Fermilab | U.S. DEPARTMENT OF ENERGY | Office of Science

QE + 2p2h (SF) …. And more

NuSTEC 2024 Summer school
CERN, June 5-13, 2024

Noemi Rocco

Practical Info

- Some of the Figures have been taken from papers published in peer reviews. The references are reported as:

  Authors, Journal's name and # of the paper

  If there is anything you find interesting, I strongly encourage you to download the paper and read it!

- I also included some suggestions for more 'pedagogical' readings. The references are indicated as

Author, Title of the Book/Journal

- Please, ask question! Now or later: nrocco@fnal.gov

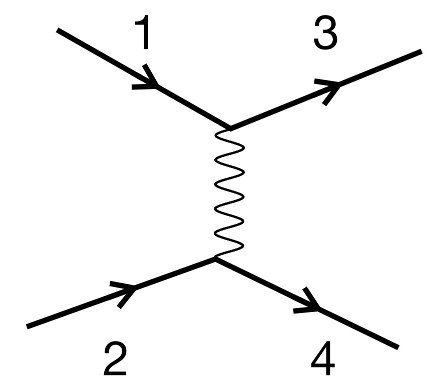
 Noemi Rocco, nrocco@fnal.gov

Outline

- Lepton - nucleon interactions
- Modeling nuclear structure
- Ab - initio description of lepton - nucleus interactions
- Factorization approach + spectral function : QE

Noemi Rocco, nrocco@fnal.gov

Electron-nucleon scattering

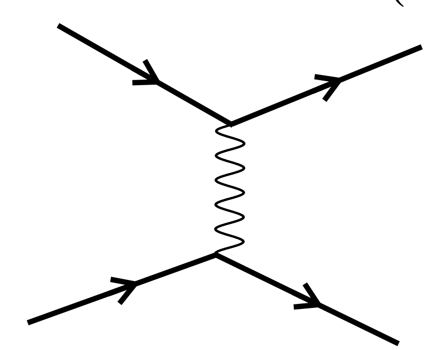


- We start from a generic process: 1+2 ➜ 3+4
- The cross section can be written as

$$d\sigma = \frac{1}{\text{flux}}\frac{1}{2E_1}\frac{1}{2E_2}|\mathcal{A}|^2\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

F. Close, An Introduction to quark and partons

- For scattering of an electron on a nucleon at rest in the lab frame:



$$\text{flux} = 2EM\,|\mathbf{v}_k - \mathbf{v}_p| = 2EM\,|\frac{k}{E}| \simeq 2EM$$

- The squared amplitude expression is given by

$$\frac{1}{(2s_e^i+1)(2s_p^i+1)}\sum_{\text{allspin}}|\mathscr{A}|^2$$

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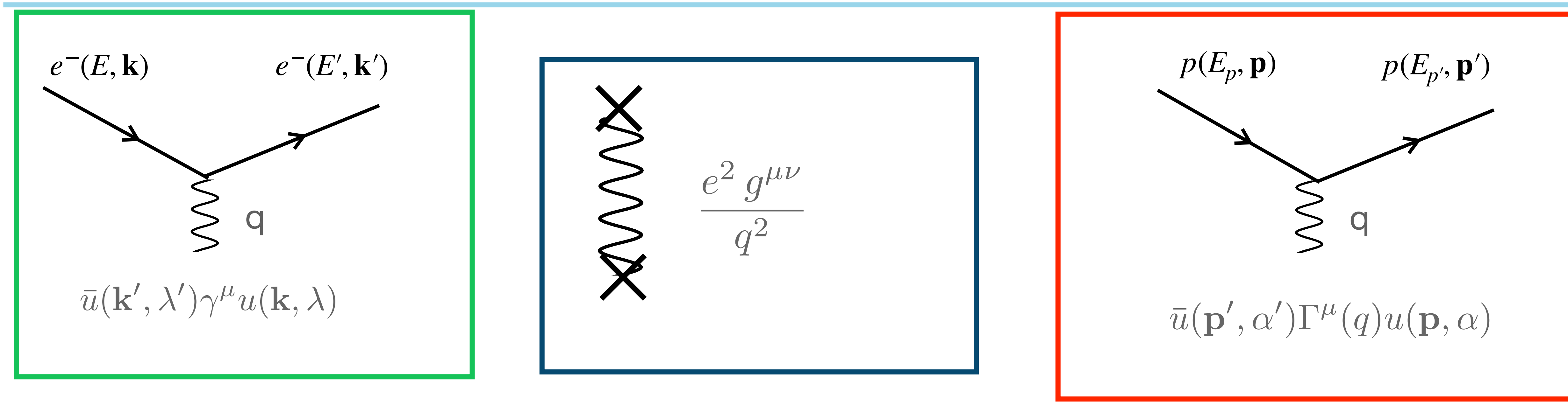
Electron-nucleon scattering



- We can put together what we learned and rewrite the squared amplitude as:

$$|\mathcal{A}|^2 = \frac{1}{4} \sum_{\lambda,\lambda',\alpha,\alpha'} \left| \bar{u}(\mathbf{k}', \lambda')\gamma^\mu u(\mathbf{k}, \lambda) \frac{e^2}{q^2} \bar{u}(\mathbf{p}', \alpha')\Gamma_\mu u(\mathbf{p}, \alpha) \right|^2 = \frac{e^4}{q^4} L_{\mu\nu} W^{\mu\nu}$$

- How do we define the leptonic and hadronic tensor? Let's start from the leptonic one

$$L_{\mu\nu} = \frac{1}{2} \mathrm{Tr}\left[\not{k}' \gamma^\mu \not{k} \gamma^\nu \right] = 2\left(k'^\mu k^\nu + k^\mu k'^\nu - g^{\mu\nu} k' \cdot k \right)$$

We neglected the electron mass in the expression of the leptonic tensor

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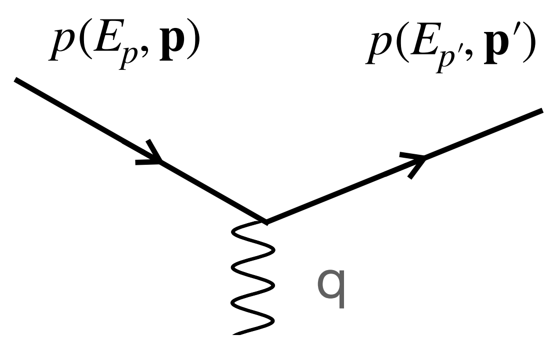
Electron-nucleon scattering



$$\bar{u}(\mathbf{p}', \alpha')\Gamma^\mu(q)u(\mathbf{p}, \alpha) = \bar{u}(\mathbf{p}', \alpha')\left[F_1\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2\right]u(\mathbf{p}, \alpha)$$

- The Dirac and Pauli form factors are **corrections to "point-like coupling"** which comes from the fact that the nucleon has an internal structure

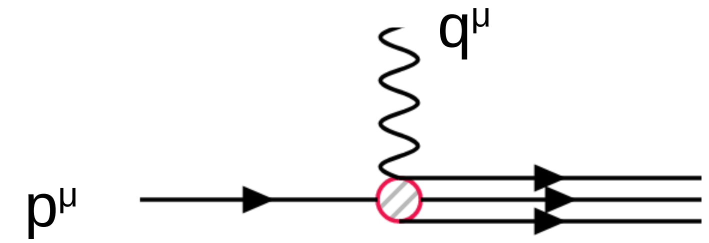
- Alternatively, F_1 and F_2 are written as a combination of the electric and magnetic form factors:

$$F_1^{p,n} = \frac{G_E^{p,n} - q^2/(4M^2)G_M^{p,n}}{1 - q^2/(4M^2)} \qquad F_2^{p,n} = \frac{G_M^{p,n} - G_E^{p,n}}{1 - q^2/(4M^2)}$$

  - The form factors are related to the **spatial distributions of the charge and magnetization** in the proton, and in the non relativistic limit are simply the Fourier transforms of these distributions.

    - The accurate determination of G_E and G_M is an important focus of both experimental and theory programs (see slide 7)

Hadronic tensor



- The most general expression for the hadronic tensor reads

$$W^{\mu\nu} = W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + i\frac{\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2M^2} W_3 + \frac{W_4}{M^2} q_\mu q_\nu + \frac{W_5}{M^2}(p^\mu q^\nu - p^\nu q^\mu)$$

- For the electromagnetic case, we can use current conservation, this allows us the rewrite the hadronic tensor as

$$W^{\mu\nu} = W_1\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \frac{W_2}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right)$$

Using this general expression of the hadronic tensor, the differential electron-proton cross section reads

$$\frac{d\sigma}{dE' d\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\left[2W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2}\right]$$

Note that for elastic scattering, the structure functions read:

$$W_1 = -\frac{q^2}{4M^2}(F_1 + F_2)^2 \delta\left(\omega + \frac{q^2}{2M}\right)$$

$$W_2 = \left(F_1^2 - \frac{q^2}{4M^2} F_2^2\right)\delta\left(\omega + \frac{q^2}{2M}\right)$$

Hadronic tensor

Suggested problem:

Use current conservation to show that the hadron tensor reduces to this expression for the electromagnetic case

- The most general expression for the had

$$W^{\mu\nu} = W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + i\frac{\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2M^2} W_3 + \ldots$$

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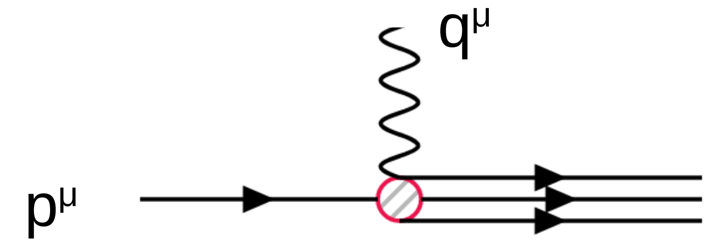
Hadronic tensor



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- For the electromagnetic case, we can use current conservation, this allows us the rewrite the hadronic tensor as

$$W^{\mu\nu} = W_1\Big(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\Big) + \frac{W_2}{M^2}\Big(p^\mu - \ldots$$

Using this general expression of the ha...

$$\frac{d\sigma}{dE' d\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\Big[2W_1 \sin^2\frac{\theta}{2} + W_2 \ldots$$

Suggested problem:

Show that the energy and momentum delta function can be rewritten in the following way for an elastic scattering on a nucleon at rest: $\delta^{(4)}(p' - p - q) \to \dfrac{1}{2m_N}\delta(\omega + q^2/(2m_N))$

Note that for elastic scattering, the structure functions read:

$$W_1 = -\frac{q^2}{4M^2}(F_1 + F_2)^2 \delta\Big(\omega + \frac{q^2}{2M}\Big)$$

$$W_2 = \Big(F_1^2 - \frac{q^2}{4M^2}F_2^2\Big)\delta\Big(\omega + \frac{q^2}{2M}\Big)$$

Noemi Rocco, nrocco@fnal.gov

Summary of electron-nucleon scattering

- We consider the process:

$$\ell^-(k) + N(p) \to \ell^-(k') + N(p')$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} = \frac{\alpha^2}{4E_k^2 \sin^4\theta/2}\cos^2\frac{\theta}{2}$$

← Scattering on a point-like spinless target

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott}\left[1 - \frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$$

← Scattering on a point-like 1/2 spin target

- Protons and neutrons have an internal structure: described by electric and magnetic form factors

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott}\left[\frac{G_E^2 - \frac{q^2}{4M^2}G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2}G_M^2\tan^2\frac{\theta}{2}\right]$$

Rosenbluth separation

Noemi Rocco, nrocco@fnal.gov

Determination of nucleon form factors

- A reduced cross section can be defined as

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \times \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1+\tau)}$$

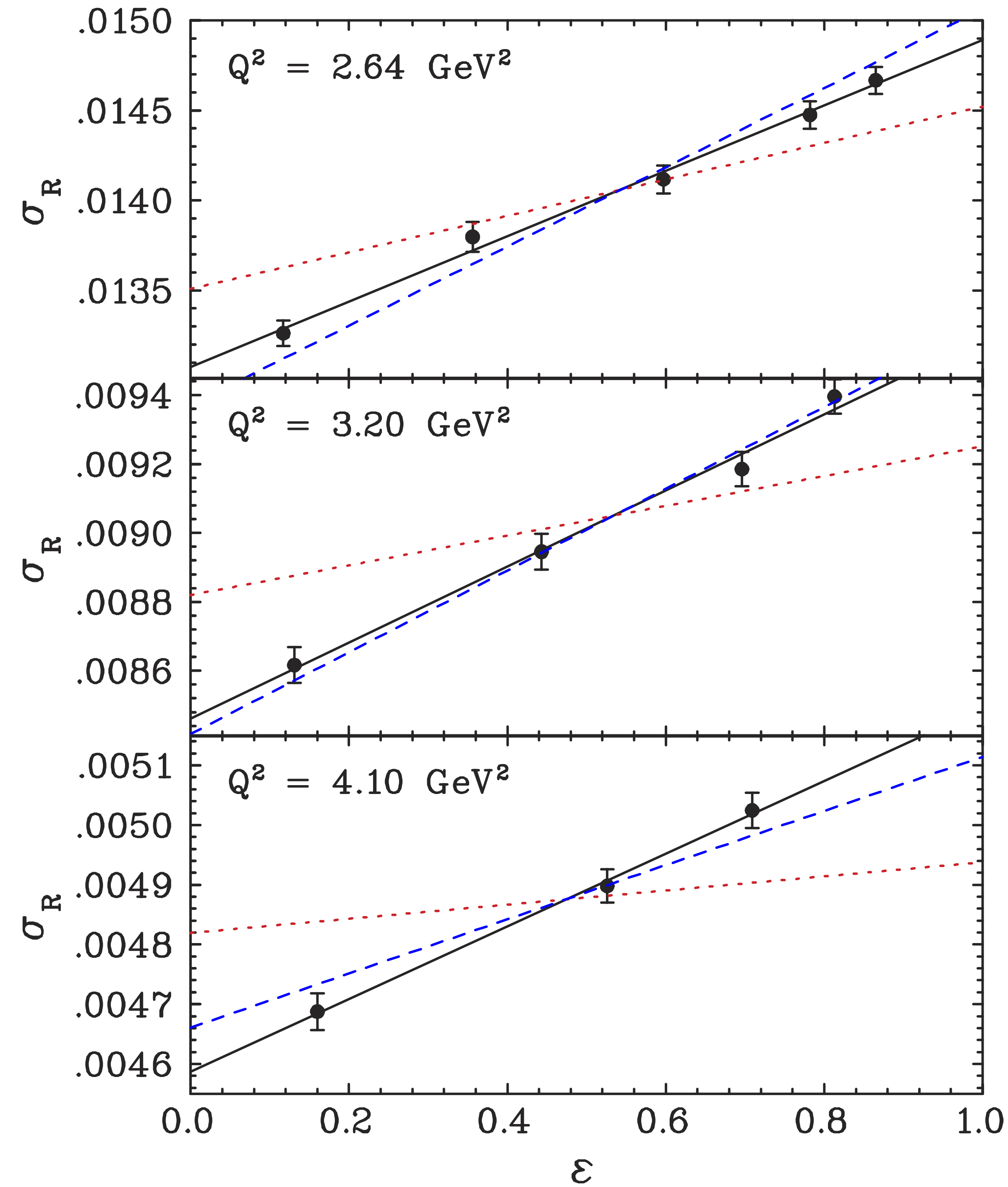
- The virtual photon polarization parameter is

$$\epsilon = \left[1 + 2(\tau+1)\tan^2\frac{\theta}{2}\right]^{1}$$

- Measuring angular dependence of the cross section at fixed Q^2

$$\sigma_R = \epsilon(1+\tau)\frac{\sigma}{\sigma_{\text{Mott}}} = \epsilon G_E^2 + \tau G_M^2$$

Qattan et al.,PRL 94, 142301 (2005)

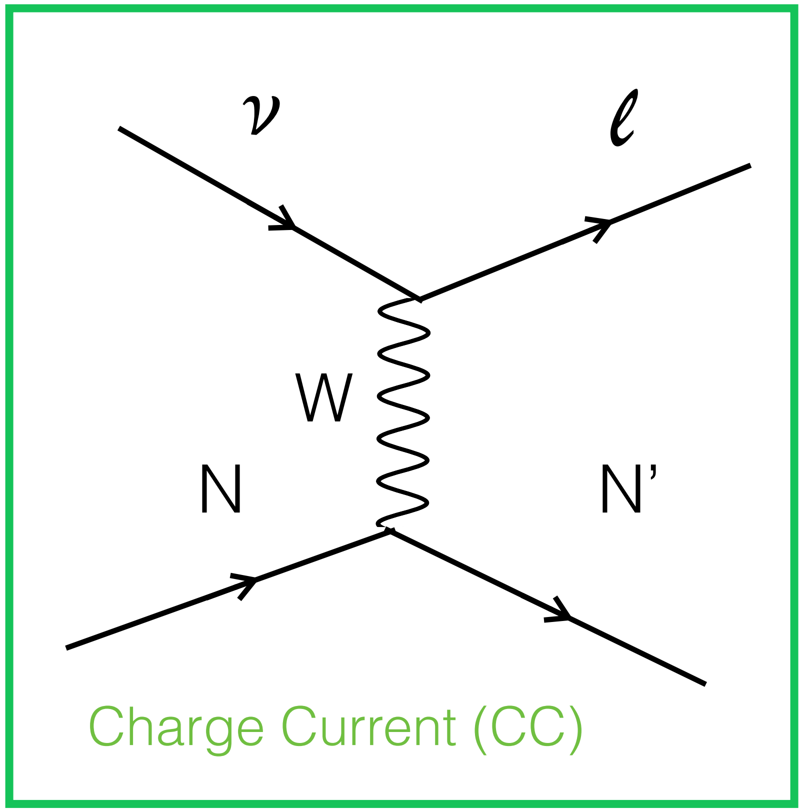


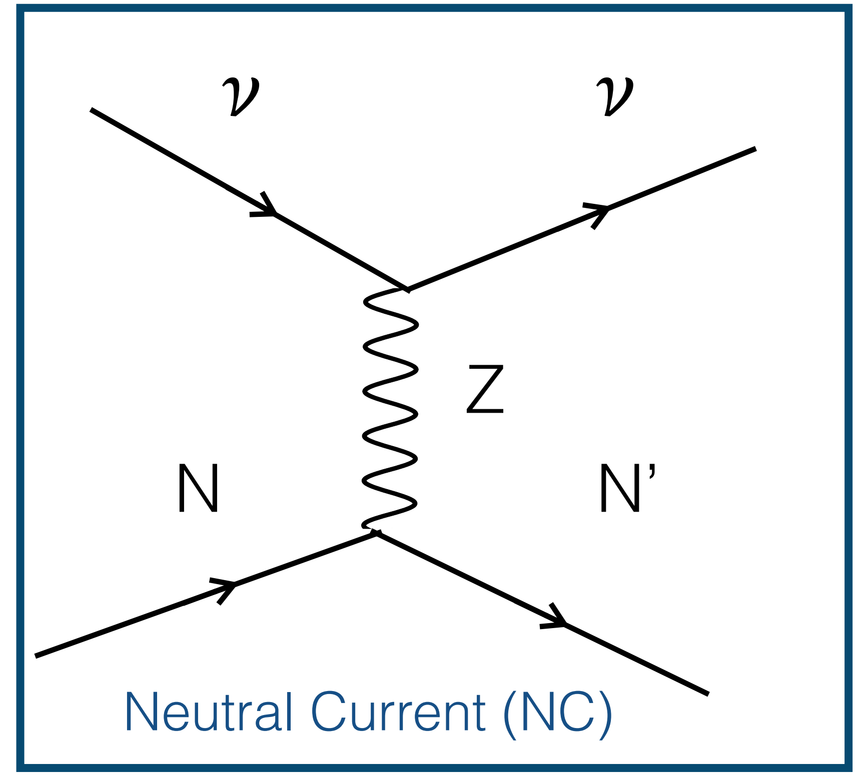
- In Born approximation: G_E^2 is the slope and the intercept is τG_M^2

··· polarization transfer experiment
- - - previous Rosenbluth

Noemi Rocco, nrocco@fnal.gov

Neutrino-Nucleon scattering





- Exchange of the W boson
- Lepton produced has the same flavor of the neutrino
- Initial and final nucleon have different isospin

- Exchange of the Z boson
- Independent of the neutrino flavor
- Initial and final nucleon have same isospin

F. Close, An Introduction to quark and partons

Noemi Rocco, nrocco@fnal.gov

Neutrino-Nucleon scattering

- Differential cross section for CC and NC processes

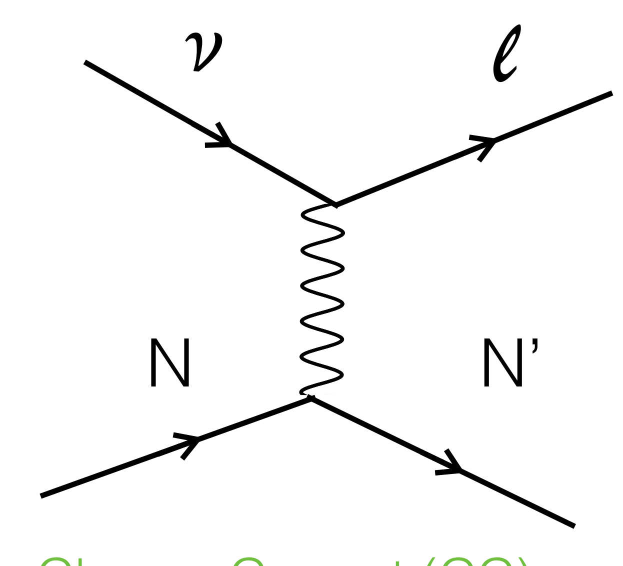
$$\frac{d^2\sigma}{dE'd\Omega'} = \frac{1}{16\pi^2}\frac{G^2}{2}L_{\mu\nu}W^{\mu\nu}$$

- For NC

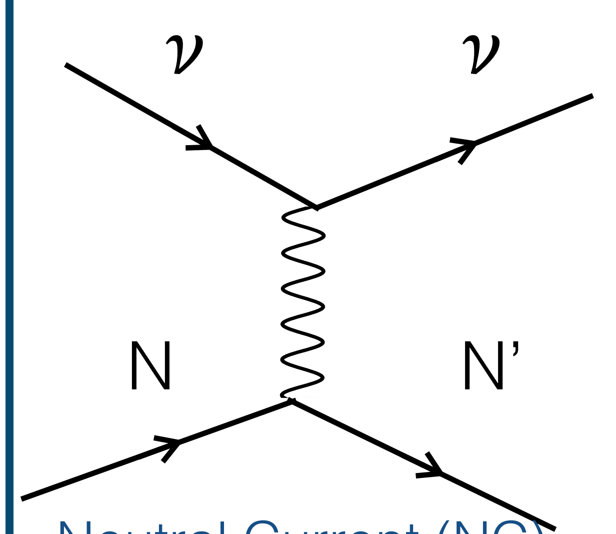
$$G = G_F$$

- For CC

$$G = G_F \cos\theta_c$$



$$G_F = 1.1803 \times 10^{-5}\ \mathrm{GeV}^{-2}\ ,\ \cos\theta_c = 0.97425$$



- Leptonic Tensor:

$$L_{\mu\nu} = 2[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}k\cdot k' \pm i\epsilon_{\mu\nu\alpha\beta}k'^\alpha k^\beta]$$

$\pm \rightarrow \nu/\bar{\nu}$

- Hadronic Tensor:

$$W^{\mu\nu} = W_1 g^{\mu\nu} + \frac{W_2}{M^2}p^\mu p^\nu + i\frac{\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2M^2}W_3 + \frac{W_4}{M^2}q_\mu q_\nu + \frac{W_5}{M^2}(p^\mu q^\nu - p^\nu q^\mu)$$

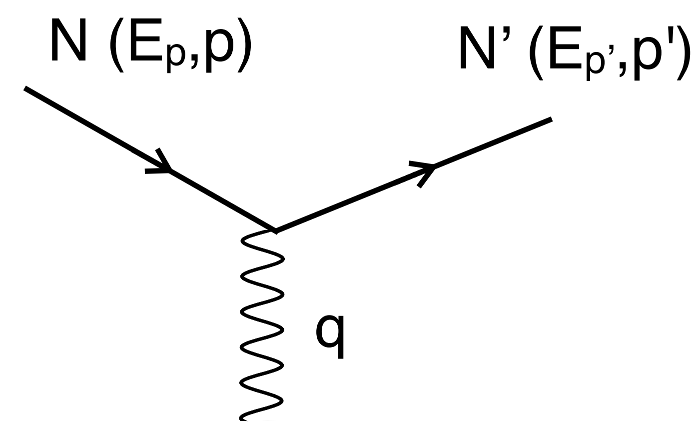
Noemi Rocco, nrocco@fnal.gov

Neutrino-Nucleon scattering

- Electroweak current operator:

$$\langle N'|J^\mu|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = J^\mu_V + J^\mu_A$$

Vector Axial



- The vector contribution is given by:

$$J^\mu_V = \mathcal{F}_1\gamma^\mu + i\sigma^{\mu\nu}q_\nu\frac{\mathcal{F}_2}{2M}$$

- The axial contribution is given by:

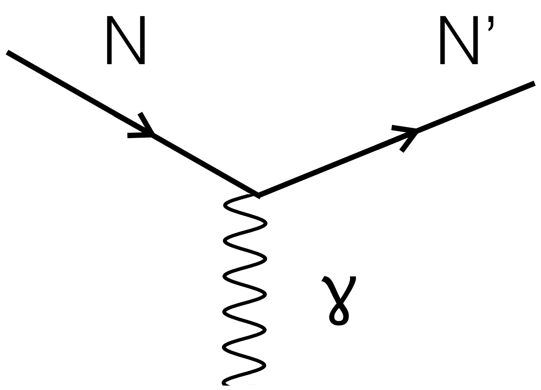
$$J^\mu_A = -\gamma^\mu\gamma_5\mathcal{F}_A - q^\mu\gamma_5\frac{\mathcal{F}_P}{M}$$

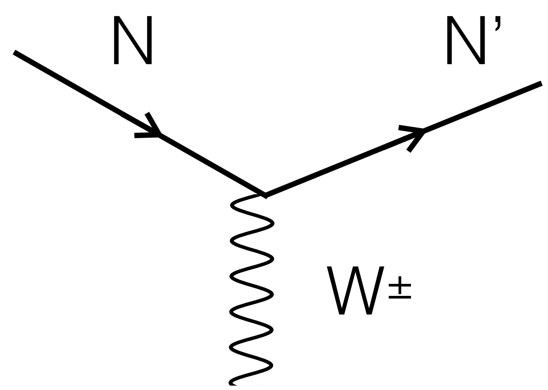
- General expression for both neutral- and charge current processes. The iso-spin dependence of these form factors is different (see next slide).

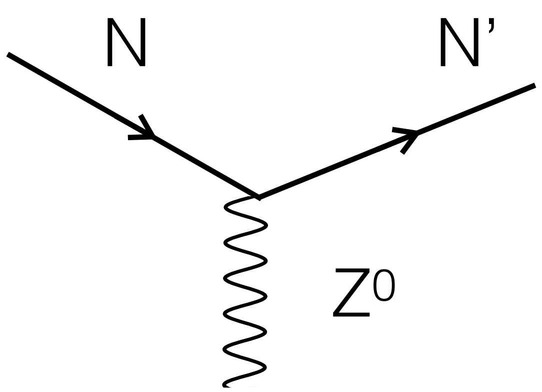
- The Vector current is the same of the electromagnetic: Conserved Vector Current hypothesis

T. Leitner, O. Buss, L. Alvarez-Ruso, and U. Mosel, Electron- and neutrino-nucleus scattering from the quasielastic to the resonance region, Phys. Rev. C 79, 034601 (2009).

Neutrino-Nucleon scattering







- EM

$$\mathcal{F}_1 = \frac{1}{2}[F_1^S + F_1^V \tau_z]$$

$$\mathcal{F}_2 = \frac{1}{2}[F_2^S + F_2^V \tau_z]$$

- CC

$$\mathcal{F}_1 = F_1^V \tau_\pm$$

$$\mathcal{F}_2 = F_2^V \tau_\pm$$

$$\mathcal{F}_A = F_A \tau_\pm$$

$$\mathcal{F}_P = F_P \tau_\pm$$

- NC

$$\mathcal{F}_1 = \frac{1}{2}[-2sin^2\theta_W F_1^S + (1 - 2sin^2\theta_W) F_1^V \tau_z]$$

$$\mathcal{F}_2 = \frac{1}{2}[-2sin^2\theta_W F_2^S + (1 - 2sin^2\theta_W) F_2^V \tau_z]$$

$$\mathcal{F}_A = \frac{1}{2} F_A \tau_z$$

$$\mathcal{F}_P = \frac{1}{2} F_p \tau_z$$

- We used the Conserved Vector Current hypothesis:

$$F_1^V \tau_z \to F_1^V \tau_\pm \ , \ F_2^V \tau_z \to F_2^V \tau_\pm$$

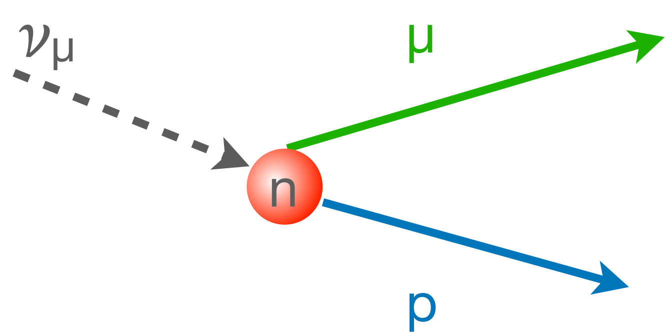
- PCAC:

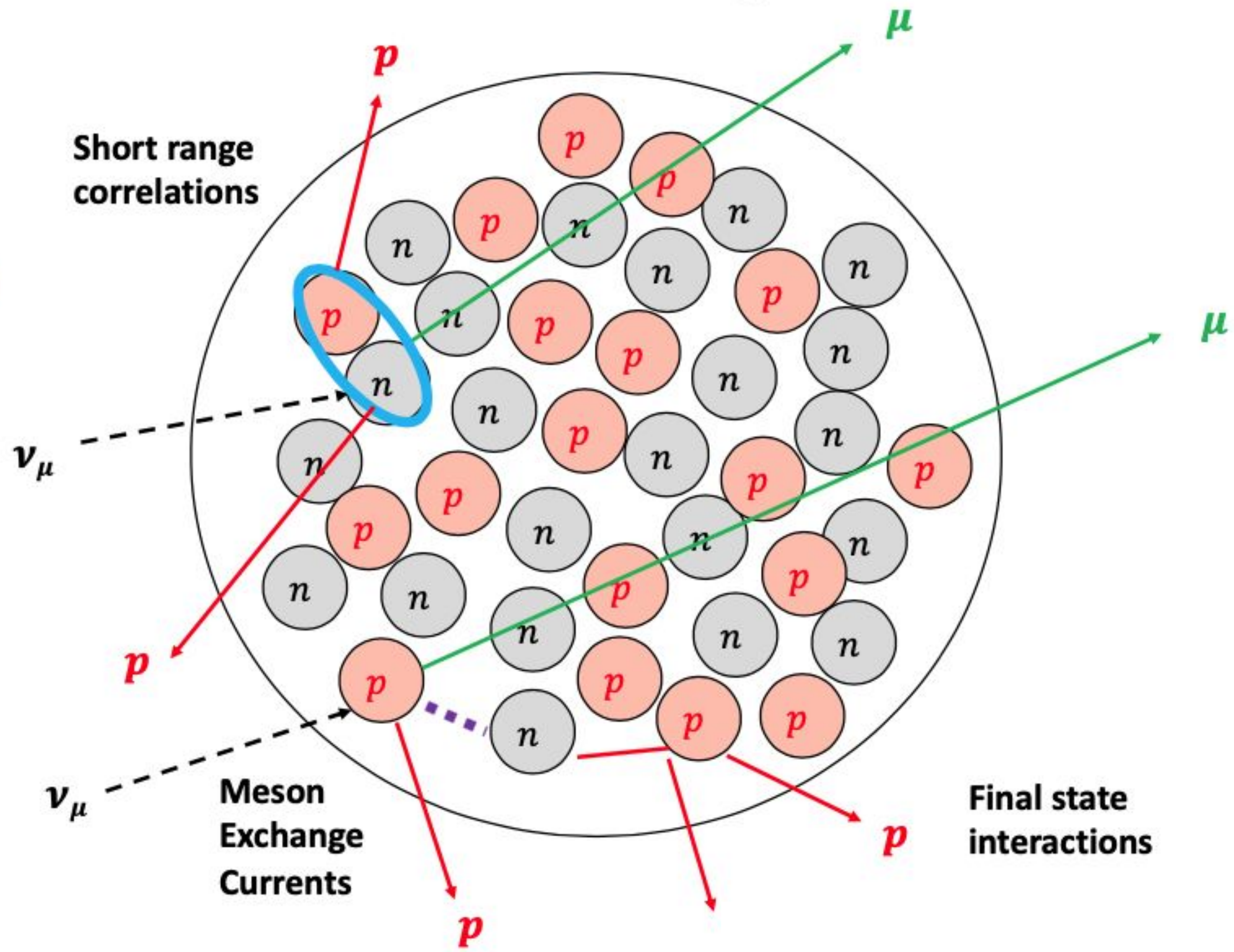
$$F_P = \frac{2m_N^2}{(m_\pi^2 - q^2)} F_A$$

Noemi Rocco, nrocco@fnal.gov

From theory to experiment

Free nucleon scattering case





Nuclear model describing the target nucleus

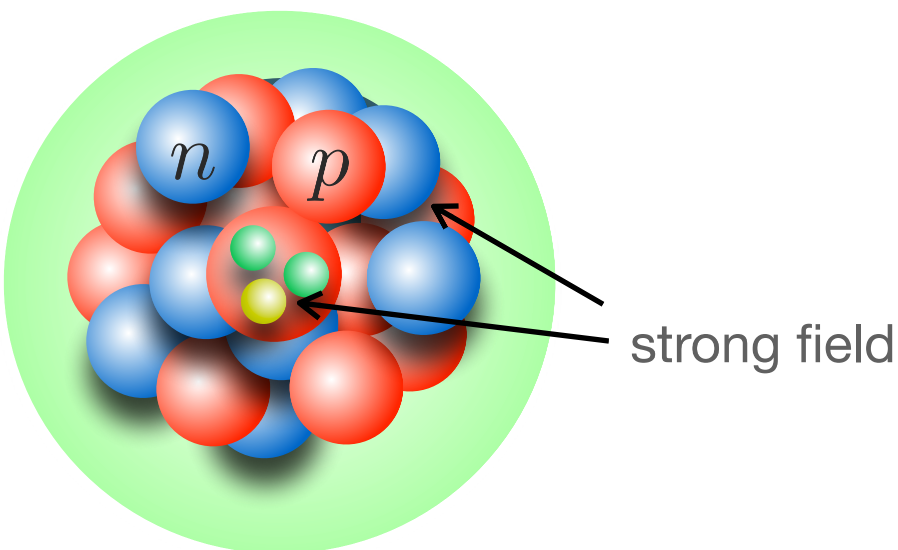
Different reaction mechanisms depending on the momentum transferred to the the nucleus

Final state interactions: describe how the particles propagate through the nuclear medium

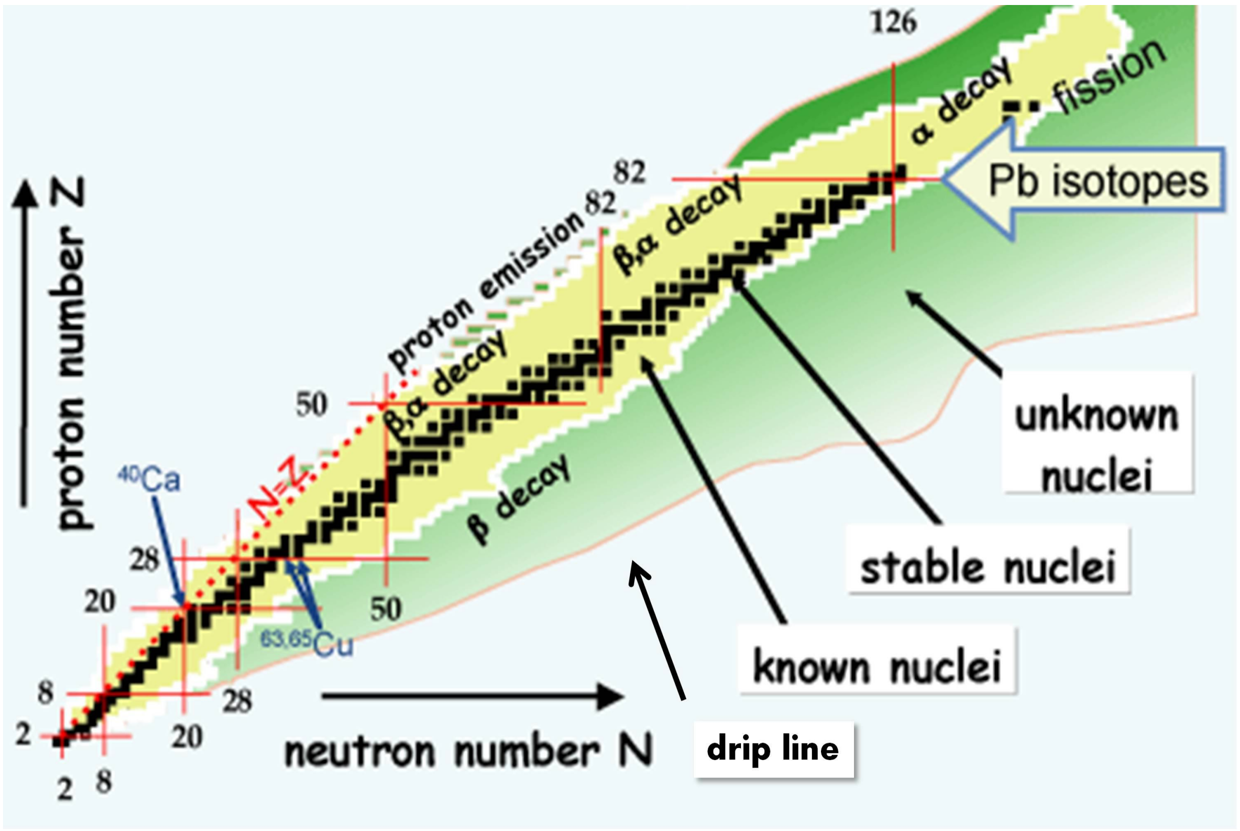
Noemi Rocco, nrocco@fnal.gov

The Nucleus internal structure

Nuclei are **strongly interacting** many body systems exhibiting fascinating properties



Nuclear chart. **Magic numbers** N or Z= 2, 8, 20, 28, 50 and 126; major shell complete and are more stable than other elements



The nucleus is formed by **protons** and **neutrons**: nucleons.

Each nucleon is made of three **quarks** held together by strong interactions→mediated by **gluons**

The nucleus is held together by the strong interactions between quark and gluons of neighboring nucleons

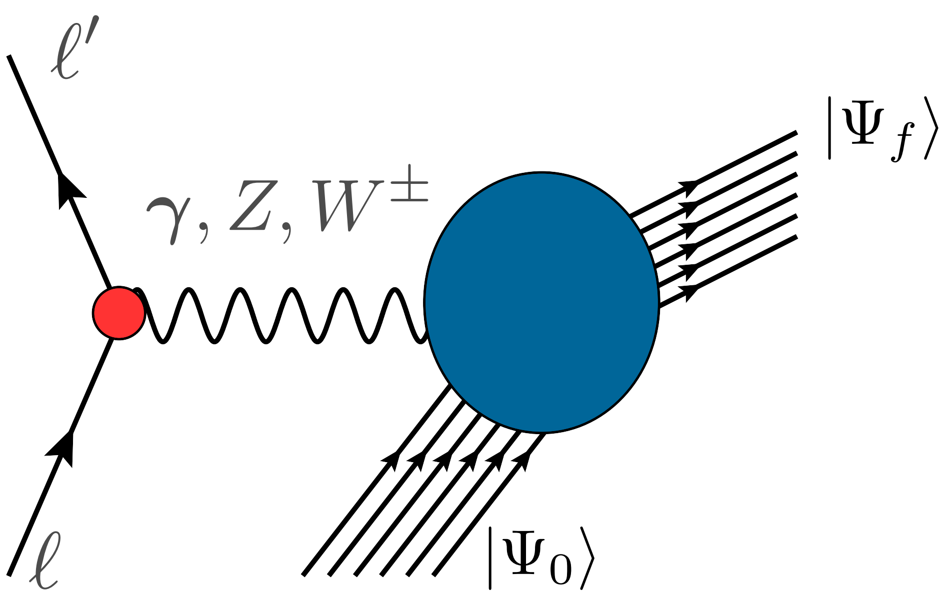
Nuclear Physicists **effectively** describe the interactions between protons and neutrons in terms of **exchange of pions**

Noemi Rocco, nrocco@fnal.gov

Theory of lepton-nucleus scattering

- The cross section of the process in which a lepton scatters off a nucleus is given by

$$d\sigma \propto L^{\alpha\beta} R_{\alpha\beta}$$



Leptonic Tensor: is the same as before, completely determined by lepton kinematics

Hadronic Tensor: nuclear response function

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J_\alpha^\dagger(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

The initial and final wave functions describe many-body states:

$$|0\rangle = |\Psi_0^A\rangle \ , |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \ldots$$

For inclusive reactions, the hadronic final state is not detected. We need to sum over all the possible ones

Noemi Rocco, nrocco@fnal.gov

Comparing electron- and neutrino-nucleus

- We start by defining the nuclear response functions, for a given value of **q** and ω

$$W^{\mu\nu}(\mathbf{q},\omega) = \sum_f \langle 0|(J^\mu)^\dagger(\mathbf{q},\omega)|f\rangle\langle f|J^\nu(\mathbf{q},\omega)|0\rangle\delta^{(4)}(p_0+\omega-p_f)$$

- Electron case we write the **inclusive** double differential cross section as:

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\mathrm{Mott}}\left[\left(\frac{q^2}{\mathbf{q}^2}\right)^2 R_L + \left(\frac{-q^2}{2\mathbf{q}^2} + tan^2\frac{\theta}{2}\right) R_T\right]$$

where: $R_L = W_{00}$, $R_T = W_{xx} + W_{yy}$

- Neutrino case:

$$\left(\frac{d\sigma}{dE'd\Omega}\right)_{\nu/\bar\nu} = \frac{G^2}{4\pi^2}\frac{k'}{2E_\nu}\left[\hat L_{CC}R_{CC} + 2\hat L_{CL}R_{CL} + \hat L_{LL}R_{LL} + \hat L_T R_T \pm 2\hat L_{T'}R_{T'}\right] ,$$

- Where the nuclear responses are given by

$$R_{CC} = W^{00}, \quad R_{CL} = -\frac{1}{2}(W^{03}+W^{30}), \quad R_{LL} = W^{33}, \quad R_T = W^{11}+W^{22}, \quad R_{T'} = -\frac{i}{2}(W^{12}-W^{21})$$

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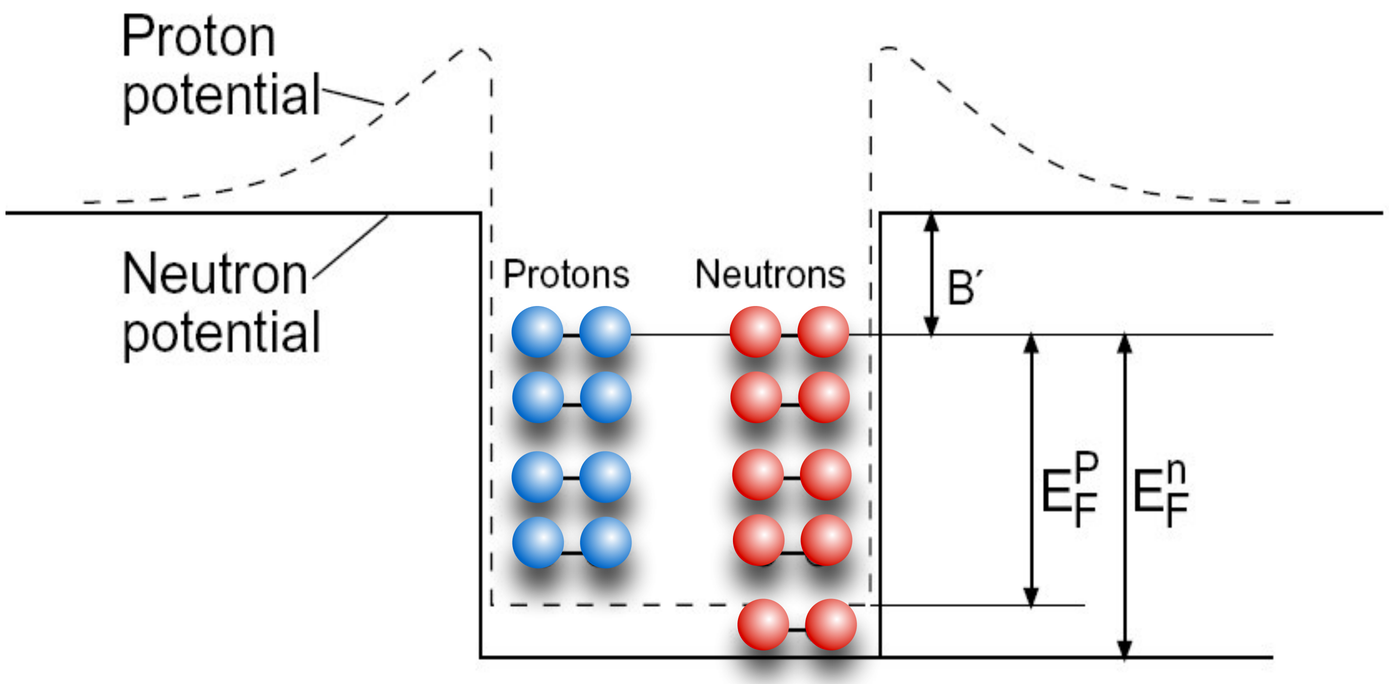
Initial state: global Fermi gas



- Simple picture of the nucleus: only statistical correlations are retained (Pauli exclusion principle)

- Protons and neutrons are considered as moving freely within the nuclear volume

- The nuclear potential wells are rectangular: constant inside the nucleus and goes sharply to zero at its edge

- The energy of the highest occupied state is the Fermi energy: E_F

- The difference B' between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon B'/A ~ 7-8 MeV

C. Bertulani, Nuclear Physics in a Nutshell

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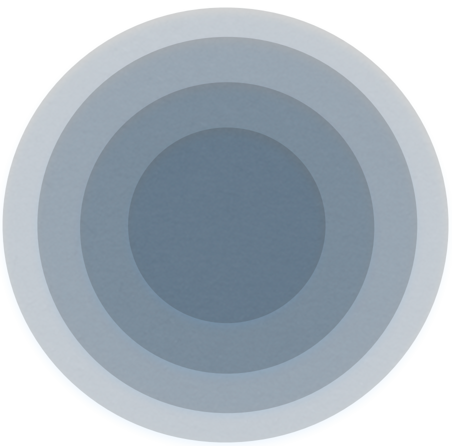
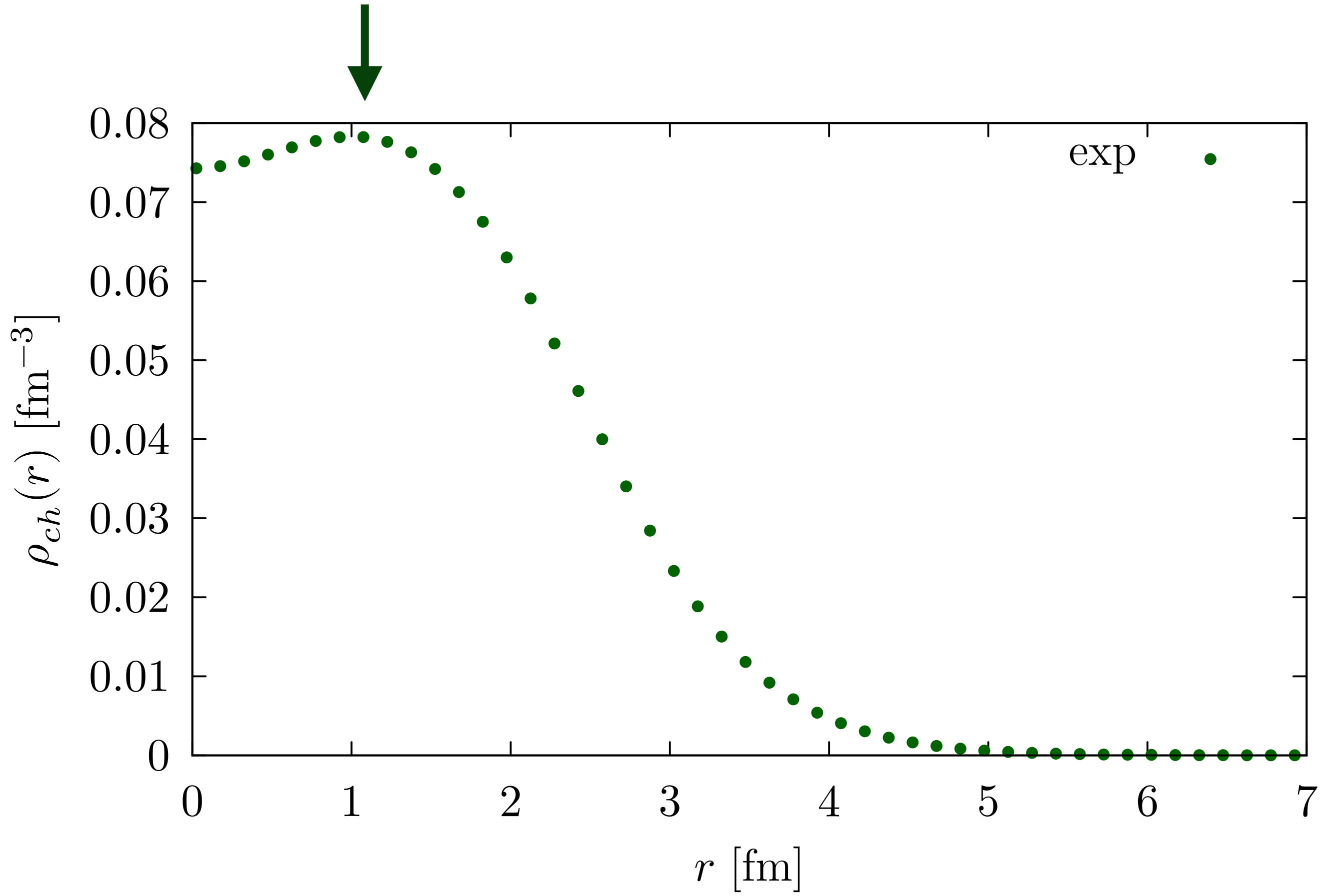
Initial state: Local Fermi gas

- A spherically symmetric nucleus can be approximated by concentric spheres of a constant density.



More likely to find a particle r ~ r_{ch}~ 2.5 fm



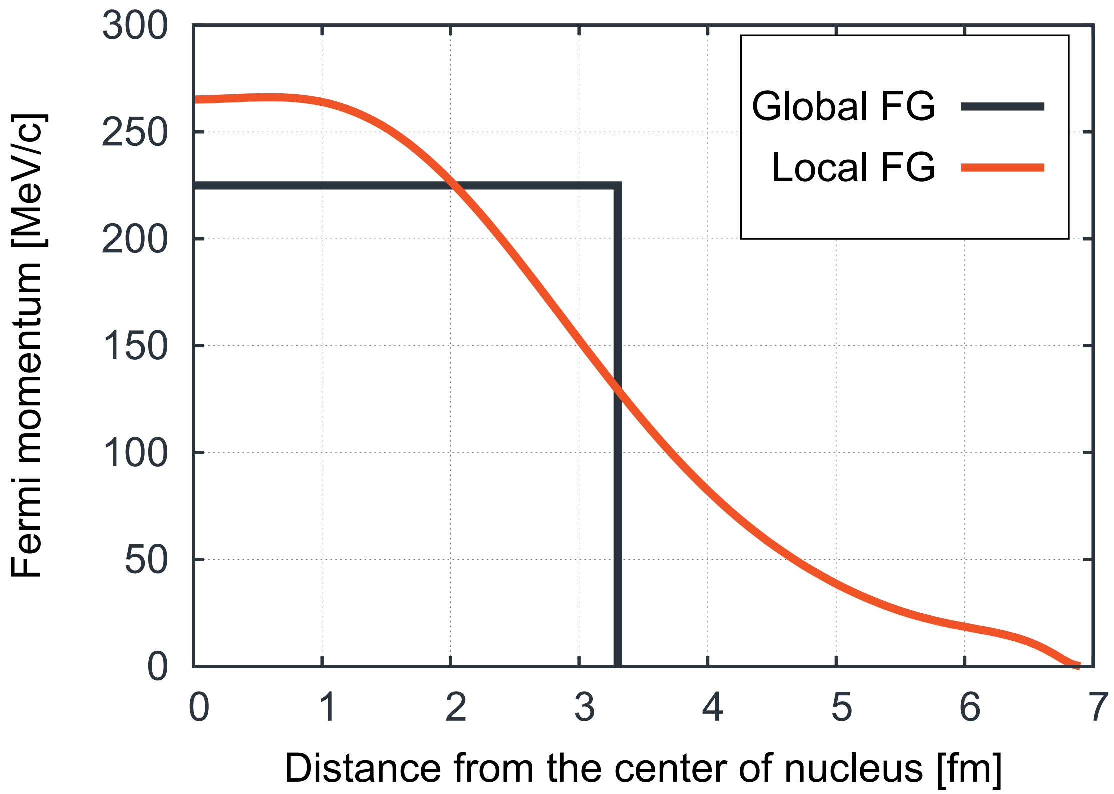


Figure by T. Golan

- **Global Fermi Gas**

$$p_F = \left(\frac{9\pi \cdot n}{4A}\right)^{1/3} \cdot \frac{\hbar}{R_0}$$

- **Local Fermi Gas**

$$p_F = \hbar\left(3\pi^2 \rho(r)\frac{n}{A}\right)^{1/3}$$

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Initial state: shell Model

- As in the Fermi Gas model: the nucleons move within the nucleus independently of each other

- Difference: the nucleons are not free: subject to a central potential

- Each nucleon moves in an average potential created by the other nucleons, the potential should be chosen to best reproduce the experimental results

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \dots \longrightarrow H = \sum_i \frac{p_i^2}{2m} + \sum_i^A U_i + H_{\rm res}$$

- We solve the Schrödinger Equation:

$$H\,\psi = E\,\psi \qquad \begin{cases} E = E_1 + E_2 + \ldots + E_A \\ \psi(1,\ldots,A) = \mathcal{A}[\phi_1(1)\ldots\phi_A(A)] \end{cases}$$

Noemi Rocco, nrocco@fnal.gov

Initial state: shell Model

- Example: Particles are subject to an harmonic oscillator potential

$$U(r) = \frac{1}{2} m \omega^2 r^2$$

The frequency should be adapted to the mass number A

- We will seek solutions of the type $\psi(r) = \frac{u(r)}{r} Y_l^m(\theta, \phi)$ ← Spherical Harmonics

- Solving the Schrödinger Equation reduces to a solution of u:

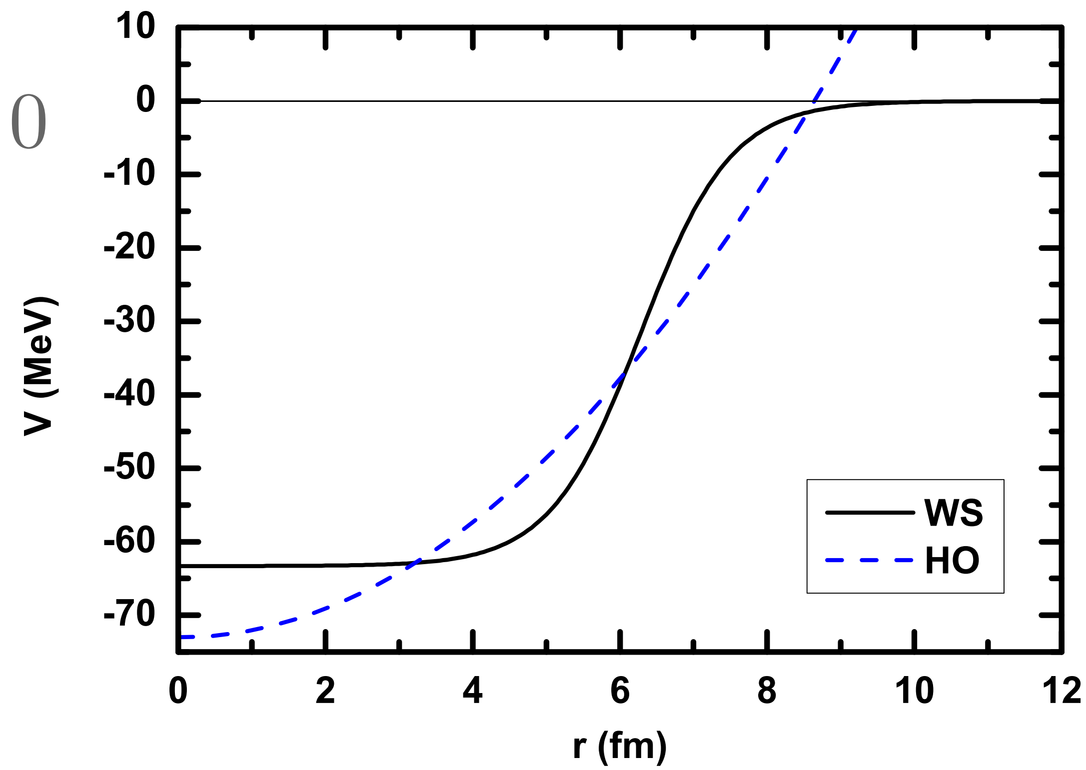
$$\frac{d^2u}{dr^2} + \left\{ \frac{2m}{\hbar^2}[E - U(r)] - \frac{l(l+1)}{r^2} \right\} u(r) = 0$$

$$E_{nl} = \hbar\omega \left(2n + l - \frac{1}{2} \right)$$ Eigenvalues

- A more realistic potential is the Wood Saxon:

$$V = V_0 / [1 + exp[(r - R)/a]$$

V_0, r, a, are adjustable parameters chosen to best reproduce the **experimental results**

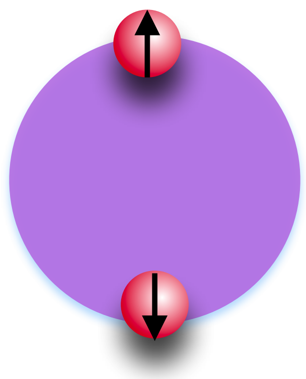


Physical Review C 87(1):014334

Noemi Rocco, nrocco@fnal.gov

Nuclear Shell Model

The lowest level, s shell, can contain 2 protons



1s (m_s=0)   Z=2

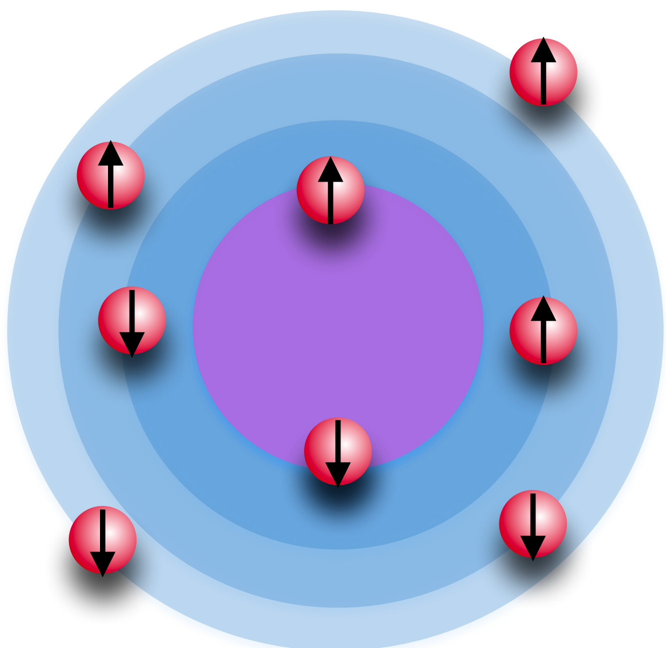
n is the principal quantum number, **l** orbital momentum, **m** magnetic quantum number

Our assumption: central potential

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

Nuclear Shell Model

The p shell can contain up to 6 protons



1s (m_s=0)
1p (m_p=-1,0,1)
Z=8

Our assumption: central potential

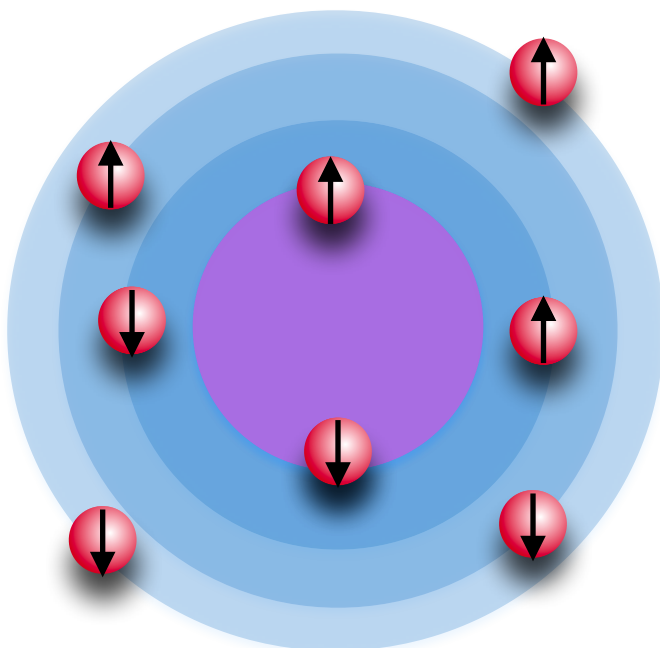
$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

We explained the first two magic numbers: 2 and 8. We can follow the same strategy for the Z=20 case; but at the next step we obtain Z=40 **while experimentally Z=50**

n is the principal quantum number, **l** orbital momentum, **m** magnetic quantum number

Nuclear Shell Model

The p shell can contain up to 6 protons



1s (m_s=0)
1p (m_p=-1,0,1)
Z=8

✏ Maria Goeppert Mayer poses with her colleagues in front of Argonne's Physics building.

Our assumption: central potential

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

We explained the first two magic numbers: 2 and 8. We can follow the same strategy for the Z=20 case; but at the next step we obtain Z=40 **while experimentally Z=50**

In 1963, Goeppert Mayer, Jensen, and Wigner shared the Nobel Prize for Physics "for their discoveries concerning nuclear shell structure."

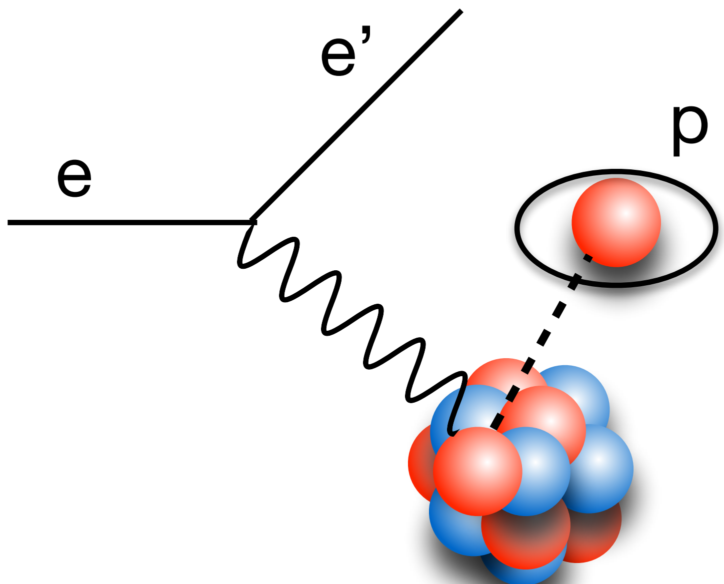
The solution to the puzzle lies in the **spin-orbit coupling**. This effect in the nuclear potential is 20 times larger then in Atomic Physics

$$V(r) \rightarrow V(r) + W(r)\mathbf{L}\cdot\mathbf{S}$$

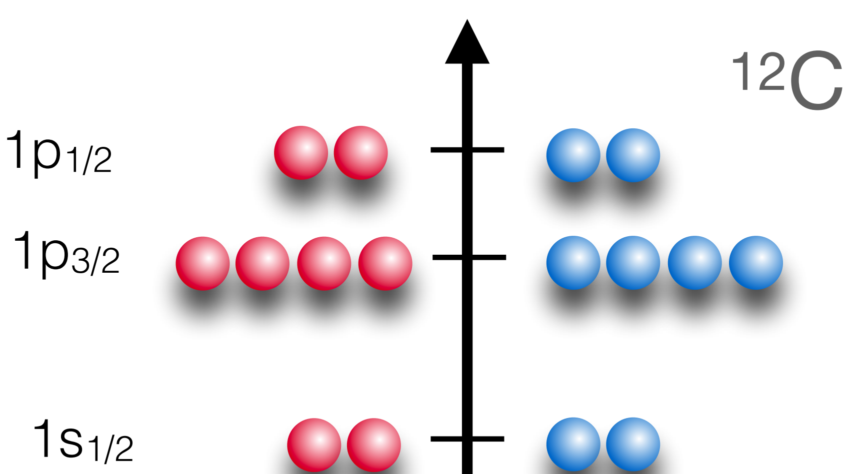
The spin-orbit introduces an energy split which modifies the shell structure and reproduces magic number up to Z=126

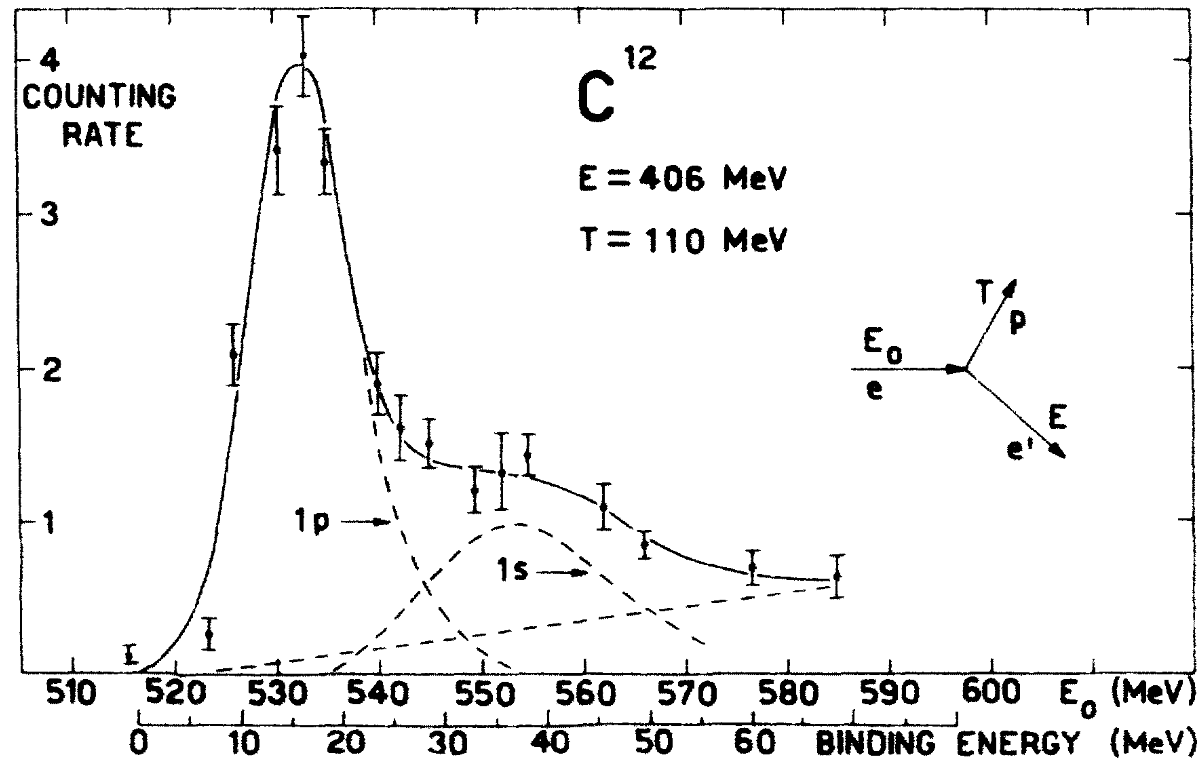
(e,e'p) scattering experiments

- (e,e'p) experiments are extremely important to investigate the internal structure of the nucleus



- Assuming NO FSI the energy and momentum of the initial nucleon can be identified with the measured p_{miss} and E_{miss}





U.Amaldi et al, Phys. Rev. Lett. 13, 10 (1964)

- The peak coming from four 1p protons is visible

- The contribution of the two 1s protons is not clearly separated with this resolution

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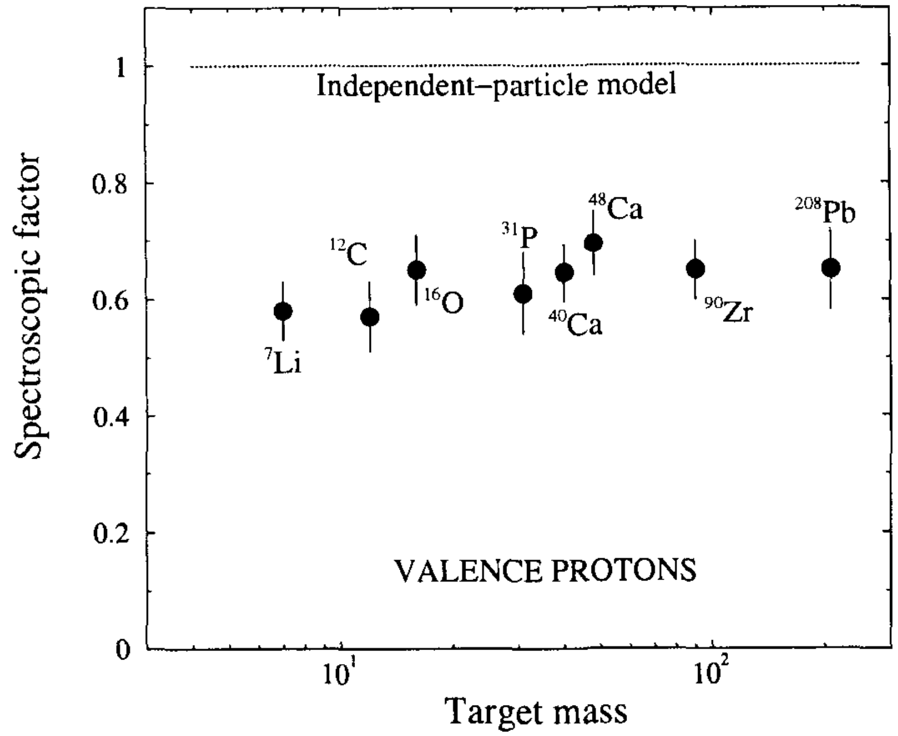
Fermilab

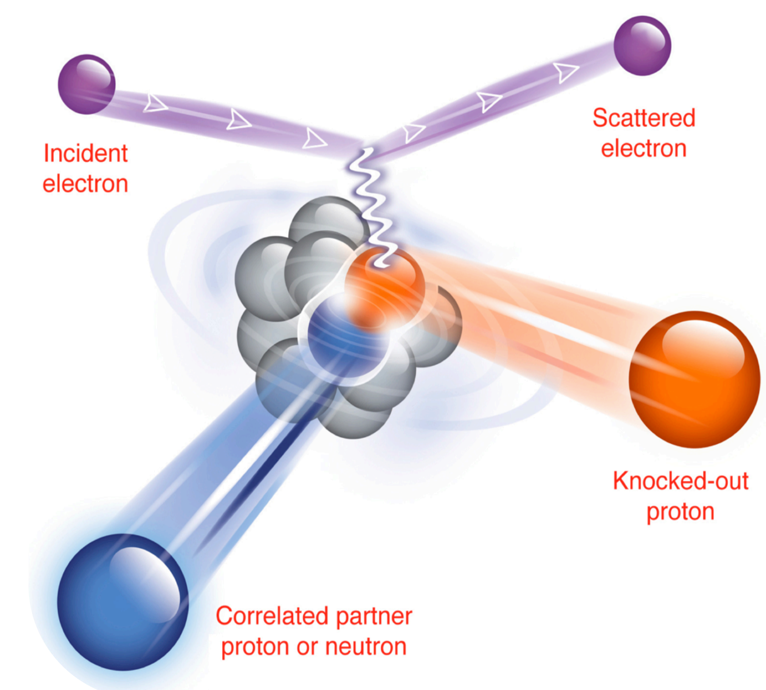
(e,e'p) scattering experiments

- Electron and proton experiments also pinned down the limitations of MF approaches



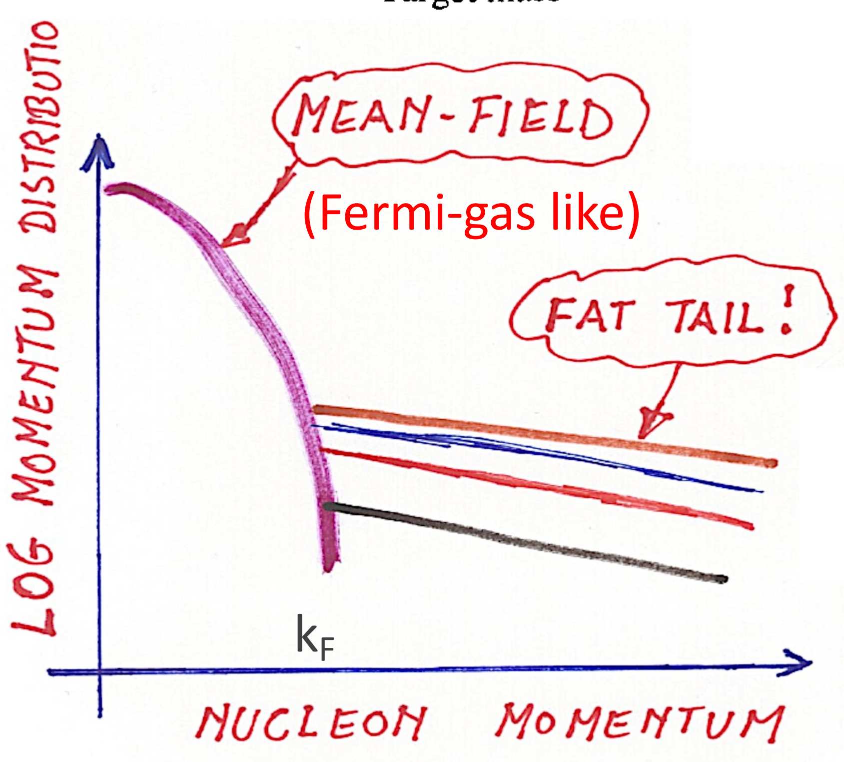
- Quenching of the spectroscopic factors of valence states has been confirmed by a number of high resolution (e,e'p) experiments

Subedi et al., Science 320, 1476 (2008)



- **Semi-exclusive 2N-SRC** experiments at x>1 allows to detect both nucleons and reconstruct the initial state

- Confirmed that the **high momentum tail** of the nuclear wave function consists mainly of 2N-SRC



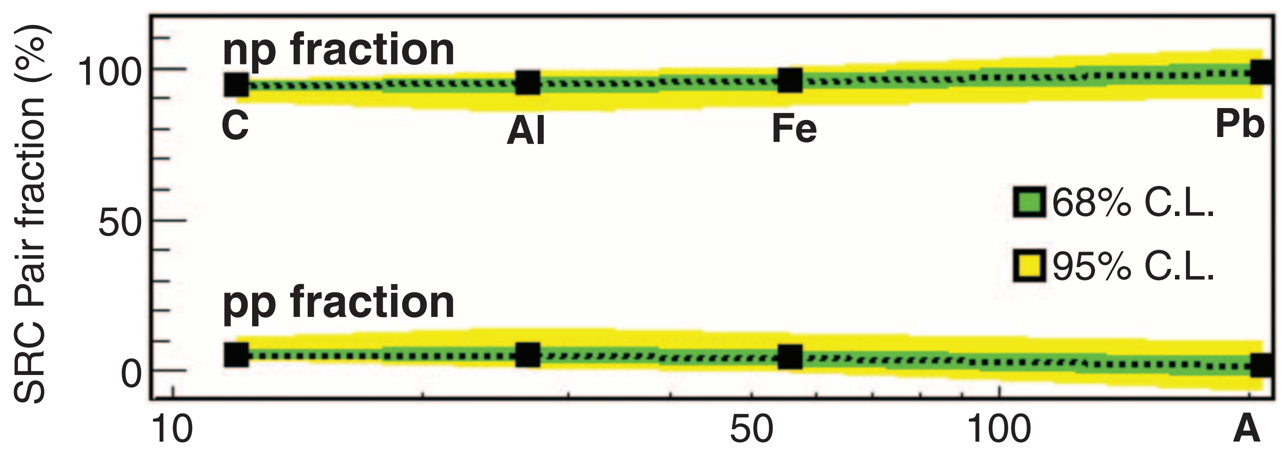
- The large-momentum (short-range) component of the wave function is dominated by the presence of Short Range Correlated (SRC) pairs of nucleons

Figure by Or Hen

Noemi Rocco, nrocco@fnal.gov

(e,e'p) scattering experiments

- Observed dominance of np-over-pp pairs for a variety of nuclei



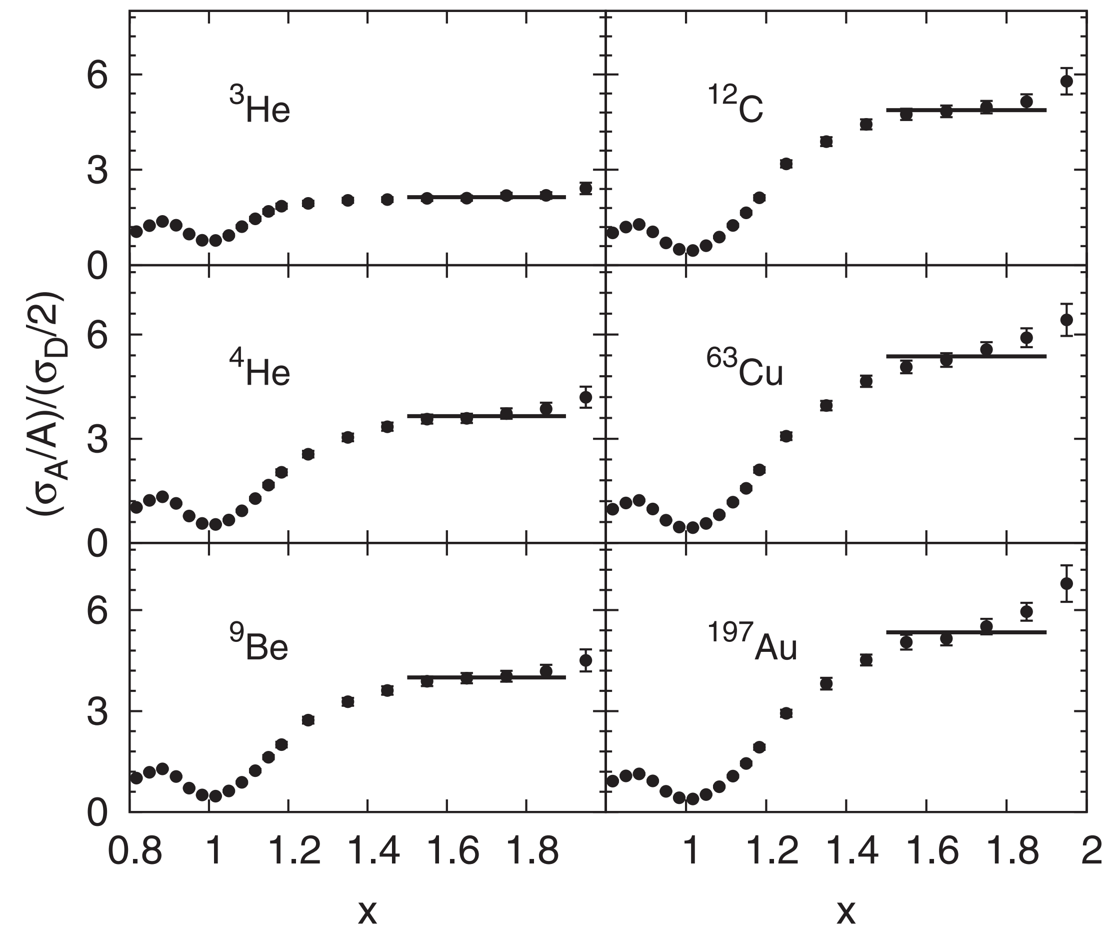
Subedi et al., Science 320, 1476 (2008)

- SRC pairs are in spin-triplet state, a consequence of the tensor part of the nucleon-nucleon interaction

Bottom Line

- Two-body Physics can not be neglected:
  - ~20% of the nucleons in nuclei
  - ~100% of the high k (>pF) nucleons
- Have large relative momentum and low center of mass momentum

- Universality of high-momentum component



N. Fomin et al., PRL 108, 092502 (2012)

- The cross section ratio: A/d, sensitive to $n_A(k)/n_d(k)$

- Observed scaling for x>1.5

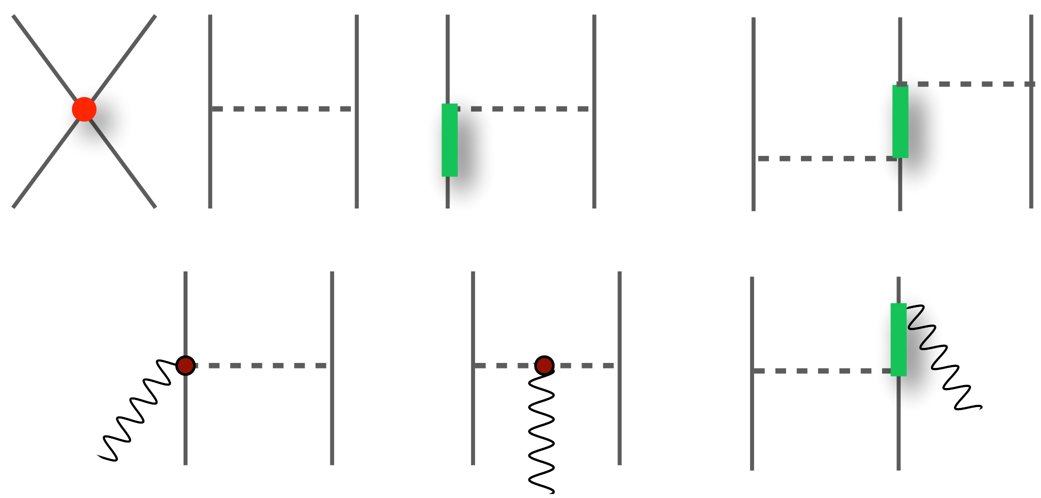
$$n_A(k > p_F) = a_2(A) \times n_d(k)$$

Noemi Rocco, nrocco@fnal.gov

Figure by Or Hen

The basic model of nuclear theory

Effective Hamiltonians and consistent currents

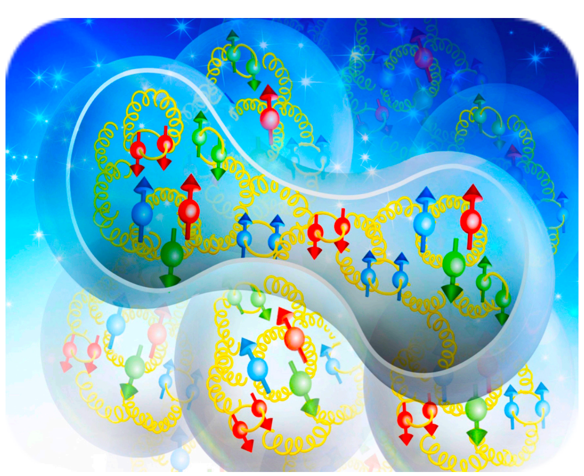


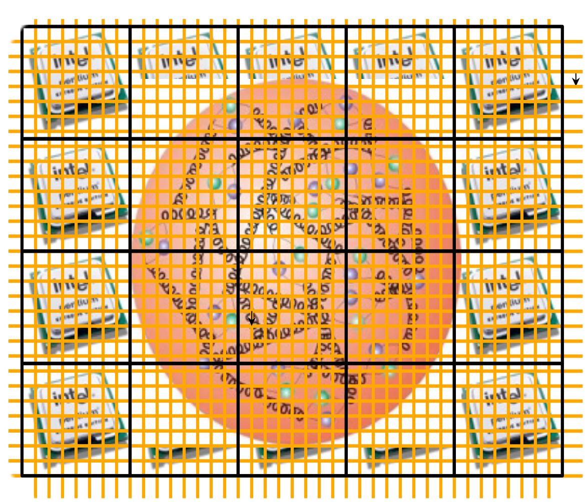
Accurate nuclear many-body methods

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

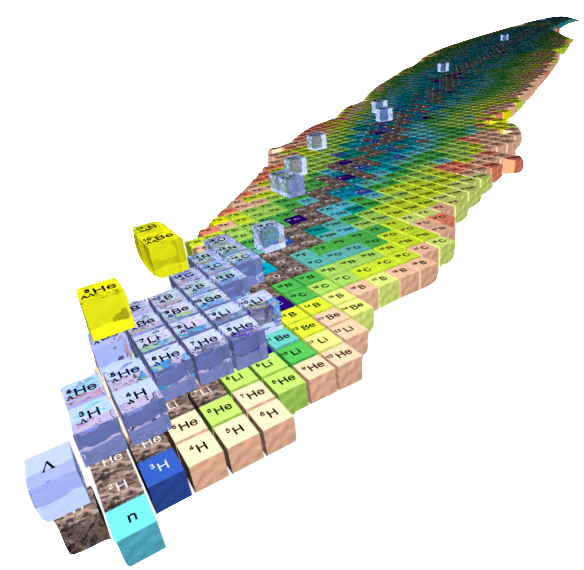
$$J_{mn} = \langle\Psi_m|J|\Psi_n\rangle$$

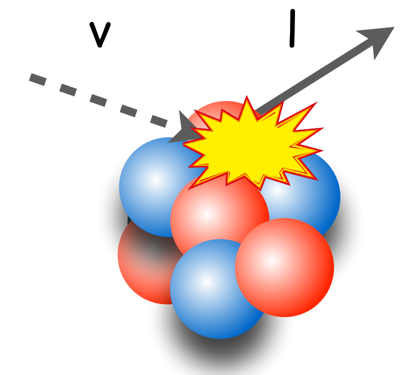
Quantum Chromodynamcs





Nuclei and electroweak interactions



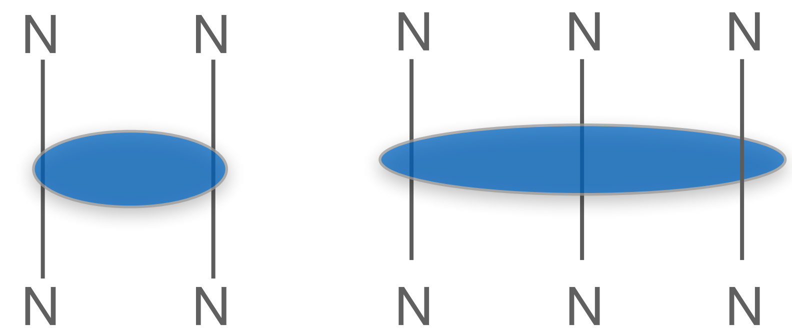


The basic model of nuclear theory

Effective field theories are the link between QCD and nuclear observables. At low energy, the effective degrees of freedom are pions and nucleons:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

1-body **2-body** **3-body**



R. Machleidt , D. R. Entem,
Phys.Rept.503:1-75,2011

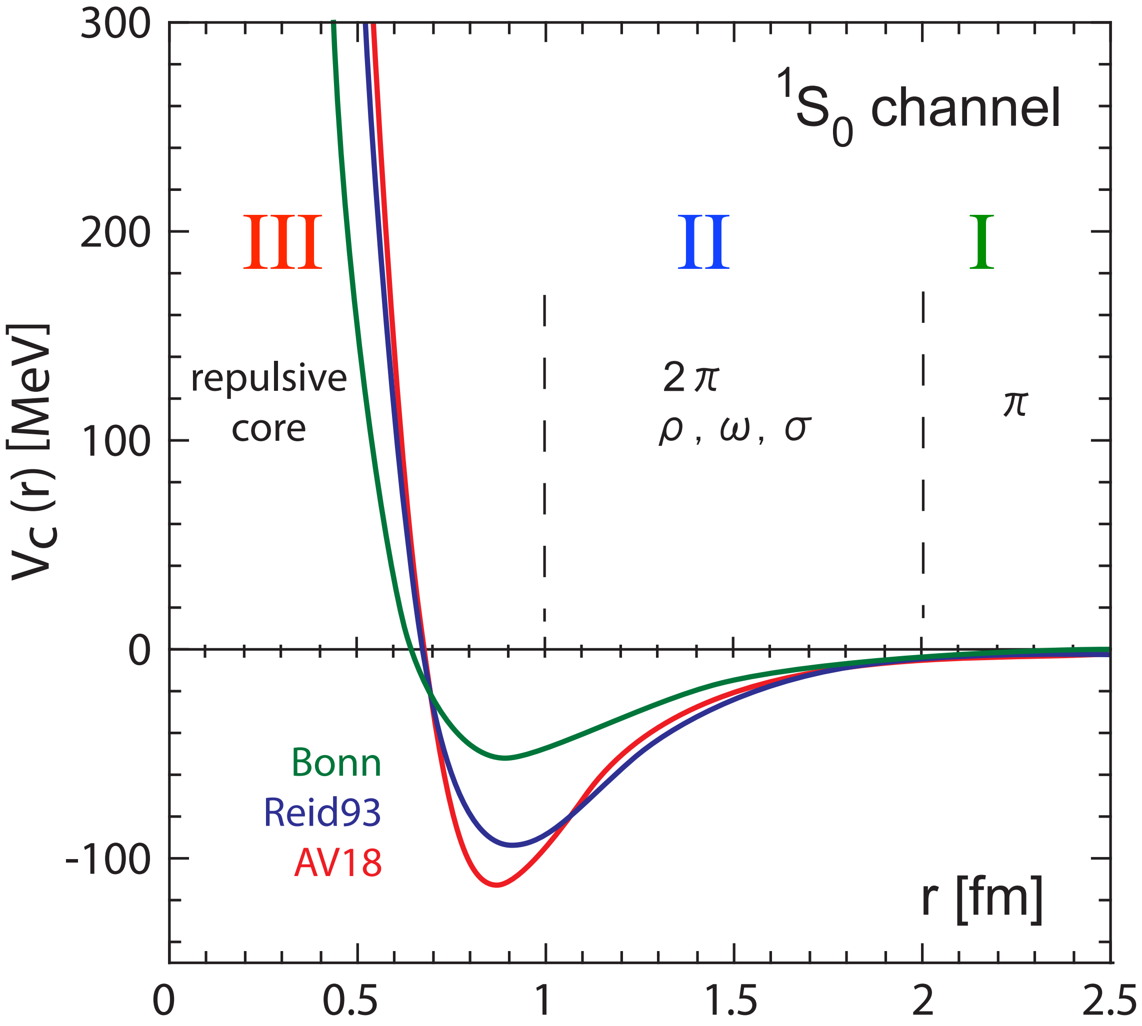
E. Epelbaum, (Lectures), arXiv:1001.3229

Different strategies to construct two- and three-body interactions

- Chiral Effective Field Theory interactions
- Phenomenological potentials

Nucleon-nucleon potential

Aoki et al. Comput.Sci.Disc.1(2008)015009



Bonn PRC 63, 024001, 2001

Reid93 PRC 49, 2950, 1994

AV18: Wiringa PRC 51, 38, 1995

Long range part: One **π** exchange

Range: $\frac{1}{m_\pi} \sim 1.4 \text{ fm}$

Medium range part: Two **π** exchange

Range: $\frac{1}{2m_\pi} \sim 0.7 \text{ fm}$

Short range part: Repulsive core

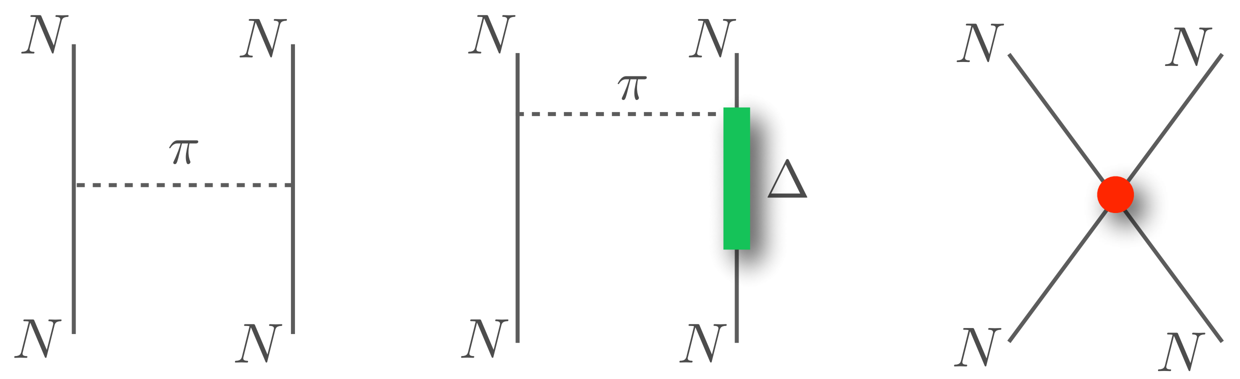
Noemi Rocco, nrocco@fnal.gov

Phenomenological potential: av18 + IL7

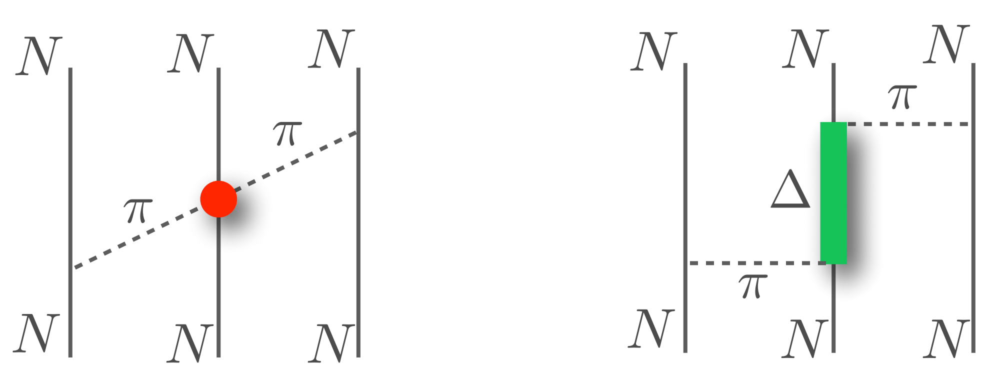
Phenomenological potentials explicitly include the **long-range one-pion exchange interaction** and a set of **intermediate- and short-range phenomenological terms**

- **Argonne v_{18}** is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij}) O^p_{ij}$$



- Phenomenological three-nucleon interactions, like the **Illinois 7**, effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects



The parameters of the AV18 + IL7 are fit to properties of **exactly solvable light nuclear systems**.

Noemi Rocco, nrocco@fnal.gov

Chiral effective field theory

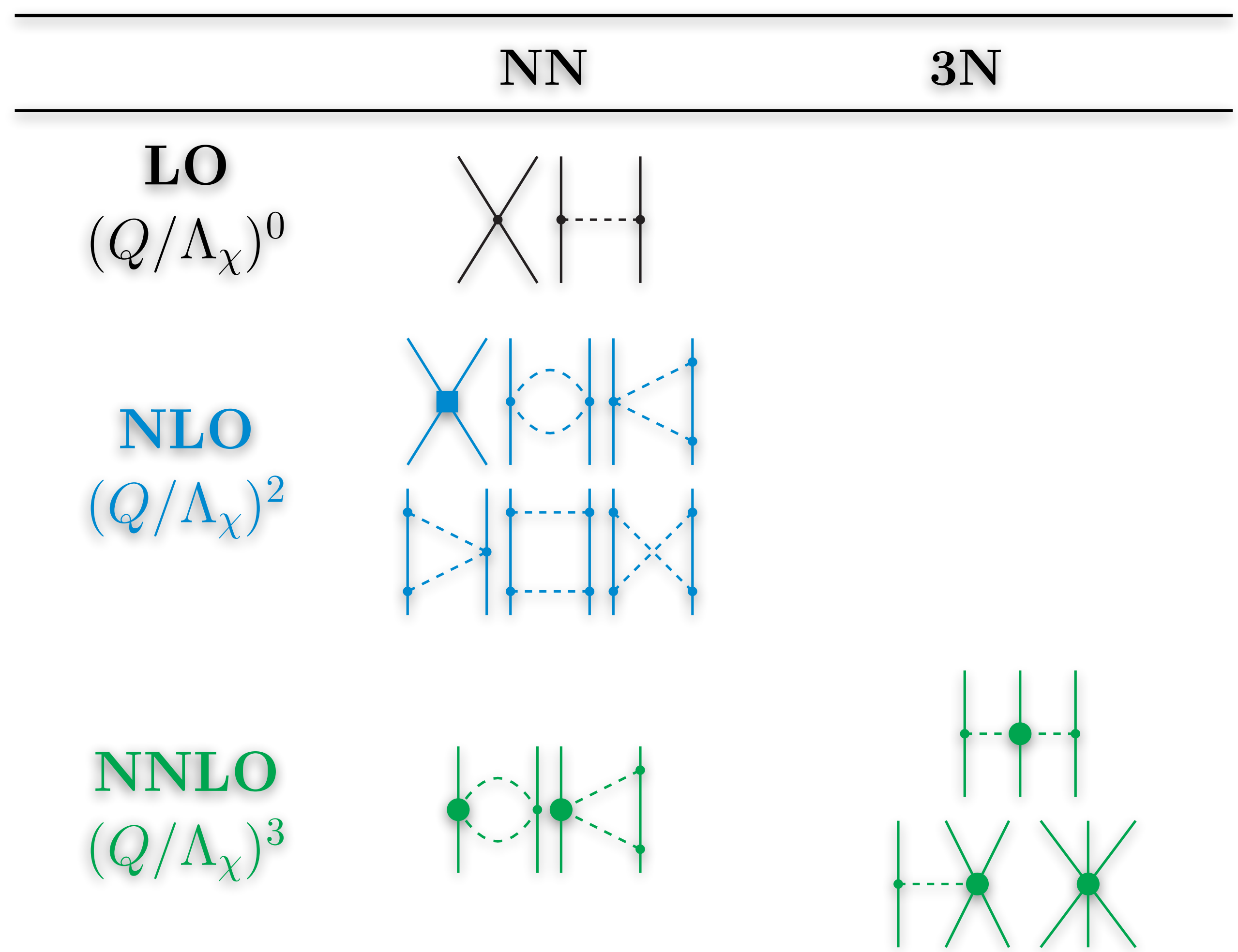
Chiral Hamiltonians exploits the (approximate) broken chiral symmetry of QCD

Identify the soft and hard scale of the problem $\mathcal{L}^{(n)} \sim \left(\frac{q}{\Lambda_b}\right)^n$ ~ 100 MeV soft scale, ~ 1 GeV hard scale

Design an organizational scheme that can distinguish between more and less important terms:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)}$$

H. Hergert, Front. in Phys. 8, 379 (2020)



Contact interactions lead to LEC:

Short range two-nucleon interaction fit to deuteron and NN scattering

Three nucleon interactions fitted on light nuclei

Long-range LEC are determined from π-nucleon scattering

Formulate statistical models for uncertainties: Bayesian estimates of EFT errors

S. Wesolowski, et al, PRC 104, 064001 (2021)

The basic model of nuclear theory

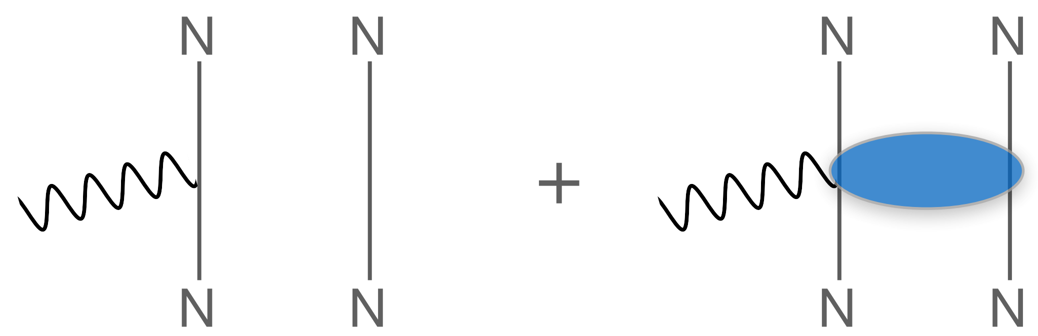
The current operator describes how the external probe interacts with nucleon, nucleons pairs, create new particles …

The structure of the current operator is constrained by the Hamiltonian through the continuity equation

$$\nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0 \qquad [v_{ij}, j^0_i] \neq 0$$

The Hamiltonian structure implies that the current operator includes one and two-body contributions

$$J^\mu(q) = \sum_i j^\mu_i + \sum_{i<j} j^\mu_{ij} + \dots$$



- Chiral Effective Field Theory Electroweak many-body currents
- "Phenomenological" Electroweak many-body currents

Noemi Rocco, nrocco@fnal.gov

Variational Monte Carlo

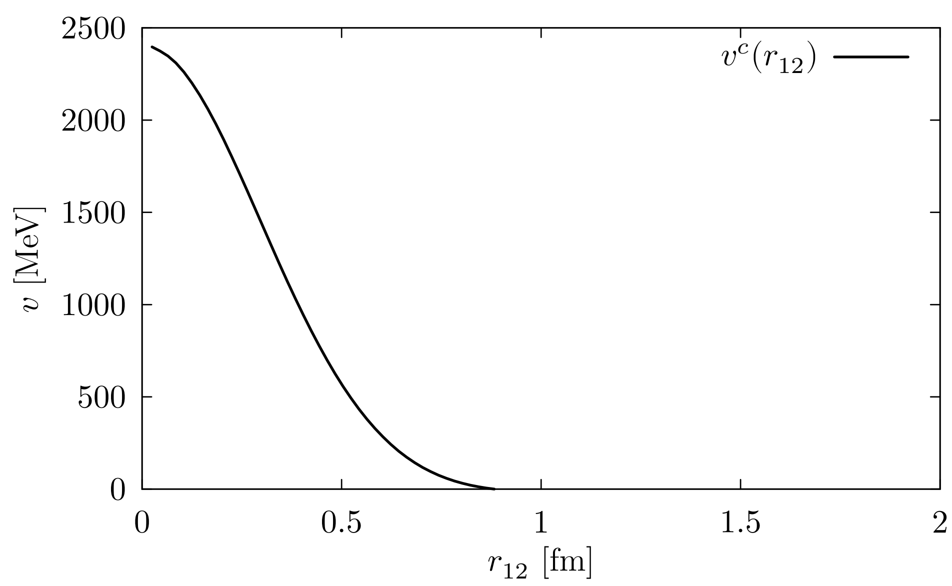
In variational Monte Carlo, one **assumes a suitable form for the trial wave function**

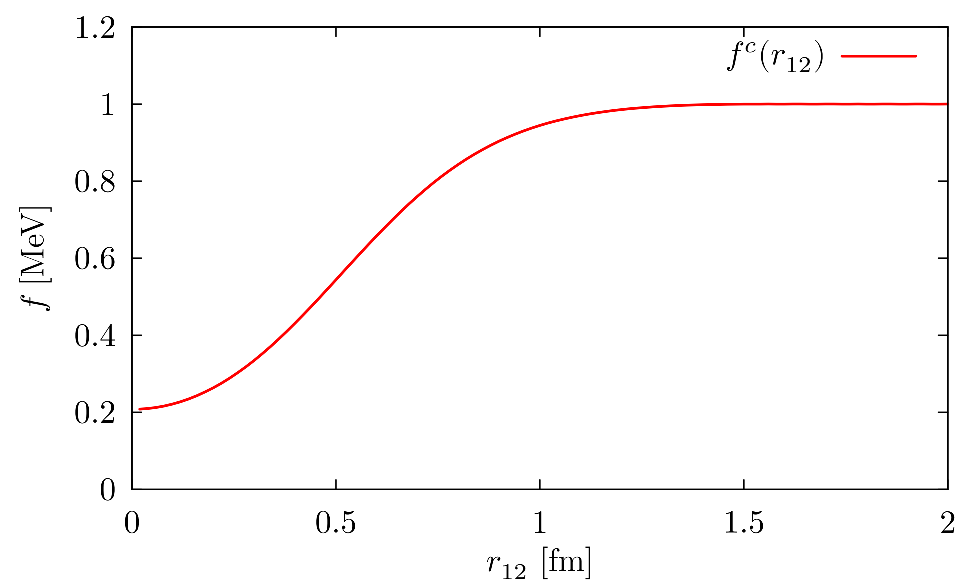
$$|\Psi_T\rangle = \mathcal{F}|\Phi\rangle \begin{cases} \Phi : \text{Mean field component; slater determinant of single-particle orbitals} \\ \mathcal{F} : \text{correlations (2b \& 3b) induced by } H \end{cases}$$

The correlation operator reflects the spin-isospin dependence of the nuclear interaction

$$\mathcal{F} \equiv \left(\mathcal{S}\prod_{i<j} F_{ij}\right) \qquad F_{ij} \equiv \sum_p f_{ij}^p O_{ij}^p$$

The best parameters are found by **optimizing the variational energy** $\dfrac{\langle\Psi_T|H|\Psi_T\rangle}{\langle\Psi_T|\Psi_T\rangle} = E_T \geq E_0$





Green's Function Monte Carlo

Any trial wave function can be expanded in the complete set of eigenstates of the the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

GFMC overcomes the limitations of the variational wave-function by using an imaginary-time projection technique to **projects out the exact lowest-energy state**

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty} \sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

The direct calculation of the imaginary-time propagator for strongly-interacting systems involves prohibitive difficulties

J. Carlson , et al. Rev. Mod. Phys. 87 (2015) 1067

The imaginary-time evolution is broken into N small imaginary-time steps, and complete sets of states are inserted

$$e^{-(H-E_0)\tau}|\Psi_V\rangle = \int dR_1 \dots dR_N |R_N\rangle\langle R_N|e^{-(H-E_0)\Delta\tau}|R_{N-1}\rangle \dots \langle R_2|e^{-(H-E_0)\Delta\tau}|R_1\rangle \Psi_V(R_1)$$

Short Time Propagator

Noemi Rocco, nrocco@fnal.gov

Green’s Function Monte Carlo

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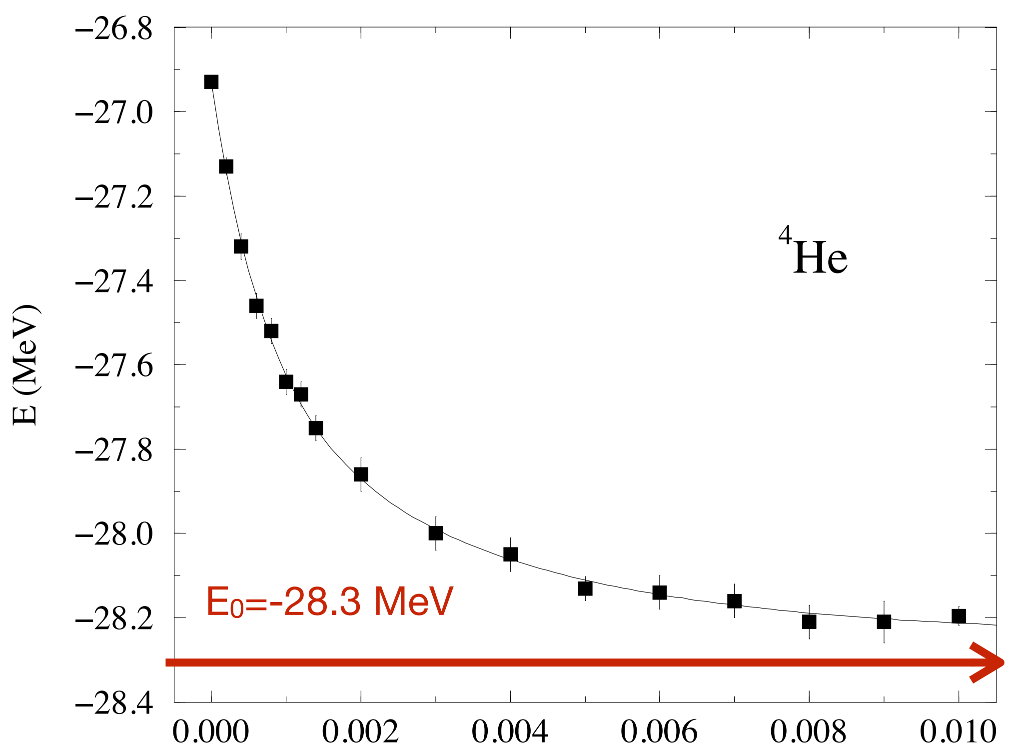


B. Pudliner et al., PRC **56**, 1720 (1997)

The computational cost of the calculation is $2^A \times A!/(Z!(A-Z)!)$

$$|S\rangle = \begin{pmatrix} s_{\uparrow\uparrow\uparrow} \\ s_{\uparrow\uparrow\downarrow} \\ s_{\uparrow\downarrow\uparrow} \\ s_{\uparrow\downarrow\downarrow} \\ s_{\downarrow\uparrow\uparrow} \\ s_{\downarrow\uparrow\downarrow} \\ s_{\downarrow\downarrow\uparrow} \\ s_{\downarrow\downarrow\downarrow} \end{pmatrix}$$

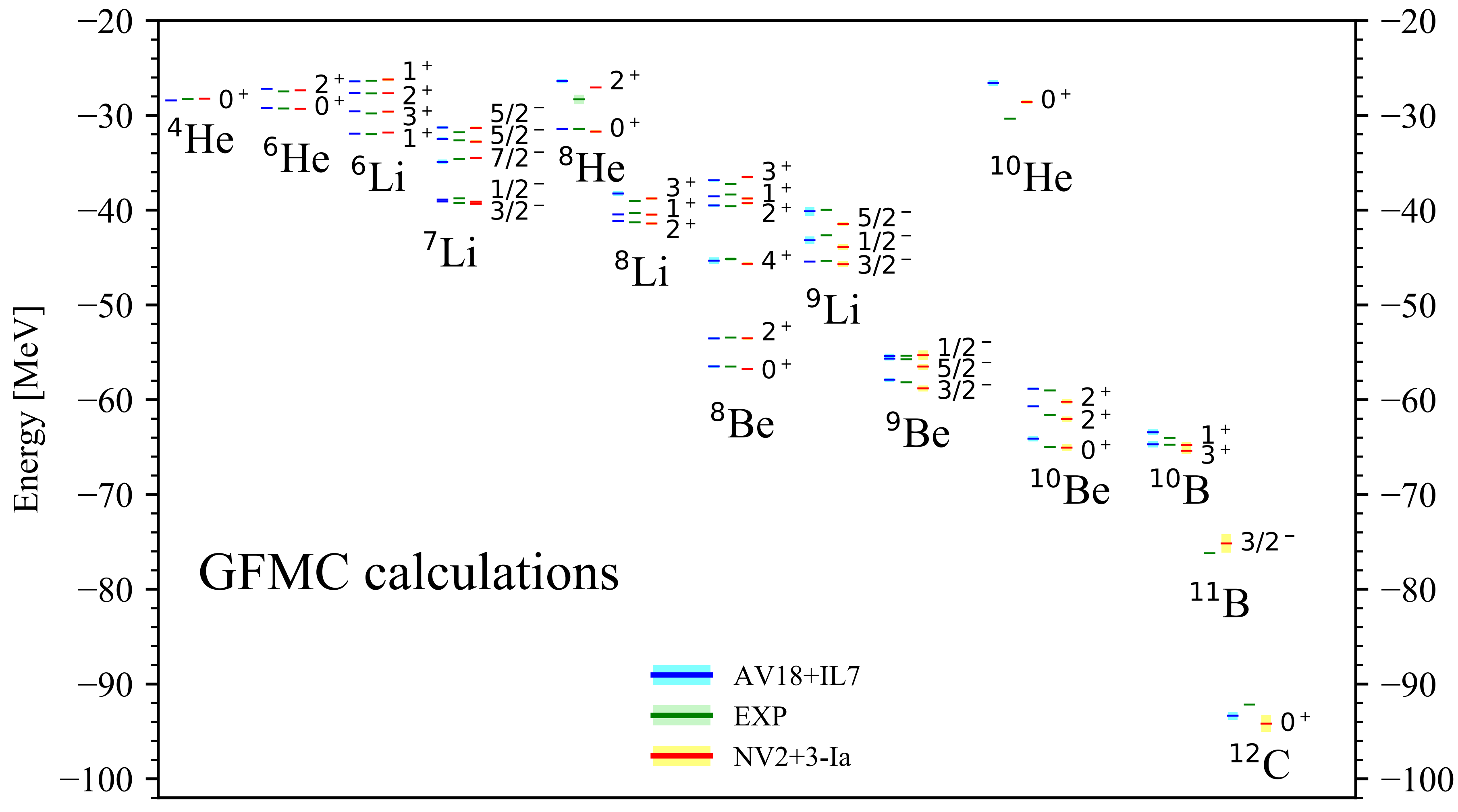
Noemi Rocco, nrocco@fnal.gov

Solve the Many Body Nuclear problem

Develop Computational Methods to solve numerically

$$H\Psi(\mathbf{R}; s_1 \dots s_A, \tau_1 \dots \tau_A) = E\Psi(\mathbf{R}; s_1 \dots s_A, \tau_1 \dots \tau_A)$$

Quantum Monte Carlo techniques are suitable to solve the Schroedinger equation of medium nuclei

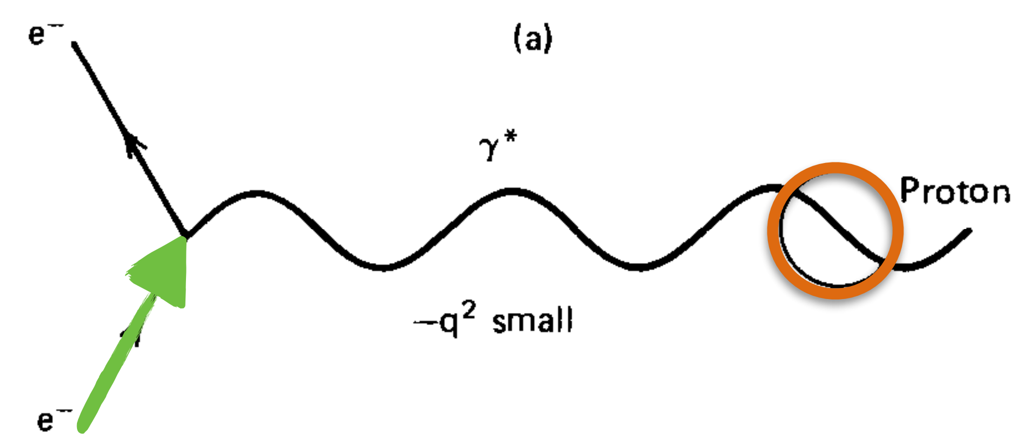


M. Piarulli, et al. Phys.Rev.Lett. 120 (2018) 5, 052503

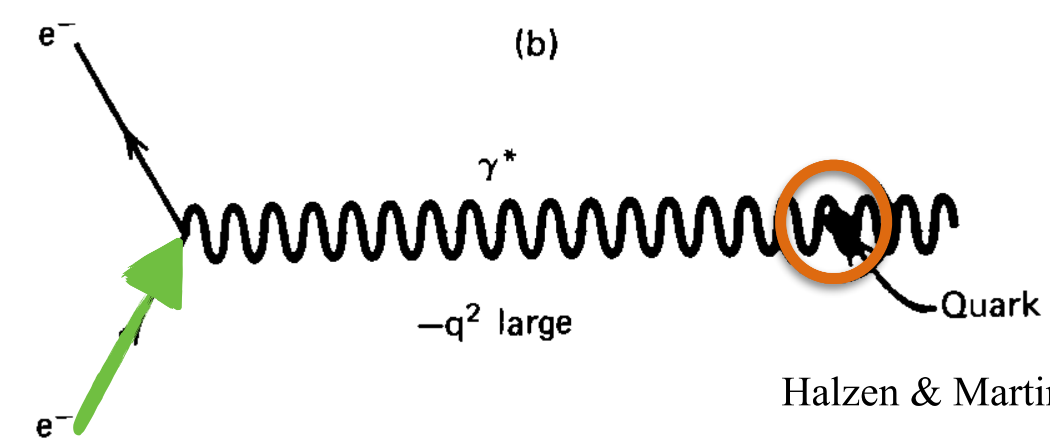
Noemi Rocco, nrocco@fnal.gov

Probing the target structure

The interaction depends on the **mediator** energy (dubbed as energy transfer ω)



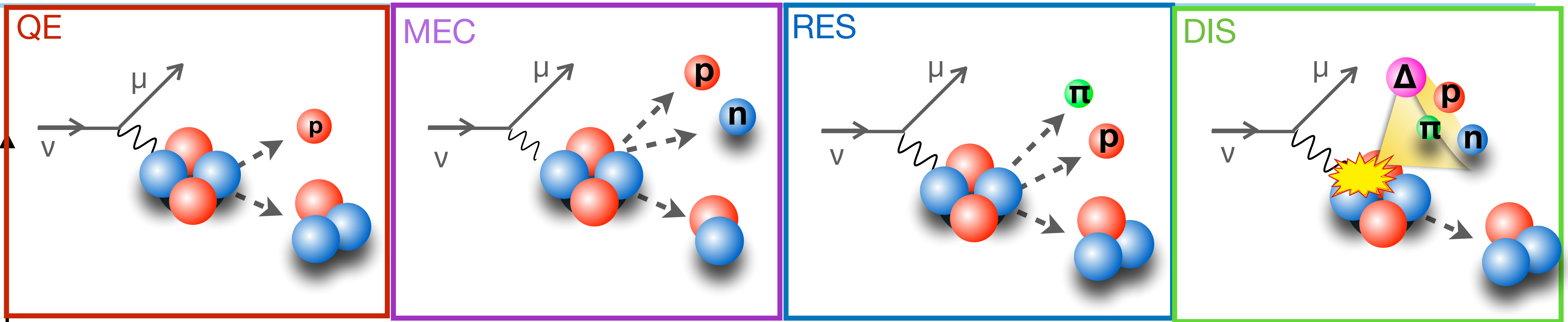
Higher energy transfer = smaller de Broglie wavelength = the probe can resolve the structure inside the nucleon

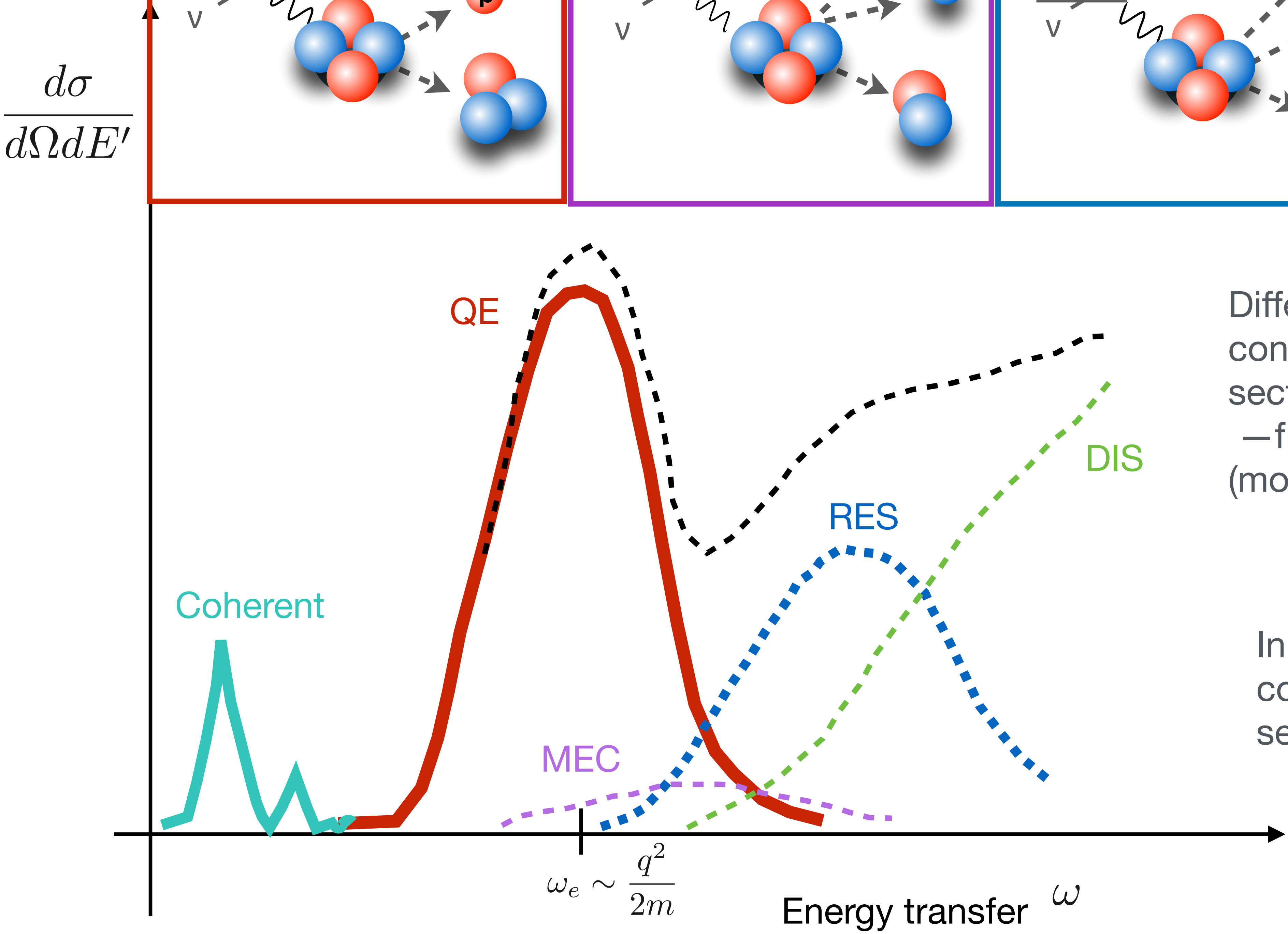


Halzen & Martin

Noemi Rocco, nrocco@fnal.gov

Lepton-nucleus cross section



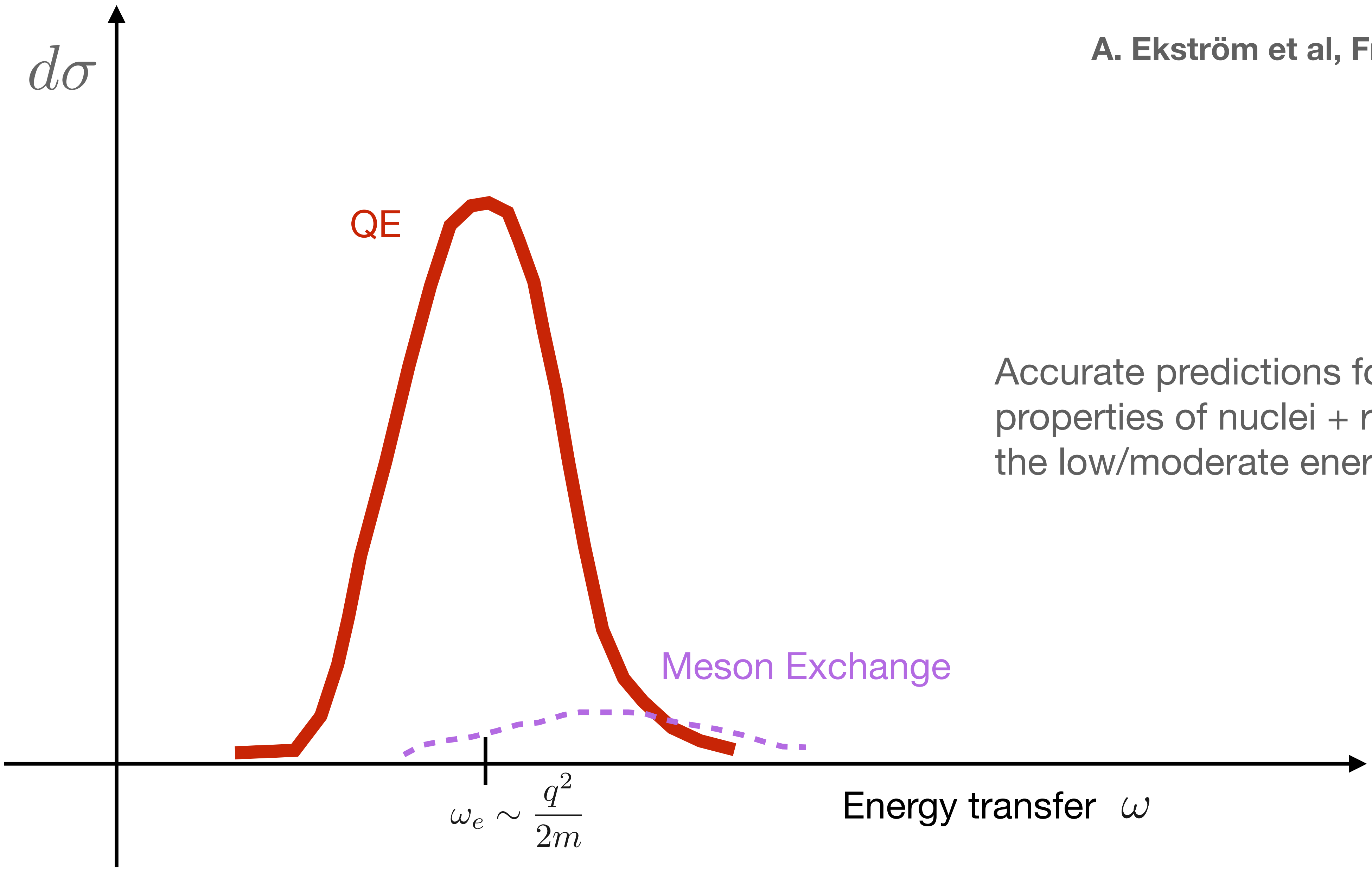


Different reaction mechanisms contributing to lepton-nucleus cross section
—fixed value of the beam energy (monochromatic)

In neutrino experiments these contributions are not nicely separated

Noemi Rocco, nrocco@fnal.gov
Fermilab

Ab initio Methods



Ab-initio methods (CC, IMSRG, SCGF, QMC, etc) are systematically improvable many-body approaches.

A. Ekström et al, Front. Phys.11 (2023) 29094

Accurate predictions for ground state properties of nuclei + response functions in the low/moderate energy region

Noemi Rocco, nrocco@fnal.gov

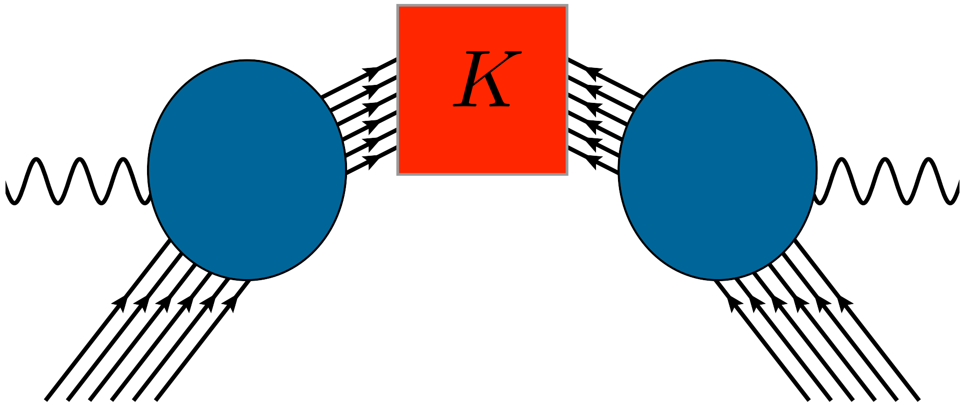
Integral transform techniques

Nuclear response function involves evaluating a number of transition amplitudes.

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J^\dagger_\alpha(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

Valuable information can be obtained from the integral transform of the response function defined as

$$E_{\alpha\beta}(\sigma, \mathbf{q}) = \int d\omega K(\sigma,\omega)R_{\alpha\beta}(\omega, \mathbf{q}) = \langle\psi_0|J^\dagger_\alpha(\mathbf{q})K(\sigma, H-E_0)J_\beta(\mathbf{q})|\psi_0\rangle$$



Using the completeness relation for the final states, we are left with a ground-state expectation value

Noemi Rocco, nrocco@fnal.gov

Green's Function Monte Carlo

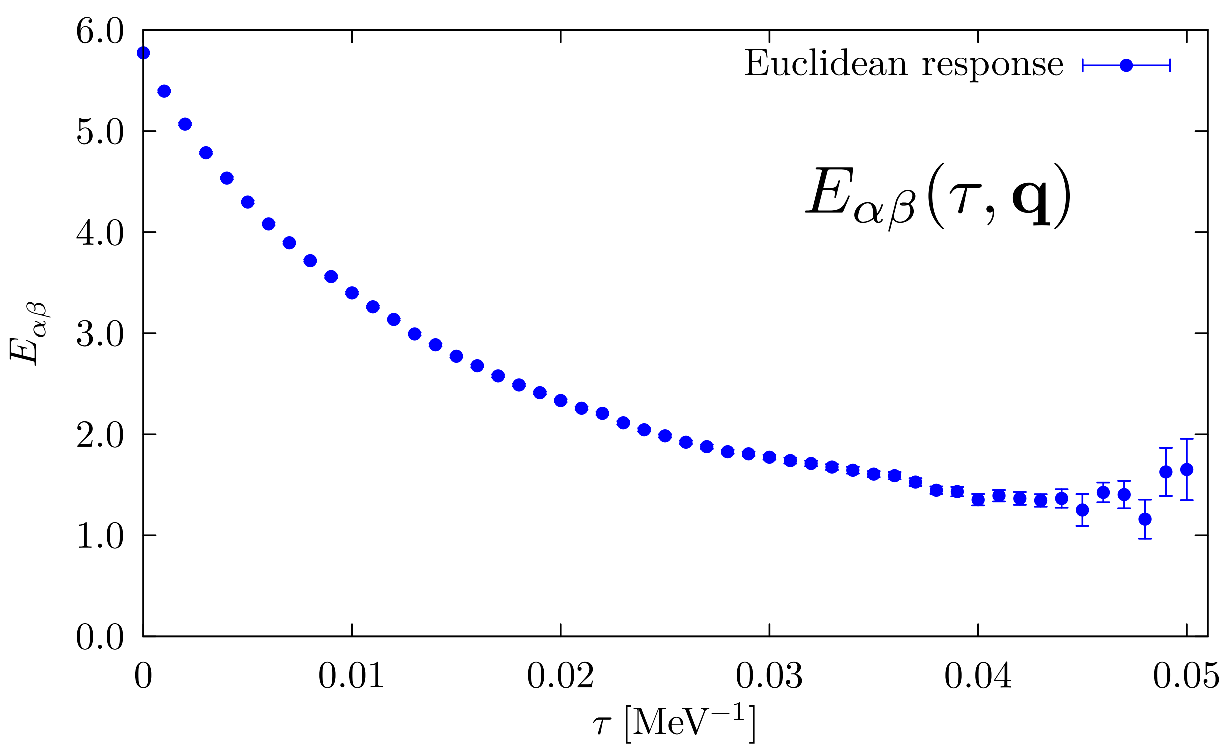
NR, **Frontiers in Phys. 8 (2020) 116**

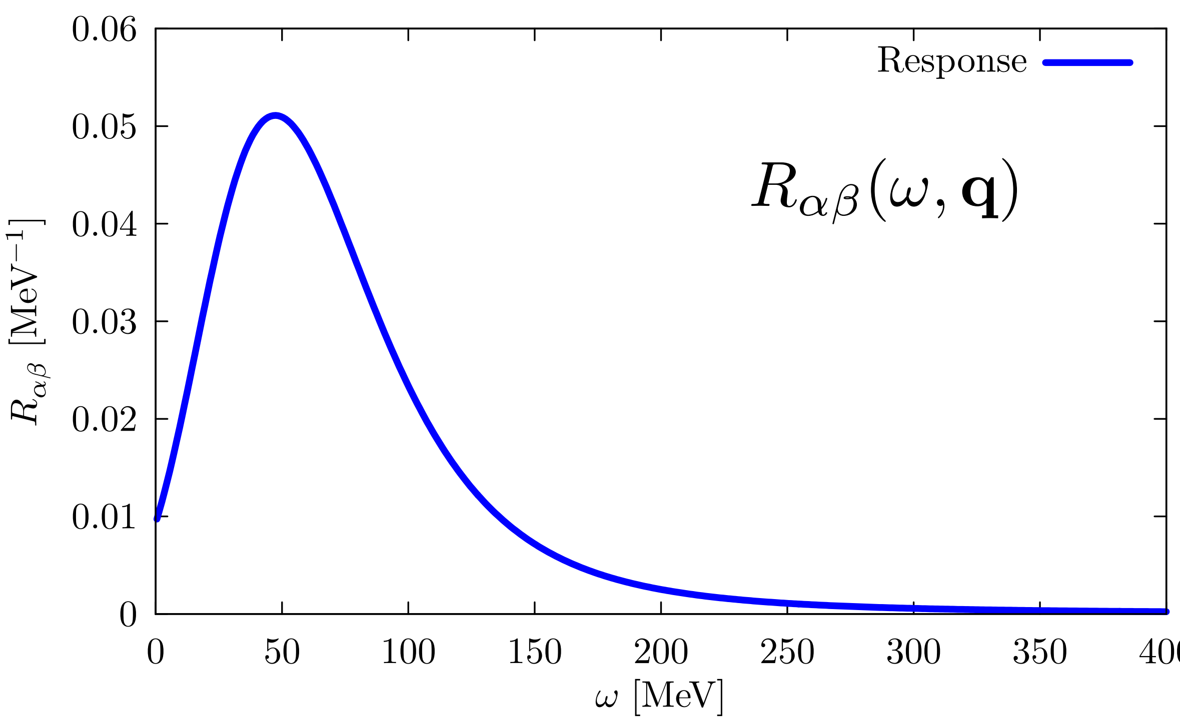
- The Laplace integral transform: $K(\sigma, \omega) = e^{-\omega\sigma}$

- Euclidean Response Function: $E_{\alpha\beta}(\sigma, \mathbf{q}) = \int d\omega e^{-\omega\sigma} R(\omega, \mathbf{q})$

Inverting the integral transform is a complicated problem
Bayesian techniques, in particular Maximum Entropy is used

A. Lovato et al, PRL117 (2016), 082501, PRC97 (2018), 022502

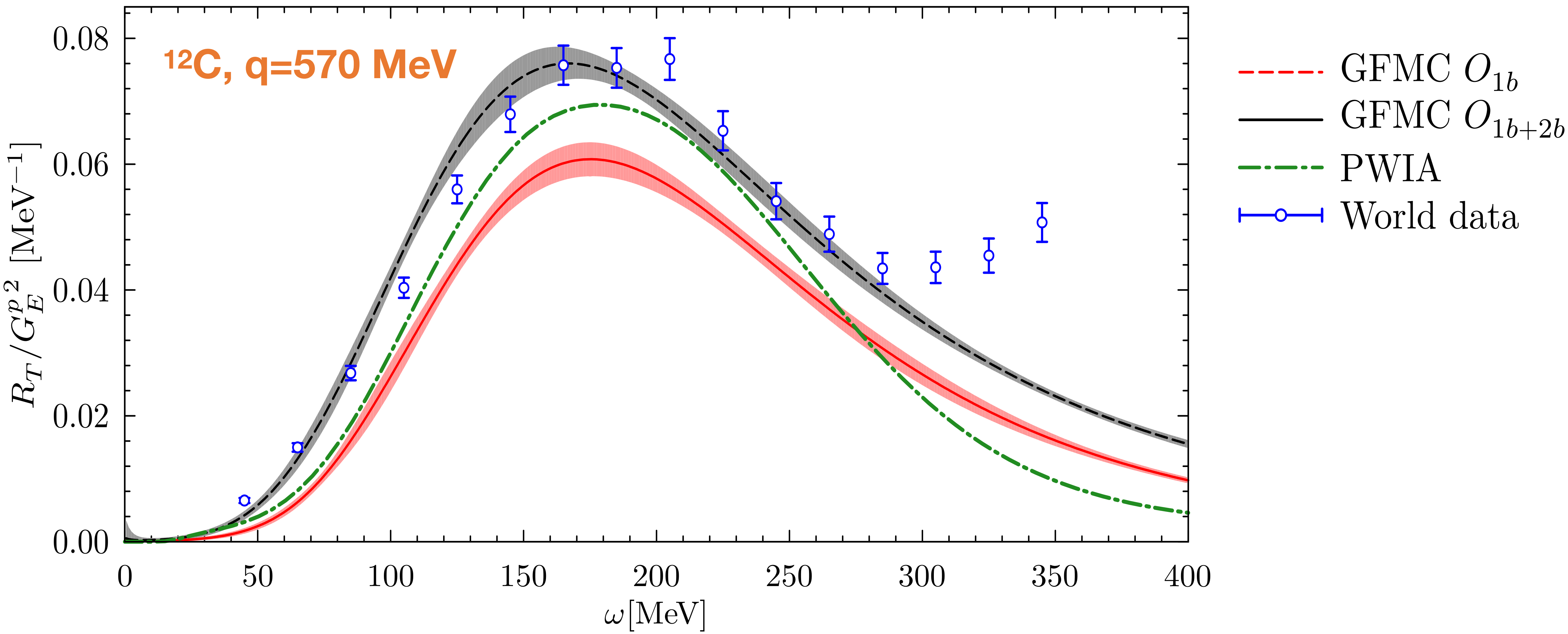




Noemi Rocco, nrocco@fnal.gov

Green's Function Monte Carlo

NR, Frontiers in Phys. 8 (2020) 116



Alessandro Lovato et al. PRL 117 082501 (2016)

Limitations:

Medium mass nuclei A < 13

Inclusive results which are virtually correct in the QE

Relies on non-relativistic treatment of the kinematics

Can not handle explicit pion degrees of freedom

Why relativity is important

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J_\alpha^\dagger(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle \delta(\omega - E_f + E_0)$$

Kinematics

Currents

Covariant expression of the e.m. current:

$$j^\mu_{\gamma,S} = \bar{u}(\mathbf{p}')\Big[\frac{G_E^S + \tau G_M^S}{2(1+\tau)}\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{4m_N}\frac{G_M^S - G_E^S}{1+\tau}\Big]u(\mathbf{p})$$

Nonrelativistic expansion in powers of $\boldsymbol{p}/m_N$

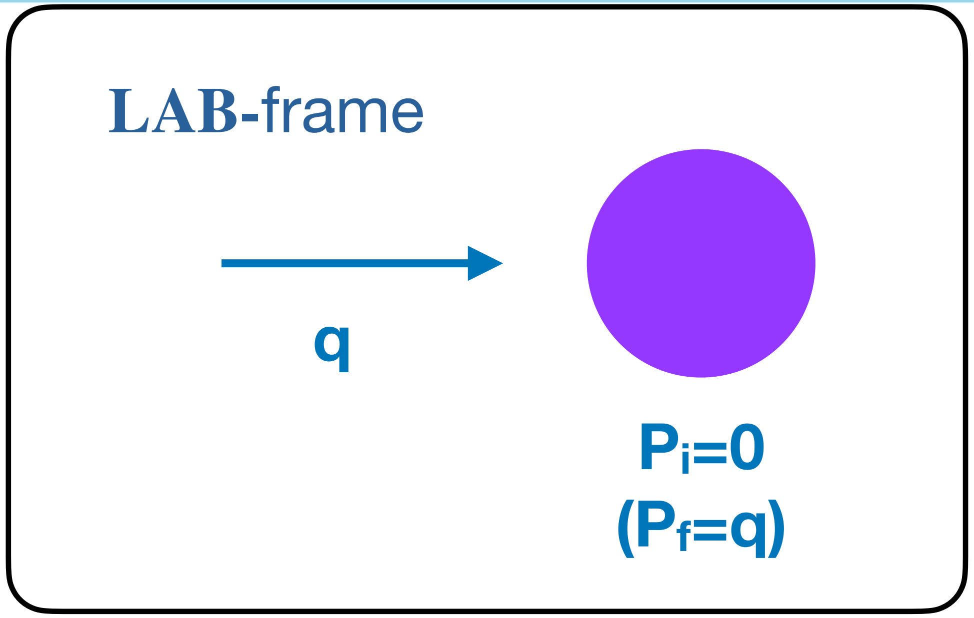
$$j^0_{\gamma,S} = \frac{G_E^S}{2\sqrt{1+Q^2/4m_N^2}} - i\frac{2G_M^S - G_E^S}{8m_N^2}\mathbf{q}\cdot(\boldsymbol{\sigma}\times\mathbf{p})$$

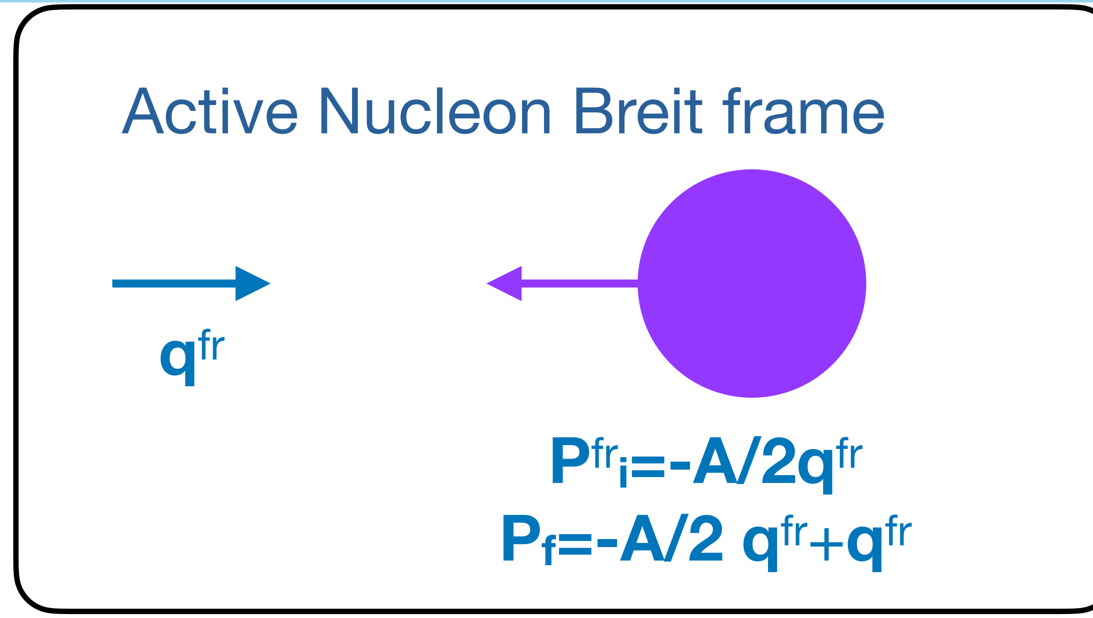
Energy transfer at the quasi-elastic peak:

$$w_{QE} = \sqrt{\mathbf{q}^2 + m_N^2} - m_N \qquad w_{QE}^{nr} = \mathbf{q}^2/(2m_N)$$

Noemi Rocco, nrocco@fnal.gov

Frame dependence





- In a generic reference frame the longitudinal non relativistic response reads

$$R_L^{fr} = \sum_f \Big| \langle \psi_i | \sum_j \rho_j(\mathbf{q}^{fr}, \omega^{fr}) | \psi_f \rangle \Big|^2 \delta(E_f^{fr} - E_i^{fr} - \omega^{fr})$$

$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) \approx \delta[e_f^{fr} + (P_f^{fr})^2/(2M_T) - e_i^{fr} - (P_i^{fr})^2/(2M_T) - \omega^{fr}]$$

- The response in the LAB frame is given by the Lorentz transformation

$$R_L(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{(\mathbf{q}^{fr})^2} \frac{E_i^{fr}}{M_0} R_L^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr})$$

where

$$q^{fr} = \gamma(q - \beta\omega), \ \omega^{fr} = \gamma(\omega - \beta q), \ P_i^{fr} = -\beta\gamma M_0, \ E_i^{fr} = \gamma M_0$$

Fermilab

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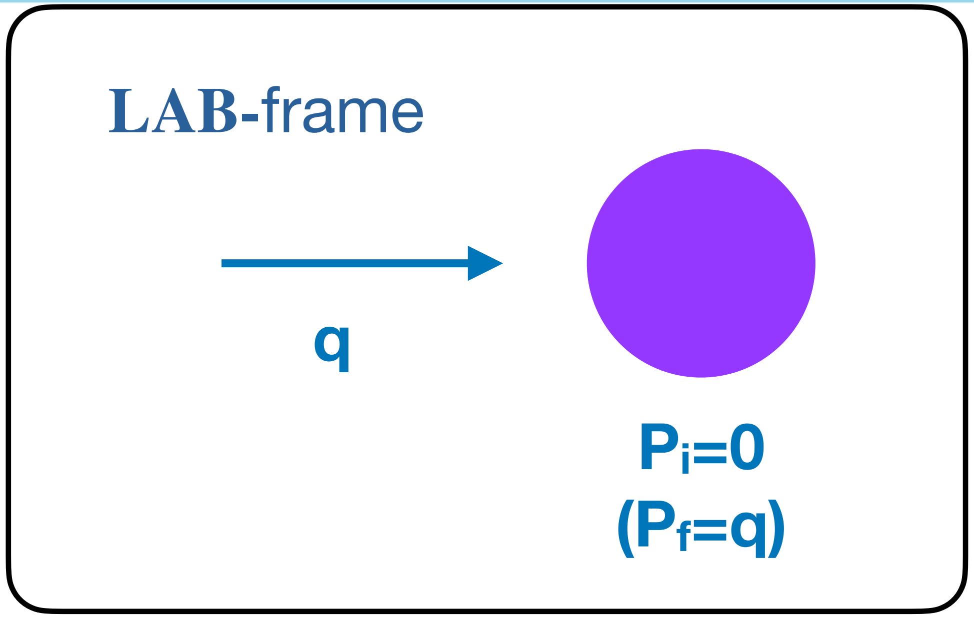
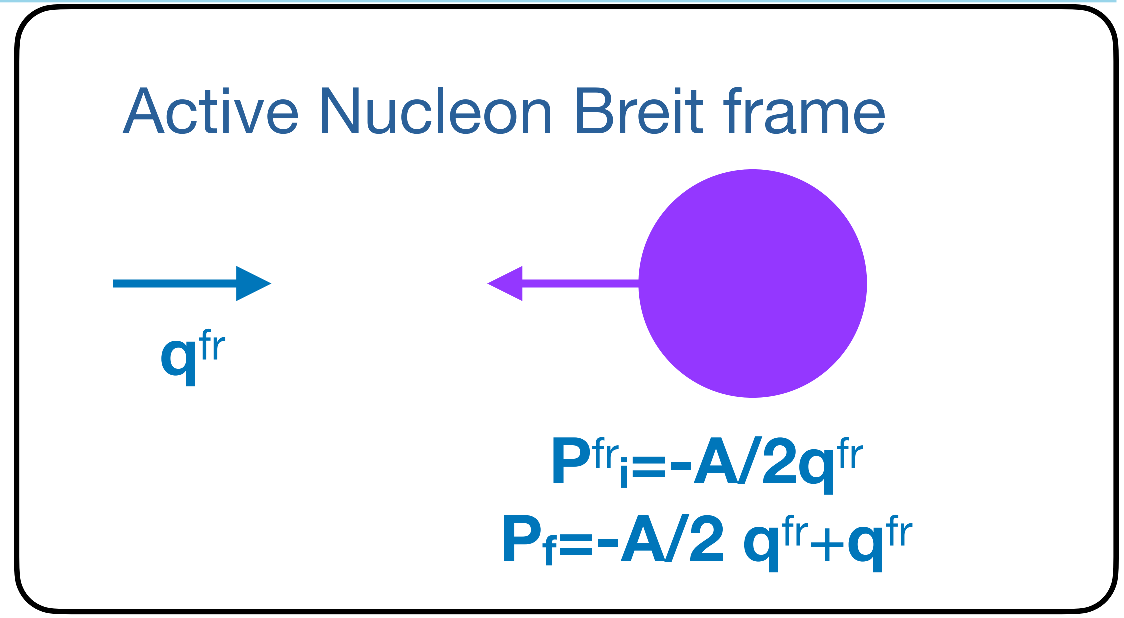
Frame dependence





- ANB @ the single nucleon level:

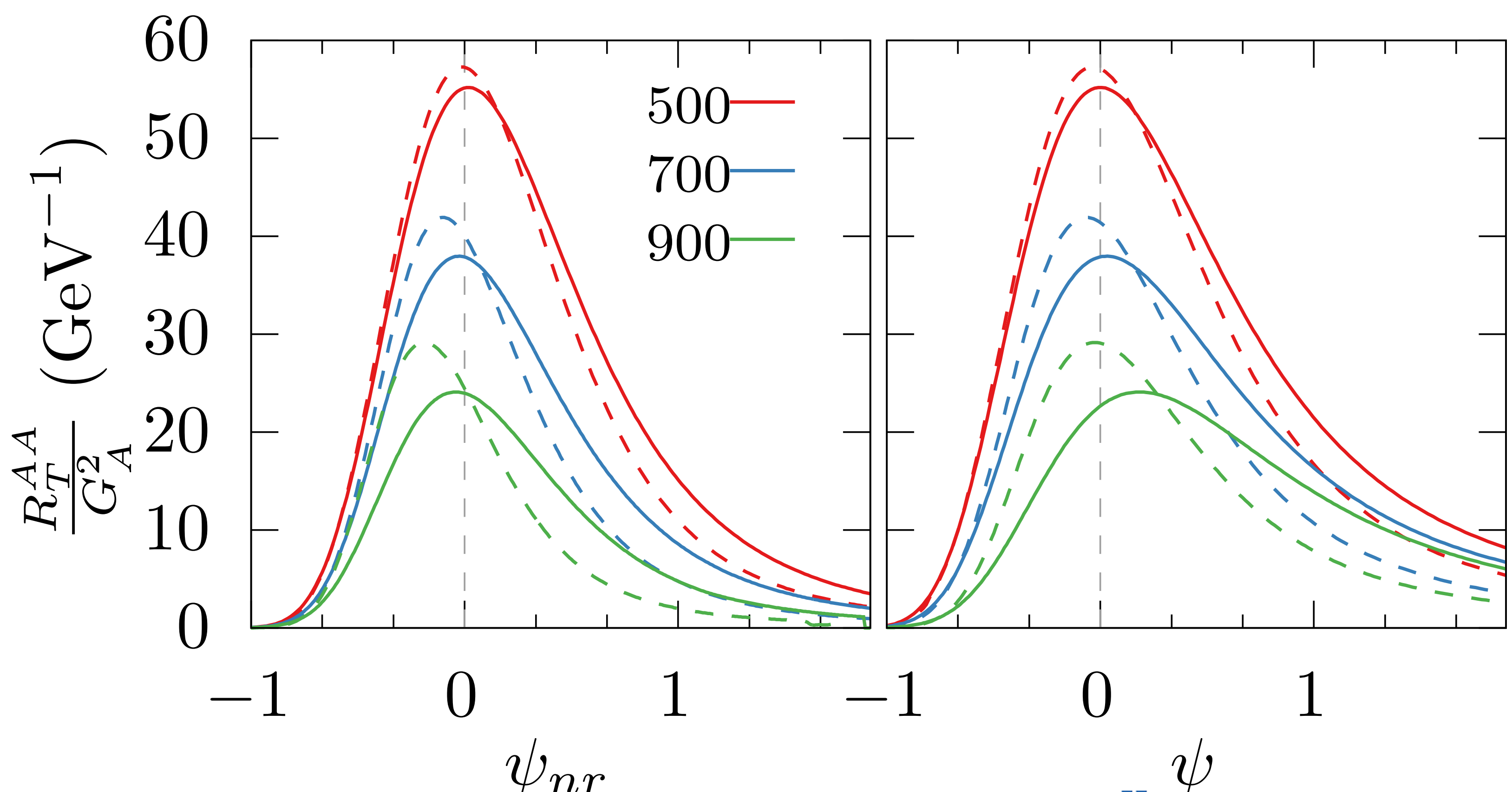
  $p^{fr}_i \simeq -q^{fr}/2$

  $p^{fr}_f \simeq q^{fr}/2$

- Same position of the quasielastic peak

$$\omega_{QE} = \omega_{QE}^{nr} = 0$$

- LAB (solid) and ANB (dashed) predictions



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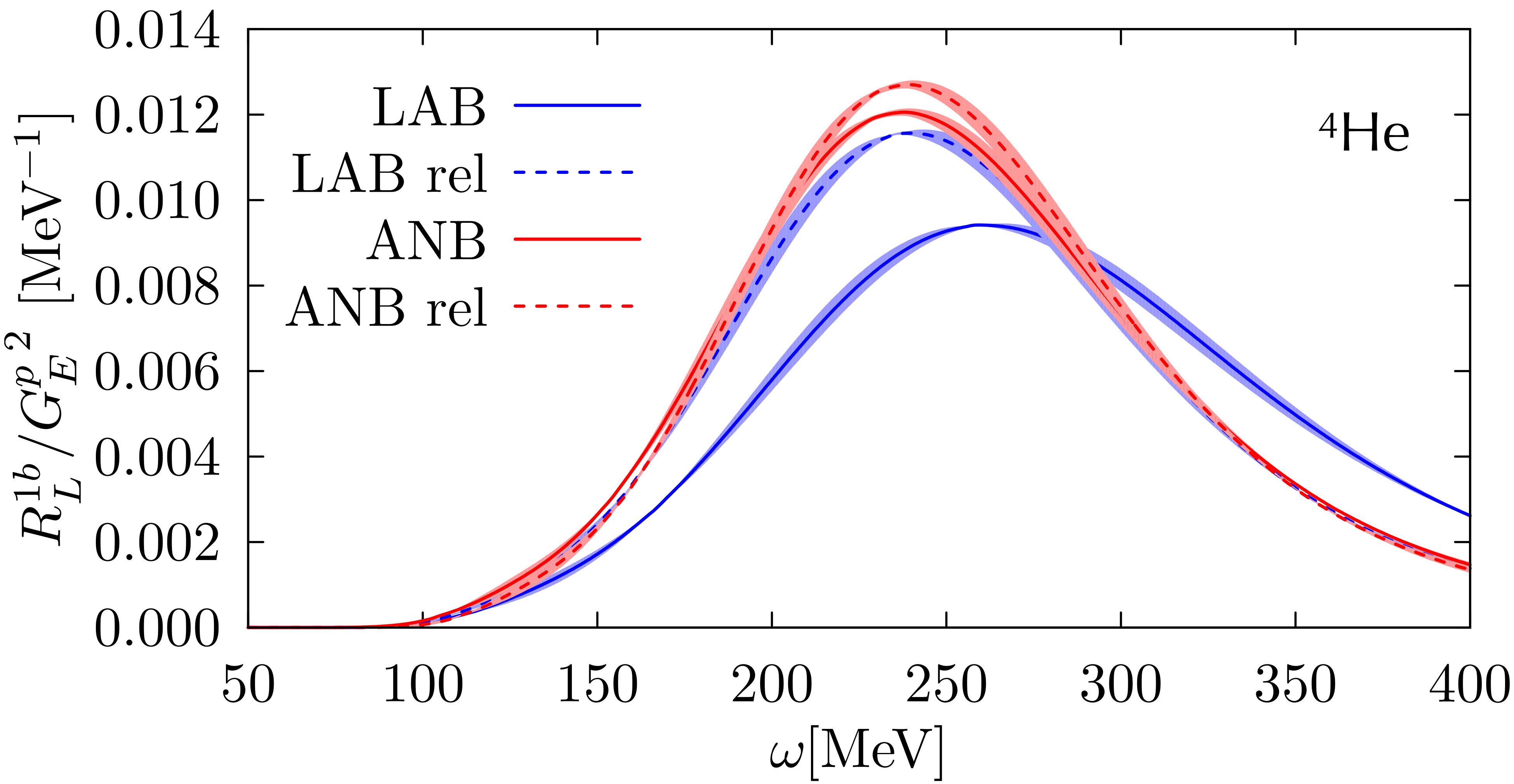
Relativistic effects in a correlated system

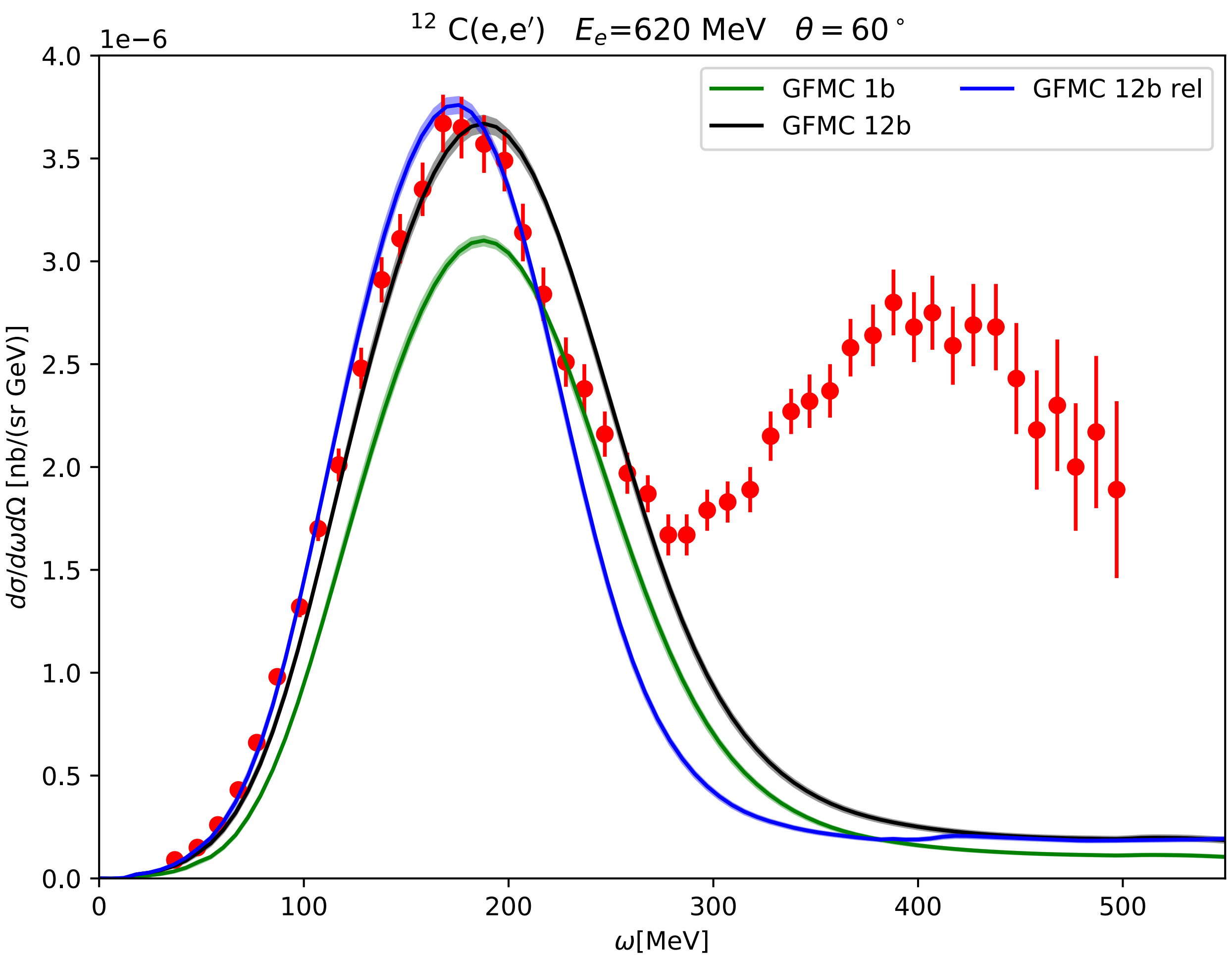


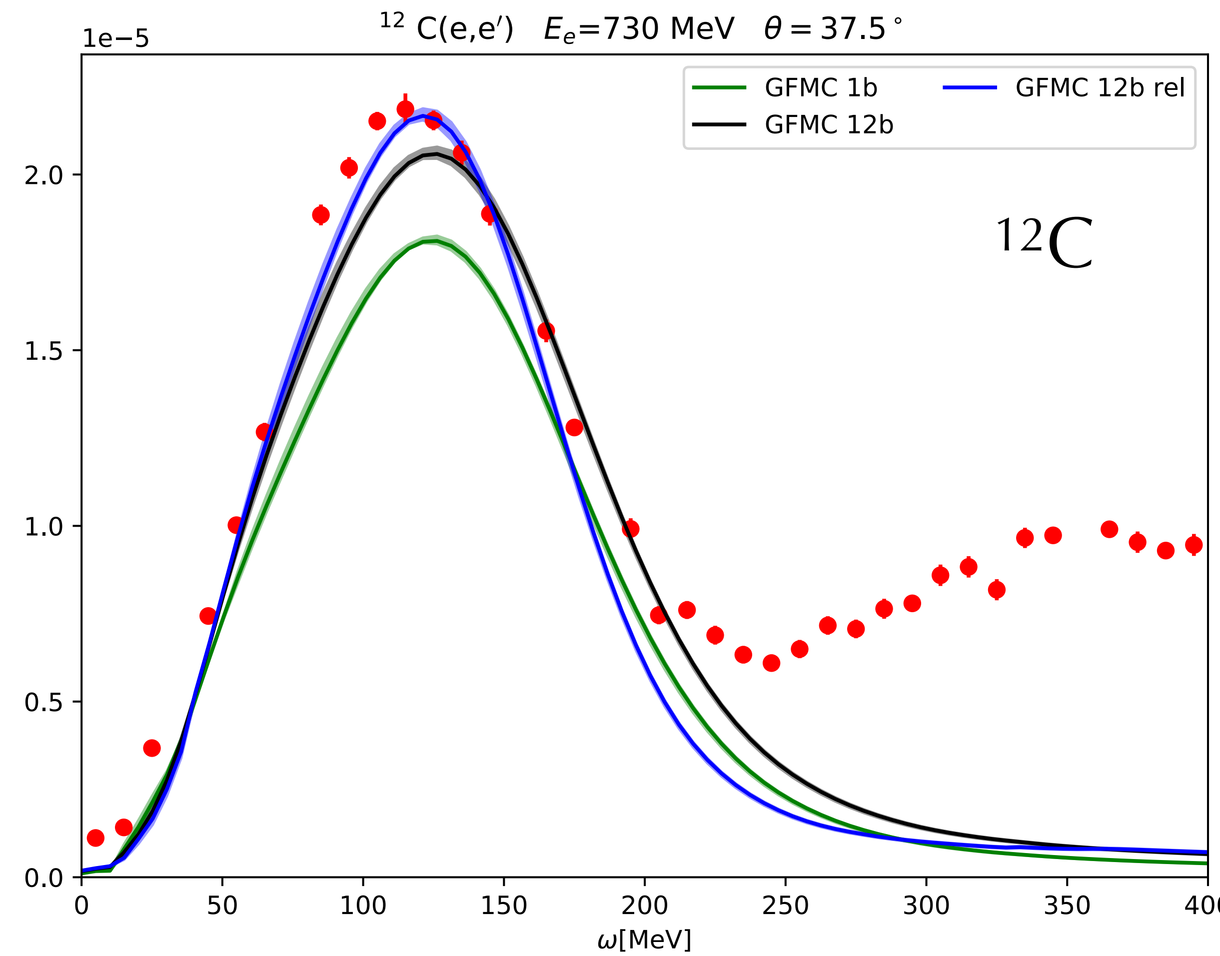
- Relativistic effects are much smaller in the ANB frame where the final nucleon momentum is ∝ q/2, the position of the peak remains almost unchanged

Noemi Rocco, nrocco@fnal.gov

Cross sections: Green's Function Monte Carlo

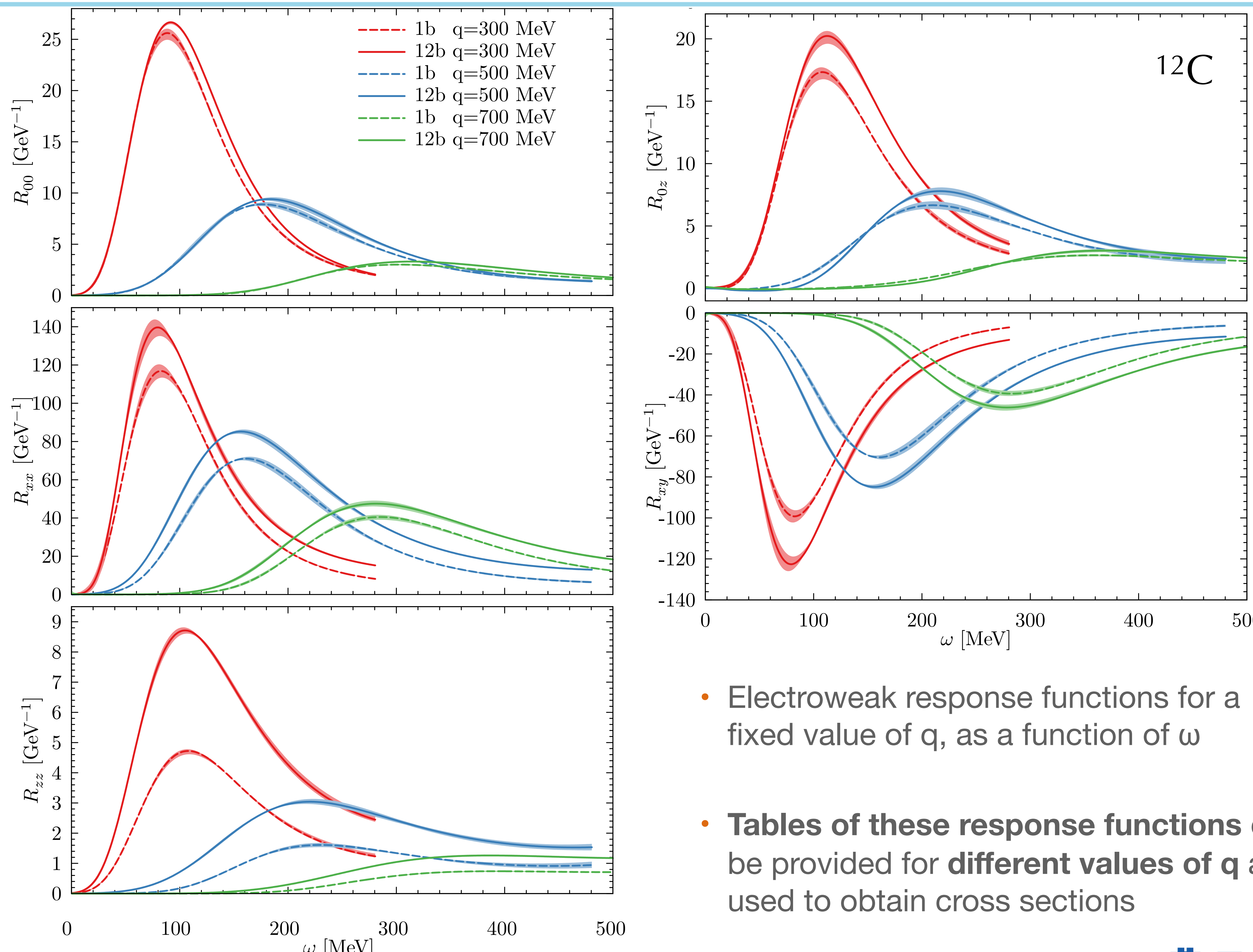
Electron scattering results including relativistic corrections for some kinematics covered by the calculated responses





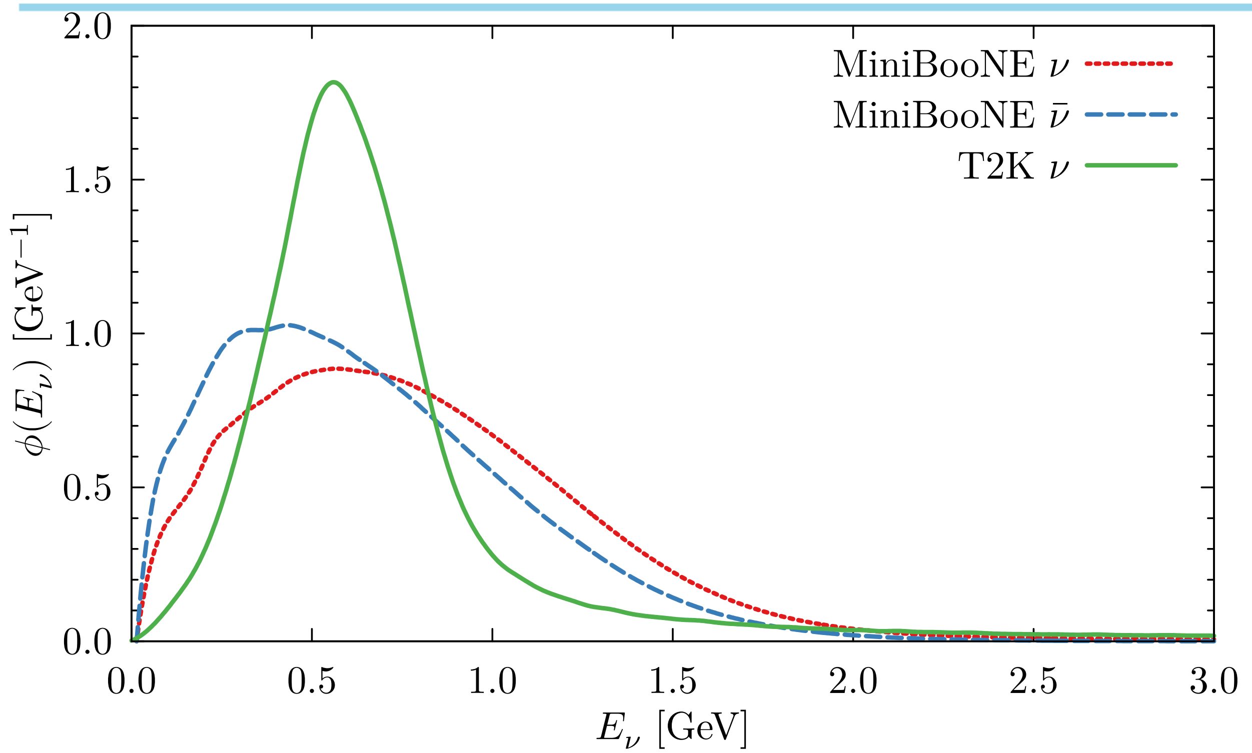
A.Lovato, **A.Nikolakopoulos**, NR, **N. Steinberg**, Universe 9 (2023) 8, 36

Cross sections: Green's Function Monte Carlo



- Electroweak response functions for a fixed value of q, as a function of ω
- **Tables of these response functions** can be provided for **different values of q** and used to obtain cross sections

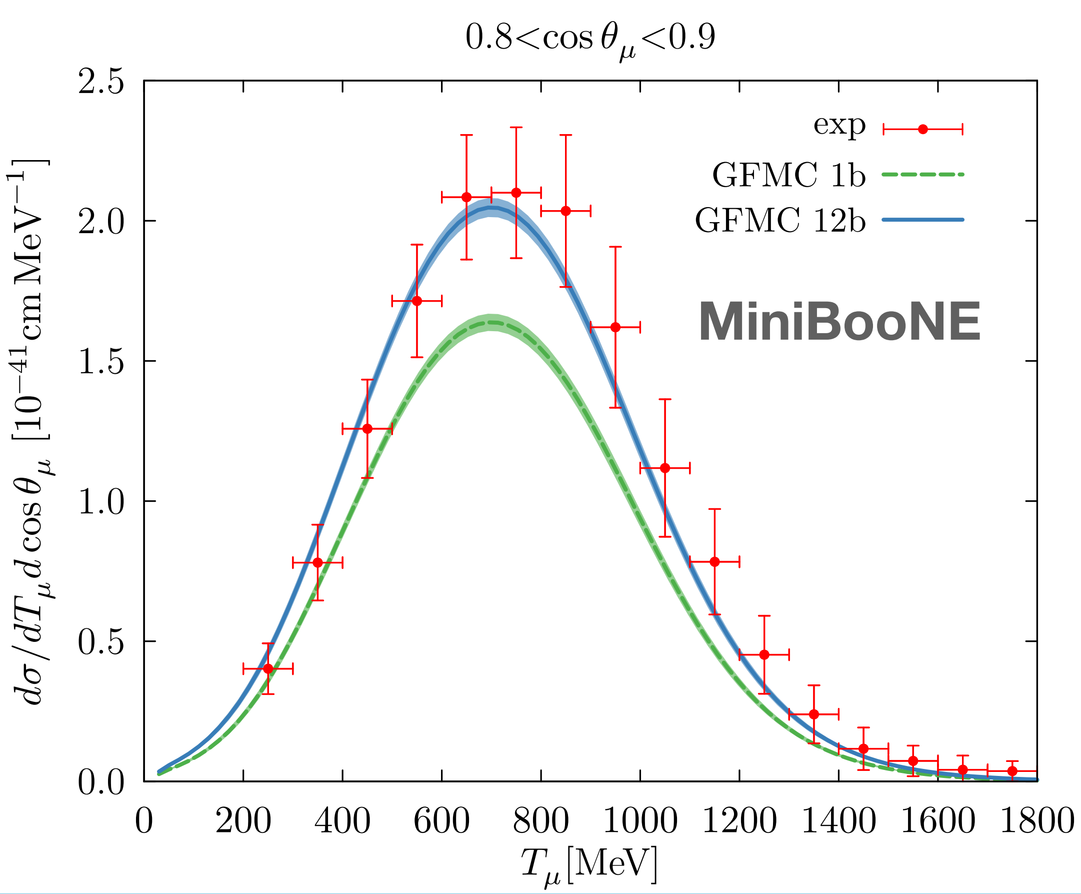
Cross sections: Green's Function Monte Carlo

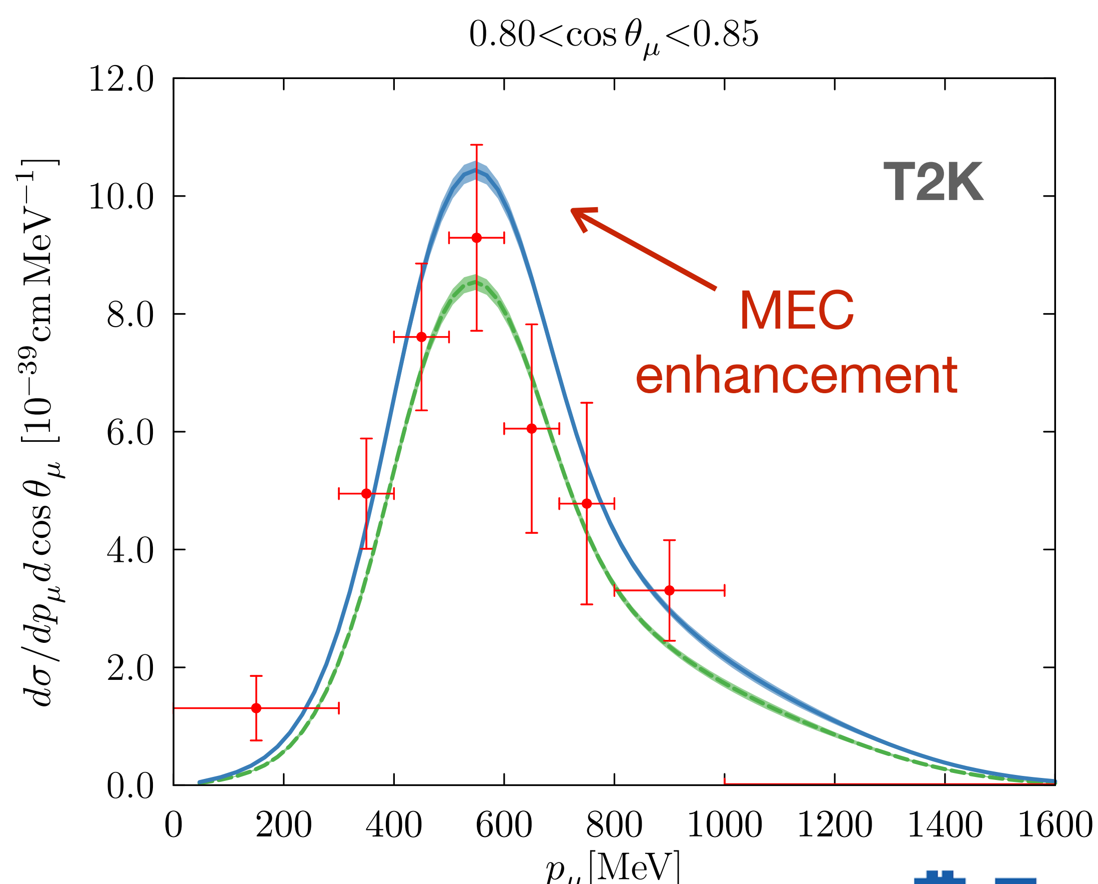


First microscopic calculation of neutrino-nucleus cross section

$$\left\langle \frac{d\sigma}{dT_\mu \, d\cos\theta_\mu} \right\rangle = \int dE_\nu \, \phi(E_\nu) \, \frac{d\sigma(E_\nu)}{dT_\mu \, d\cos\theta_\mu}$$

A. Lovato, NR et al , Phys. Rev. X. 10 (2020) 3, 031068





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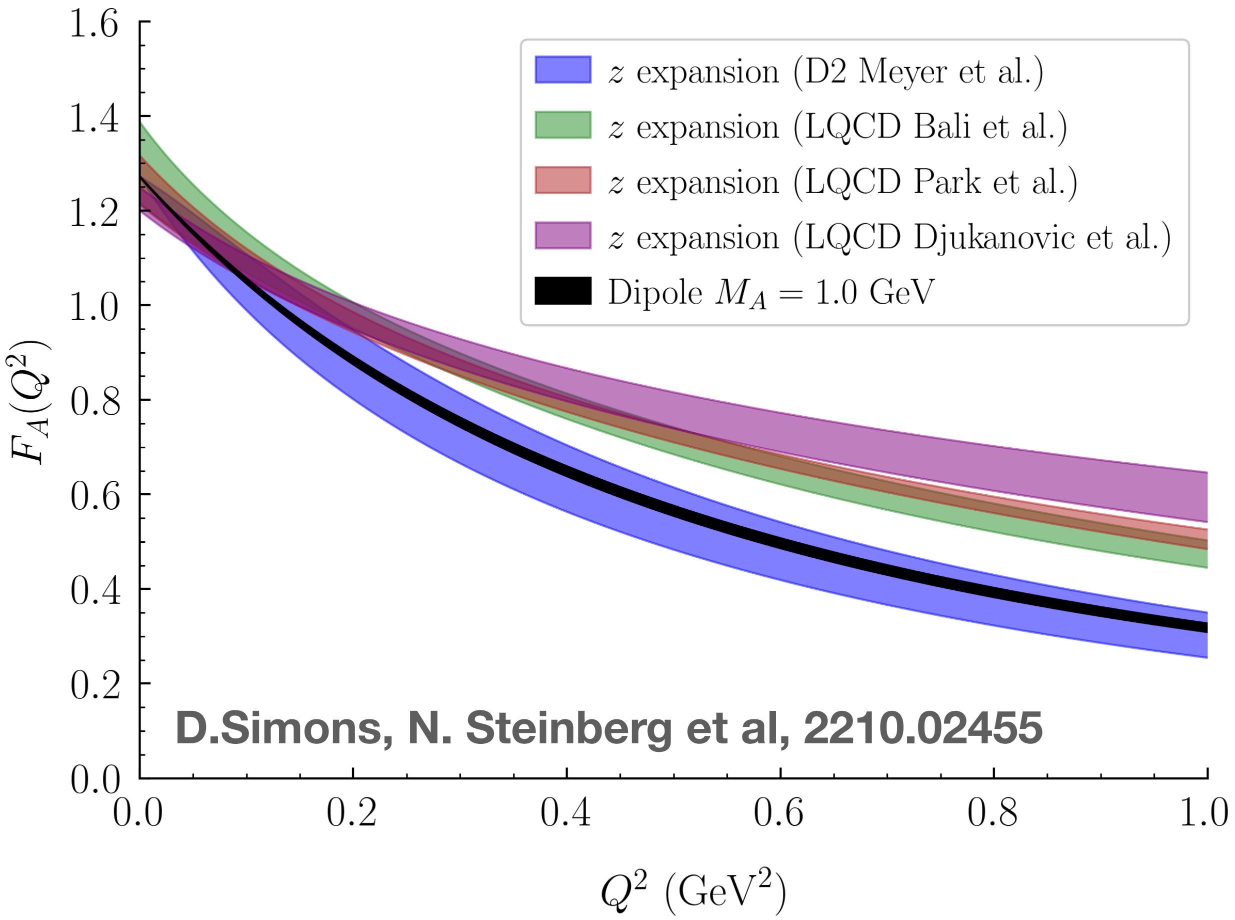
Axial form factor determination



A. Meyer, A. Walker-Loud, C. Wilkinson, 2201.01839

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations

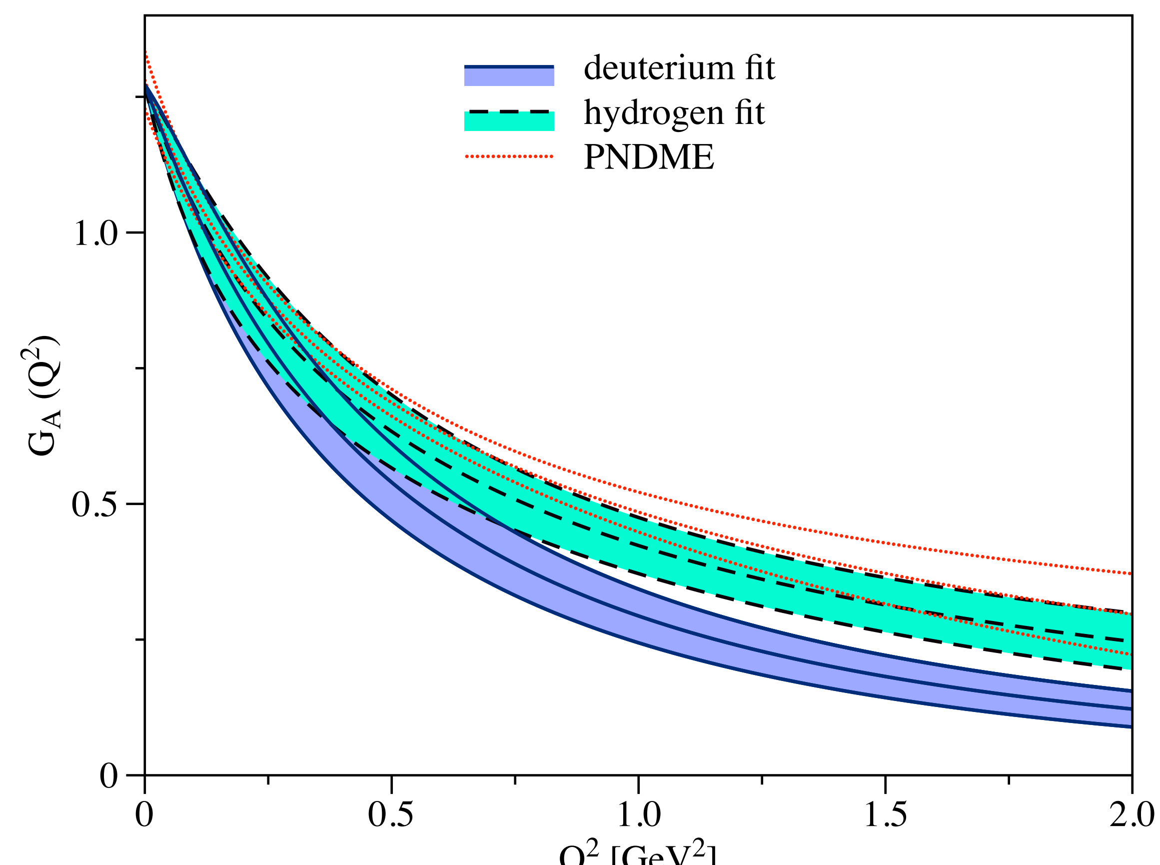
LQCD results are 2-3σ larger than D2 Meyer ones for $Q^2 > 0.3$ GeV^2

O. Tomalak, R. Gupta, T. Battacharaya, 2307.14920

Comparison with recent MINERvA antineutrino-hydrogen charged-current measurements

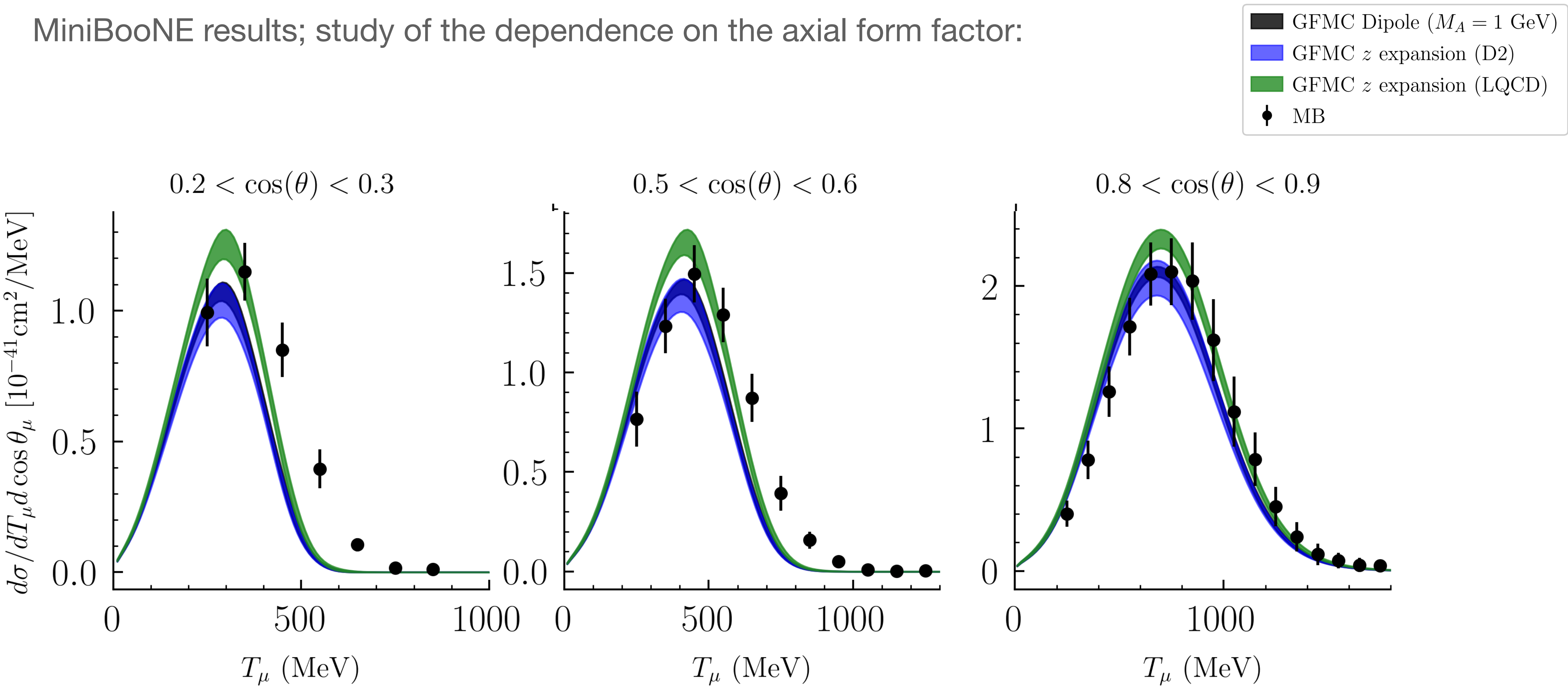
1-2σ agreement with MINERvA data and LQCD prediction by PNDME Collaboration

Novel methods are needed to remove excited-state contributions and discretization errors



Study of model dependence in neutrino predictions

MiniBooNE results; study of the dependence on the axial form factor:



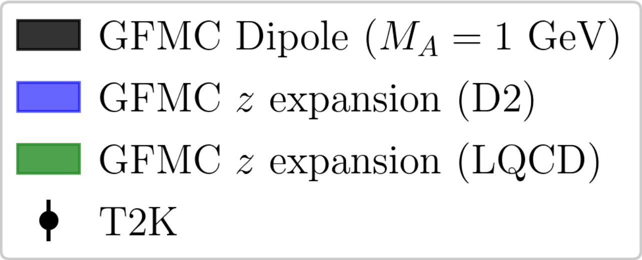
D.Simons, N. Steinberg et al, 2210.02455

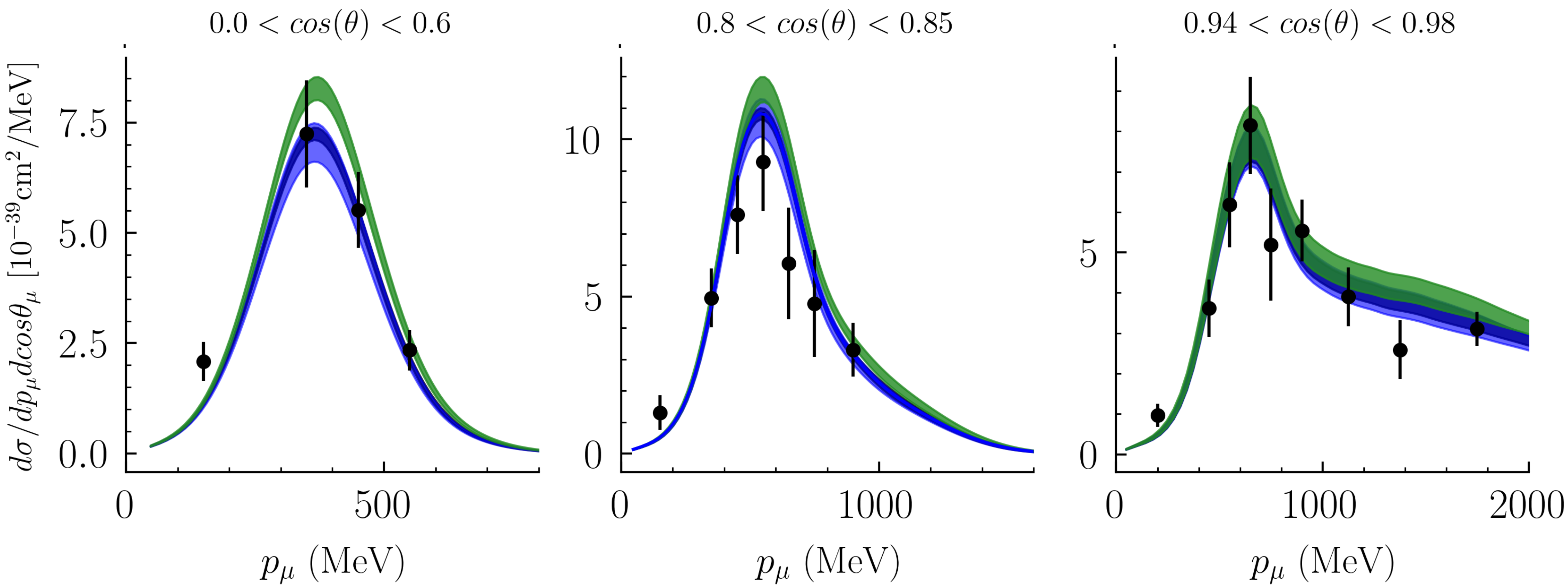
MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
GFMC Difference in $d\sigma_{\text{peak}}$ (%)	18.6	17.1	12.2

Noemi Rocco, nrocco@fnal.gov

Study of model dependence in neutrino predictions

T2K results; study of the dependence on the axial form factor:





D.Simons, N. Steinberg et al, 2210.02455

T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
GFMC difference in $d\sigma_{\text{peak}}$ (%)	15.8	8.0	4.6

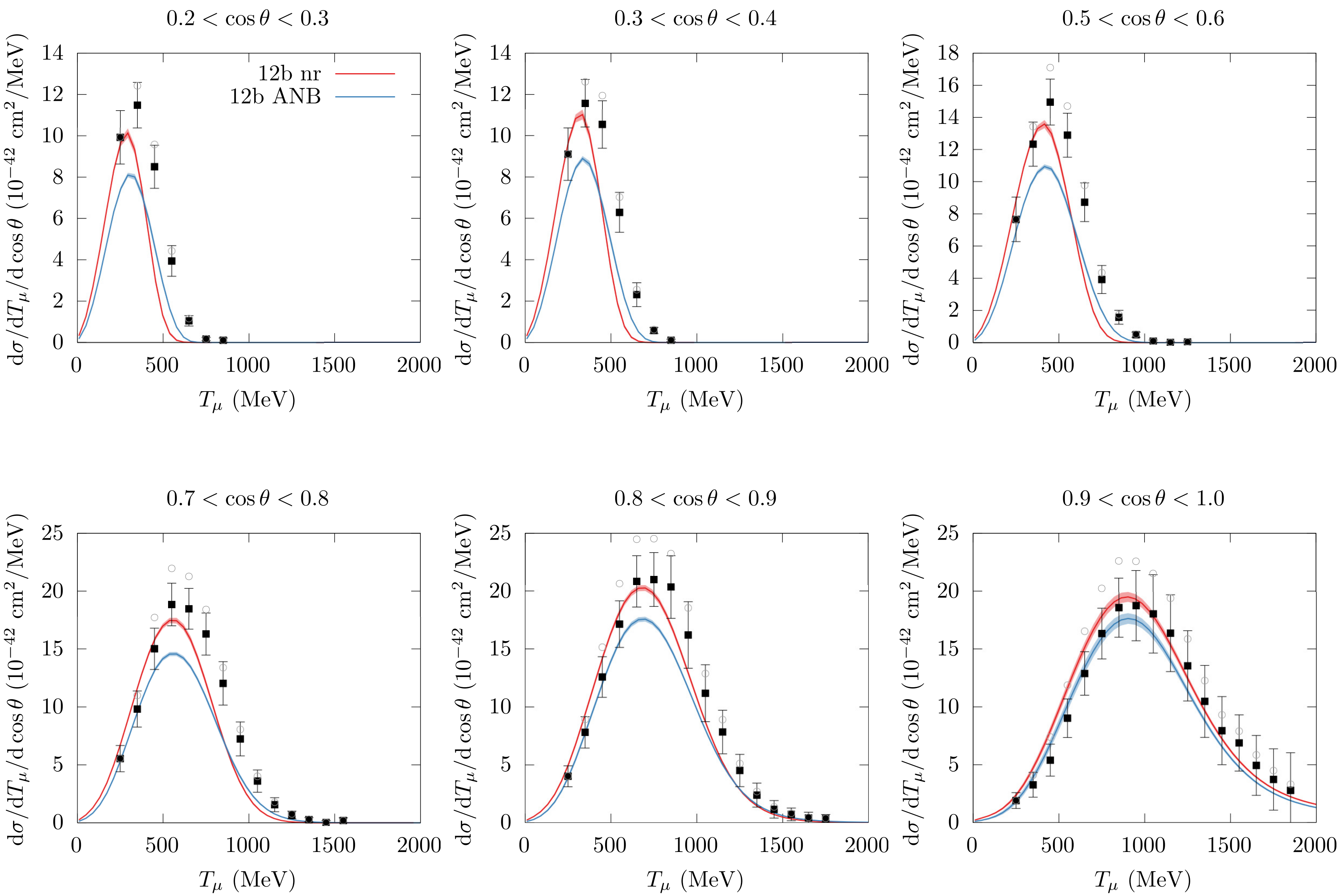
Noemi Rocco, nrocco@fnal.gov

Fermilab

Cross sections: Green's Function Monte Carlo

MiniBooNE results including relativistic corrections

A.Nikolakopoulos, A.Lovato, NR, Universe 9 (2023) 8, 367



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Coupled Cluster Method

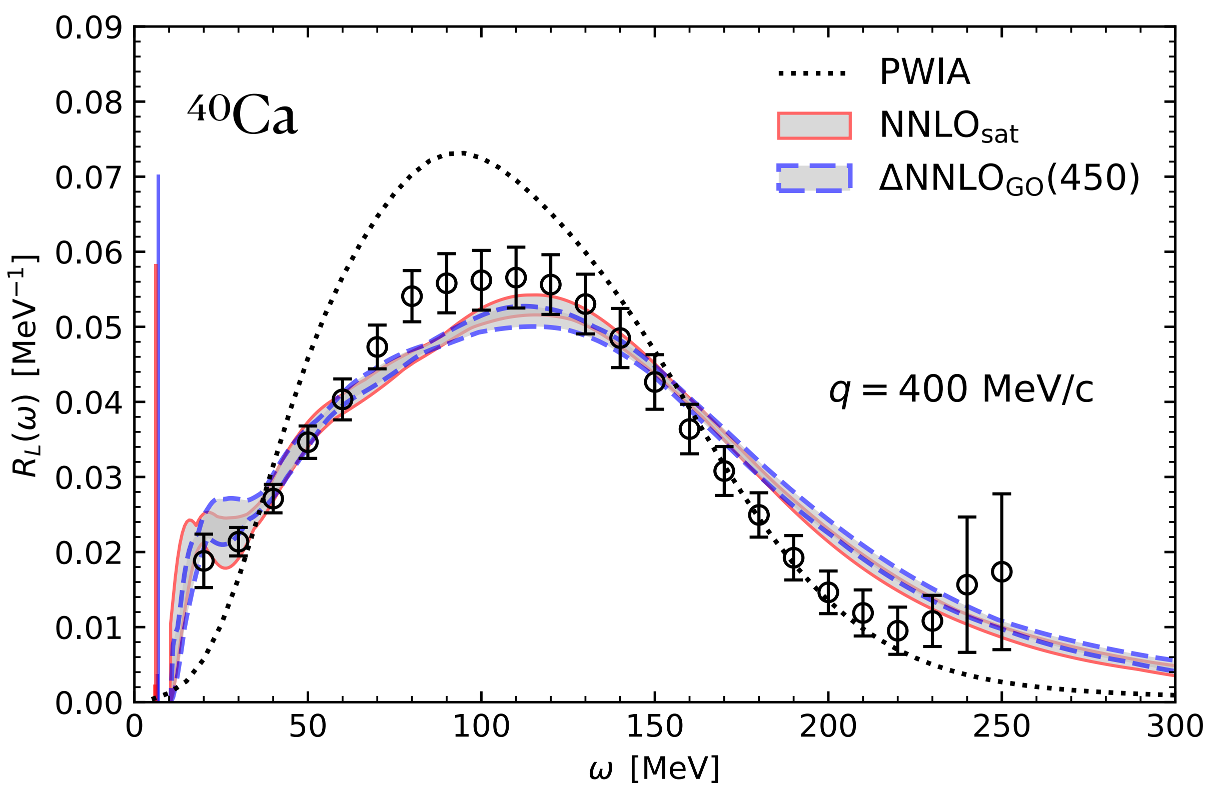
Reference state Hartree Fock: $|\Psi\rangle$

Include correlations through e^T operator

Similarity transformed Hamiltonian $e^{-T}He^{T}|\Psi\rangle = \bar{H}|\Psi\rangle = E|\Psi\rangle$

Expansion in second quantization single + doubles:

$$T = \sum t_a^i a_a^\dagger a_i + \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \dots$$



Polynomial scaling with the number of nucleons (predictions for ^{132}Sn and ^{208}Pb)

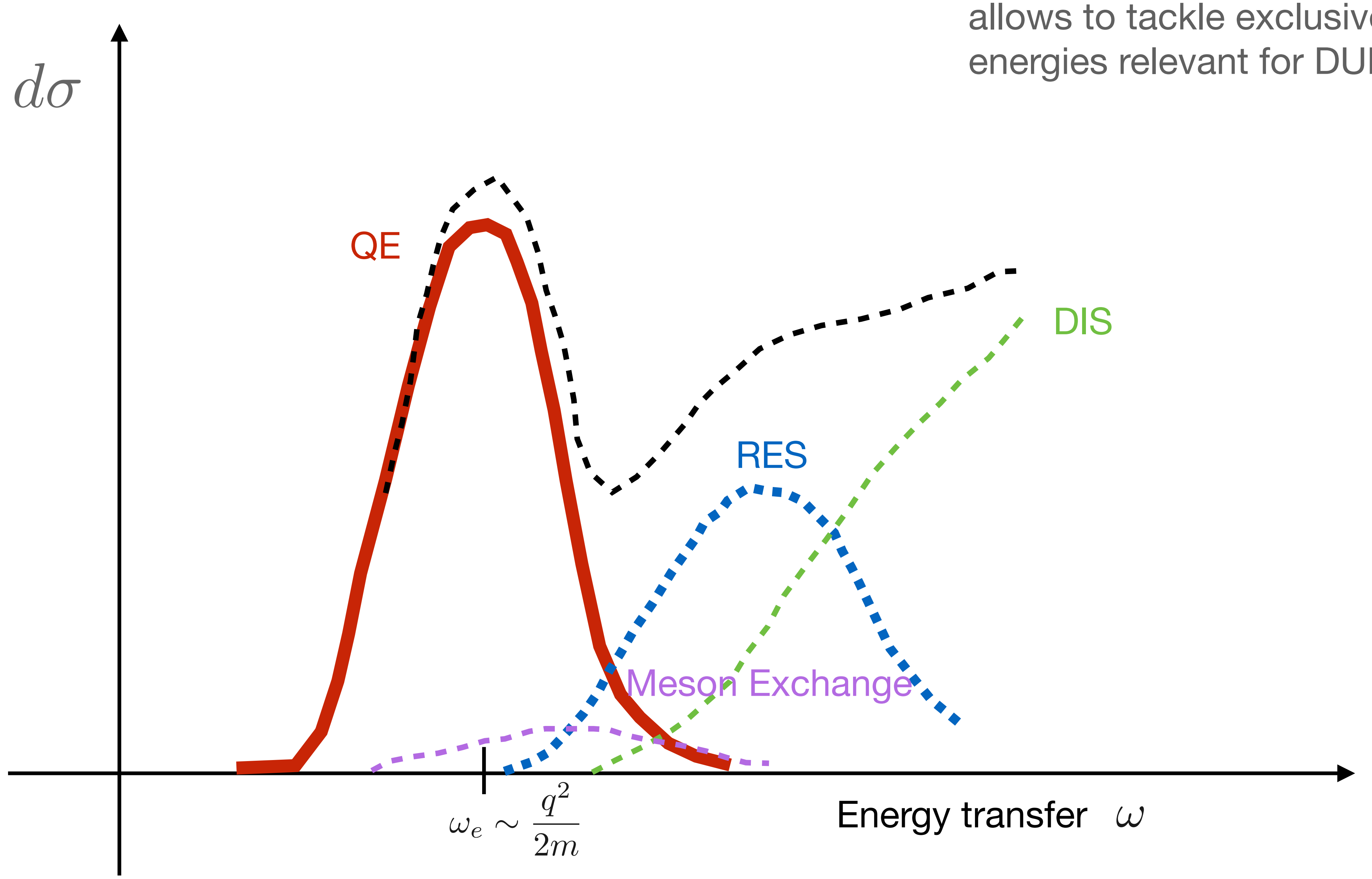
Electroweak response functions obtained using **LIT**

$$K_\Gamma(\omega, \sigma) = \frac{1}{\pi}\frac{\Gamma}{\Gamma^2 + (\omega-\sigma)^2}$$

JES, B. Acharya, S. Bacca, G. Hagen; PRL 127 (2021) 7, 072501

Factorization Based Approaches

Factorization of the hadronic final states: allows to tackle exclusive channels + higher energies relevant for DUNE

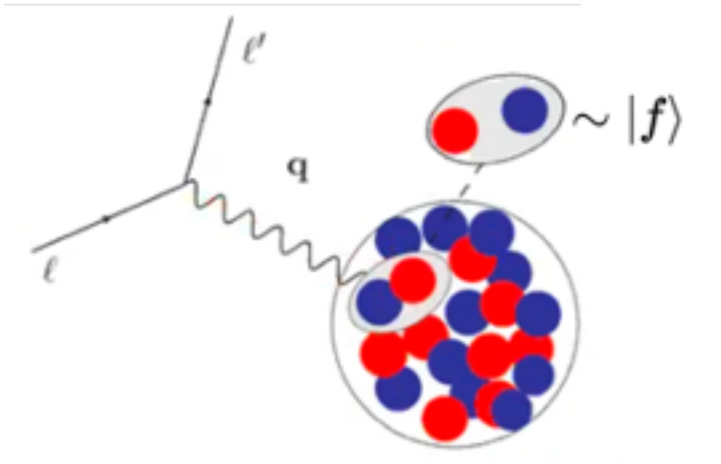


Noemi Rocco, nrocco@fnal.gov

Short-Time Approximation

NR, **Frontiers in Phys. 8 (2020) 116**

- Based on Factorization
- Retains two-body physics
- Response functions are given by the scattering from pairs of fully interacting nucleons that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities



The sum over all final states is replaced by a two nucleon propagator

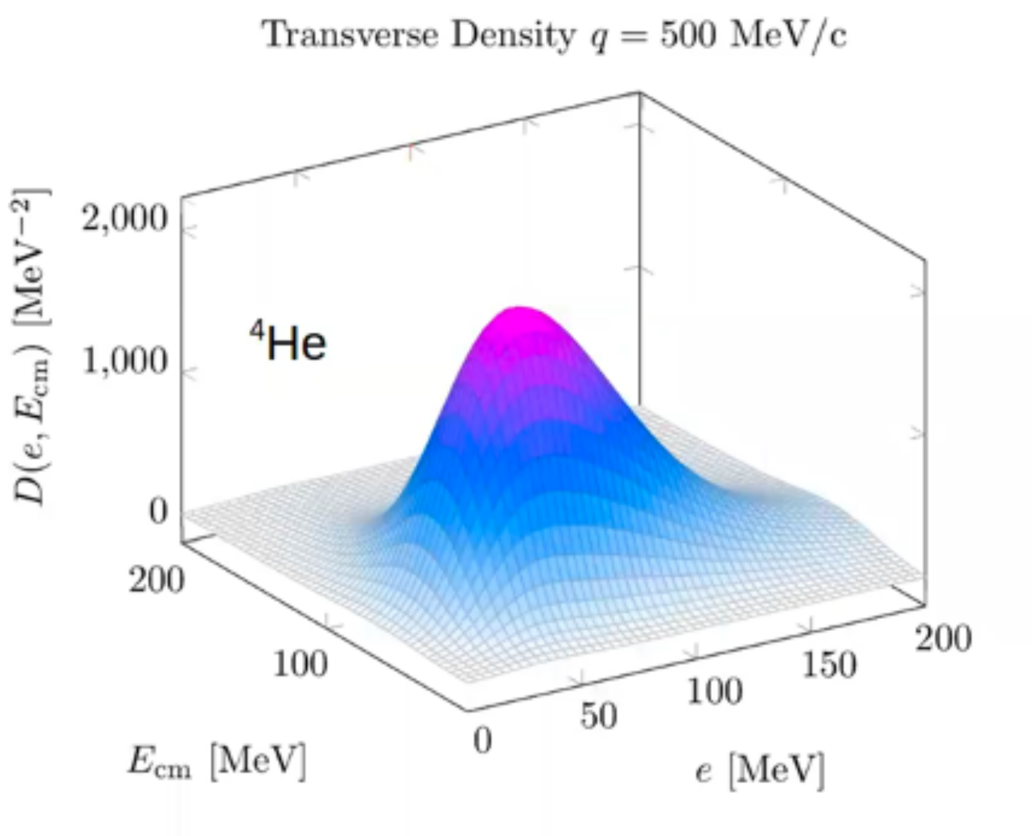
$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

The STA restricts the propagation to two active nucleons and allows to compute density functions of the CoM and relative momentum of the pair

$$R^{\rm STA}(q,\omega) \sim \int \delta(\omega + E_0 - E_f) de\, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

Noemi Rocco, nrocco@fnal.gov

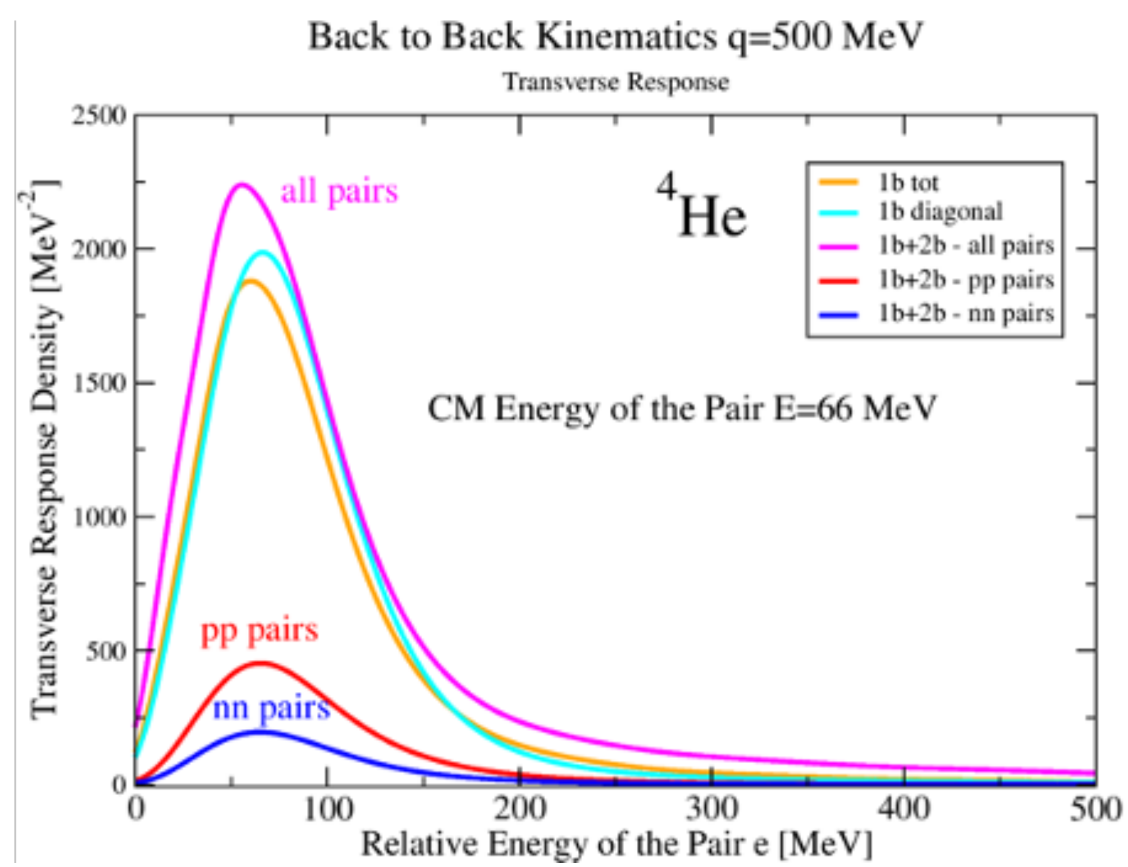
Short-Time Approximation



$$R^{STA}(q,\omega) \sim \int \delta(\omega + E_0 - E_f) de\, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

Electron scattering from 4He:

- Response density as a function of (E,e)
- Give access to particular kinematics for the struck nucleon pair



Pastore et al. PRC101(2020)044612

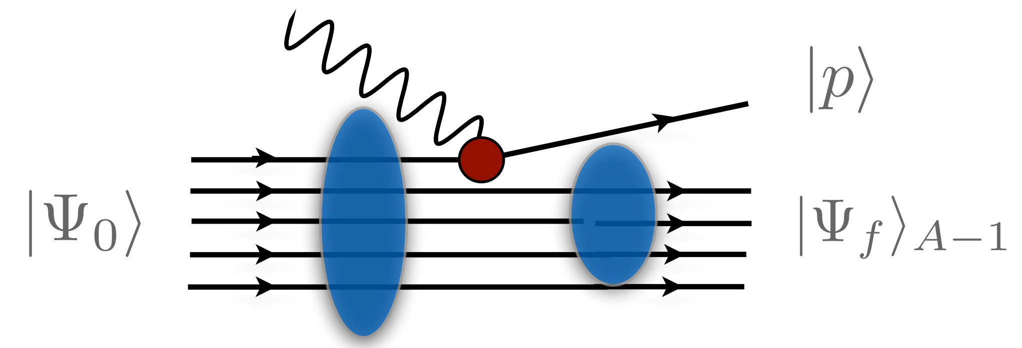
- pp pairs
- nn pairs
- 1 body tot
- all pairs tot

Noemi Rocco, nrocco@fnal.gov
Fermilab

Spectral function approach

The general expression for the hadronic tensor reads

$$R^{\mu\nu}(\mathbf{q},\omega) = \sum_f \langle \Psi_0 | j^{\mu\dagger} | \Psi_f \rangle \langle \Psi_f | j^\nu | \Psi_0 \rangle \delta(E_0 + \omega - E_f)$$



At large momentum transfer, the scattering reduces to the sum of individual terms

$$J_\alpha = \sum_i j^i_\alpha \qquad\qquad |\Psi_f\rangle \to |p\rangle \otimes |\Psi_f\rangle_{A-1}$$

We employ the factorization ansatz and insert a single- nucleon completeness relation

$$\langle \Psi_f | j^\mu | \Psi_0 \rangle \to \sum_k \left[{}_{A-1}\langle \Psi_f | \otimes \langle k | \right] | \Psi_0 \rangle \langle p | \sum_i j^\mu_i | k \rangle$$

The incoherent contribution of the one-body response reads

$$R^{\mu\nu}(\mathbf{q},\omega) = \sum_{p,k,f} \sum_i \langle k | j_i^{\mu\dagger} | p \rangle \langle p | j_i^\nu | k \rangle \Big[\langle \Psi_0 | | \Psi_f \rangle_{A-1} \otimes | k \rangle \Big]^2$$
$$\times \delta(\omega - e(\mathbf{p}) - E_f^{A-1} + E_0^A)$$

Noemi Rocco, nrocco@fnal.gov

Spectral function approach

We can rewrite the delta using the identity:

$$\delta(\omega - e(\mathbf{p}) - E_f^{A-1} + E_0) = \int dE \delta(\omega + E - e(\mathbf{p}))\delta(E + E_f^{A-1} - E_0)$$

The response tensor is given by

$$R^{\mu\nu}(\mathbf{q}, \omega) = \int d^3k dE\, P_h(\mathbf{k}, E) \frac{m_N^2}{e(\mathbf{k})e(\mathbf{k}+\mathbf{q})} \sum_i \langle k | j_i^{\mu\dagger} | p \rangle \langle p | j_i^{\nu} | k \rangle \delta(\omega + E - e(\mathbf{k}+\mathbf{q}))$$

Spectral Function   Implicit covariant normalization of the four-spinors

The hole spectral function reads

$$P_h(\mathbf{k}, E) = \sum_f \left| \langle \Psi_0 | \left[|k\rangle \otimes |\Psi_f^{A-1}\rangle \right] \right|^2 \times \delta(E + E_f^{A-1} - E_0)$$

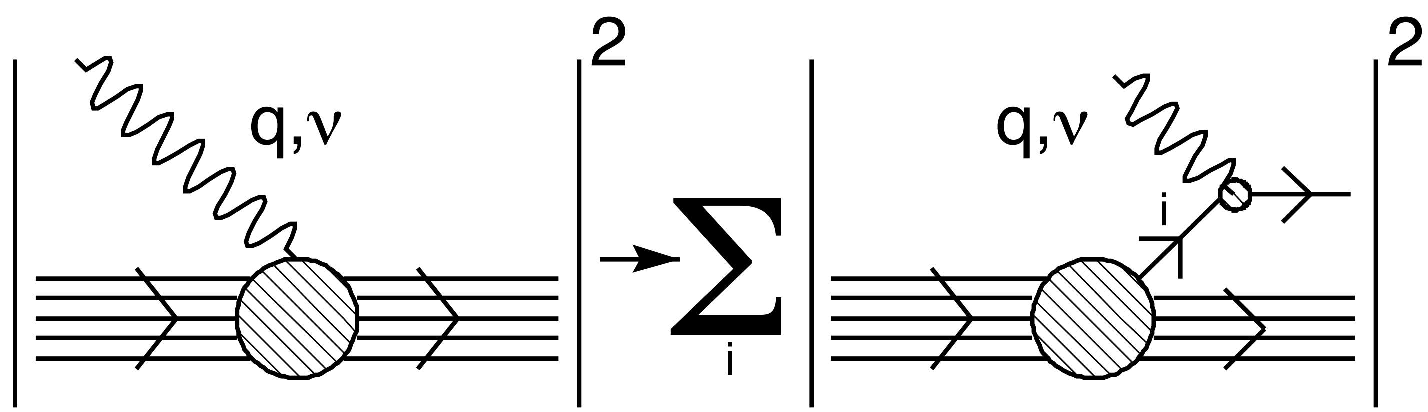
The proton spectral function $P_h(\mathbf{k}, E)$ describes the **probability distribution** of removing a proton of momentum **k** from the target nucleus, leaving the residual system with excitation energy E

Noemi Rocco, nrocco@fnal.gov

Spectral function approach

The response tensor is given by

$$R^{\mu\nu}(\mathbf{q},\omega) = \int d^3k dE\, P_h(\mathbf{k},E) \frac{m_N^2}{e(\mathbf{k})e(\mathbf{k}+\mathbf{q})} \sum_i \langle k\,|\,j_i^{\mu\dagger}\,|\,p\rangle\langle p\,|\,j_i^{\nu}\,|\,k\rangle \delta(\omega + E - e(\mathbf{k}+\mathbf{q}))$$



The nuclear cross section is replaced by the incoherent sum of cross sections describing scattering off individual nucleons, the recoiling (A – 1)-nucleon system acting as a spectator.

The scattering takes place on a bound nucleon, how do we introduce this effect?

Noemi Rocco, nrocco@fnal.gov

Off shellness effects

The nucleon tensor can be rewritten as

$$r_N^{\mu\nu} = \sum_i \langle k \,|\, j_i^{\mu\dagger} \,|\, p \rangle \langle p \,|\, j_i^{\nu} \,|\, k \rangle \delta(\tilde{\omega} + e(\mathbf{k}) - e(\mathbf{k}+\mathbf{q}))$$

Where: $\tilde{\omega} = e(\mathbf{k}+\mathbf{q}) - e(\mathbf{k}) = E_0 + \omega - E_f^{A-1} - e(\mathbf{k}) = \omega - E + m - e(\mathbf{k})$

We can identify the nucleon tensor as describing the scattering off a **free nucleon** at four momentum

$$q = (\omega, \mathbf{q}) \rightarrow \tilde{q} = (\tilde{\omega}, \mathbf{q})$$

The quantity $\delta\omega = \omega - \tilde{\omega}$ is the amount of energy going into the recoiling spectator system

Let's rewrite the explicit expression of the nucleon tensor in the case of quasi elastic scattering:

$$r_N^{\mu\nu} = w_1^N\Big(- g^{\mu\nu} + \frac{\tilde{q}^{\mu}\tilde{q}^{\nu}}{\tilde{q}^2}\Big) + \frac{w_2^N}{m^2}\Big(k^{\mu} - \frac{k\tilde{q}}{\tilde{q}^2}\tilde{q}^{\mu}\Big)\Big(k^{\nu} - \frac{k\tilde{q}}{\tilde{q}^2}\tilde{q}^{\nu}\Big)$$

Noemi Rocco, nrocco@fnal.gov

Off shellness effects

While the replacement of $\omega \to \tilde{\omega}$ is reasonable on physics grounds, it poses a considerable conceptual problem in that it leads to a violation of current conservation, which requires

$$q_\mu r_N^{\mu\nu} = 0$$

A possibility is to adopt a convention from de Forest, where the different components of the tensor are defined taking z along the **q** directions

$$\tilde{r}_N^{\mu\nu} = r_N^{\mu\nu}(\tilde{q})$$

For μ and/or $\nu = 0$

$$\tilde{r}_N^{3\nu} = \frac{\omega}{|\mathbf{q}|} r_N^{0\nu}(\tilde{q})$$

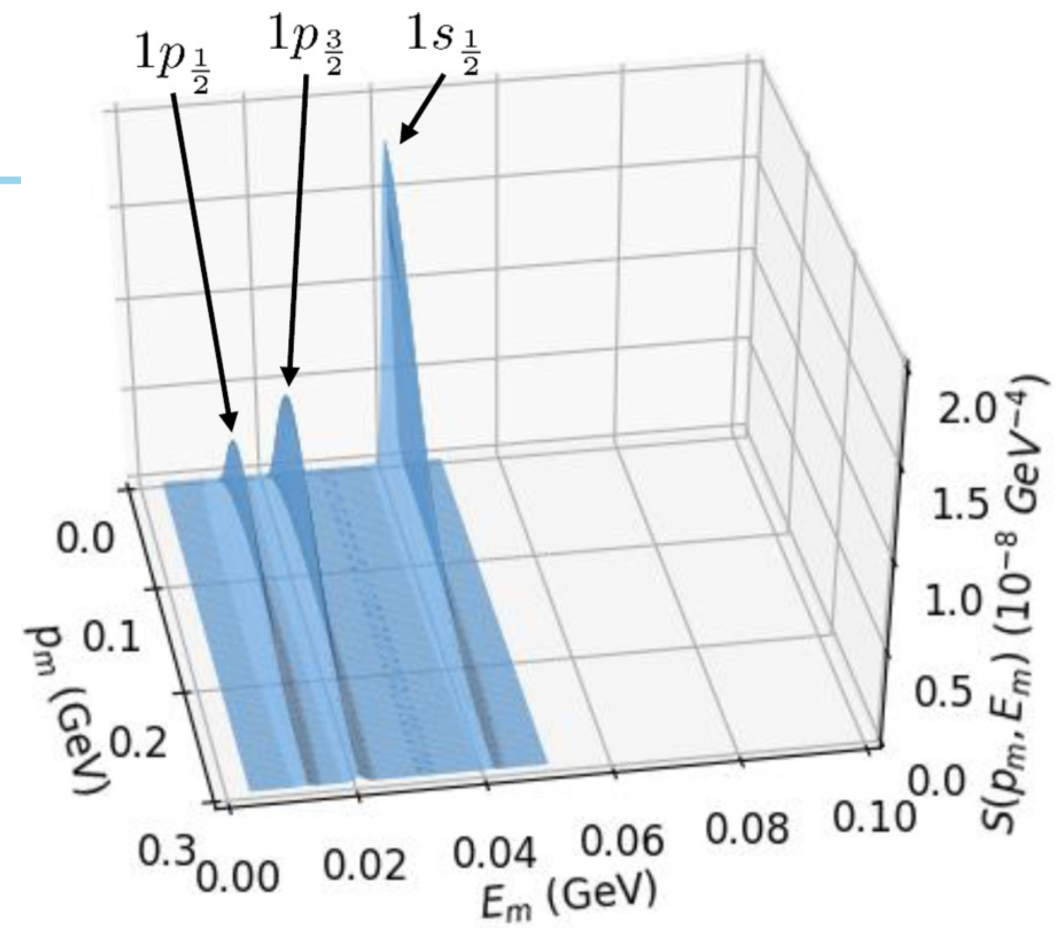
Different procedures can be used to restore gauge invariance. However, this only affects the longitudinal response. As a consequence, they are expected to become less and less important as the momentum transfer increases.

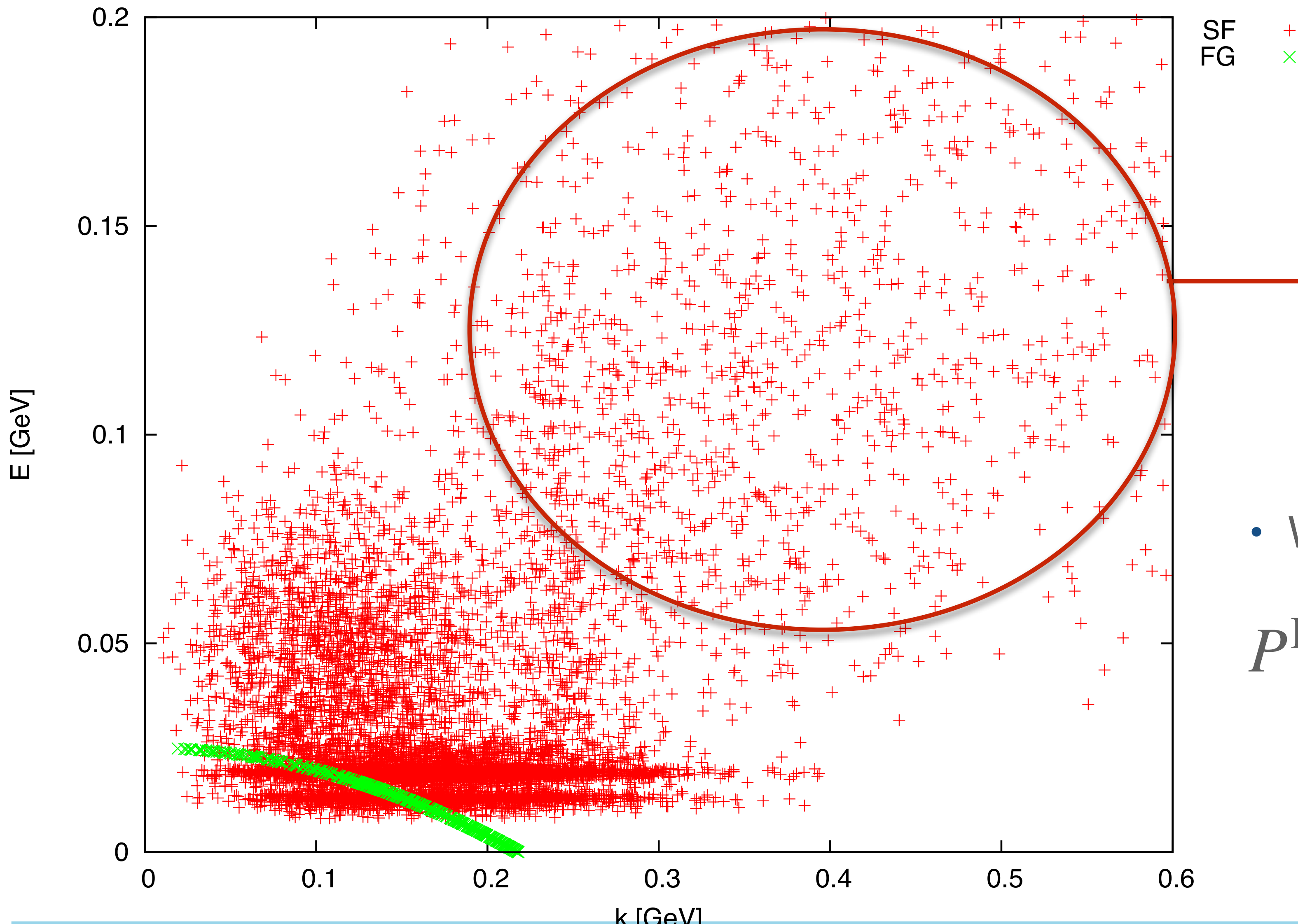
Noemi Rocco, nrocco@fnal.gov

Spectral function approach

- Single-nucleon spectral function:

$$P_h(\mathbf{k}, E) = P_{\rm MF}(\mathbf{k}, E) + P_{\rm corr}(\mathbf{k}, E)$$





High energy and momentum correlated pairs

- Within the Global Fermi gas, one can write

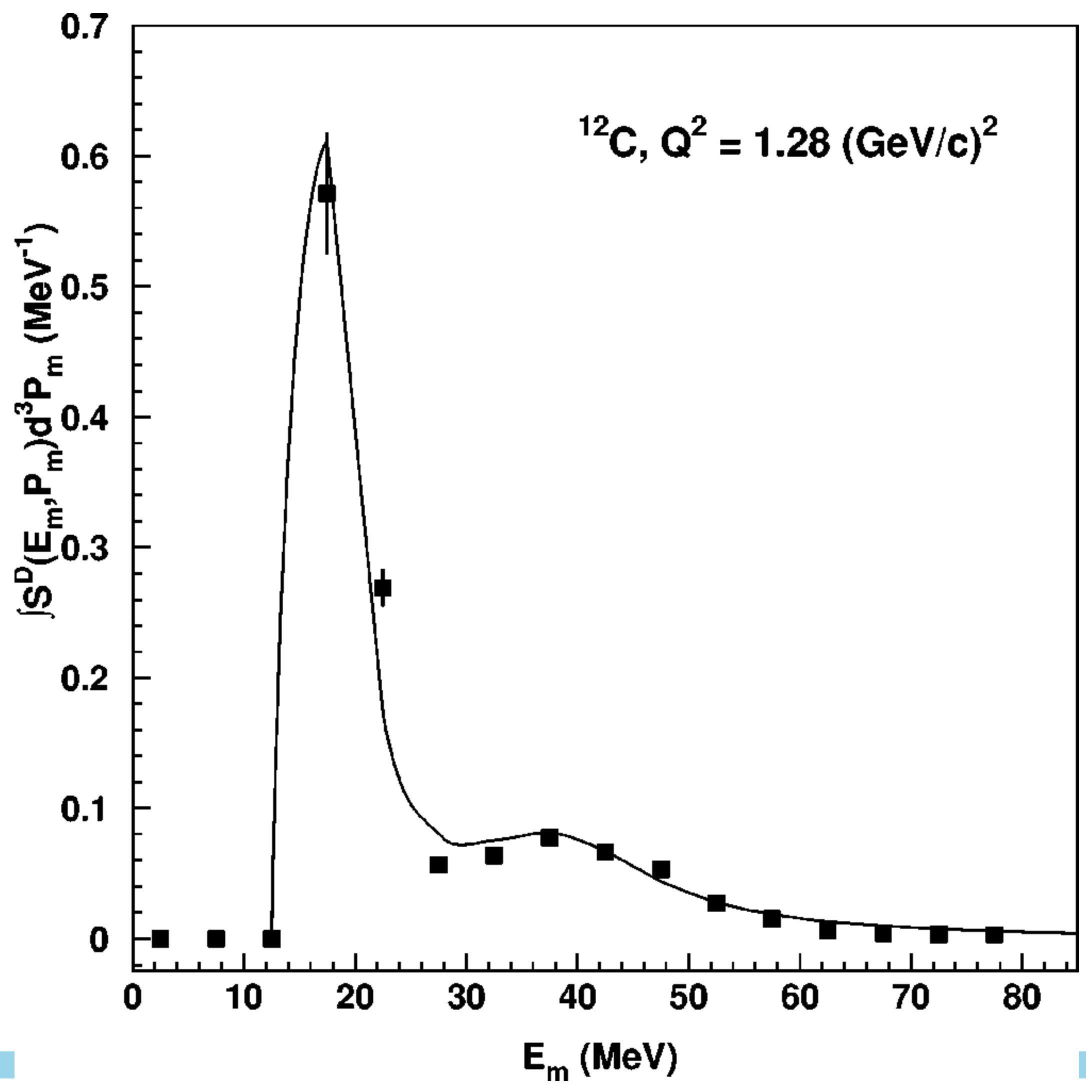
$$P^{\rm FG}(\mathbf{k}, E) = \delta(E - \epsilon_B)\theta(k_F - |\,\mathbf{k}\,|)$$

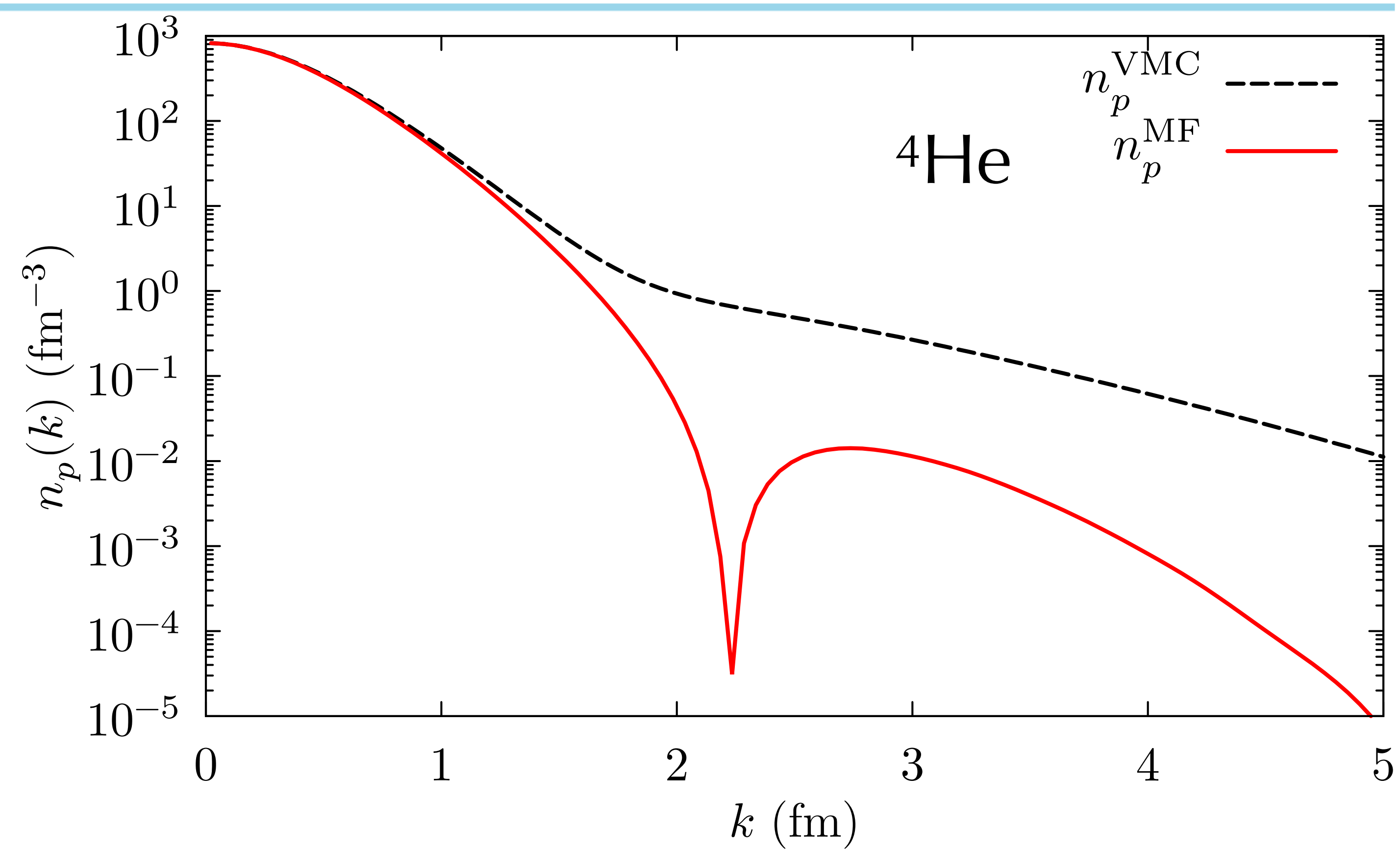
Spectral function approach

From the Spectral function we can obtain

- Single-nucleon momentum distribution:

$$n(k) = \int dE P_h(\mathbf{k}, E)$$





- Missing energy distribution

$$S(E) = \int d^3k P_h(\mathbf{k}, E)$$

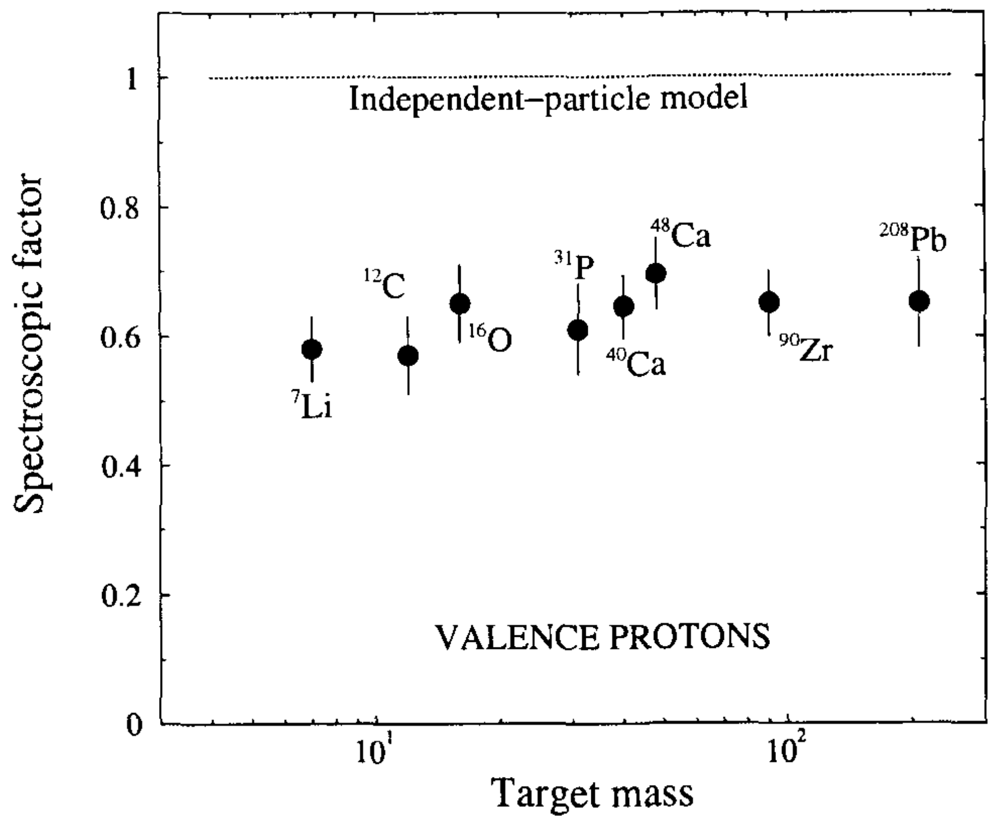
D. Dutta, et al., Phys. Rev. C 68, 064603 (2003).

Correlated Basis Function Approach

- ^{16}O Spectral Function obtained within CBF and using the Local Density Approximation

$$P_{LDA}(\mathbf{k}, E) = P_{MF}(\mathbf{k}, E) + P_{corr}(\mathbf{k}, E)$$

$$\sum_n Z_n |\phi_n(\mathbf{k})|^2 F_n(E - E_n)$$



O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989).

O. Benhar, A. Fabrocini, S. Fantoni, and I. Sick, Nucl. Phys. A579, 493 (1994)

Correlated Basis Function Approach

- ^{16}O Spectral Function obtained within CBF and using the Local Density Approximation

$$P_{LDA}(\mathbf{k}, E) = P_{MF}(\mathbf{k}, E) + P_{corr}(\mathbf{k}, E)$$

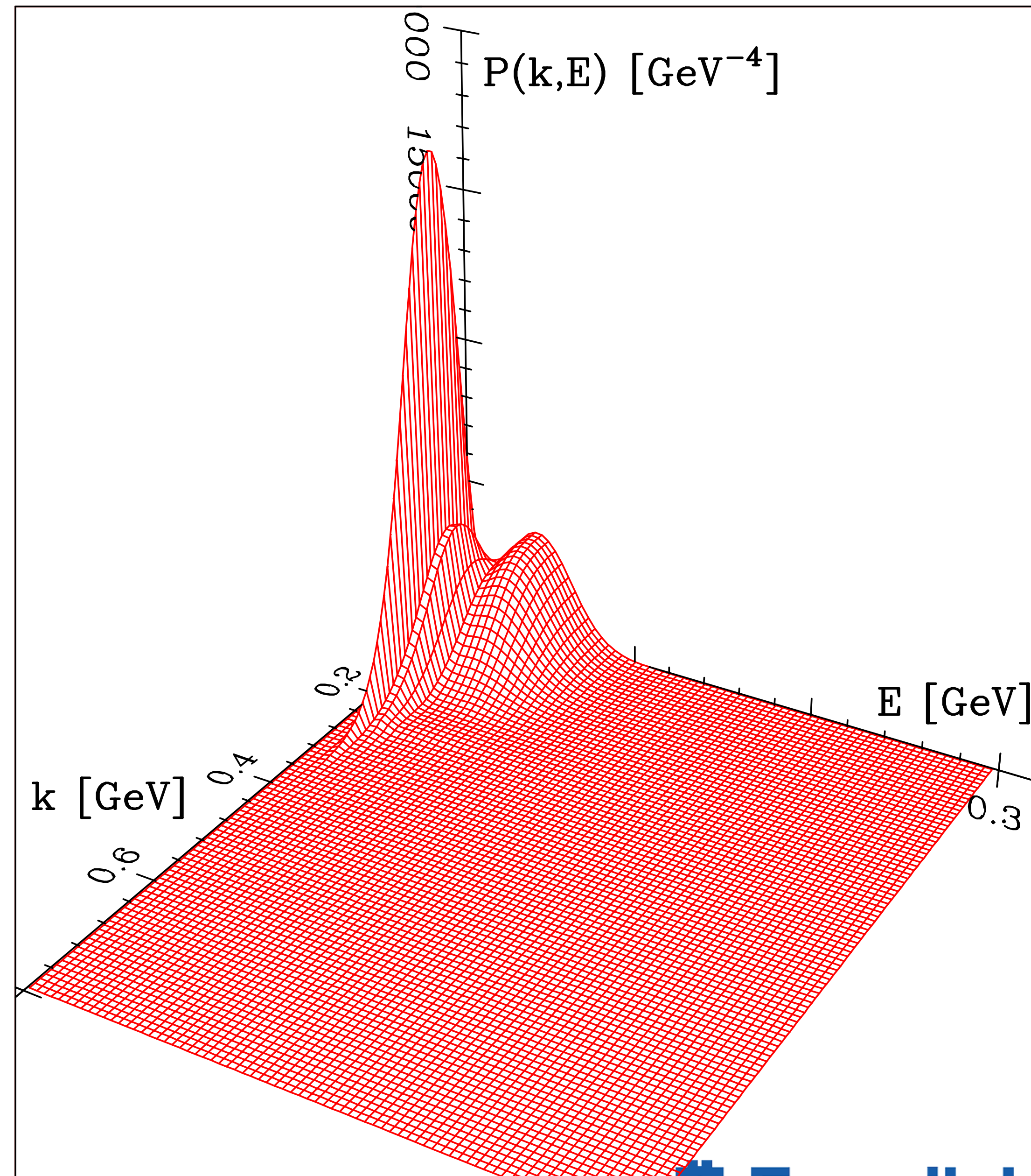
$$\int d^3r P^{NM}_{corr}(\mathbf{k}, E; \rho = \rho_A(r))$$

- The one-body Spectral function of nuclear matter:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

Argonne v18 (v_{ij})

UIX, IL7 (V_{ijk})



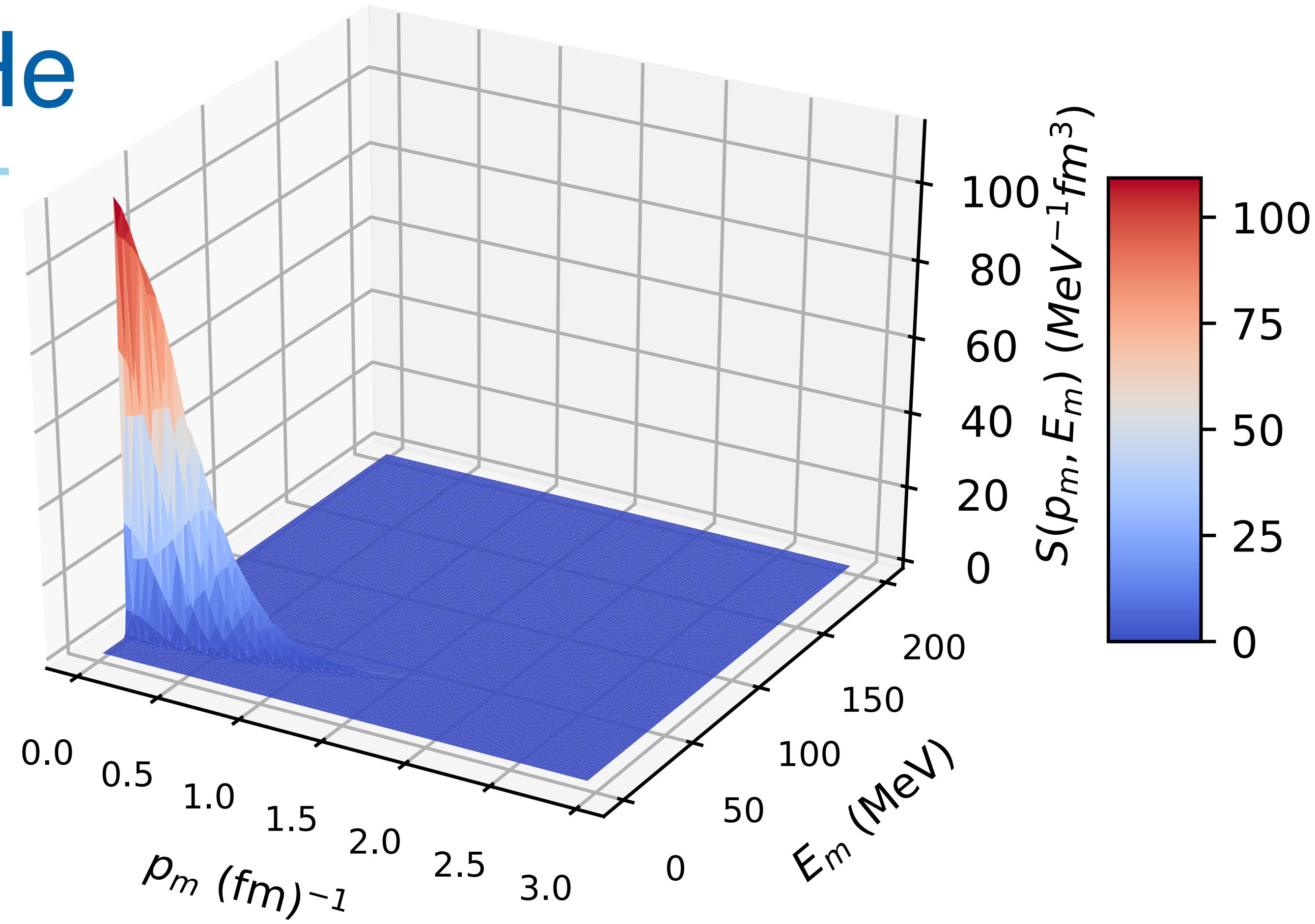
- The Correlated Basis Function approach accounts for correlations induced by the nuclear interactions

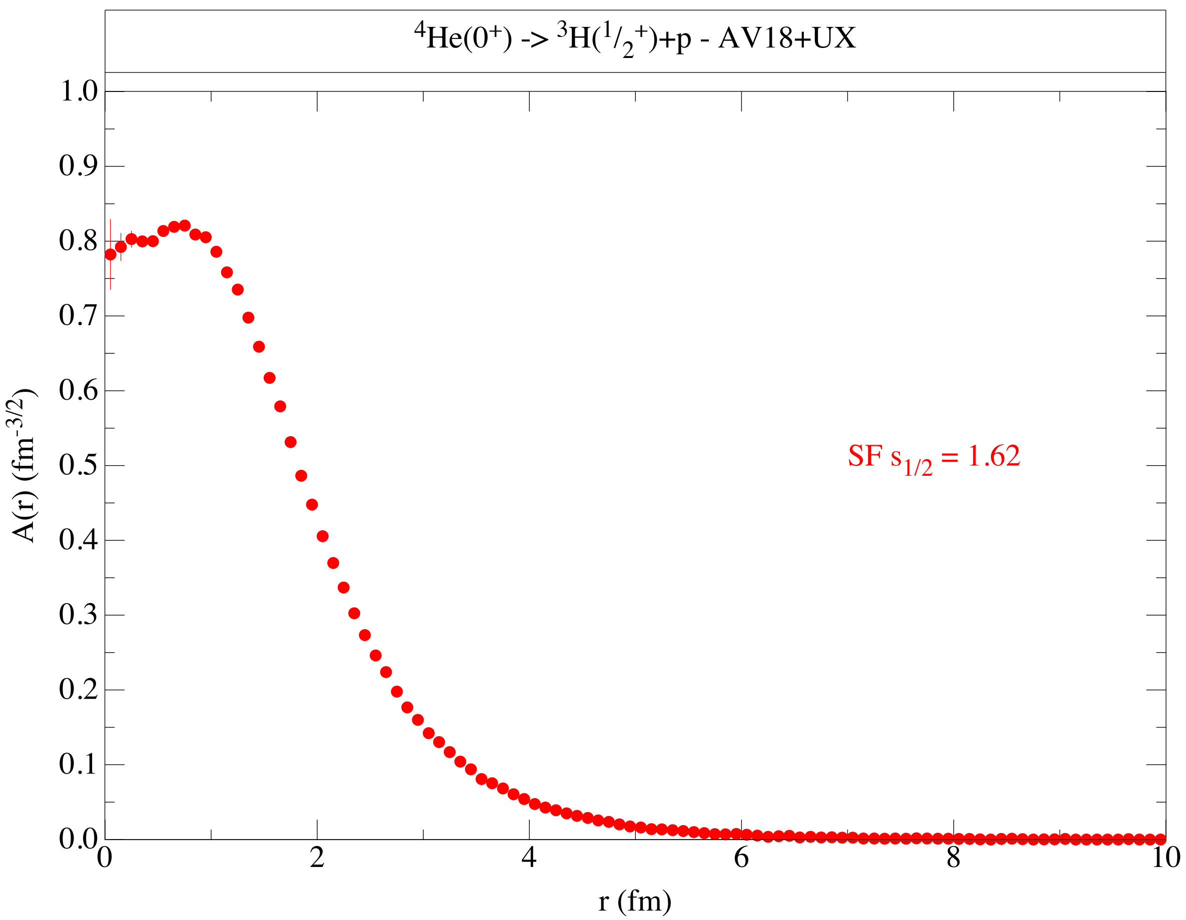
$$\Phi_n(x_1 \dots x_A) \longrightarrow \mathcal{F}\,\Phi_n(x_1 \dots x_A)$$

O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989).

QMC Spectral Function of ^{4}He

$$P_{\tau_k}(\mathbf{k}, E) = \sum_n |\langle \Psi_0^A | [|k\rangle \, |\Psi_n^{A-1}\rangle]|^2 \times \delta(E + E_0^A - E_n^{A-1})$$





$$P_p^{\rm MF}(\mathbf{k}, E) = n_p^{\rm MF}(\mathbf{k})\delta\left(E - B_{^4\rm He} + B_{^3\rm H} - \frac{k^2}{2m_{^3\rm H}}\right)$$

$$|\langle \Psi_0^{^4\rm He} | [|k\rangle \otimes |\Psi_0^{^3\rm H}\rangle]|^2$$

- The single-nucleon overlap has been computed within VMC (center of mass motion fully accounted for)

Noemi Rocco, nrocco@fnal.gov

QMC Spectral Function of ^{4}He

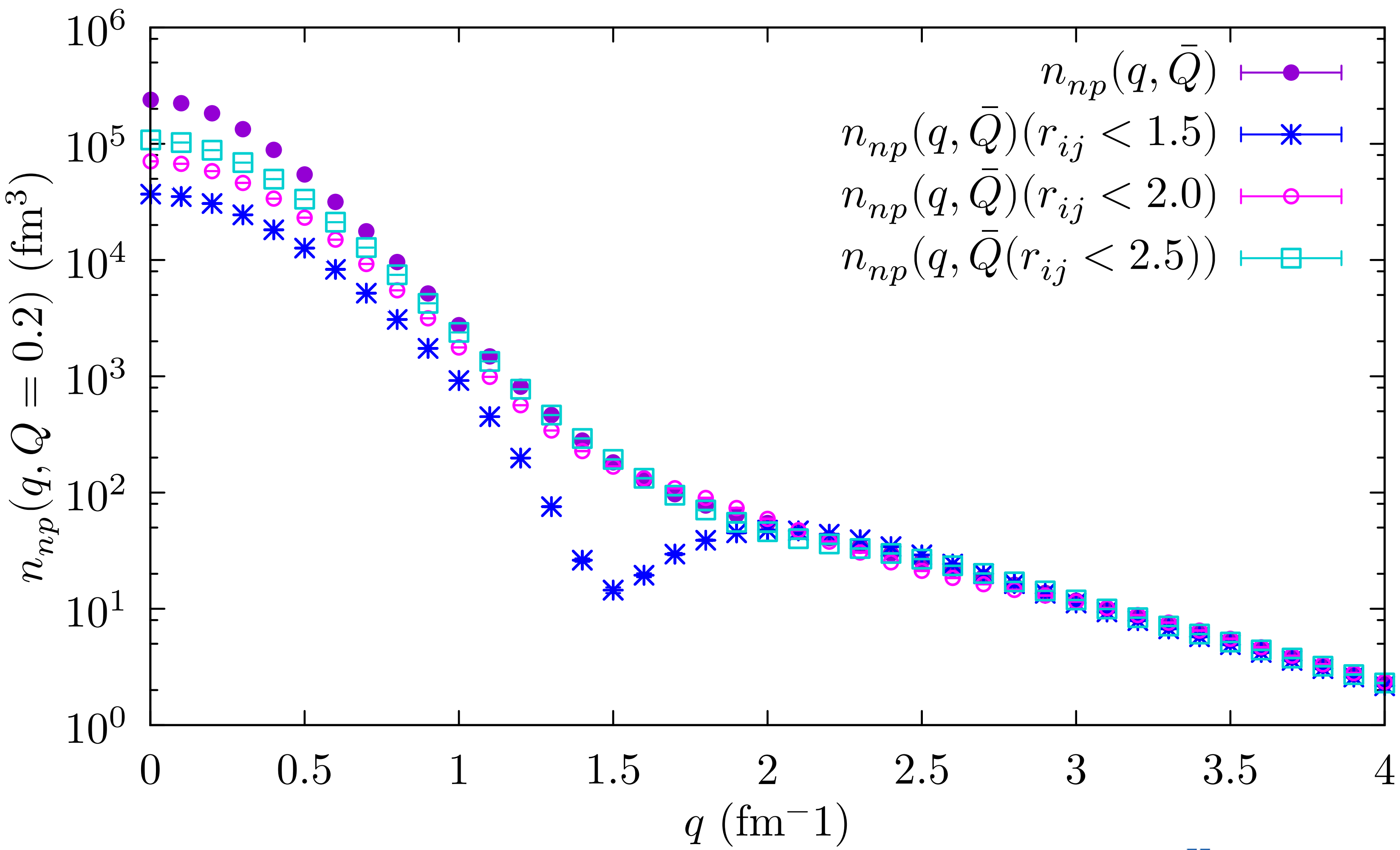
$$P_p^{\rm corr}(\mathbf{k},E) = \sum_n \int \frac{d^3k'}{(2\pi)^3} |\langle \Psi_0^A | [|k\rangle \, |k'\rangle \, |\Psi_n^{A-2}\rangle]|^2 \delta(E + E_0^A - e(\mathbf{k}') - E_n^{A-2})$$

Using QMC techniques

$$\sum_{\tau_{k'}=p,n} n_{p,\tau_{k'}}(\mathbf{k},\mathbf{k}')\delta\Big(E - B_A \quad - e(\mathbf{k}') + B_{A-2} - \frac{(\mathbf{k}+\mathbf{k}')^2}{2m_{A-2}}\Big)$$

Only SRC pairs should be considered: $|\Psi_0^{A-1}\rangle$ and $|k'\rangle|\psi_n^{A-2}\rangle$ be orthogonalized

One can introduce **cuts** on the **relative distance** between the particles in the two-body momentum distribution



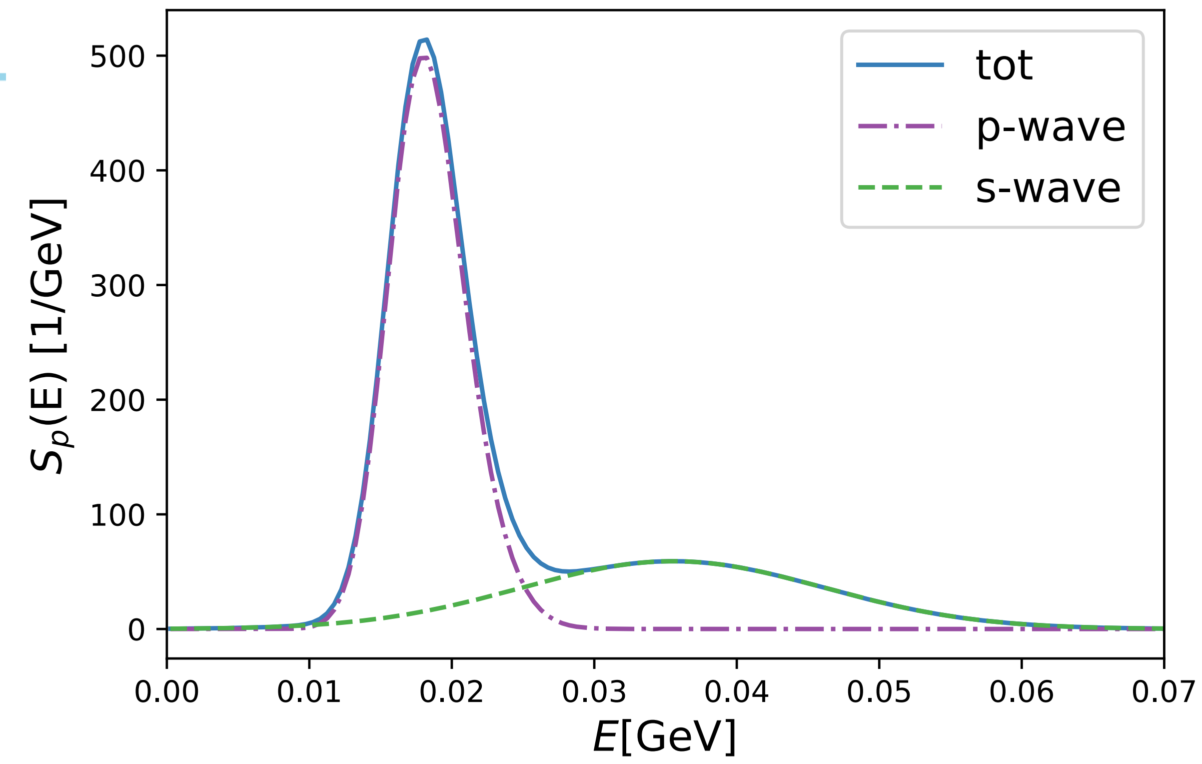
Noemi Rocco, nrocco@fnal.gov
Fermilab

QMC Spectral Function of ^{12}C

- The p-shell contribution has been obtained by FT the radial overlaps:

$$^{12}\mathrm{C}(0^+) \to ^{11}\mathrm{B}(3/2^-) + p$$
$$^{12}\mathrm{C}(0^+) \to ^{11}\mathrm{B}(1/2^-) + p$$
$$^{12}\mathrm{C}(0^+) \to ^{11}\mathrm{B}(3/2^-)^* + p\,.$$

R. Crespo, et al, Phys.Lett.B 803 (2020) 135355

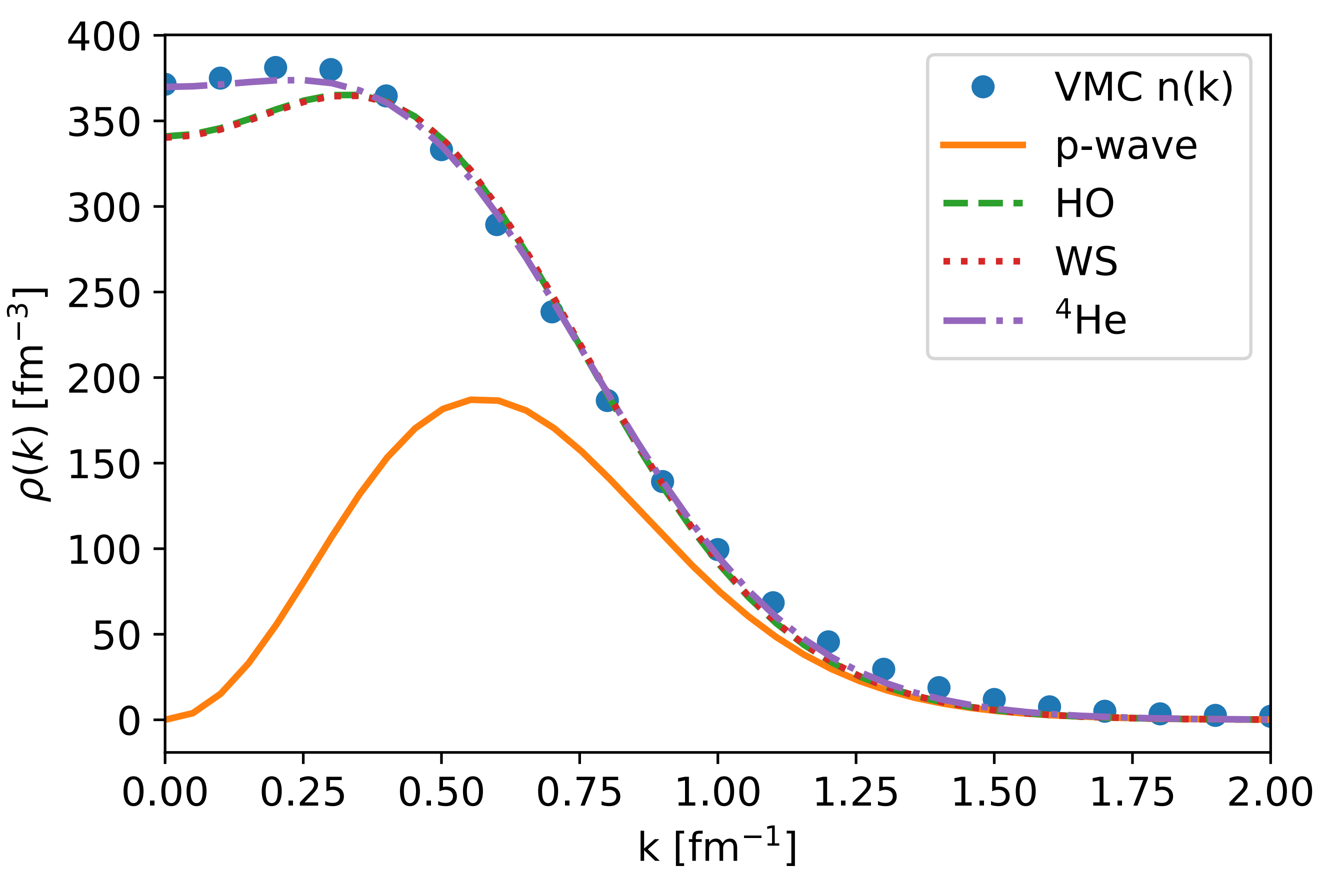


- The quenching of the spectroscopic factors automatically emerges from the VMC calculations

Computing the s-shell contribution is non trivial within VMC. We explored different alternatives:

- Quenched Harmonic Oscillator
- Quenched Wood Saxon
- VMC overlap associated for the $^4\mathrm{He}(0^+) \to\ ^3\mathrm{H}(1/2^+) + p$ transition

Korover, et al, CLAS collaboration submitted (2021)

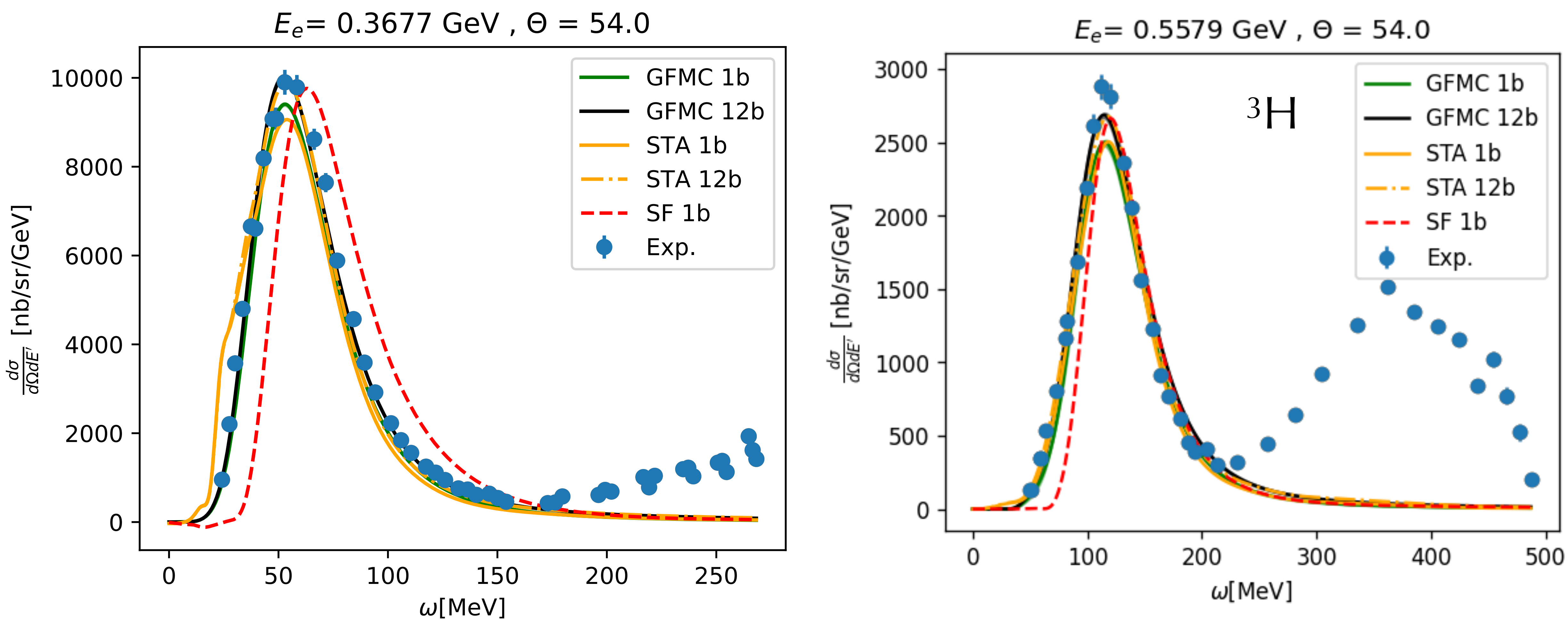


Noemi Rocco, nrocco@fnal.gov

Comparing different many-body methods

- e -^{3}H: inclusive cross section

L. Andreoli, NR, et al, PRC 105 (2022) 1, 014002



- Comparisons among GFMC, SF, and STA approaches: first step to precisely **quantify the uncertainties i**nherent to the factorization of the final state.

- Gauge the role of **relativistic effects** in the energy region relevant for neutrino experiments.

Noemi Rocco, nrocco@fnal.gov

Two point Green's function

- The nuclear matrix element can be rewritten in terms of the transition amplitude

$$[\langle\psi_f^{A-1}|\otimes\langle k|]|\psi_0^A\rangle = \sum_\alpha \mathcal{Y}_{f,\alpha}\tilde{\Phi}_\alpha(\mathbf{k}) = \sum_\alpha \tilde{\Phi}_\alpha(\mathbf{k})\langle\psi_f^{A-1}|a_\alpha|\psi_0^A\rangle\,,$$

- The Spectral Function gives the probability distribution of removing a nucleon with momentum **k**, leaving the spectator system with an excitation energy E

$$P_h(\mathbf{k},E) = \sum_f |\langle\psi_0^A|[|\mathbf{k}\rangle\otimes|\psi_f^{A-1}\rangle]|^2\delta(E+E_f^{A-1}-E_0^A)$$

$$= \frac{1}{\pi}\sum_{\alpha\beta}\tilde{\Phi}^*_\beta(\mathbf{k})\tilde{\Phi}_\alpha(\mathbf{k})\mathrm{Im}\langle\psi_0^A|a_\beta^\dagger\frac{1}{E+(H-E_0^A)-i\epsilon}a_\alpha|\psi_0^A\rangle\,.$$

- The two points Green's Function describes nucleon propagation in the nuclear medium

$$G_{h,\alpha\beta}(E) = \langle\psi_0^A|a_\beta^\dagger\frac{1}{E+(H-E_0^A)-i\epsilon}a_\alpha|\psi_0^A\rangle$$

Noemi Rocco, nrocco@fnal.gov

Self Consistent Green's Function

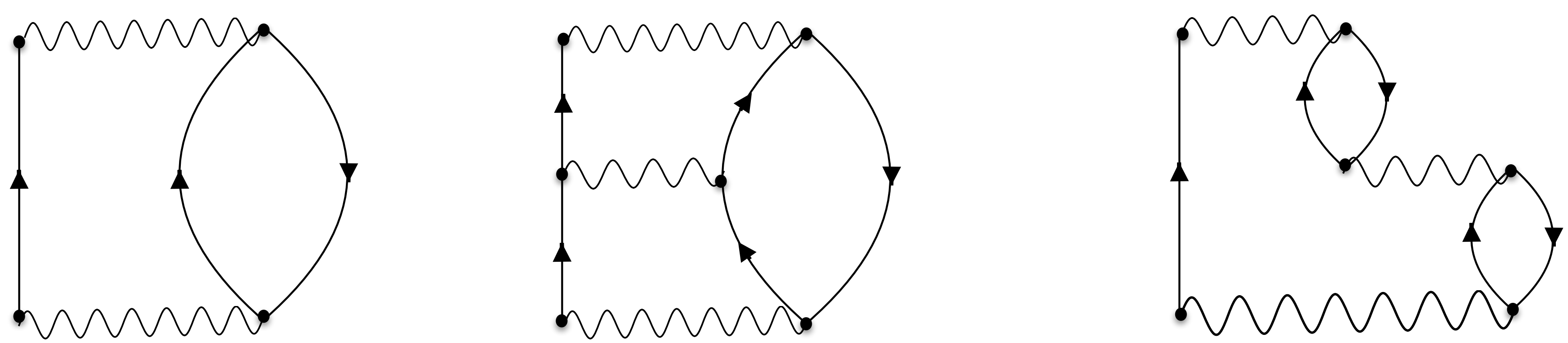
- The one-body Green's function is completely determined by solving the Dyson equation

$$G_{\alpha\beta}(E) = G^0_{\alpha\beta}(E) + \sum_{\gamma\delta} G^0_{\alpha\gamma}\Sigma^*_{\gamma\delta}(E)G_{\delta\beta}(E)$$

initial reference state, HF

- $\Sigma^* = \Sigma^*[G(E)]$, an iterative procedure is required to solve the Dyson equation self-consistently

- The self-energy is systematically calculated in a non-perturbative fashion within the Algebraic Diagrammatic Construction (ADC). The saturating chiral interaction at NNLO ($NNLO_{sat}$) is used.



2nd and 3rd order diagrams with 2h1p (and 2p1h) intermediate configurations

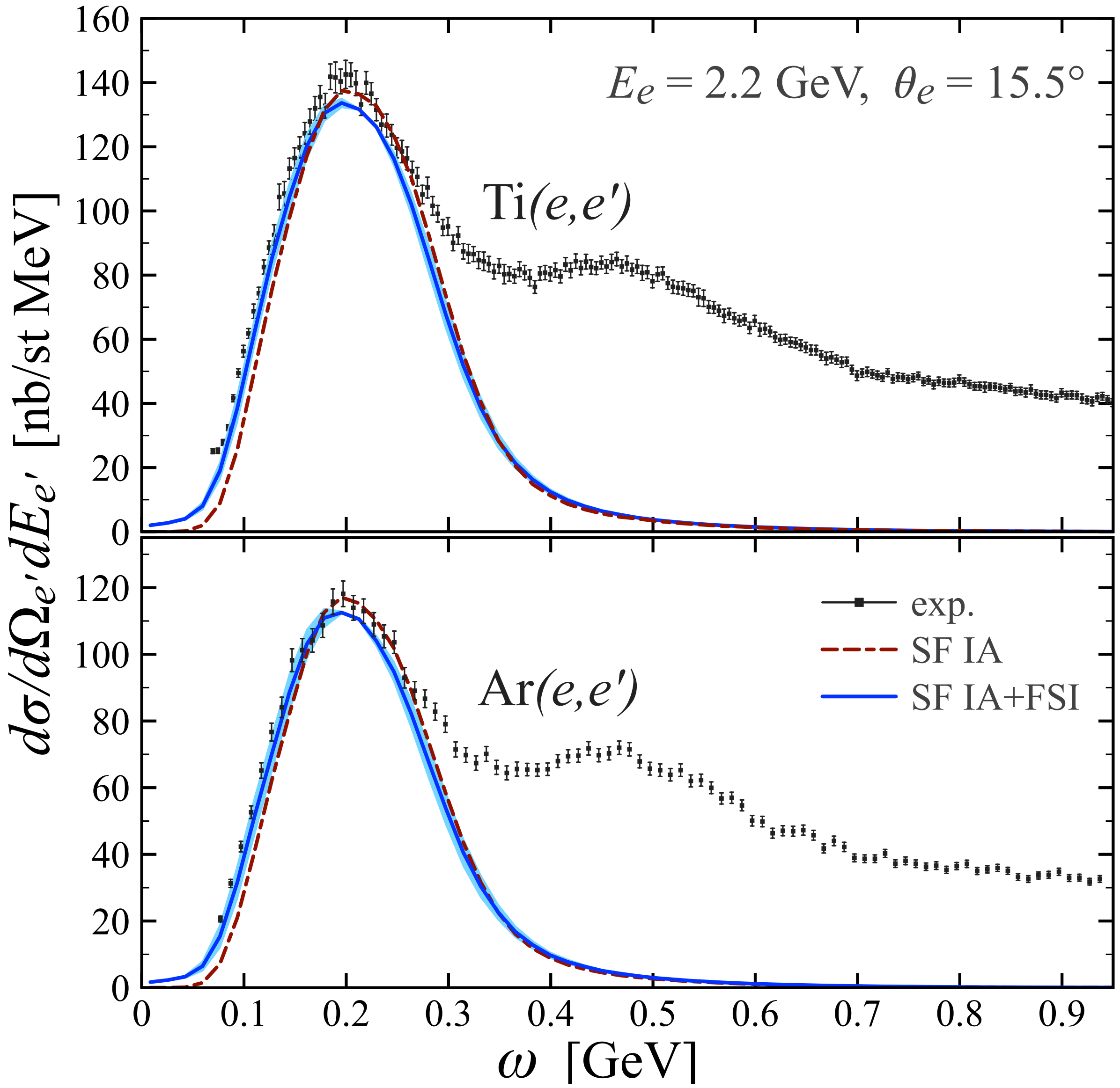
✤ V. Somà et al, Phys.Rev. C87 (2013) no.1, 011303 : generalization of this formalism within Gorkov theory allows to describe open-shell nuclei such as Ar^{40}, Ti^{48} …

Noemi Rocco, nrocco@fnal.gov

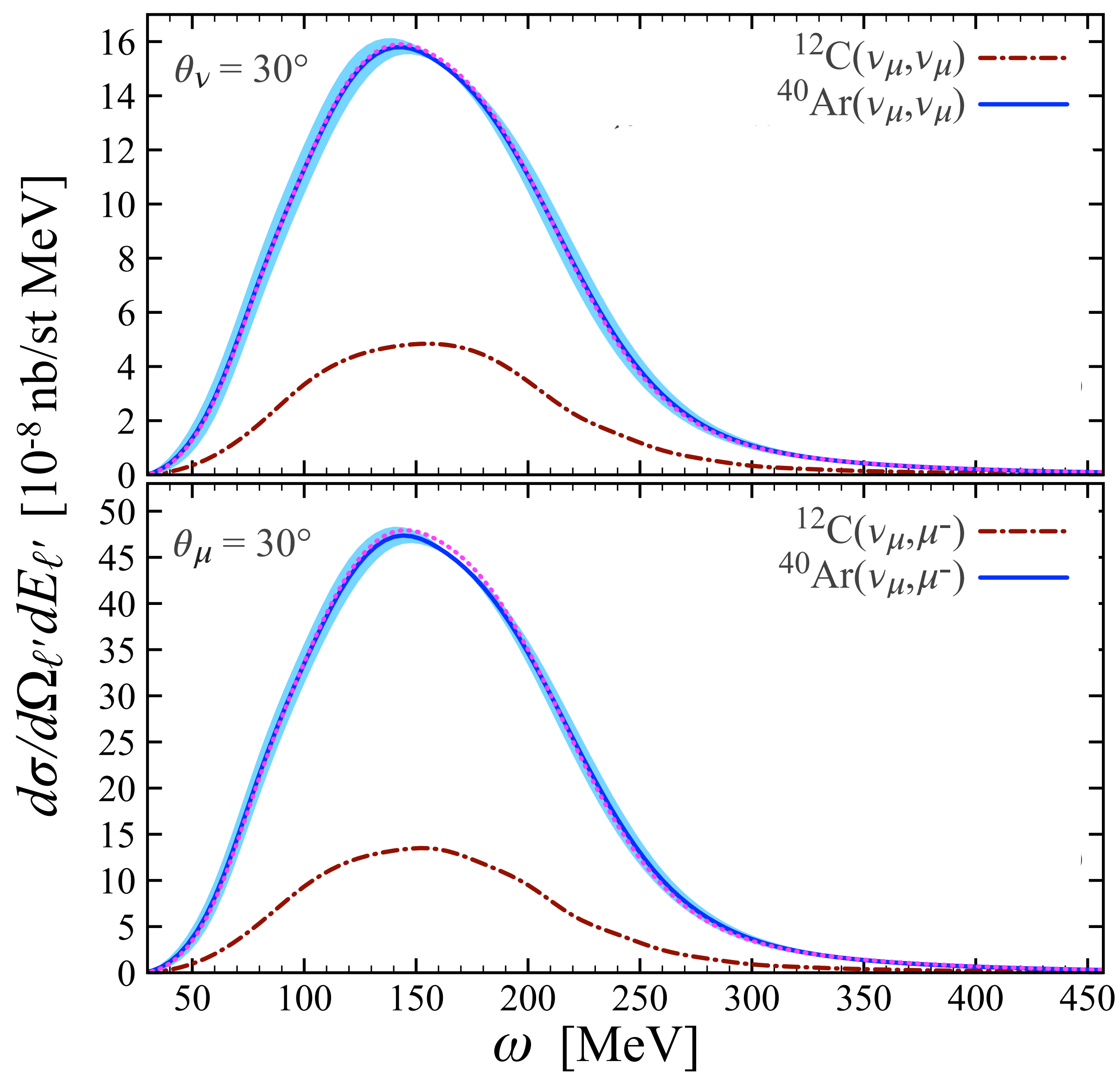
Self Consistent Green's Function

^{40}Ar(e,e') and ^{48}Ti(e,e') cross sections

✐ C. Barbieri, NR, and V. Somà, PRC 100, no.6, 062501 (2019)



Charge current and neutral current ν_μ scattering on ^{12}C and Ar for $E\nu_\mu$ =1 GeV



The band comes from a **first estimate of the uncertainty on the spectral function** calculation obtained by varying the model-space and the harmonic oscillator frequency

Outline

- Lepton - nucleon interactions

- Modeling nuclear structure

- Ab - initio description of lepton - nucleus interactions

  - Factorization approach + spectral function : QE

  - More spectral function

- Factorization approach + spectral function : MEC + interference (+pion)

  - Scaling properties

  - ACHILLES, BSM studies with the spectral function approach

Noemi Rocco, nrocco@fnal.gov

Thank you for your attention!

Extra — Structure function contributions

$$\frac{W^{\mu\nu}}{2M_i} = -g^{\mu\nu}W_1 + \frac{P^\mu P^\nu}{M_i^2}W_2 + i\frac{\epsilon^{\mu\nu\gamma\delta}P_\gamma q_\delta}{2M_i^2}W_3 + \frac{q^\mu q^\nu}{M_i^2}W_4 + \frac{P^\mu q^\nu + P^\nu q^\mu}{2M_i^2}W_5 + i\frac{P^\mu q^\nu - P^\nu q^\mu}{2M_i^2}W_6 .$$

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE_l'} = \frac{|\vec{k}'|E_l' M_i G^2}{\pi^2}\Bigg\{ 2W_1 \sin^2\frac{\theta'}{2} + W_2\cos^2\frac{\theta'}{2} - W_3\frac{E_\nu + E_l'}{M_i}\sin^2\frac{\theta'}{2} + \frac{m_l^2}{E_l'(E_l' + |\vec{k}'|)}\Bigg[W_1\cos\theta' - \frac{W_2}{2}\cos\theta' + \frac{W_3}{2}\left(\frac{E_l' + |\vec{k}'|}{M_i} - \frac{E_\nu + E_l'}{M_i}\cos\theta'\right) + \frac{W_4}{2}\left(\frac{m_l^2}{M_i^2}\cos\theta' + \frac{2E_l'(E_l' + |\vec{k}'|)}{M_i^2}\sin^2\theta'\right) - W_5\frac{E_l' + |\vec{k}'|}{2M_i}\Bigg]\Bigg\}$$

Noemi Rocco, nrocco@fnal.gov