

AXEL-2023

Introduction to Particle Accelerators



Transverse instabilities:

- ✓ *How do they arise*
- ✓ *Single-bunch effects (“head-tail” instability)*
- ✓ *Multi-bunch modes (very brief)*
- ✓ *Possible cures*
- ✓ *Space charge effects*

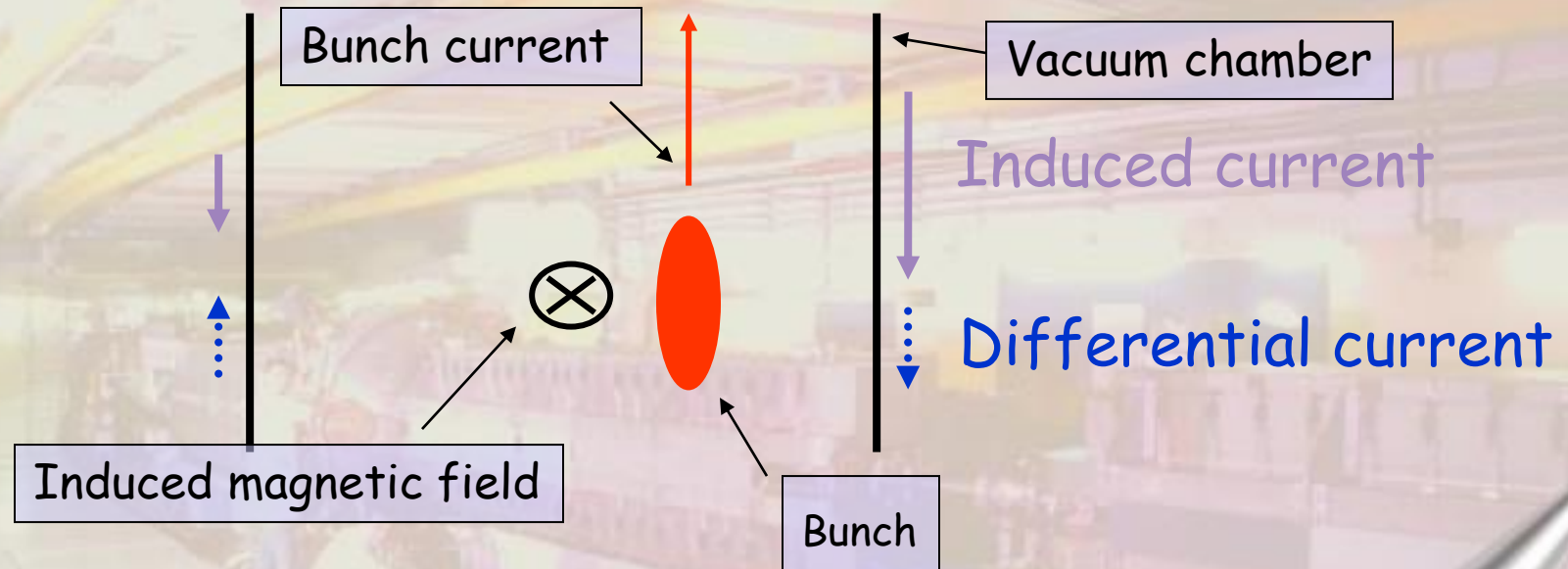
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29 November 2023

Coherent Transverse Oscillation (1)

- # The complete bunch is displaced from side to side (or up and down)
- # A bunch of charged particles induces a charge in the vacuum chamber
- # This creates an image current in the vacuum chamber walls
- # How can these currents affect transverse motion?

Coherent Transverse Oscillation (2)



- # If the bunch is displaced from the centre of the vacuum chamber it will drive a differential wall current
- # This leads to a magnetic field, which deflects the bunch

Transverse coupling impedance (1)

- # We characterize the electromagnetic response to the bunch by a “transverse coupling impedance” (as for longitudinal case)

$$\int (Z_{\perp}(\omega) \times I(\omega)) d\omega = \int_0^S (E + v \times B) ds$$

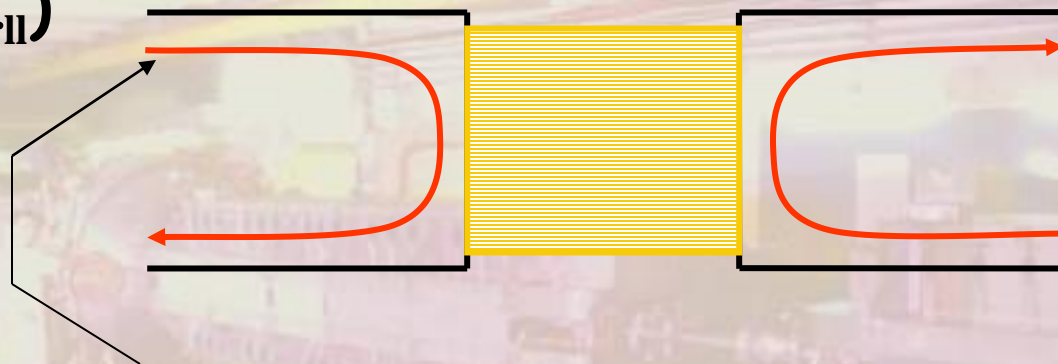
Frequency spectrum
of bunch current

Transverse E & B fields
summed around the machine

- # Z_{\perp} (exactly as Z_{\parallel}) is also a function of frequency
- # Z_{\perp} also has resistive, capacitive and inductive components
- # However, there is one big difference between Z_{\perp} & Z_{\parallel}

Transverse coupling impedance (2)

- # For a vacuum chamber with a short non-conducting section the direct image current sees a high impedance (large Z_{\parallel})



- # For the differential current (current loops) is not greatly affected so Z_{\perp} is unchanged by the non-conducting section
- # Thus:
 - Any interruption to a smooth vacuum chamber increases Z_{\parallel}
 - Any structure that will support current loops increases Z_{\perp}

Relationship with the longitudinal plane

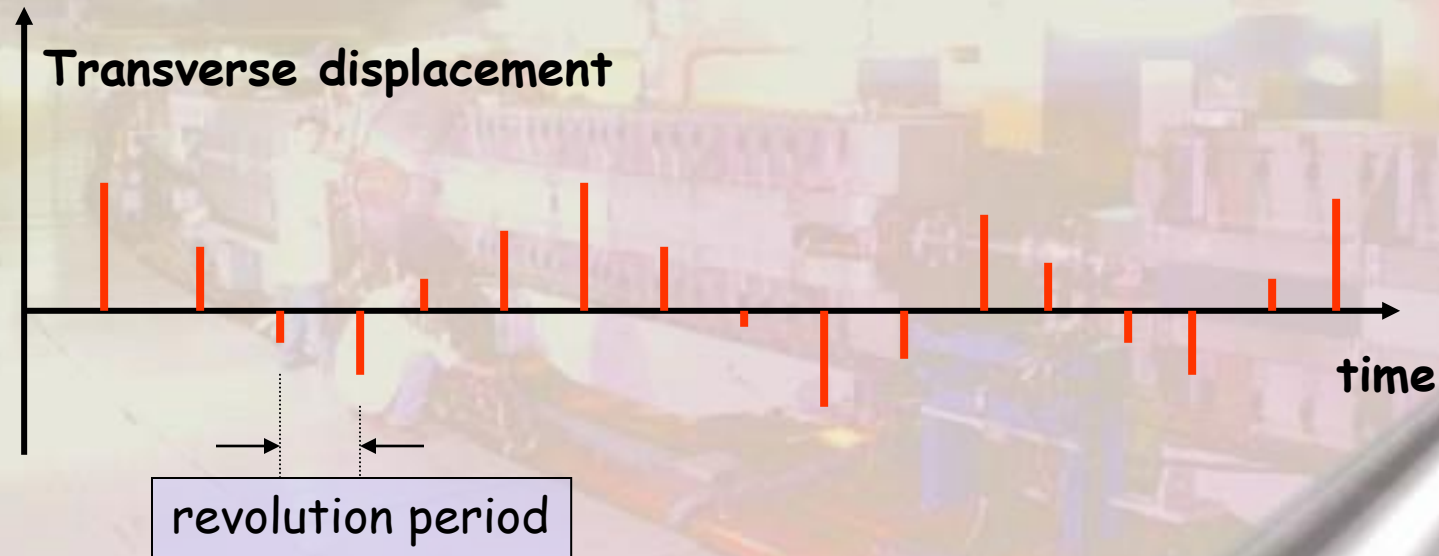
- # **Longitudinal instabilities** are related to **synchrotron oscillations**
- # **Transverse instabilities** are related to **synchrotron and betatron oscillations**
- # Why....?....
- # Particles move around the machine and execute synchrotron and betatron oscillations
- # If the chromaticity $\left(\xi = \frac{\Delta Q}{Q} / \frac{\Delta p}{p} \right)$ is non-zero
- # Then the changing energy, due to synchrotron oscillations will also change the betatron oscillation frequency (Q)

Single bunch modes

- # As for longitudinal oscillation there are different **modes** for single bunch transverse oscillations
- # We can observe the transverse bunch motion from the difference signal on a position monitor

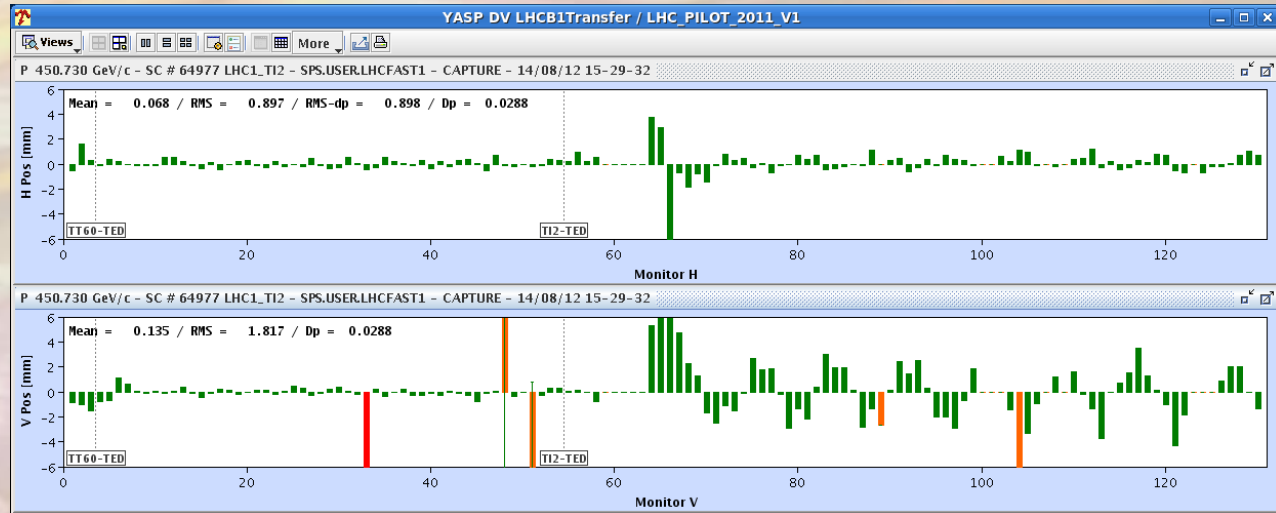
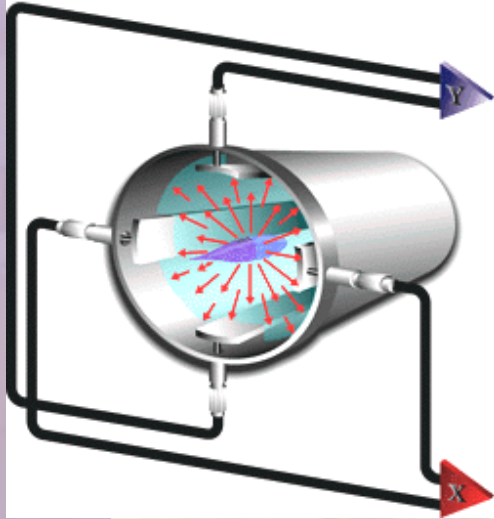
Rigid bunch mode (1)

- # The bunch oscillates transversely as a rigid unit
- # On a single position sensitive pick-up we can observe the following:



Change in position/turn \Rightarrow betatron phase advance/turn

Beam Position Measurement



$$x = a \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} \equiv a \cdot \frac{\Delta}{\Sigma}$$

Rigid bunch mode (2)

Transverse displacement



Lets superimpose successive turns

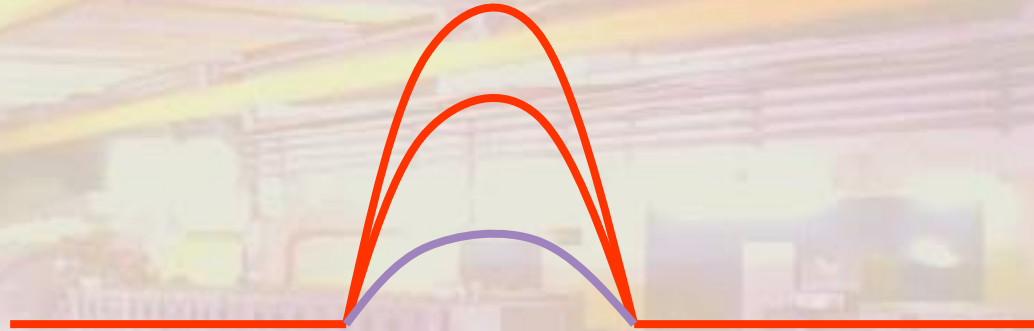
Rigid bunch mode (3)

Transverse displacement



Rigid bunch mode (4)

Transverse displacement



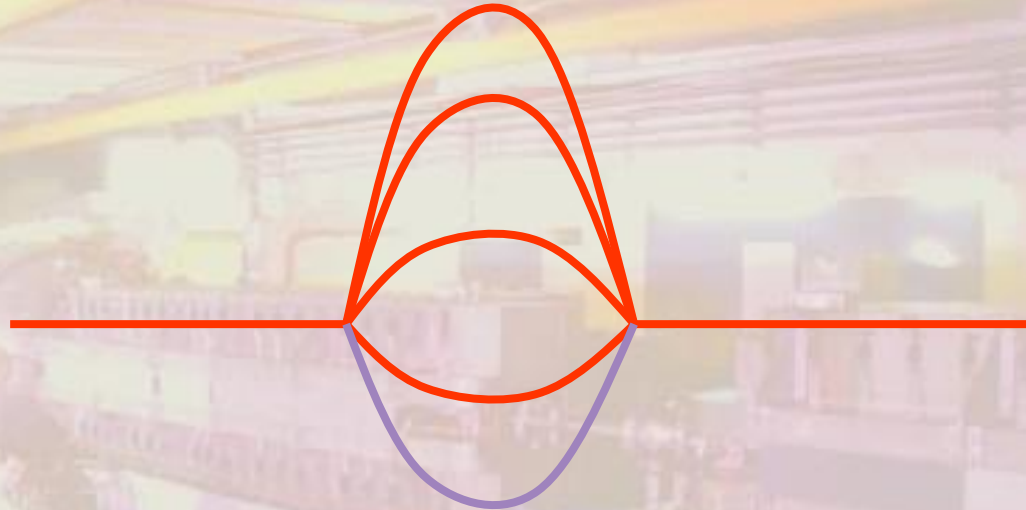
Rigid bunch mode (5)

Transverse displacement



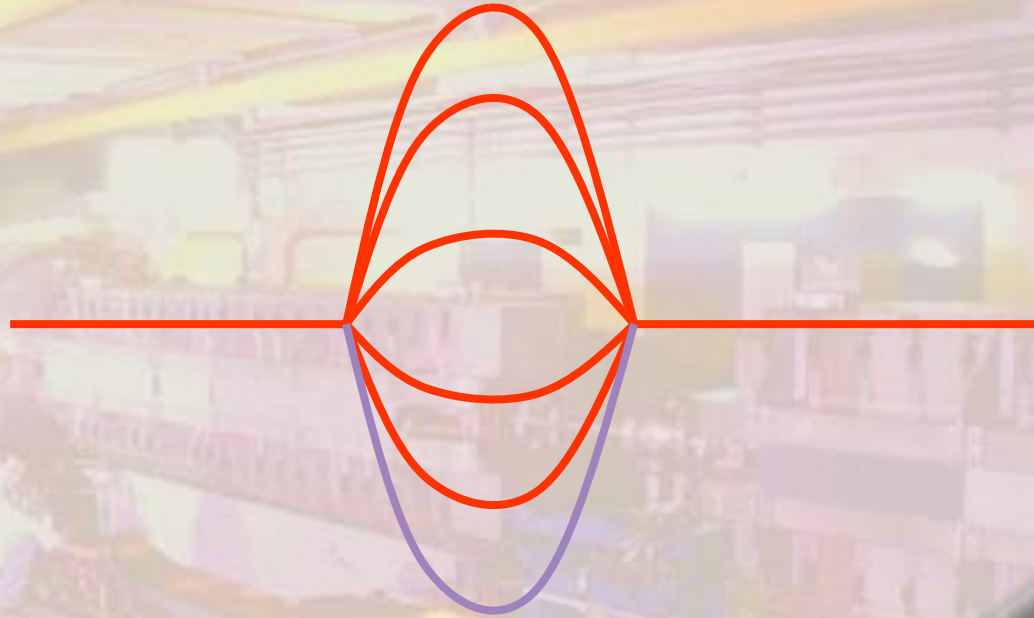
Rigid bunch mode (6)

Transverse displacement



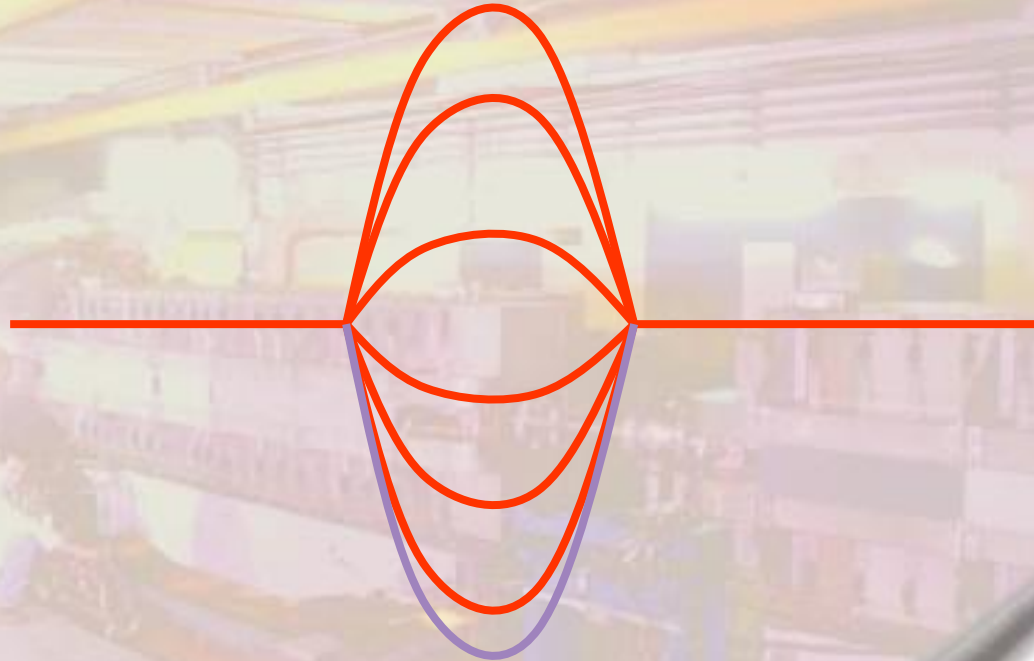
Rigid bunch mode (7)

Transverse displacement



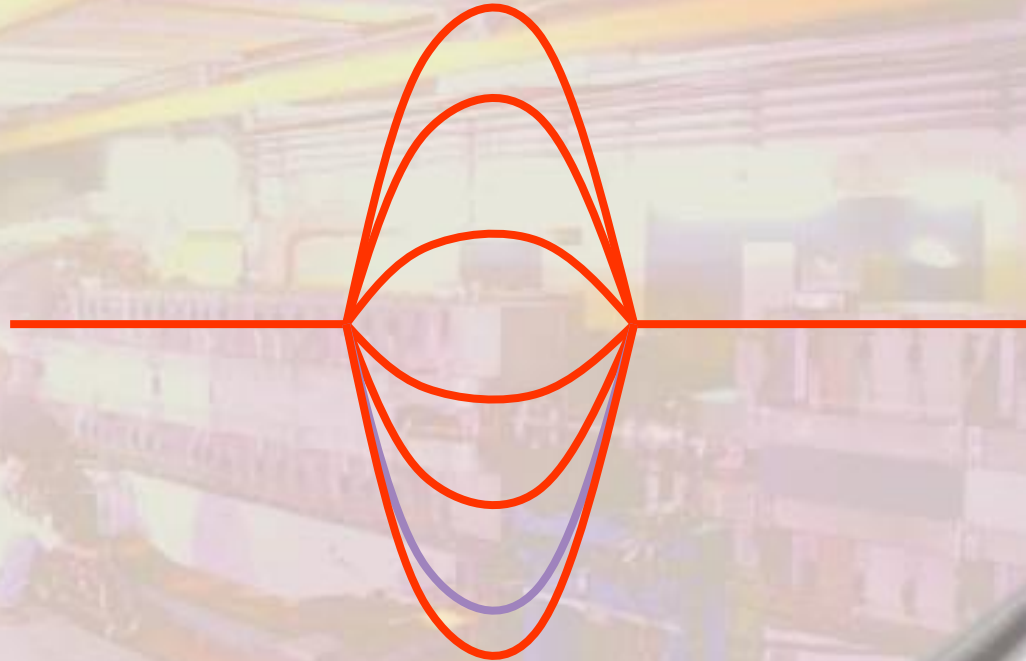
Rigid bunch mode (8)

Transverse displacement



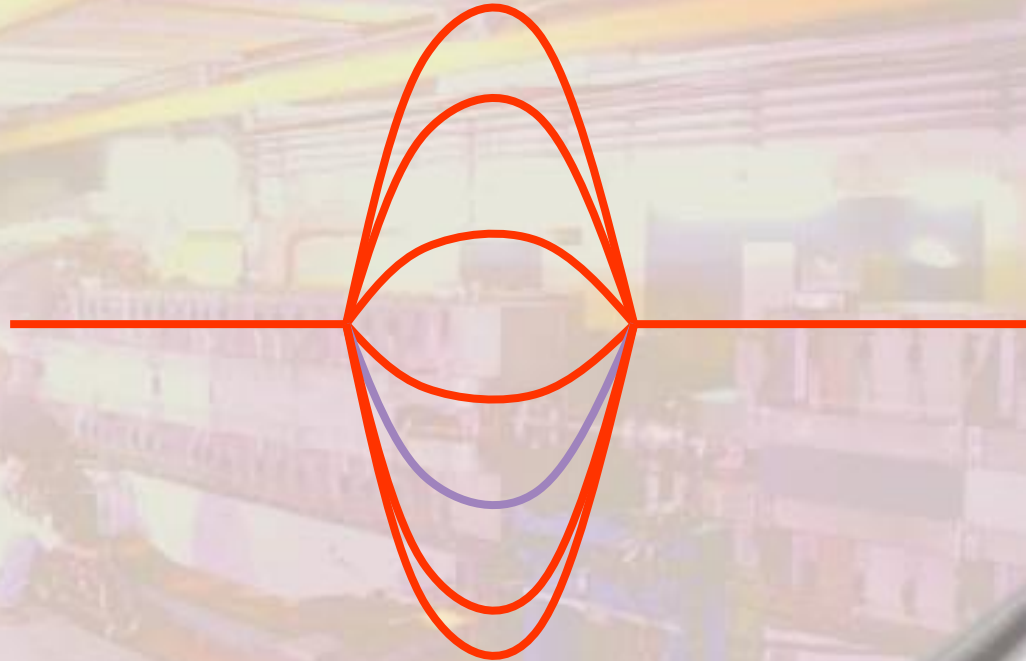
Rigid bunch mode (9)

Transverse displacement



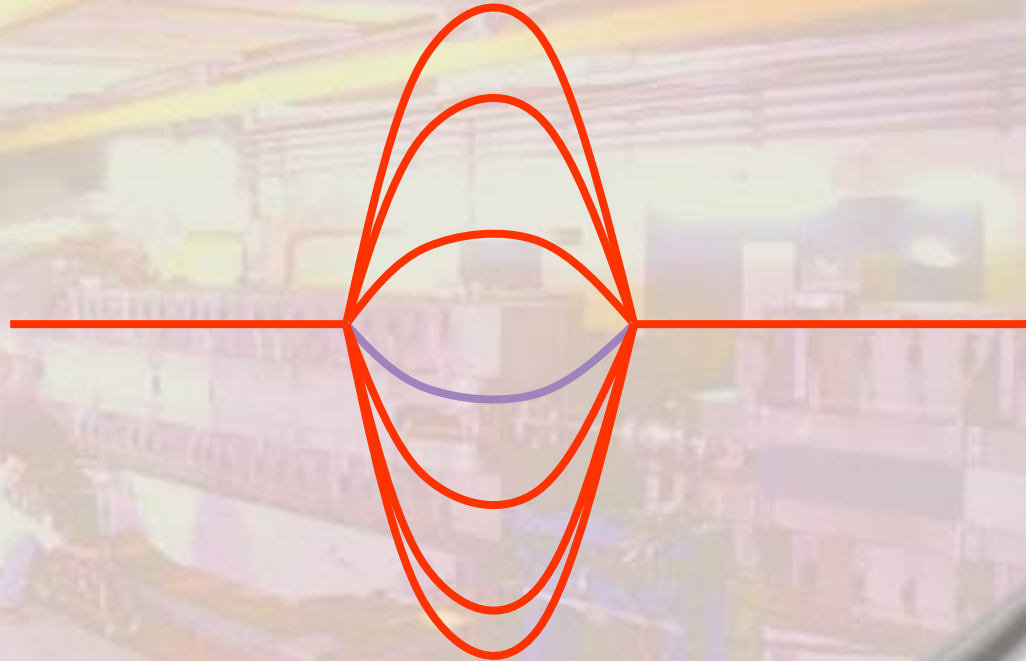
Rigid bunch mode (10)

Transverse displacement



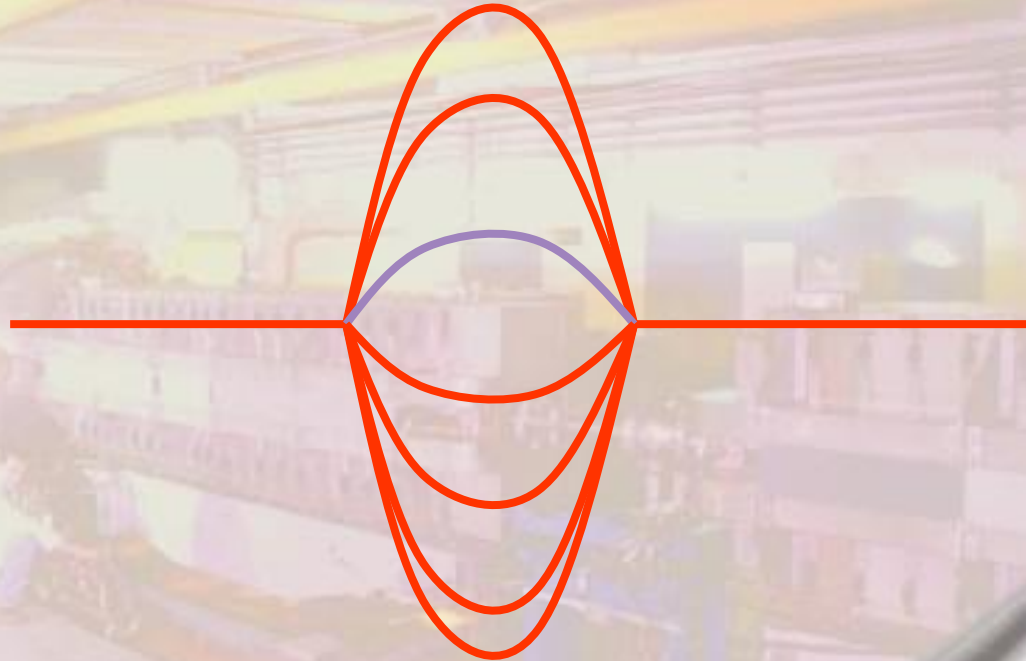
Rigid bunch mode (11)

Transverse displacement



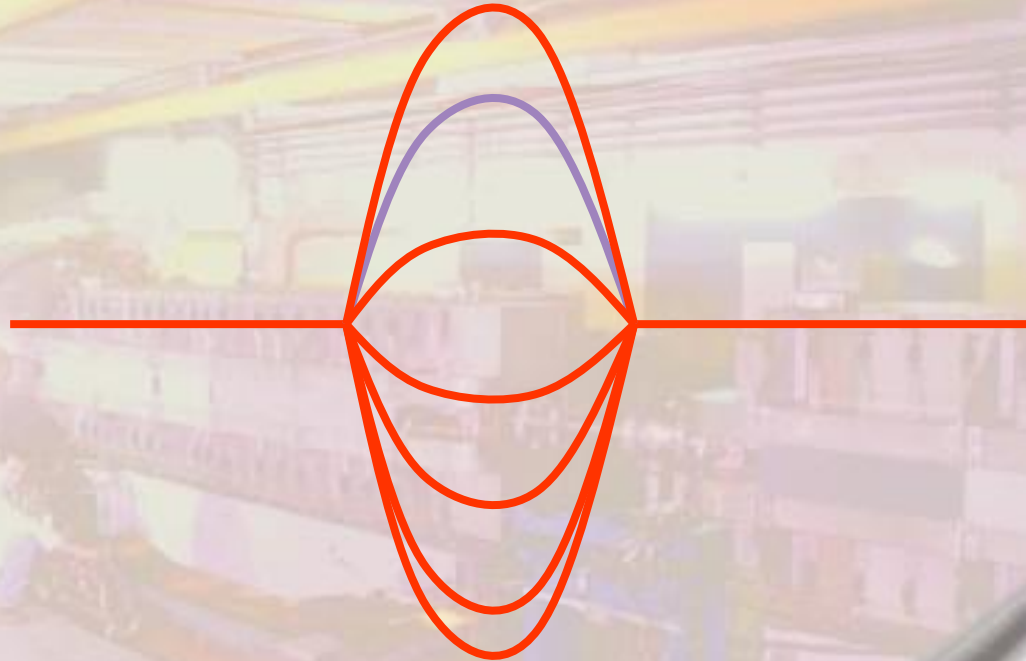
Rigid bunch mode (12)

Transverse displacement



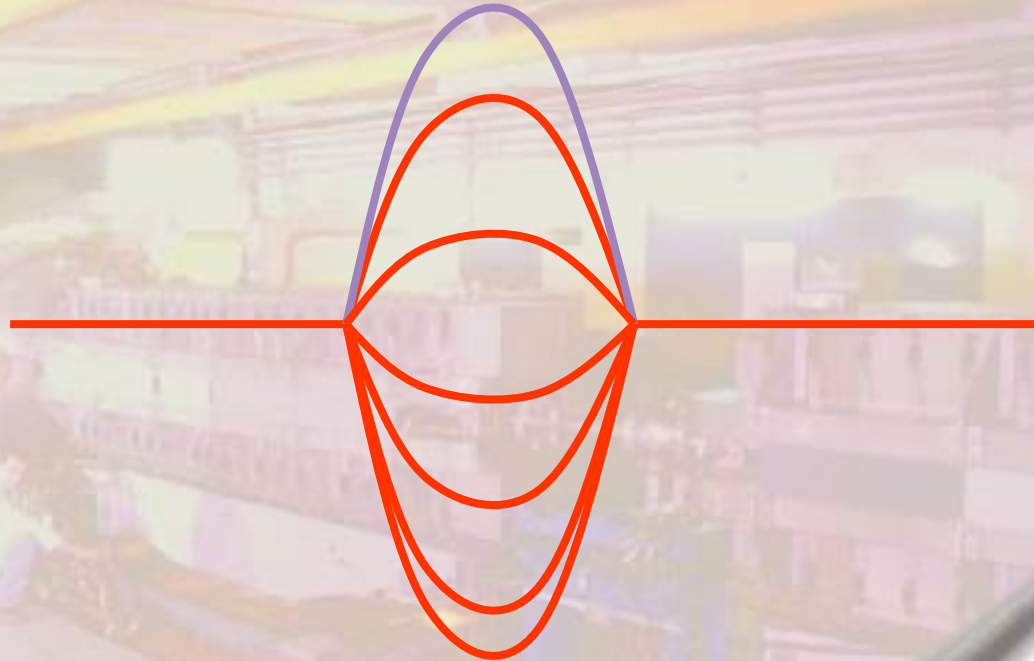
Rigid bunch mode (13)

Transverse displacement



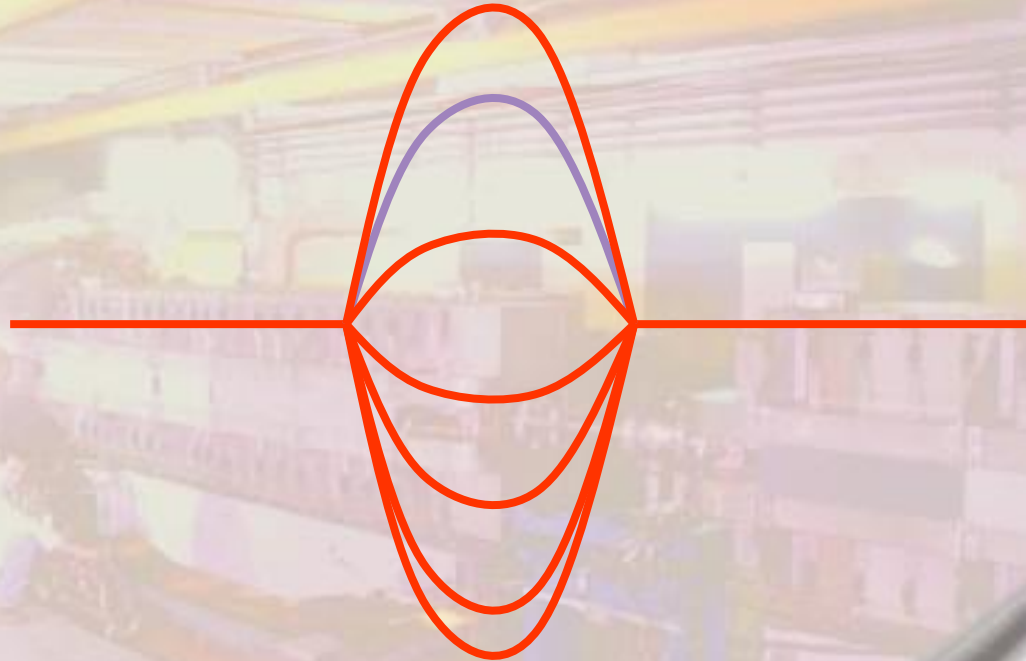
Rigid bunch mode (14)

Transverse displacement



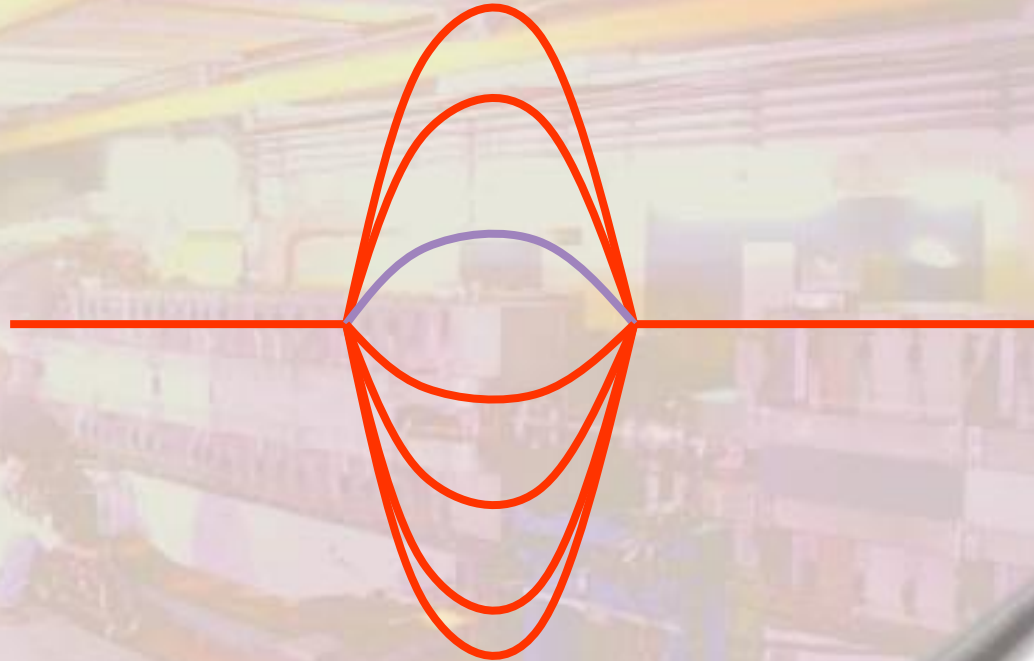
Rigid bunch mode (15)

Transverse displacement



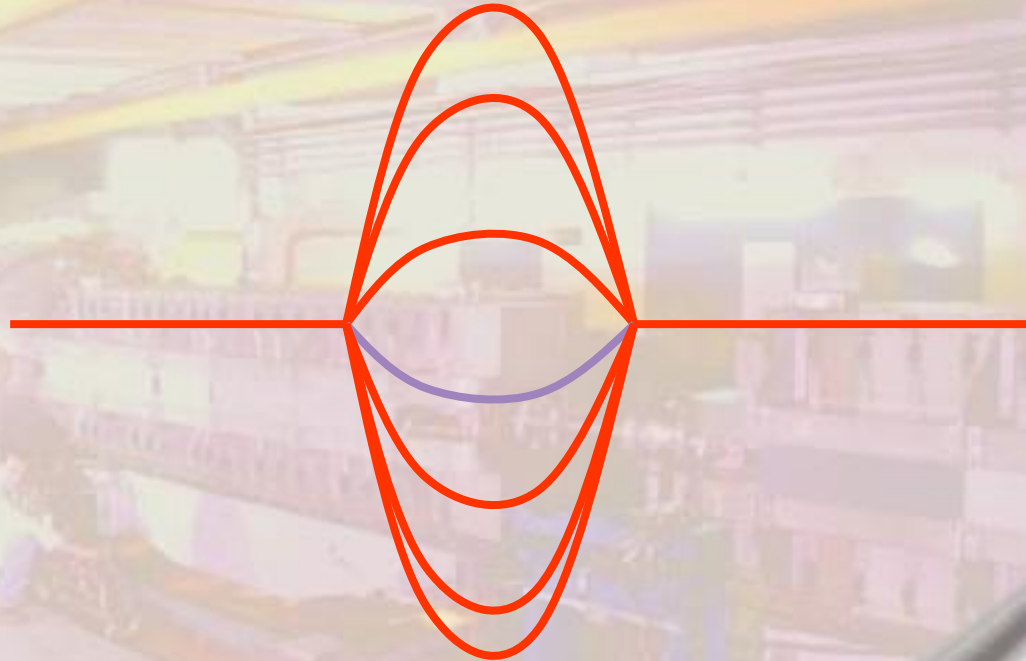
Rigid bunch mode (16)

Transverse displacement



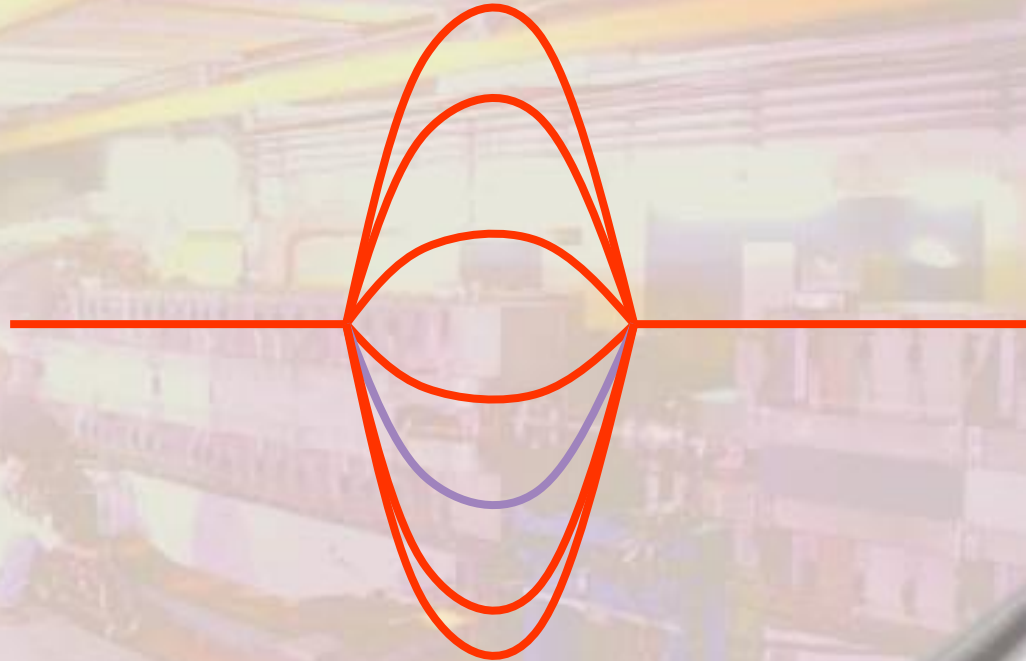
Rigid bunch mode (17)

Transverse displacement



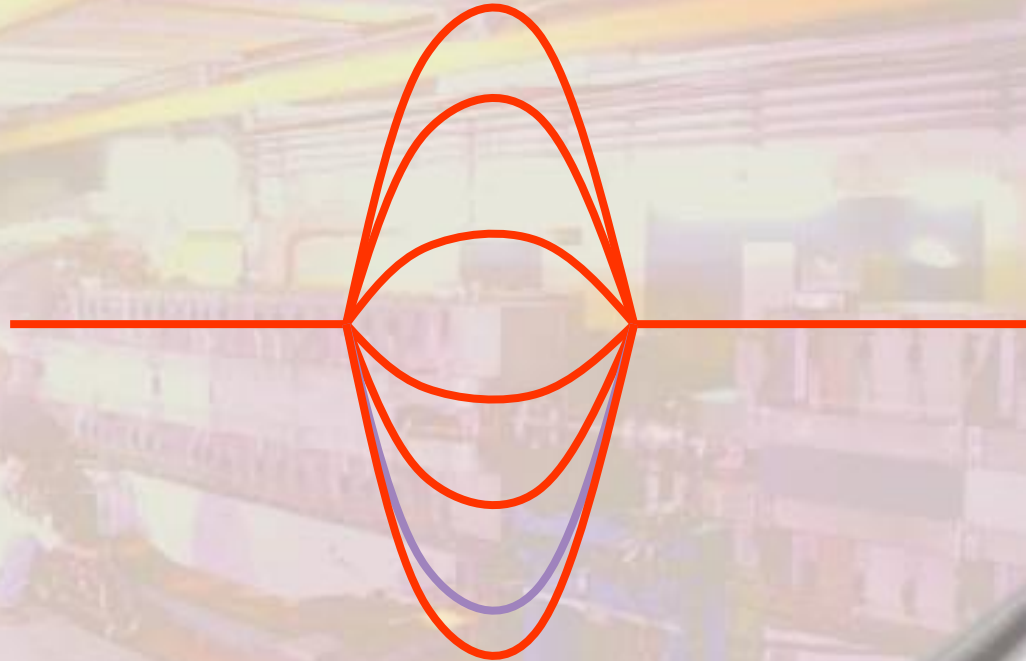
Rigid bunch mode (18)

Transverse displacement



Rigid bunch mode (19)

Transverse
displacement



Standing wave without node \Rightarrow Mode $M=0$

Cure for rigid bunch mode instability

- # To help avoid this instability we need a **non-zero chromaticity**

$$\left(\xi = \frac{\Delta Q}{Q} / \frac{\Delta p}{p} \right)$$

- # The bunch has an **energy/momentum spread**

- # The Particles will have a **spread in betatron frequencies**

- # A **spread in betatron frequencies** will mean that any coherent transverse oscillation (all particles moving together) will very quickly become **incoherent** again.

Higher order bunch modes

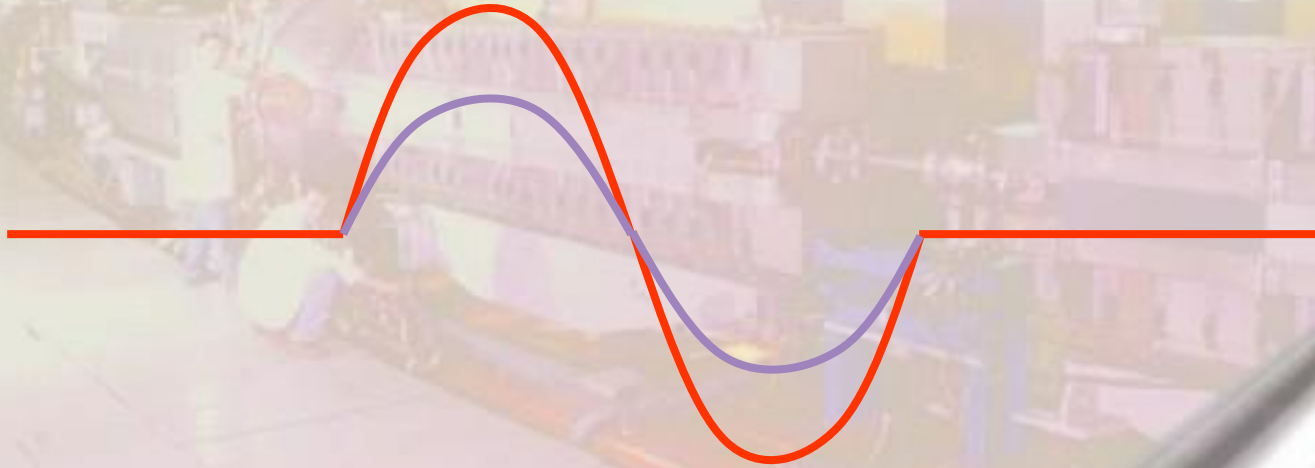
- # Higher order modes are called “Head-tail” modes as the electro-magnetic fields induced by the head of the bunch excite oscillation of the tail
- # However, these modes may be harder to observe as the centre of gravity on the bunch may not move.....
- # Nevertheless, they are very important and cannot be neglected

Head-tail modes (1)

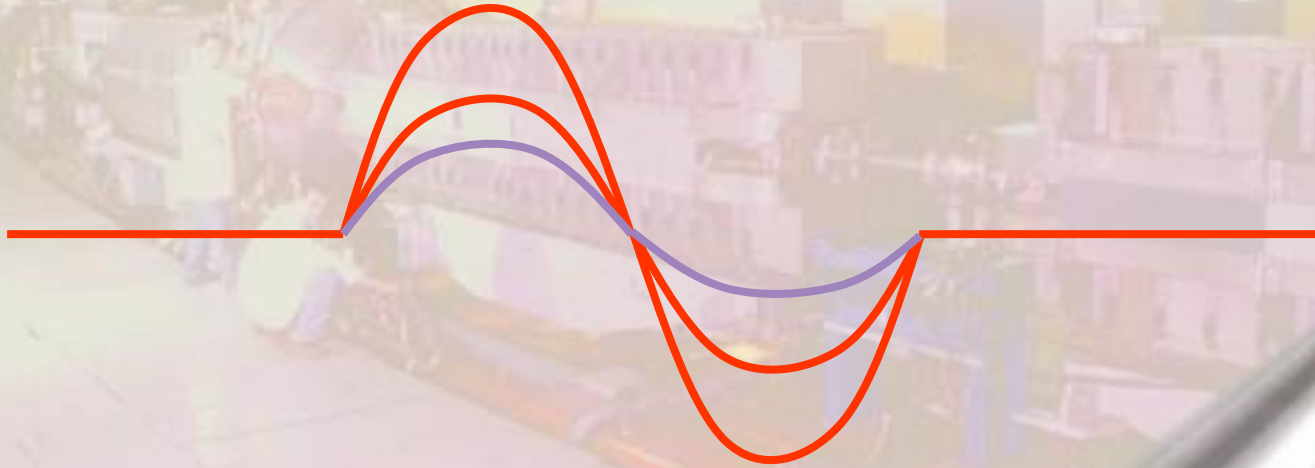
- # Head & Tail of bunch move π out of phase with each other
- # Again, lets superimpose successive turns



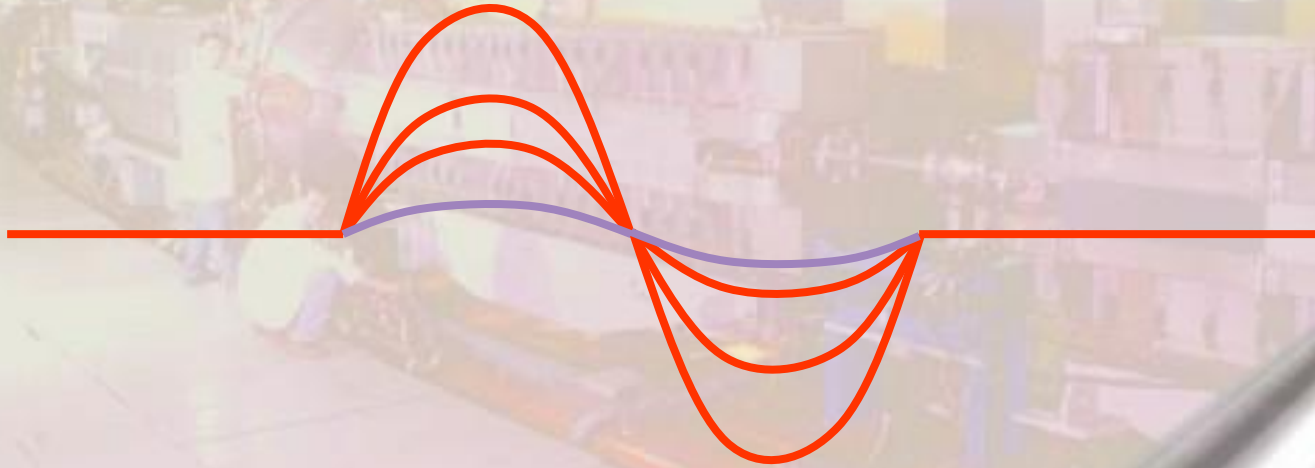
Head-tail modes (2)



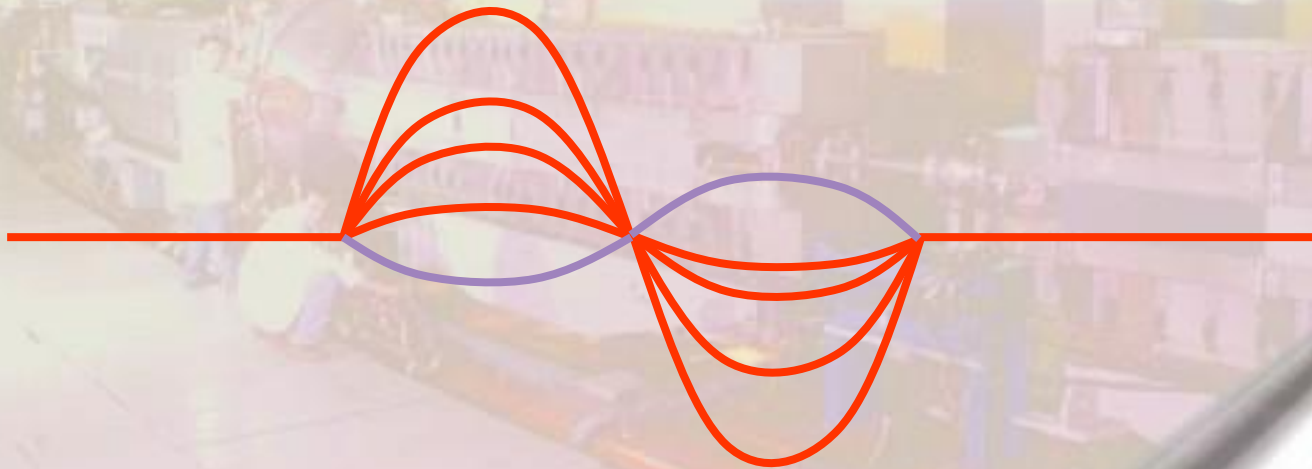
Head-tail modes (3)



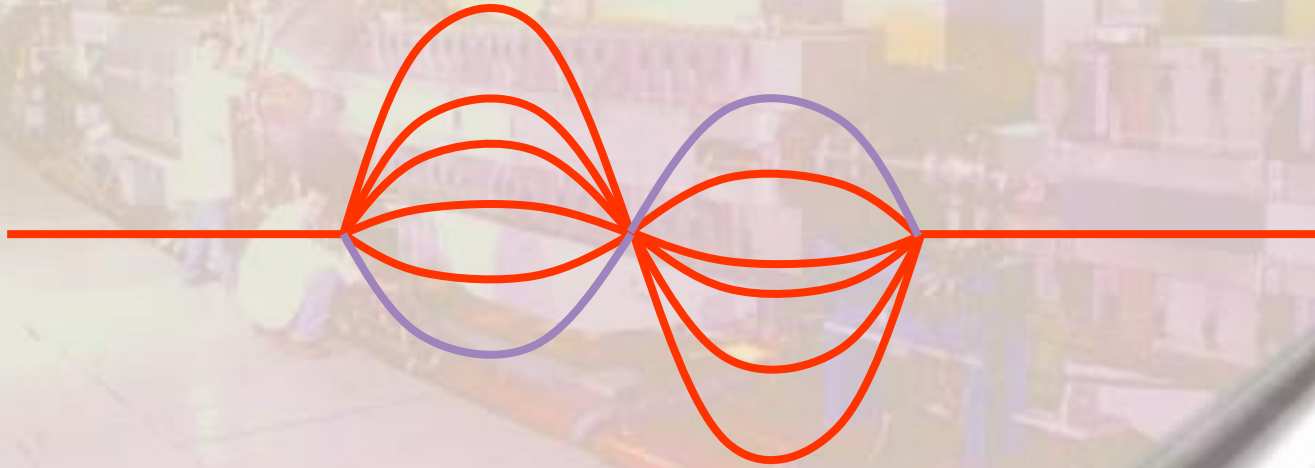
Head-tail modes (4)



Head-tail modes (5)

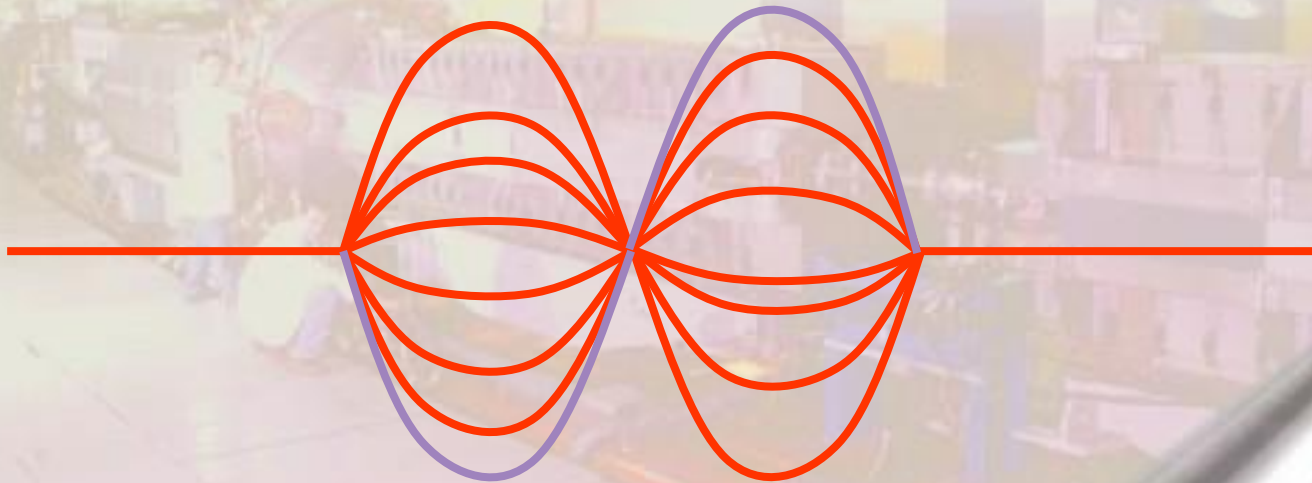


Head-tail modes (6)



Head-tail modes (7)

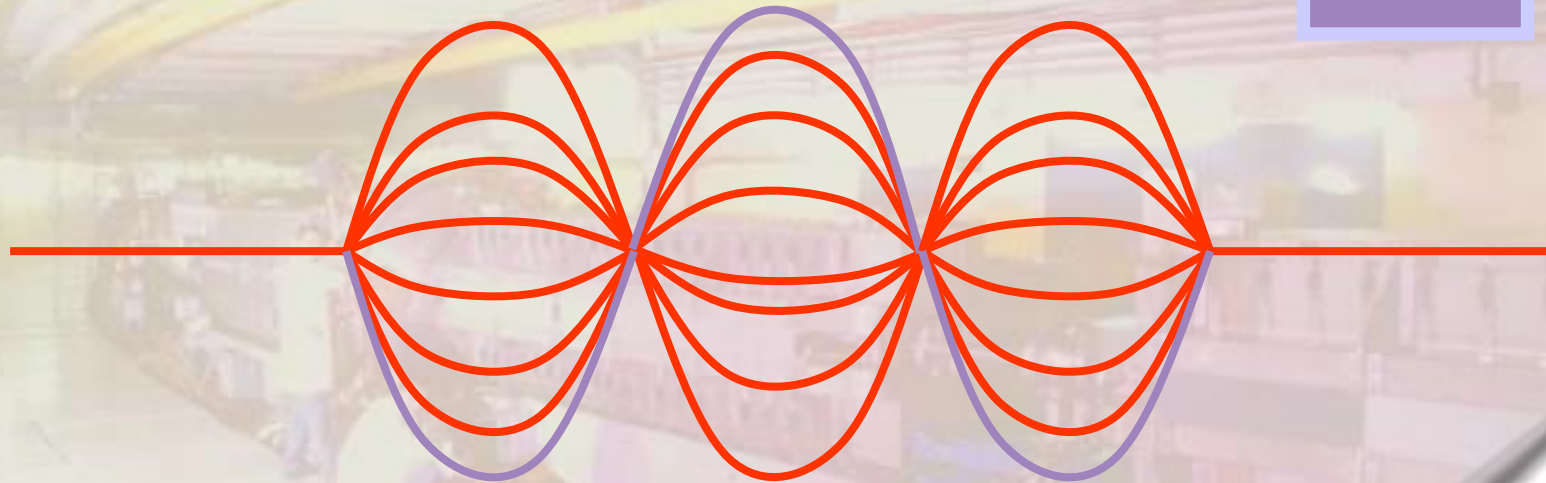
- # This is a standing wave with one node
- # Thus: Mode $M=1$



Head-tail modes (8)

This is (obviously!) Mode:

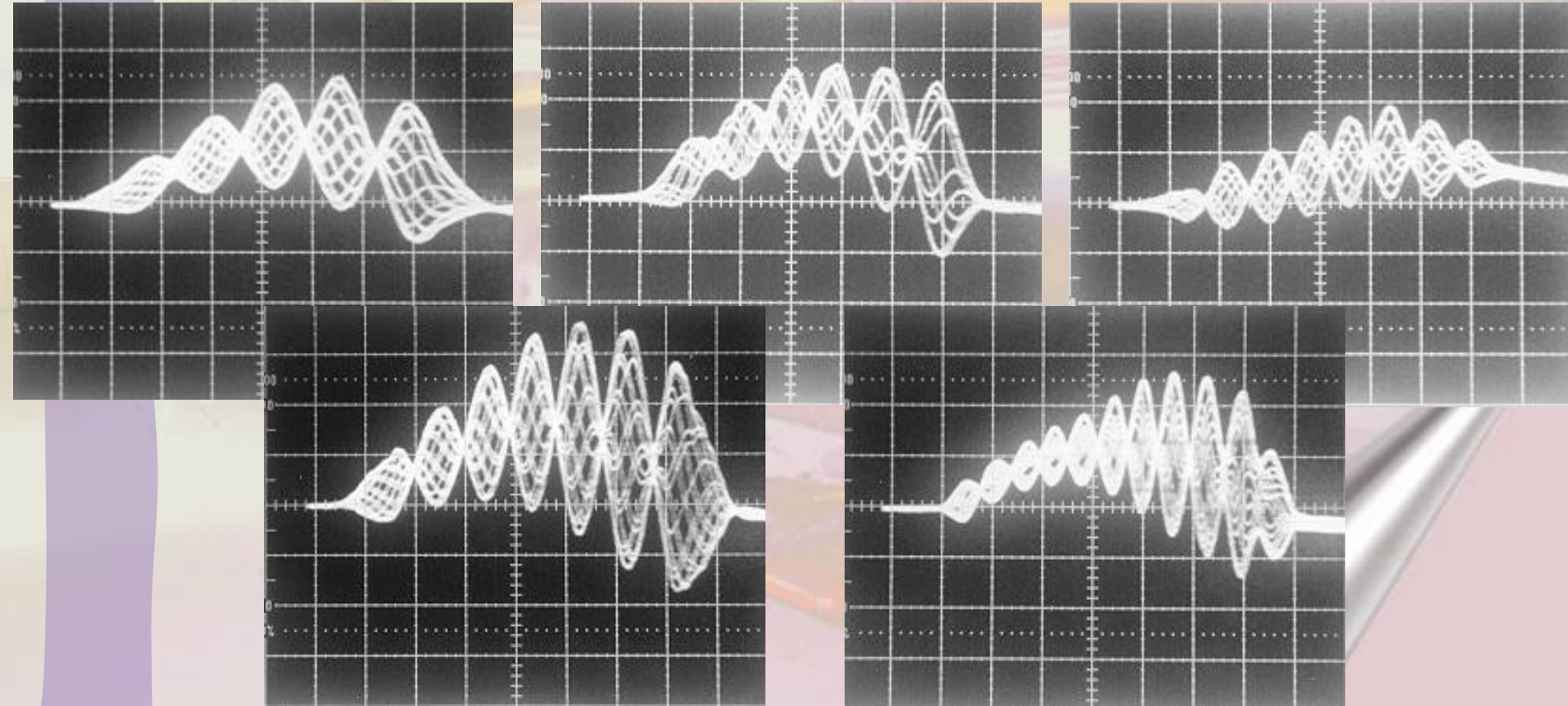
M=2



- # Let's look more in detail at the **M=1** “head-tail” mode
- # But first some real-life examples.....

Head-tail modes (8)

Some real life examples:



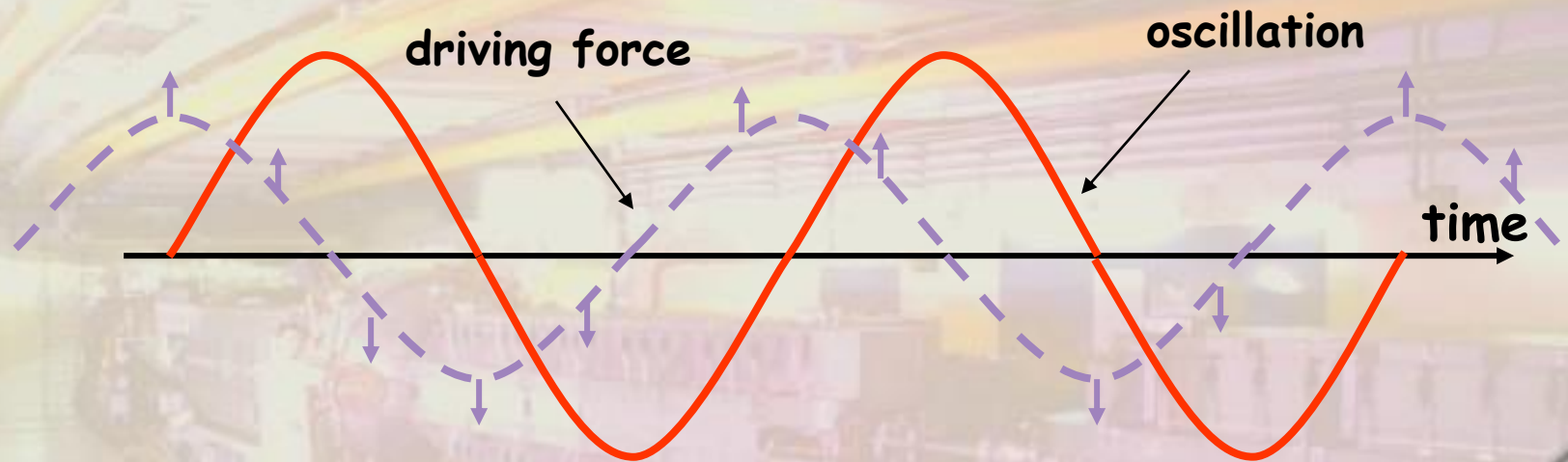
Oscillation and the driving force (1)

- # Before continuing, first a memory refresher....



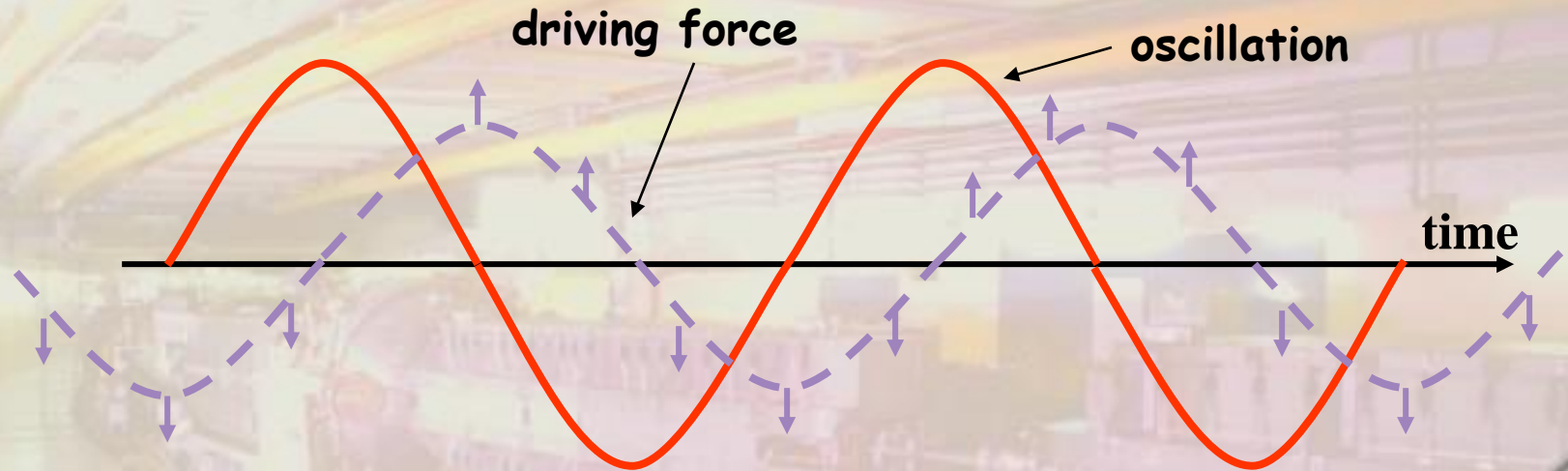
- # In order to increase the amplitude of a driven oscillator the driving force must be ahead (in phase) of the motion
- # Anyone who has pushed a child on a swing will know this.....

Oscillation and the driving force (2)



Driving force ahead of oscillation \Rightarrow increasing amplitude
Makes children happy but the beam unstable
INSTABILITY

Oscillation and the driving force (3)

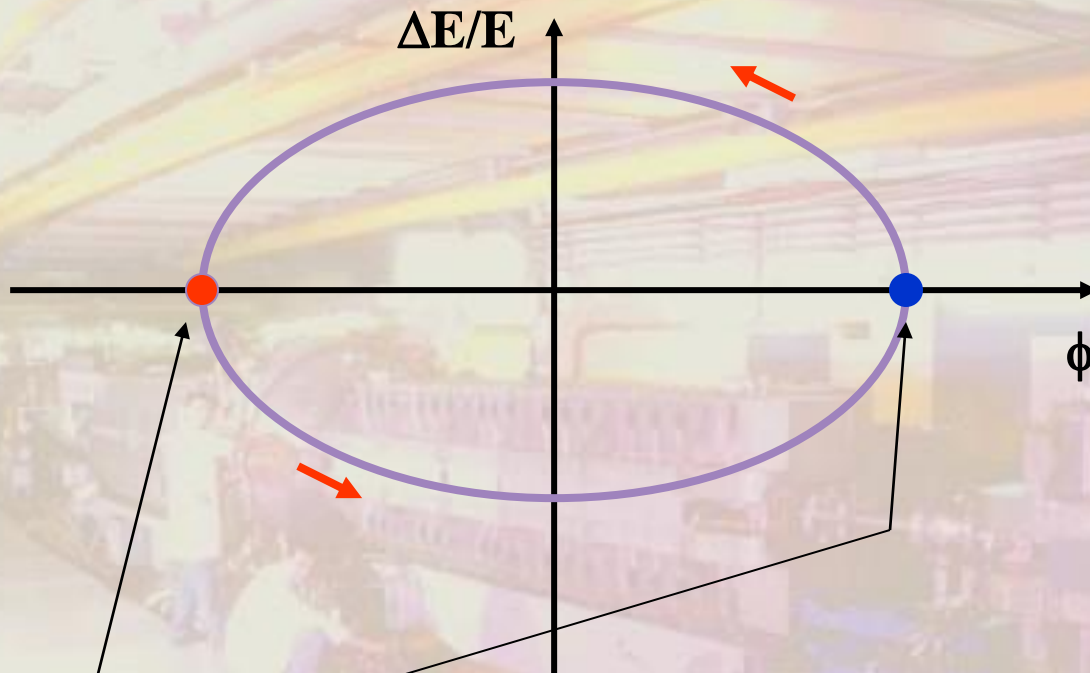


Driving force behind the oscillation \Rightarrow decreasing amplitude
Makes children unhappy but the beam stable
DAMPING

M=1 Head-tail mode (1)

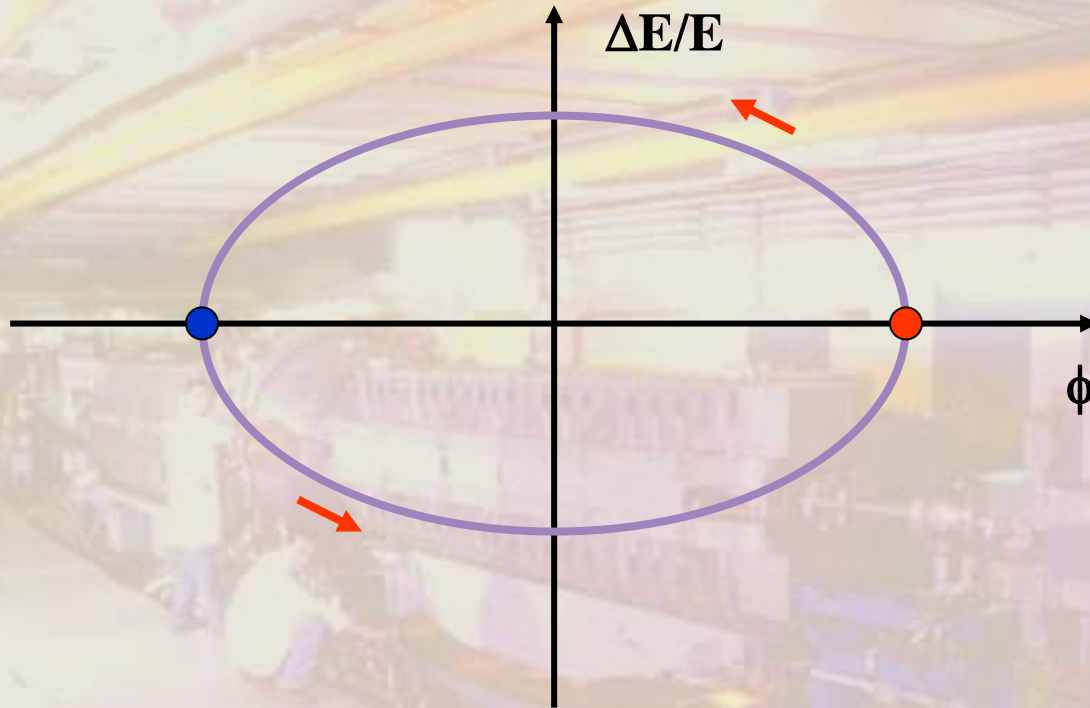
- # The M=1 head tail mode includes both **betatron** and **synchrotron** oscillations
- # There are many betatron oscillations during one synchrotron oscillation
- # Thus: $Q_s \ll Q_h$ and also $Q_s \ll Q_v$
- # Let's set up an M=1 mode transverse bunch oscillation
- # This means that the particles in the tail of the bunch are deflected by the electro-magnetic field left behind by the head of the bunch

M=1 Head-tail mode (2)



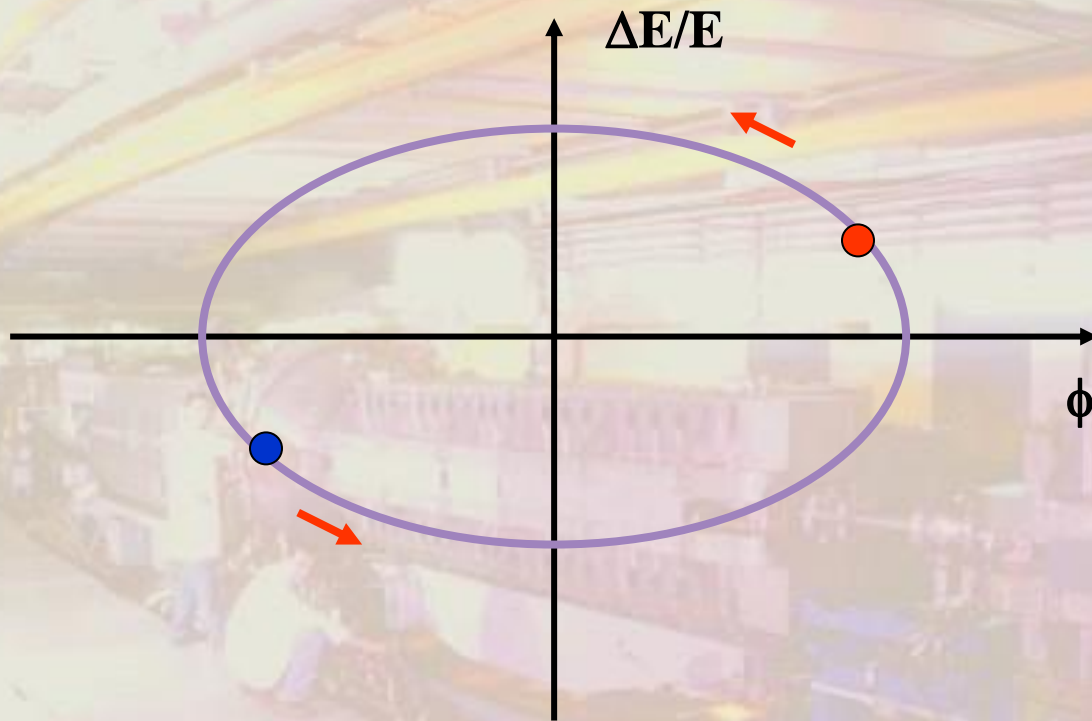
Two particles in longitudinal phase space:
Transverse oscillation of the blue particle is exactly out of phase with red one \Rightarrow red particle is exactly out of phase with the field left by the blue particle
NO EXCITATION

M=1 Head-tail mode (3)

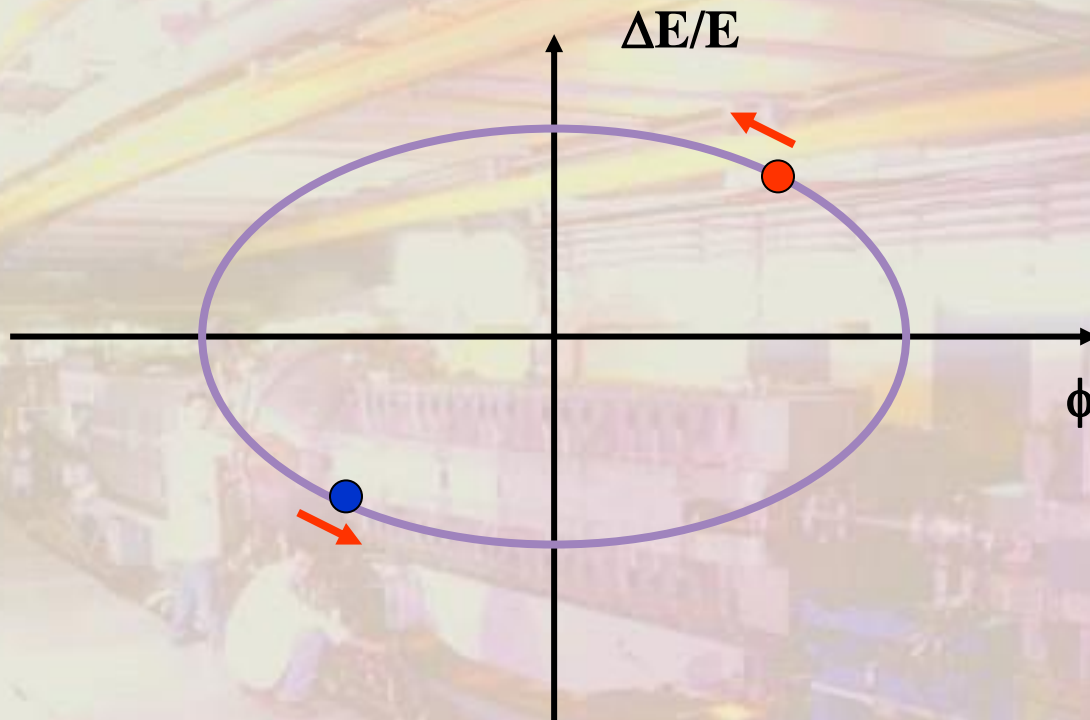


However, in 1/2 of a synchrotron period the particles will change places

M=1 Head-tail mode (4)

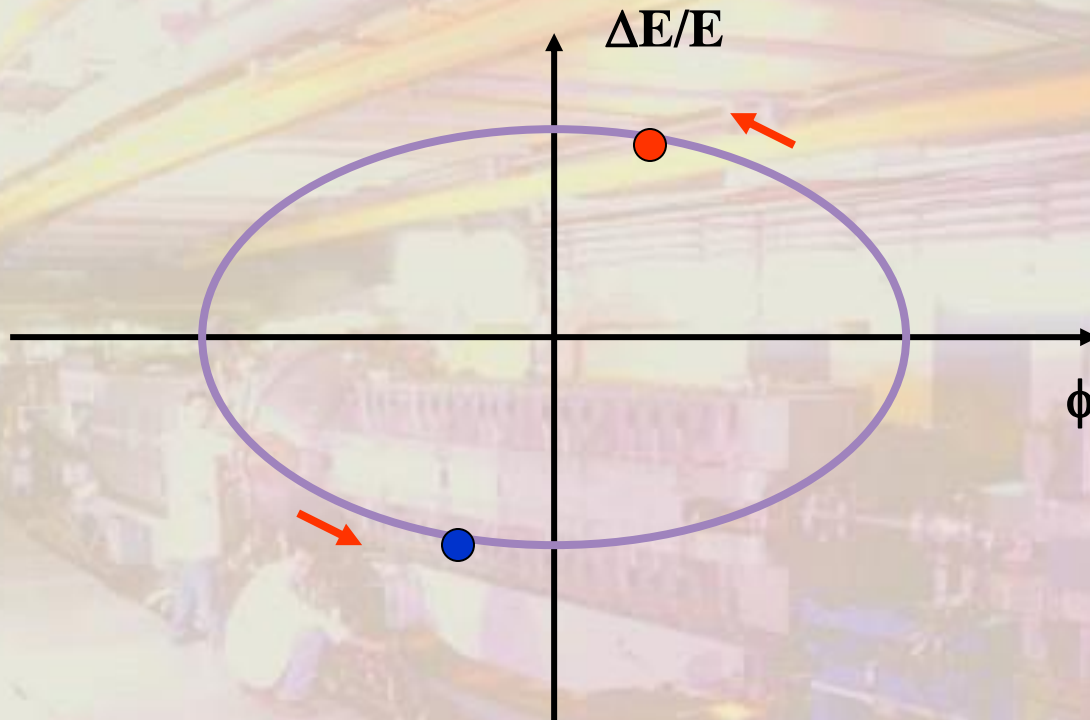


M=1 Head-tail mode (5)

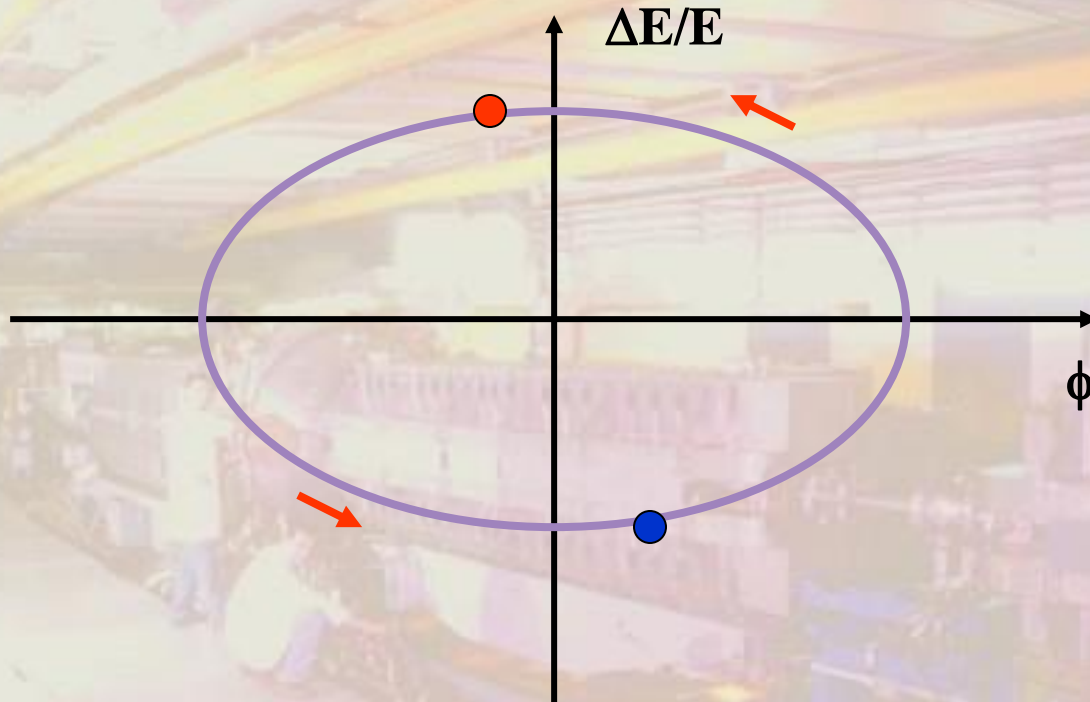


The energy of **red** particle is increasing
The energy of **blue** particle is decreasing

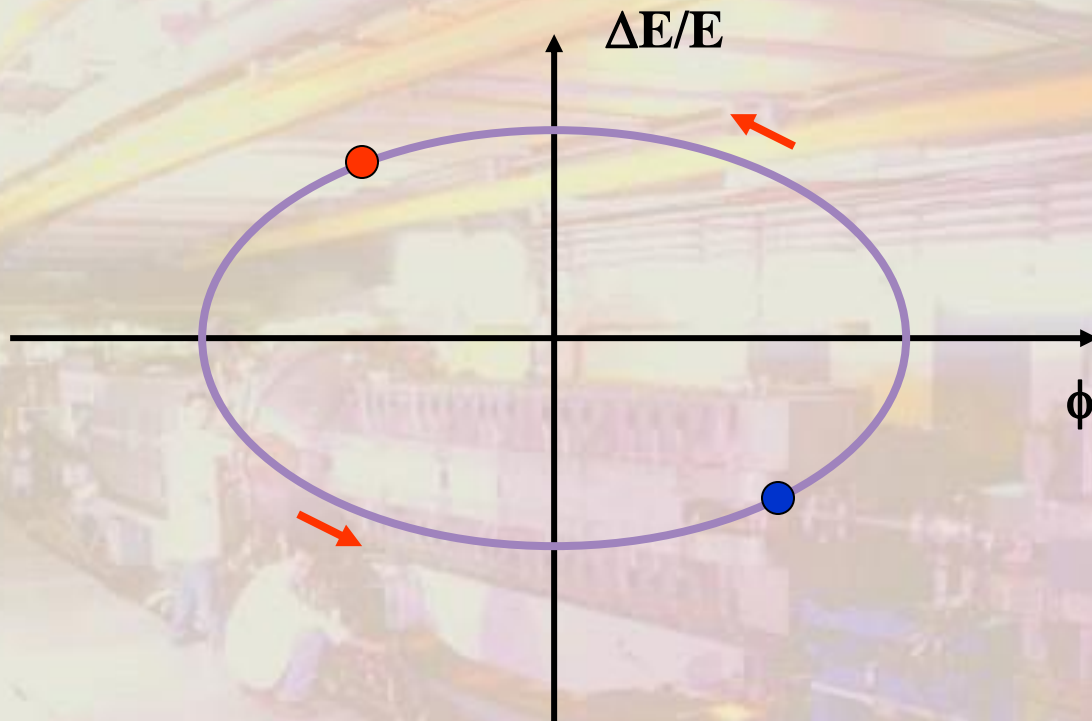
M=1 Head-tail mode (6)



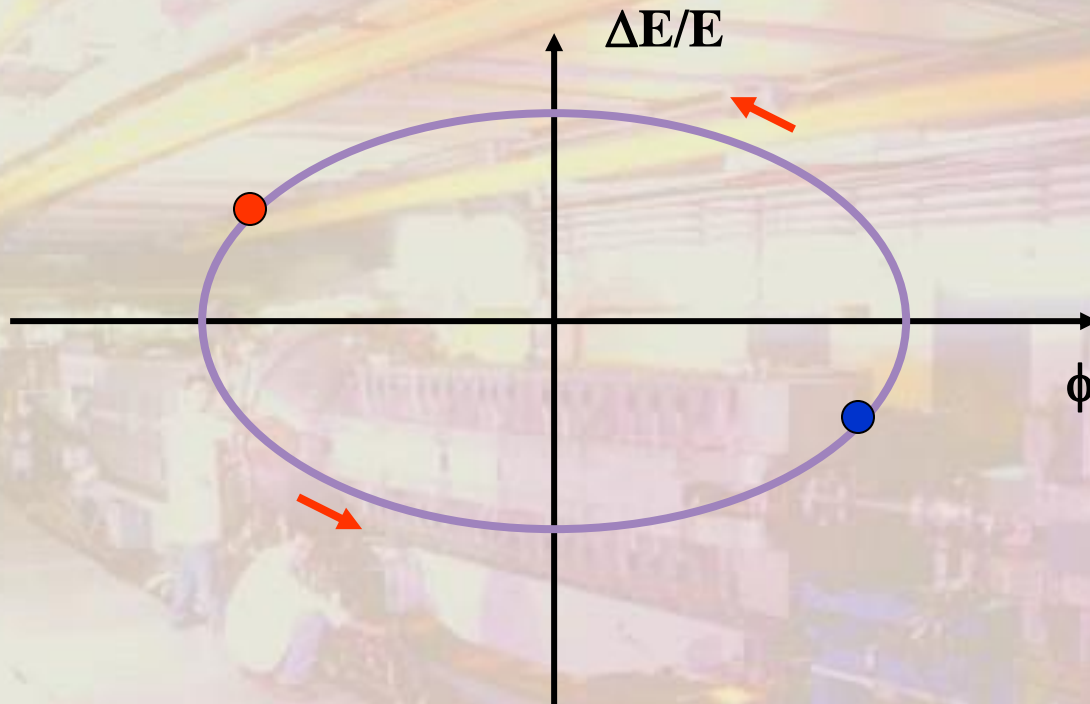
M=1 Head-tail mode (7)



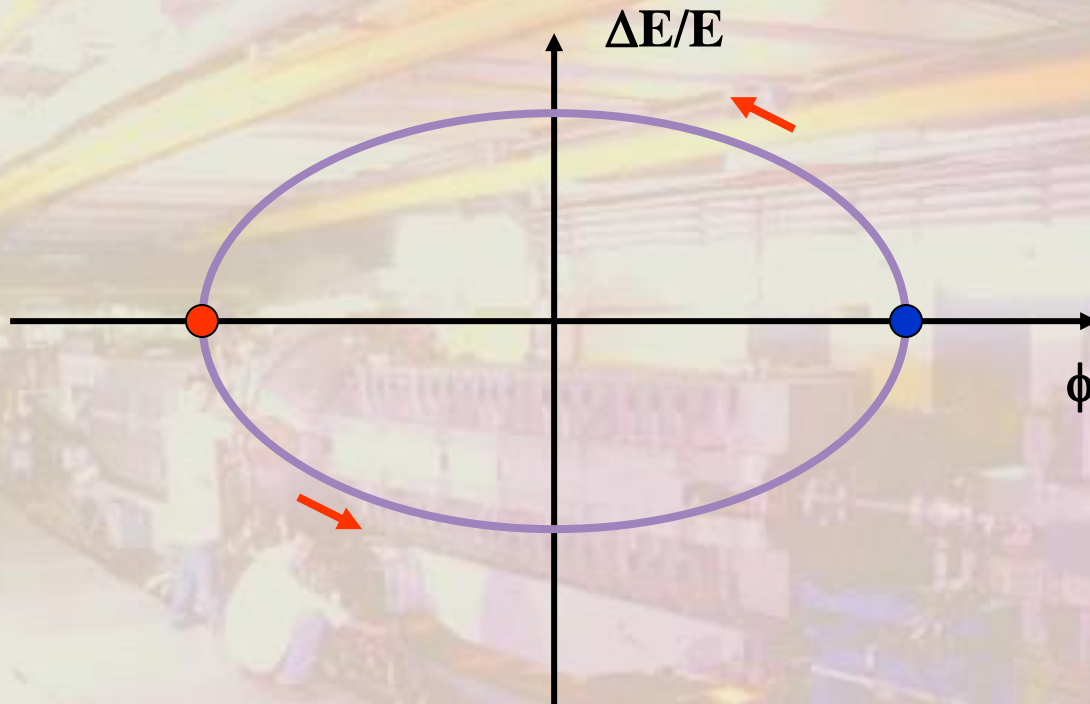
M=1 Head-tail mode (8)



M=1 Head-tail mode (9)

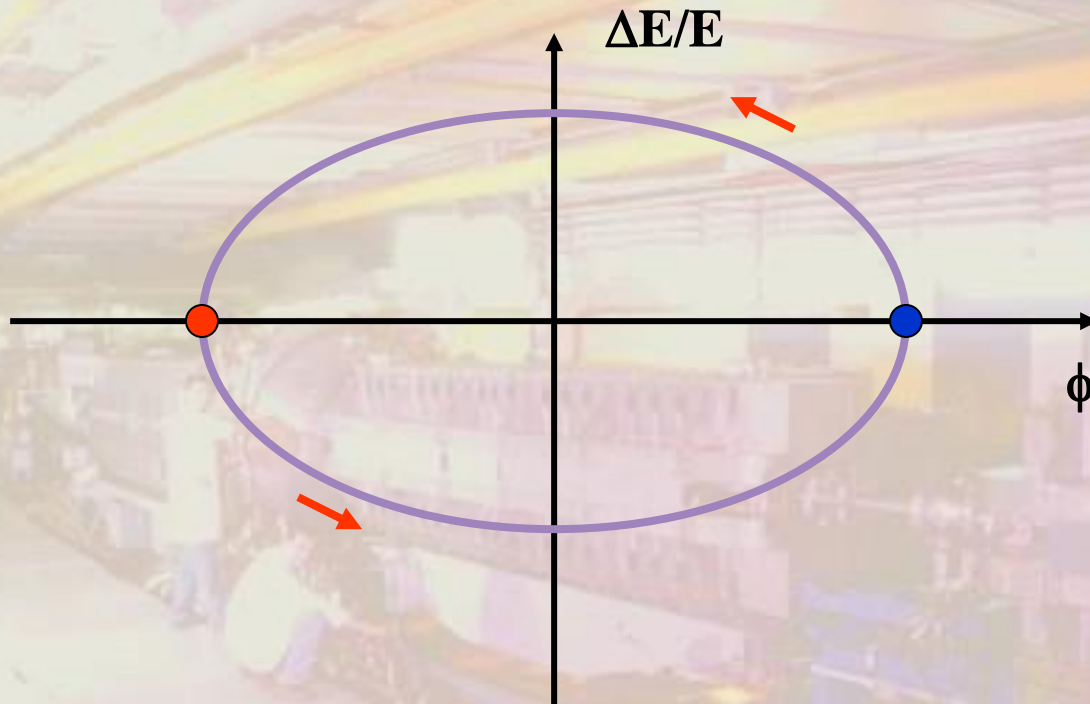


M=1 Head-tail mode (10)



Now they have changed places and have returned to their original energies

M=1 Head-tail mode (11)



If the chromaticity is zero **red** will still be exactly out of phase with the wake field left behind by **blue**

STABLE CONDITION

M=1 Head-tail mode (12)



If Chromaticity is negative **red** would have made slightly less betatron oscillations than **blue**
Then **red's** transverse oscillation would **lag slightly behind the wake field** left by **blue**

INSTABLE

M=1 Head-tail mode (13)



If Chromaticity is positive **red** would have made slightly more betatron oscillations than **blue**
Then **red's** transverse oscillation would be **slightly ahead of the wake field** left by **blue**

STABLE

M=1 Head-tail mode (14)

Conclusion:

- **Above transition** we must have a **positive chromaticity** to avoid the M=1 mode Head-Tail instability.
 - **Below transition** we must have a **negative chromaticity**.
- # The natural chromaticity of the machine without sextupoles is normally negative ($E\uparrow \rightarrow Q\downarrow$)
- # We therefore we need sextupoles to be able to correct the chromaticity.

Transverse multi-bunch modes

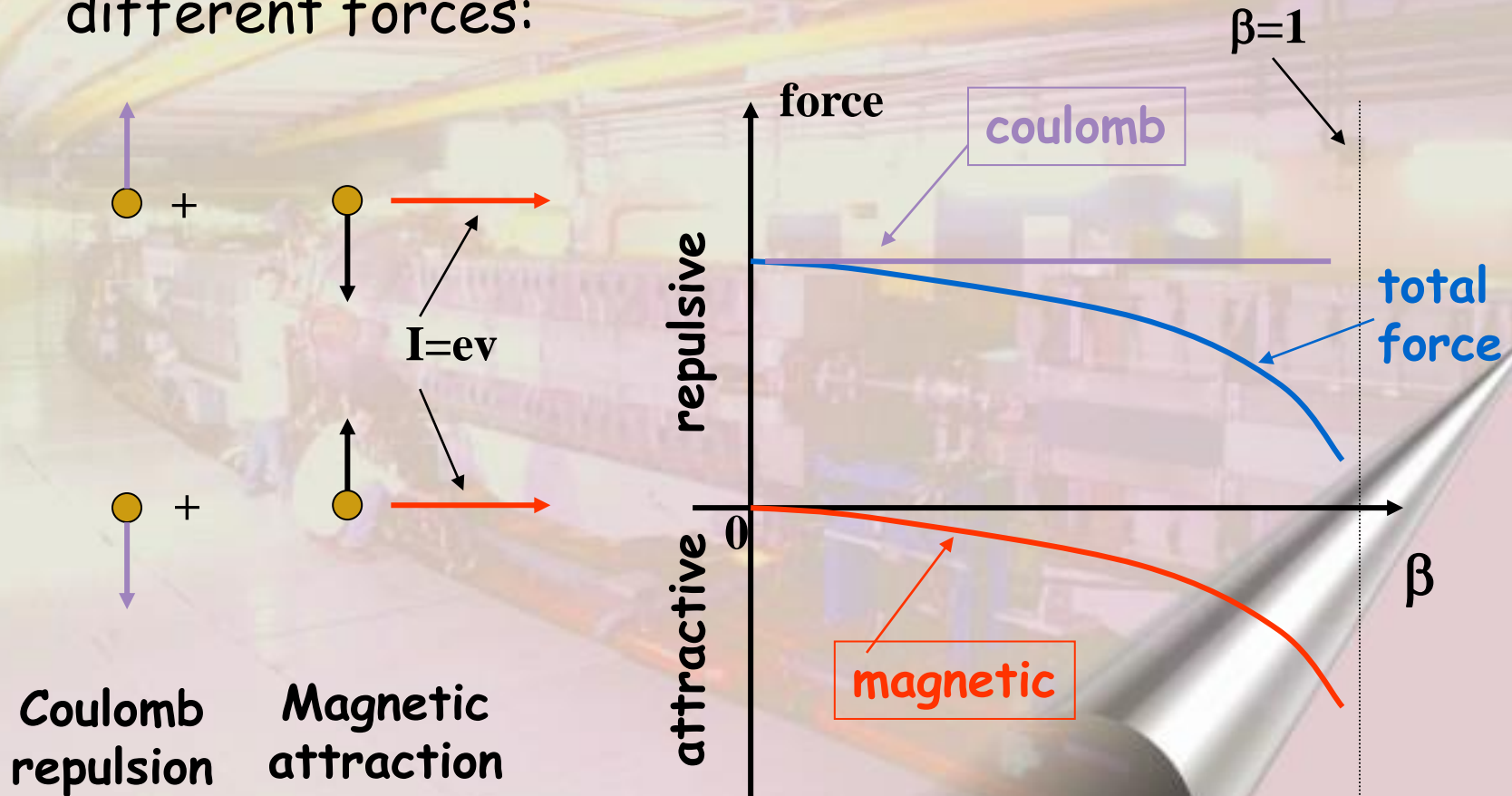
- # Longitudinal multi-bunch instabilities limit the bunch intensity before the transverse modes become a problem
- # However, once a longitudinal feed back system has been built, one may need to consider a transverse feed back system too.....

Cures

- # Correct the natural chromaticity of the machine (set chromaticity negative below transition and positive above transition, but not zero)
- # Install a feed-back system.
 - ▣ Detect a coherent oscillation and damp it using a transverse kicker
- # Damp transverse modes in cavities, where they will remain longest, using a damping antenna

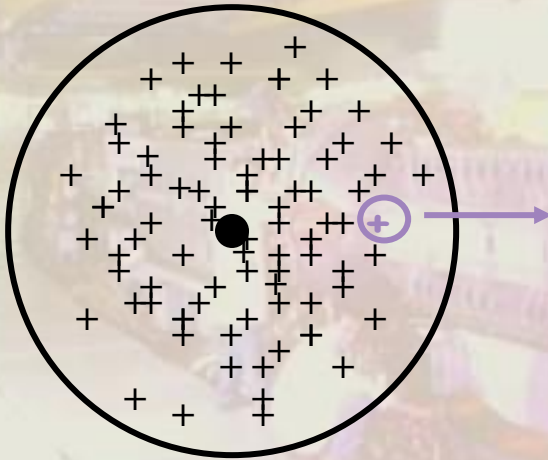
Space Charge effects (1)

Between two charged particles in a beam we have different forces:

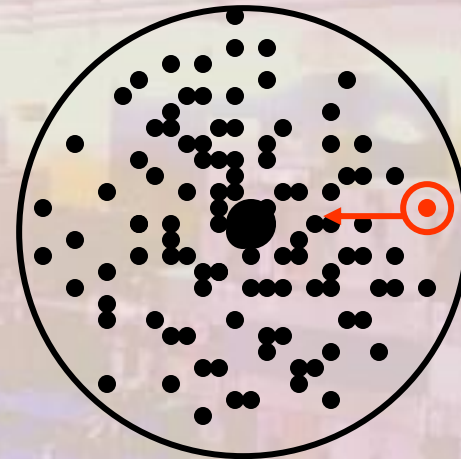


Space Charge effects (2)

- # For many particles in a beam we can represent it as following:



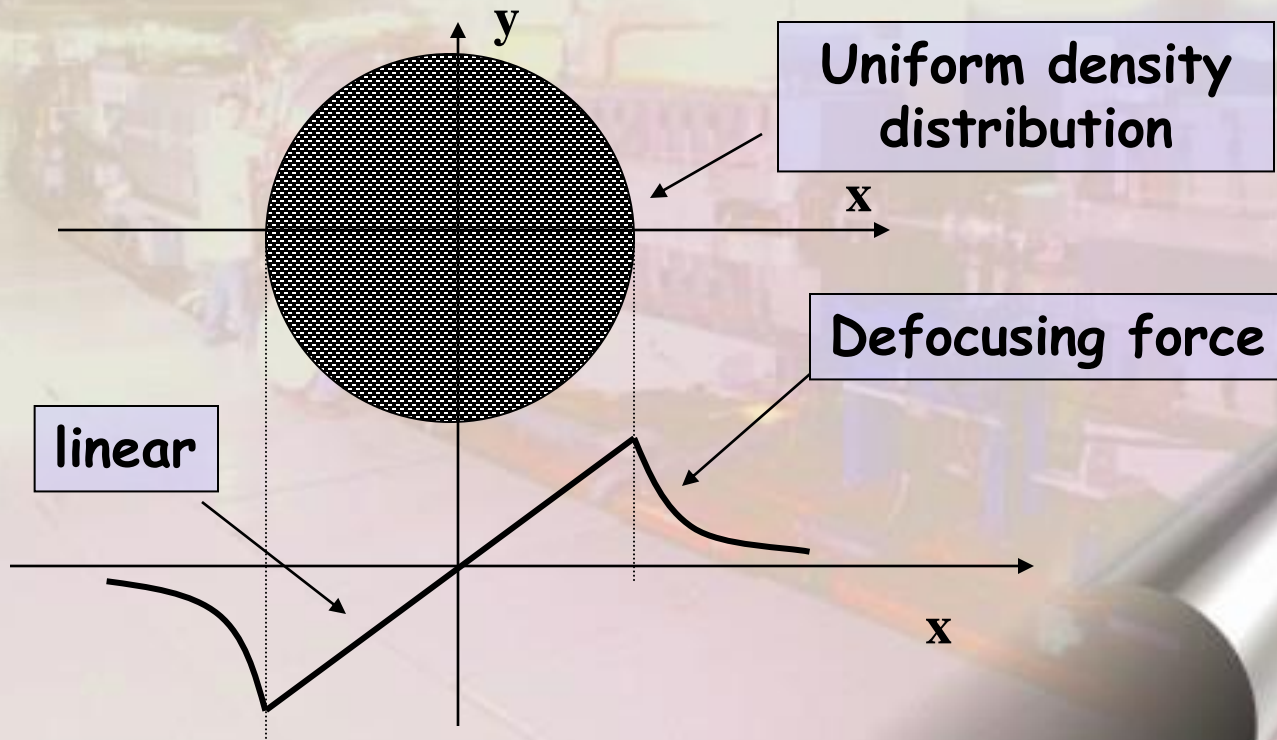
Charges \Rightarrow repulsion



Parallel currents \Rightarrow attraction

Space Charge effects (2)

- # At low energies, which means $\beta \ll 1$, the force is mainly repulsive \Rightarrow defocusing
- # It is zero at the centre of the beam and maximum at the edge of the beam



Space Charge effects (3)

- # For the uniform beam distribution, this linear defocusing leads to a tune shift given by:

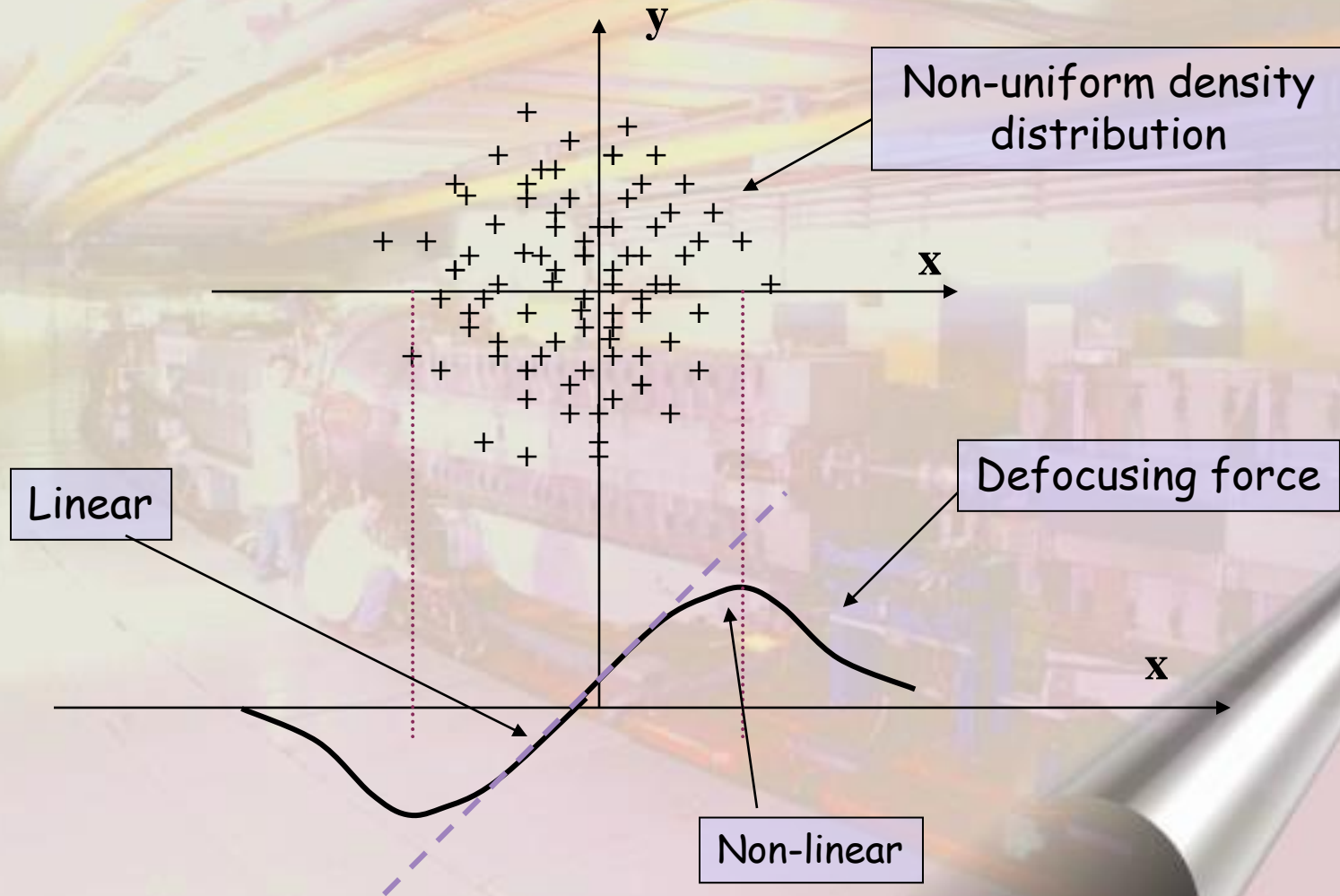
$$\Delta Q_{h,v} = - \frac{r_0 N}{2\pi \epsilon_{h,v} \beta^2 \gamma^3}$$

Diagram illustrating the components of the tune shift equation:

- Machine radius (points to r_0)
- Number of particles in the beam (points to N)
- Transverse emittance (points to $\epsilon_{h,v}$)
- Relativistic parameters (points to $\beta^2 \gamma^3$)

- # This tune shift is the **same for all particles** and vanishes at high momenta ($\beta=1$, $\gamma \gg 1$)
- # However, in reality the beam distribution is not uniform....

Space charge effects (4)



Laslett tune shift (1)

- # For the non-uniform beam distribution, this non-linear defocusing means the ΔQ is a function of x (transverse position)
- # This leads to a spread of tune shift across the beam
- # This tune shift is called the 'LASLETT tune shift'

$$\Delta Q_{h,v} \approx -\frac{r_0 N}{4\pi \epsilon_{h,v} \beta^2 \gamma^3}$$

half of the
uniform tune shift

- # This tune spread cannot be corrected and does get very large at high intensity and low momentum

Laslett tune shift (2)

Tune Shift

$$\Delta Q_{h,v} \approx -\frac{r_0 N}{4\pi \varepsilon_{h,v} \beta^2 \gamma^3}$$

Large necktie
in tune diagram

- # At injection into the PS Booster
 - $E = 0.988 \text{ GeV}$, $\gamma = 1.053$, $\beta = 0.313 \Rightarrow \Delta Q \approx 0.3$
- # For the same beam at injection into the PS
 - $E = 2.3826 \text{ GeV}$, $\gamma = 2.475$, $\beta = 0.915 \Rightarrow \Delta Q \approx 0.005$
- # For the same beam at injection into the SPS
 - $E = 14 \text{ GeV}$, $\gamma = 14.93$, $\beta = 0.998 \Rightarrow \Delta Q \approx 0.00001$
- # We accelerate the beam in the PSB as quickly as possible to avoid problems of blow-up due to betatron resonances

Questions....,Remarks...?

*Single bunch
modes*

Head-tail modes

Space charge

Tune shift



Beam Break-up around transition....

