Lecture 2

Outline

DIS

↳ Structure fn's (Compton tensor, ν-DIS)

↳ Analytic Structure

↳ OPE

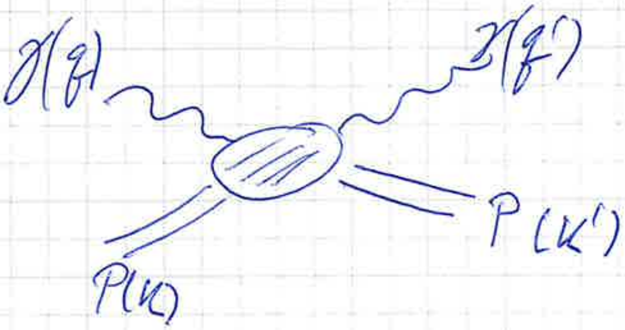
Form Factors

↳ High-Q^2 pQCD

↳ Low-Q^2; CEνNS, etc.

↳ CVC

↳ PCAC



$\gamma p \to \gamma p$

$$i\mathcal{M}_{\gamma\gamma} = (+ie)^2 \int d^4x\, e^{i(k'-k)\cdot x} \langle P(k')|T\{\hat{J}_\mu(x), J_\nu(0)\}|P(k)\rangle \epsilon^{\mu}(\epsilon^{\nu})^*$$

This is the matrix el. for Compton Scattering

Notice

$$\cancel{2\,\mathrm{Im}\int\langle P(k)|T}$$

$$2\,\mathrm{Im}\int d^4x\, e^{\cancel{i(k'-k)\cdot x}\; iq\cdot x} \langle P(k')|T\{J_\mu(x)\, J_\nu(0)\}|P(k)\rangle$$

$$= \int d^4x\, e^{\cancel{i(k'-k)\cdot x}\; iq\cdot x} \langle P(k')|[J_\mu(x), J_\nu(0)]|P(k)\rangle$$

$$= \int d^4x \; \cancel{e^{i[(k'-k)-p_X]\cdot x} \langle P(k')| J_\mu |X\rangle\langle X| J_\nu |P(k)\rangle}$$

$$+$$

Now let's set $\langle P(k')| = \langle P(k)|$

$\longrightarrow$ FWD Scattering

Disc  $= \displaystyle\int_X$  $(2\pi)^4 \, \delta^{(4)}(-)$

$$2\,\mathrm{Im}\,\mathrm{Disc}\; T_{\mu\nu} = W_{\mu\nu}$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1 + \left(P_\mu - \frac{P\cdot q \, q_\mu}{q^2}\right)\frac{F_2}{P\cdot q}$$

sym.

Satisfies $q_\alpha g^{\mu\alpha} W^\alpha_{\nu\nu} = q^\nu W_{\mu\nu} = 0$

The Neutrino Case Is More Involved

Five Inel. Structure F'ns

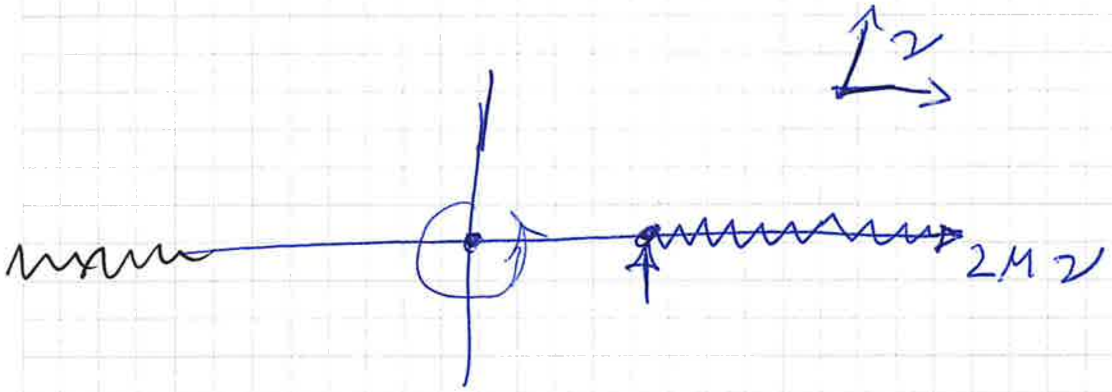
$$W_{\mu\nu} = -g_{\mu\nu} F_1 + P_{1\mu} P_{1\nu} F_2$$

$$-i \epsilon_{\mu\nu\rho\sigma} \frac{P_1^\rho q^\sigma}{2 P_1 \cdot q} F_3$$

$$\frac{q_\mu q_\nu}{P_1 \cdot q} F_4 + (P_{1\mu} q_\nu + P_{1\nu} q_\mu) F_5$$

Connection to Short Distance Physics

↳ what can we say about $\lim_{Q^2\to\infty}$?



complex ν-plane

see also ITEP sum rules

for $e^+e^- \to$ Hadrons

Fig 8 of Manohar

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$\boxed{\text{Back to Form Factors}}$

↳ we just learned how pQCD helps us learn about structure fns

• what about FF's ?

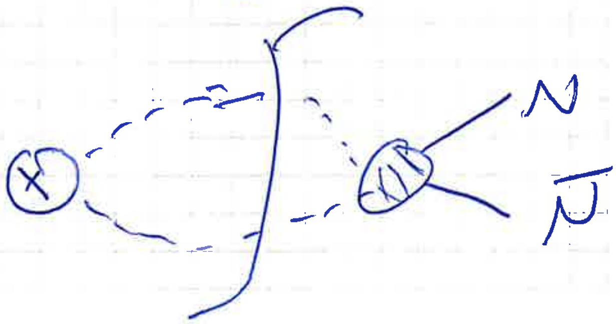
$$\langle N | J_\mu | N \rangle \qquad \text{Space like}$$

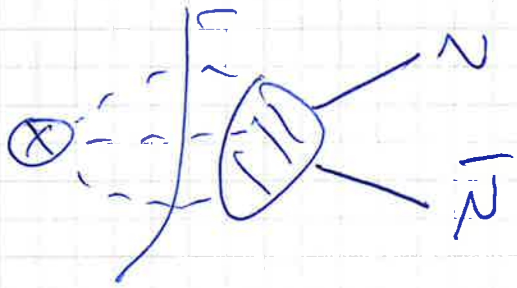
$$\langle N \bar{N} | J_\mu | 0 \rangle \qquad \text{time like}$$

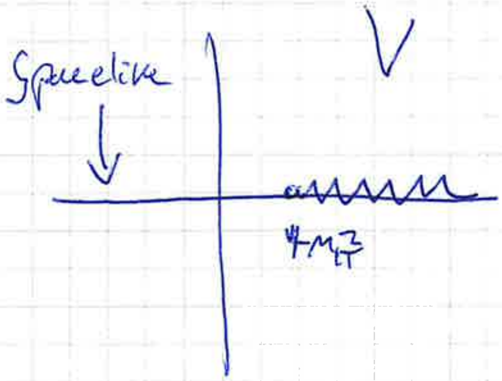
See e.g. Lin, Hammer, Meißner (2021) Eur Phys J. A

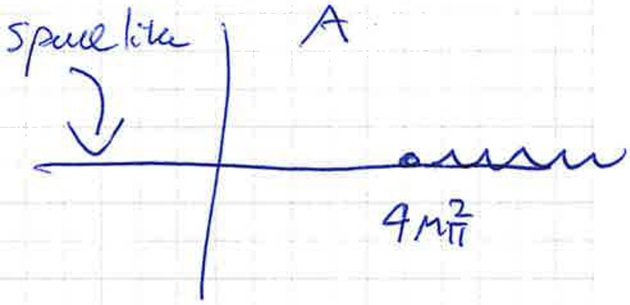
$$= \int d^4x \, (\not\partial + M_N) \langle \bar{N} | T\{ \bar{\Psi}(x), J_\mu(0) \} | 0 \rangle$$

Can insert complete set of states





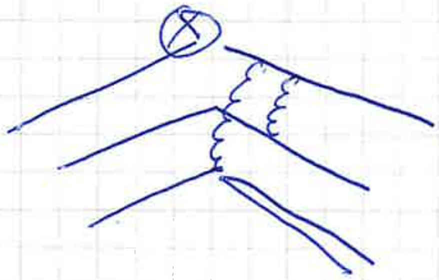


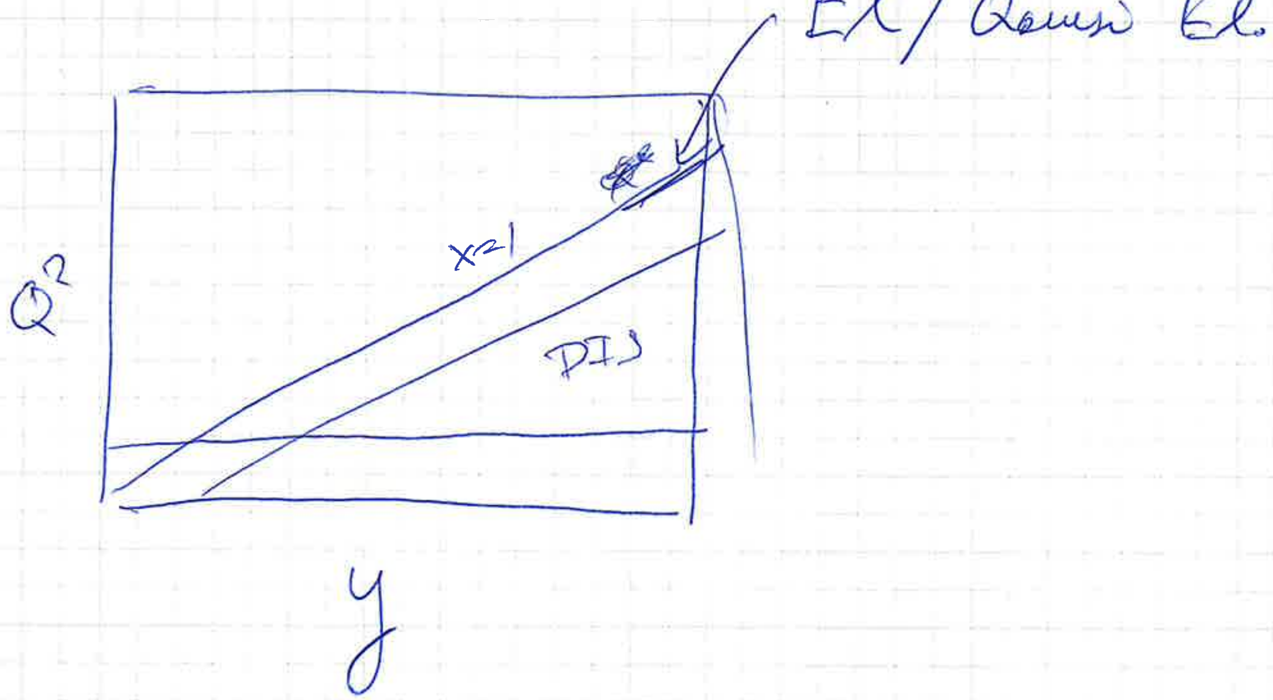


Branch cut ~~inhibits~~ restricts radius of conv. for Q^2

High Q^2 Asym.

PQCD





Q^2 is large $\Rightarrow$ PQCD still helps

$$G_M(Q^2) \sim \frac{32\pi^2}{9} C^2 \frac{\alpha_s^2(Q^2)}{Q^4} \left[\log\left(\frac{Q^2}{\Lambda^2}\right)\right]^{-\frac{4}{3}\beta}$$

Low-Q^2 Behaviour

↳ Currents can be constrained at low momentum transfer

$$\lim_{q\to 0} \langle A(p+q)| \hat{J}_\mu | A(p)\rangle \propto P_\mu$$

as $q \to 0$ only 4 vector

$$\hat{Q} = \int_\Sigma \hat{J}_0(\vec{x})\, d^3x$$

$$\hat{Q}\,|A\rangle = Q_A\,|A\rangle$$

$$\int d^3x \langle A|J_0(\vec{x})|A\rangle \to \int d^3x\, e^{i\vec{p}\cdot\vec{x}} \langle A|\hat{J}_0|A\rangle$$

$$= (2\pi)^3 \delta^{(3)}(\vec{p}) \langle A|\hat{J}_0|A\rangle$$

$$\langle A|\hat{Q}|A\rangle = Q_A\, 2E_A (2\pi)^3 \delta^{(3)}(\vec{p})$$

conserved current normalized to $\hat{Q}$

Current Cons.

Current cons. supplies valuable constraints

$$\partial^\mu \hat{J}_\mu = 0 \implies q^\mu \langle \beta | \hat{J}_\mu | \alpha \rangle = 0$$

CVC

$$\cancel{\partial^\mu} \; \partial_\mu \hat{V}^\mu \simeq 0$$

very good approx

PCAC

$$\partial_\mu \hat{A}^\mu \propto M_\pi^2$$

CEνNS

we can ∴ reduce scattering at low momentum transfer to point-like charge

Ex:

$$\langle A | \hat{J}^{(0)}_\mu | A \rangle \cancel{\propto Q_W}$$

$$\langle A | \hat{Q}_W | A \rangle V_\mu$$

Entire nucleus

$$\frac{d\sigma}{d|t|} \simeq \frac{G_F^2 M_A}{4\pi}\left(1 - \frac{|t|}{E_\nu} - \frac{M_A |t|}{2E_\nu}\right) Q_W^2$$

$$\boxed{Q_W = N - (1 - 4\sin^2\theta_W) Z} + \mathcal{O}($$

Interesting probe of light ν-philic mediators!

PCAC Goldberger Treiman

1) $\cancel{\partial_\mu} \langle \pi | A_\mu | 0 \rangle = i f_\pi q_\mu$

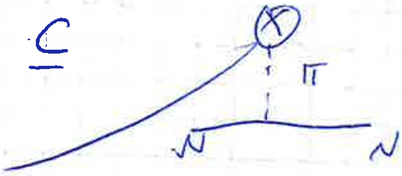
$\Rightarrow \quad q^\mu \langle \pi | A_\mu | 0 \rangle = i f_\pi m_\pi^2$

$\partial^\mu A_\mu$ = good pion interpolating field

$$\langle N | A_\mu | N \rangle \doteq \bar{u} \left[F_A \gamma_\mu \gamma_5 + \gamma_5 q_\mu F_p \right] u$$

~~Suppose~~

$$q^\mu \langle N | A_\mu | N \rangle = f_\pi \cancel{m_\pi^2} \frac{i}{\cancel{q^2}} \, 0$$

$\langle N | \cancel{A_\mu} | N \rangle \subseteq$  $= \cancel{\frac{i f_\pi \, q_\mu}{q^2 - m_\pi^2} \, g_{\pi NN}}$ $\bar{u} \gamma_5 u$

~~$i f_\pi q_\mu$~~

But $q^\mu \langle N|A_\mu|N\rangle = \bar{u}\left[F_A \not{q}\gamma_5 + \gamma_5 F_P q^2\right]u$

$$= \bar{u}\left[F_A 2M + F_P q^2\right]u$$

$$\therefore \quad F_P(q^2) \sim \frac{1}{q^2}$$

$$2M g_A = f_\pi g_{\pi NN}$$

Goldberger Treiman rel.

Generalization w/ massive pion

$$F_P \simeq \frac{f_\pi g_{\pi NN}}{q^2 - m_\pi^2}$$

$\bar{q}\gamma_5 q$ $\quad$ F_P

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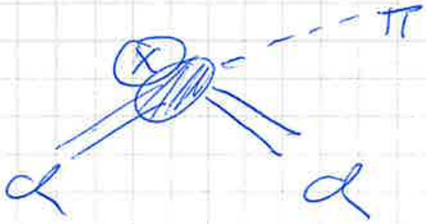
Another fun Example

coh π prod.

$$\langle \alpha\, \pi | A_\mu | \alpha \rangle = \frac{f_\pi\, q_\mu}{q^2 - M_\pi^2} T_{A\pi} + A_\mu^{NP}$$



$$\langle \alpha\, \pi | \partial^\mu A_\mu | \alpha \rangle = \frac{f_\pi M_\pi^2}{q^2 - M_\pi^2} T_{A\pi} + f_\pi M_\pi^2 B^{NP}$$

$$T_{A\pi} = -q^\mu A_\mu^{NP} + B^{NP} M_\pi^2 f_\pi$$

Now consider

$$\nu A \to \nu A \pi$$

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Decompose A_μ into $\underbrace{L, T}_{\text{small}}, q_{\parallel}, q_{\perp}$

$$i\mathcal{M} \simeq \frac{(q^\mu A_\mu)\cdot \underbrace{q_\mu \cdot L^\mu}_{=0}}{q^2} + \frac{(\tilde{q}^\mu \cdot A_\mu)\,\tilde{q}^\mu \cdot L_\mu}{\tilde{q}^2}$$

$\uparrow$ conserved lepton current

$$\tilde{q}\cdot q = 0$$

$$\tilde{q}^\mu A_\mu = \tilde{q}^\mu A_\mu^{NP} \simeq q^\mu A_\mu^{NP} + \mathcal{O}\left(\frac{m_\pi^2}{E_\pi^2}\right)$$

$$\simeq \underline{T_{A\pi}} + \mathcal{O}\left(\frac{m_\pi^2}{E_\pi^2}\right)$$

Notice $\mathcal{M}$ dominated by $\tilde{q}$ (transverse to q) but PCAC still fixes ampl.