Lecture 1: Neutrino-Nucleon Scattering

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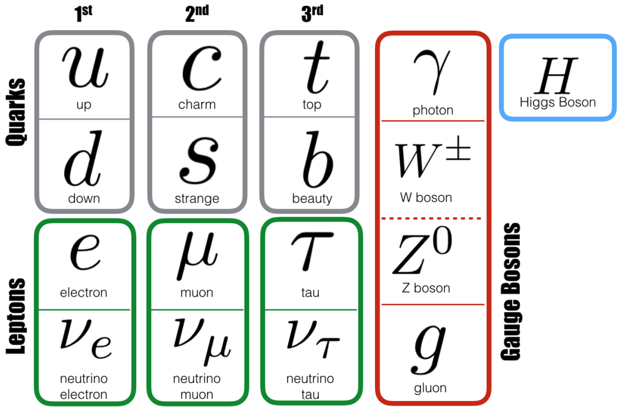
Outline

- Introduction
- Measuring Neutrinos
- Nucleon Interaction Topologies
- Axial Mass Problem
- Conclusions

Note: all references in online slides are hyperlinked

Introduction

The Standard Model



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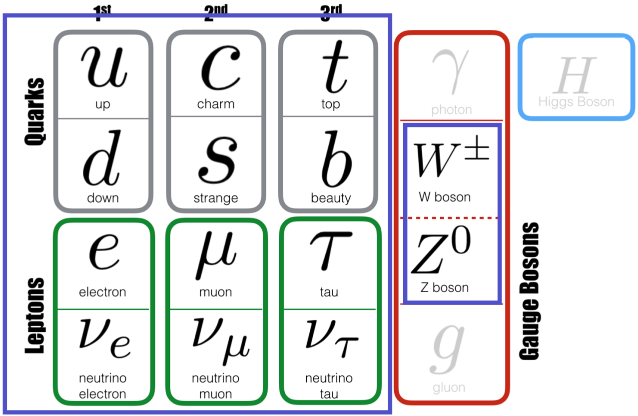
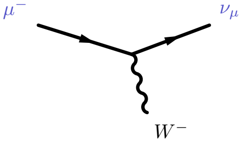
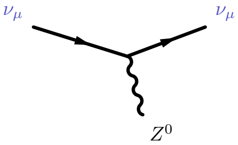




charged current (CC)



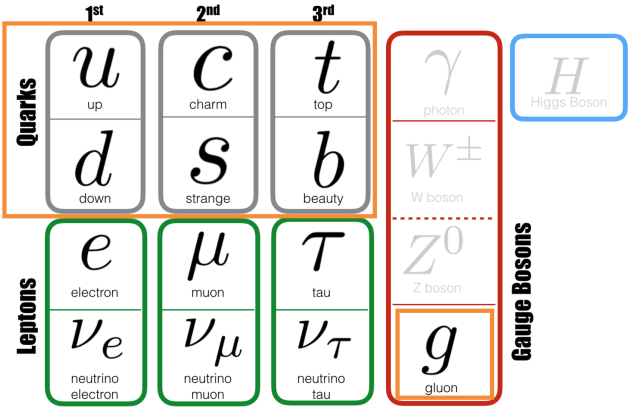
neutral current (NC)

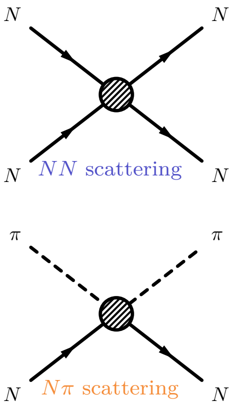
"Weak" force – W, Z bosons → quarks and leptons

Weakly coupled – perturbative methods work, but small interaction rates

$\implies$ Only achievable way to interact with neutrinos

The Standard Model



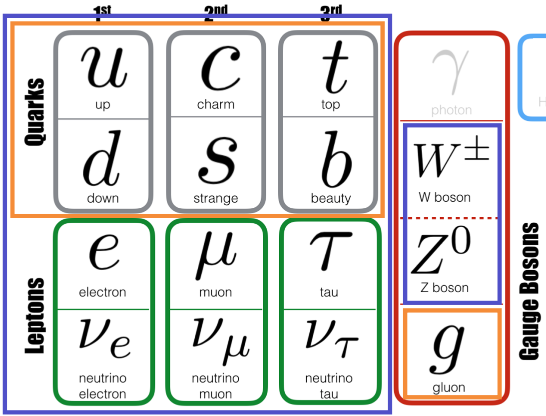


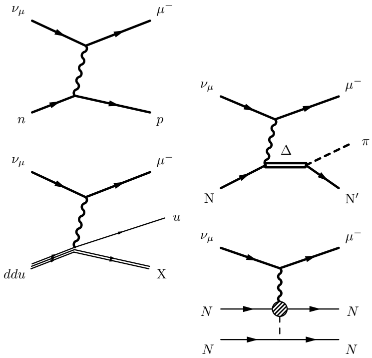
"Strong" force: Quantum ChromoDynamics (QCD)

Tightly bound – perturbation theory fails, quarks bind into hadrons

$\implies$ Need advanced techniques and/or experimental data to constrain

The Standard Model



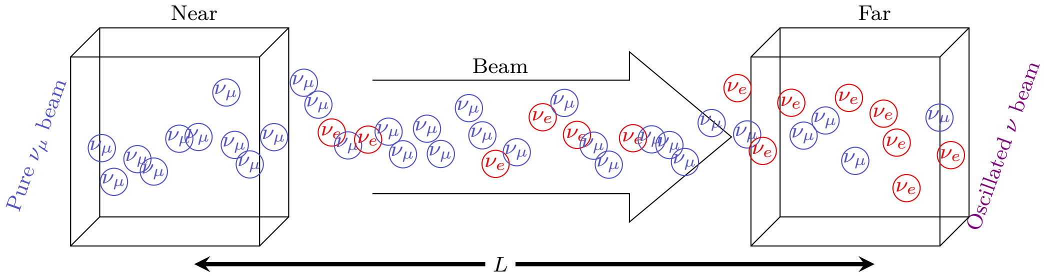


Seek to understand neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$

Probe ν physics by scattering ν with nuclei (protons, neutrons)

$\Longrightarrow$ Utilize weak interactions on hadronic matter

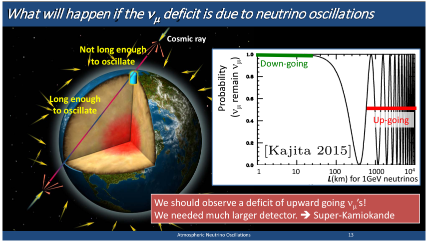
Neutrino Oscillation



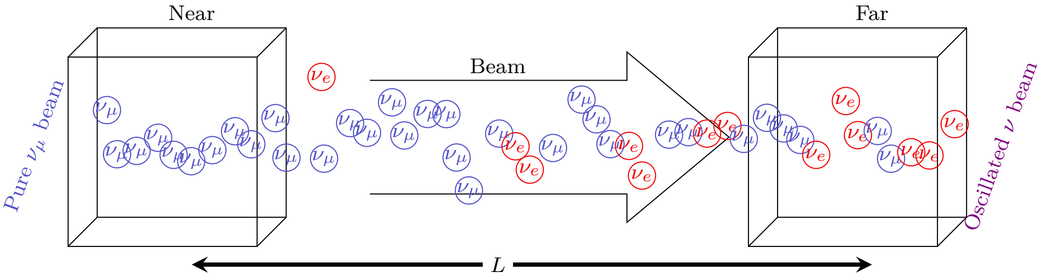
Neutrino oscillation: ν spontaneously change flavor

Oscillation parameters not completely known

$\implies$ δ_{CP}, mass hierarchy



Neutrino Oscillation



Neutrino oscillation: ν spontaneously change flavor

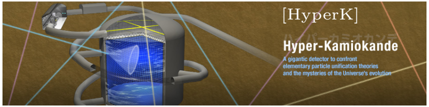
Oscillation parameters not completely known

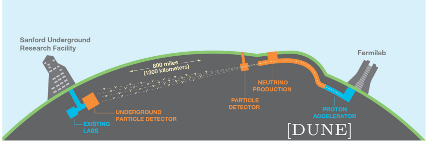
$\implies$ δ_{CP}, mass hierarchy

Flagship experimental programs upcoming

$\implies$ DUNE, HyperK

This talk: accelerator experiments





Neutrino Oscillation

Neutrinos created in flavor eigenstates, propagate as mass eigenstates

Mass eigenstates $\neq$ flavor eigenstates

$$|i\rangle = \begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix} \qquad |\alpha\rangle = \begin{pmatrix} |e\rangle \\ |\mu\rangle \end{pmatrix} \implies |i\rangle = \begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix} = \overbrace{\begin{pmatrix} U_{1e} & U_{1\mu} \\ U_{2e} & U_{2\mu} \end{pmatrix}}^{\text{mixing matrix}} \begin{pmatrix} |e\rangle \\ |\mu\rangle \end{pmatrix} = U|\alpha\rangle$$

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Mass eigenstates obey Hamiltonian time evolution: $\frac{\partial}{\partial t}\left|i\right\rangle = -\frac{i\hat{H}}{\hbar}\left|i\right\rangle$, $(\hbar = c = 1)$

$m_\nu \ll 1 \quad \implies \quad t \approx L/c \quad \text{and} \quad |p| \approx E$

$$\implies E_i = |p|\sqrt{1 + \frac{m_i^2}{p^2}} \approx E\left(1 + \frac{m_i^2}{2E^2}\right) \qquad \implies \Delta E \equiv E_2 - E_1 \approx \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m^2}{2E}$$

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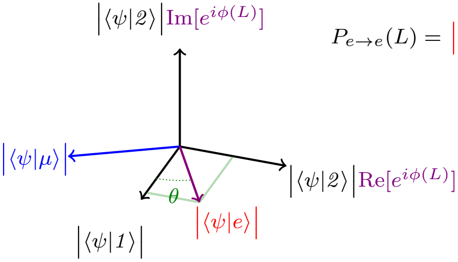
Putting it all together:

$$\frac{\partial}{\partial t}\begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix} = -i\begin{pmatrix} E_1 & 0 \\ 0 & E_1 + \Delta E \end{pmatrix}\begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix}$$

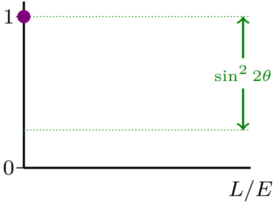
$$\Longrightarrow \begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix}(t) = (\text{phase}) \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\frac{\Delta m^2 L}{2E}} \end{pmatrix}\begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix}(0)$$

$$\Longrightarrow \begin{pmatrix} |e\rangle \\ |\mu\rangle \end{pmatrix}(t) = (\text{phase}) \cdot \begin{pmatrix} U_{1e}^* & U_{2e}^* \\ U_{1\mu}^* & U_{2\mu}^* \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{-i\frac{\Delta m^2 L}{2E}} \end{pmatrix}\begin{pmatrix} U_{1e} & U_{1\mu} \\ U_{2e} & U_{2\mu} \end{pmatrix}\begin{pmatrix} |e\rangle \\ |\mu\rangle \end{pmatrix}(0)$$

Two-Flavor Neutrino Oscillation



$P_{e\to e}(L) = \big|\langle e|\psi_L\rangle\big|^2$



flavor eigenstate (e,μ) $\quad |\psi_0\rangle = |e\rangle = \big(\cos\theta|1\rangle + \sin\theta|2\rangle\big)$

mass eigenstate $(1,2)$ $\quad |\psi_L\rangle \propto \big(\cos\theta|1\rangle + e^{-i\phi(L)}\sin\theta|2\rangle\big)$

$$\Delta m^2 \equiv m_2^2 - m_1^2, \quad \phi(L) = \tfrac{1}{4}\Delta m^2 \tfrac{L}{E}$$

"Survival probability": $\big|\langle e|\psi_L\rangle\big|^2 = 1 - \sin^2 2\theta \sin^2 \frac{\phi(L)}{2}$

"Disappearance probability": $\big|\langle \mu|\psi_L\rangle\big|^2 = \quad + \sin^2 2\theta \sin^2 \frac{\phi(L)}{2}$

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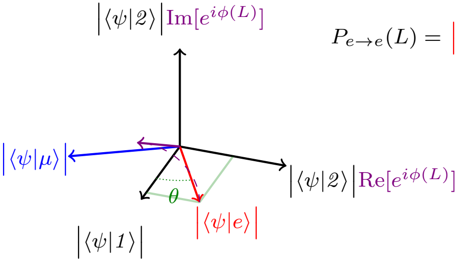

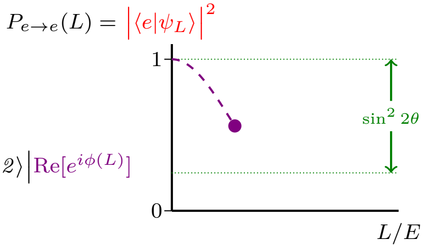

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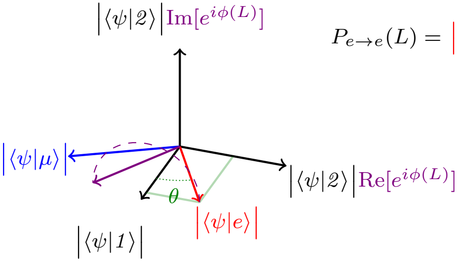

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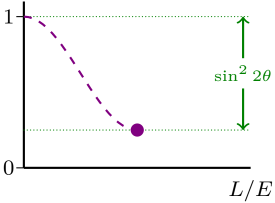

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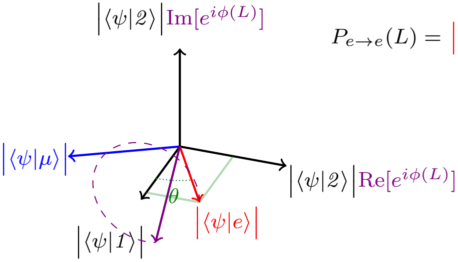

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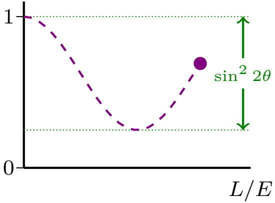

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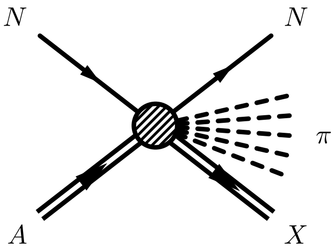
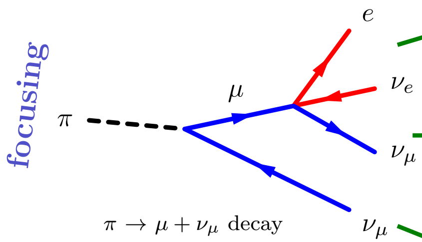
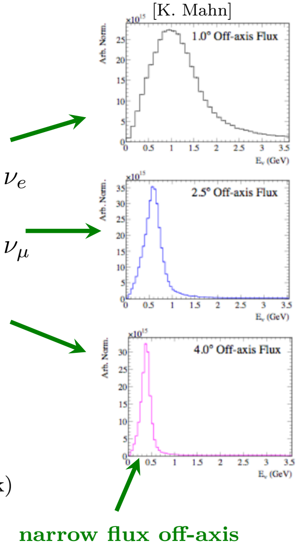
Measuring Neutrinos

Creating a Neutrino Beam







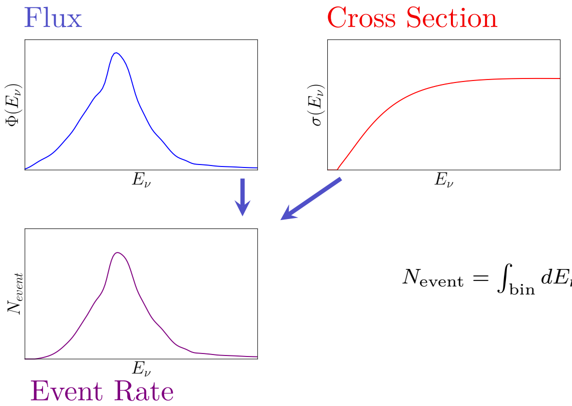
proton scatters on target
$\Longrightarrow$ π production

$\pi \to \mu + \nu_\mu$ decay
$\mu \to e + \nu_e + \nu_\mu$ decay

Beam production mechanisms prevent monoenergetic beam
$\Longrightarrow$ broad flux of ν

Tradeoff between peak width vs beam intensity (more ν, broader peak)

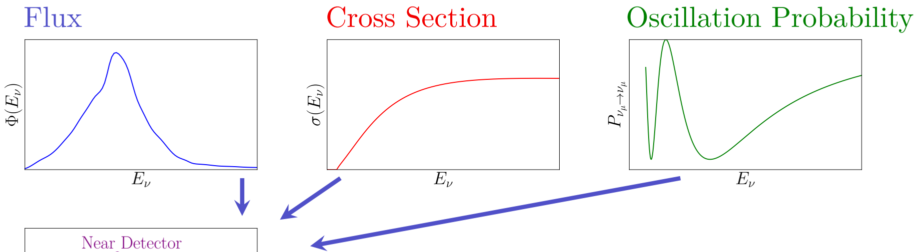
Measuring Oscillation Probability

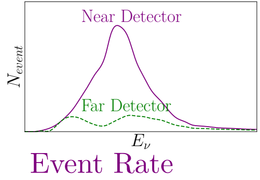


$$N_{\text{event}} = \int_{\text{bin}} dE_\nu \, \Phi(E_\nu)\sigma(E_\nu)$$

Broad flux & distribution of event E_ν

Measuring Oscillation Probability





$$N_{\text{event}} = \int_{\text{bin}} dE_\nu \, \Phi(E_\nu)\sigma(E_\nu) \underbrace{\left[\times P(L/E_\nu)\right]}_{\text{far detector only}}$$

$$\left[P(L/E_\nu)\right]_{\text{bin}} \approx \left[\frac{N_{\text{event}}[\text{FAR}]}{N_{\text{event}}[\text{NEAR}]}\right]_{\text{bin}}$$

Broad flux & distribution of event E_ν

far/near $\implies$ oscillation probability, assuming we can get the E_ν dependence correct...

Determining E_ν

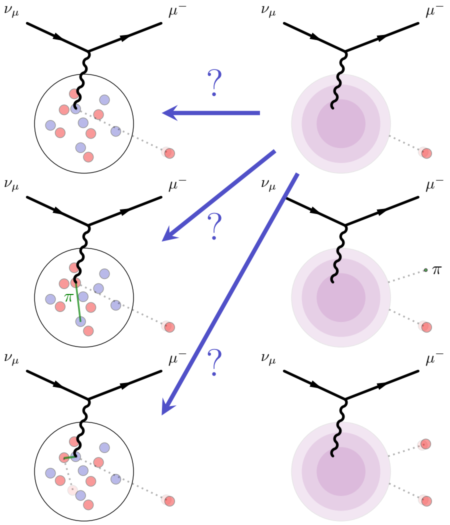
Need to know kinematics of events to get E_ν,
which determines oscillation probability

Not possible event-by-event:

- diffuse flux of E_ν:
  $\Longrightarrow$ do not know E_ν a priori from beam
- some particles escape detection:
  $\Longrightarrow$ cannot measure total energy of all events
- reinteractions within nucleus:
  $\Longrightarrow$ cannot determine primary interaction type

Detector efficiencies, thresholds must also be considered

$\Longrightarrow$ E_ν must be inferred from statistical distributions



Monte Carlo Generators

Predict event rates using generator outputs based on:

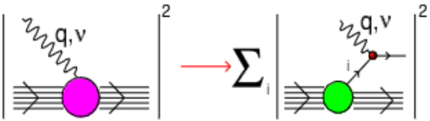
- interaction physics
- detector geometry
- beam flux

Generators constructed from both empirical or theoretical sources

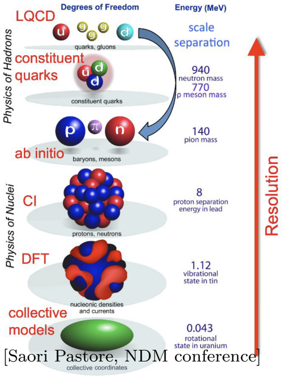
Often messy – trade-off between speed (statistics) and accuracy (systematics)

Simplified by impulse approximation –
increasing momentum probes smaller distance scales

Use single-nucleon for high-energy interactions,
low-energy nuclear model built from nucleon otherwise



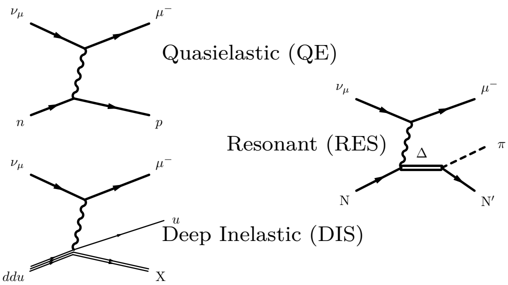
[Phys.Rev.D 72 (2005)]

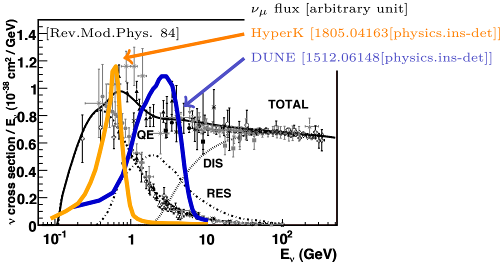


[Saori Pastore, NDM conference]

Neutrino-Nucleon Interactions

Neutrino-Nucleon Interaction Topologies





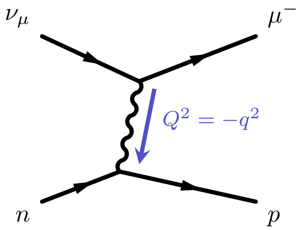
Energy range of flux in accelerator experiments span several *nucleon* interaction topologies

HyperK: mostly QE, some RES; DUNE: roughly equal QE, RES, DIS

Need constraints on *Nucleon* amplitudes used to build *nuclear* cross sections

$\Longrightarrow$ inputs to nuclear models, used in Monte Carlo simulations, to perform E_ν reconstruction

Quasielastic Scattering



$$\mathcal{M}_{\rm CCQE} \sim \langle \ell | \mathcal{J}^\mu | \nu_\ell \rangle \langle p | \mathcal{J}_\mu | n \rangle \qquad \frac{d\sigma_{CCQE}}{dQ^2} \sim |\mathcal{M}_{\rm CCQE}|^2$$

$$\langle p_k | (\mathcal{V}_\mu - \mathcal{A}_\mu) | n_p \rangle$$
$$= \bar{u}_k^{(p)} \Big[\gamma_\mu F_1(Q^2) + \frac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(Q^2) + \gamma_\mu \gamma_5 F_A(Q^2) + \frac{1}{2M_N} q_\mu \gamma_5 F_P(Q^2) \Big] u_p^{(n)}$$

Simplest topology, lowest E_ν – nucleon scatters elastically with neutrino

Nucleon response described by *form factors*, "parameterization of ignorance"

- F_1, F_2: vector form factors, constrained by eN scattering
- F_P: "induced pseudoscalar" form factor, subleading in cross section
  related to F_A by pion pole dominance constraint
- F_A: axial form factor

Nucleon cross section uncertainty dominated by axial form factor F_A

CCQE Cross section

Full cross section obtained from square $|\mathcal{M}_{\text{CCQE}}|^2$...

+(−) for (anti)neutrino

$$\frac{d\sigma_{CCQE}}{dQ^2}(E_\nu, Q^2) \propto \frac{1}{E_\nu^2}\left(A(Q^2) \boxed{\mp} \left(\frac{s-u}{M_N^2}\right) B(Q^2) + \left(\frac{s-u}{M_N^2}\right)^2 C(Q^2)\right) \qquad s-u = 4M_N E_\nu - Q^2 - m_\ell^2$$

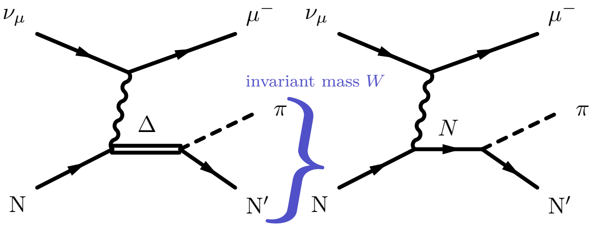
$$A(Q^2) = \left(\frac{m_\ell^2}{M_N^2} + 4\eta\right)\left[(1+\eta)F_A^2 - (1-\eta)(F_1^2 + \eta F_2^2) + 4\eta F_1 F_2 - \frac{m_\ell^2}{4M_N^2}\left((F_1+F_2)^2 + (F_A + 2F_P)^2 - 4(1+\eta)F_P^2\right)\right]$$

$$B(Q^2) = 4\eta F_A\,(F_1 + F_2) \qquad C(Q^2) = \frac{1}{4}\left(F_A^2 + F_1^2 + \eta F_2^2\right) \qquad \eta \equiv \frac{Q^2}{4M_N^2}$$

[Llewellyn Smith, 1972]
see also[Rev.Mod.Phys. 84 (2012)]

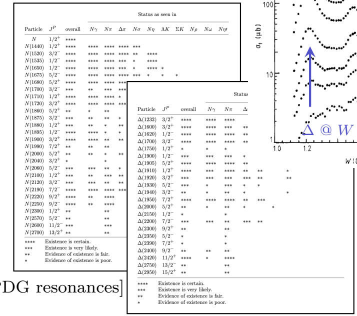
We will explore this in more detail...

Resonant Production



Resonant
(temporary bound state)

Nonresonant
(off-shell nucleon)



Particle	J^P	overall	$N\gamma$	$N\pi$	$\Delta\pi$	$N\sigma$	$N\eta$	ΛK	ΣK	$N\rho$	$N\omega$	$N\eta'$
			Status as seen in									
N	$1/2^+$	****										
$N(1440)$	$1/2^+$	****	****	****	****	***						
$N(1520)$	$3/2^-$	****	****	****	****	**	****					
$N(1535)$	$1/2^-$	****	****	****	***	*	****					
$N(1650)$	$1/2^-$	****	****	****	***	*	****	*				
$N(1675)$	$5/2^-$	****	****	****	****	***	*	*	*			
$N(1680)$	$5/2^+$	****	****	****	***							
$N(1700)$	$3/2^-$	***	**	***	***							
$N(1710)$	$1/2^+$	****	****	****	*							
$N(1720)$	$3/2^+$	****	****	****	***							
$N(1860)$	$5/2^+$	**	*	**								
$N(1875)$	$3/2^-$	***	**	**	*							
$N(1880)$	$1/2^+$	***	**	*	**							
$N(1895)$	$1/2^-$	****	****	*	*							
$N(1900)$	$3/2^+$	****	****	**	**							
$N(1990)$	$7/2^+$	**	**	**								
$N(2000)$	$5/2^+$	**	**	*	**							
$N(2040)$	$3/2^+$	*		*								
$N(2060)$	$5/2^-$	***	***	**	*							
$N(2100)$	$1/2^+$	***	**	***	**							
$N(2120)$	$3/2^-$	***	***	**	**							
$N(2190)$	$7/2^-$	****	****	****	***							
$N(2220)$	$9/2^+$	****	**	****								
$N(2250)$	$9/2^-$	****	**	****								
$N(2300)$	$1/2^+$	**		**								
$N(2570)$	$5/2^-$	**		**								
$N(2600)$	$11/2^-$	***		***								
$N(2700)$	$13/2^+$	**		**								

**** Existence is certain.
*** Existence is very likely.
** Evidence of existence is fair.
* Evidence of existence is poor.

Particle	J^P	overall	$N\gamma$	$N\pi$	Δ			
			Status					
$\Delta(1232)$	$3/2^+$	****	****	****				
$\Delta(1600)$	$3/2^+$	****	****	***	**			
$\Delta(1620)$	$1/2^-$	****	****	****	**			
$\Delta(1700)$	$3/2^-$	****	****	****	**			
$\Delta(1750)$	$1/2^+$	*	*	*				
$\Delta(1900)$	$1/2^-$	***	***	***	*			
$\Delta(1905)$	$5/2^+$	****	****	****	**			
$\Delta(1910)$	$1/2^+$	****	***	****	**	**		*
$\Delta(1920)$	$3/2^+$	***	***	***	***	**		**
$\Delta(1930)$	$5/2^-$	***	*	***	*	*		
$\Delta(1940)$	$3/2^-$	**	*	**	*			*
$\Delta(1950)$	$7/2^+$	****	****	****	**	***		
$\Delta(2000)$	$5/2^+$	**	*	**	*		*	
$\Delta(2150)$	$1/2^-$	*		*				
$\Delta(2200)$	$7/2^-$	***	***	**	***	**		
$\Delta(2300)$	$9/2^+$	**		**				
$\Delta(2350)$	$5/2^-$	*		*				
$\Delta(2390)$	$7/2^+$	*		*				
$\Delta(2400)$	$9/2^-$	**	**	**				
$\Delta(2420)$	$11/2^+$	****	*	****				
$\Delta(2750)$	$13/2^-$	**		**				
$\Delta(2950)$	$15/2^+$	**		**				

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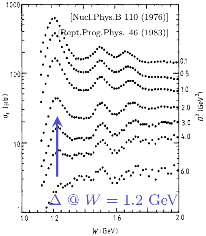
[PDG resonances]



Neutrino interaction excites nucleon to different state
produces $\geq 1\pi$ in general

Resonances can have different quantum numbers than nucleon (isospin, angular momentum)
$\Longrightarrow$ internal configuration of quarks, gluons changed

Resonances are would-be bound states, if physical world was different (revisit tomorrow)

Lots of resonances + nonresonant! Complicated problem with quantum interference

Deep Inelastic Scattering

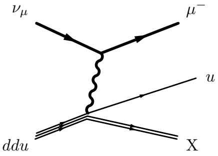
Deep	–	$Q^2 \gg M_N^2 \sim 1\ \text{GeV}^2$	(momentum transfer vs nucleon mass)
Inelastic	–	$W^2 \gtrsim M_N^2$	(hadronic invariant mass vs nucleon mass)

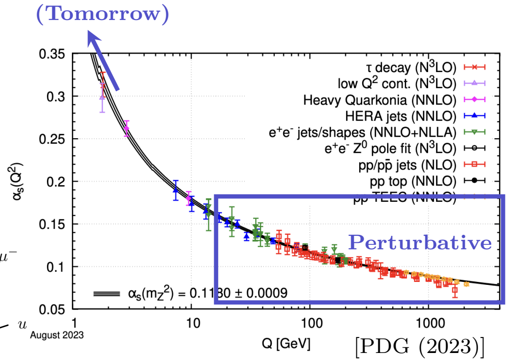
Neutrino interaction exchanges enough energy
to resolve individual quarks

Treated perturbatively at high Q^2, like EM

Quark-hadron duality $\implies$ smooth transition
between resonance region and DIS region

Highly inelastic interactions undergo hadronization
produce chains of mesons along path of parton

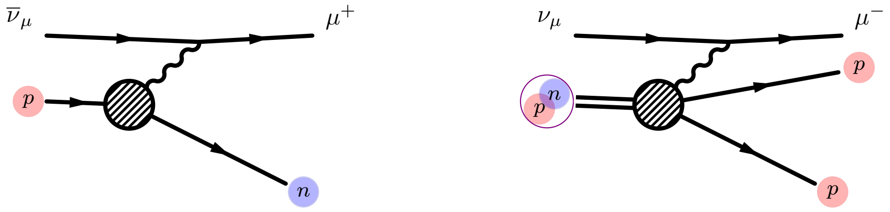




[PDG (2023)]

The Axial Mass Problem

Hydrogen vs Deuterium



Scattering off hydrogen – free nucleon matrix elements, but experimentally challenging:

- must detect neutrons
- $\bar{\nu}$ cross section $\approx \frac{1}{3}$ of ν cross section

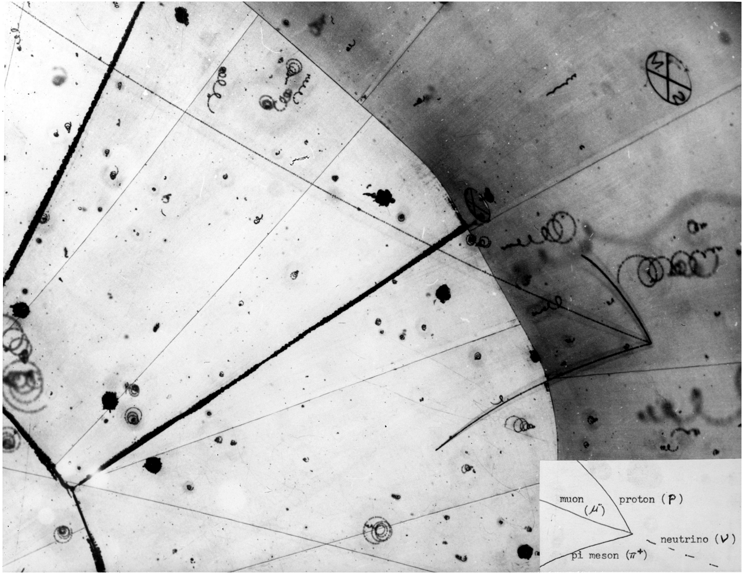
As of 2002: < 10 neutrons from $\bar{\nu}$–p scattering events detected

Deuterium scattering – interact with bound neutron to create pp system

Both protons and charged lepton detectable (in principle)

- Total $O(10^3)$ ν_μQE events
- Digitized event distributions only
- Unknown corrections to data
- Insufficient deuterium correction

Deuterium Bubble Chamber Experiments

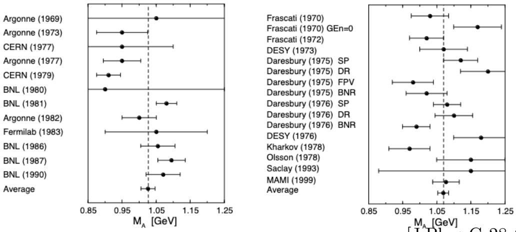


The Dipole Parameterization

Dipole ansatz — $F_A(Q^2) = g_A \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}$

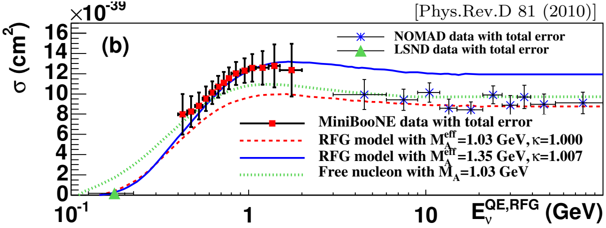
Fourier transform of exponential decay in 3 dimensions

- g_A: axial vector coupling
  Fixed precisely by neutron decay measurements, $g_A \approx 1.2754 \pm 0.0013$ [0.1%]
- m_A: axial mass
  As of 2002, world average $m_A \approx 1.026 \pm 0.021$[2%]



[J.Phys.G 28 (2002)]

The Axial Mass Problem



2010 bombshell result from MiniBooNE: $M_A^{\text{eff}} \approx 1.35$ GeV

Lots of misleading info here –

- Different definitions of quasielastic (MiniBooNE $\mu + \not\pi$; NOMAD μ or $\mu + p$)
- MiniBooNE effective M_A, including binding energy corrections
- E_ν is NOT an observable (baked-in nuclear modeling)

Lots of physics here –

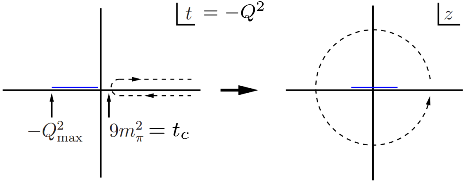
- Effects of Meson Exchange Currents (MEC)
- Effects of Short Range Correlations
- Nucleon Matrix Elements

z Expansion

Conformal mapping (change of variables, using complex analyticity): [Phys.Rev.D 84 (2011)]

$$z(Q^2; t_0, t_c) = \frac{\sqrt{t_c + Q^2} - \sqrt{t_c - t_0}}{\sqrt{t_c + Q^2} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad t_c = 9m_\pi^2$$

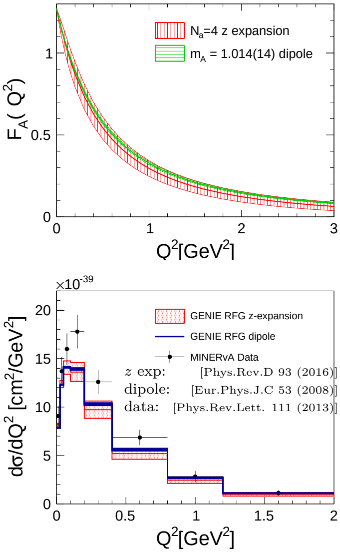
$-Q^2 \leq 0$ kinematically allowed $\quad \rightarrow \quad |z| < 1$



- Rapidly converging expansion
- Controlled procedure for introducing new parameters

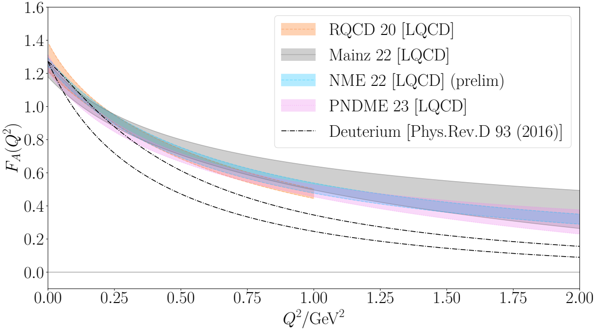
Deuterium Constraints on F_A

- Dipole overconstrained by data
  underestimated uncertainty $\times O(10)$
- Form factor uncertainties similar size
  to theory–data discrepancies
- Prediction discrepancies could be from
  nucleon and/or nuclear origins



LQCD Axial Form Factor Summary

More on LQCD tomorrow!
Black box for now

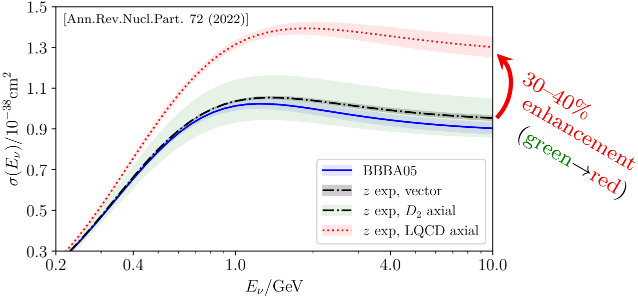


LQCD results becoming available late 2010s:

- Many results, all physical M_π: *independent data & different methods*
- Small systematic effects observed (expectation: largest at $Q^2 \to 0$)
- Nontrivial consistency checks

Evidence of slow Q^2 falloff, consistent with $M_A \sim 1.3$ GeV

Free Nucleon Cross Section



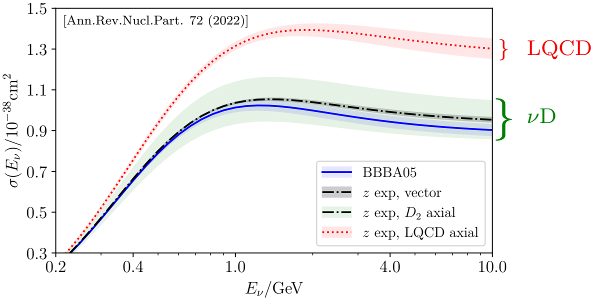
Integrate over Q^2-dependent function to get QE cross section $\sigma(E_\nu)$

$\Longrightarrow$ large-Q^2 discrepancy enhances cross section 30–40%!

$\Longrightarrow$ recent Monte Carlo tunes prefer ~20% enhancement of QE

[Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]

Free Nucleon Cross Section



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Current-generation LQCD about $\times 2$ more precise than νD scattering

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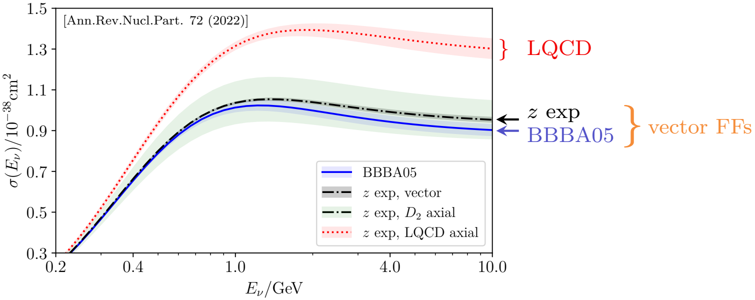

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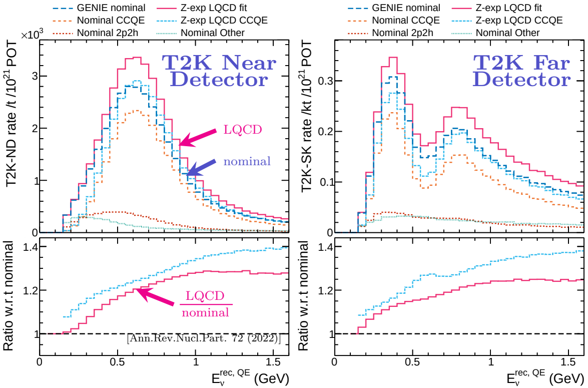
[Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]

Current-generation LQCD about $\times 2$ more precise than νD scattering

LQCD precision small enough to be sensitive to vector form factor discrepancies

T2K Implications



Insert new form factor into Monte Carlo event generator

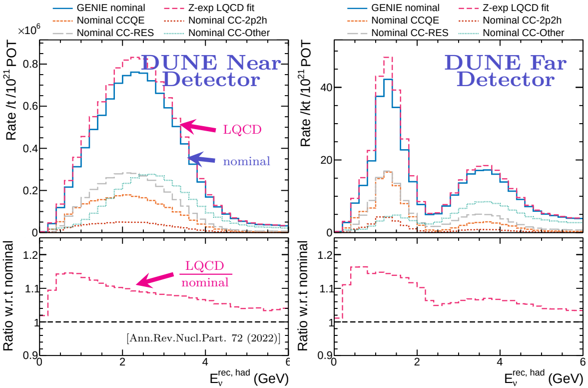
Convolve with realistic flux, nuclear model; compute neutrino event rates

Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)

E_ν-dependent event rate changes, different for near/far detectors

$\Longrightarrow$ Potential source of bias – caution!

DUNE Implications



Insert new form factor into Monte Carlo event generator

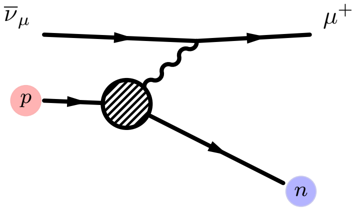
Convolve with realistic flux, nuclear model; compute neutrino event rates

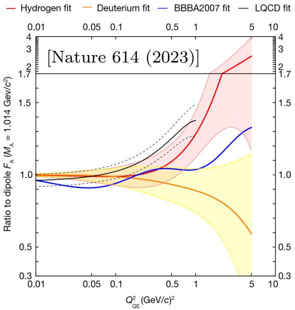
Solid dark blue (GENIE nominal) vs dashed magenta (z exp LQCD fit)

Similar story, different E_ν dependence $\implies$ different potential bias

Moving target $\implies$ other topologies adjusted to soften QE changes

Hydrogen vs Deuterium





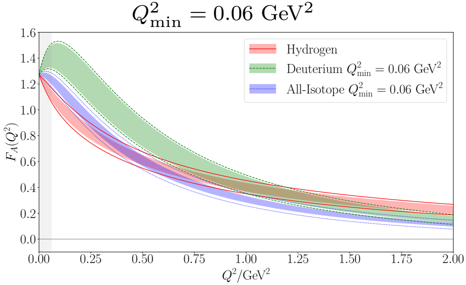
Work done with MINERνA collaboration on published data
Special thanks: Tejin Cai, Kevin McFarland, Miriam Moore

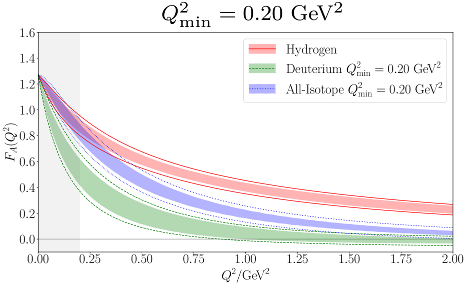
MINERνA result for $\overline{\nu}$–p scattering in plastic scintillator

Test consistency between hydrogen, deuterium fit together

Some visible disagreements between hydrogen, deuterium
$\implies$ how does this manifest in combined fit?

Isotope Fit Comparisons

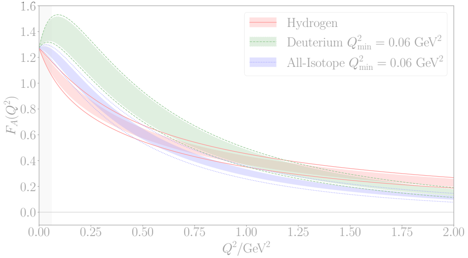


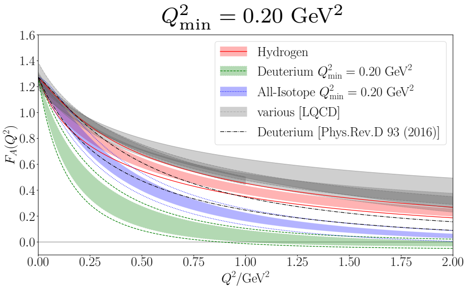


Inner band – uncertainty from axial only
Outer band – uncertainty from axial + vector [Phys.Rev.D 102 (2020)]

Cut low Q^2 in deuterium to avoid systematics (nominal $Q^2_{\mathrm{min}} = 0.20\ \mathrm{GeV}^2$)

Isotope Fit Comparisons





Tension in fits:

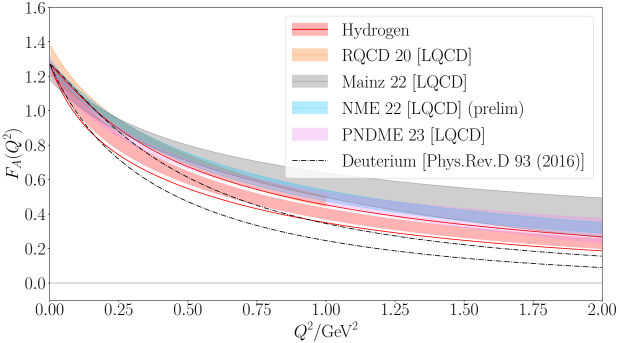
$$\Delta\chi^2 = \chi^2_{\mathrm{H+D}} - \chi^2_{\mathrm{D}} - \chi^2_{\mathrm{H}} \approx 8.8 \quad \Longrightarrow \quad \Delta\chi^2 \ /\ 1 \text{ DoF yields } p\text{-Value} \approx 3.0 \times 10^{-3}$$

Test compatibility by fixing axial parameters (marginalize deuterium nuisance parameters):

	$\{a_k\}_{\mathrm{D}}$	p_{D}	$\{a_k\}_{\mathrm{H}}$	p_{H}
$\chi^2_{\mathrm{D}}/\mathrm{DoF}_{\mathrm{D}}$	94.9/94	0.45	167.7/96	8.3×10^{-6}
$\chi^2_{\mathrm{H}}/\mathrm{DoF}_{\mathrm{H}}$	23.3/15	0.08	10.0/13	0.69

Deuterium is incompatible with hydrogen, LQCD

Hydrogen–Deuterium Comparison Summary



Today – LQCD “prediction” that deuterium fits underestimate axial form factor at high Q^2

Hydrogen & deuterium shapes mutually incompatible

We need more modern hydrogen data!

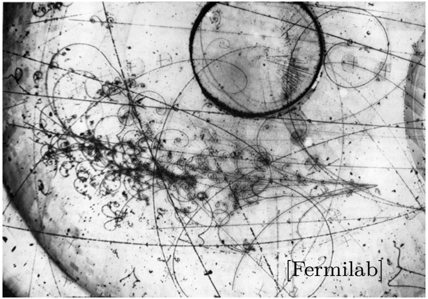
Concluding Remarks

What have we learned?

1. Be careful about signal definitions (NOMAD vs MiniBooNE)
2. Be careful about where nuclear model dependence appears (plotting vs E_ν)
   $\implies$ use things that are directly measureable!
3. Be careful about overconstraining parameterizations (dipole)
4. Nucleon form factor uncertainties are not small (z expansion)
   (the central value might be wrong too) (LQCD)
5. Don't mix single-nucleon effects with multinucleon effects (MiniBooNE)

Bonus Multinucleon effects are not negligible (MEC, 2p2h)

Outlook



- Learned about *nucleon* amplitudes that contribute to *nuclear* predictions
- Nucleon form factor uncertainty is significantly underestimated
- Case study of historical overview of axial mass problem
- Tomorrow: Lattice QCD for nonpractitioners

Homework

Let's explore the z expansion:

1. Rewrite the dipole parameterization as a power series in z. What happens to the expansion coefficients as the order increases? What does this say about the dipole?

2. Perform a principle component analysis (PCA) on the z expansion with the following parameter values:

$$g_A = 1.2723 \qquad t_0 = -0.28\ \mathrm{GeV}^2 \qquad t_c = 9 \cdot (0.14)^2\ \mathrm{GeV}^2 \qquad \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} -2.29629615 \\ 0.57023535 \\ 3.78739562 \\ -2.31277022 \end{pmatrix}$$

$$\text{covariance} = \begin{pmatrix} 0.01545818 & 0.04518361 & -0.21564118 & 0.20647022 \\ 0.04518361 & 1.08090689 & -2.38701609 & 1.03860128 \\ -0.21564118 & -2.38701609 & 6.53567566 & -4.76577493 \\ 0.20647022 & 1.03860128 & -4.76577493 & 7.39831933 \end{pmatrix}$$

To do this, you will need to solve for a_0 using the condition $\sum_k a_k \left[z(Q^2 = 0)\right]^k = g_A$.

How does this PCA compare to a dipole with axial mass $M_A = 1.0 \pm 0.1$ GeV? Try different linear combinations of 1σ shifts in the PCA components to distort the z expansion result within its uncertainty and see how much you can alter the shape of the form factor.

Backup