# NUSTEC 2024: Problem set 1

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## 1 Neutrino scattering with elementary fermions

Compute the differential cross section $\mathrm{d}\sigma/\mathrm{d}t$ (where $t$ is the Mandelstam variable) for

$$\nu_\ell d \to (u,c,t)\ell^- \tag{1}$$

$$\bar{\nu}_\ell u \to (d,s,b)\ell^- \tag{2}$$

treating the quarks as elementary fermions.

You may find the following identity useful

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{1}{16\pi s^2}\langle|\mathcal{M}|^2\ \rangle_{\text{spins}}\ , \tag{3}$$

Be careful to perform the spin-sum/average correctly. Re-express your result in terms of the inelasticity $y$.

What behaviour distinguishes neutrinos from anti-neutrinos? How does this relate to neutrino nucleon scattering?

## 2 Isospin and form factors

It is much easier to perform scattering experiments with electrons than it is with neutrinos. Therefore, the electromagnetic form factors of the neutron and proton are well measured.

These form factors are defined by

$$\langle p(k+q)|\hat{\mathscr{J}}_\mu|p(k)\rangle = \bar{u}_p(k+q)\left[F_{1,p}\gamma_\mu - \frac{\mathrm{i}}{2m_p}\sigma_{\mu\nu}q^\nu F_{2,p}\right]u_p(k)\ , \tag{4}$$

$$\langle n(k+q)|\hat{\mathscr{J}}_\mu|n(k)\rangle = \bar{u}_n(k+q)\left[F_{1,n}\gamma_\mu - \frac{\mathrm{i}}{2m_p}\sigma_{\mu\nu}q^\nu F_{2,n}\right]u_n(k)\ , \tag{5}$$

This current contains both an isoscalar and isovector contribution.

In neutrino scattering we are (often) interested in the isovector charged current. Let us focus on the vector current

$$\langle n(k+q)|\hat{V}^-_\mu|p(k)\rangle = \bar{u}_n(k+q)\left[F_{1,\mathrm{iV}}\gamma_\mu - \frac{\mathrm{i}}{2m_p}\sigma_{\mu\nu}q^\nu F_{2,\mathrm{iV}}\right]u_p(k)\ . \tag{6}$$

How do we extract the isovector form factors from electromagnetic data? What assumptions are inherent in the relation between form factors?

Hint: Use the Wigner-Eckhart theorem.

## 3 Time ordered products and response functions

When considering scattering amplitudes, we use the Dyson series which involves a time-ordered exponential.

$$T\{\mathrm{e}^{\mathrm{i}\int \mathcal{L}_{\mathrm{int}}\mathrm{d}^4x}\} = 1 + \mathrm{i}\int \mathrm{d}^4x\mathcal{L}_{\mathrm{int}}(x) - \frac{1}{2}\int\int \mathrm{d}^4x\mathrm{d}^4y(\mathcal{L}_{\mathrm{int}}(y)\mathcal{L}_{\mathrm{int}}(x)\Theta(y_0 - x_0) + \mathcal{L}_{\mathrm{int}}(x)\mathcal{L}_{\mathrm{int}}(y)\Theta(x_0 - y_0)) \tag{7}$$

This naturally leads to time-ordered products of currents (for instance taking $\mathcal{L}_{\mathrm{int}} \to \mathscr{J}_\mu A^\mu$ and absorbing the photon into propagators and/or external spinors. ).

1) Show (or convince yourself) that the matrix element for Compton scattering from a proton $\gamma p \to \gamma p$ naturally leads to the correlator

$$T_{\mu\nu} = \int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}q\cdot x}\,\langle p|T\{\mathscr{J}_\mu(x)\mathscr{J}_\nu(0)\}|p\rangle \tag{8}$$

where $q$ is the momentum transfer to the protons.

2) Using the optical theorem, show (or convince yourself) that

$$2\ \mathrm{Im}T_{\mu\nu} \propto \sum_X \int \mathrm{d}\Pi_X (2\pi)^4\delta^{(4)}k + q - k_X\ \langle p|\mathscr{J}_\mu(0)|X\rangle\ \langle X|\mathscr{J}_\nu(0)|p\rangle \tag{9}$$

where $X$ is a complete set of states.

Hint: At an operator level you will encounter $[J_\mu(x), J_\nu(0)]$ and can then insert a complete set of states.

3) Convince yourself that this same object naturally appears in deep inelastic $ep \to eX$ scattering where we are interested in the inclusive cross section summed over all final states $X$.