Mainz Institute for Theoretical Physics

Lecture 1

From quarks

currents

Response Fns

Form Factors

References

- Georgi (1984)
- Manohar 9204208 (DIS)
- Formaggio & Zeller 1305.7513
- Donoghue, Golowich, Holstein "Red Book"
  Dynamics of the Standard Model

Mainz Institute for Theoretical Physics

Outline

Motivation: Use σ-theory as a tool to learn about nature

i) # of events $\Leftrightarrow$ Statistical uncertainty

ii) final state $\Leftrightarrow$ Experimental systematics

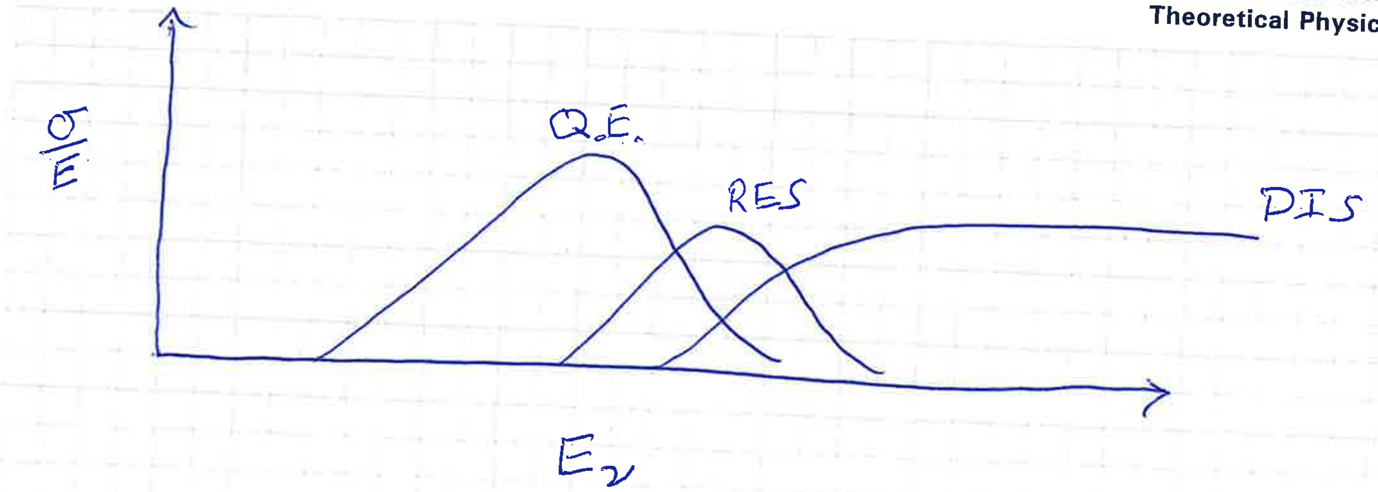
iii) Process of interest $\Leftrightarrow$ theoretical uncertainty

Depending on application, any of (i), (ii), or (iii) can be the limiting factor.

Examples:

$\nu e \to \nu e (\gamma)$	$\nu N \to \ell X$	$\nu A \to \nu A$
(i) ✗ (poor)	(i) ✓✓✓	(i) ✓✓
(ii) ✓	(ii) ✗ ?	(ii) ✗
(iii) ✓✓✓	(iii) Depends	(iii) ✓✓
$\sigma \sim 10^{-42}\,cm^2 \left(\frac{E_\nu}{GeV}\right)$	$\sigma \sim 10^{-38}\,cm^2 \left(\frac{E_\nu}{GeV}\right)$	$\sigma \sim 3.4 \times 10^{-39} \left(\frac{E_\nu}{30\,MeV}\right)$
• Flux normalization • Test SM	ν-osc, Expt'l cons.	• Test SM

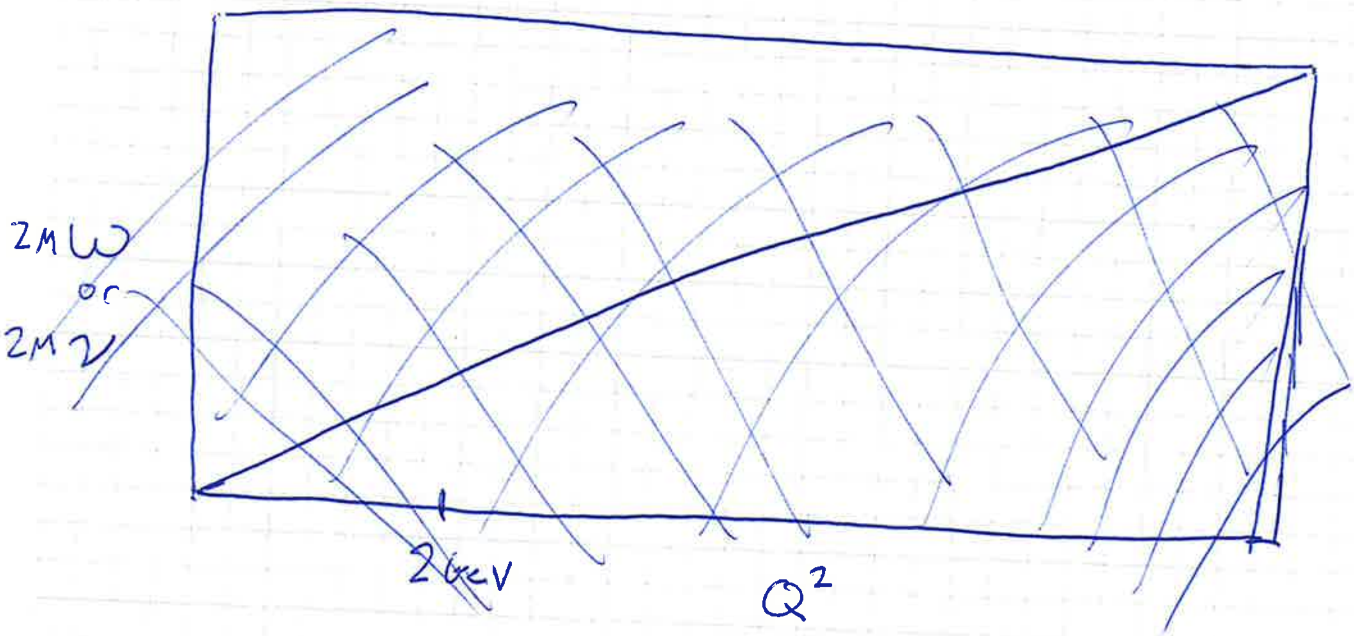
Other Important Channels | IBD, $\nu d \to \nu p n$, $\nu A \to \ell A \pi$, etc..



Reality is <u>more complicated</u>

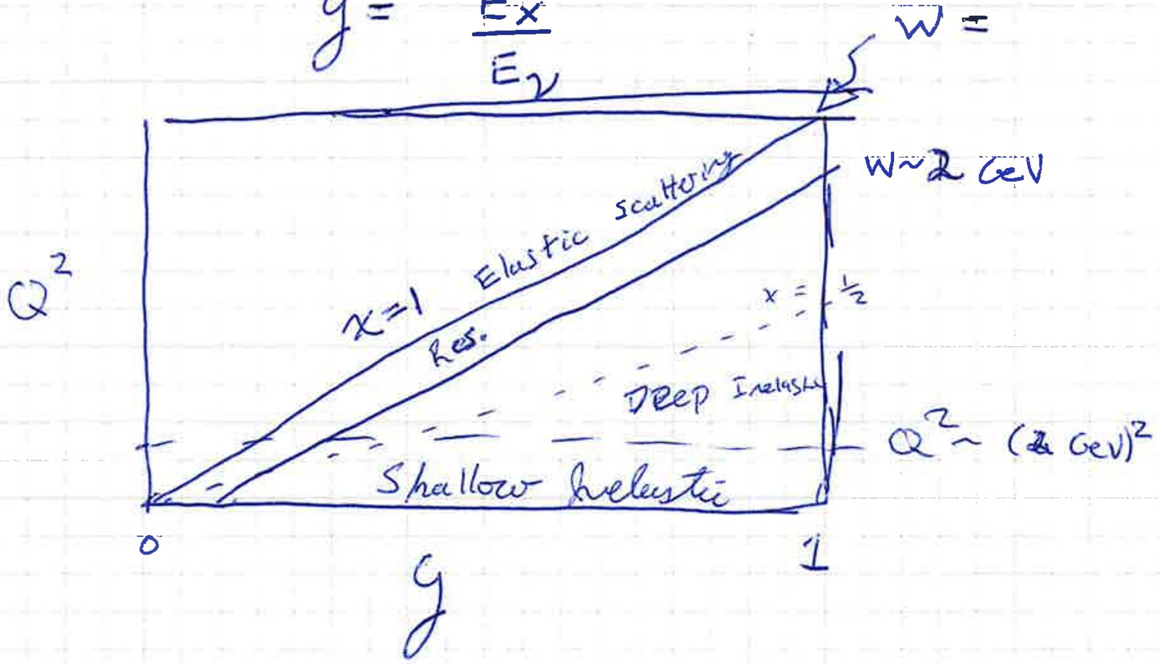
↳ Relevant theory depends on more kinematic variables

↳ Example: Inclusive Scattering



Inelasticity: $\nu A \to \ell X$

$$y = \frac{E_X}{E_\nu}$$



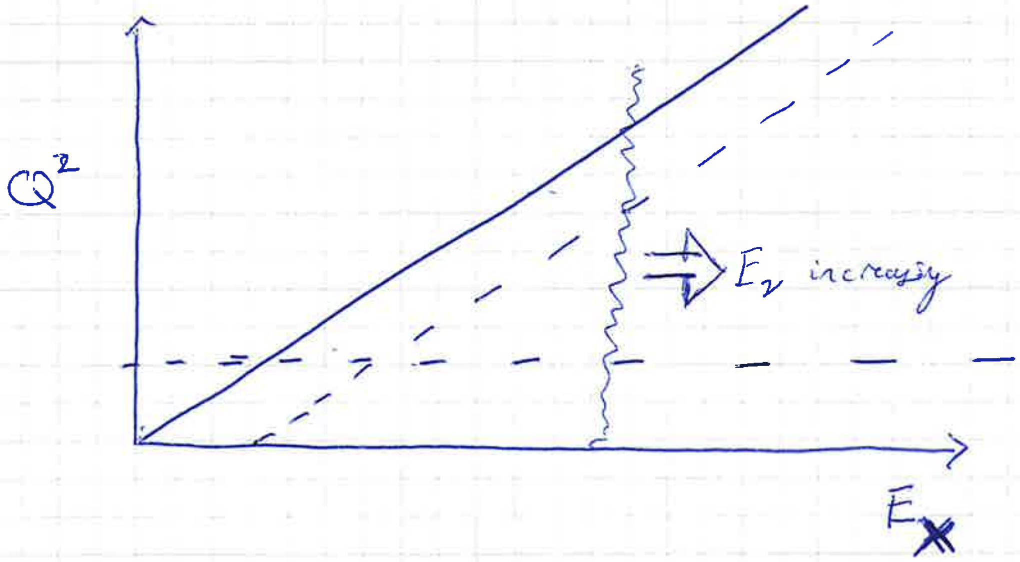
$$Q^2 = (2M E_X)x = (2M E_\nu\, y)\, x$$

Invariant mass: $W^2 = P_X^2$

$$P_X^2 = (P+Q)^2 = M^2 + 2P\cdot Q + Q^2$$
$$= M^2 + 2ME_\nu y + \cancel{2ME_\nu x y}\; Q^2$$

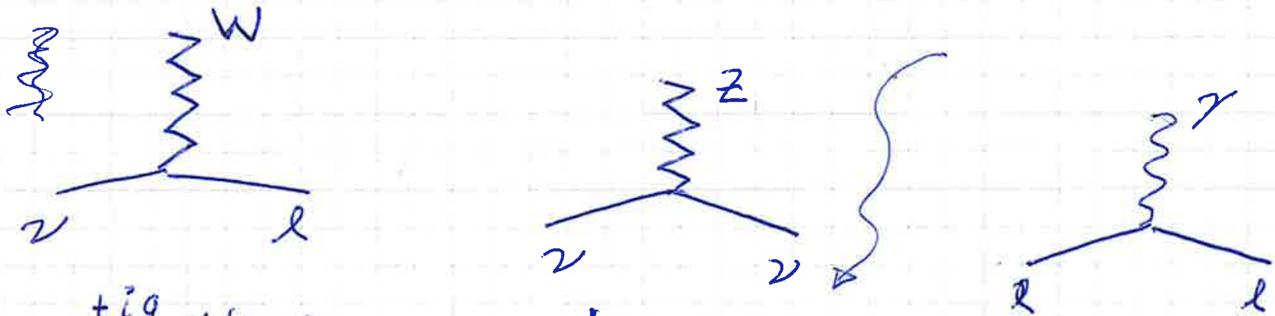
$$\therefore\ Q^2 = W^2 - M^2 + 2ME_\nu y$$

$$\mathcal{L} \subseteq \mathcal{L}_{quarks} + \mathcal{L}_{gluons} + \mathcal{L}_{leptons} + \mathcal{L}_{weak} + \dots$$



Just from phase space can understand basic kinds in phase space.

Neutrino interactions in the S.M.



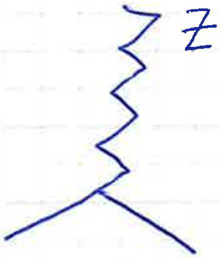
$+\frac{ig}{\sqrt{2}}\gamma_\mu\frac{(1-\gamma_5)}{2}$ $\qquad$ $\frac{ig}{\cos\theta_W}\gamma_\mu(g_V - g_A\gamma_5)$ $\qquad$ $-ie\gamma_\mu$

<u>Unitary Gauge</u>

 $= \dfrac{-i\left(\eta_{\mu\nu} - p_\mu p_\nu / M^2\right)}{p^2 - M^2}$

 $= \dfrac{-i(\eta_{\mu\nu})}{p^2}$ ← Feynman Gauge

Quarks

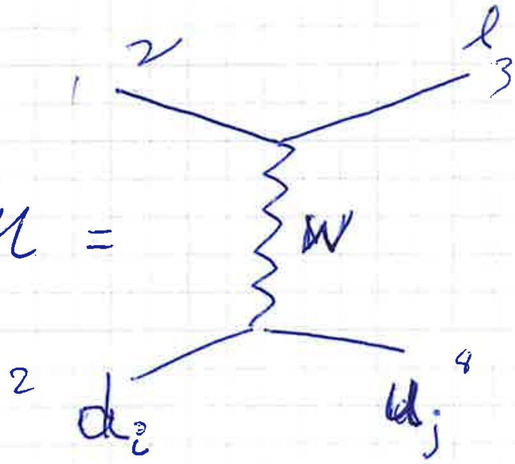


$\frac{ig}{\sqrt{2}} V_{ij} \left(\frac{1-\gamma_5}{2}\right)$

Same as leptons

$-ie \to Q_i\,(ie)$

$i\mathcal{M}$ =



$$= \left(\frac{ig}{\sqrt{2}}\right)\left(\frac{ig}{\sqrt{2}} V_{ud}^*\right) \frac{-i(g_{\mu\nu} - p_\mu p_\nu/M_W^2)}{p^2 - M_W^2}$$

$$\bar{u}_3 \gamma^\mu \frac{(1-\gamma_5)}{2} u_1 \;\bar{u}_4 \gamma^\nu (1-\gamma_5) u_2$$

Lepton tensor

$$|\mathcal{M}|^2 \subset \sum_{\text{spins}} \bar{u}_1 \gamma^\alpha \left(\frac{1-\gamma_5}{2}\right) u_3 \bar{u}_3 \gamma^\beta \left(\frac{1-\gamma_5}{2}\right) u_1$$

$$= \mathrm{Tr}\left(\sum_{\text{spins}} u_1 \bar{u}_1 \gamma^\alpha \left(\frac{1-\gamma_5}{2}\right) u_3 \bar{u}_3 \gamma^\beta \left(\frac{1-\gamma_5}{2}\right)\right)$$

$$\equiv L^{\alpha\beta}$$

Notion of Currents

It is convenient to write SM as

$$\mathcal{L} \supseteq W^\mu_\pm \hat{J}^\pm_\mu + Z^\mu \hat{J}^{(0)}_\mu + A^\mu \hat{j}_\mu$$

charged current   Neutral-current   EM-current

Currents have unambiguous def'n in terms of quarks   ← Field re-def. ?

$$\hat{J}^+_\mu = \sum_{ij} \bar{u}_i \gamma_\mu (1-\gamma_5) V_{ij} d_j$$

$$\hat{J}^{(0)}_\mu = \frac{e}{\sin\theta\cos\theta} \sum_j \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta\right) \bar{u}_i \gamma \bar{u}_i$$

$$\hat{j}_\mu = -\cancel{\bar{e}\gamma^\mu e} + \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d + \dots$$

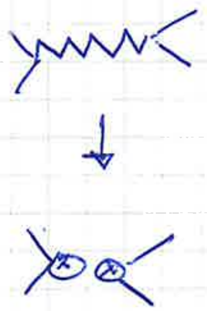
Isospin: $\begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{pmatrix} p \\ n \end{pmatrix}$ $\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$ etc.

$$\hat{J}^{(0)}_\mu = \frac{e}{\sin\theta\cos\theta} \sum_j \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta\right) \bar{u}_j \gamma_\mu \frac{(1-\gamma_5)}{2} u_j + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta\right) \bar{d}_j \gamma^\mu \frac{(1-\gamma_5)}{2} d_j$$

$$- \frac{2}{3}\sin^2\theta\, \bar{u}_j \gamma^\mu P_R u_j + \frac{1}{3}$$

4-Fermi Theory

↳ When $E_\nu \ll M_W$ it is very convenient to eliminate W from theory



or

"current-current" Interaction

The advantage of this will be stressed on Friday

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

Hadrons in QCD

↪ Currents have simple def'n in QCD

↪ What about hadrons?

A1) Do everything in QCD on lattice

A2) Hadrons a complicated but constrained by Lorentz Invariance.

$$\underbrace{|P\rangle_{QCD} = |uud\rangle + |uudg\rangle + |uud\bar{u}u\rangle + \dots}_{\text{complicated gloop}}$$

Draw a picture

Matrix Elements

$\cancel{\langle n | J_\mu^{+} | p\rangle}$

$\langle p | \hat{J}_\mu | p\rangle =$ ~~EM elastic~~ EM El

$\langle n | \hat{J}_\mu^{-} | p\rangle =$ ~~QE C.C.~~ C.C. QE

Mainz Institute for Theoretical Physics

Matrix Elements of Currents are constrained by Lorentz Inv.

- Use Dirac spinors for spin - 1/2
  - ↳ Encodes spin, Lorentz cov of state
- use exhaustive list of Dirac Bilinears
- use all independent 4-vectors in problem

E.g.

$$\langle P| \hat{J}_\mu |P\rangle = \bar{u}(\sigma';p')\left(F^{(1)}\Gamma^{(1)}_\mu + F^{(2)}\Gamma^{(2)}_\mu\right)u(\sigma,p)$$

↳ conventional choice $\Gamma^{(1)}_\mu = \gamma_\mu$

$$\Gamma^{(2)}_\mu = \frac{i}{2M_p} q^\nu \sigma_{\mu\nu}$$

$F^{(1)}$ depends on $(p'-p)^2 \equiv q^2$

$F^{(2)}$ depends on $(p'-p)^2 \equiv q^2$

Mainz Institute for Theoretical Physics

Quasi Elastic Scattering

$$\langle N' | J_\mu | N \rangle = \langle N' | V_\mu - A_\mu | N \rangle$$

$$\langle N' | V_\mu | N \rangle = \bar{u}(p', \sigma') \left[F_1(q^2) \gamma_\mu + \frac{i F_2}{2M_N} \sigma_{\mu\nu} q^\nu + \frac{i q_\mu}{M_N} F_S(q^2) \right]$$

$$\langle N' | A_\mu | N \rangle = \bar{u}(p', \sigma') \left[F_A \gamma_\mu \gamma_5 + \frac{F_T}{M_N} \sigma_{\mu\nu} q^\nu \gamma_5 + \frac{F_P}{M_N} (q^2) q_\mu \gamma_5 \right] u(p, \sigma)$$

Isospin limit $\Rightarrow$ $F_S = F_T = 0$

follows from hermiticity of current

At low q^2 form factors just become #'s.

Structure Fns

- Suppose we only measure leptonic d.o.f.

$$\sum_{\beta} (\nu\ \alpha \to \ell\ \beta)$$

e.g. $\nu_\mu\ {}^{40}Ar \to \mu^-\ X^+$

$\nu_\mu\ p \to \mu^-\ X^+$

$\bar{\nu}_e\ n \to e^+\ X^-$

$\vdots$

etc.

$$d\sigma \sim \underbrace{\left(\sum_{spins} \bar{u}\gamma^\mu P_L u \bar{u}\gamma^\nu P_L u\right)}_{L^{\mu\nu}} \underbrace{\sum_\beta \int d\Pi_\beta\ (2\pi)^4 \delta^{(4)}(\ )\ \langle\alpha| J_\mu^+ |\beta\rangle\langle\beta| J_\nu |\alpha\rangle}_{W_{\mu\nu}}$$

Hadronic tensor

Can rewrite this as

$$\int d^4x\ e^{i[(p_\alpha+q)-p_\beta]\cdot x} \langle\alpha| J^{\dagger}_\mu(x)\ \cancel{J_\nu(0)}\ \underbrace{\sum_\beta |\beta\rangle\langle\beta|}_{\text{completeness}}\ J_\nu(0)|\alpha\rangle$$