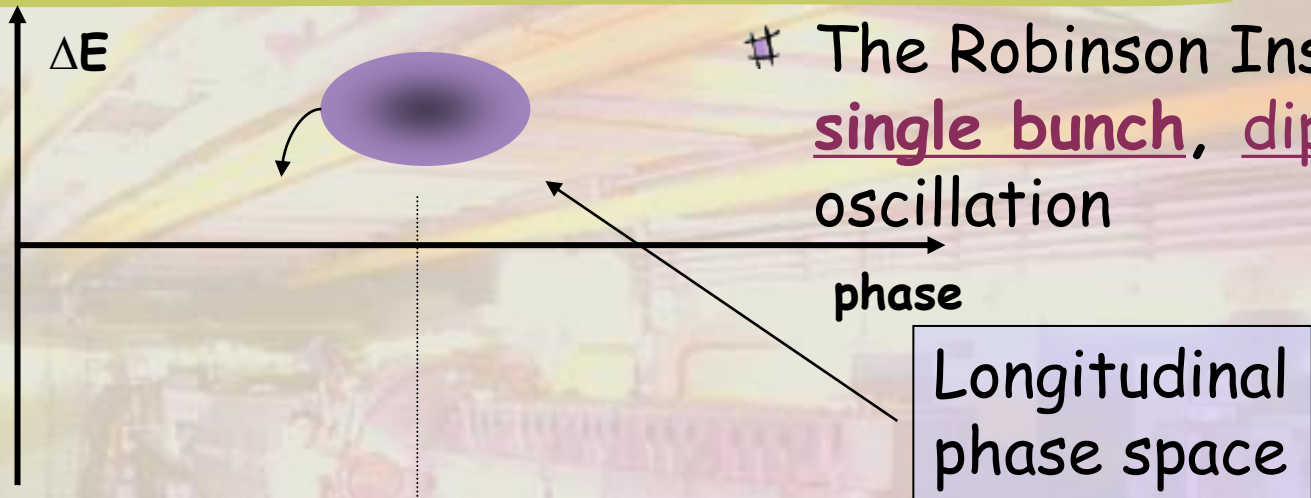
Single bunch Longitudinal Instabilities (3)

Case: $\omega_R < h\omega$



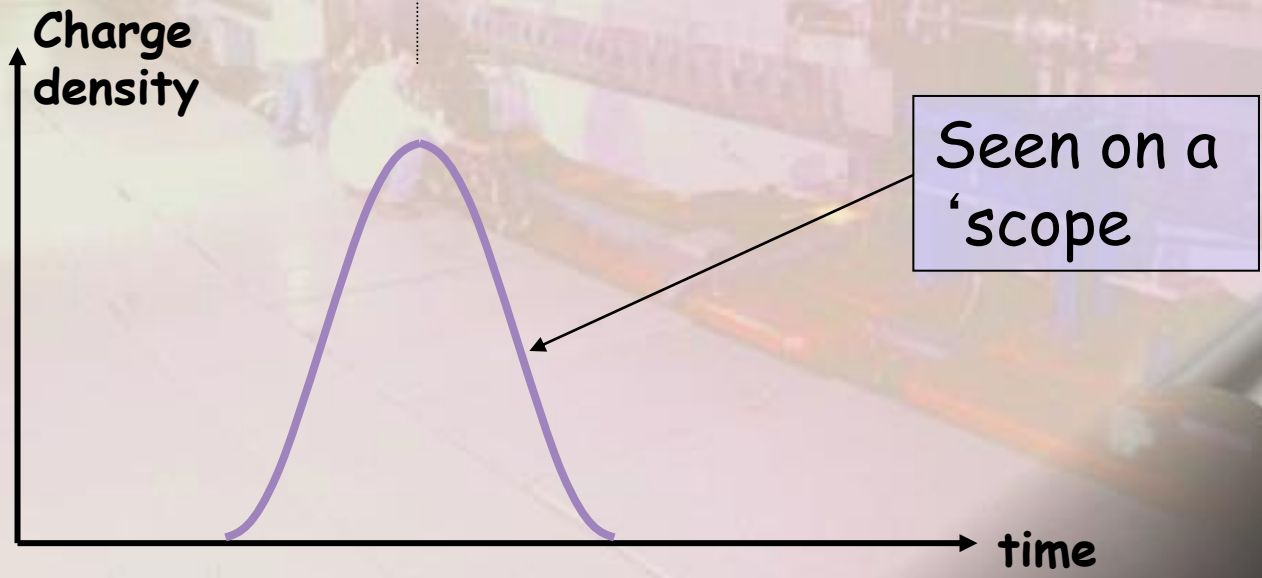
- # This is known as the 'Robinson Instability'
- # To damp this instability one should retune the cavity so that $\omega_R < h\omega$

Robinson Instability (1)

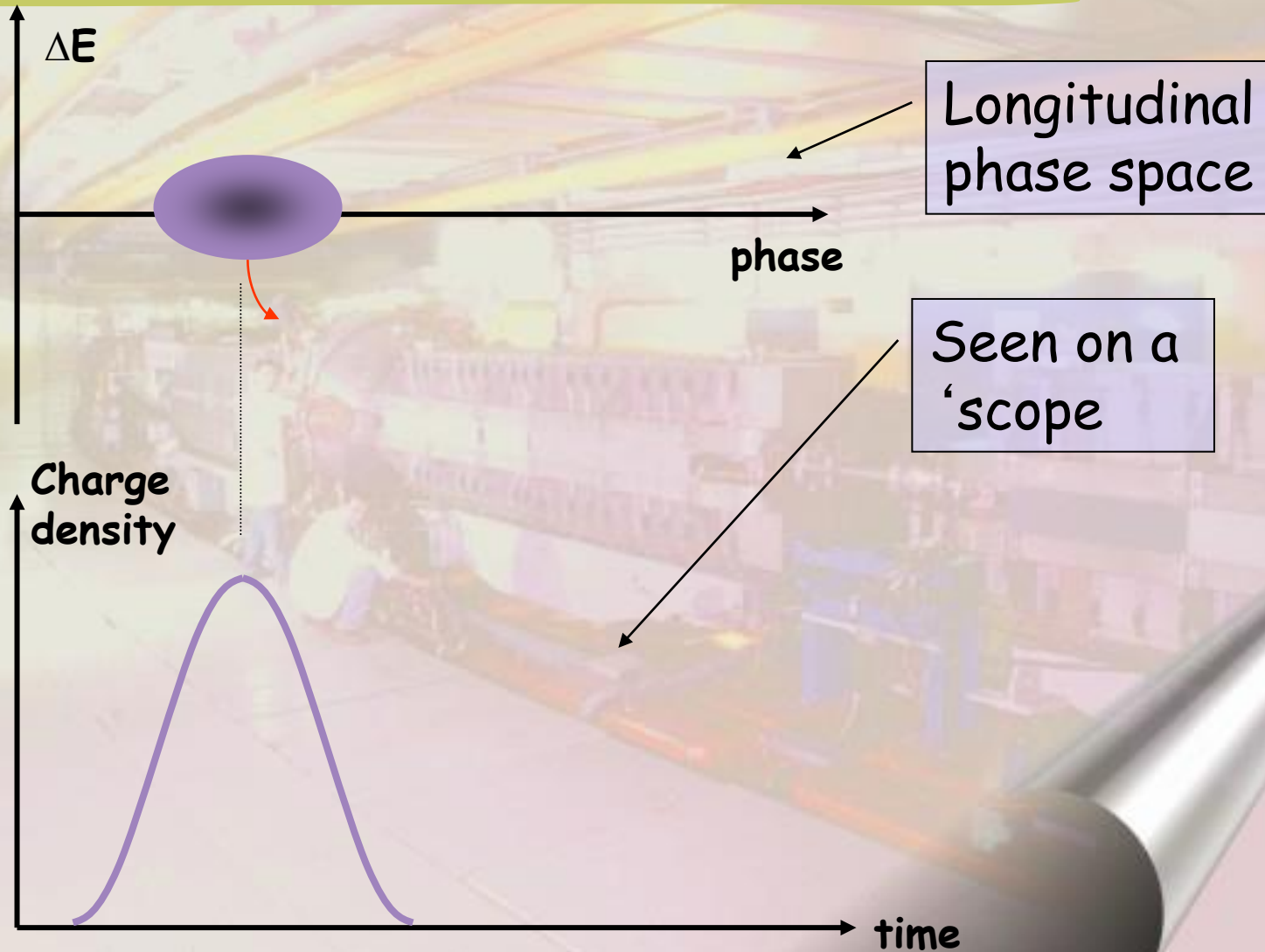


The Robinson Instability is a single bunch, dipole mode oscillation

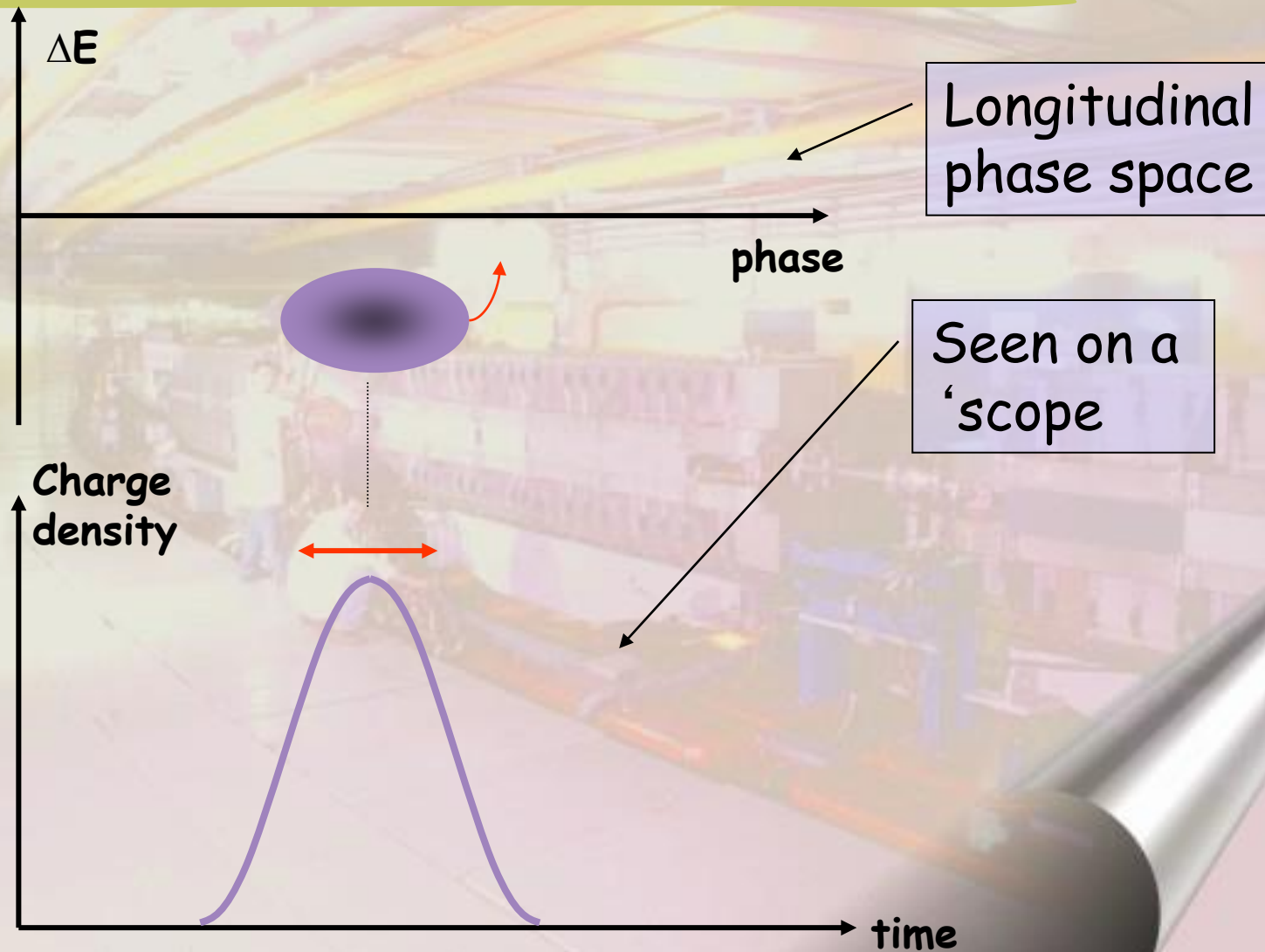
Longitudinal phase space



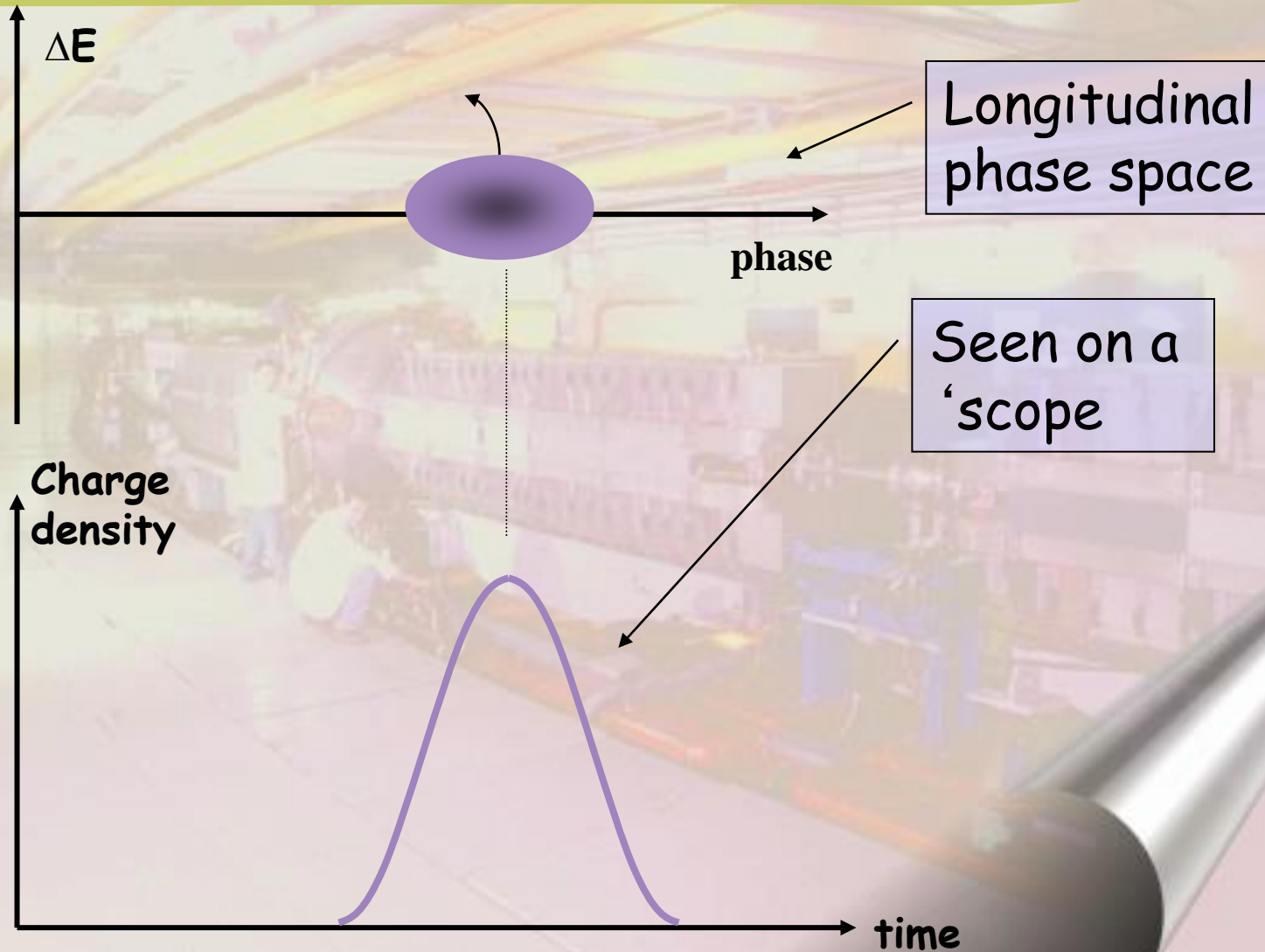
Robinson Instability (2)



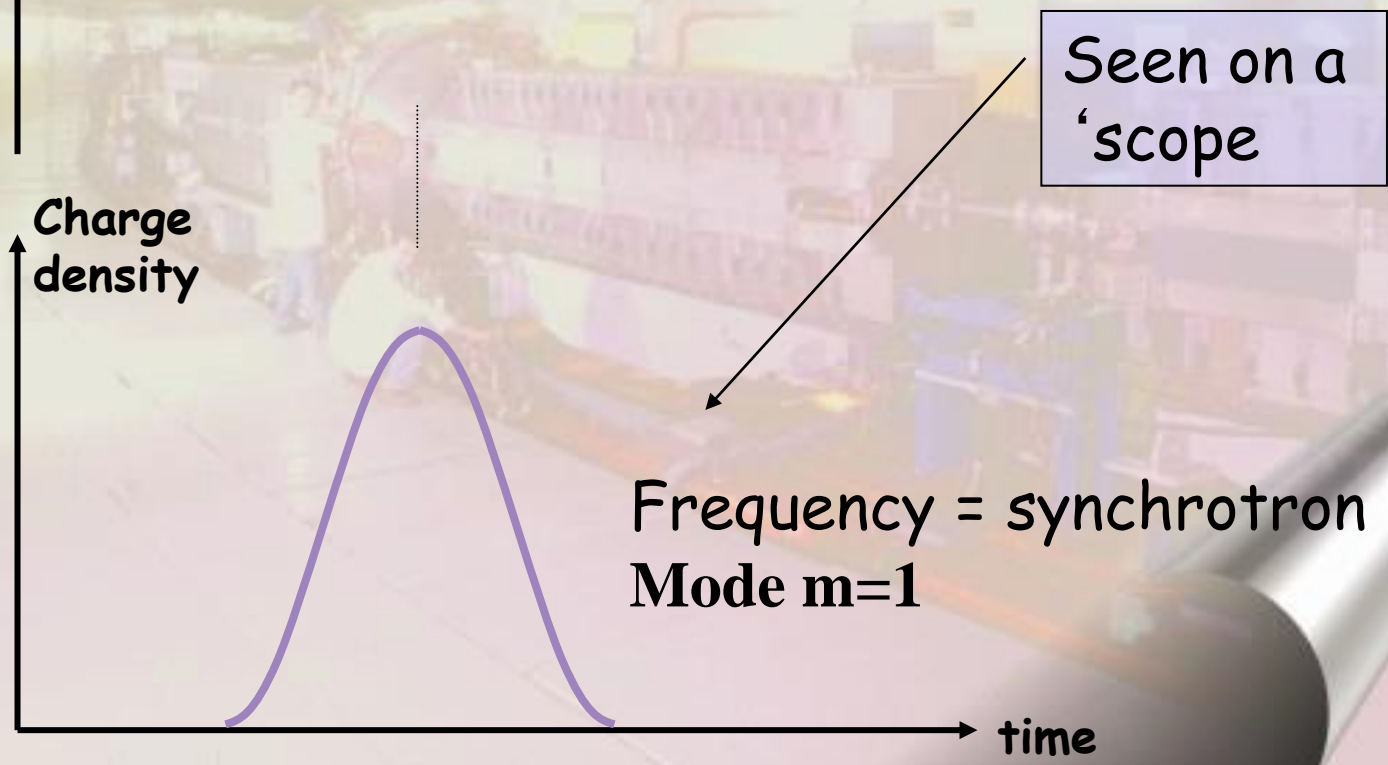
Robinson Instability (3)



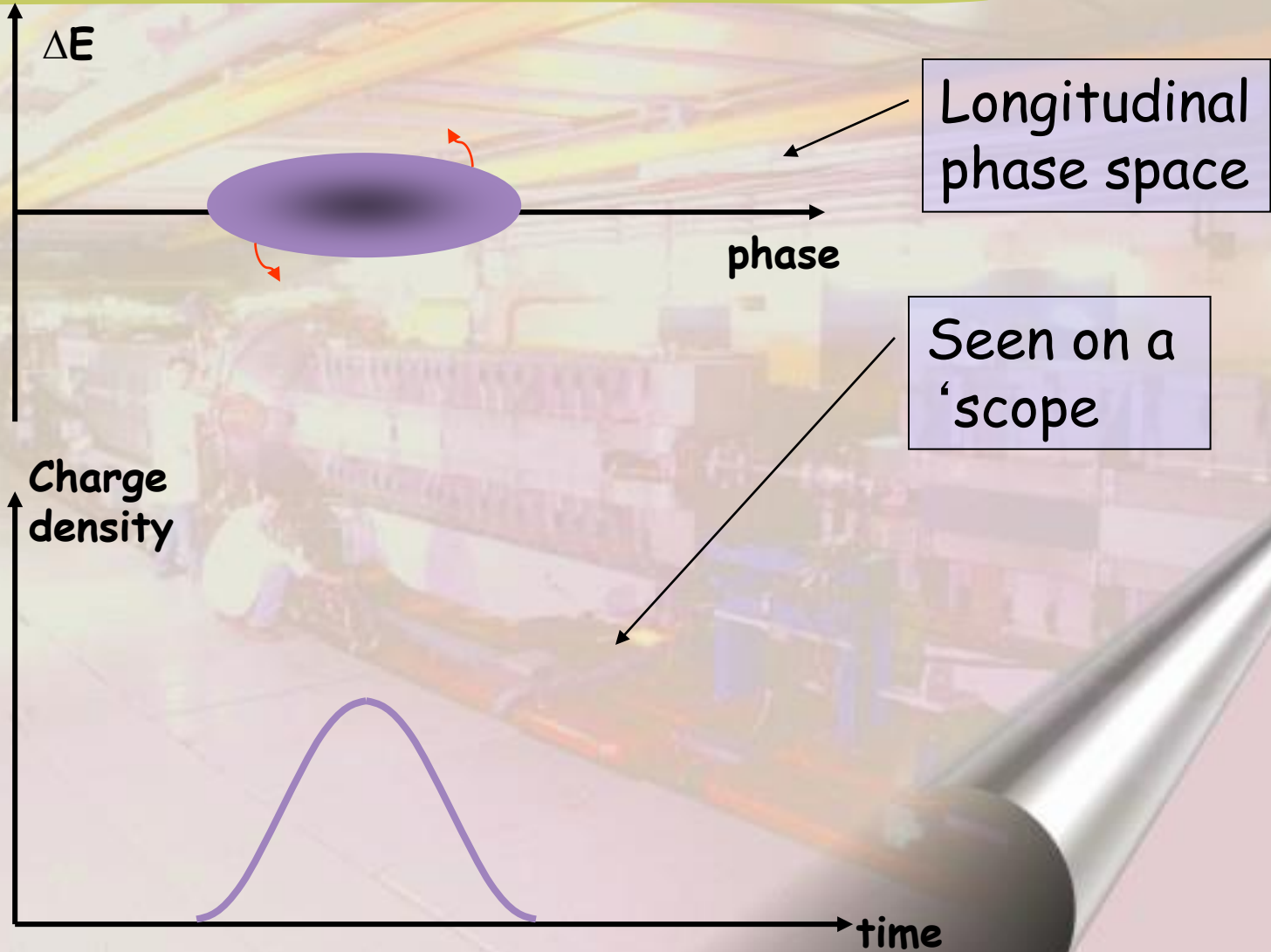
Robinson Instability (4)



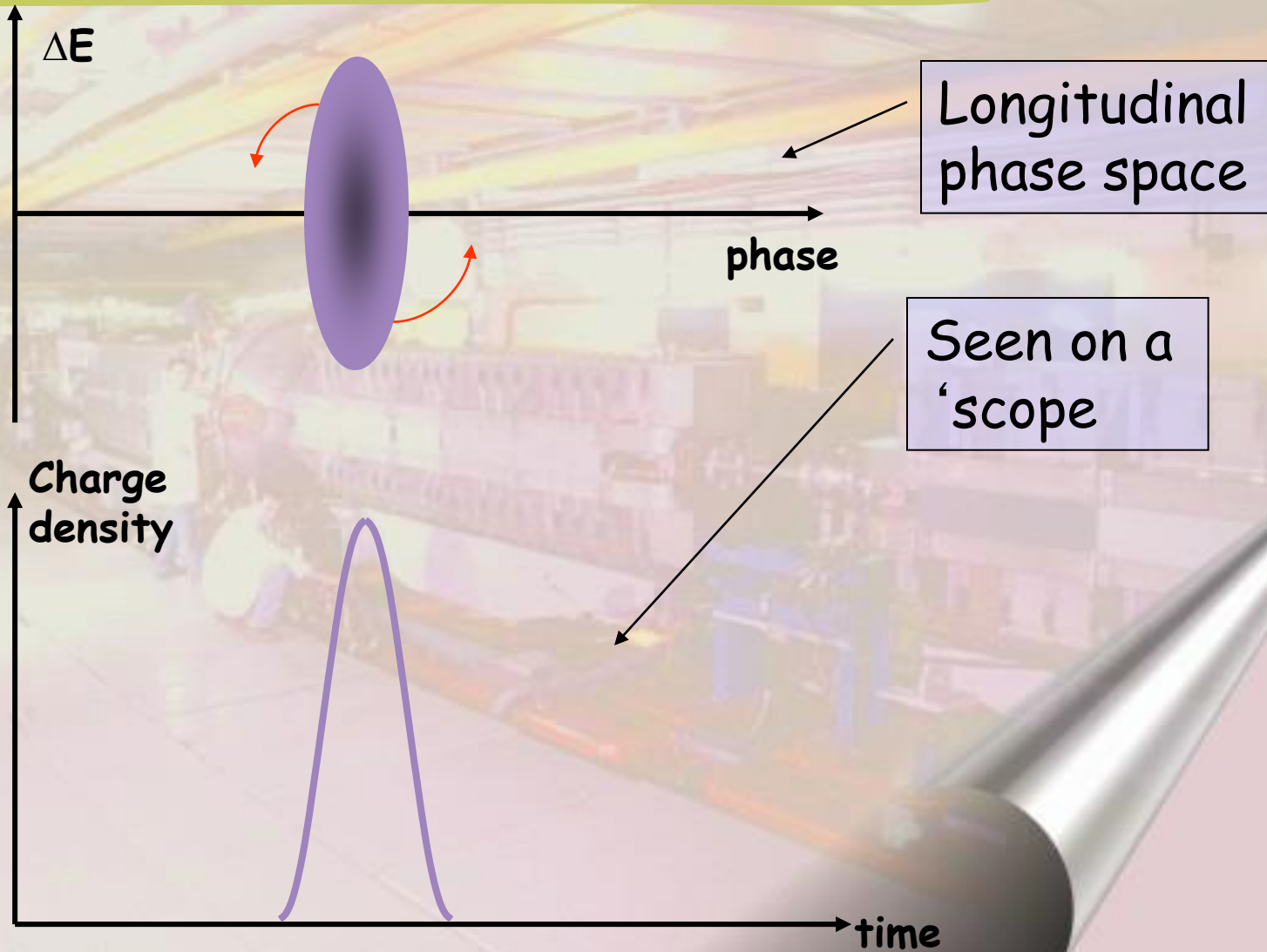
Robinson Instability (5)



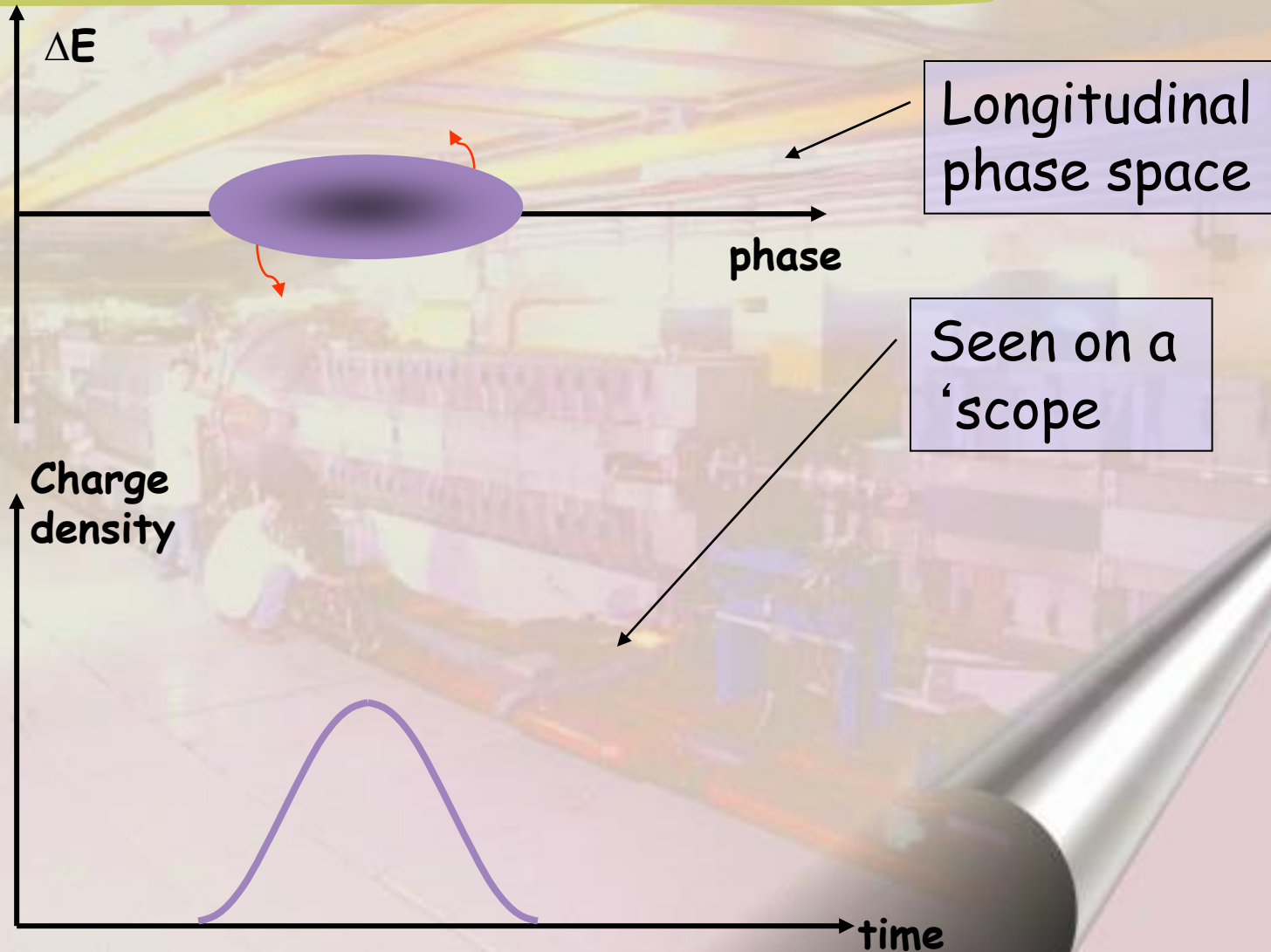
Higher order modes $m=2$ (1)



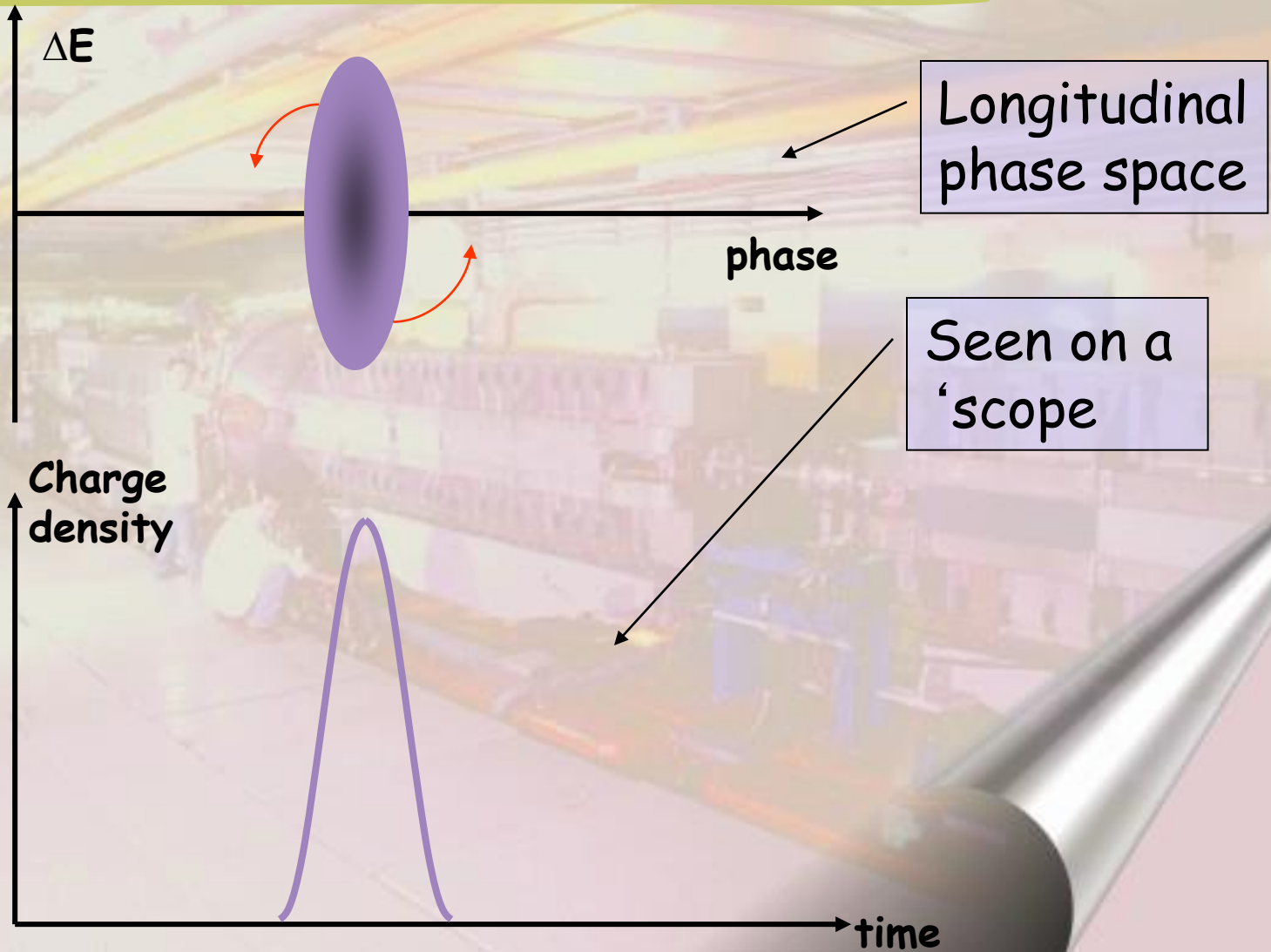
Higher order modes $m=2$ (2)



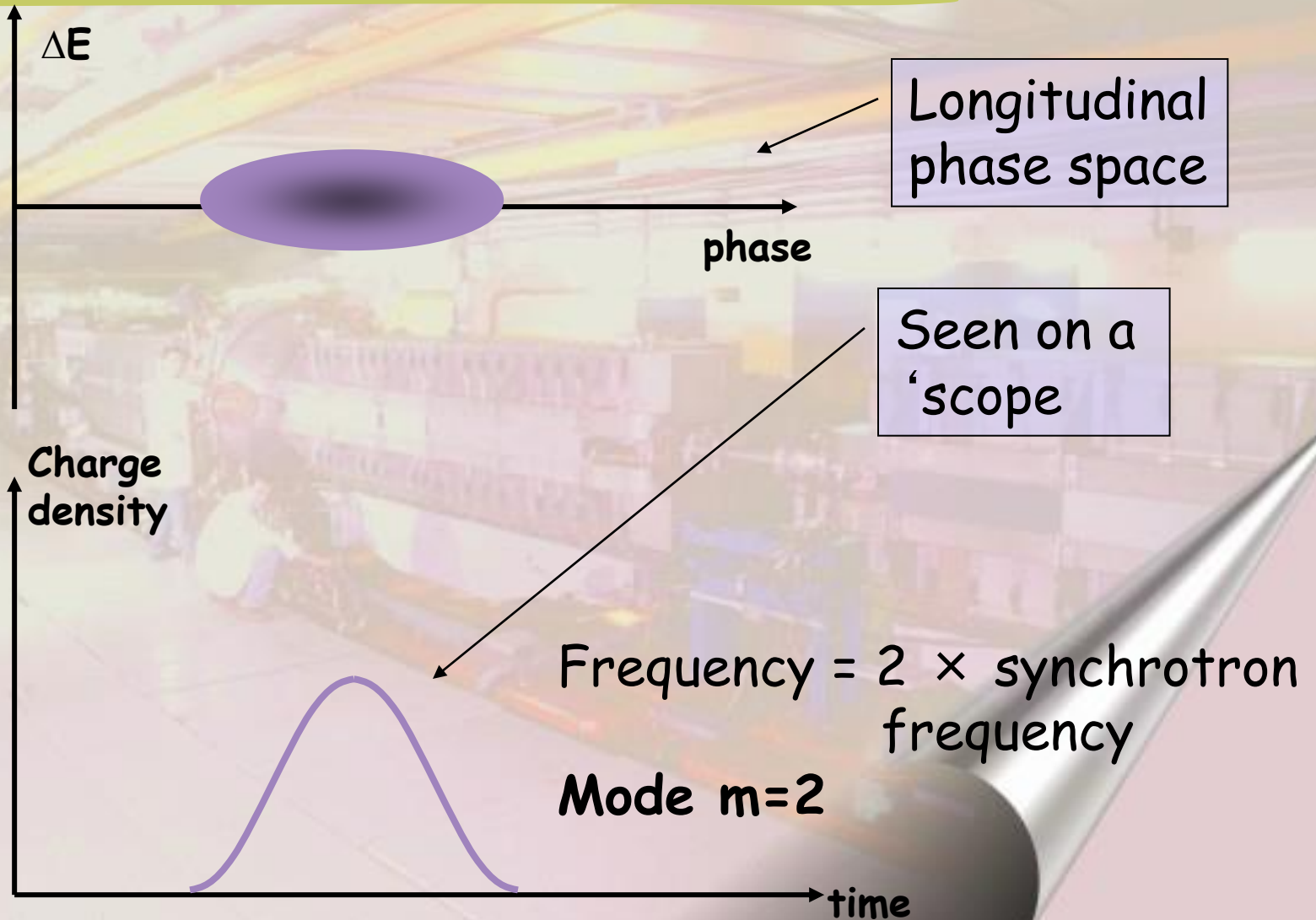
Higher order modes $m=2$ (3)



Higher order modes $m=2$ (4)



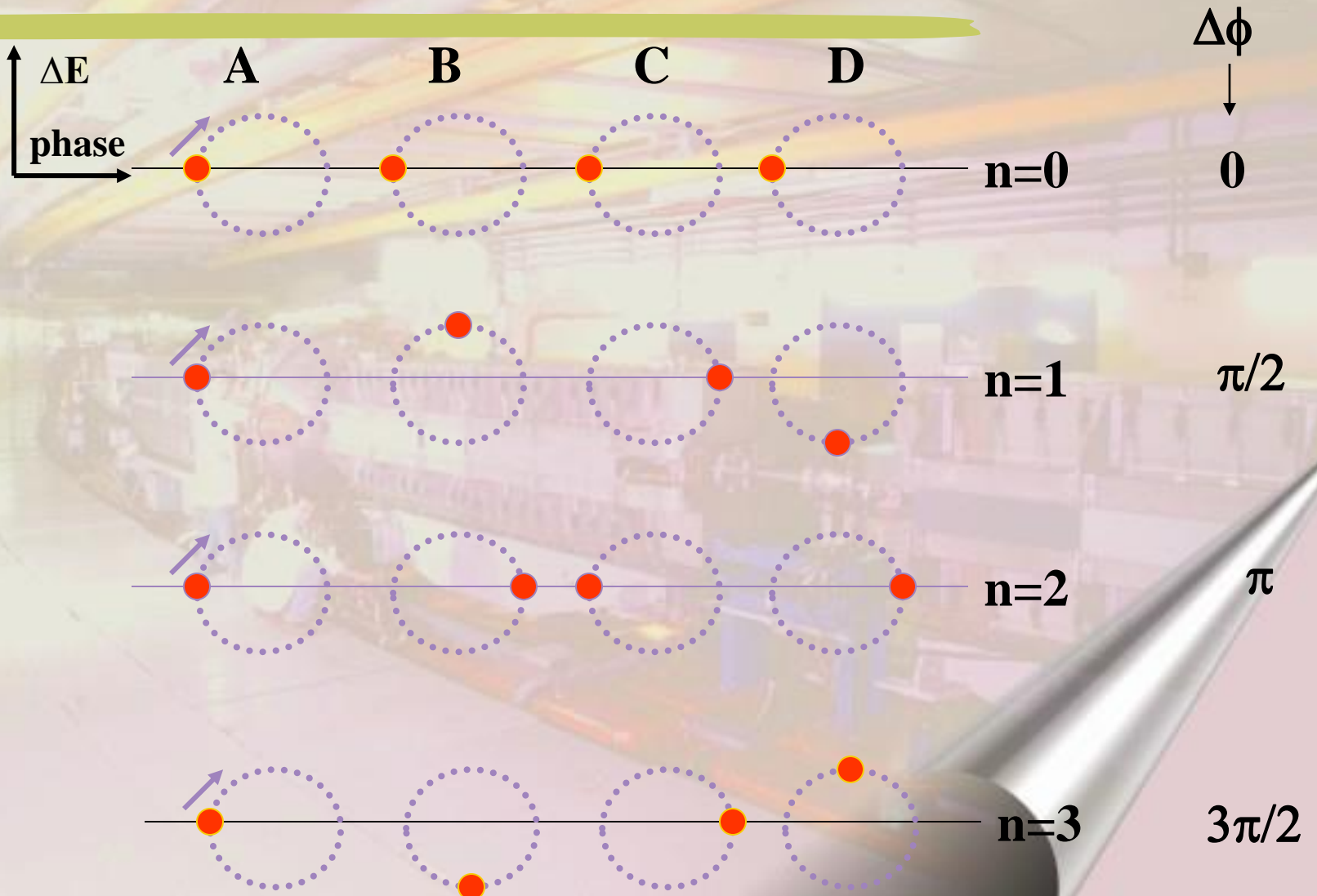
Higher order modes $m=2$ (5)



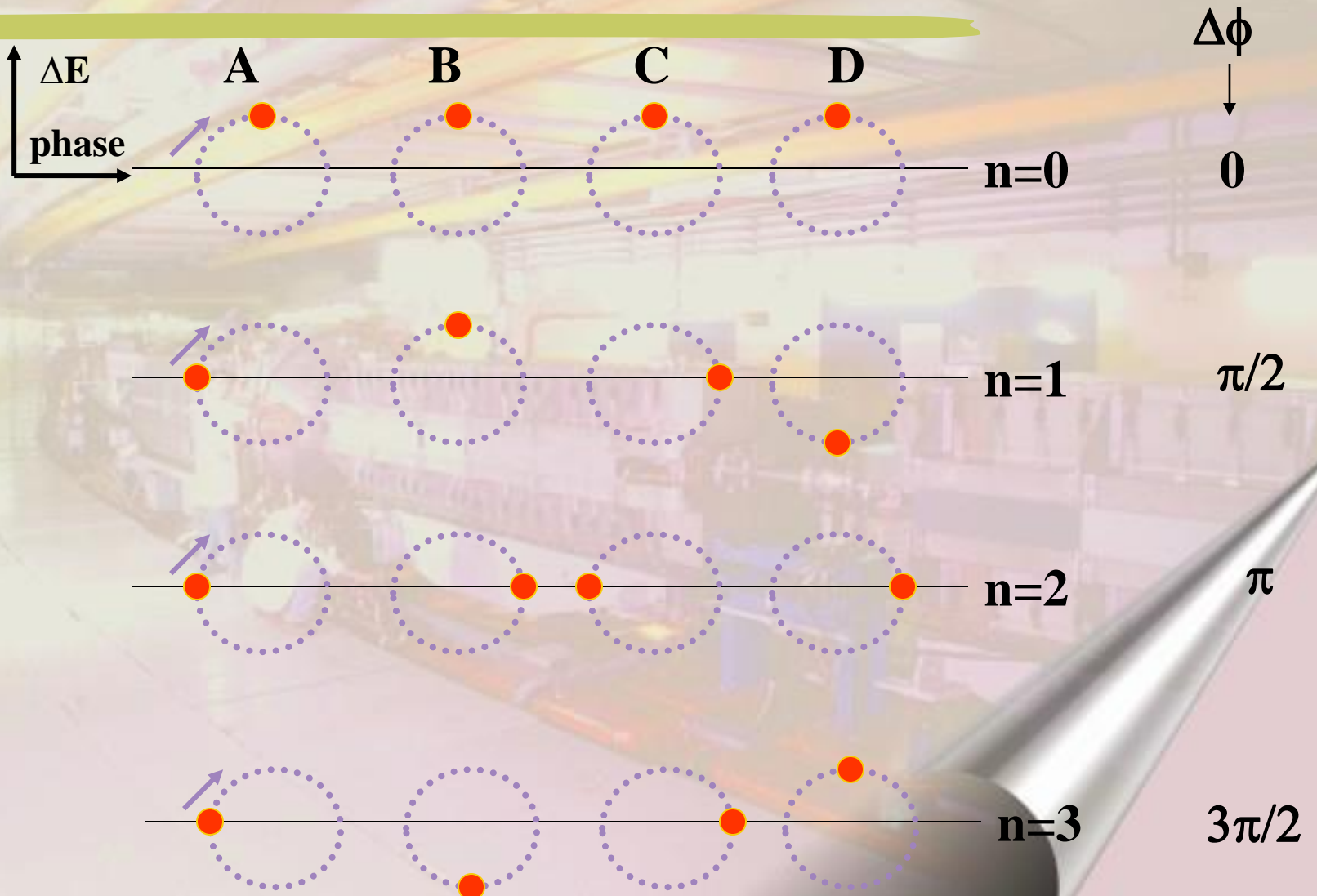
Multi-bunch instabilities (1)

- # What if we have more than one bunch in our ring.....?
- # Lets take 4 equidistant bunches **A**, **B**, **C** & **D**
- # The field left in the cavity by bunch **A** alters the coherent synchrotron oscillation motion of **B**, which changes field left by bunch **B**, which alters bunch **C**.....to bunch **D**, etc...etc..
- # Until we get back to bunch **A**.....
- # For 4 bunches there are 4 possible modes of coupled bunch longitudinal oscillation

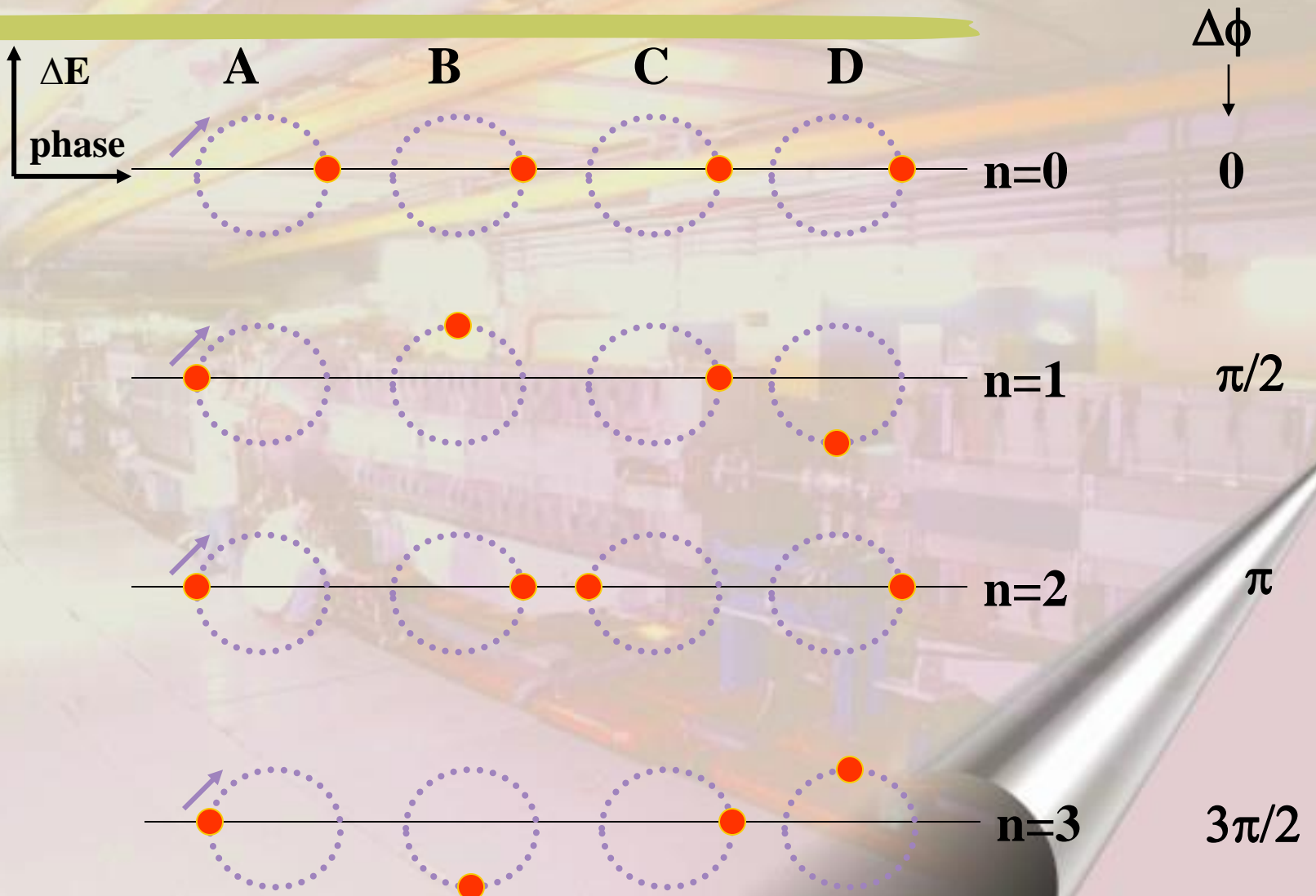
Multi-bunch instabilities (2)



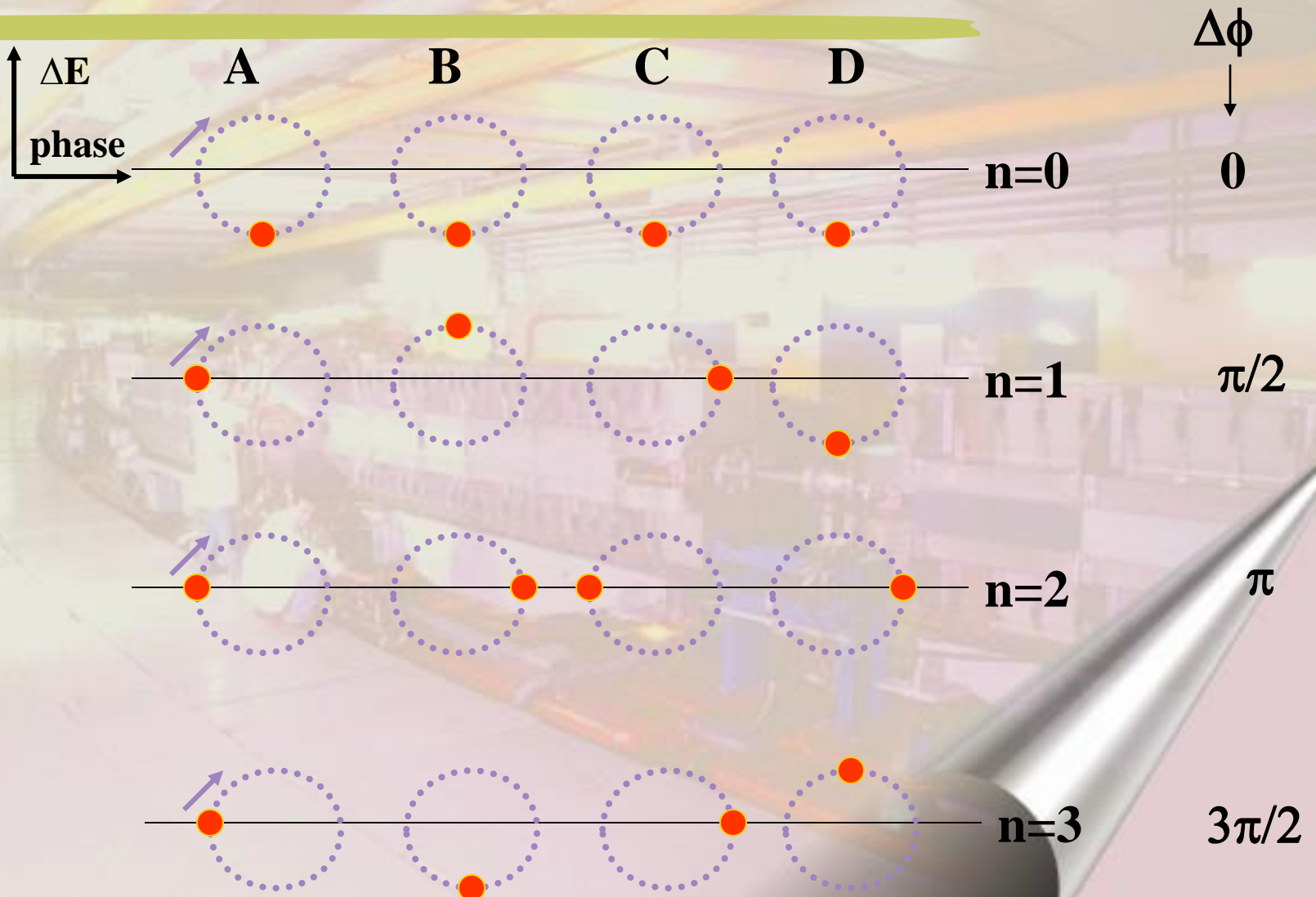
Multi-bunch instabilities (3)



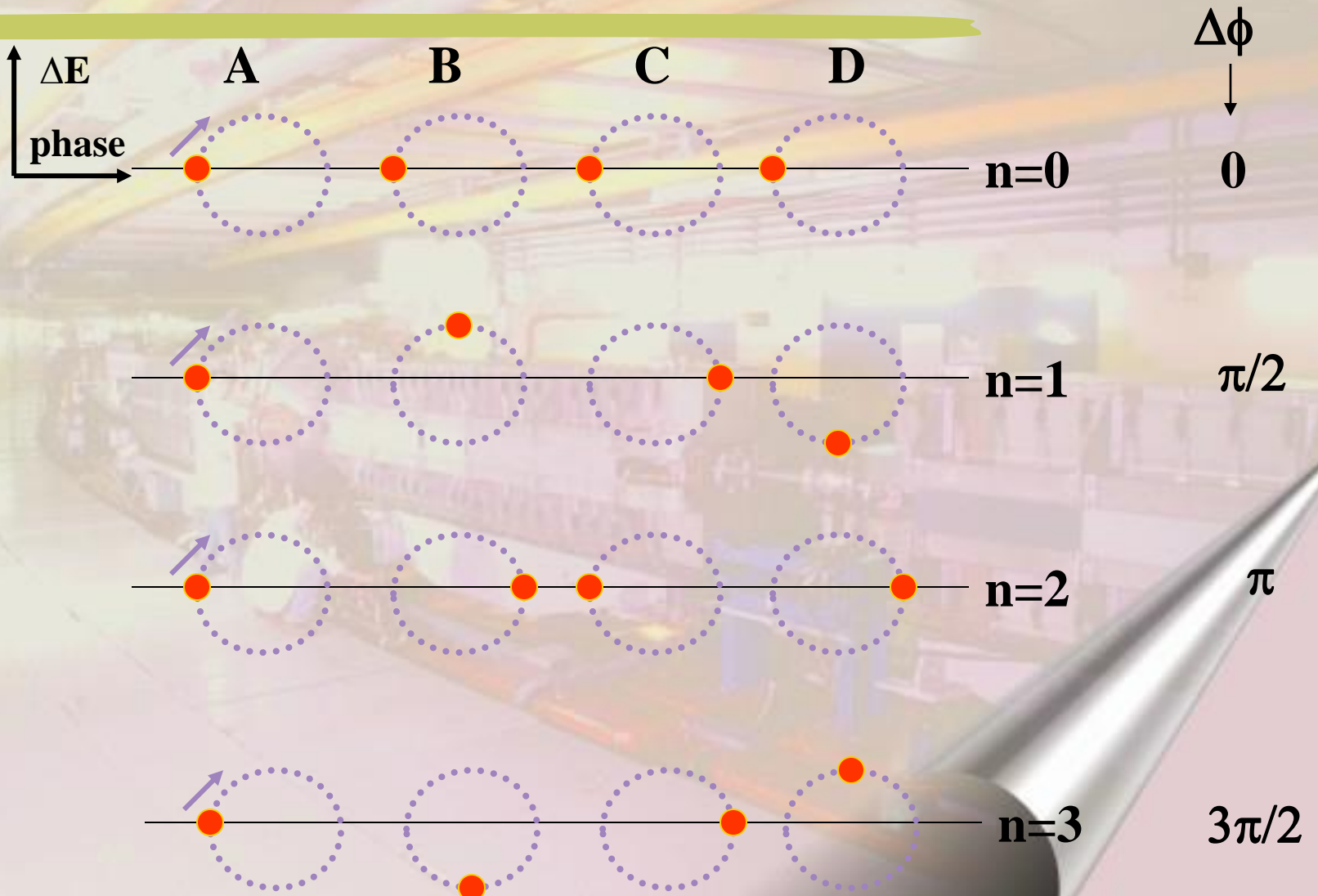
Multi-bunch instabilities (4)



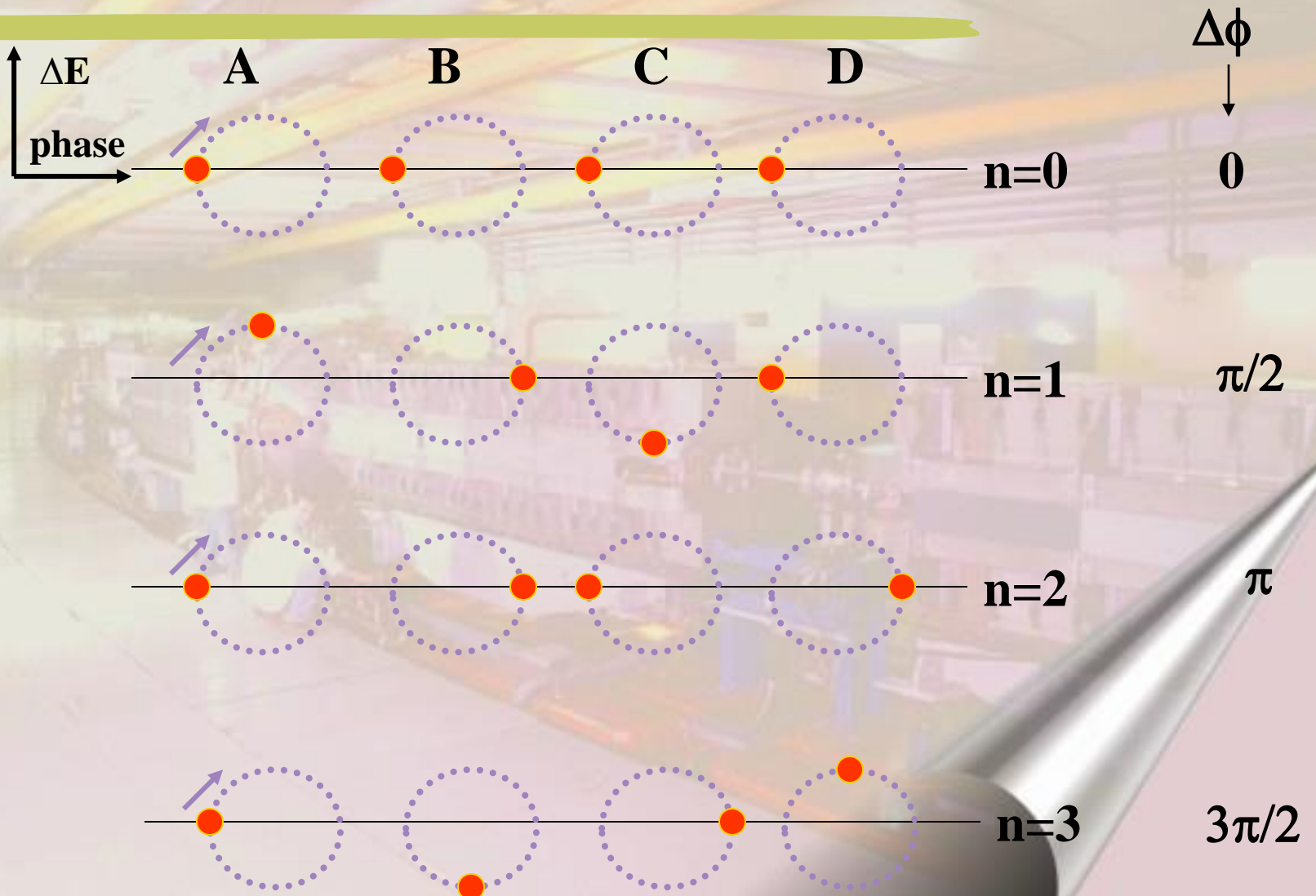
Multi-bunch instabilities (5)



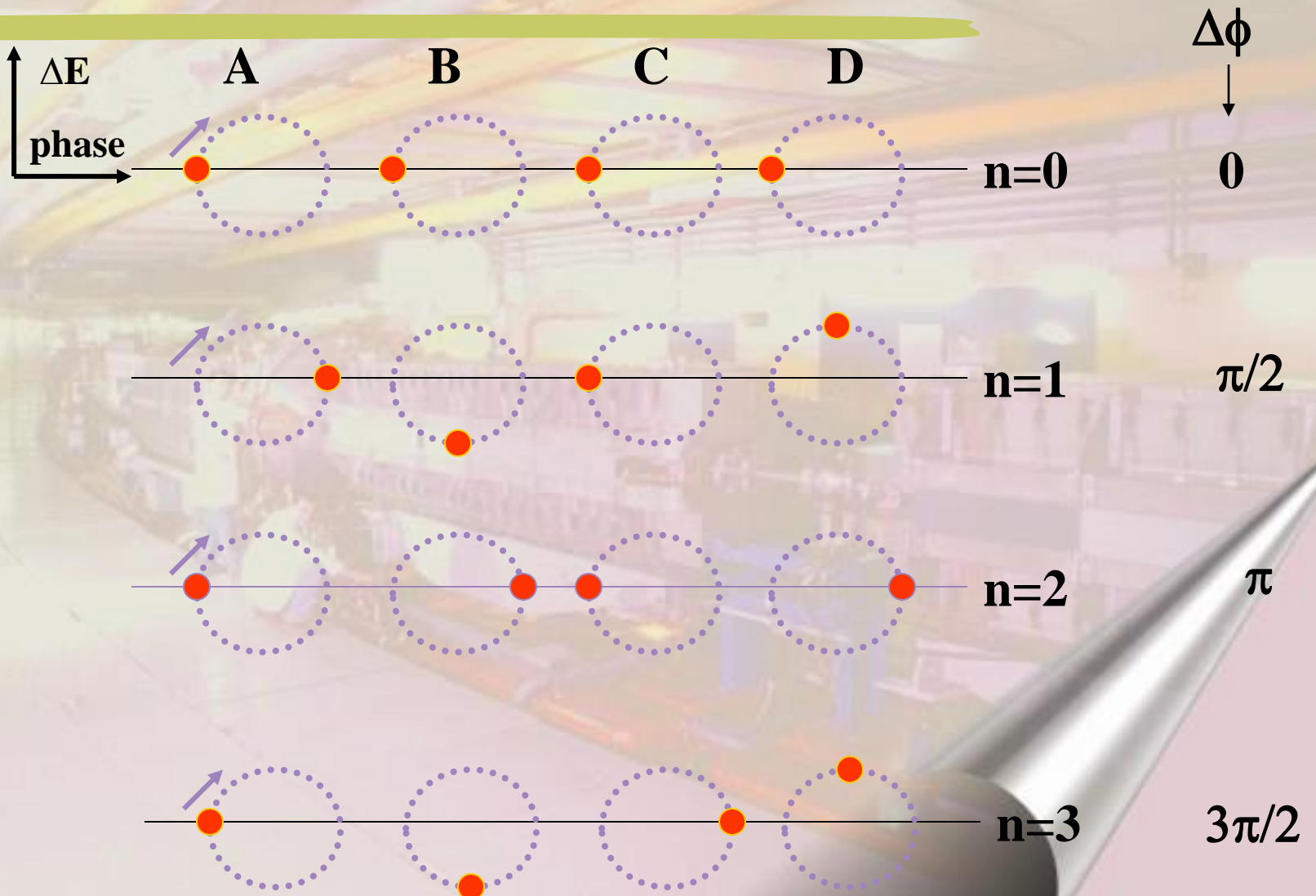
Multi-bunch instabilities (6)



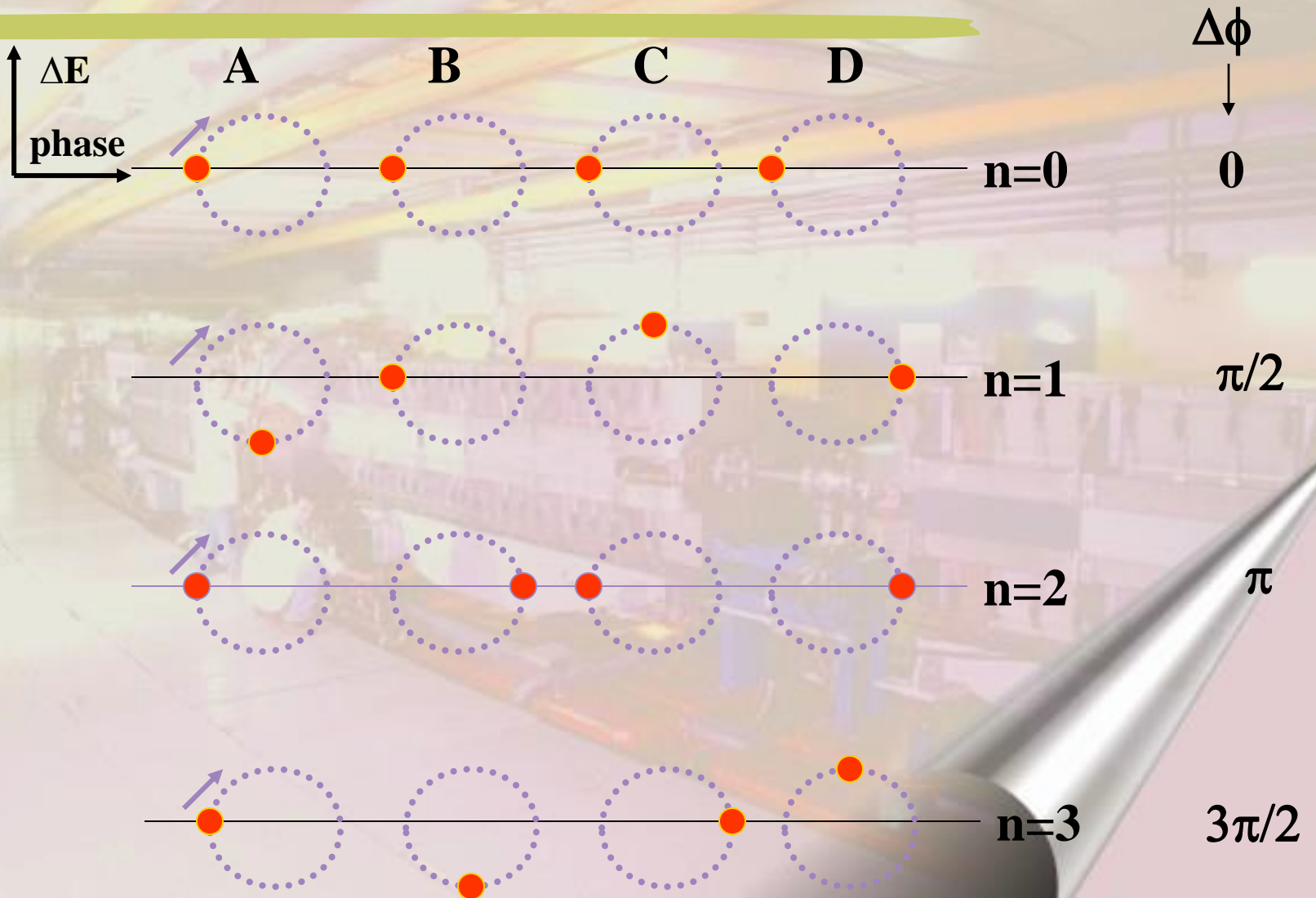
Multi-bunch instabilities (7)



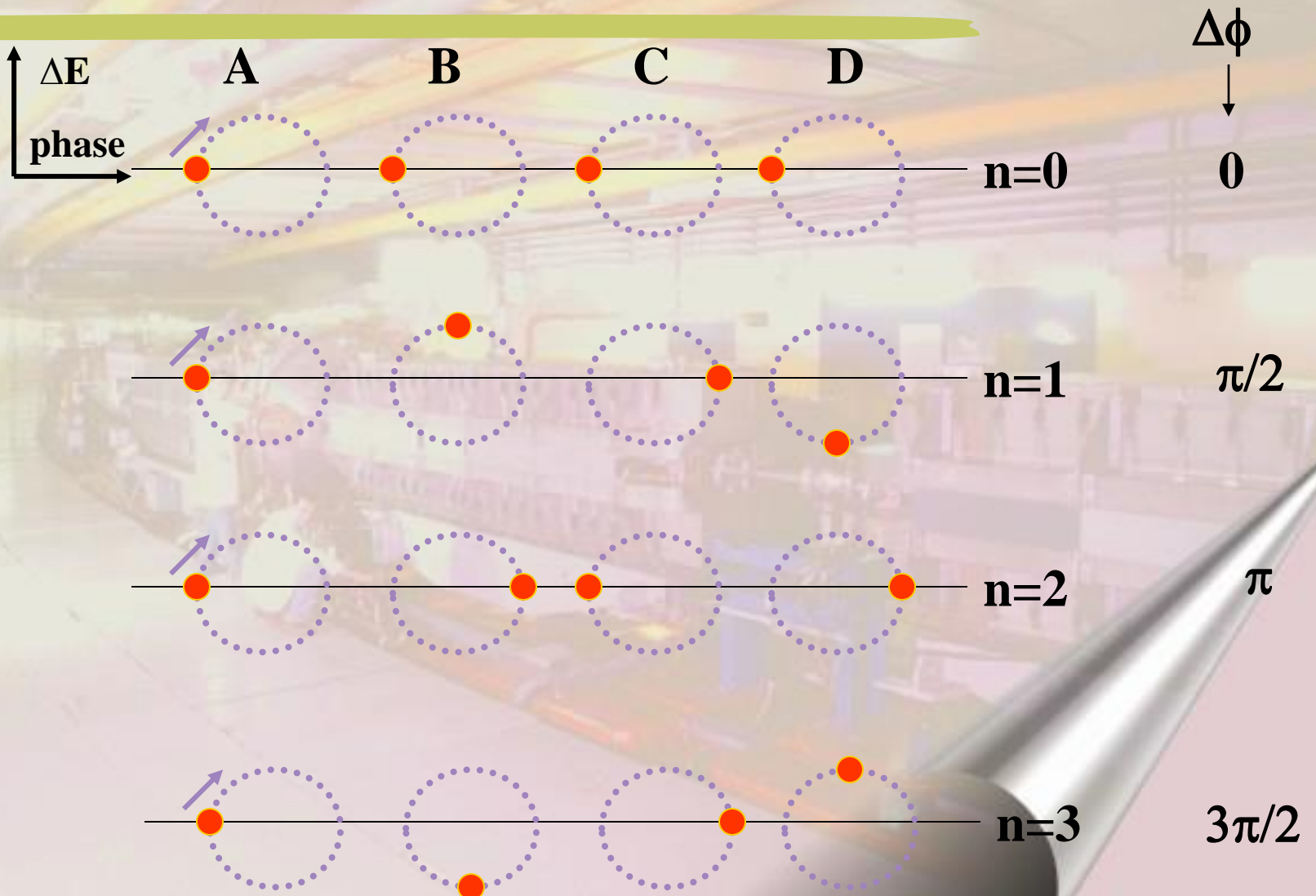
Multi-bunch instabilities (8)



Multi-bunch instabilities (9)

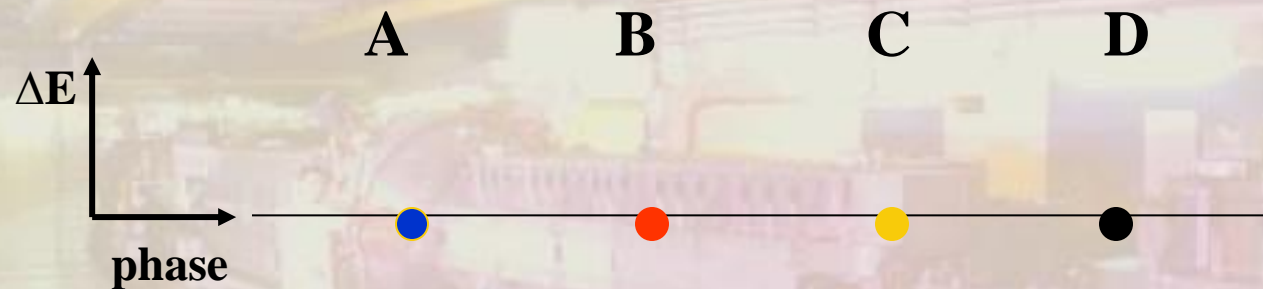


Multi-bunch instabilities (10)



Multi-bunch instabilities (11)

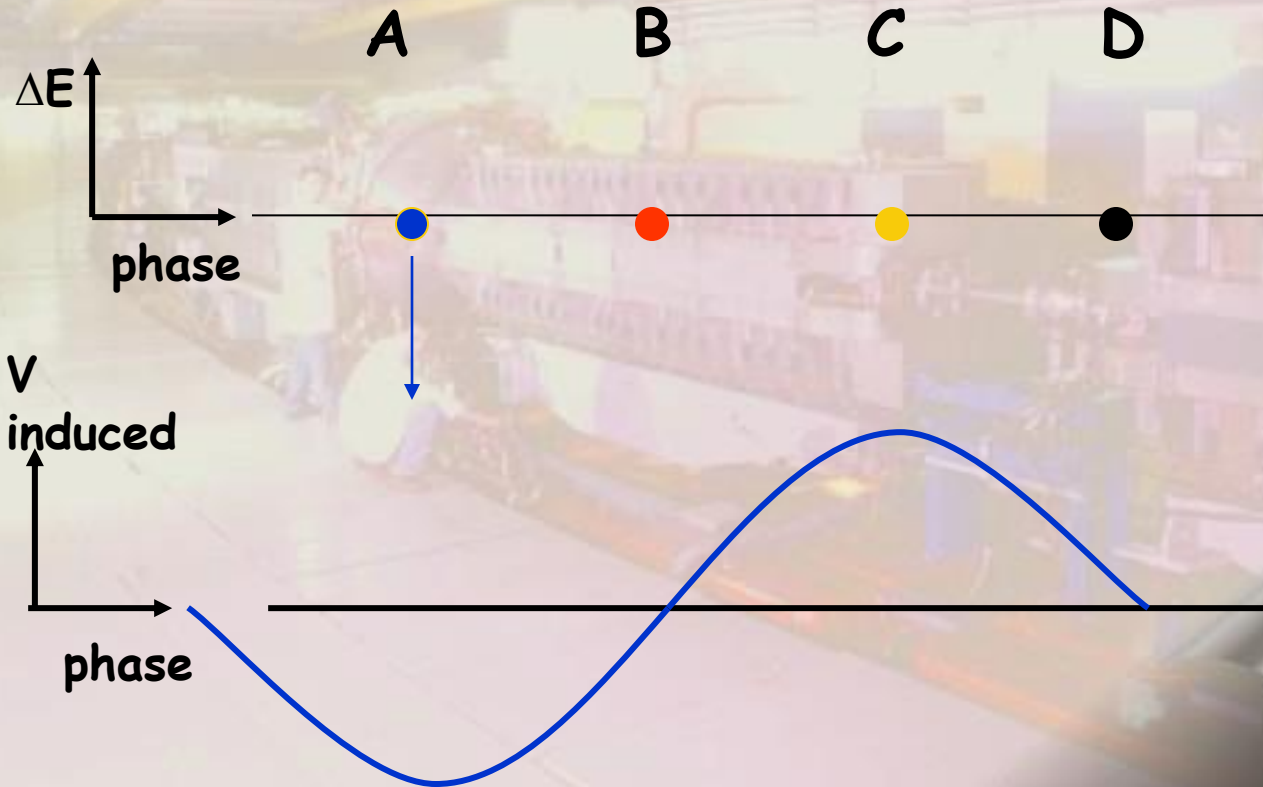
- # For simplicity assume we have a single cavity which resonates at the revolution frequency
- # With no coherent synchrotron oscillation we have:



- # Lets have a look at the voltage induced in a cavity by each bunch

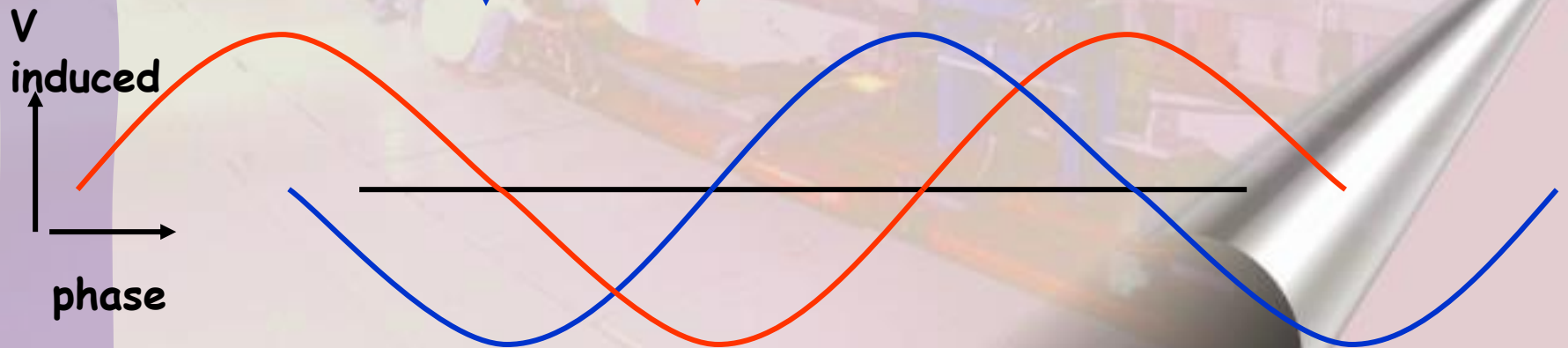
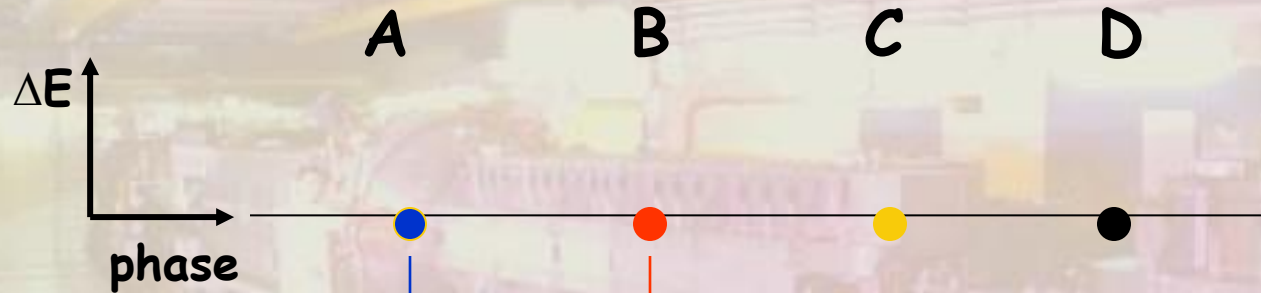
Multi-bunch instabilities (12)

Bunch A



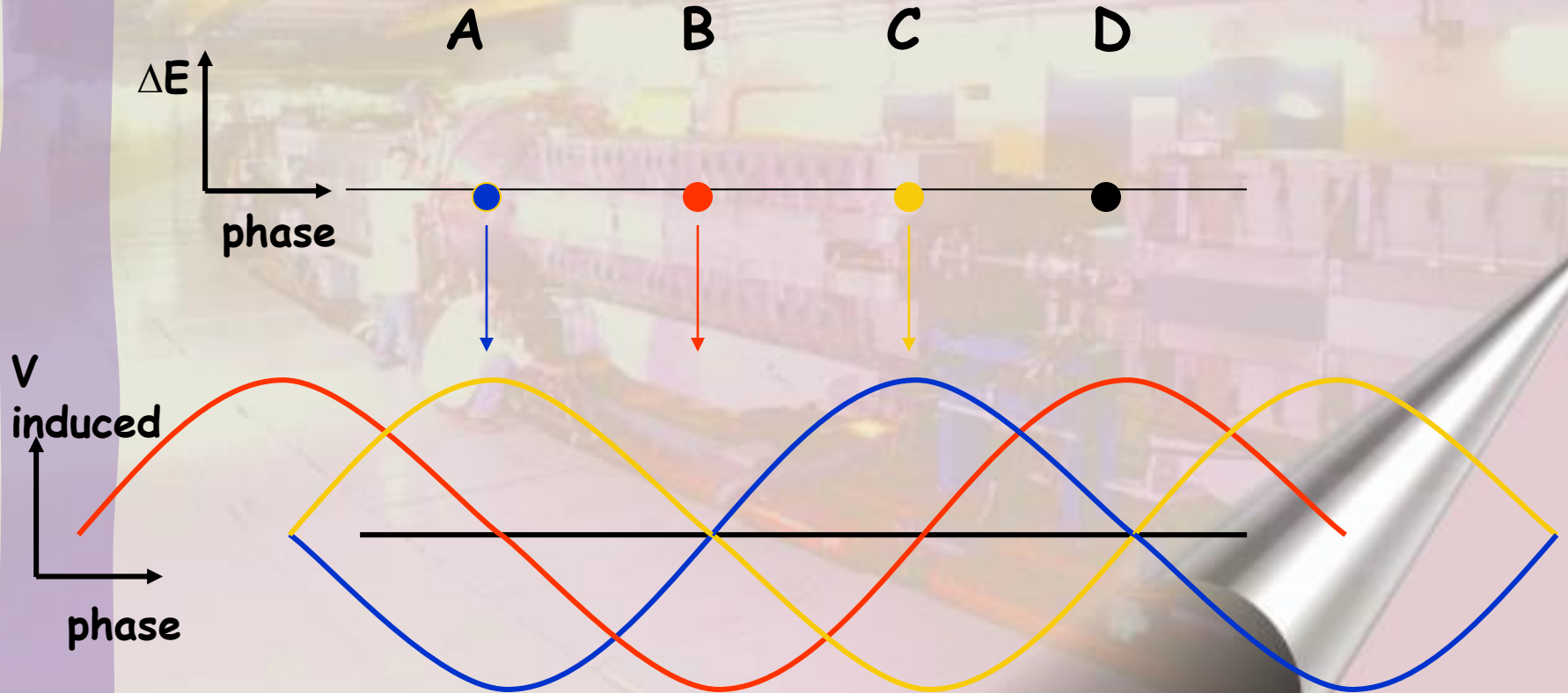
Multi-bunch instabilities (13)

Bunch B



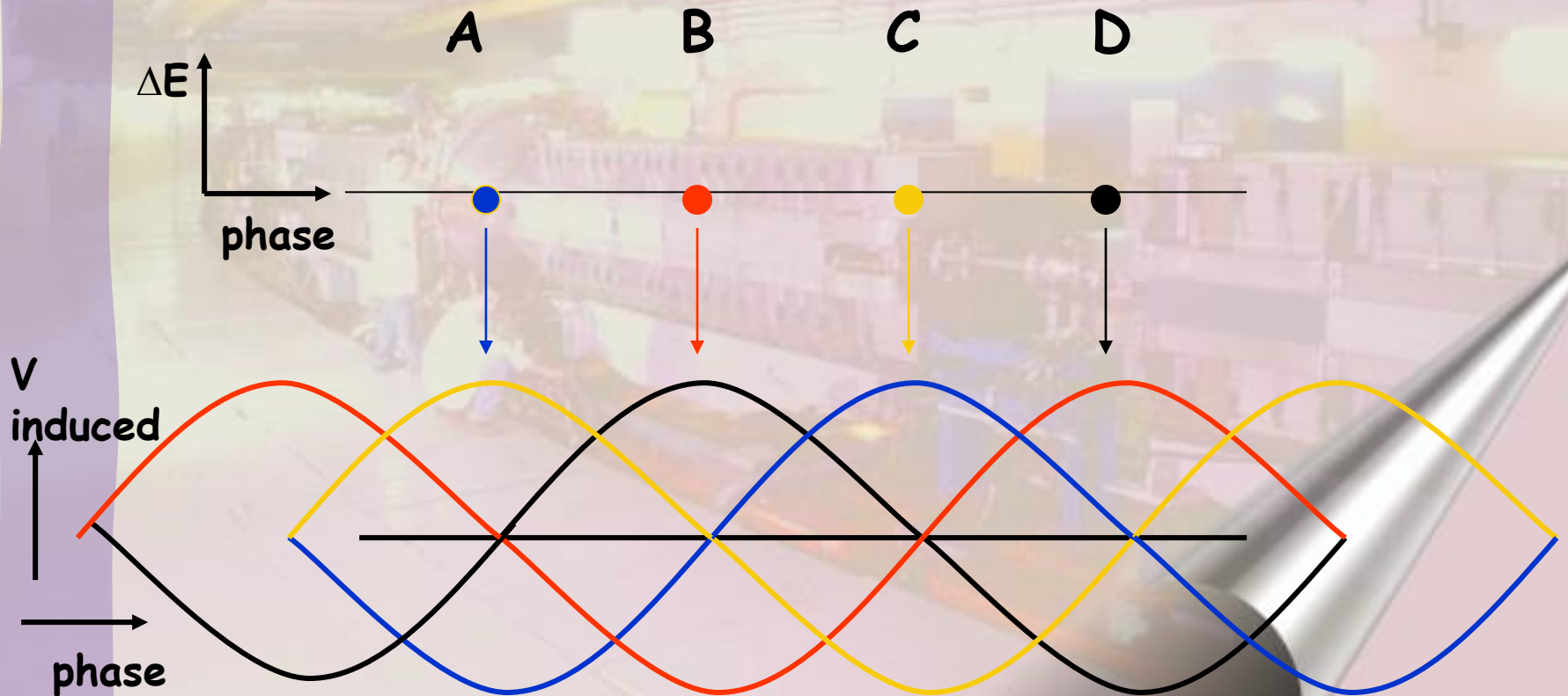
Multi-bunch instabilities (14)

Bunch C



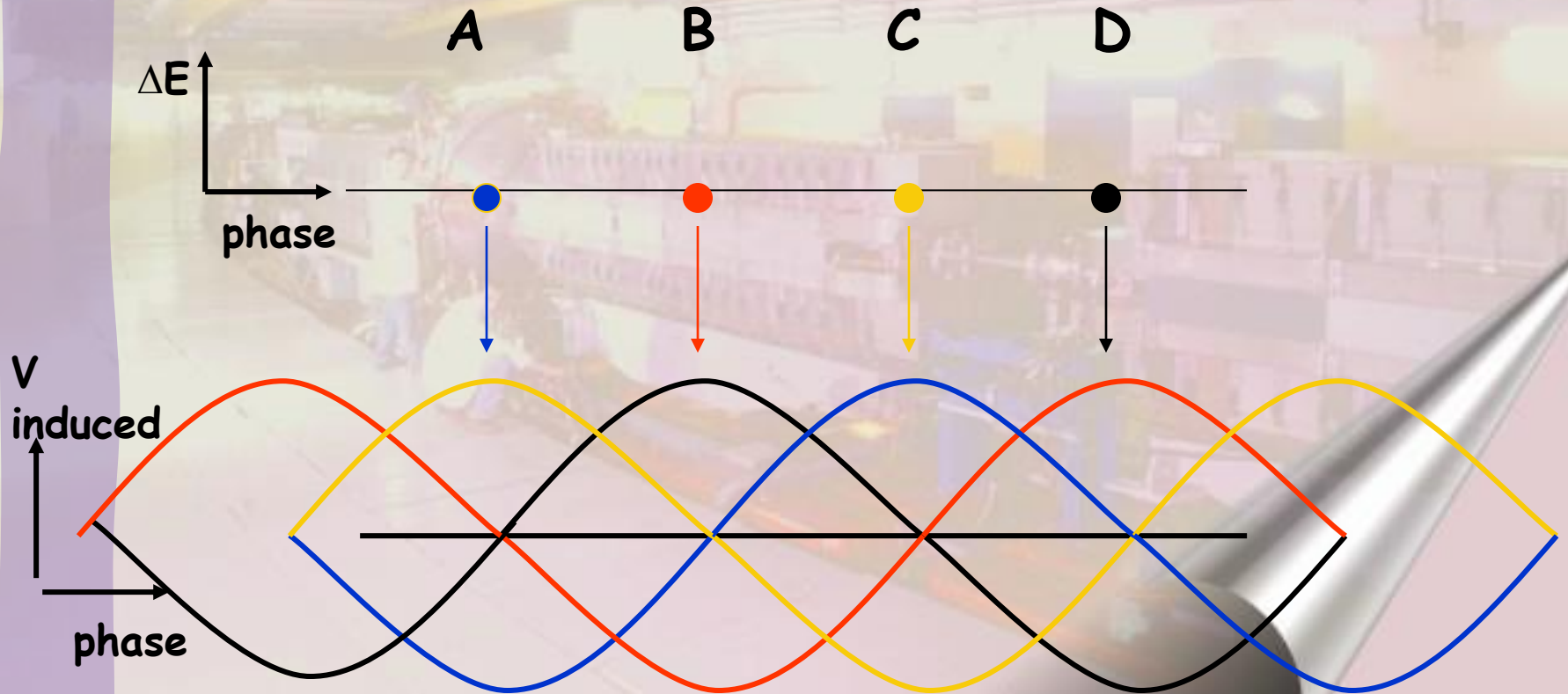
Multi-bunch instabilities (15)

Bunch D



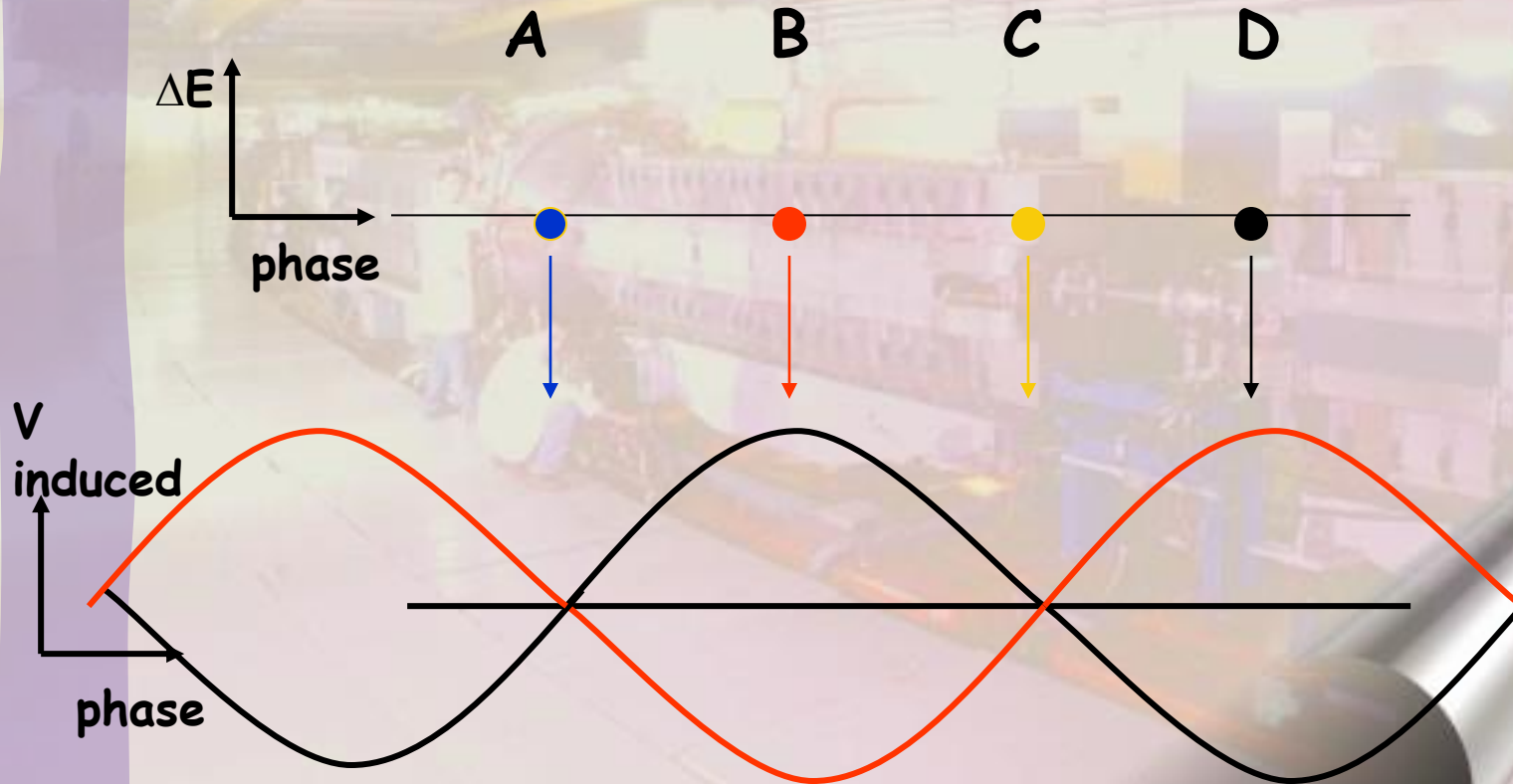
Multi-bunch instabilities (16)

A & C induced voltages cancel



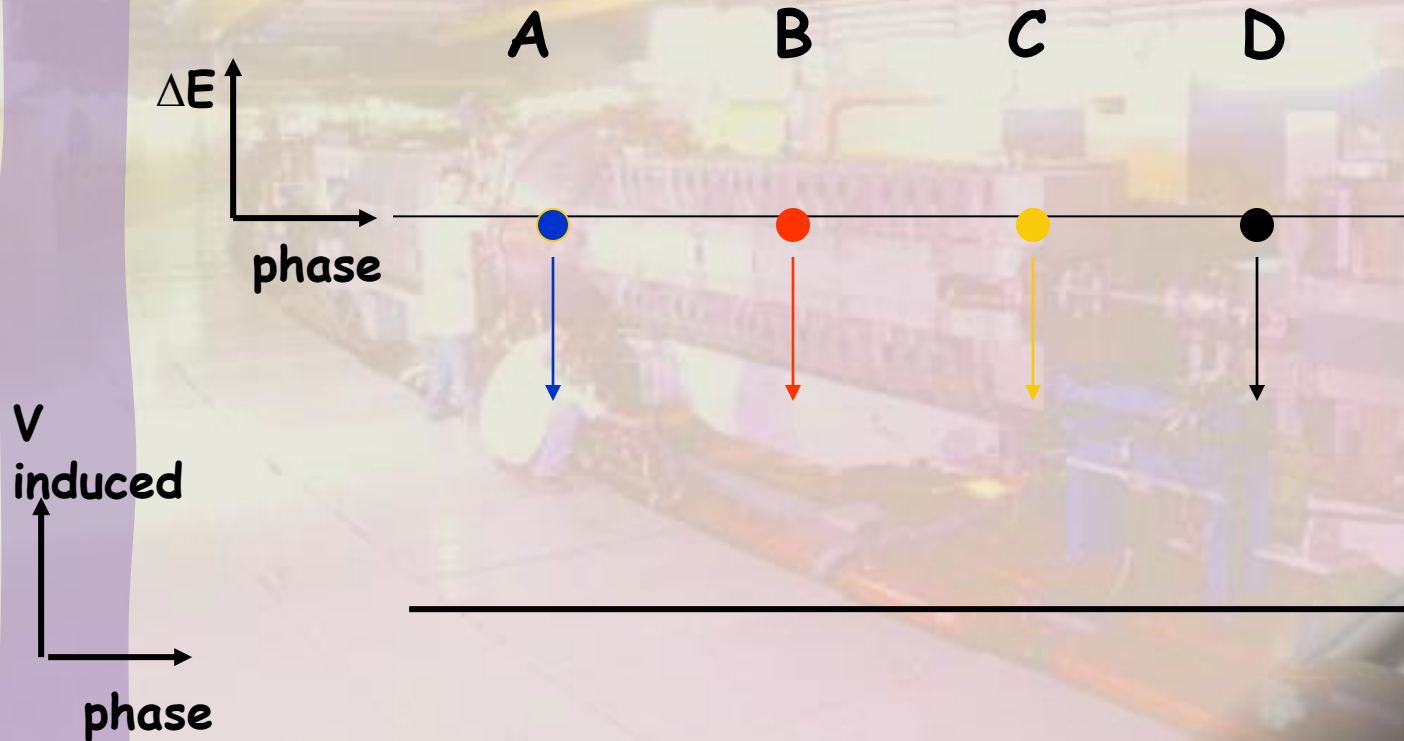
Multi-bunch instabilities (17)

B & D induced voltages cancel

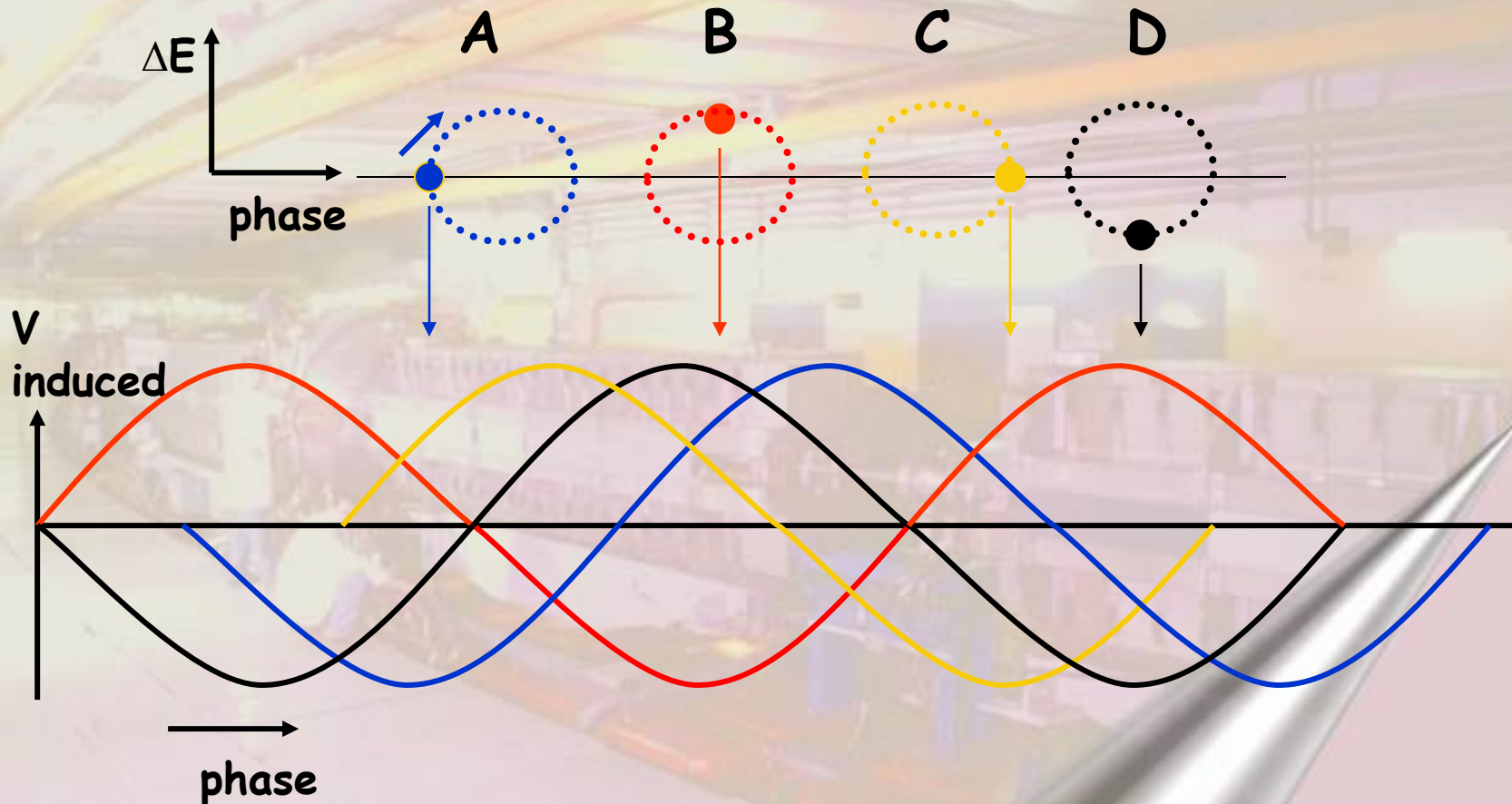


Multi-bunch instabilities (18)

All voltages cancel \Rightarrow no residual effect



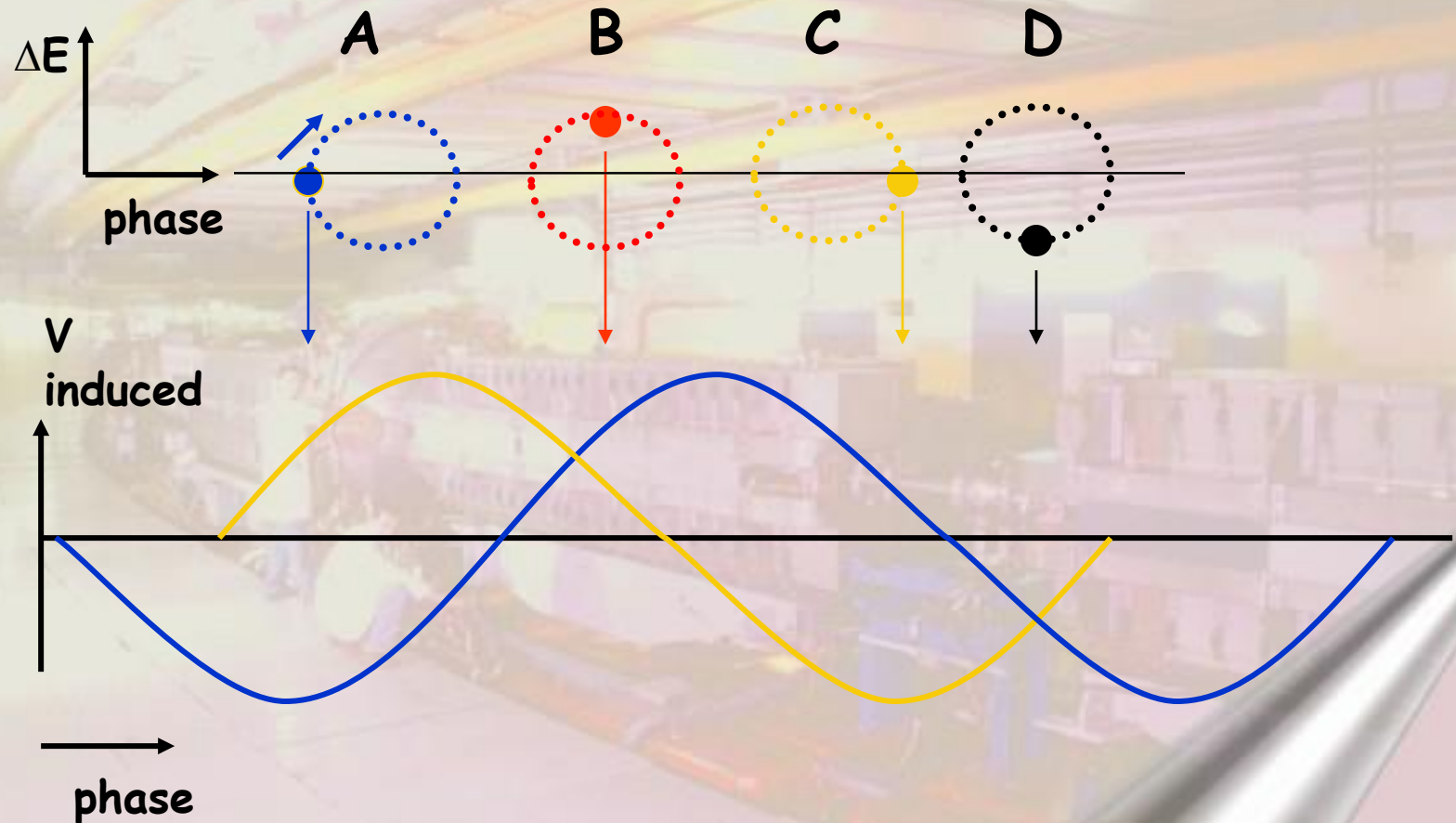
Multi-bunch instabilities (19)



Lets Introduce an $n=1$ mode coupled bunch oscillation

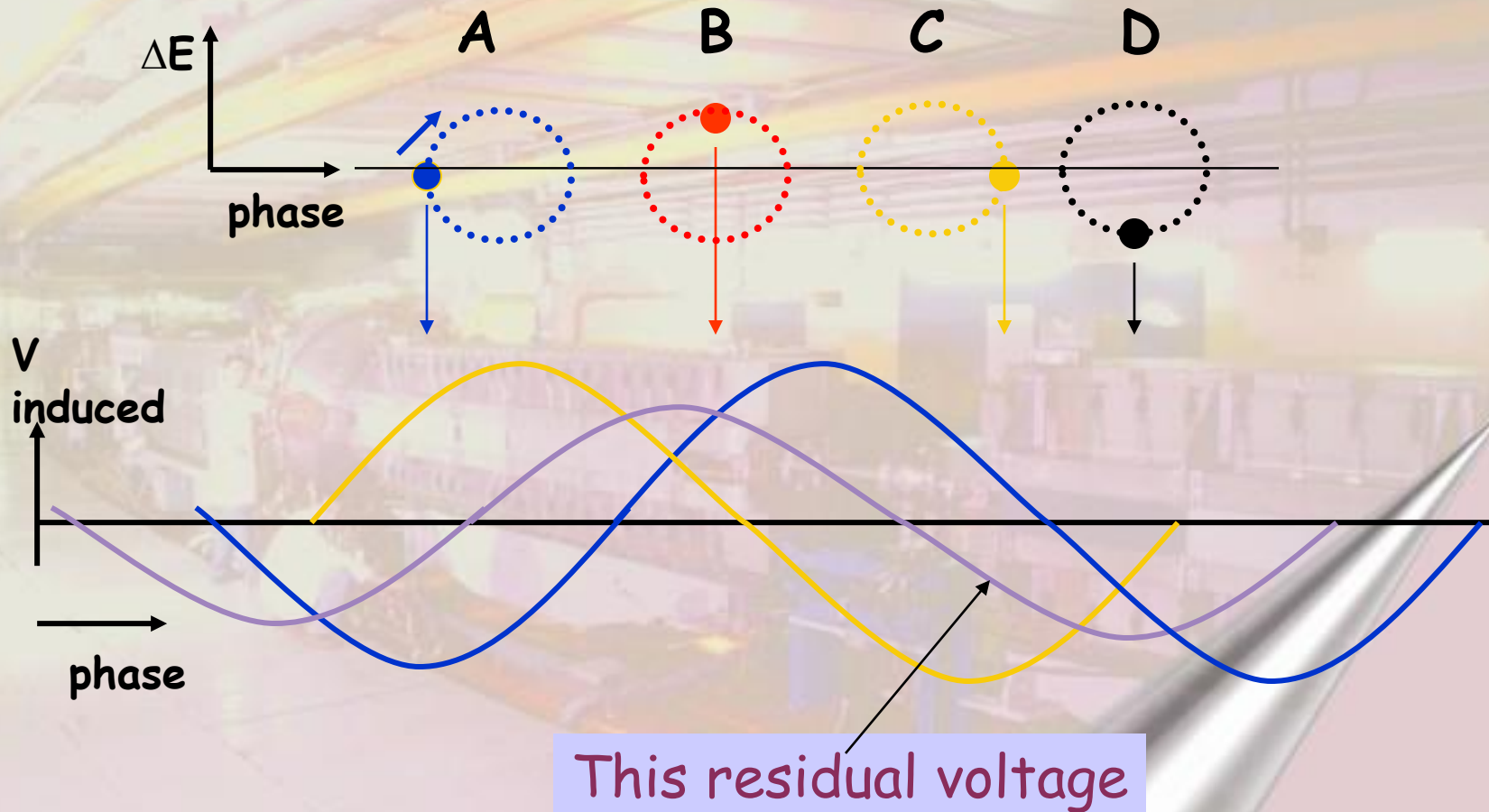
B & D induced voltages cancel

Multi-bunch instabilities (20)

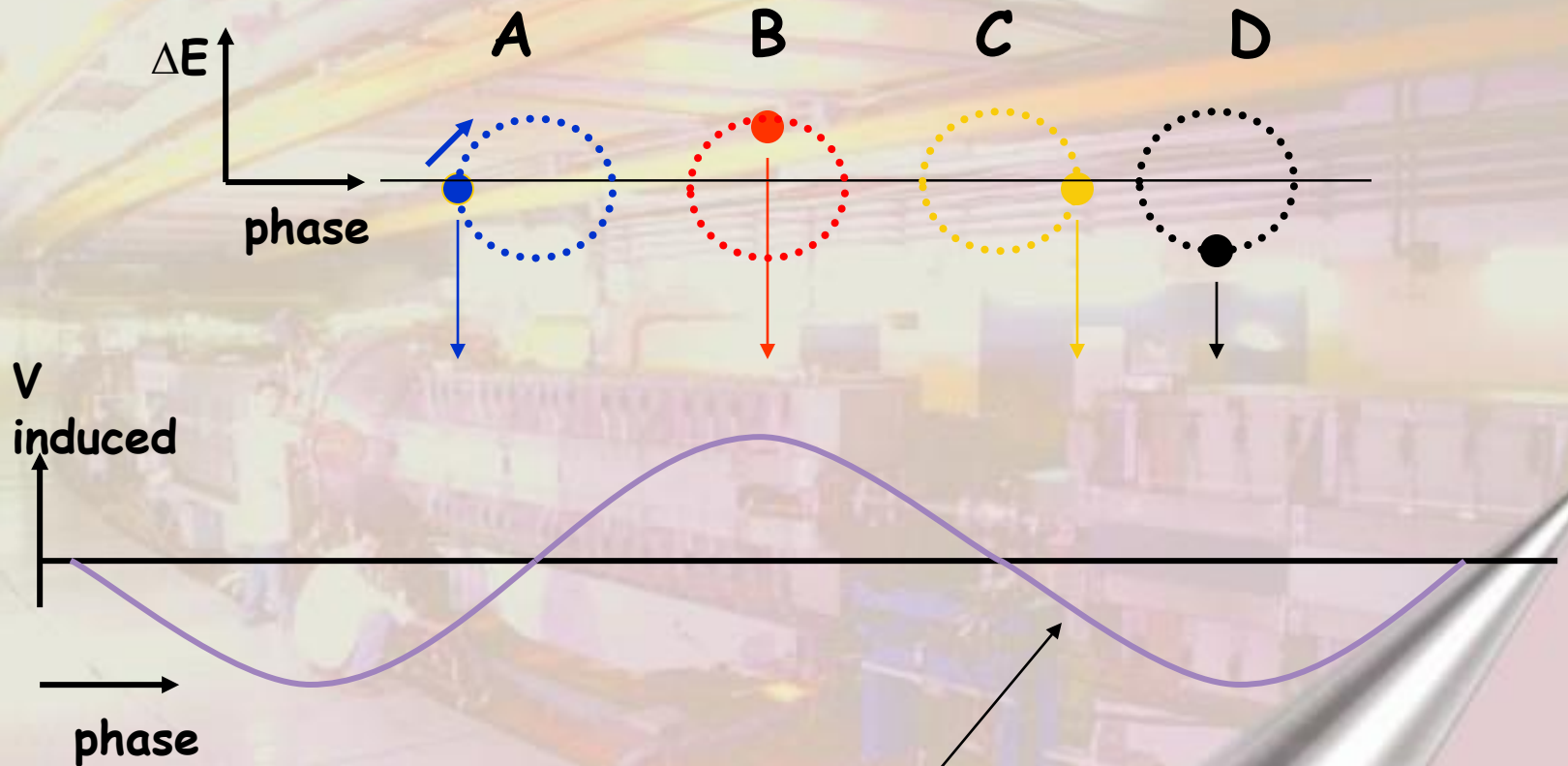


A & C induced voltages do not cancel

Multi-bunch instabilities (21)

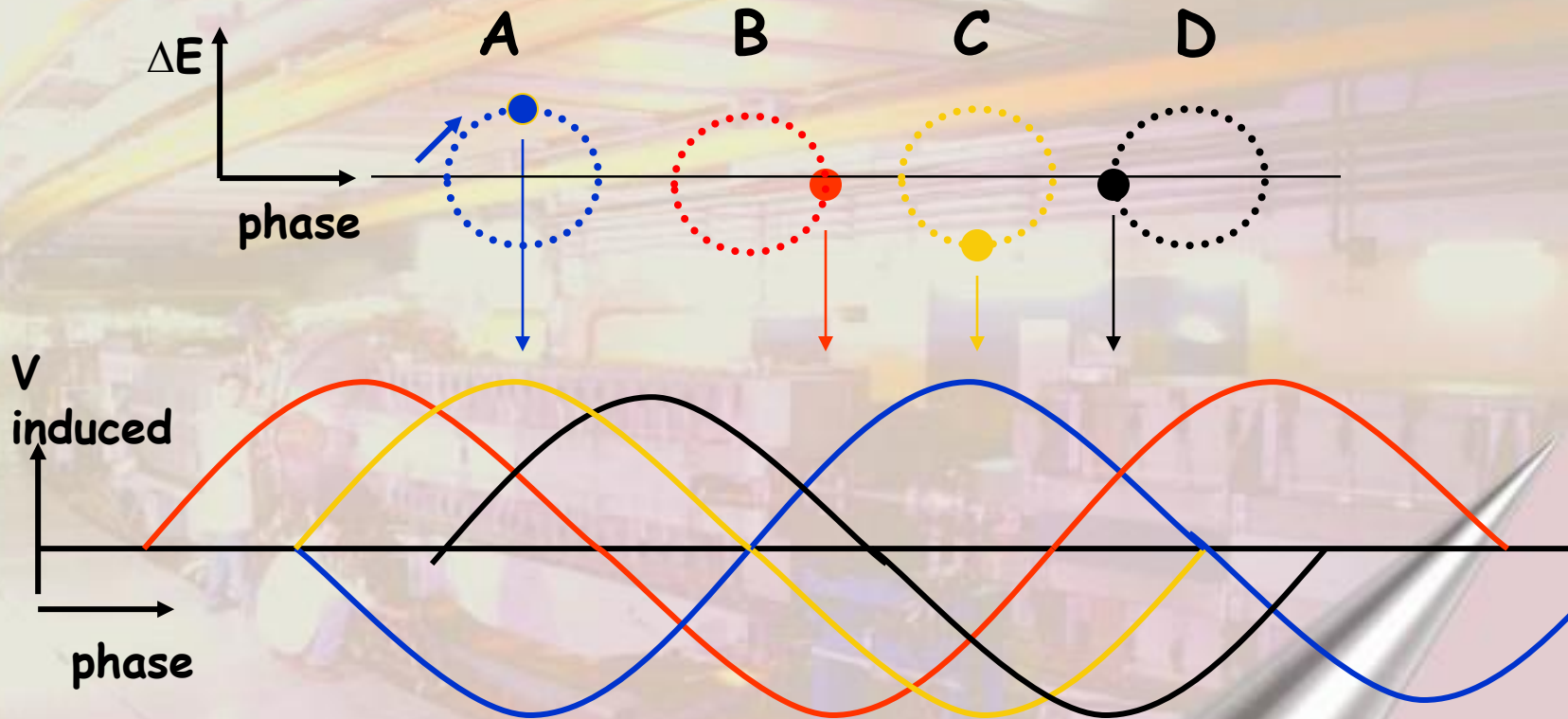


Multi-bunch instabilities (22)



This residual voltage will accelerate B and decelerate D
This increase the oscillation amplitude

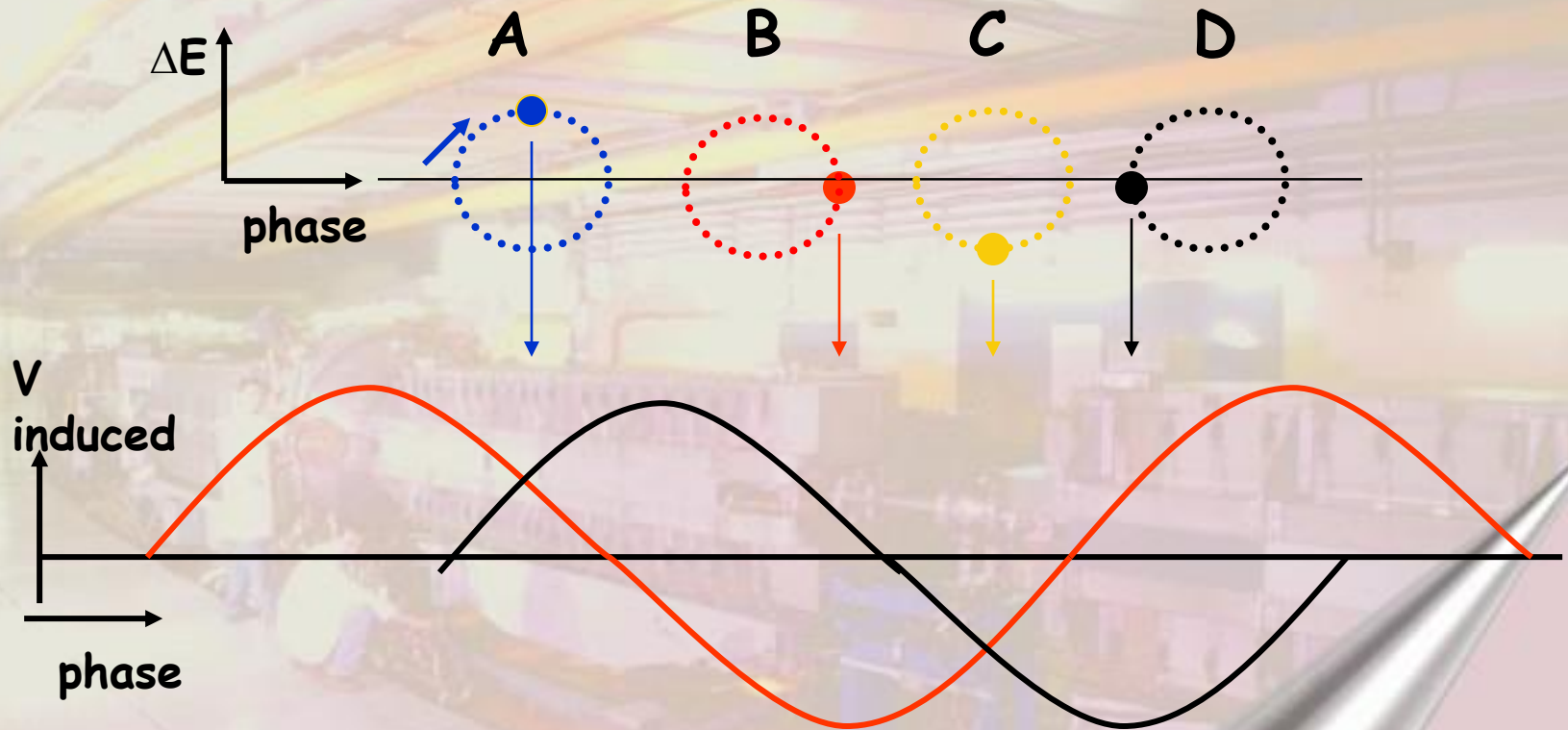
Multi-bunch instabilities (23)



1/4 of a synchrotron
period later

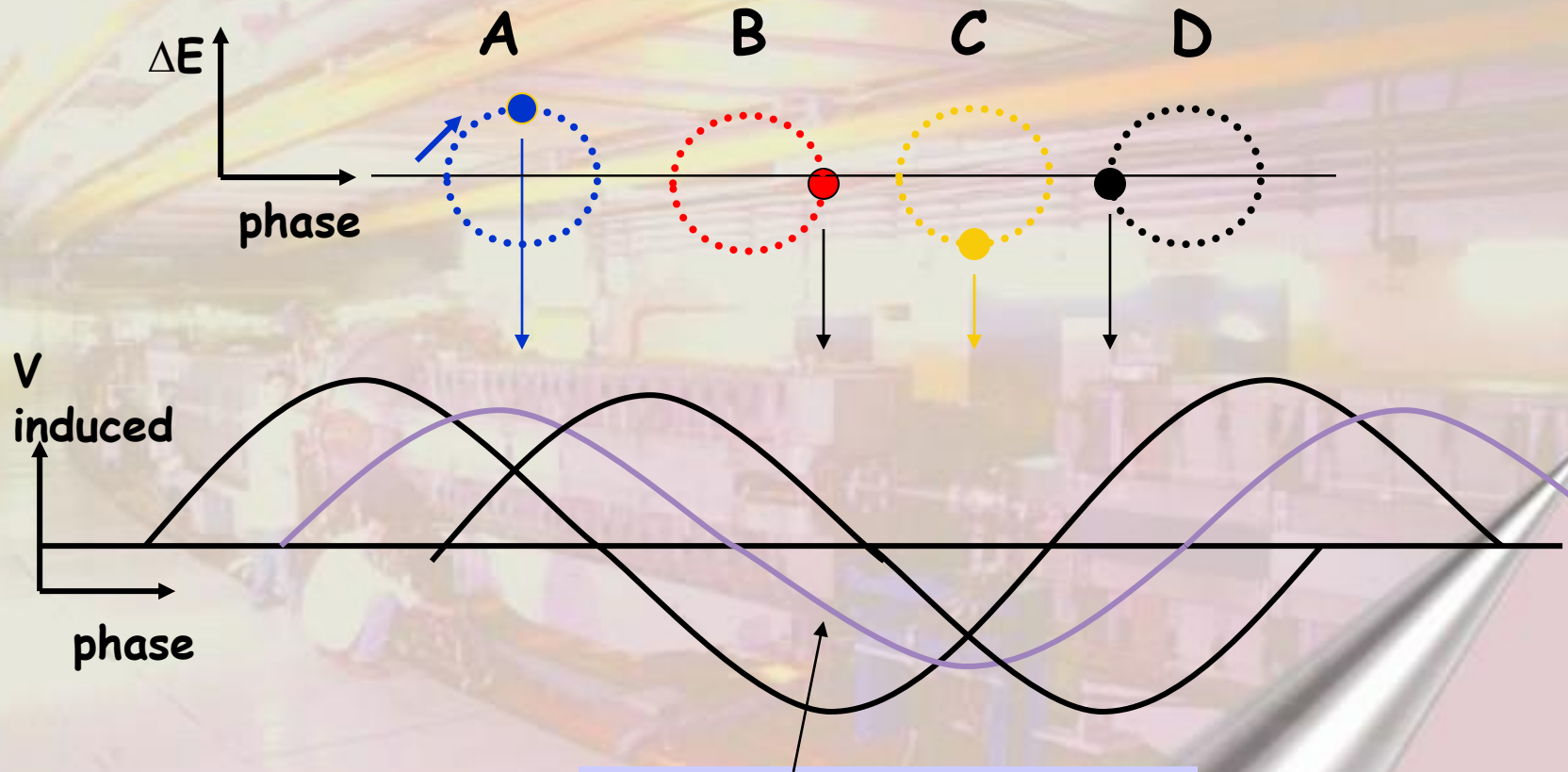
A & C induced voltages now cancel

Multi-bunch instabilities (24)



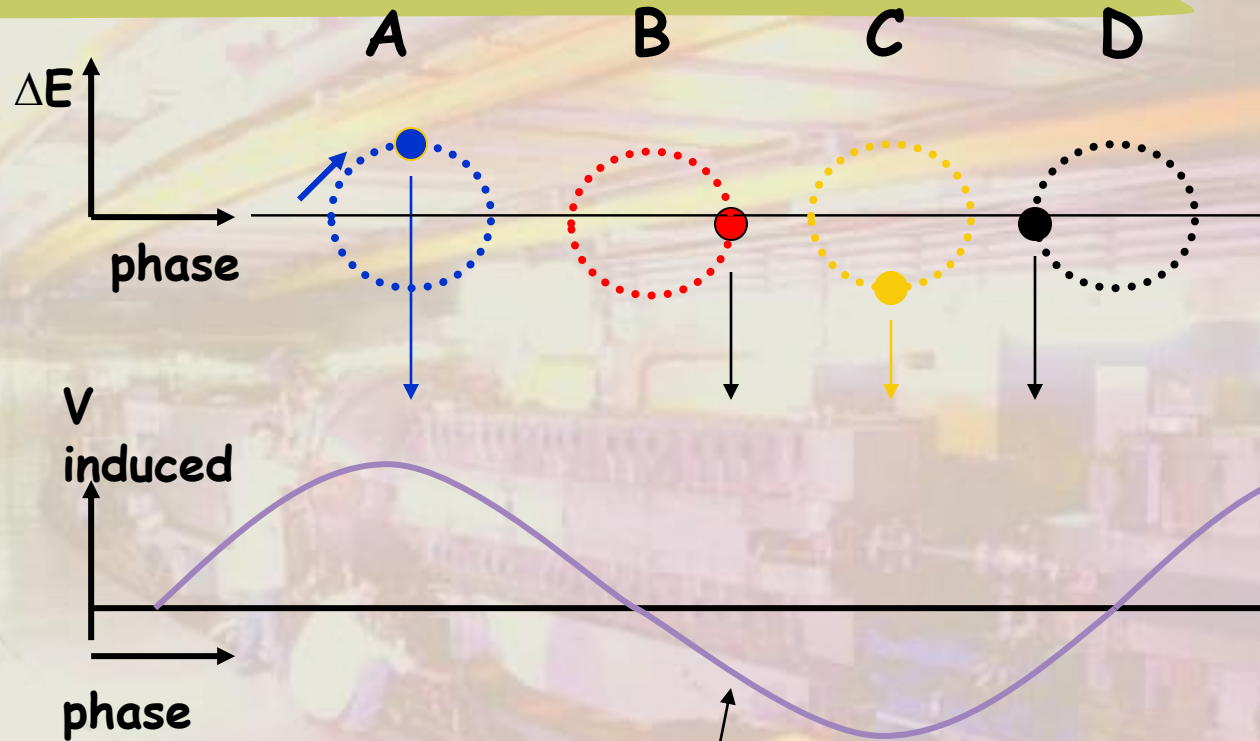
B & D induced voltages do not cancel

Multi-bunch instabilities (25)



This residual voltage

Multi-bunch instabilities (26)

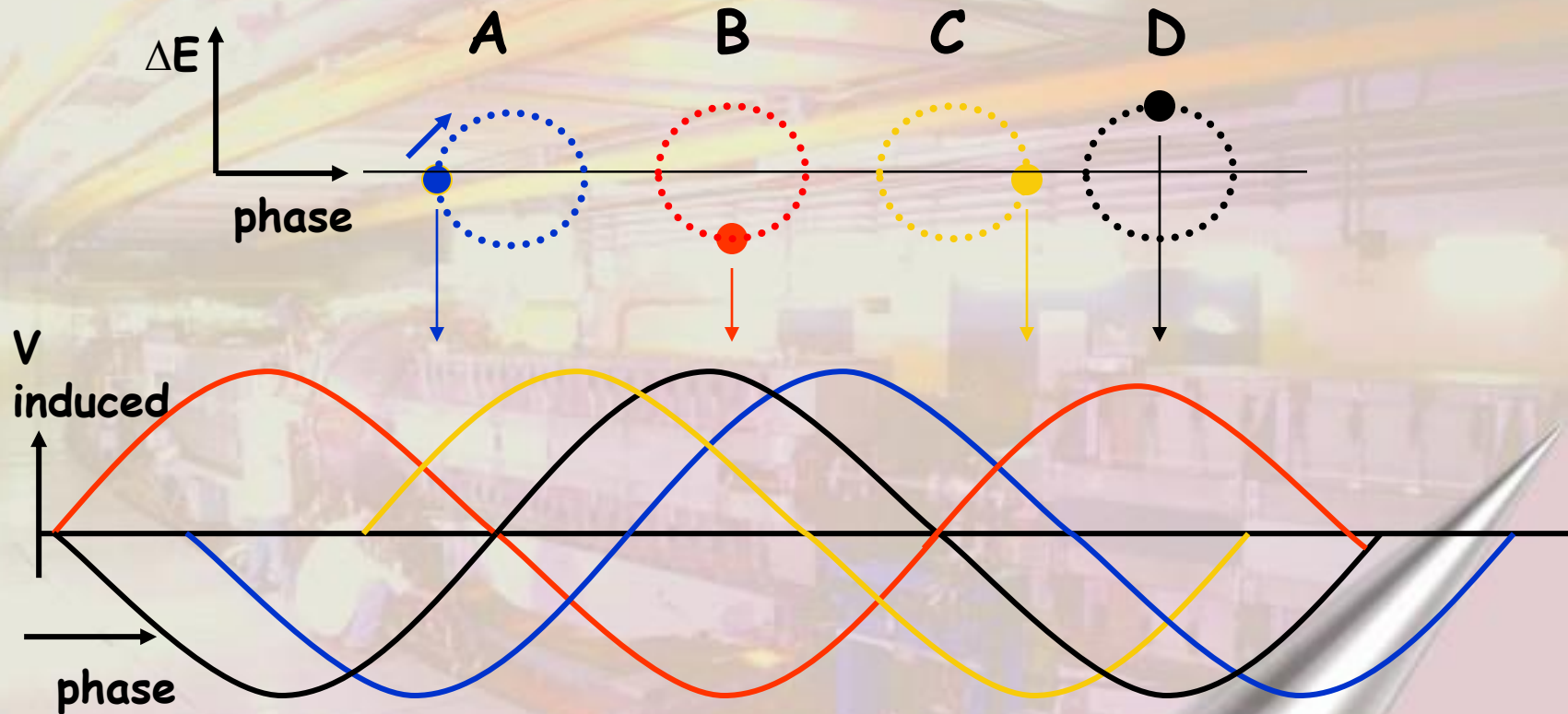


This residual voltage will accelerate A and decelerate C
Again \Rightarrow increase the oscillation amplitude

Multi-bunch instabilities (27)

- # Hence the $n=1$ mode coupled bunch oscillation is unstable
- # Not all modes are unstable look at $n=3$

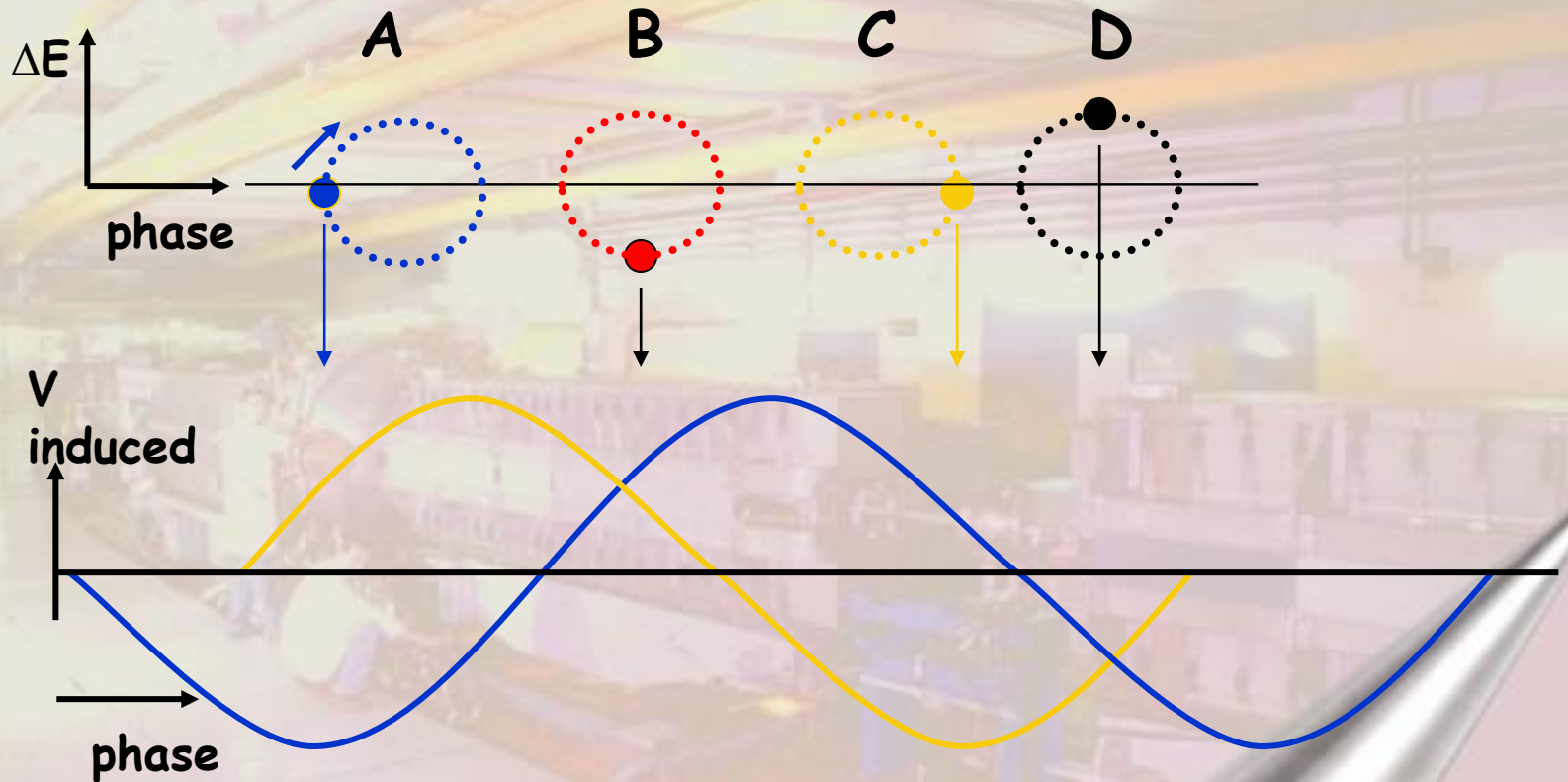
Multi-bunch instabilities (28)



Introduce an $n=3$ mode coupled bunch oscillation

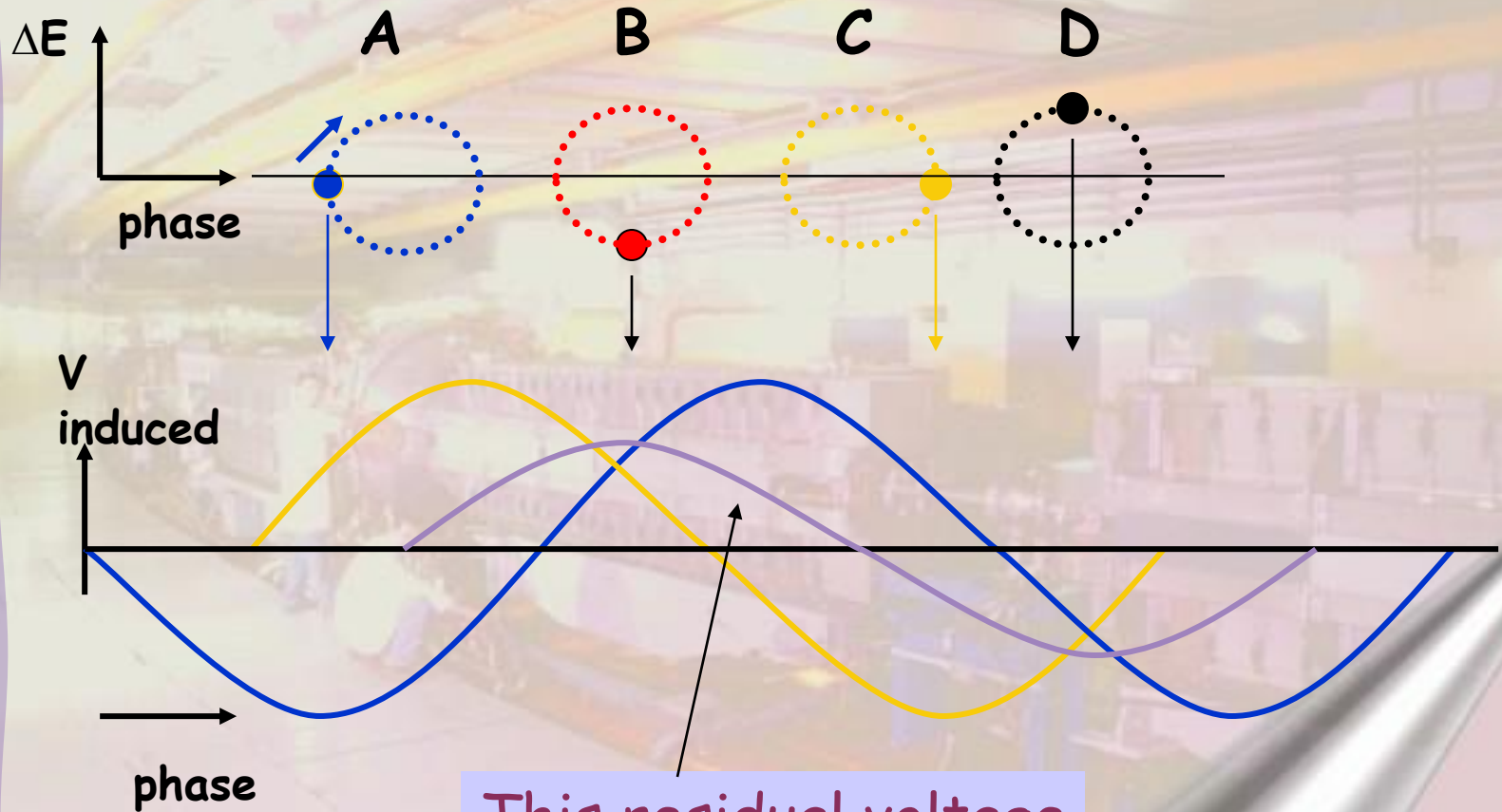
B & D induced voltages cancel

Multi-bunch instabilities (29)

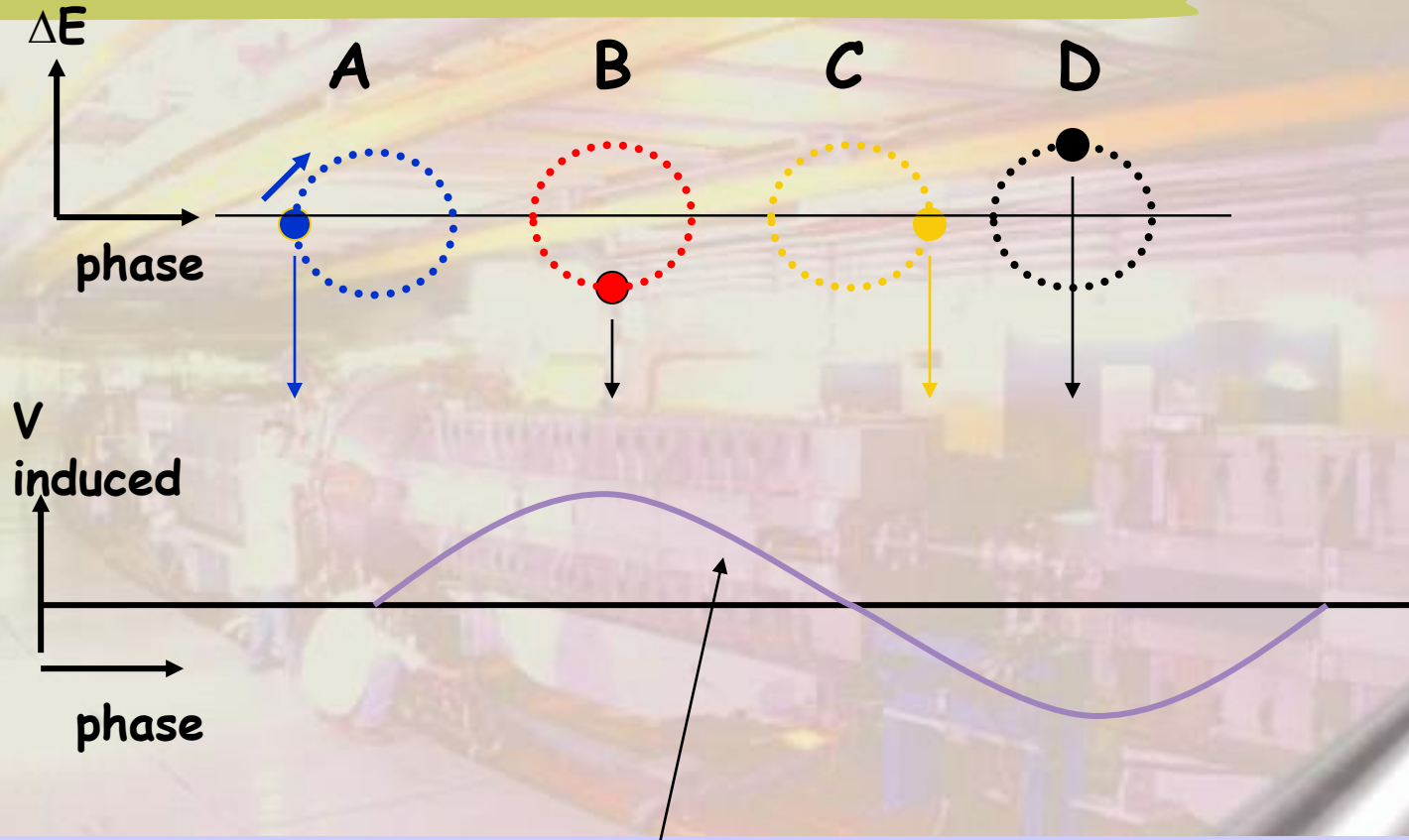


A & C induced voltages do not cancel

Multi-bunch instabilities (30)



Multi-bunch instabilities (31)



This residual voltage will accelerate B and decelerate D
⇒ decrease the oscillation amplitude

Multi-bunch instabilities on a 'scope (1)



“Mountain range display”

Multi-bunch instabilities on a 'scope (2)



Add snapshot images some turns later

Multi-bunch instabilities on a 'scope (3)



Multi-bunch instabilities on a 'scope (4)



Multi-bunch instabilities on a 'scope (5)



Multi-bunch instabilities on a 'scope (6)



Multi-bunch instabilities on a 'scope (7)



Multi-bunch instabilities on a 'scope (8)



Multi-bunch instabilities on a 'scope (9)



Multi-bunch instabilities on a 'scope (10)



Multi-bunch instabilities on a 'scope (11)



Multi-bunch instabilities on a 'scope (12)



Multi-bunch instabilities on a 'scope (13)



Multi-bunch instabilities on a 'scope (14)



Multi-bunch instabilities on a 'scope (15)



Multi-bunch instabilities on a 'scope (16)

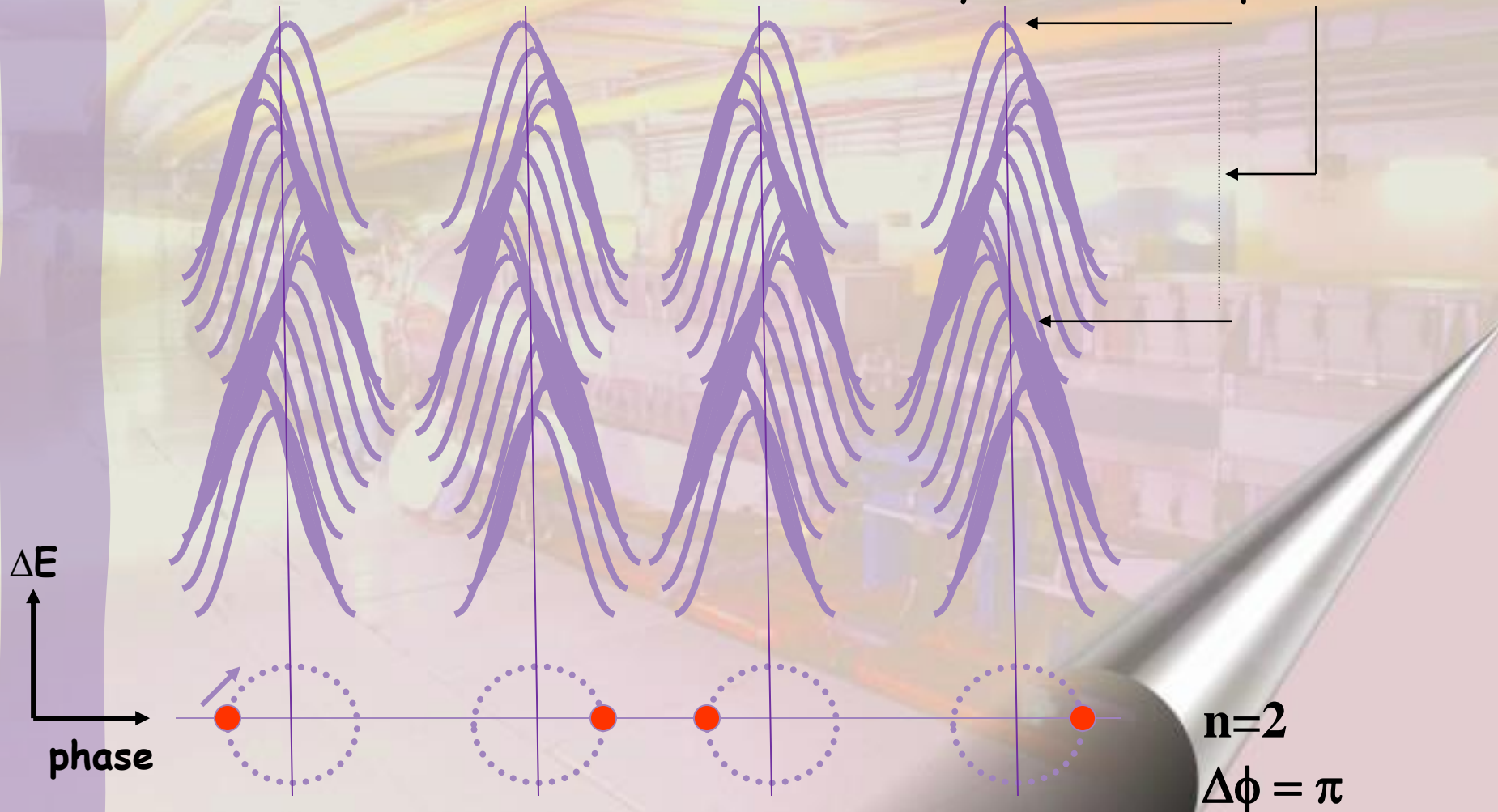
- # What mode is this ?
- # What is the synchrotron period?



Multi-bunch instabilities on a 'scope (17)

This is Mode $n = 2$

One Synchrotron period



Possible cures for single bunch modes

- # Tune the RF cavities correctly in order to avoid the Robinson Instability
- # Have a phase lock system, this is a feedback on phase difference between RF and bunch
- # Have correct Longitudinal matching
- # Radiation damping (Leptons)
- # Damp higher order resonant modes in cavities
- # Reduce machine impedance as much as possible

Possible cures for multi-bunch modes

- # Reduce machine impedance as far as possible
- # Feedback systems - correct bunch phase errors with high frequency RF system
- # Radiation damping
- # Damp higher order resonant modes in cavities

Bunch lengthening (1)

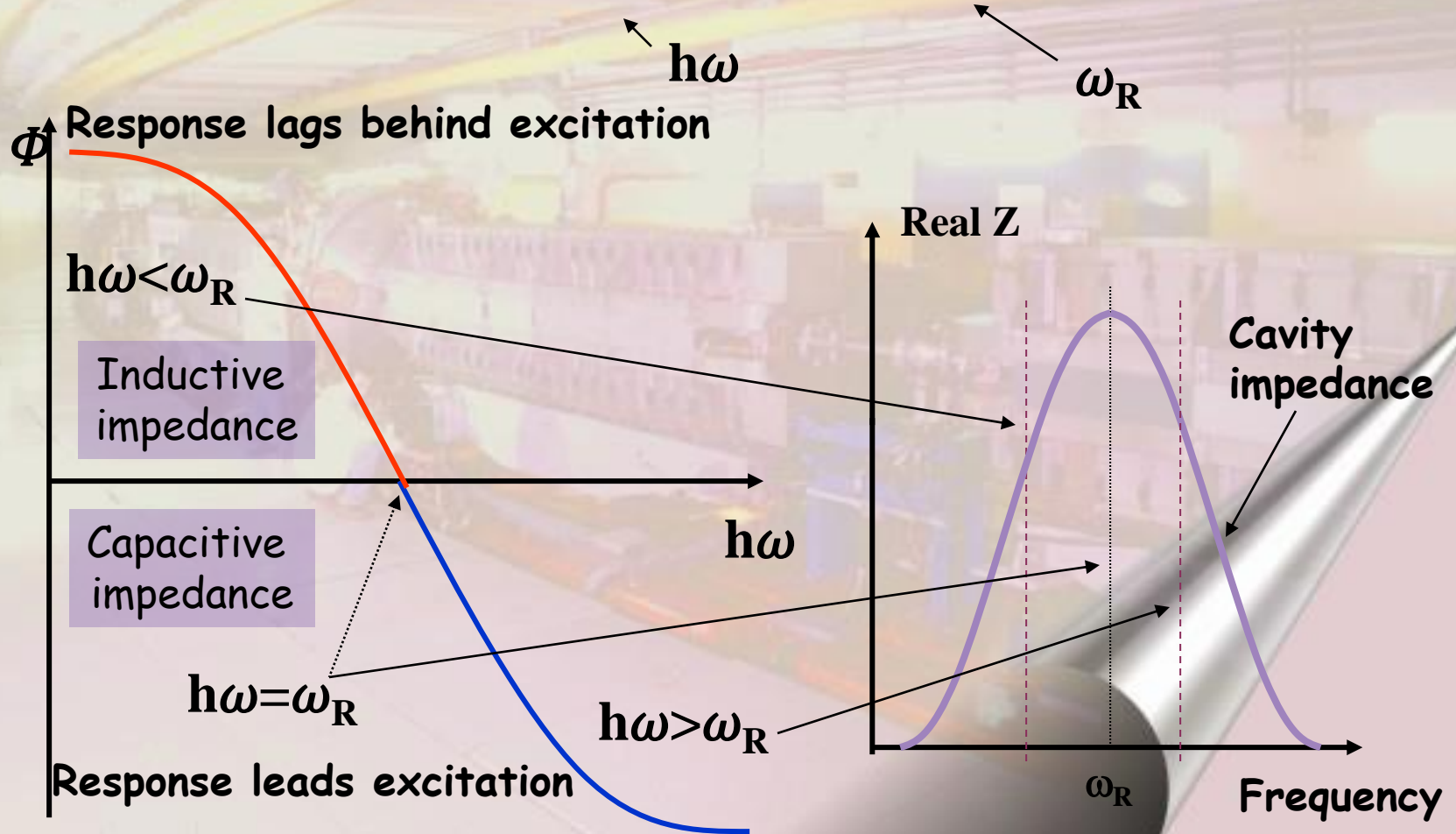
- # Now we controlled all longitudinal instabilities, but
- # It seems that we are unable to increase peak bunch current above a certain level
- # The bunch gets longer as we add more particles.

- # Why..?
- # What happens....?

- # Lets look at the behaviour of a cavity resonator as we change the driving frequency.

Bunch lengthening (2)

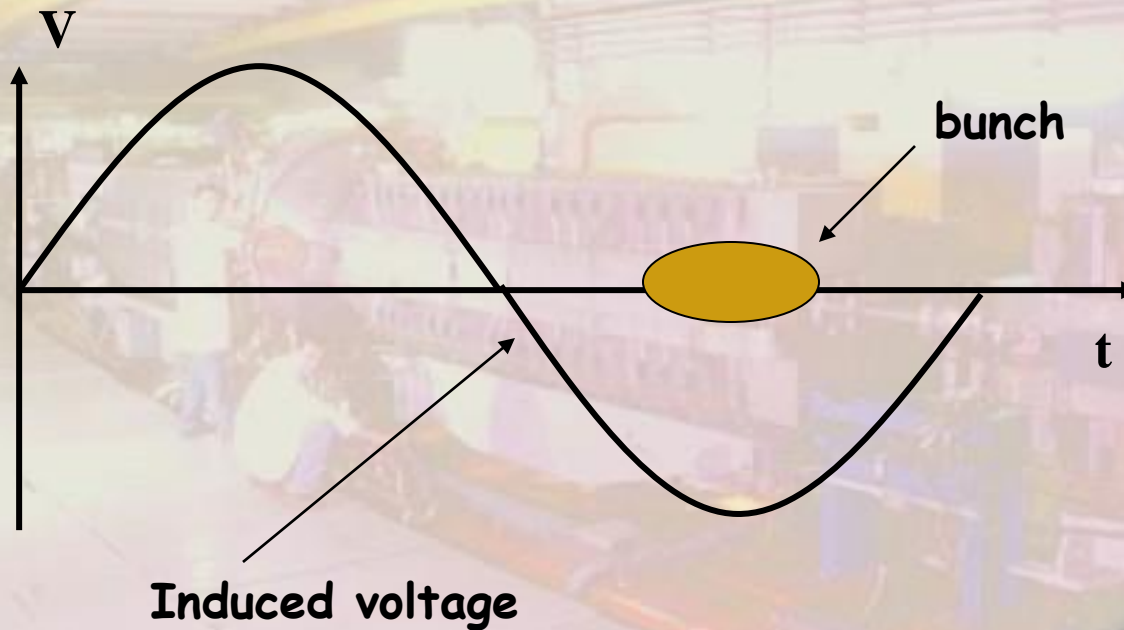
The **phase** of the response of a resonator depends on the difference between the **driving** and the **resonant** frequencies



Bunch lengthening (3)

Cavity driven on resonance

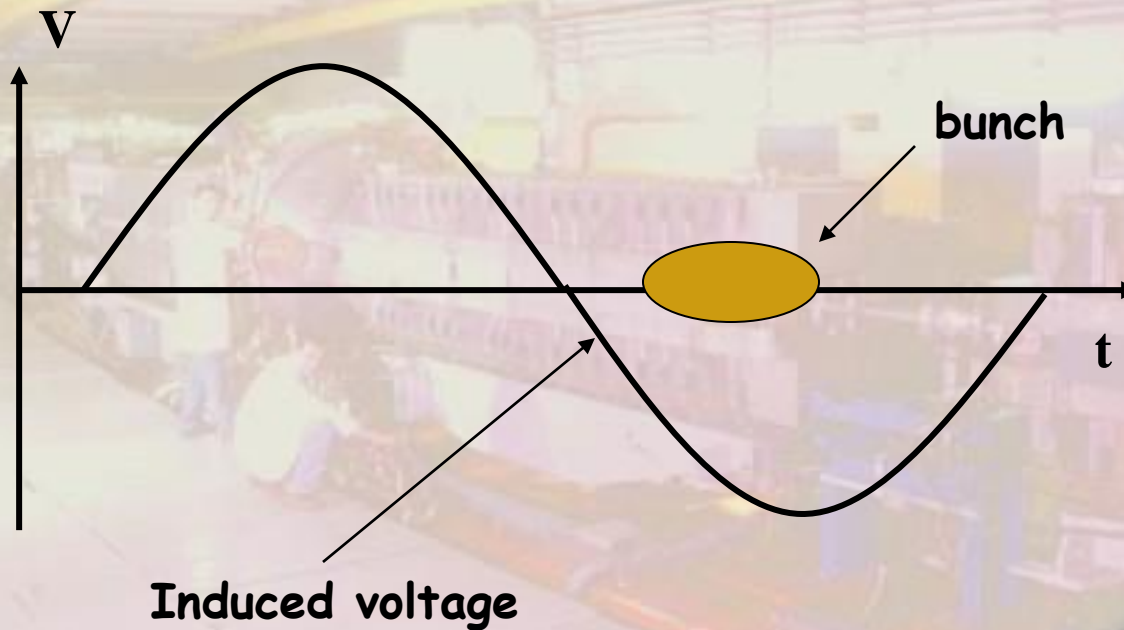
$h\omega = \omega_R \Rightarrow$ resistive impedance



Bunch lengthening (4)

Cavity driven above resonance

$h\omega > \omega_R \Rightarrow$ capacitive impedance

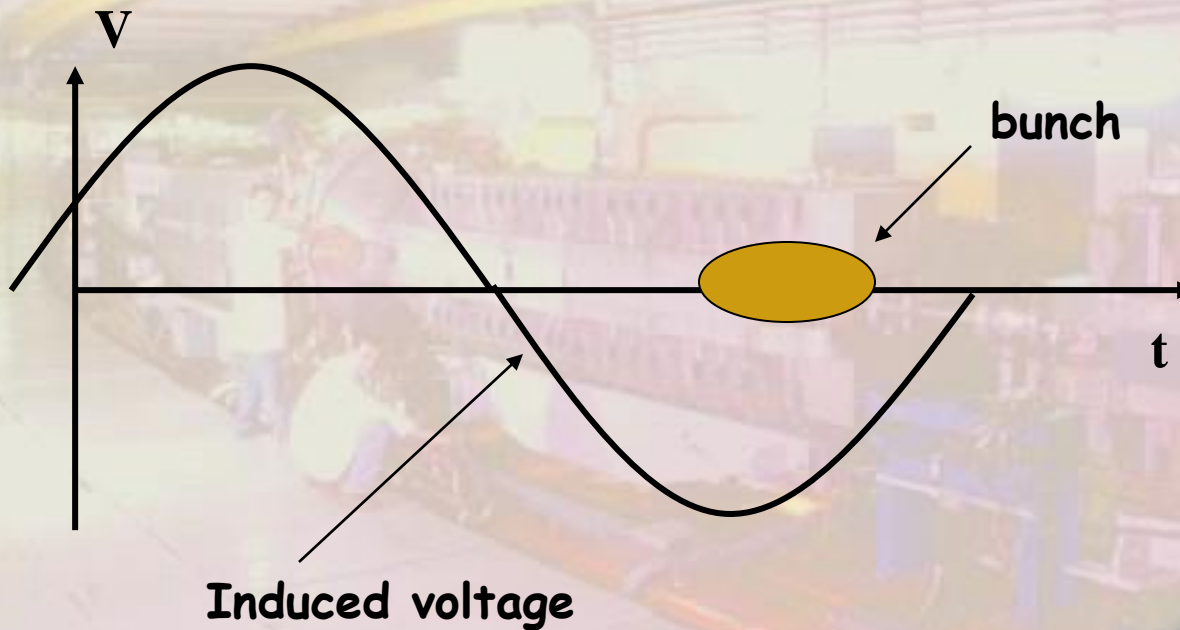


Response leads excitation

Bunch lengthening (5)

Cavity driven below resonance

$h\omega < \omega_R \Rightarrow$ inductive impedance



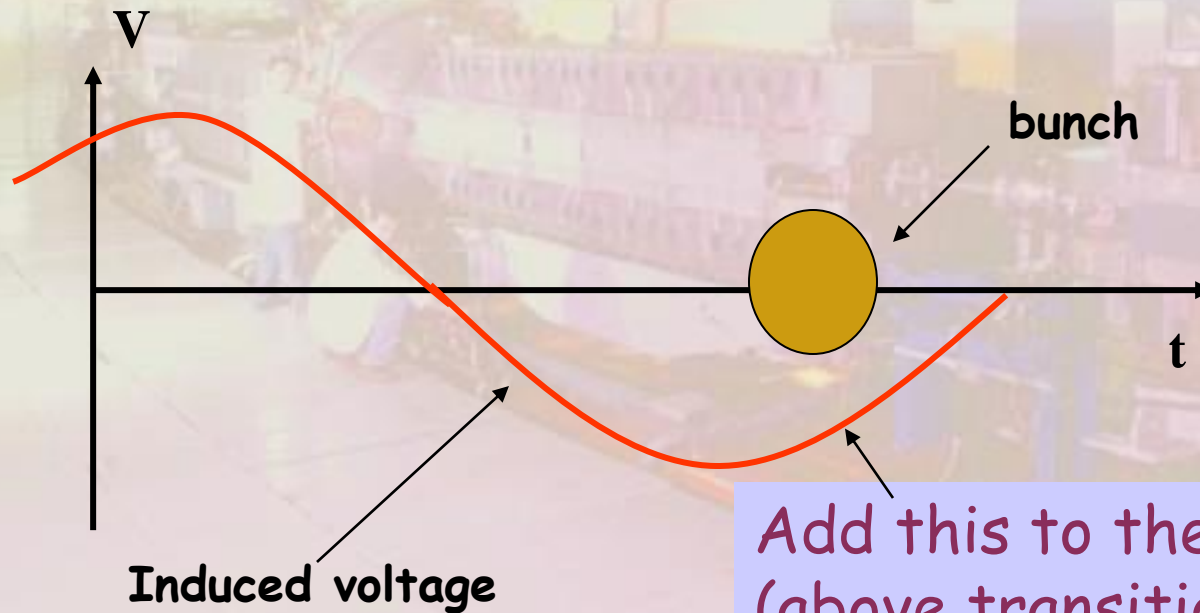
Response lags behind excitation

Bunch lengthening (6)

- # In general the Broad Band impedance of the machine, vacuum pipe etc (other than the cavities) is inductive
- # The bellows etc. represent very high frequency resonators, which resonate mostly at frequencies above the bunch spectrum

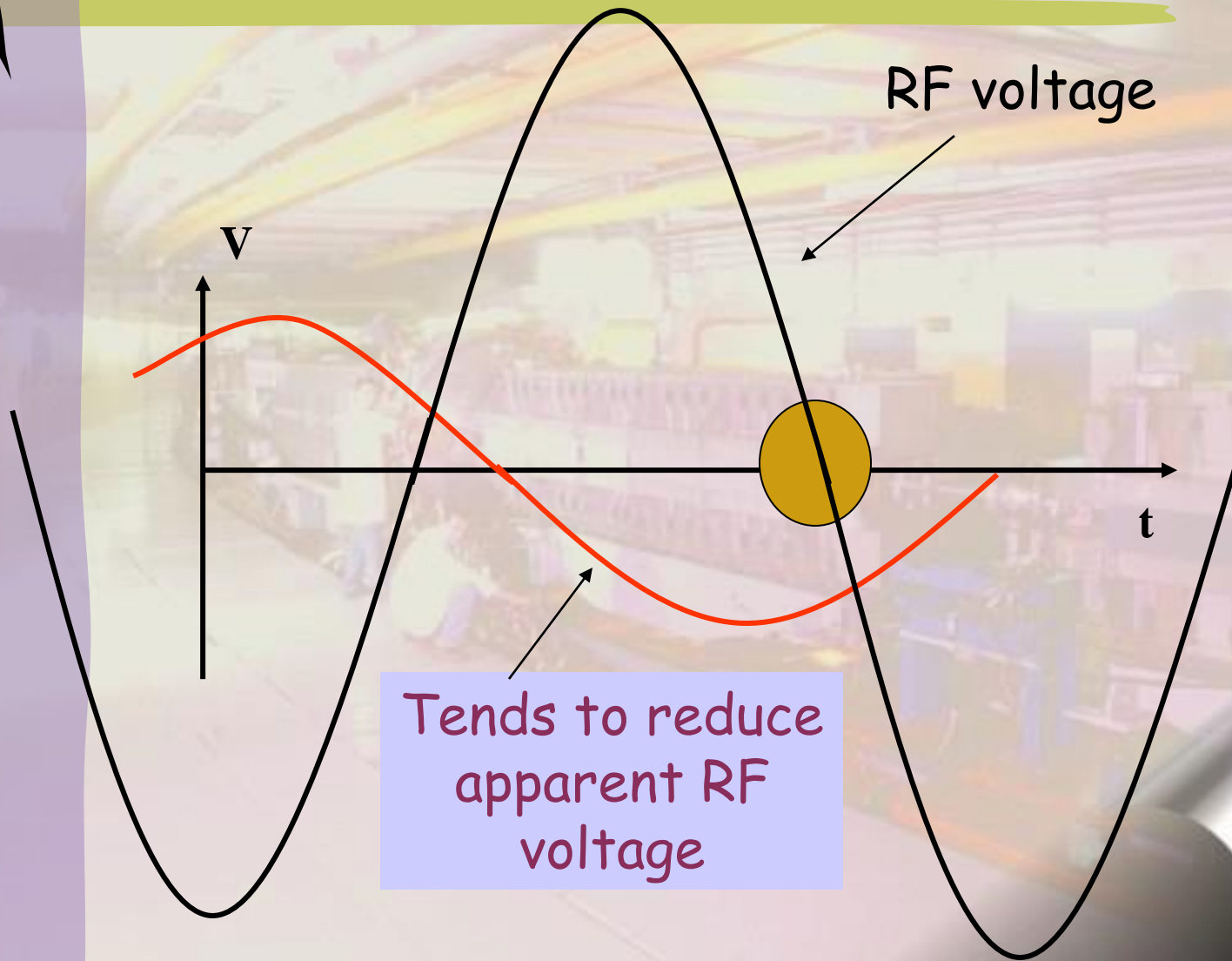
Bunch lengthening (7)

- # Since the Broad Band impedance of the machine is predominantly inductive, the **response lags behind excitation**

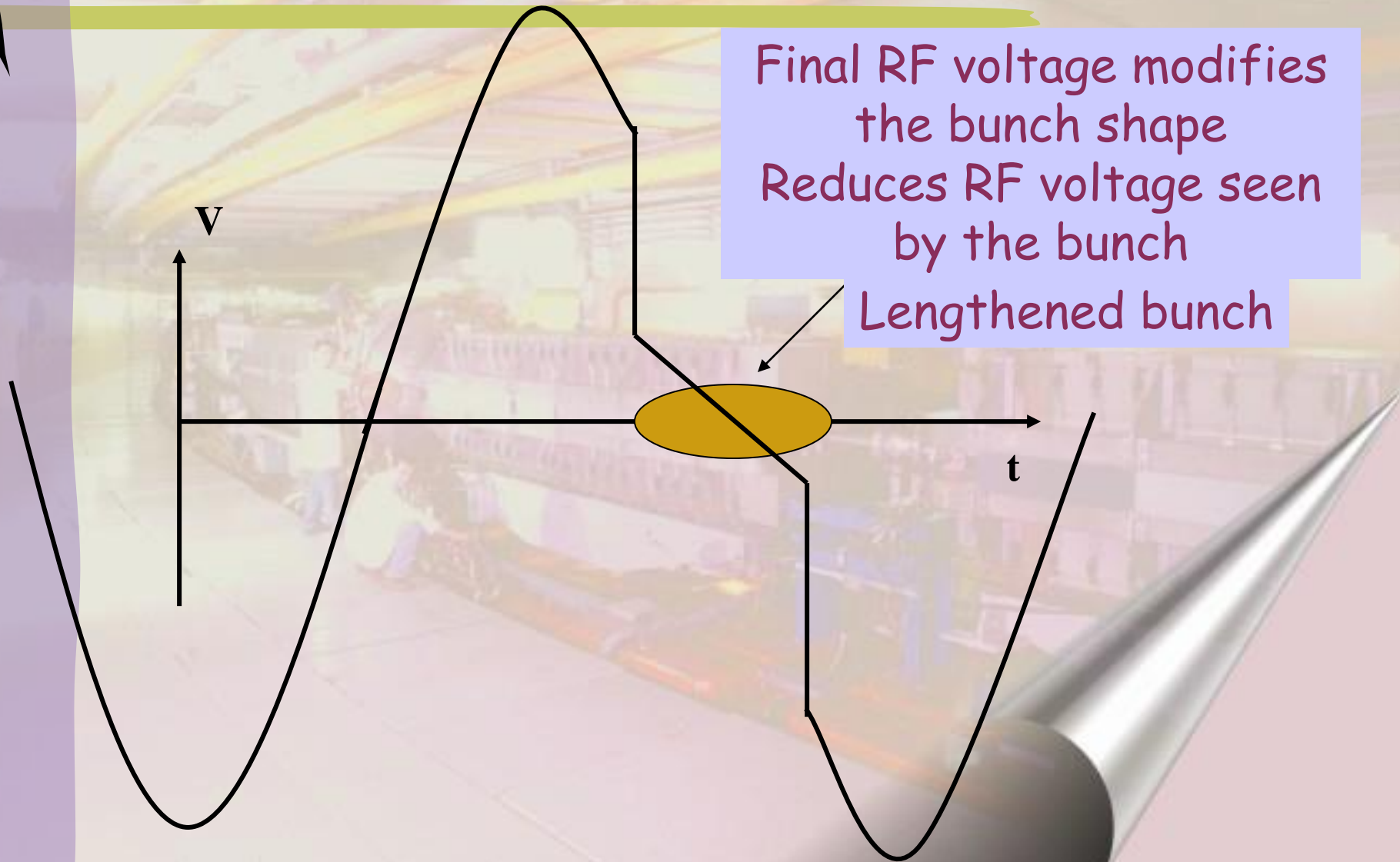


Add this to the RF voltage
(above transition)

Bunch lengthening (8)



Bunch lengthening (10)



Questions....,Remarks...?

*Single bunch
instabilities*

*Multi bunch
instabilities*

*Cures for
instabilities*

*Bunch
lengthening*

