QE + 2p2h (SF) …. And more

NuSTEC 2024 Summer school
CERN, June 5-13, 2024

Noemi Rocco

Practical Info

- Some of the Figures have been taken from papers published in peer reviews. The references are reported as:

  <u>Authors</u>, Journal's name and # of the paper

  If there is anything you find interesting, I strongly encourage you to download the paper and read it!

- I also included some suggestions for more 'pedagogical' readings. The references are indicated as

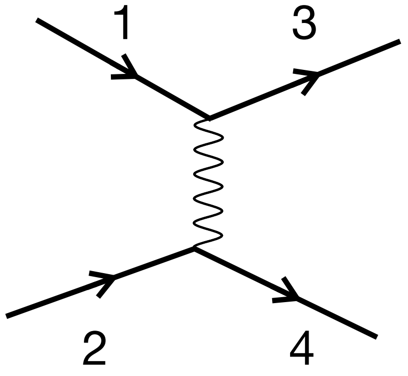
Author, Title of the Book/Journal

- Please, ask question! Now or later: nrocco@fnal.gov

Outline

- Lepton - nucleon interactions
- Modeling nuclear structure
- Ab - initio description of lepton - nucleus interactions

Electron-nucleon scattering



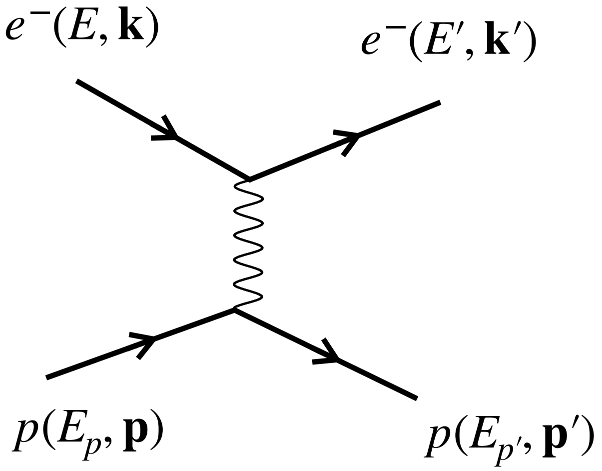
- We start from a generic process: 1+2 ➜ 3+4

- The cross section can be written as

$$d\sigma = \frac{1}{\text{flux}}\frac{1}{2E_1}\frac{1}{2E_2}|\mathcal{A}|^2\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

F. Close, An Introduction to quark and partons

- For scattering of an electron on a nucleon at rest in the lab frame:



$$\text{flux} = 2EM\,|\mathbf{v}_k - \mathbf{v}_p| = 2EM\,|\frac{k}{E}| \simeq 2EM$$

- The squared amplitude expression is given by

$$\frac{1}{(2s_e^i+1)(2s_p^i+1)}\sum_{\text{allspin}}|\mathscr{A}|^2$$

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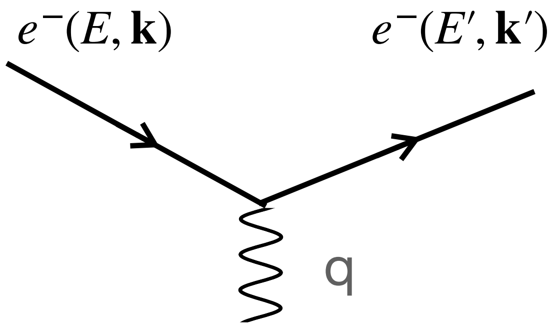
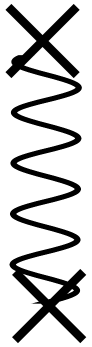
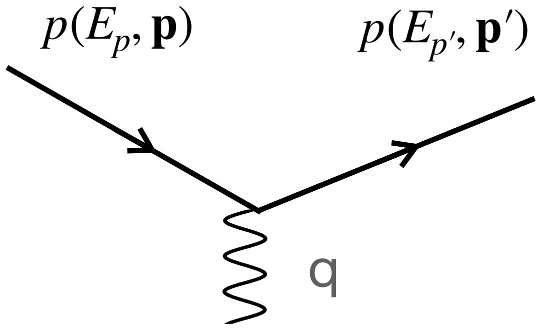
Electron-nucleon scattering



$\bar{u}(\mathbf{k}',\lambda')\gamma^\mu u(\mathbf{k},\lambda)$



$$\frac{e^2\, g^{\mu\nu}}{q^2}$$



$\bar{u}(\mathbf{p}',\alpha')\Gamma^\mu(q)u(\mathbf{p},\alpha)$

- We can put together what we learned and rewrite the squared amplitude as:

$$|\mathcal{A}|^2 = \frac{1}{4}\sum_{\lambda,\lambda',\alpha,\alpha'}\left|\bar{u}(\mathbf{k}',\lambda')\gamma^\mu u(\mathbf{k},\lambda)\frac{e^2}{q^2}\bar{u}(\mathbf{p}',\alpha')\Gamma_\mu u(\mathbf{p},\alpha)\right|^2 = \frac{e^4}{q^4}L_{\mu\nu}W^{\mu\nu}$$

- How do we define the leptonic and hadronic tensor? Let's start from the leptonic one

$$L_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\not{k}'\gamma^\mu\not{k}\gamma^\nu\right] = 2\left(k'^\mu k^\nu + k^\mu k'^\nu - g^{\mu\nu}k'\cdot k\right)$$

We neglected the electron mass in the expression of the leptonic tensor

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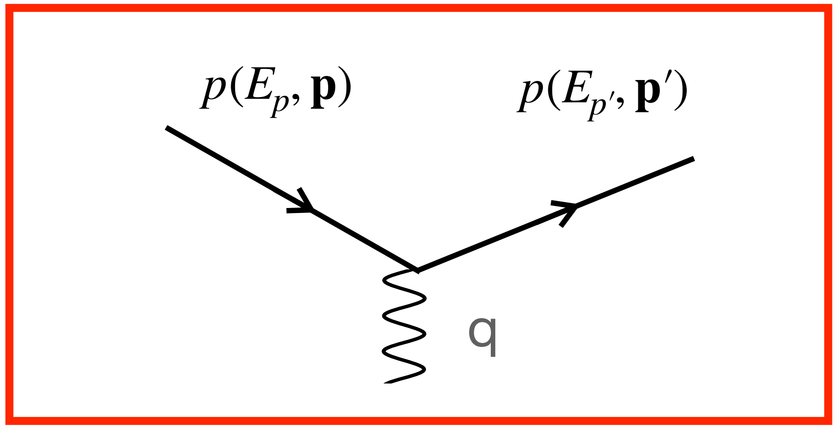
Electron-nucleon scattering



$$\bar{u}(\mathbf{p}', \alpha')\Gamma^\mu(q)u(\mathbf{p}, \alpha) = \bar{u}(\mathbf{p}', \alpha')\left[F_1\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2\right]u(\mathbf{p}, \alpha)$$

- The Dirac and Pauli form factors are **corrections to "point-like coupling"** which comes from the fact that the nucleon has an internal structure

- Alternatively, F_1 and F_2 are written as a combination of the electric and magnetic form factors:

$$F_1^{p,n} = \frac{G_E^{p,n} - q^2/(4M^2)G_M^{p,n}}{1 - q^2/(4M^2)} \qquad F_2^{p,n} = \frac{G_M^{p,n} - G_E^{p,n}}{1 - q^2/(4M^2)}$$

- The form factors are related to the **spatial distributions of the charge and magnetization** in the proton, and in the non relativistic limit are simply the Fourier transforms of these distributions.

  - The accurate determination of G_E and G_M is an important focus of both experimental and theory programs (see slide 7)

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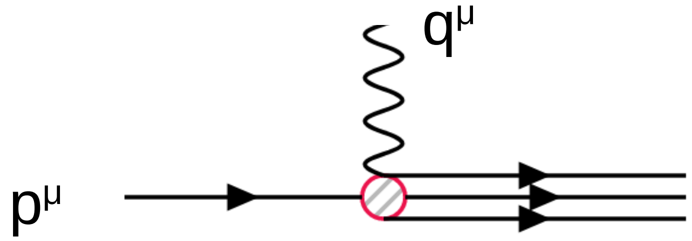
Hadronic tensor



- The most general expression for the hadronic tensor reads

$$W^{\mu\nu} = W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + i\frac{\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2M^2} W_3 + \frac{W_4}{M^2} q_\mu q_\nu + \frac{W_5}{M^2}(p^\mu q^\nu - p^\nu q^\mu)$$

- For the electromagnetic case, we can use current conservation, this allows us the rewrite the hadronic tensor as

$$W^{\mu\nu} = W_1\Big(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\Big) + \frac{W_2}{M^2}\Big(p^\mu - \frac{p\cdot q}{q^2} q^\mu\Big)\Big(p^\nu - \frac{p\cdot q}{q^2} q^\nu\Big)$$

Using this general expression of the hadronic tensor, the differential electron-proton cross section reads

$$\frac{d\sigma}{dE' d\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\Big[2W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2}\Big]$$

Note that for elastic scattering, the structure functions read:

$$W_1 = -\frac{q^2}{4M^2}(F_1 + F_2)^2 \delta\Big(\omega + \frac{q^2}{2M}\Big)$$

$$W_2 = \Big(F_1^2 - \frac{q^2}{4M^2} F_2^2\Big)\delta\Big(\omega + \frac{q^2}{2M}\Big)$$

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Hadronic tensor

- The most general expression for the had

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Suggested problem:

Use current conservation to show that the hadron tensor reduces to this expression for the electromagnetic case

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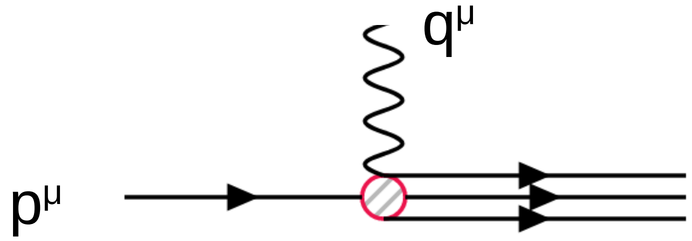
Hadronic tensor



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Suggested problem:

Show that the energy and momentum delta function can be rewritten in the following way for an elastic scattering on a nucleon at rest: $\delta^{(4)}(p' - p - q) \to \frac{1}{2m_N}\delta(\omega + q^2/(2m_N))$

Using this general expression of the ha

$$\frac{d\sigma}{dE' d\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\Big[2W_1 \sin^2\frac{\theta}{2} + W_2$$

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Summary of electron-nucleon scattering

- We consider the process:

$$\ell^-(k) + N(p) \to \ell^-(k') + N(p')$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4E_k^2 \sin^4\theta/2} \cos^2\frac{\theta}{2}$$

← Scattering on a point-like spinless target

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[1 - \frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$$

← Scattering on a point-like 1/2 spin target

- Protons and neutrons have an internal structure: described by electric and magnetic form factors

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{G_E^2 - \frac{q^2}{4M^2}G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2}G_M^2 \tan^2\frac{\theta}{2}\right]$$

Rosenbluth separation

Determination of nucleon form factors

- A reduced cross section can be defined as

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \times \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1+\tau)}$$

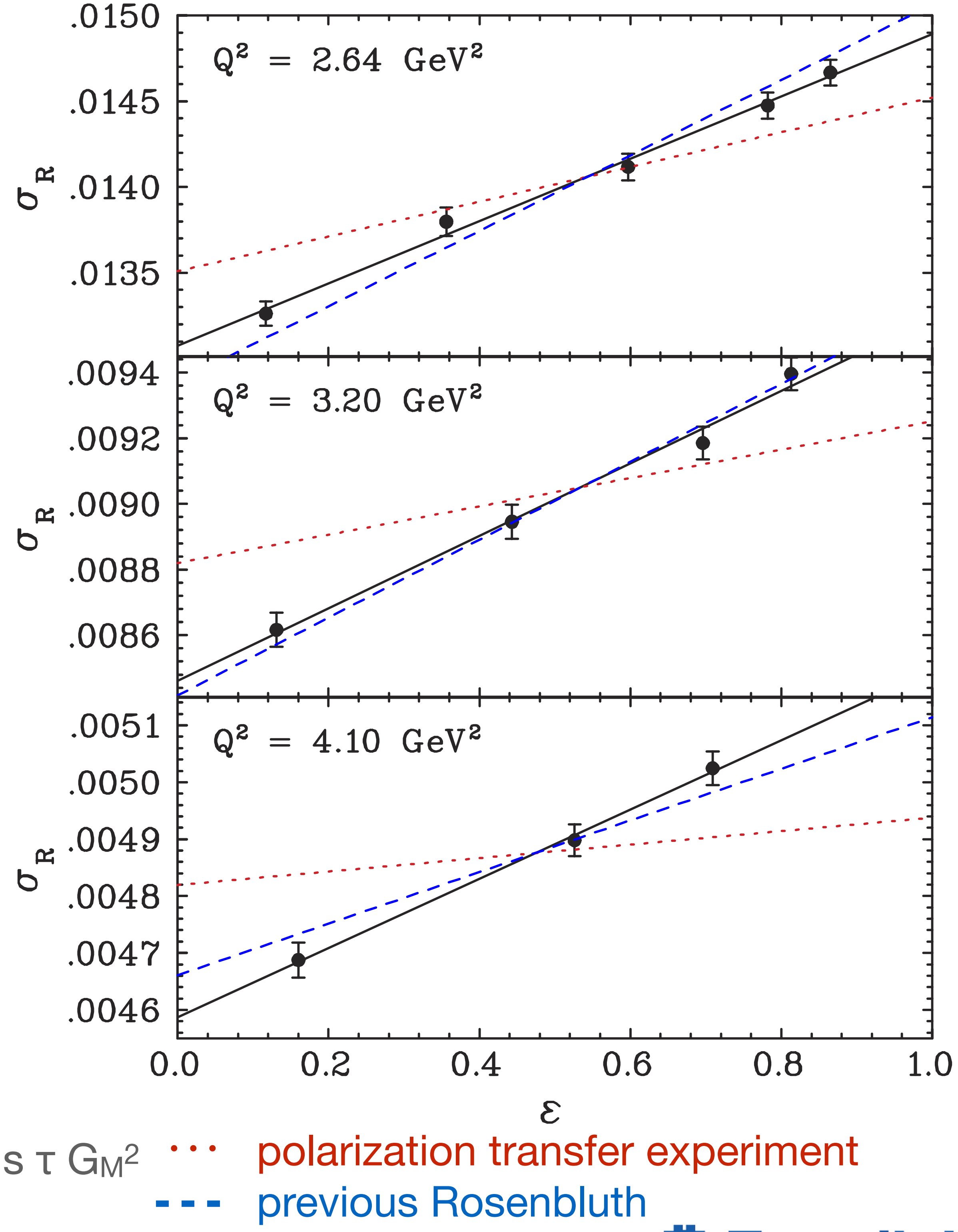
- The virtual photon polarization parameter is

$$\epsilon = \left[1 + 2(\tau+1)\tan^2\frac{\theta}{2}\right]^{1}$$

- Measuring angular dependence of the cross section at fixed Q^2

$$\sigma_R = \epsilon(1+\tau)\frac{\sigma}{\sigma_{\text{Mott}}} = \epsilon G_E^2 + \tau G_M^2$$

✐ Qattan et al.,PRL 94, 142301 (2005)



- In Born approximation: G_E^2 is the slope and the intercept is τ G_M^2

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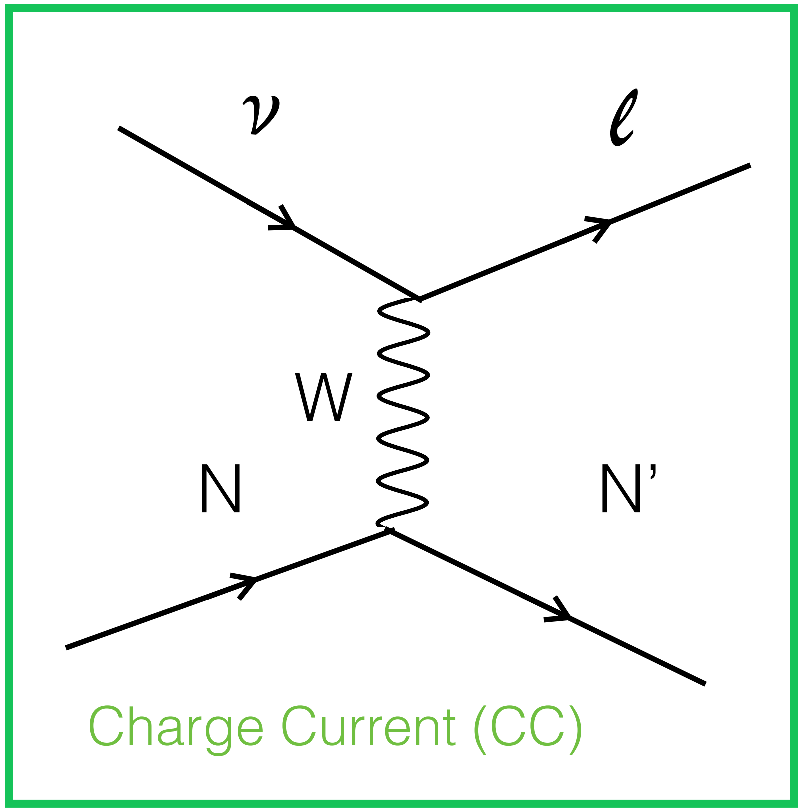
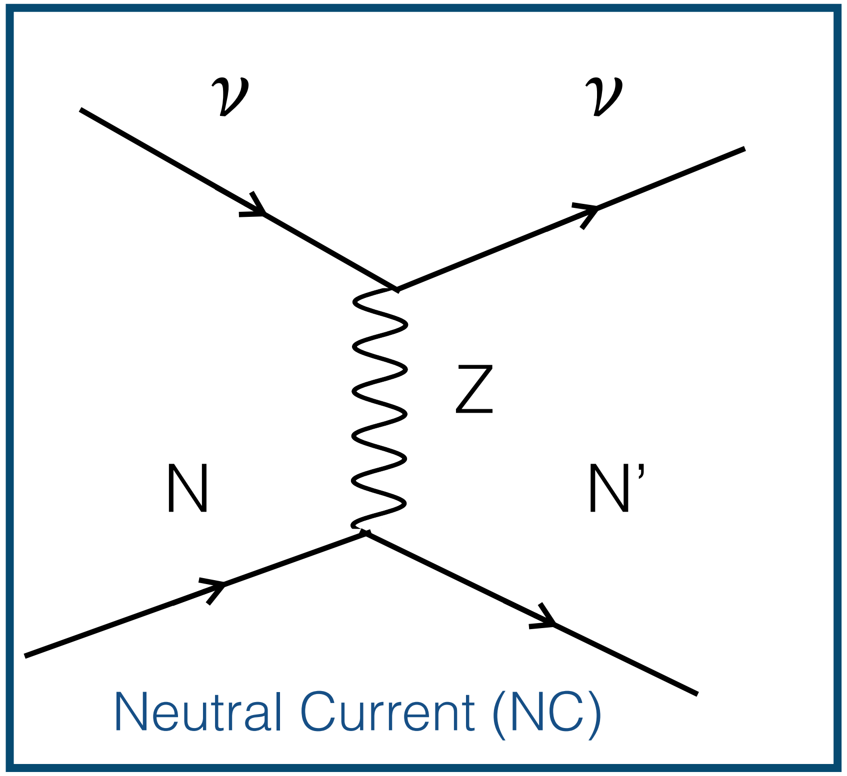
Neutrino-Nucleon scattering



Charge Current (CC)



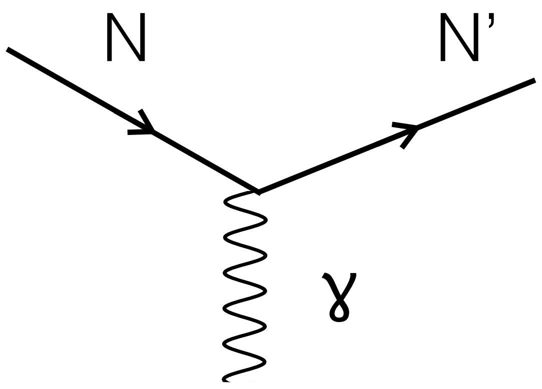
Neutral Current (NC)

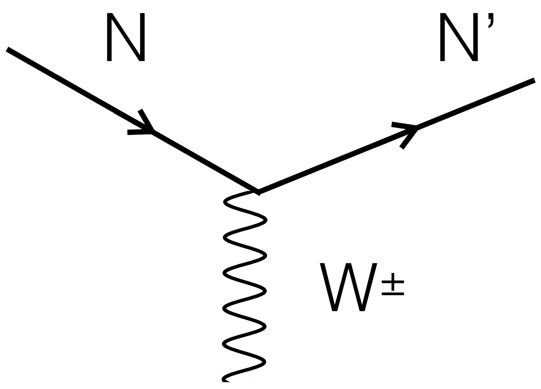
- Exchange of the W boson
- Lepton produced has the same flavor of the neutrino
- Initial and final nucleon have different isospin

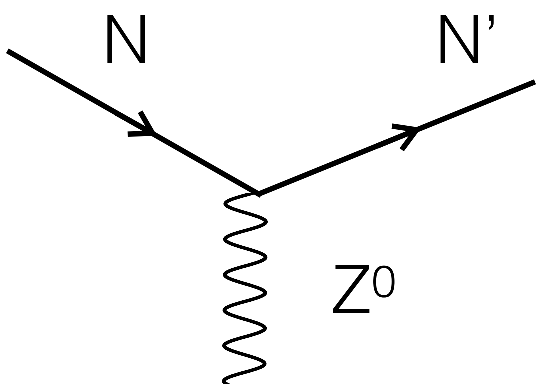
- Exchange of the Z boson
- Independent of the neutrino flavor
- Initial and final nucleon have same isospin

F. Close, An Introduction to quark and partons

Neutrino-Nucleon scattering







- EM

$$\mathcal{F}_1 = \frac{1}{2}[F_1^S + F_1^V \tau_z]$$

$$\mathcal{F}_2 = \frac{1}{2}[F_2^S + F_2^V \tau_z]$$

- CC

$$\mathcal{F}_1 = F_1^V \tau_\pm$$

$$\mathcal{F}_2 = F_2^V \tau_\pm$$

$$\mathcal{F}_A = F_A \tau_\pm$$

$$\mathcal{F}_P = F_P \tau_\pm$$

- NC

$$\mathcal{F}_1 = \frac{1}{2}[-2sin^2\theta_W F_1^S + (1-2sin^2\theta_W)F_1^V \tau_z]$$

$$\mathcal{F}_2 = \frac{1}{2}[-2sin^2\theta_W F_2^S + (1-2sin^2\theta_W)F_2^V \tau_z]$$

$$\mathcal{F}_A = \frac{1}{2}F_A \tau_z$$

$$\mathcal{F}_P = \frac{1}{2}F_p \tau_z$$

- We used the Conserved Vector Current hypothesis:

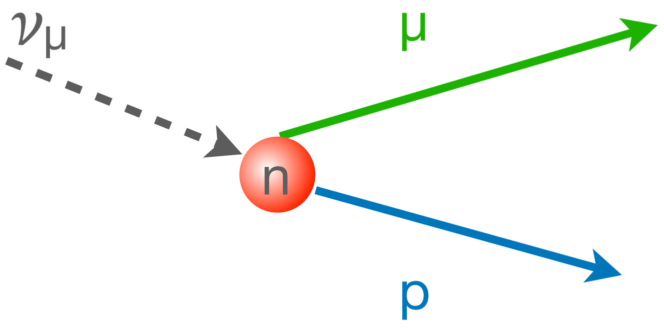
$$F_1^V \tau_z \to F_1^V \tau_\pm \ , \ F_2^V \tau_z \to F_2^V \tau_\pm$$

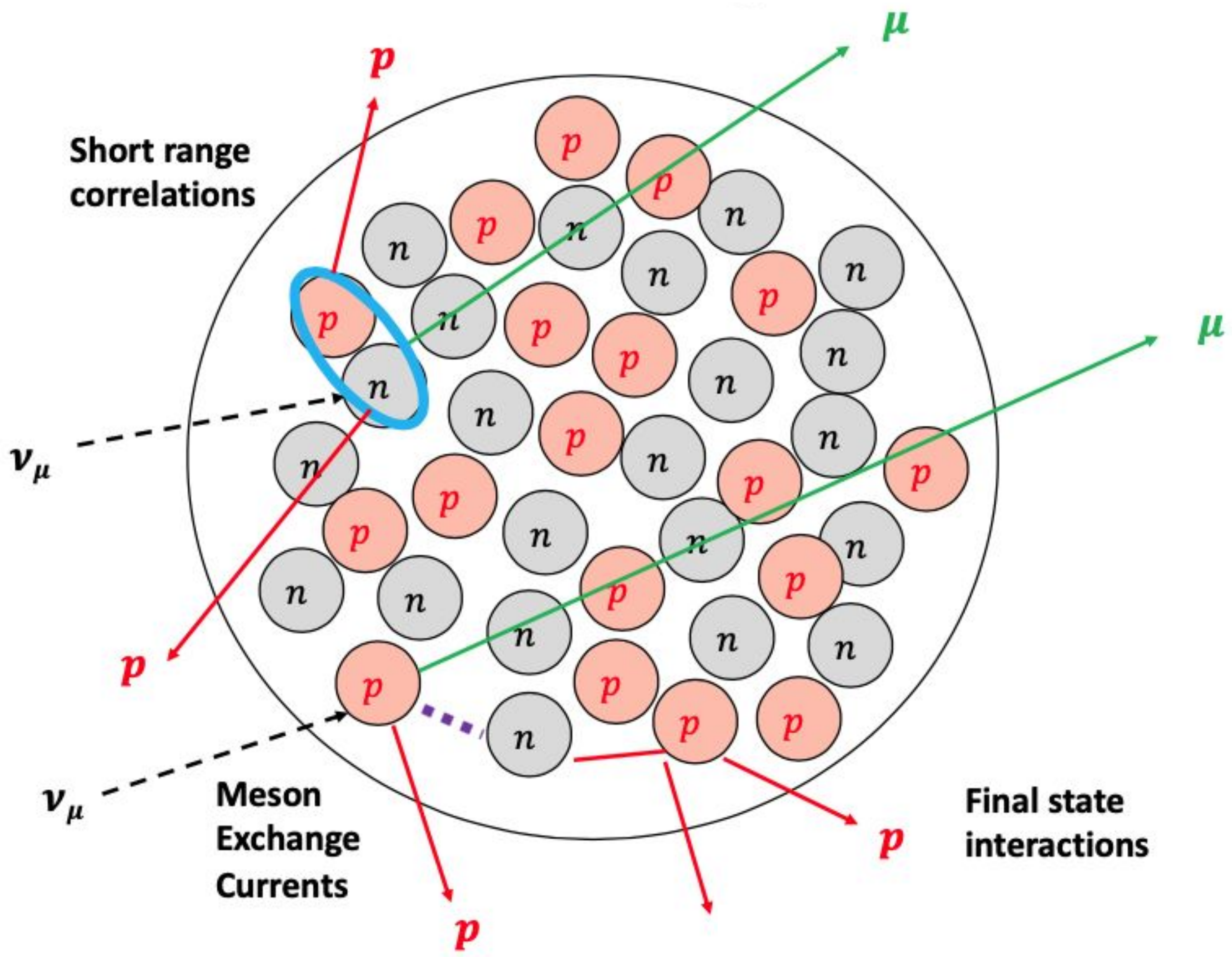
- PCAC:

$$F_P = \frac{2m_N^2}{(m_\pi^2 - q^2)} F_A$$

From theory to experiment

Free nucleon scattering case





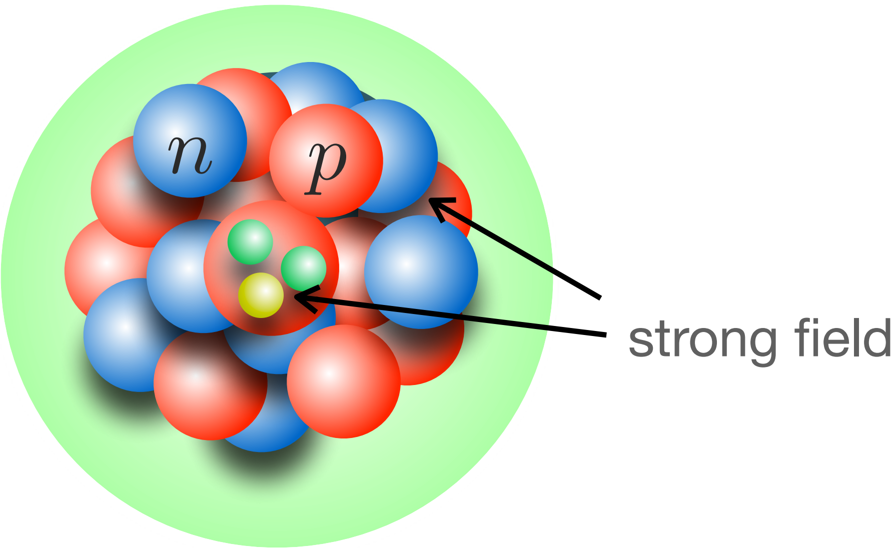
Nuclear model describing the target nucleus

Different reaction mechanisms depending on the momentum transferred to the the nucleus

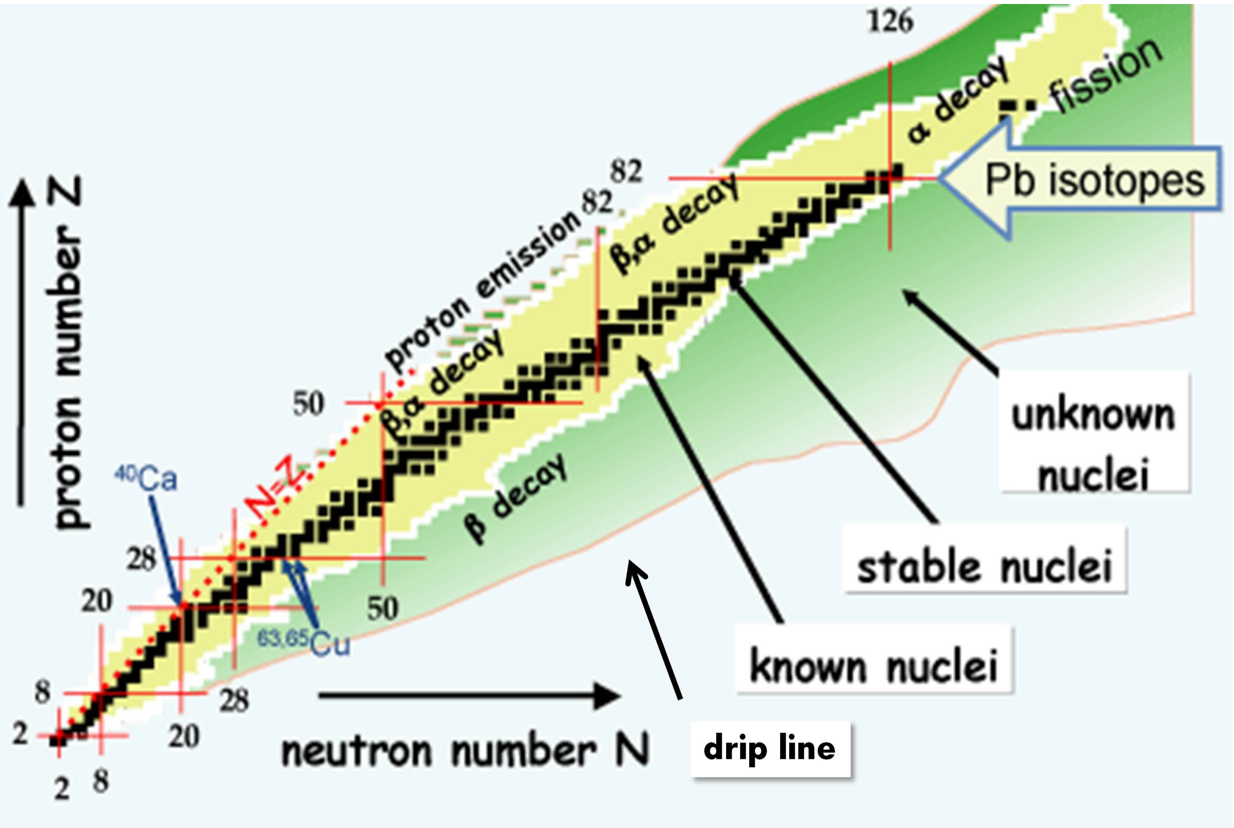
Final state interactions: describe how the particles propagate through the nuclear medium

The Nucleus internal structure

Nuclei are **strongly interacting** many body systems exhibiting fascinating properties



Nuclear chart. **Magic numbers** N or Z= 2, 8, 20, 28, 50 and 126; major shell complete and are more stable than other elements



The nucleus is formed by **protons** and **neutrons**: nucleons.

Each nucleon is made of three **quarks** held together by strong interactions→mediated by **gluons**

The nucleus is held together by the strong interactions between quark and gluons of neighboring nucleons

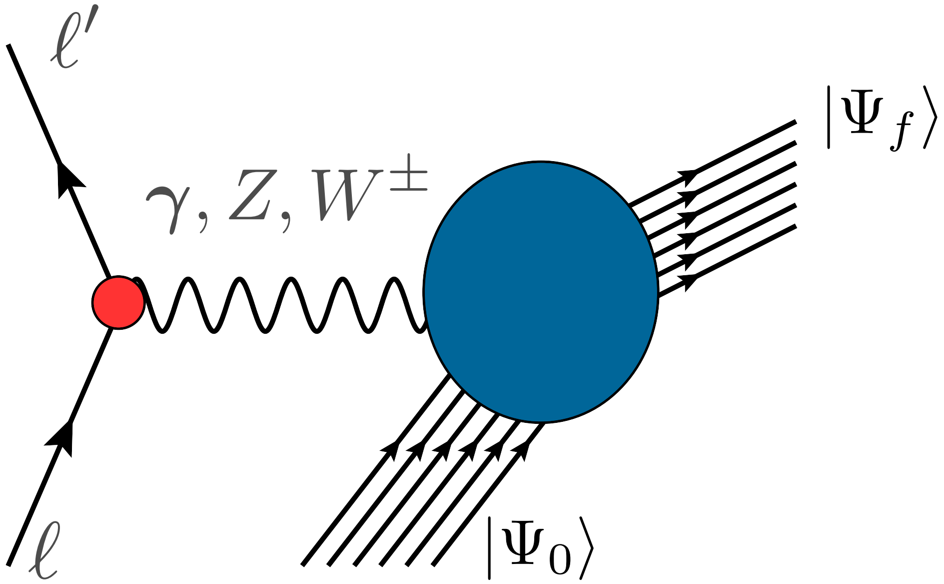
Nuclear Physicists **effectively** describe the interactions between protons and neutrons in terms of **exchange of pions**

Theory of lepton-nucleus scattering

- The cross section of the process in which a lepton scatters off a nucleus is given by

$$d\sigma \propto L^{\alpha\beta} R_{\alpha\beta}$$

Leptonic Tensor: is the same as before, completely determined by lepton kinematics



Hadronic Tensor: nuclear response function

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J_\alpha^\dagger(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

The initial and final wave functions describe many-body states:

$$|0\rangle = |\Psi_0^A\rangle \ , |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \ldots$$

For inclusive reactions, the hadronic final state is not detected. We need to sum over all the possible ones

Comparing electron- and neutrino-nucleus

- We start by defining the nuclear response functions, for a given value of **q** and ω

$$W^{\mu\nu}(\mathbf{q},\omega)=\sum_f \langle 0|(J^\mu)^\dagger(\mathbf{q},\omega)|f\rangle\langle f|J^\nu(\mathbf{q},\omega)|0\rangle\delta^{(4)}(p_0+\omega-p_f)$$

- Electron case we write the **inclusive** double differential cross section as:

$$\frac{d\sigma}{dE'd\Omega}=\sigma_{\text{Mott}}\left[\left(\frac{q^2}{\mathbf{q}^2}\right)^2\boxed{R_L}+\left(\frac{-q^2}{2\mathbf{q}^2}+tan^2\frac{\theta}{2}\right)\boxed{R_T}\right]$$

where: $R_L=W_{00}\quad,\quad R_T=W_{xx}+W_{yy}$

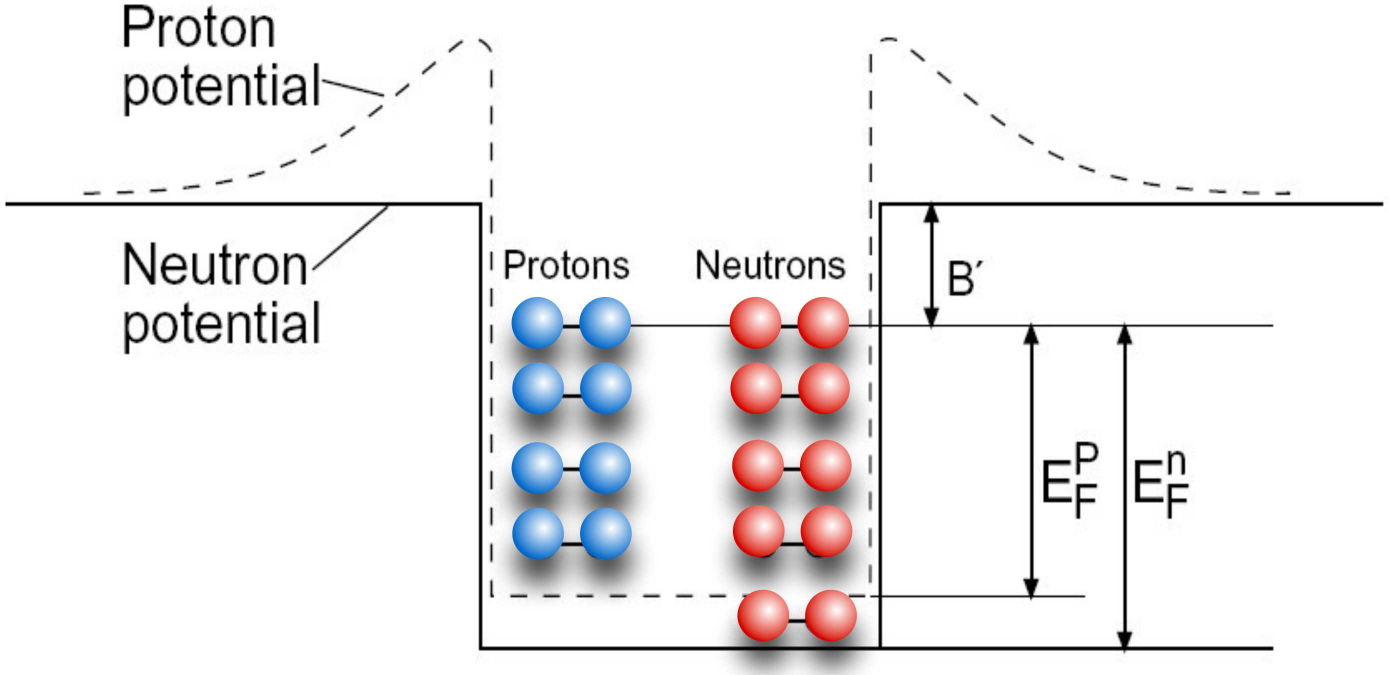
- Neutrino case:

$$\left(\frac{d\sigma}{dE'd\Omega}\right)_{\nu/\bar\nu}=\frac{G^2}{4\pi^2}\frac{k'}{2E_\nu}\left[\hat{L}_{CC}\boxed{R_{CC}}+2\hat{L}_{CL}\boxed{R_{CL}}+\hat{L}_{LL}\boxed{R_{LL}}+\hat{L}_T\boxed{R_T}\pm 2\hat{L}_{T'}\boxed{R_{T'}}\right],$$

- Where the nuclear responses are given by

$$R_{CC}=W^{00}\qquad R_{CL}=-\frac{1}{2}(W^{03}+W^{30})\quad,\quad R_{LL}=W^{33}\qquad R_T=W^{11}+W^{22}\quad,\quad R_{T'}=-\frac{i}{2}(W^{12}-W^{21})$$

Initial state: global Fermi gas



- Simple picture of the nucleus: only statistical correlations are retained (Pauli exclusion principle)

- Protons and neutrons are considered as moving freely within the nuclear volume

- The nuclear potential wells are rectangular: constant inside the nucleus and goes sharply to zero at its edge

- The energy of the highest occupied state is the Fermi energy: E_F

- The difference B' between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon B'/A ~ 7-8 MeV

C. Bertulani, Nuclear Physics in a Nutshell

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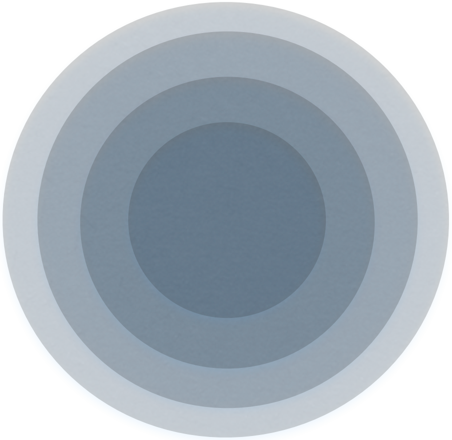
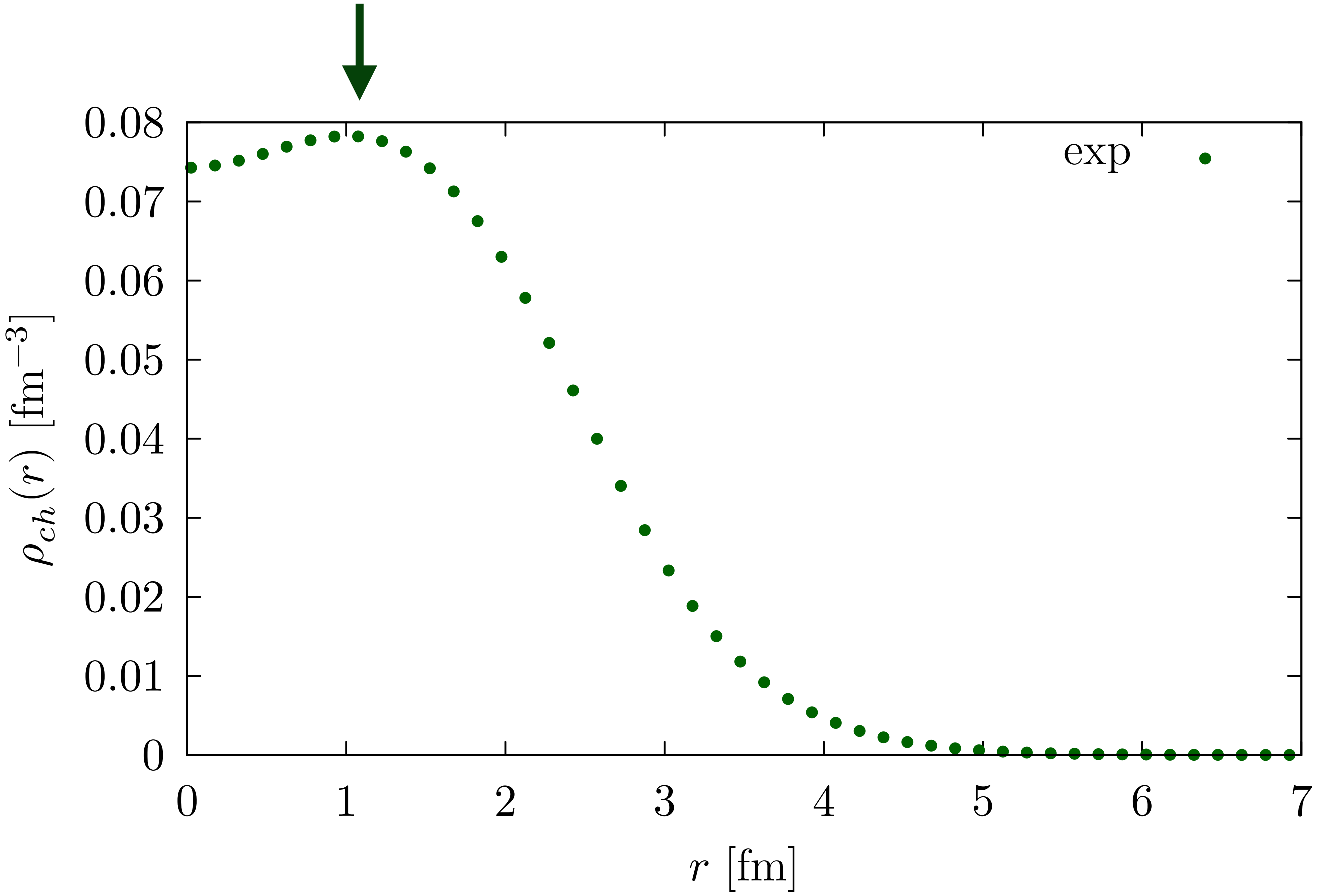
Initial state: Local Fermi gas

- A spherically symmetric nucleus can be approximated by concentric spheres of a constant density.



More likely to find a particle r ~ r_{ch}~ 2.5 fm



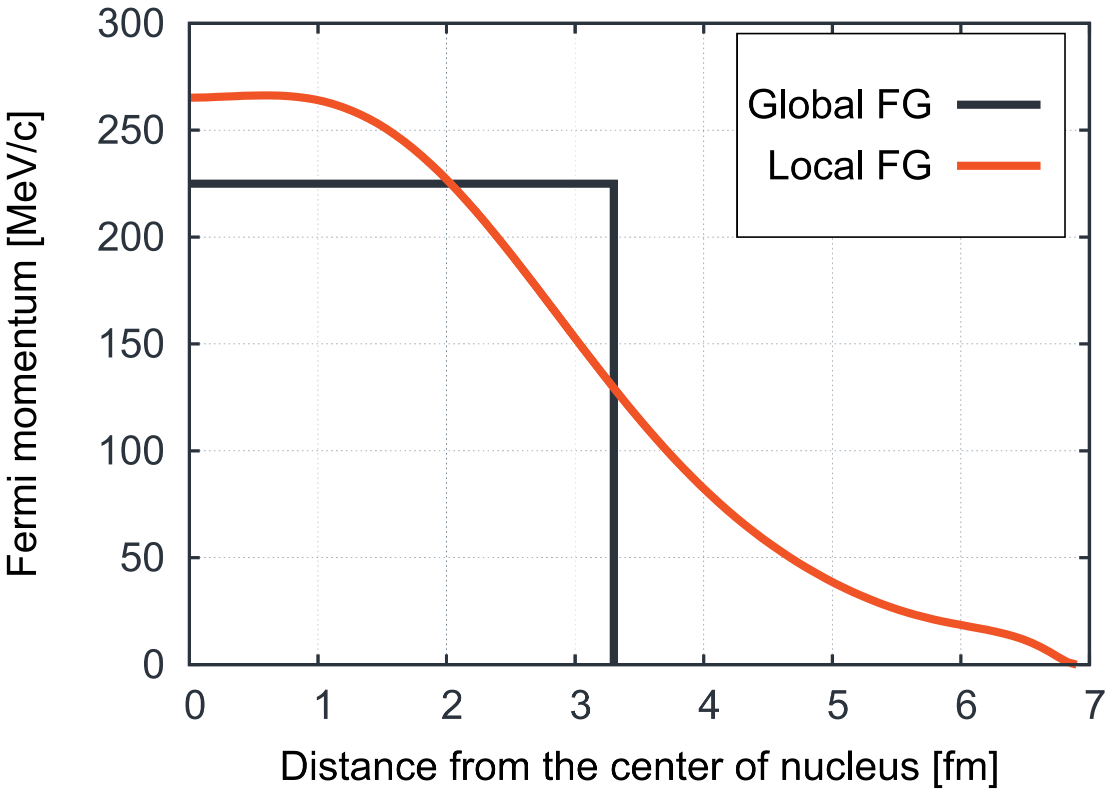


Figure by T. Golan

- **Global Fermi Gas**

$$p_F = \left(\frac{9\pi \cdot n}{4A}\right)^{1/3} \cdot \frac{\hbar}{R_0}$$

- **Local Fermi Gas**

$$p_F = \hbar\left(3\pi^2 \rho(r)\frac{n}{A}\right)^{1/3}$$

Initial state: shell Model

- As in the Fermi Gas model: the nucleons move within the nucleus independently of each other
- Difference: the nucleons are not free: subject to a central potential

- Each nucleon moves in an average potential created by the other nucleons, the potential should be chosen to best reproduce the experimental results

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \dots \longrightarrow H = \sum_i \frac{p_i^2}{2m} + \sum_i^A U_i + H_{\text{res}}$$

- We solve the Schrödinger Equation:

$$H\,\psi = E\,\psi \qquad \begin{cases} E = E_1 + E_2 + \ldots + E_A \\ \psi(1, \ldots, A) = \mathcal{A}[\phi_1(1) \ldots \phi_A(A)] \end{cases}$$

Initial state: shell Model

- Example: Particles are subject to an harmonic oscillator potential

$$U(r) = \frac{1}{2} m \omega^2 r^2$$

The frequency should be adapted to the mass number A

- We will seek solutions of the type $\psi(r) = \frac{u(r)}{r} Y_l^m(\theta, \phi)$ ← Spherical Harmonics

- Solving the Schrödinger Equation reduces to a solution of u:

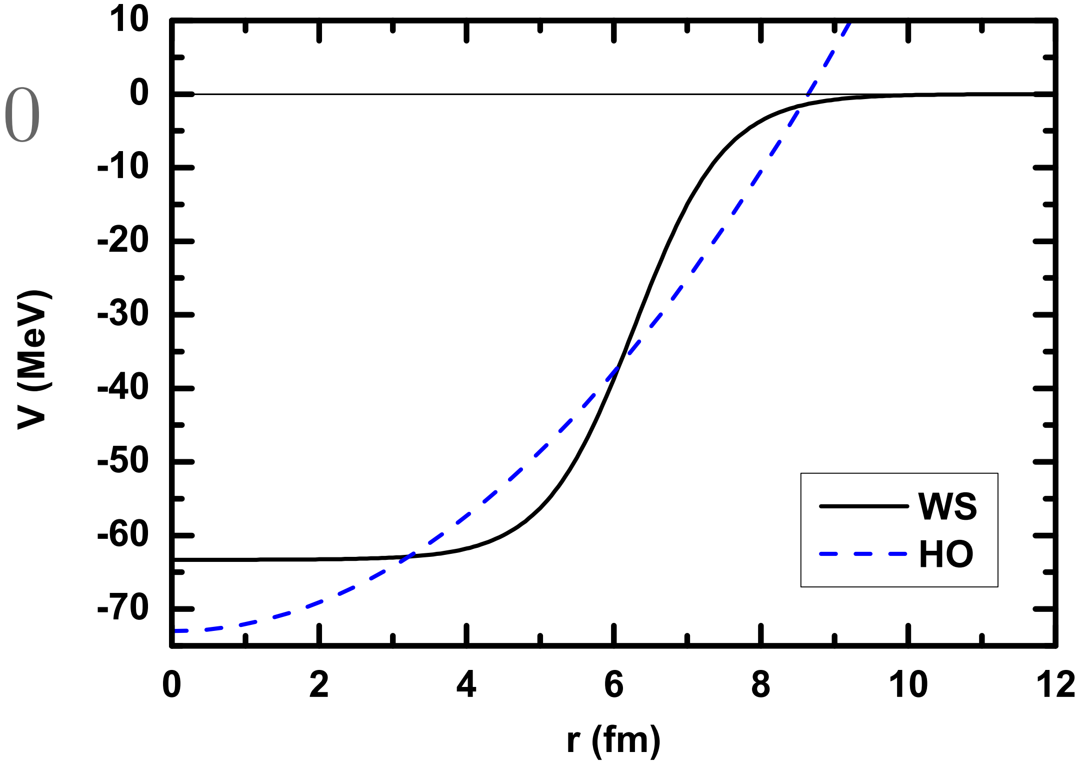
$$\frac{d^2u}{dr^2} + \left\{ \frac{2m}{\hbar^2}[E - U(r)] - \frac{l(l+1)}{r^2} \right\} u(r) = 0$$

$$E_{nl} = \hbar\omega \left(2n + l - \frac{1}{2} \right)$$ Eigenvalues

- A more realistic potential is the Wood Saxon:

$$V = V_0/[1 + exp[(r - R)/a]$$

V_0, r, a, are adjustable parameters chosen to best reproduce the **experimental results**



Physical Review C 87(1):014334

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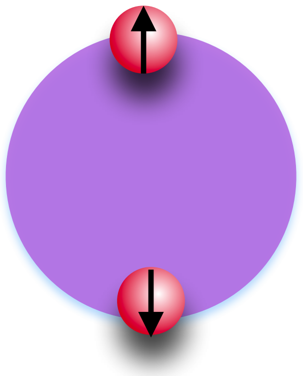
Nuclear Shell Model

The lowest level, s shell, can contain 2 protons



1s (m_s=0)   Z=2

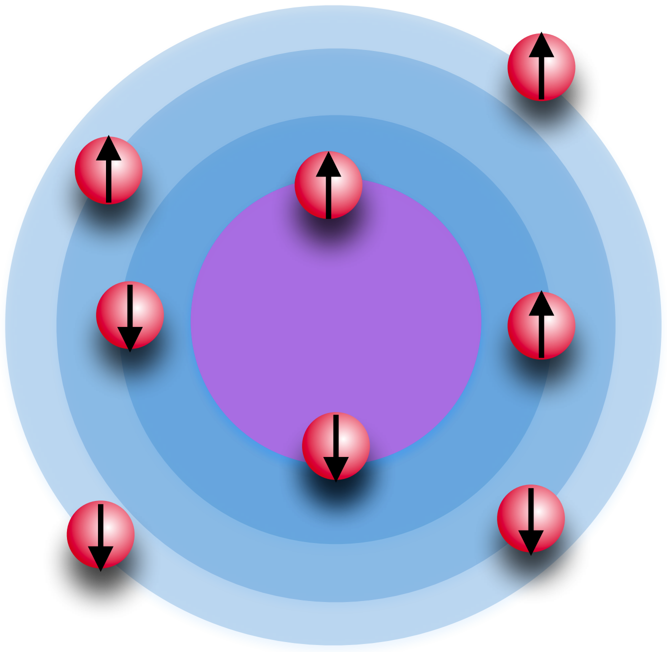
n is the principal quantum number, **l** orbital momentum, **m** magnetic quantum number

Our assumption: central potential

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

Nuclear Shell Model

The p shell can contain up to 6 protons



1s (m_s=0)
1p (m_p=-1,0,1)
Z=8

Our assumption: central potential

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

We explained the first two magic numbers: 2 and 8. We can follow the same strategy for the Z=20 case; but at the next step we obtain Z=40 **while experimentally Z=50**

n is the principal quantum number, **l** orbital momentum, **m** magnetic quantum number

Nuclear Shell Model

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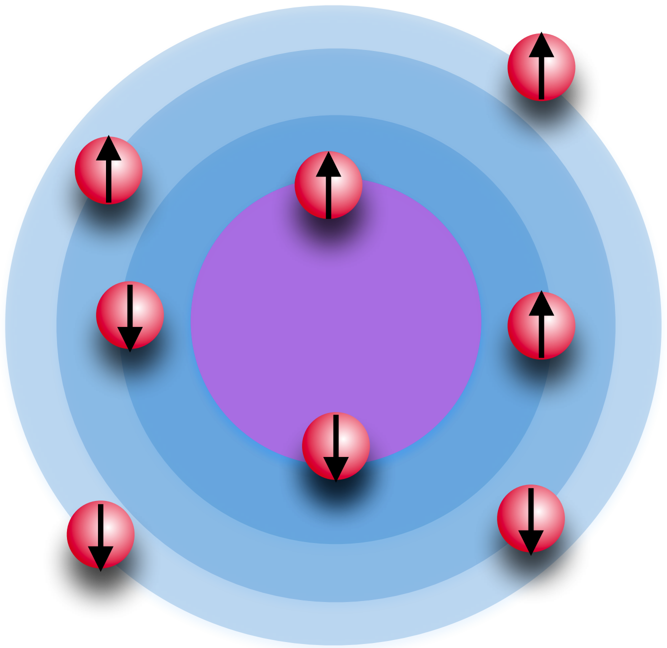

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Z=8

✐ Maria Goeppert Mayer poses with her colleagues in front of Argonne's Physics building.

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In 1963, Goeppert Mayer, Jensen, and Wigner shared the Nobel Prize for Physics "for their discoveries concerning nuclear shell structure."

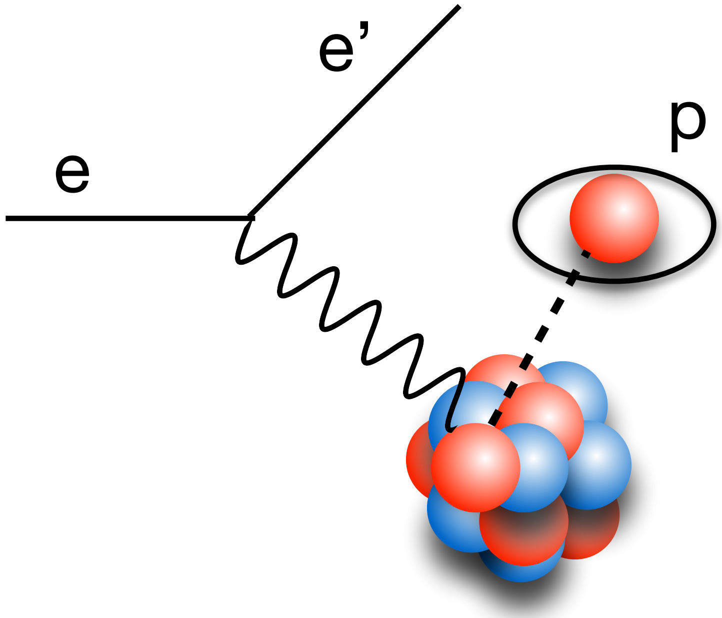
The solution to the puzzle lies in the **spin-orbit coupling**. This effect in the nuclear potential is 20 times larger then in Atomic Physics

$$V(r) \rightarrow V(r) + W(r)\mathbf{L}\cdot\mathbf{S}$$

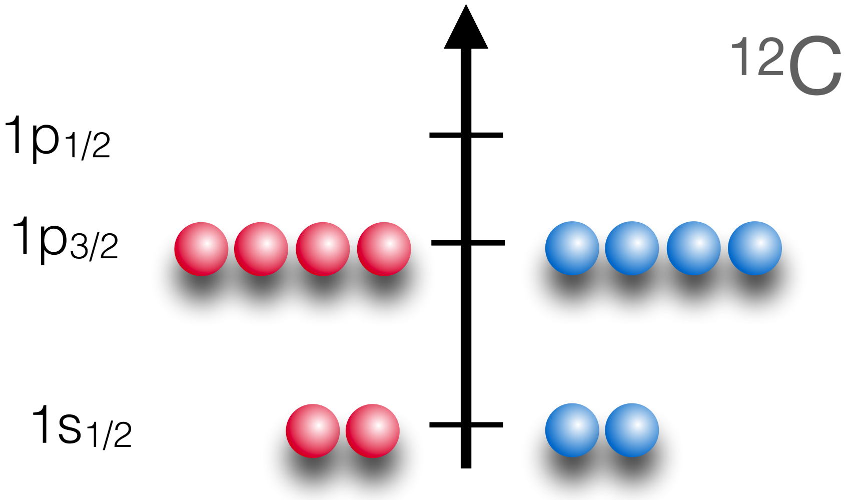
The spin-orbit introduces an energy split which modifies the shell structure and reproduces magic number up to Z=126

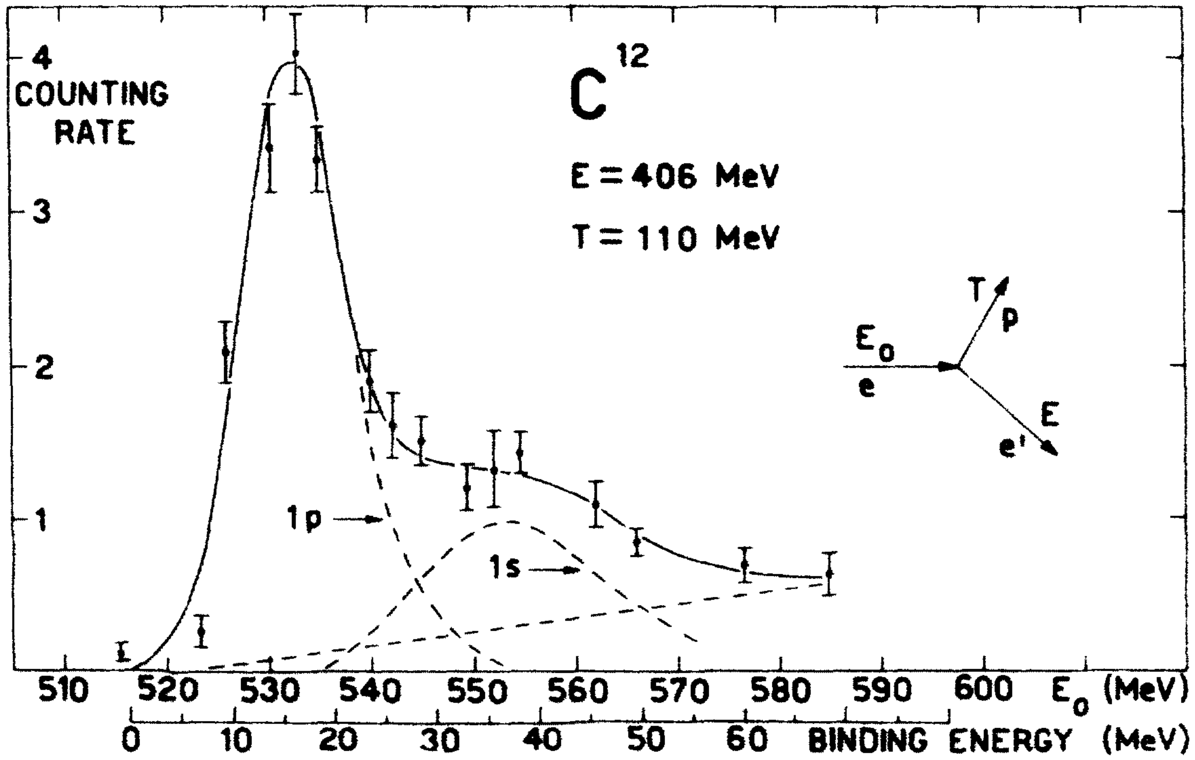
(e,e’p) scattering experiments

- (e,e’p) experiments are extremely important to investigate the internal structure of the nucleus



- Assuming NO FSI the energy and momentum of the initial nucleon can be identified with the measured p_{miss} and E_{miss}





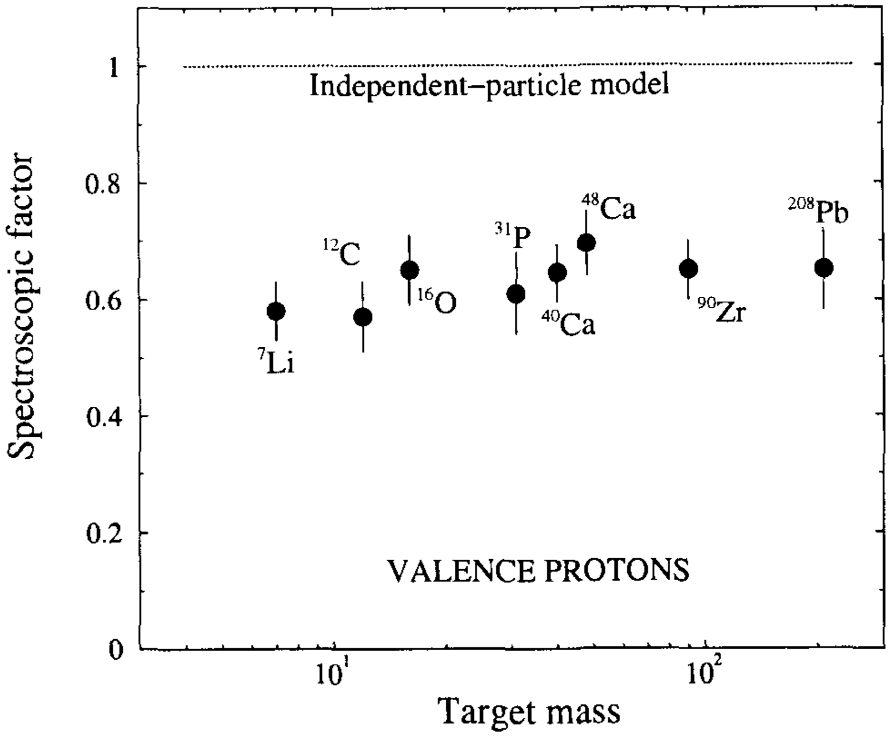
U.Amaldi et al, Phys. Rev. Lett. 13, 10 (1964)

- The peak coming from four 1p protons is visible

- The contribution of the two 1s protons is not clearly separated with this resolution

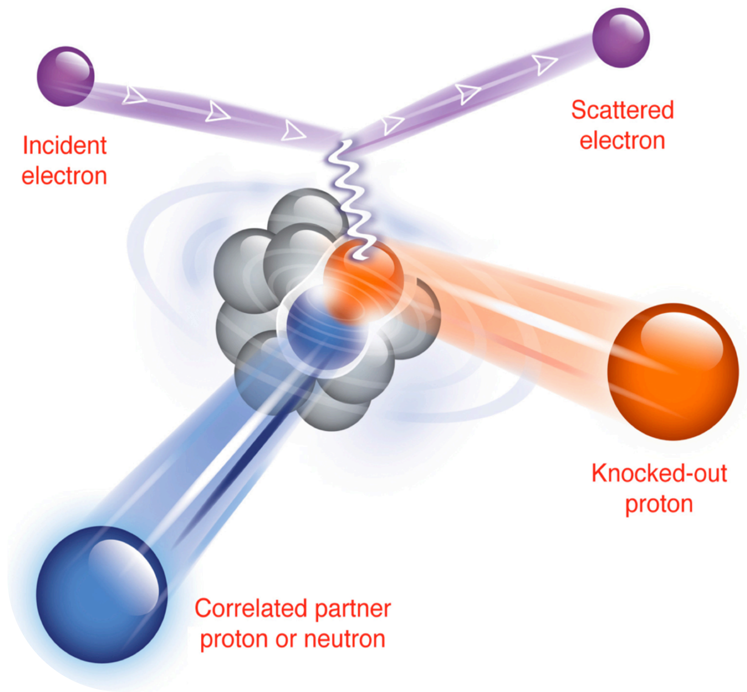
(e,e’p) scattering experiments

- Electron and proton experiments also pinned down the limitations of MF approaches



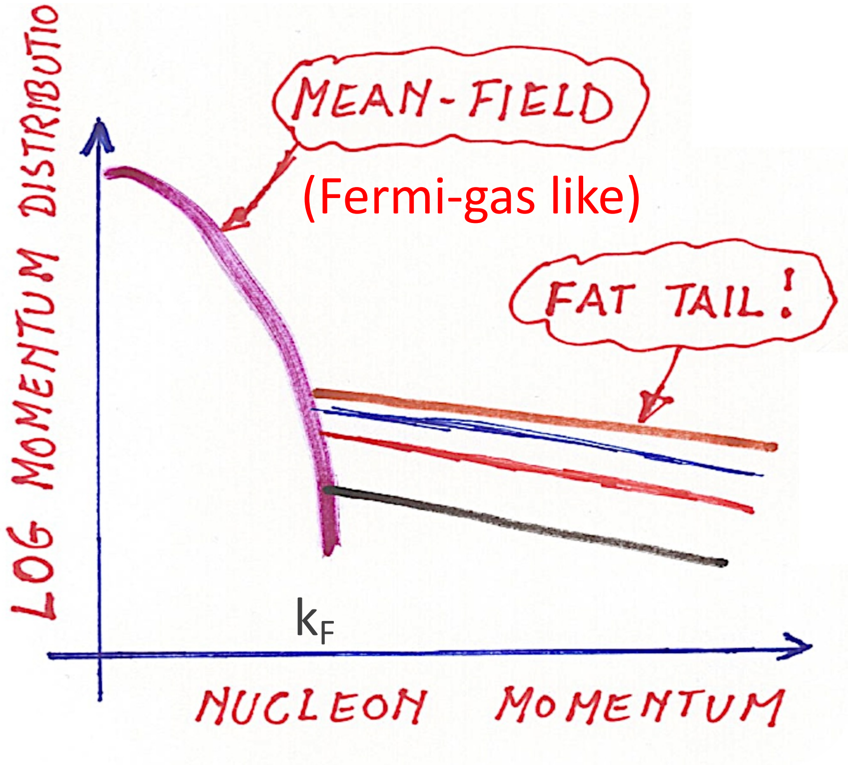
- Quenching of the spectroscopic factors of valence states has been confirmed by a number of high resolution (e,e’p) experiments

Subedi et al., Science 320, 1476 (2008)



- **Semi-exclusive 2N-SRC** experiments at x>1 allows to detect both nucleons and reconstruct the initial state

- Confirmed that the **high momentum tail** of the nuclear wave function consists mainly of 2N-SRC

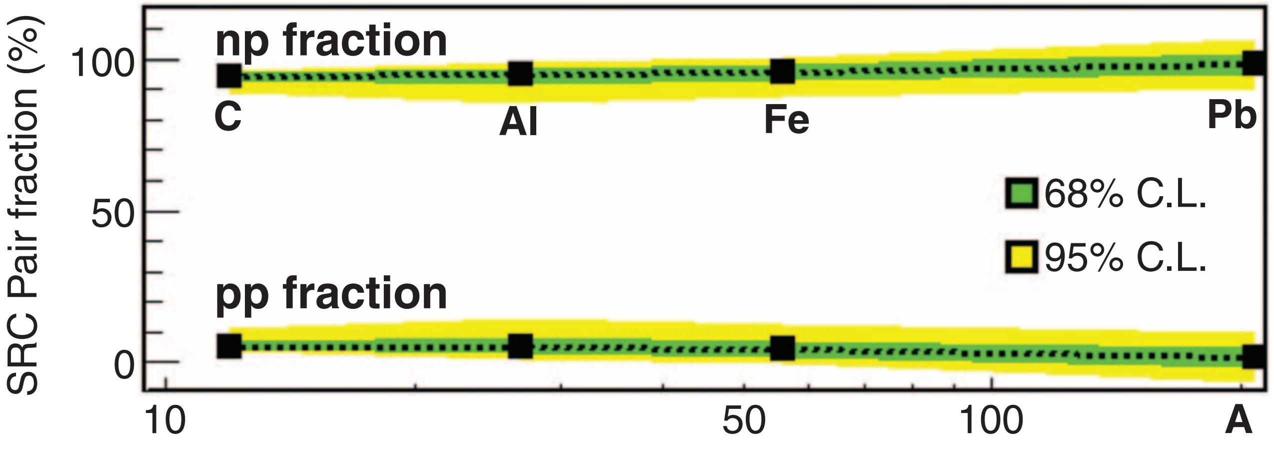


- The large-momentum (short-range) component of the wave function is dominated by the presence of Short Range Correlated (SRC) pairs of nucleons

Figure by Or Hen

(e,e'p) scattering experiments

- Observed dominance of np-over-pp pairs for a variety of nuclei



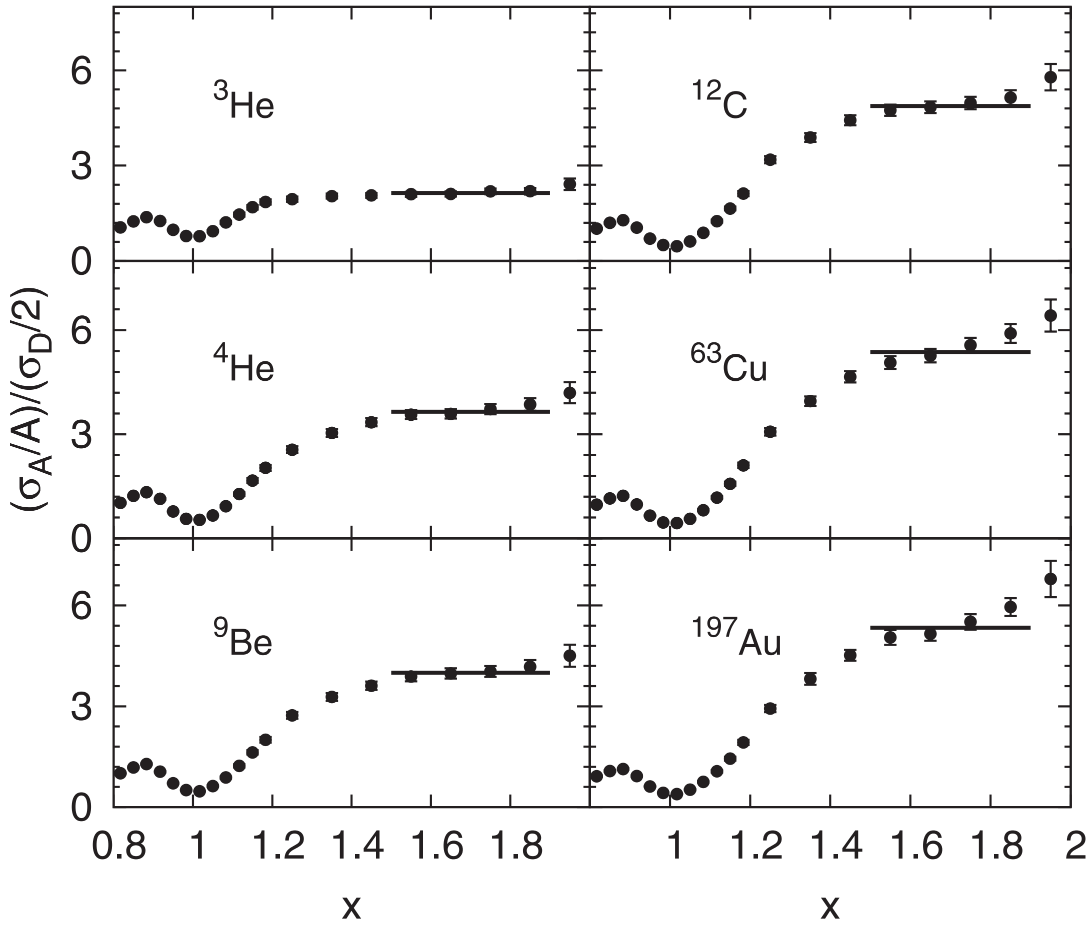
Subedi et al., Science 320, 1476 (2008)

- SRC pairs are in spin-triplet state, a consequence of the tensor part of the nucleon-nucleon interaction

Bottom Line

- Two-body Physics can not be neglected:
  - ~20% of the nucleons in nuclei
  - ~100% of the high k (>pF) nucleons
- Have large relative momentum and low center of mass momentum

- Universality of high-momentum component



N. Fomin et al., PRL 108, 092502 (2012)

- The cross section ratio: A/d, sensitive to $n_A(k)/n_d(k)$

- Observed scaling for x>1.5

$$n_A(k > p_F) = a_2(A) \times n_d(k)$$

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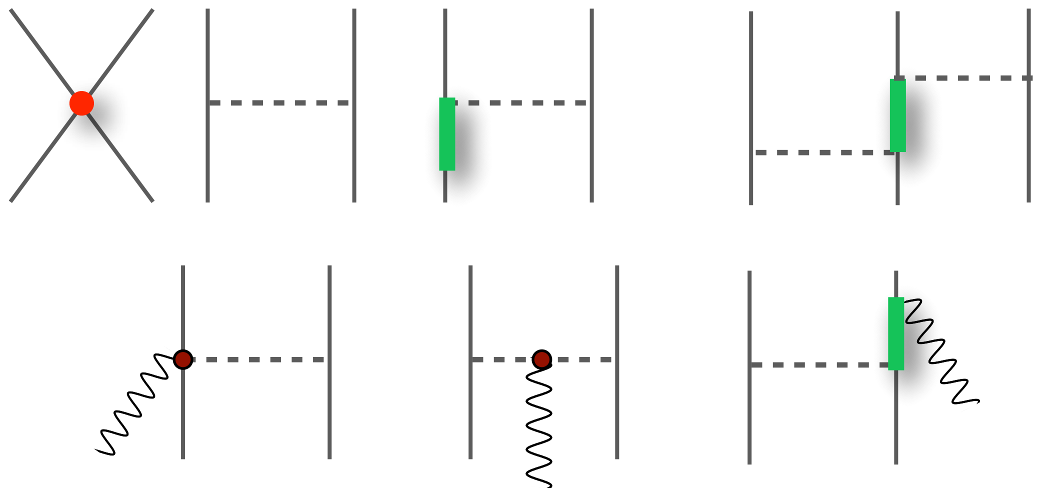
Figure by Or Hen

The basic model of nuclear theory

Effective Hamiltonians and consistent currents

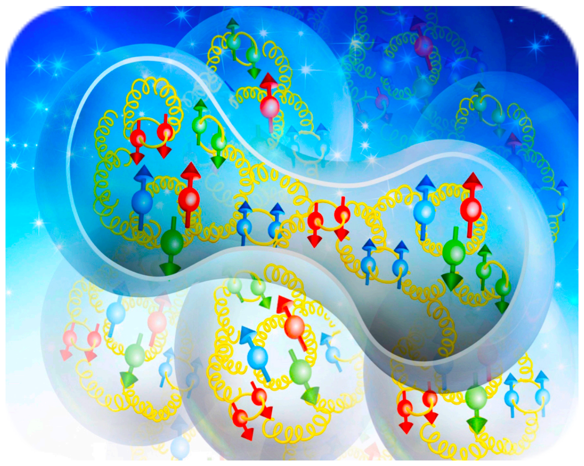


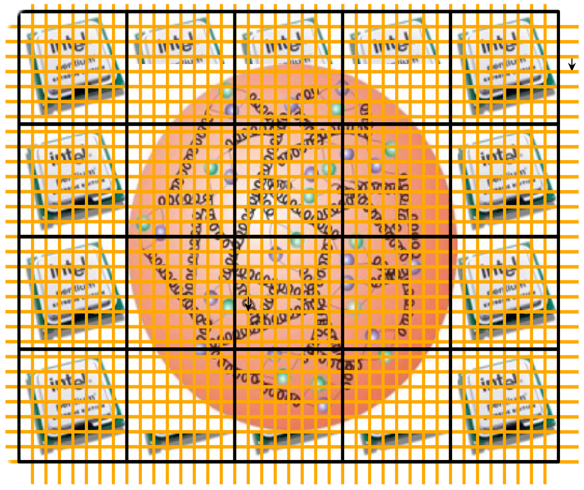
Accurate nuclear many-body methods

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

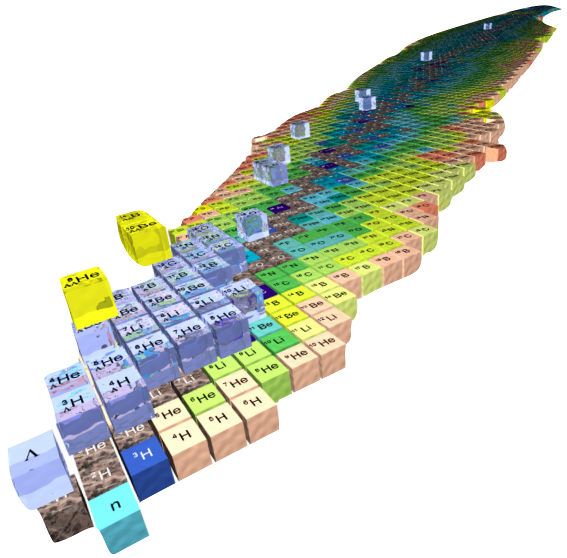
$$J_{mn} = \langle\Psi_m|J|\Psi_n\rangle$$

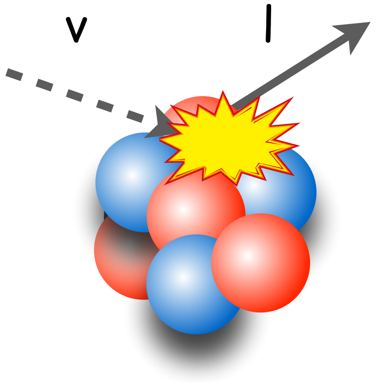
Quantum Chromodynamcs





Nuclei and electroweak interactions



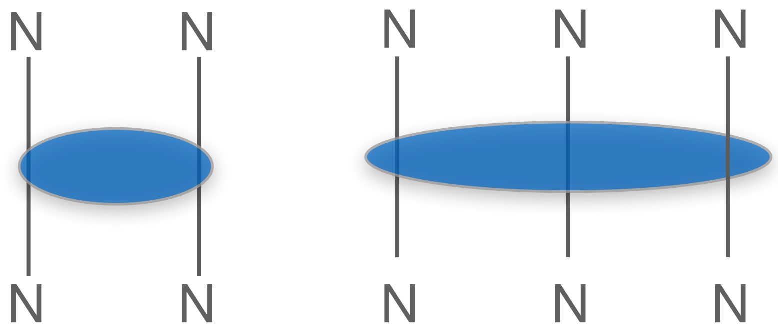


The basic model of nuclear theory

Effective field theories are the link between QCD and nuclear observables. At low energy, the effective degrees of freedom are pions and nucleons:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

1-body **2-body** **3-body**



R. Machleidt , D. R. Entem, Phys.Rept.503:1-75,2011

E. Epelbaum, (Lectures), arXiv:1001.3229

Different strategies to construct two- and three-body interactions

- Chiral Effective Field Theory interactions
- Phenomenological potentials

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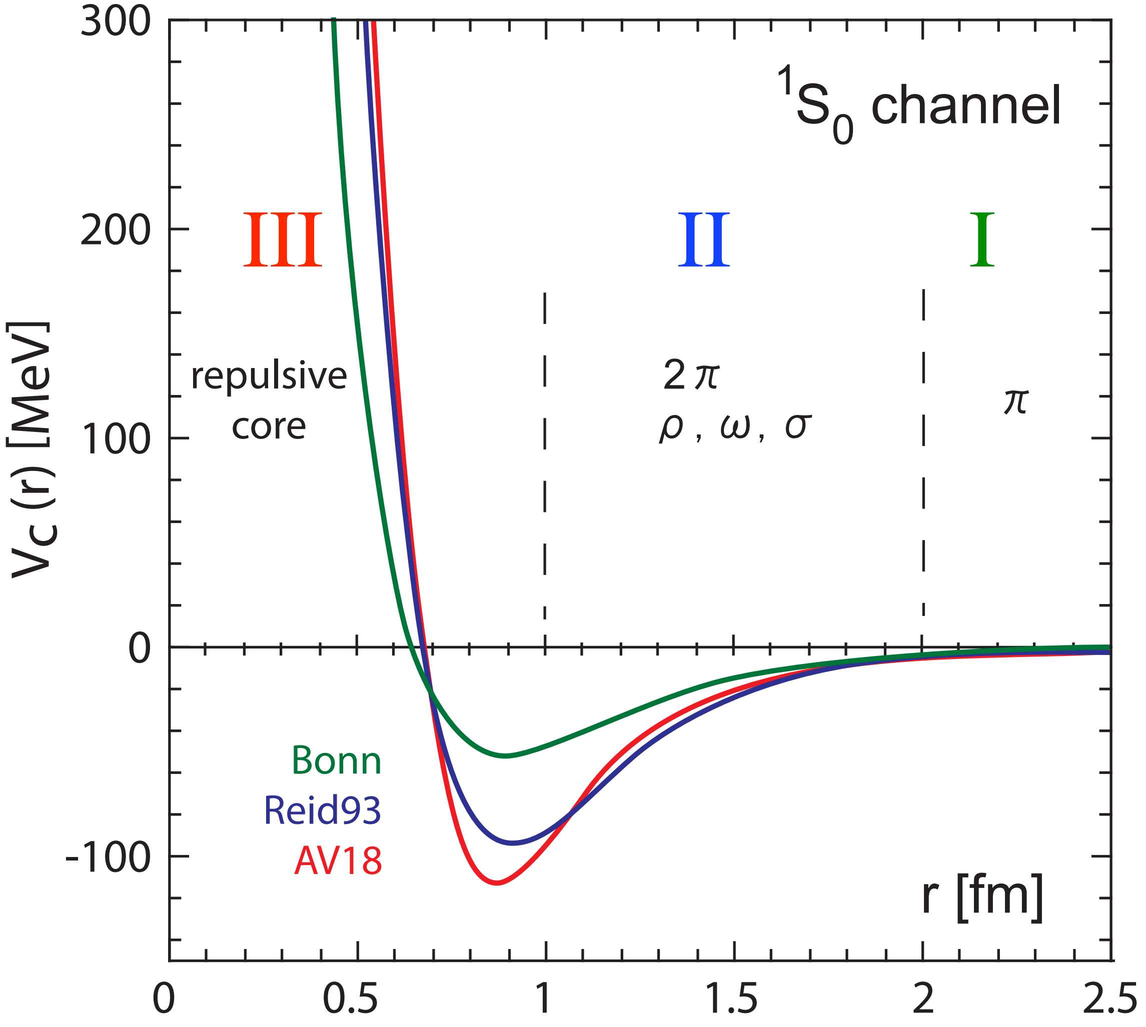
Nucleon-nucleon potential

Aoki et al. Comput.Sci.Disc.1(2008)015009



Bonn PRC 63, 024001, 2001

Reid93 PRC 49, 2950, 1994

AV18: Wiringa PRC 51, 38, 1995

Long range part: One **π** exchange

Range: $\frac{1}{m_\pi} \sim 1.4 \text{ fm}$

Medium range part: Two **π** exchange

Range: $\frac{1}{2m_\pi} \sim 0.7 \text{ fm}$

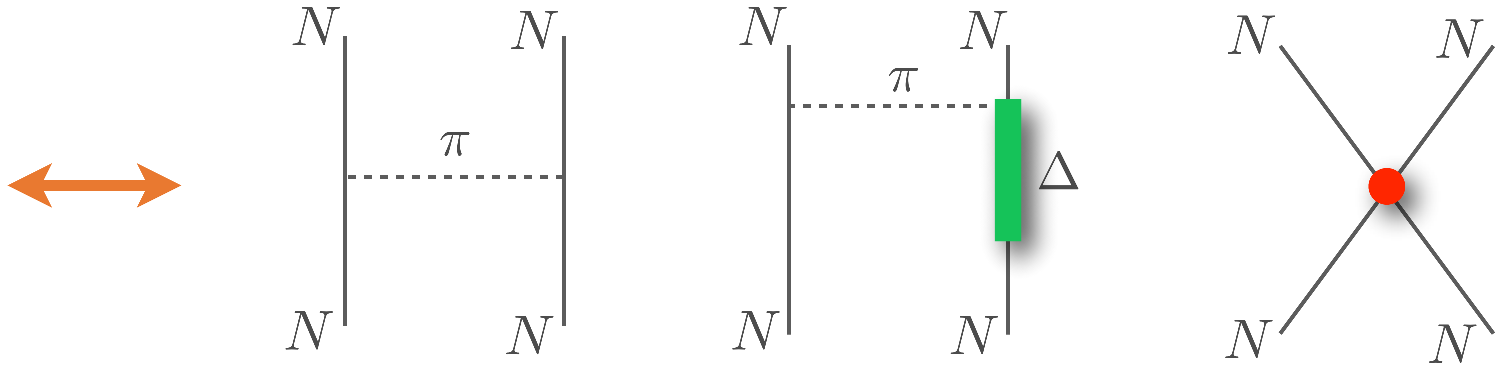
Short range part: Repulsive core

Phenomenological potential: av18 + IL7

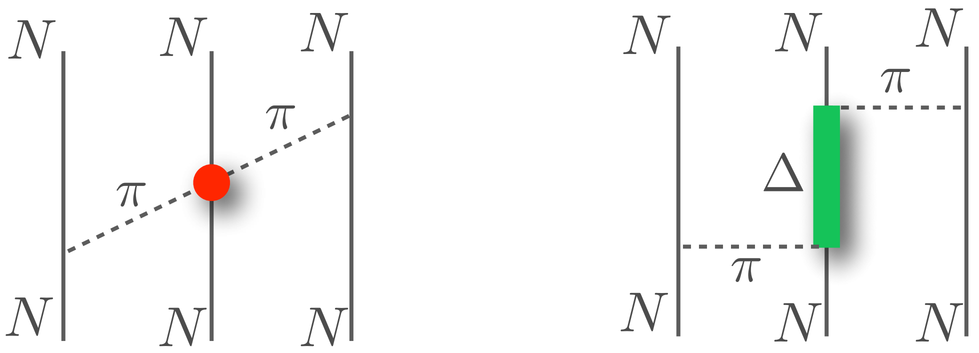
Phenomenological potentials explicitly include the **long-range one-pion exchange interaction** and a set of **intermediate- and short-range phenomenological terms**

- **Argonne v$_{18}$** is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij}) O^p_{ij}$$



- Phenomenological three-nucleon interactions, like the **Illinois 7**, effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects



The parameters of the AV18 + IL7 are fit to properties of **exactly solvable light nuclear systems**.

Chiral effective field theory

Chiral Hamiltonians exploits the (approximate) broken chiral symmetry of QCD

Identify the soft and hard scale of the problem $\mathcal{L}^{(n)} \sim \left(\frac{q}{\Lambda_b}\right)^n$

~ 100 MeV soft scale

~ 1 GeV hard scale

Design an organizational scheme that can distinguish between more and less important terms:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)}$$

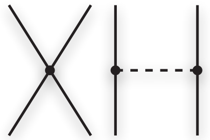
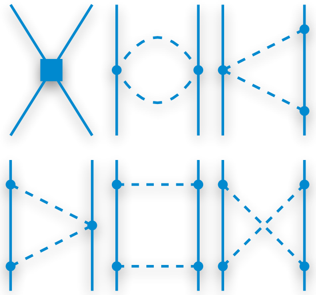
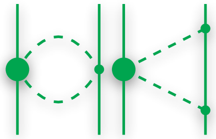
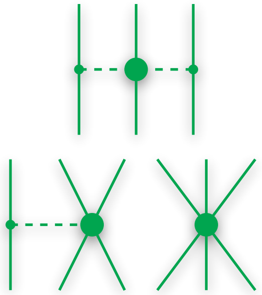
Contact interactions lead to LEC:

Short range two-nucleon interaction fit to deuteron and NN scattering

Three nucleon interactions fitted on light nuclei

Long-range LEC are determined from π-nucleon scattering

H. Hergert, Front. in Phys. 8, 379 (2020)

	NN	3N
LO $(Q/\Lambda_\chi)^0$		
NLO $(Q/\Lambda_\chi)^2$		
NNLO $(Q/\Lambda_\chi)^3$		

Formulate statistical models for uncertainties: Bayesian estimates of EFT errors

S. Wesolowski, et al, PRC 104, 064001 (2021)

The basic model of nuclear theory

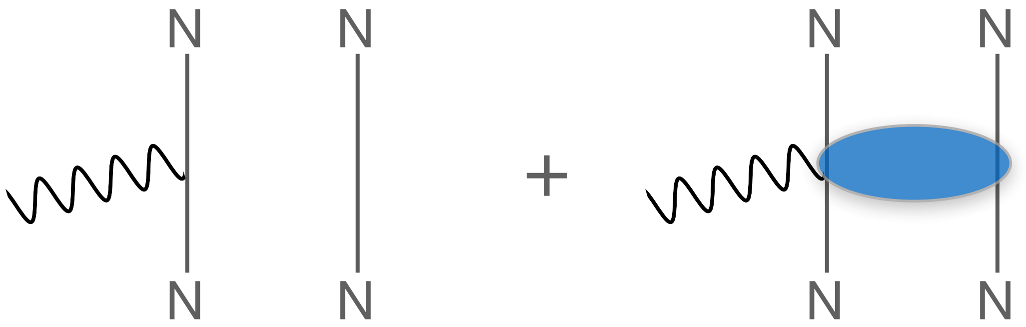
The current operator describes how the external probe interacts with nucleon, nucleons pairs, create new particles …

The structure of the current operator is constrained by the Hamiltonian through the continuity equation

$$\nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0 \qquad [v_{ij}, j^0_i] \neq 0$$

The Hamiltonian structure implies that the current operator includes one and two-body contributions

$$J^\mu(q) = \sum_i j^\mu_i + \sum_{i<j} j^\mu_{ij} + \ldots$$



- ❖ Chiral Effective Field Theory Electroweak many-body currents
- ❖ “Phenomenological” Electroweak many-body currents

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Variational Monte Carlo

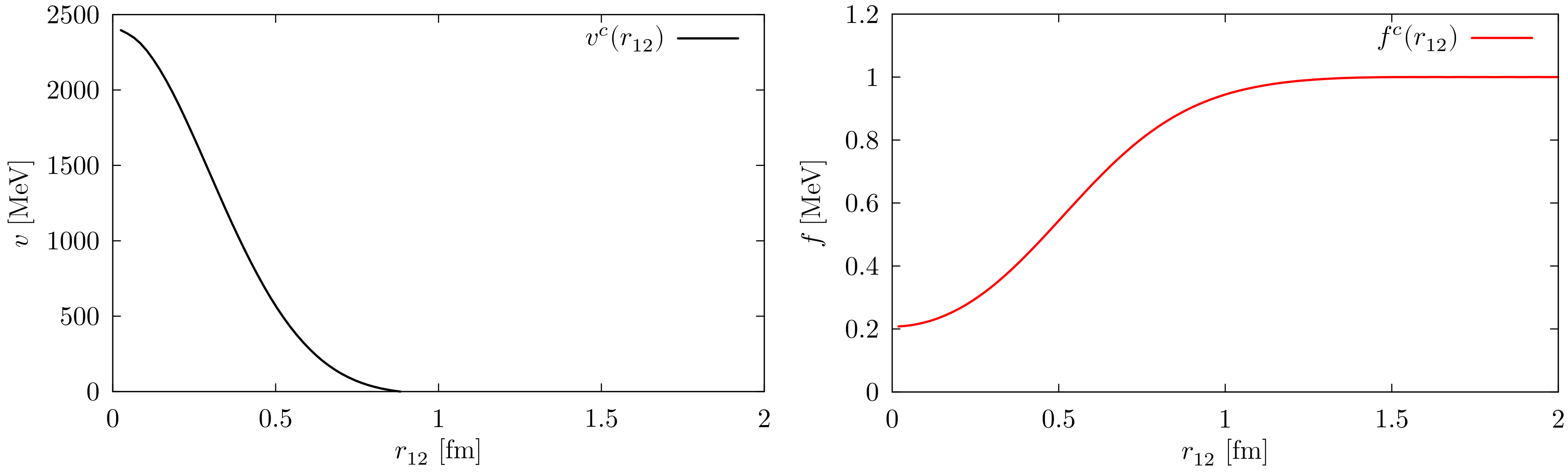
In variational Monte Carlo, one **assumes a suitable form for the trial wave function**

$$|\Psi_T\rangle = \mathcal{F}|\Phi\rangle \begin{cases} \Phi : \text{Mean field component; slater determinant of single-particle orbitals} \\ \mathcal{F} : \text{correlations (2b \& 3b) induced by } H \end{cases}$$

The correlation operator reflects the spin-isospin dependence of the nuclear interaction

$$\mathcal{F} \equiv \left(\mathcal{S}\prod_{i<j} F_{ij}\right) \qquad F_{ij} \equiv \sum_p f_{ij}^p O_{ij}^p$$

The best parameters are found by **optimizing the variational energy** $\dfrac{\langle\Psi_T|H|\Psi_T\rangle}{\langle\Psi_T|\Psi_T\rangle} = E_T \geq E_0$



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Green’s Function Monte Carlo

Any trial wave function can be expanded in the complete set of eigenstates of the the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n|\Psi_n\rangle \qquad\qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

GFMC overcomes the limitations of the variational wave-function by using an imaginary-time projection technique to **projects out the exact lowest-energy state**

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

The direct calculation of the imaginary-time propagator for strongly-interacting systems involves prohibitive difficulties

J. Carlson , et al. Rev. Mod. Phys. 87 (2015) 1067

The imaginary-time evolution is broken into N small imaginary-time steps, and complete sets of states are inserted

$$e^{-(H-E_0)\tau}|\Psi_V\rangle = \int dR_1\ldots dR_N|R_N\rangle\langle R_N|e^{-(H-E_0)\Delta\tau}|R_{N-1}\rangle\ldots\langle R_2|e^{-(H-E_0)\Delta\tau}|R_1\rangle\Psi_V(R_1)$$

Short Time Propagator

Green’s Function Monte Carlo

Any trial wave function can be expanded in the complete set of eigenstates of the the Hamiltonian according to

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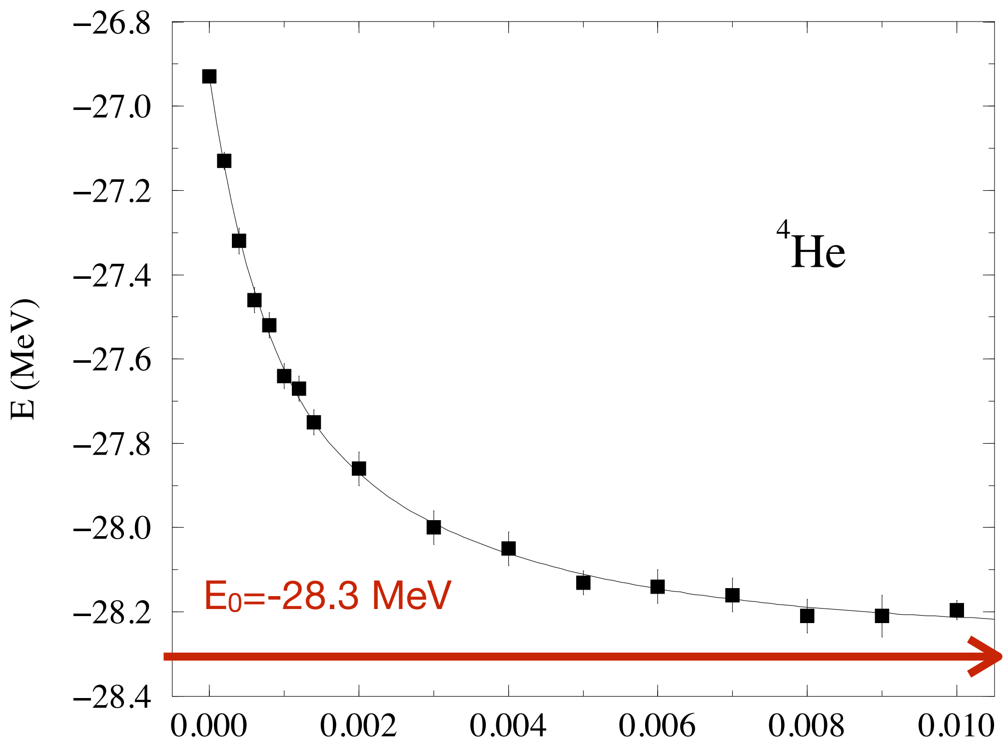


B. Pudliner et al., PRC **56**, 1720 (1997)

The computational cost of the calculation is
2^A x A!/(Z!(A-Z)!)

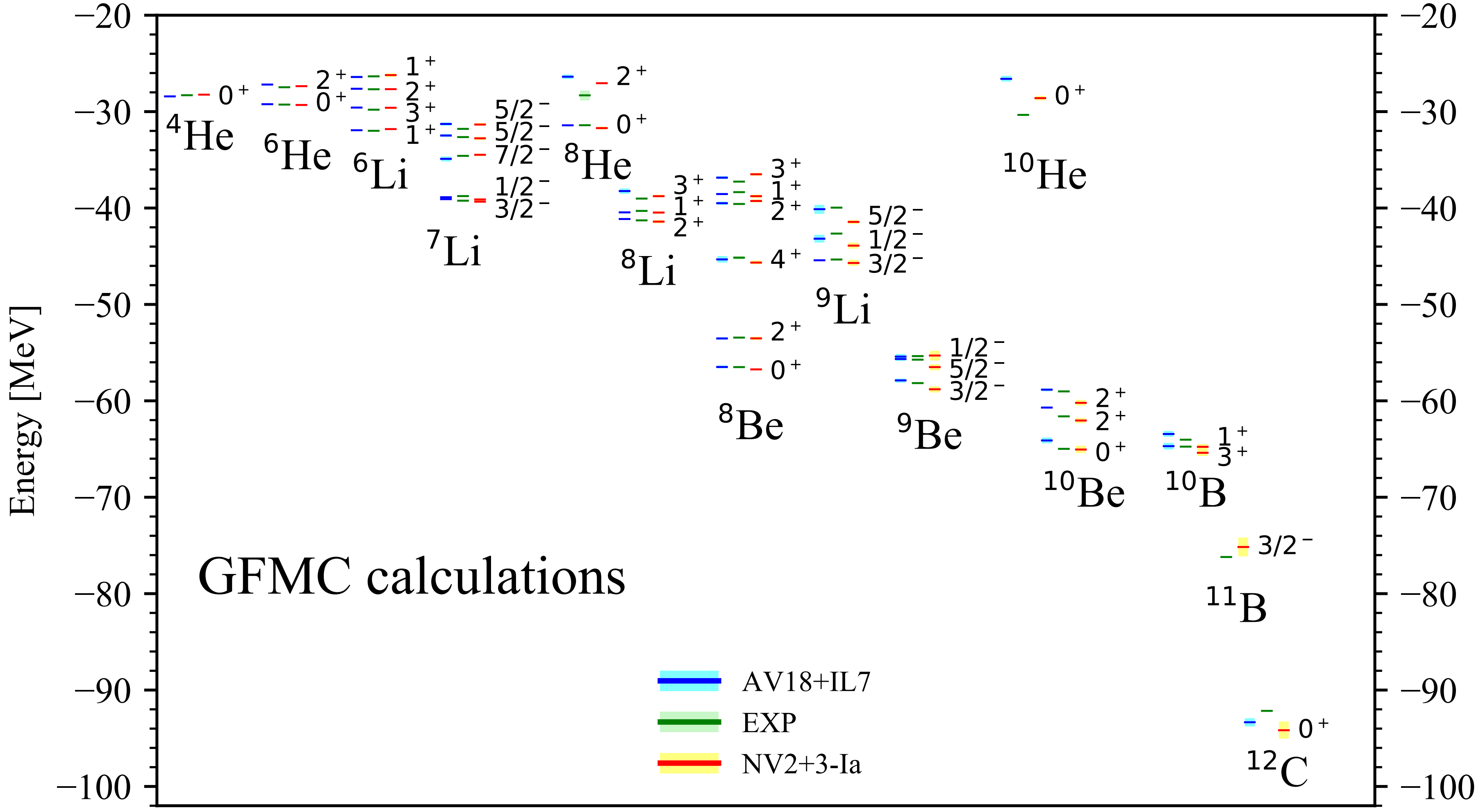
$$|S\rangle = \begin{pmatrix} s_{\uparrow\uparrow\uparrow} \\ s_{\uparrow\uparrow\downarrow} \\ s_{\uparrow\downarrow\uparrow} \\ s_{\uparrow\downarrow\downarrow} \\ s_{\downarrow\uparrow\uparrow} \\ s_{\downarrow\uparrow\downarrow} \\ s_{\downarrow\downarrow\uparrow} \\ s_{\downarrow\downarrow\downarrow} \end{pmatrix}$$

Solve the Many Body Nuclear problem

Develop Computational Methods to solve numerically

$$H\Psi(\mathbf{R}; s_1 \ldots s_A, \tau_1 \ldots \tau_A) = E\Psi(\mathbf{R}; s_1 \ldots s_A, \tau_1 \ldots \tau_A)$$

Quantum Monte Carlo techniques are suitable to solve the Schroedinger equation of medium nuclei

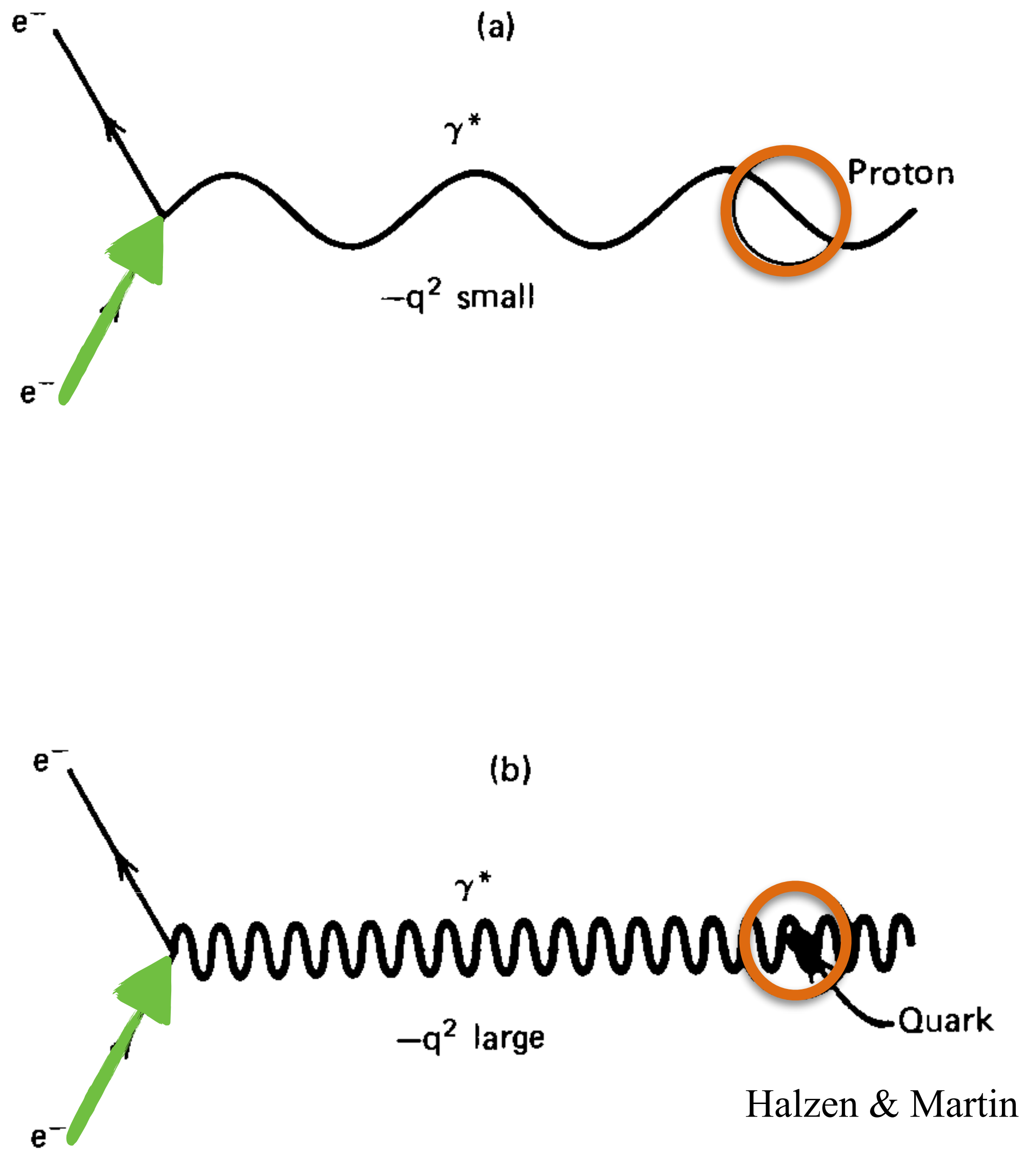


M. Piarulli, et al. Phys.Rev.Lett. 120 (2018) 5, 052503

Probing the target structure

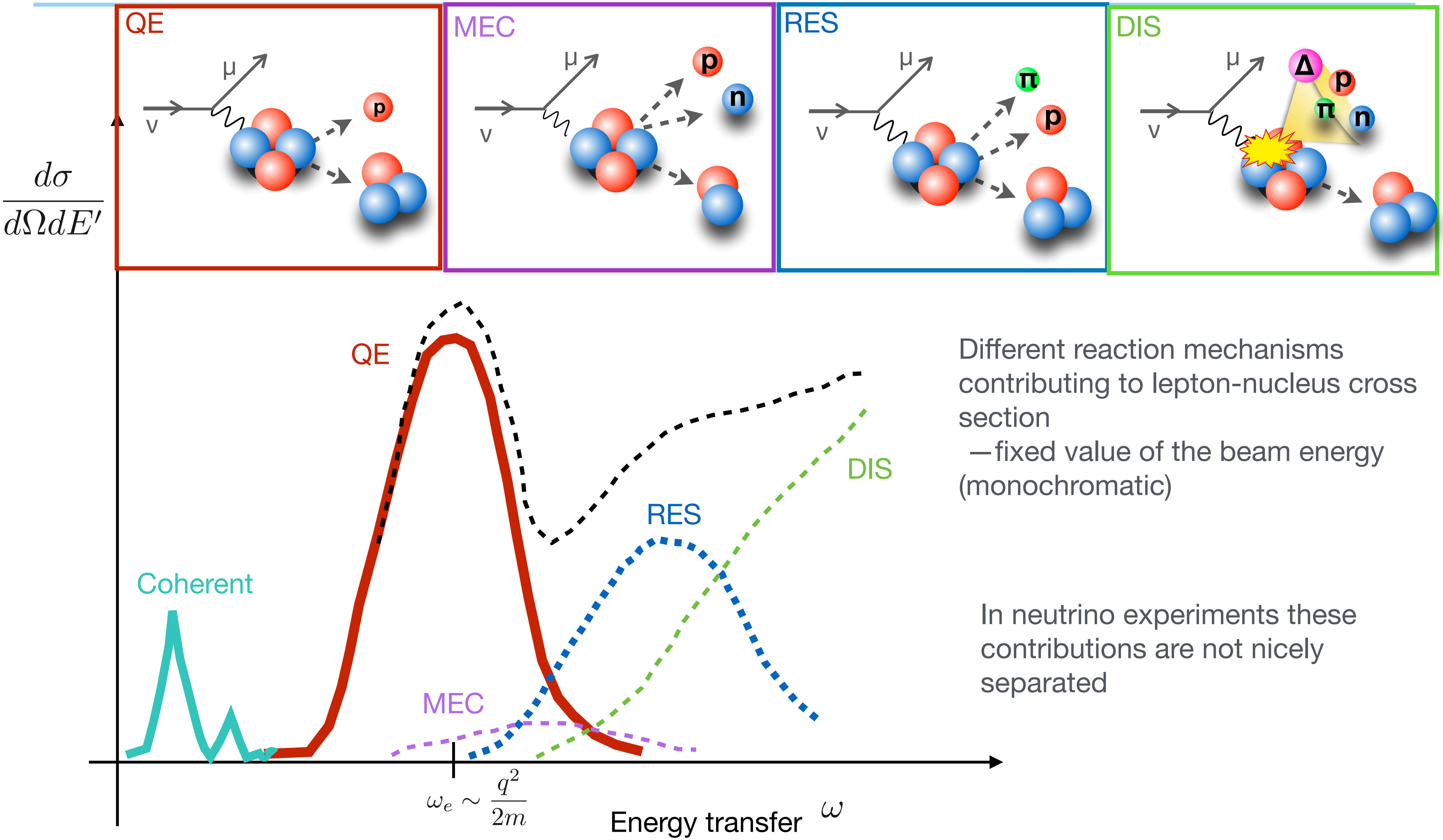
The interaction depends on the **mediator** energy (dubbed as energy transfer ω)

Higher energy transfer = smaller de Broglie wavelength = the probe can resolve the structure inside the nucleon



Halzen & Martin

Lepton-nucleus cross section



Different reaction mechanisms contributing to lepton-nucleus cross section

—fixed value of the beam energy (monochromatic)

In neutrino experiments these contributions are not nicely separated

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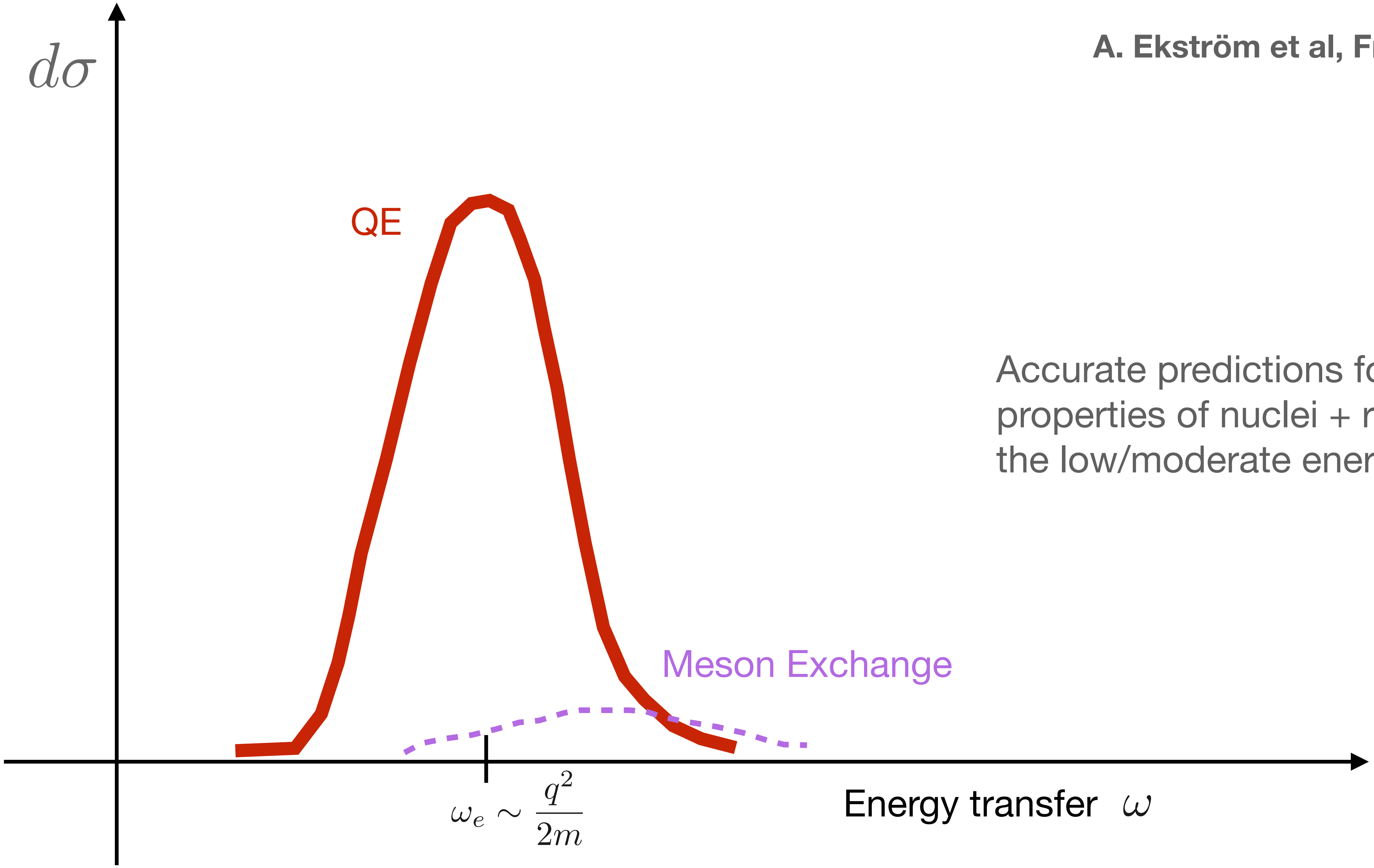
Ab initio Methods

Ab-initio methods (CC, IMSRG, SCGF, QMC, etc) are systematically improvable many-body approaches.

A. Ekström et al, Front. Phys.11 (2023) 29094



Accurate predictions for ground state properties of nuclei + response functions in the low/moderate energy region

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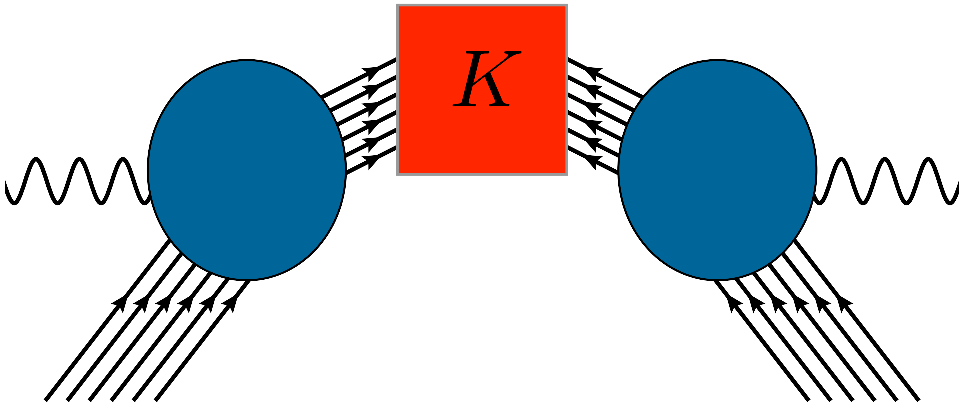
Integral Transform Techniques

✤ Accurate GFMC calculations of the electroweak responses of ^{4}He and ^{12}C have been recently performed:

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle 0|J_\alpha^\dagger(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

Valuable information can be obtained from the integral transform of the response function defined as

$$E_{\alpha\beta}(\sigma,\mathbf{q}) = \int d\omega K(\sigma,\omega)R_{\alpha\beta}(\omega,\mathbf{q}) = \langle\psi_0|J_\alpha^\dagger(\mathbf{q})K(\sigma,H-E_0)J_\beta(\mathbf{q})|\psi_0\rangle$$



Using the completeness relation for the final states, we are left with a ground-state expectation value

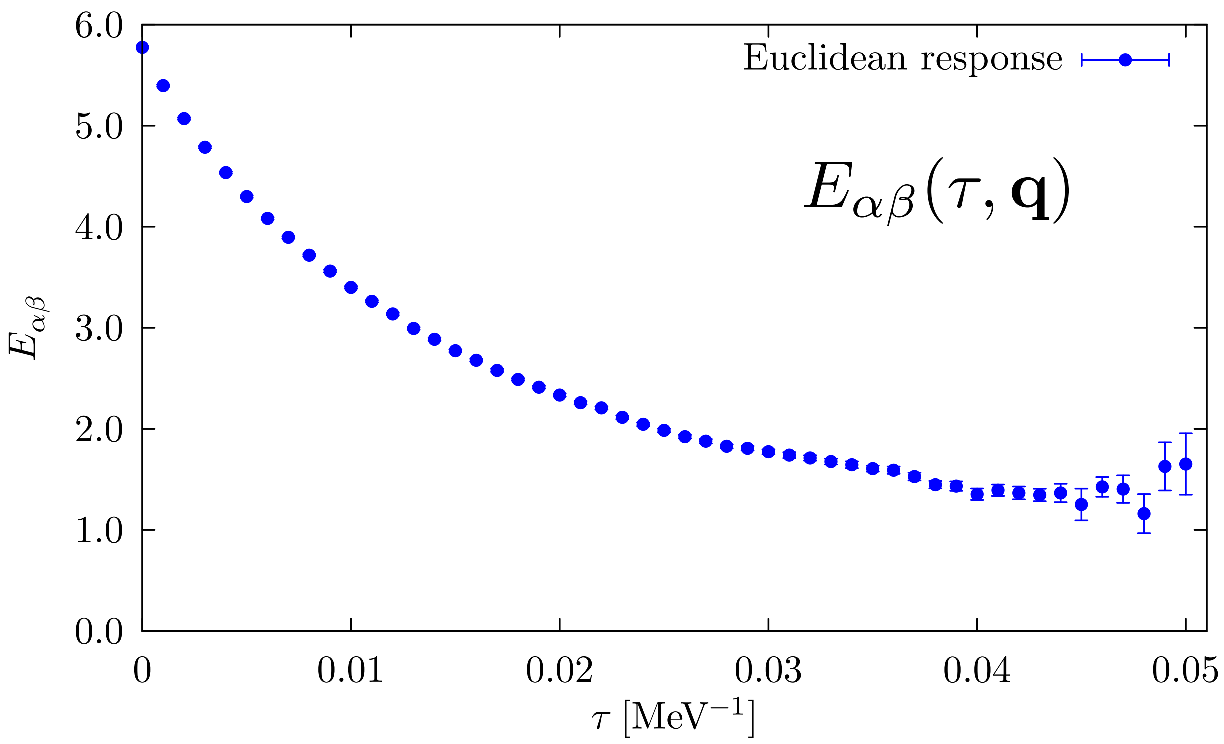
Green's Function Monte Carlo

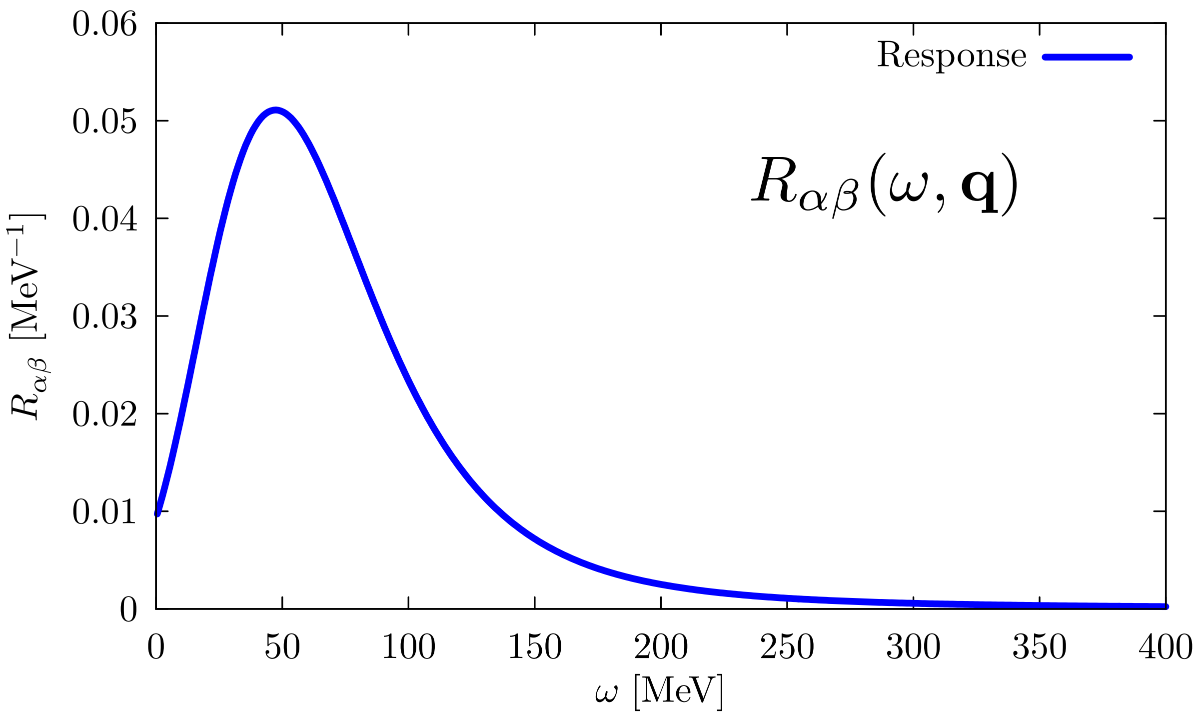
***NR,* Frontiers in Phys. 8 (2020) 116**

- The Laplace integral transform: $K(\sigma, \omega) = e^{-\omega\sigma}$
- Euclidean Response Function: $E_{\alpha\beta}(\sigma, \mathbf{q}) = \int d\omega e^{-\omega\sigma} R(\omega, \mathbf{q})$

Inverting the integral transform is a complicated problem
Bayesian techniques, in particular Maximum Entropy is used

A. Lovato et al, PRL117 (2016), 082501, PRC97 (2018), 022502





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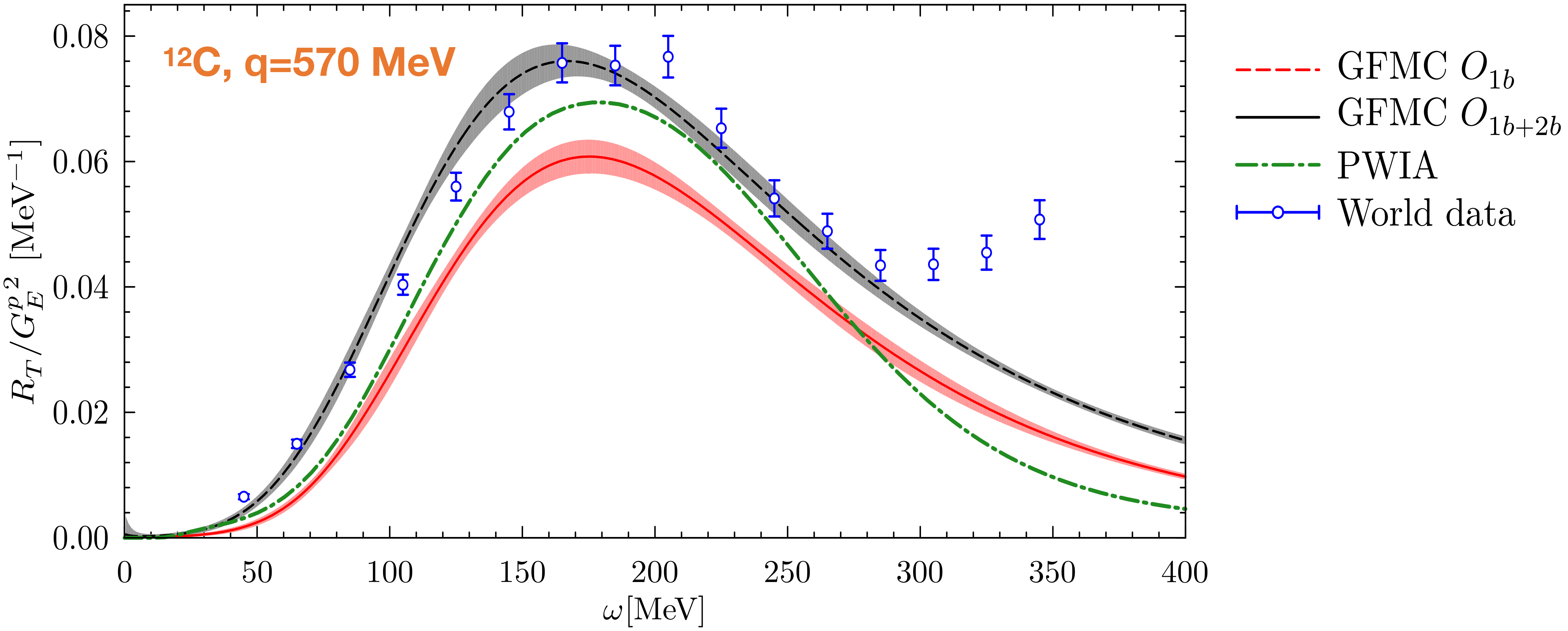
Green’s Function Monte Carlo

NR, **Frontiers in Phys. 8 (2020) 116**



Alessandro Lovato et al. PRL 117 082501 (2016)

Limitations:

Medium mass nuclei A < 13

Inclusive results which are virtually correct in the QE

Relies on non-relativistic treatment of the kinematics

Can not handle explicit pion degrees of freedom

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Why relativity is important

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \underbrace{\langle 0|J^\dagger_\alpha(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle}_{\text{Currents}}\,\delta(\omega - E_f + E_0) \longrightarrow \text{Kinematics}$$

Covariant expression of the e.m. current:

$$j^\mu_{\gamma,S} = \bar{u}(\mathbf{p}')\Big[\frac{G_E^S + \tau G_M^S}{2(1+\tau)}\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{4m_N}\frac{G_M^S - G_E^S}{1+\tau}\Big]u(\mathbf{p})$$

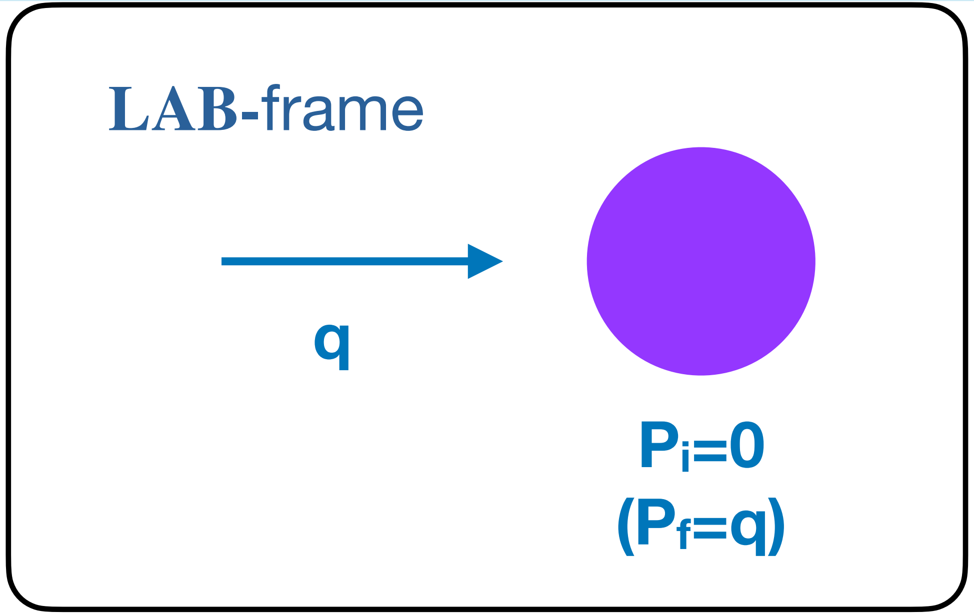
Nonrelativistic expansion in powers of $\boldsymbol{p}/m_N$

$$j^0_{\gamma,S} = \frac{G_E^S}{2\sqrt{1+Q^2/4m_N^2}} - i\frac{2G_M^S - G_E^S}{8m_N^2}\mathbf{q}\cdot(\boldsymbol{\sigma}\times\mathbf{p})$$

Energy transfer at the quasi-elastic peak:

$$w_{QE} = \sqrt{\mathbf{q}^2 + m_N^2} - m_N \qquad w^{nr}_{QE} = \mathbf{q}^2/(2m_N)$$

Frame dependence

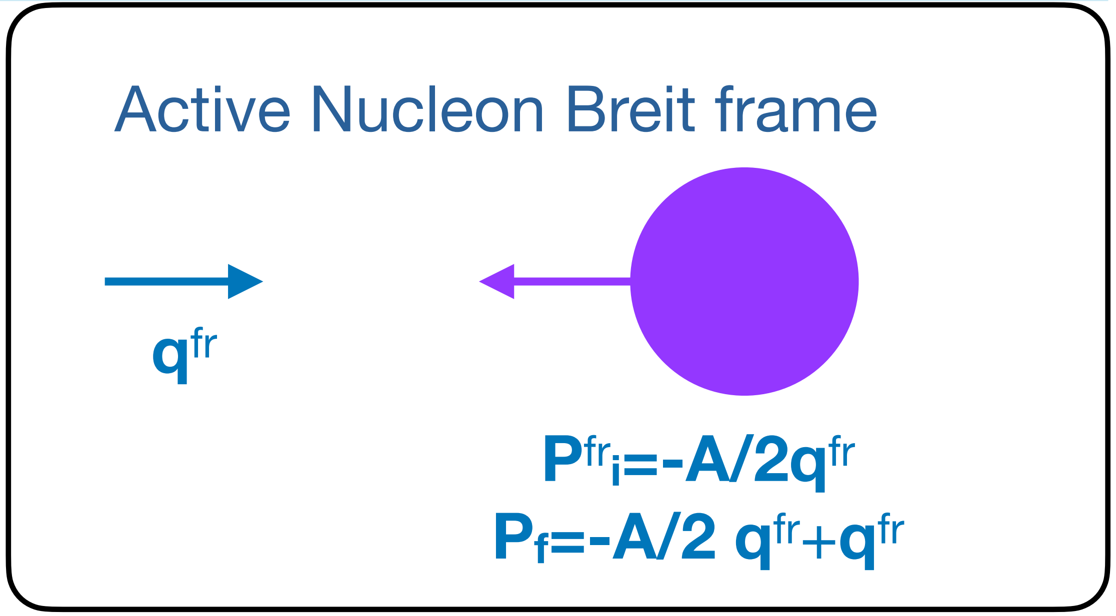


LAB-frame

q

P_i=0

(P_f=q)



Active Nucleon Breit frame

q^{fr}

P^{fr}_i=-A/2q^{fr}

P_f=-A/2 q^{fr}+q^{fr}

- In a generic reference frame the longitudinal non relativistic response reads

$$R_L^{fr} = \sum_f \Big| \langle \psi_i | \sum_j \rho_j(\mathbf{q}^{fr}, \omega^{fr}) | \psi_f \rangle \Big|^2 \delta(E_f^{fr} - E_i^{fr} - \omega^{fr})$$

$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) \approx \delta[e_f^{fr} + (P_f^{fr})^2/(2M_T) - e_i^{fr} - (P_i^{fr})^2/(2M_T) - \omega^{fr}]$$

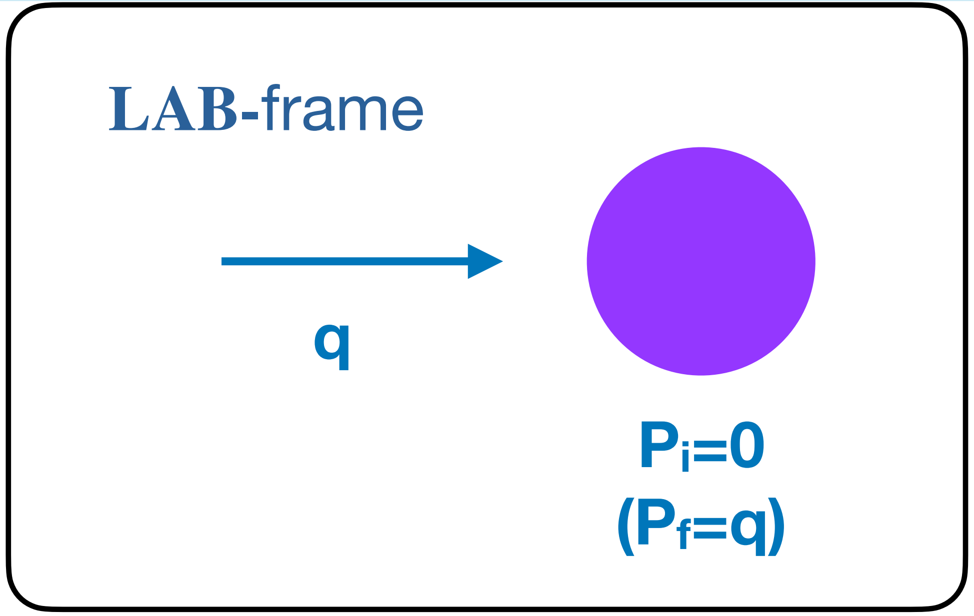
- The response in the LAB frame is given by the Lorentz transformation

$$R_L(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{(\mathbf{q}^{fr})^2} \frac{E_i^{fr}}{M_0} R_L^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr})$$

where

$$q^{fr} = \gamma(q - \beta\omega),\ \omega^{fr} = \gamma(\omega - \beta q),\ P_i^{fr} = -\beta\gamma M_0,\ E_i^{fr} = \gamma M_0$$

Frame dependence

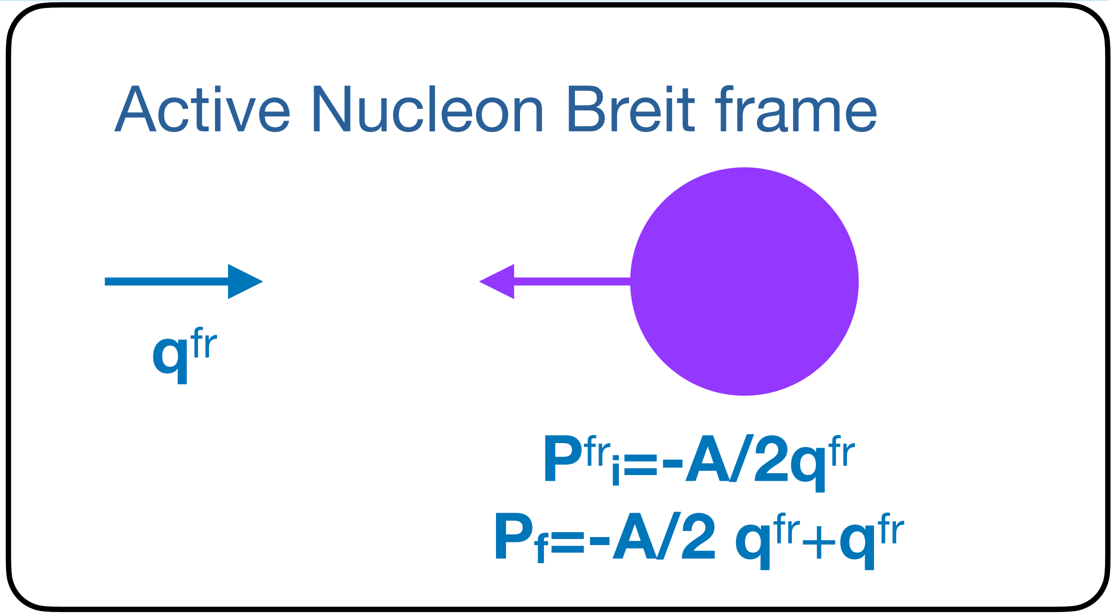


LAB-frame

q

$P_i=0$

$(P_f=q)$



Active Nucleon Breit frame

q^{fr}

$P^{fr}_i=-A/2q^{fr}$

$P_f=-A/2\ q^{fr}+q^{fr}$

- ANB @ the single nucleon level:

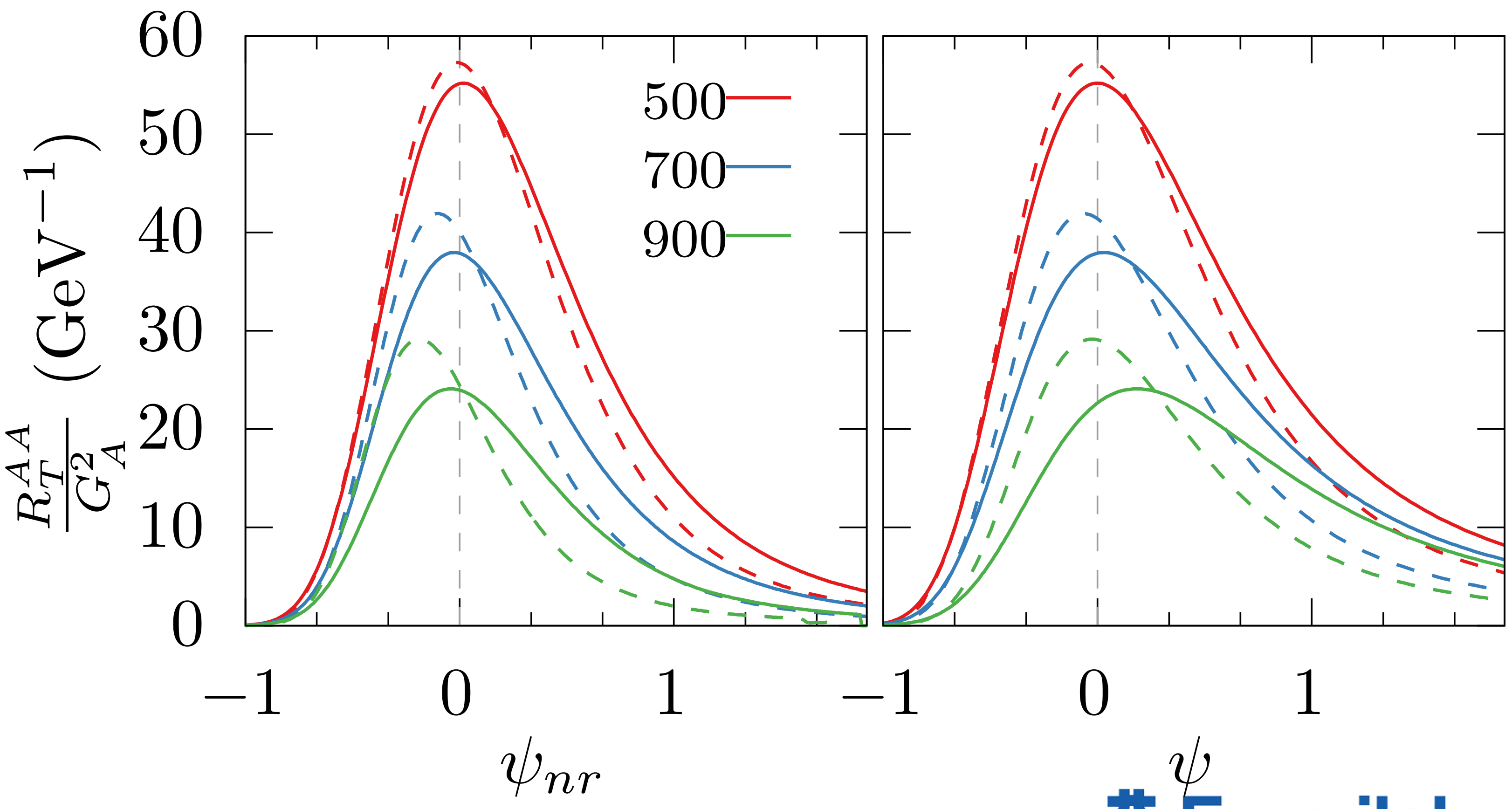
$p^{fr}_i \simeq -q^{fr}/2$

$p^{fr}_f \simeq q^{fr}/2$

- Same position of the quasielastic peak

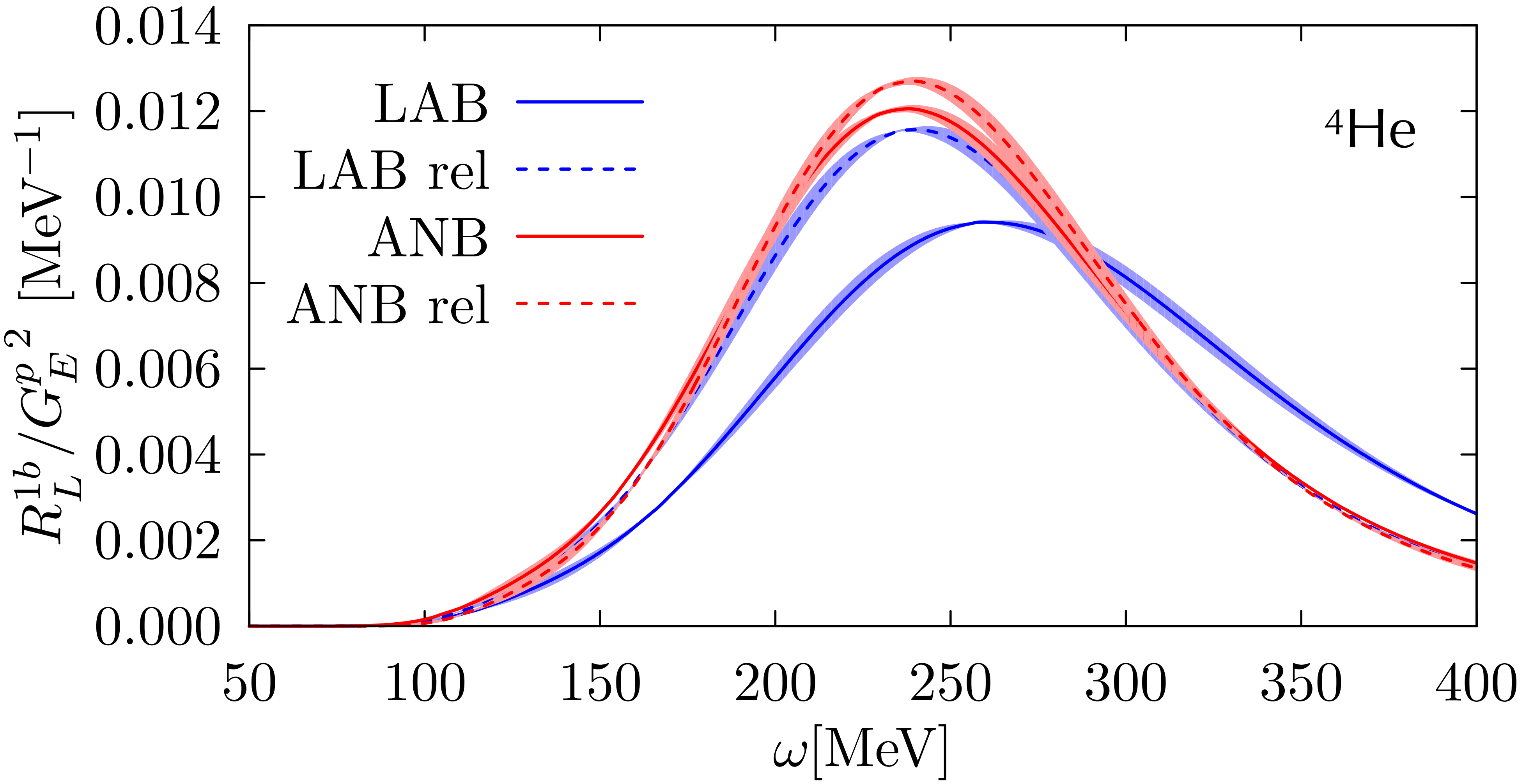
$$\omega_{QE} = \omega_{QE}^{nr} = 0$$

- LAB (solid) and ANB (dashed) predictions



Noemi Rocco, nrocco@fnal.gov

Relativistic effects in a correlated system

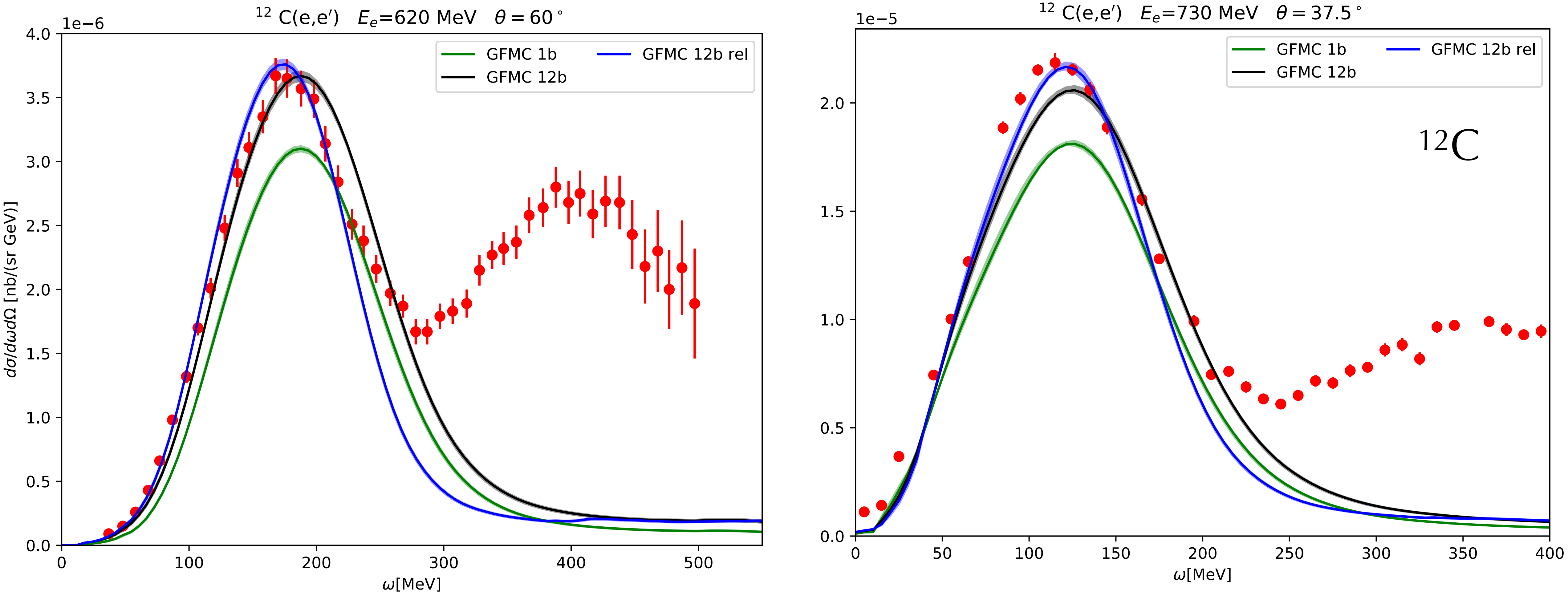


- Relativistic effects are much smaller in the ANB frame where the final nucleon momentum is $\propto$ q/2, the position of the peak remains almost unchanged

Noemi Rocco, nrocco@fnal.gov

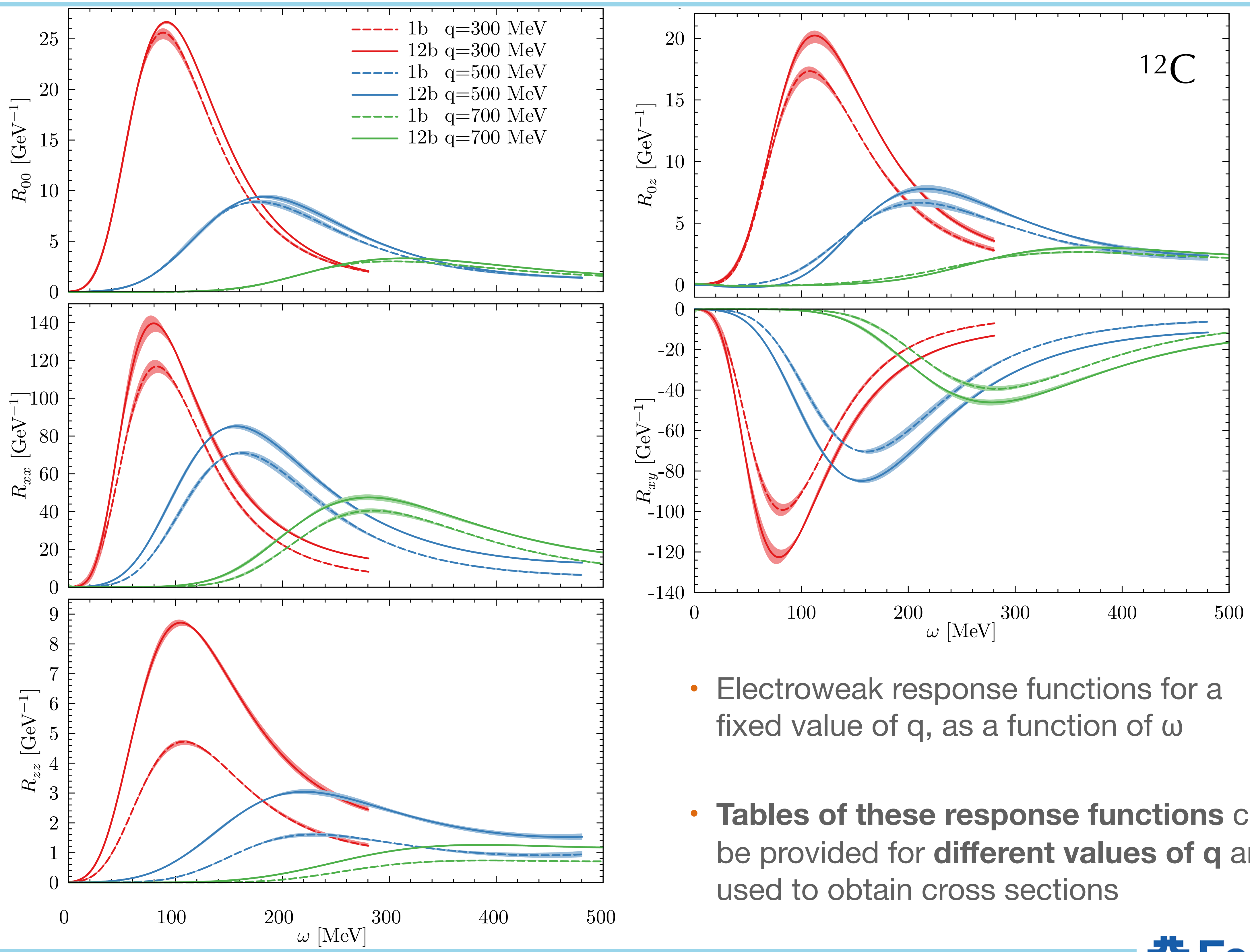
Cross sections: Green’s Function Monte Carlo

Electron scattering results including relativistic corrections for some kinematics covered by the calculated responses



A.Lovato, **A.Nikolakopoulos**, NR, **N. Steinberg**, Universe 9 (2023) 8, 36

Cross sections: Green's Function Monte Carlo



- Electroweak response functions for a fixed value of q, as a function of ω

- **Tables of these response functions** can be provided for **different values of q** and used to obtain cross sections

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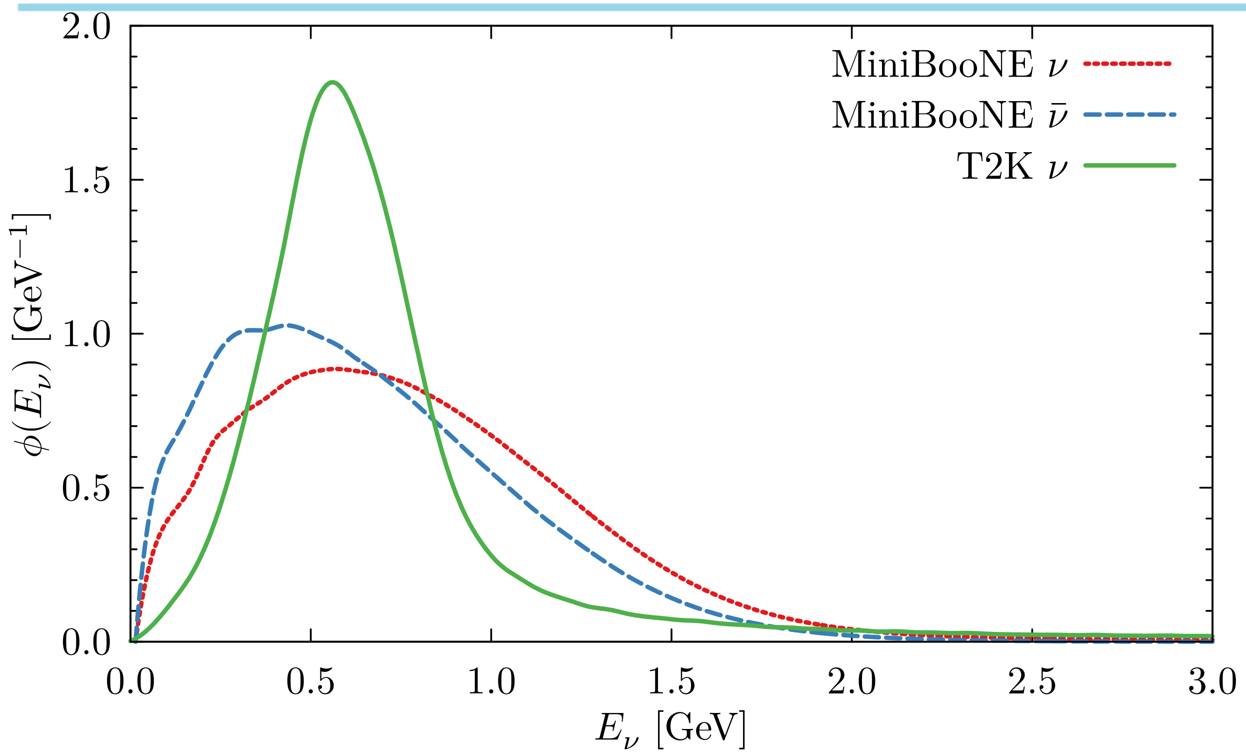
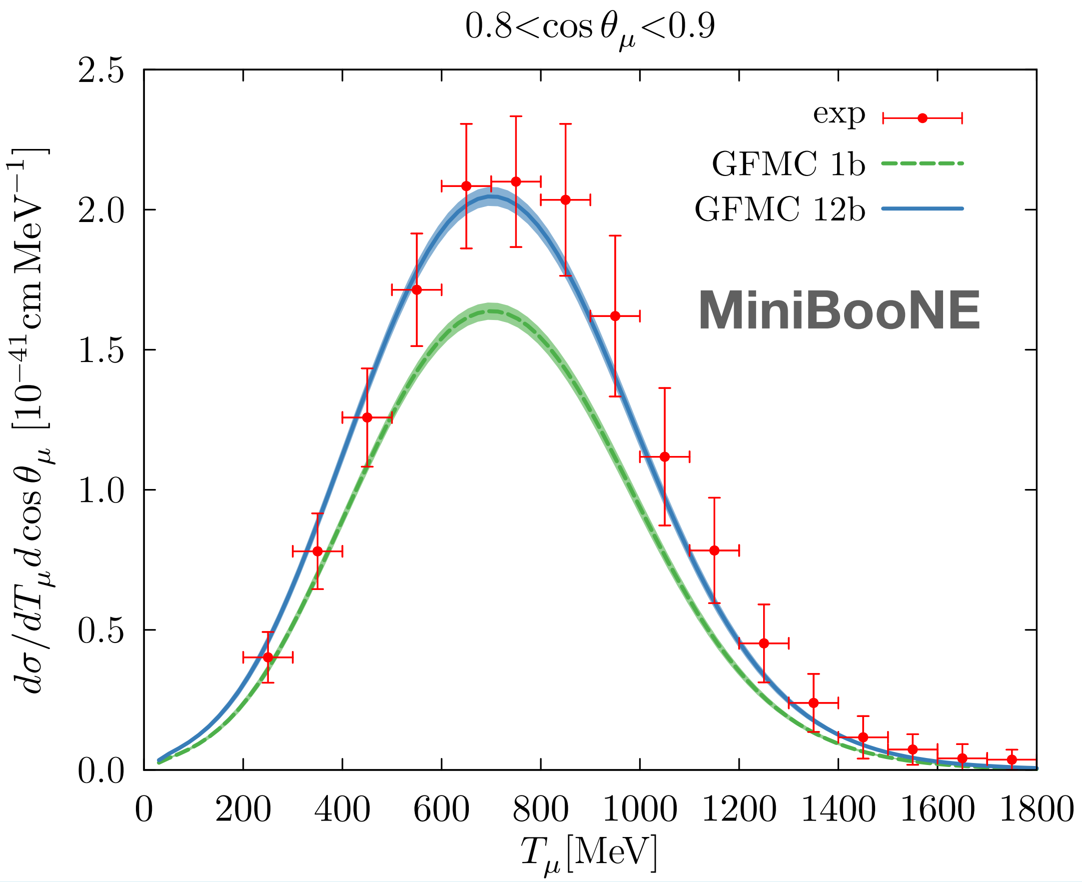
Cross sections: Green's Function Monte Carlo

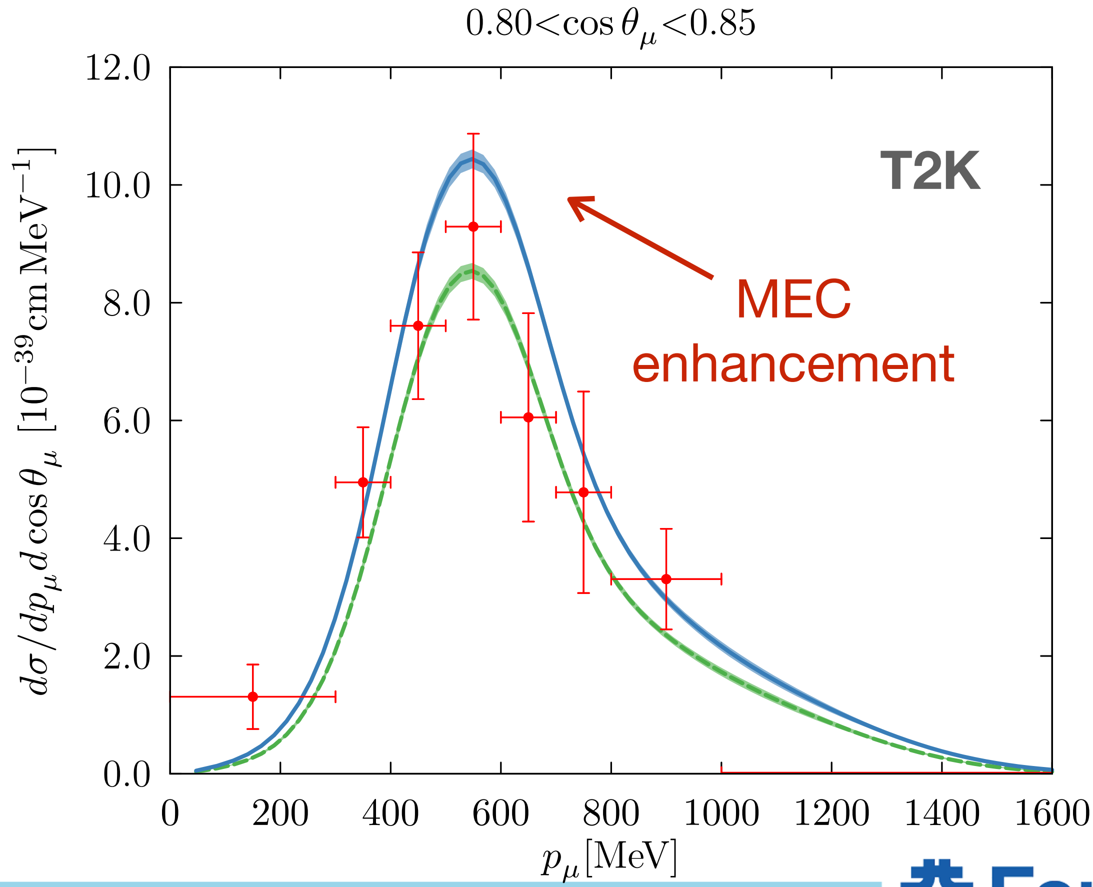


First microscopic calculation of neutrino-nucleus cross section

$$\left\langle \frac{d\sigma}{dT_\mu \, d\cos\theta_\mu} \right\rangle = \int dE_\nu \, \phi(E_\nu) \, \frac{d\sigma(E_\nu)}{dT_\mu \, d\cos\theta_\mu}$$

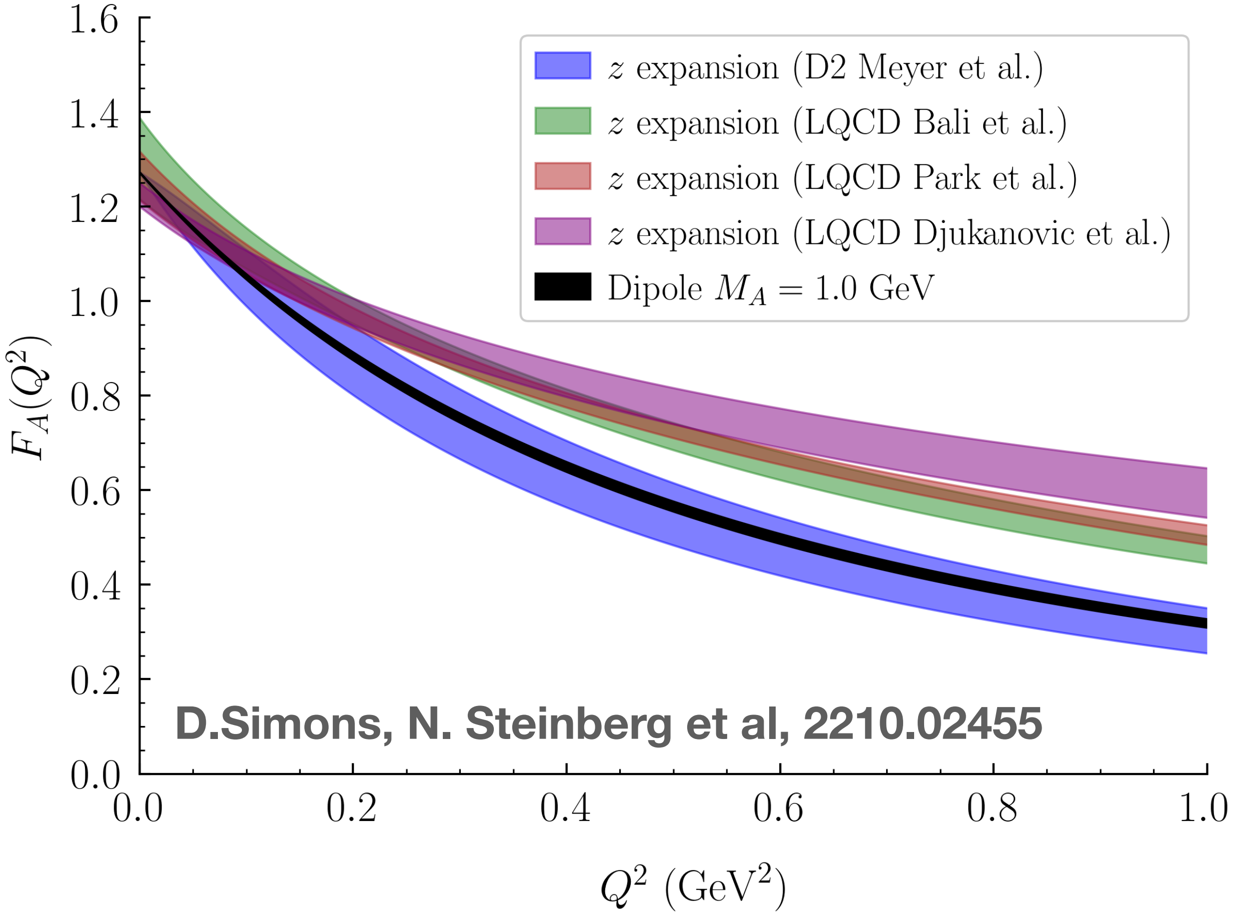
A. Lovato, NR et al , Phys. Rev. X. 10 (2020) 3, 031068





MEC enhancement

Axial form factor determination



D.Simons, N. Steinberg et al, 2210.02455

Comparison with recent MINERvA antineutrino-hydrogen charged-current measurements

1-2σ agreement with MINERvA data and LQCD prediction by PNDME Collaboration

Novel methods are needed to remove excited-state contributions and discretization errors

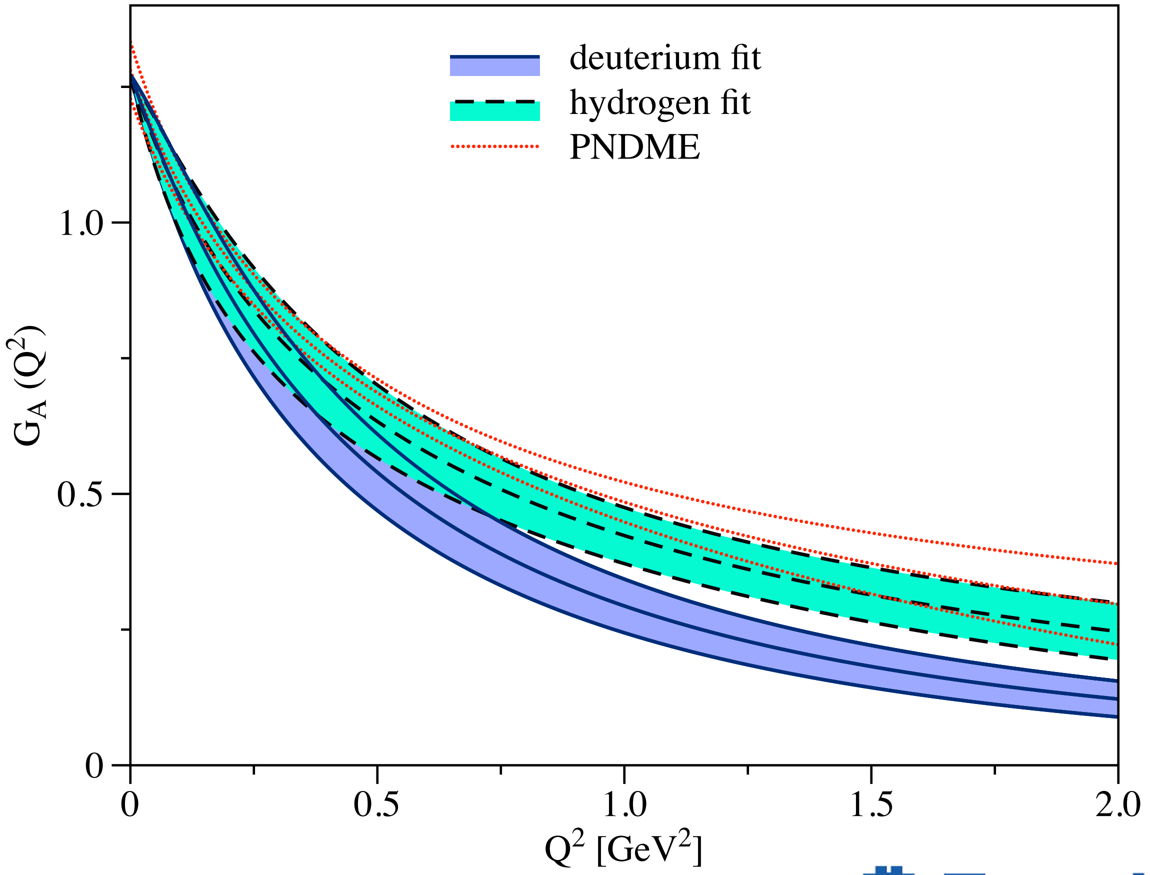
A. Meyer, A. Walker-Loud, C. Wilkinson, 2201.01839

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations

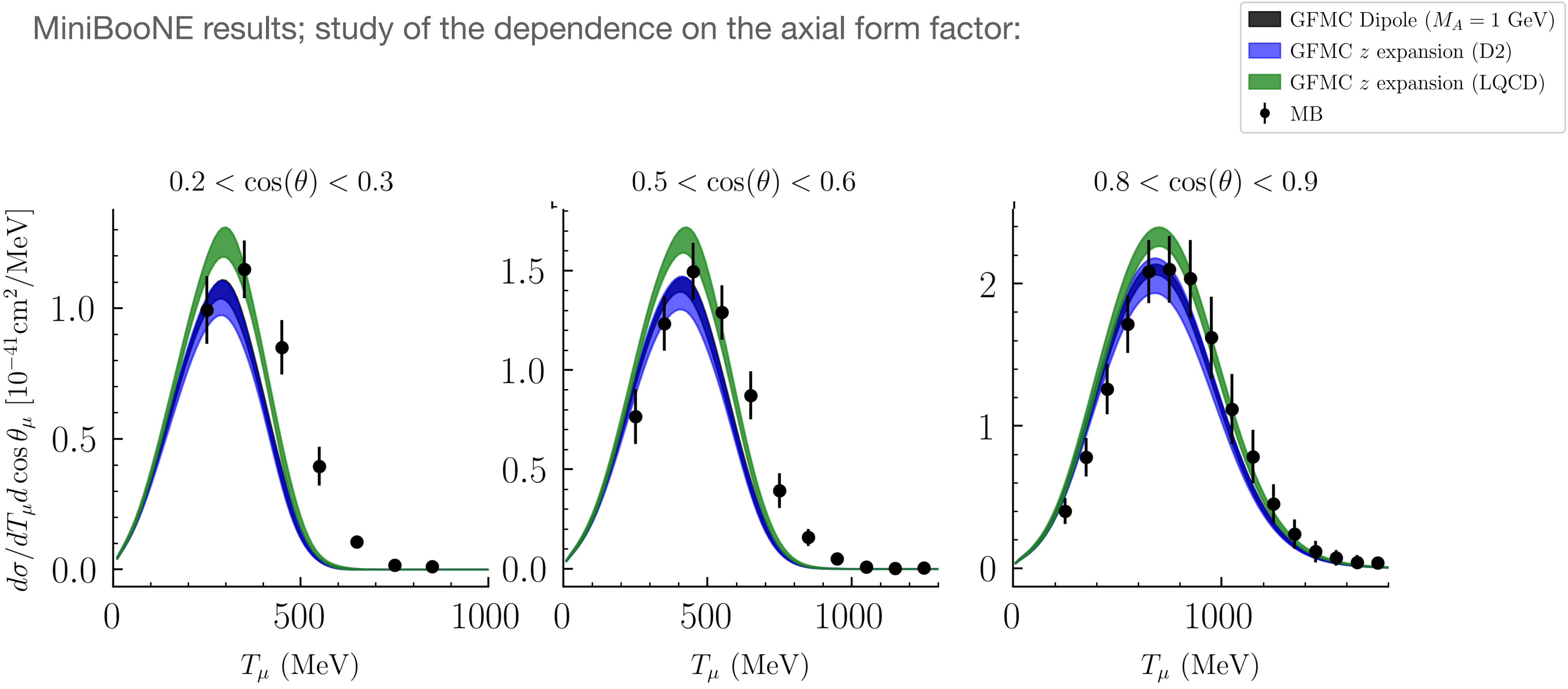
LQCD results are 2-3σ larger than D2 Meyer ones for $Q^2 > 0.3$ GeV^2

O. Tomalak, R. Gupta, T. Battacharaya, 2307.14920



Study of model dependence in neutrino predictions

MiniBooNE results; study of the dependence on the axial form factor:

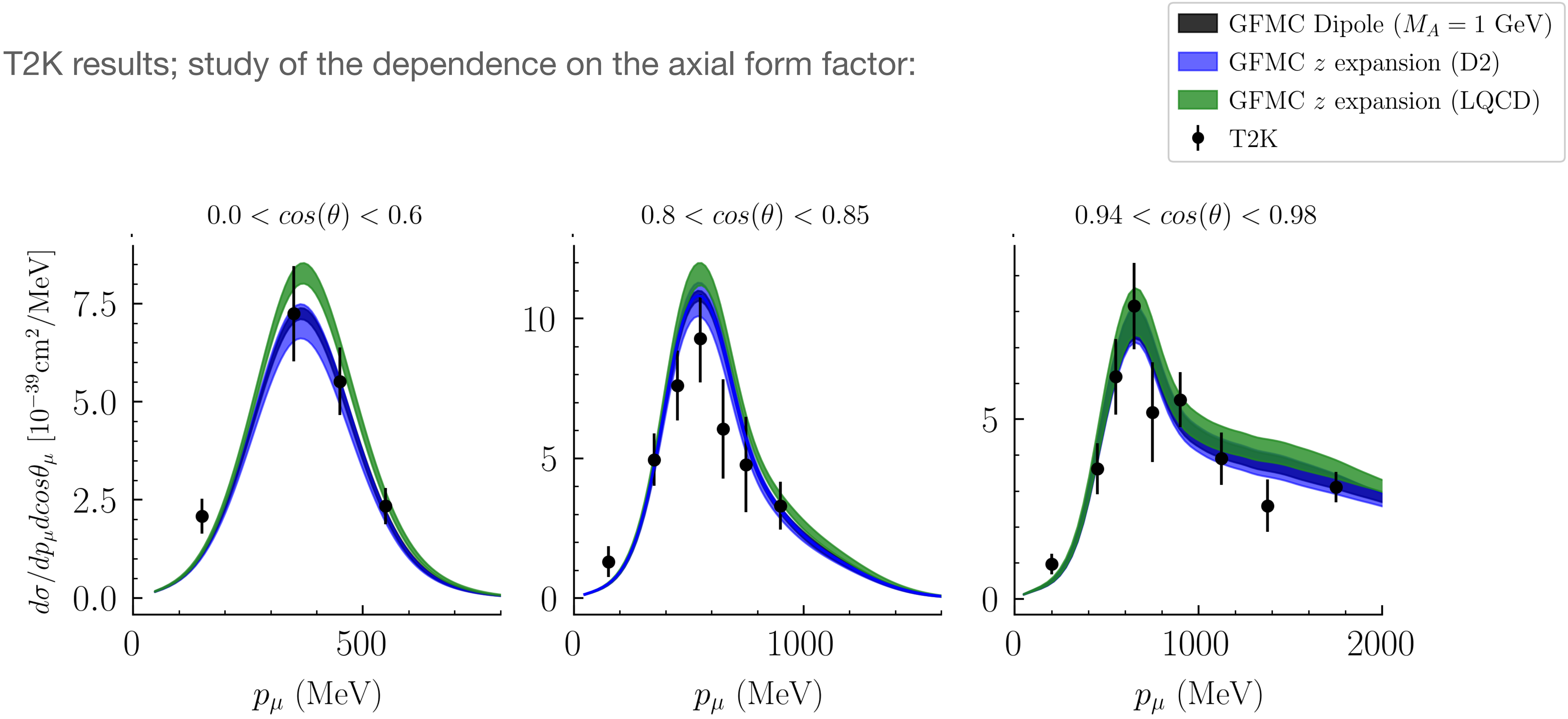


D.Simons, N. Steinberg et al, 2210.02455

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
GFMC Difference in $d\sigma_{\text{peak}}$ (%)	18.6	17.1	12.2

Study of model dependence in neutrino predictions

T2K results; study of the dependence on the axial form factor:



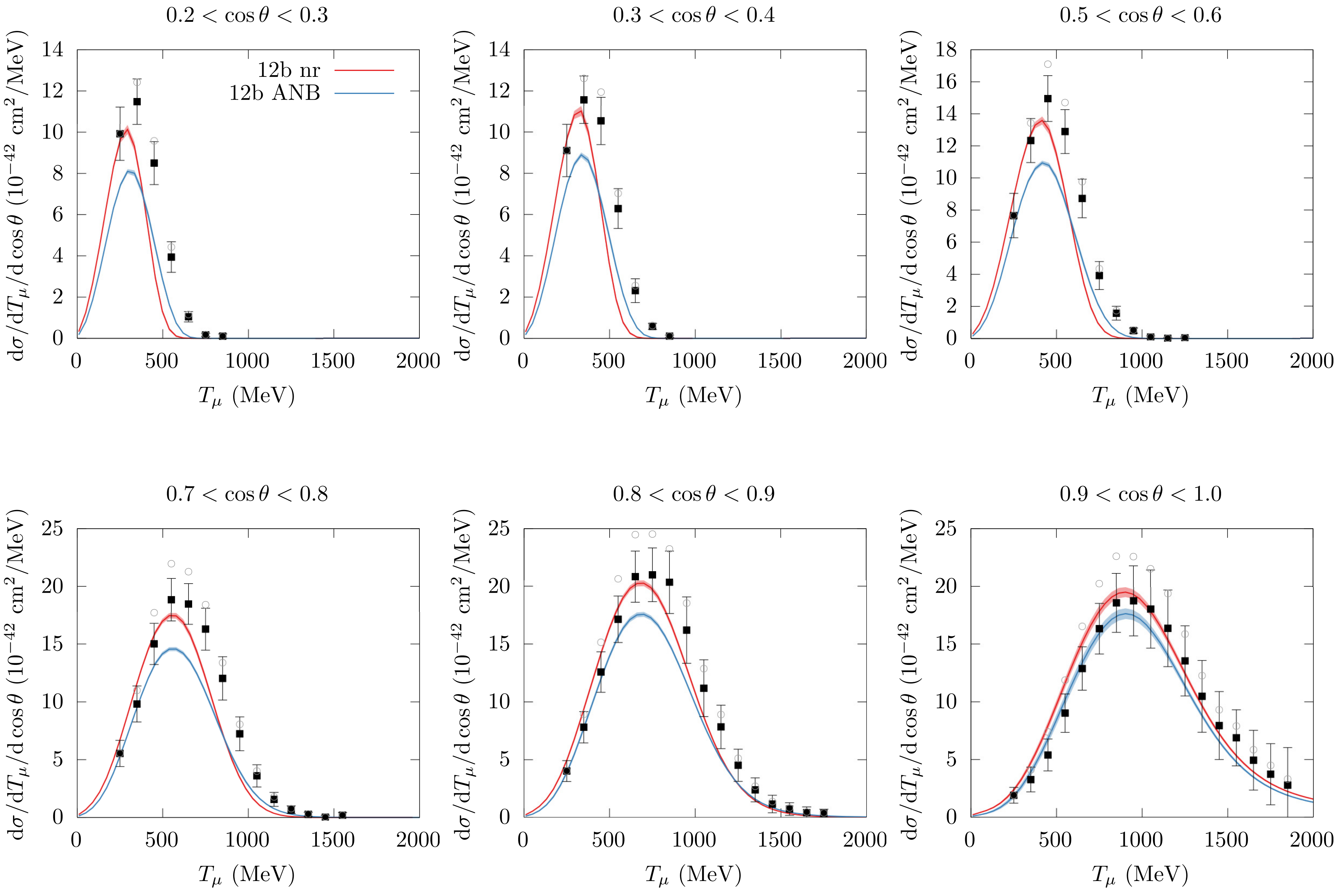
D.Simons, N. Steinberg et al, 2210.02455

T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
GFMC difference in $d\sigma_{\text{peak}}$ (%)	15.8	8.0	4.6

Cross sections: Green’s Function Monte Carlo

MiniBooNE results including relativistic corrections

A.Nikolakopoulos, A.Lovato, NR, Universe 9 (2023) 8, 367



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Coupled Cluster Method

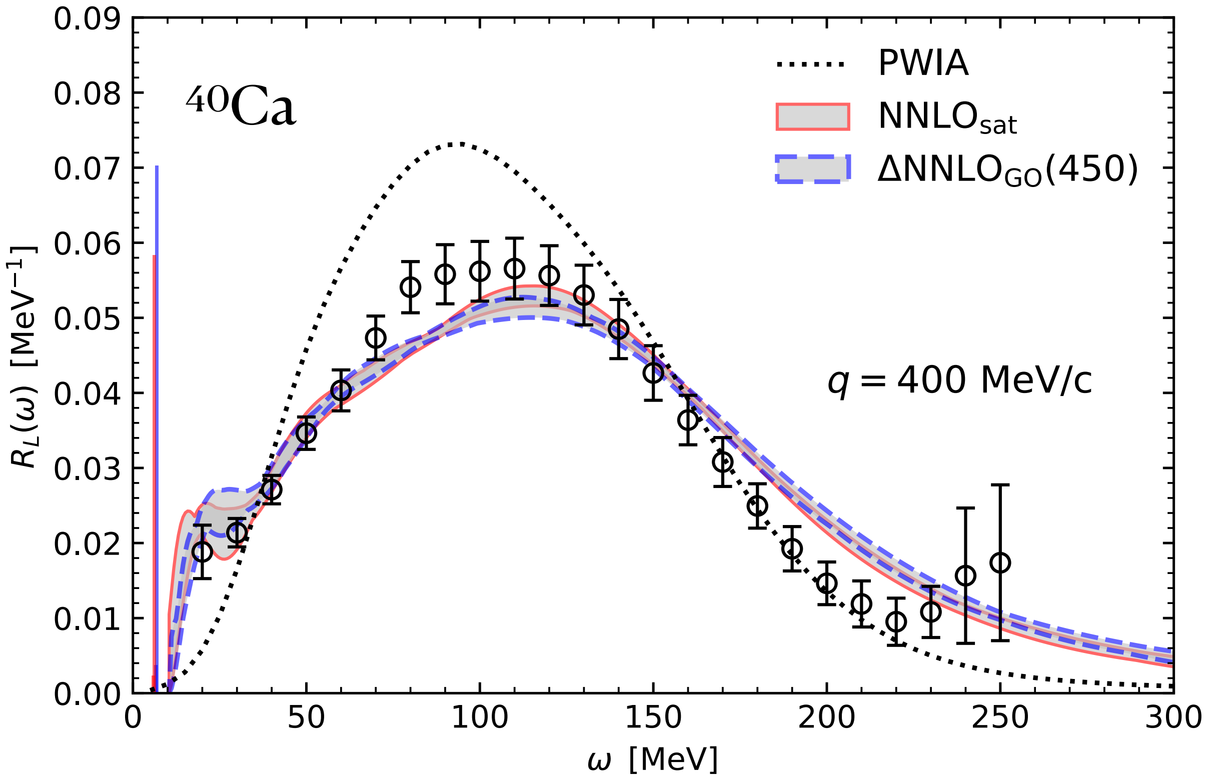
Reference state Hartree Fock: $|\Psi\rangle$

Include correlations through e^{T} operator

Similarity transformed Hamiltonian $e^{-T}He^{T}|\Psi\rangle = \bar{H}|\Psi\rangle = E|\Psi\rangle$

Expansion in second quantization single + doubles:

$$T = \sum t_a^i a_a^\dagger a_i + \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \ldots$$



Polynomial scaling with the number of nucleons (predictions for ^{132}Sn and ^{208}Pb)

Electroweak response functions obtained using **LIT**

$$K_\Gamma(\omega,\sigma) = \frac{1}{\pi}\frac{\Gamma}{\Gamma^2 + (\omega-\sigma)^2}$$

JES, B. Acharya, S. Bacca, G. Hagen; PRL 127 (2021) 7, 072501

Outline

Lepton - nucleon interactions

Modeling nuclear structure

Ab - initio description of lepton - nucleus interactions

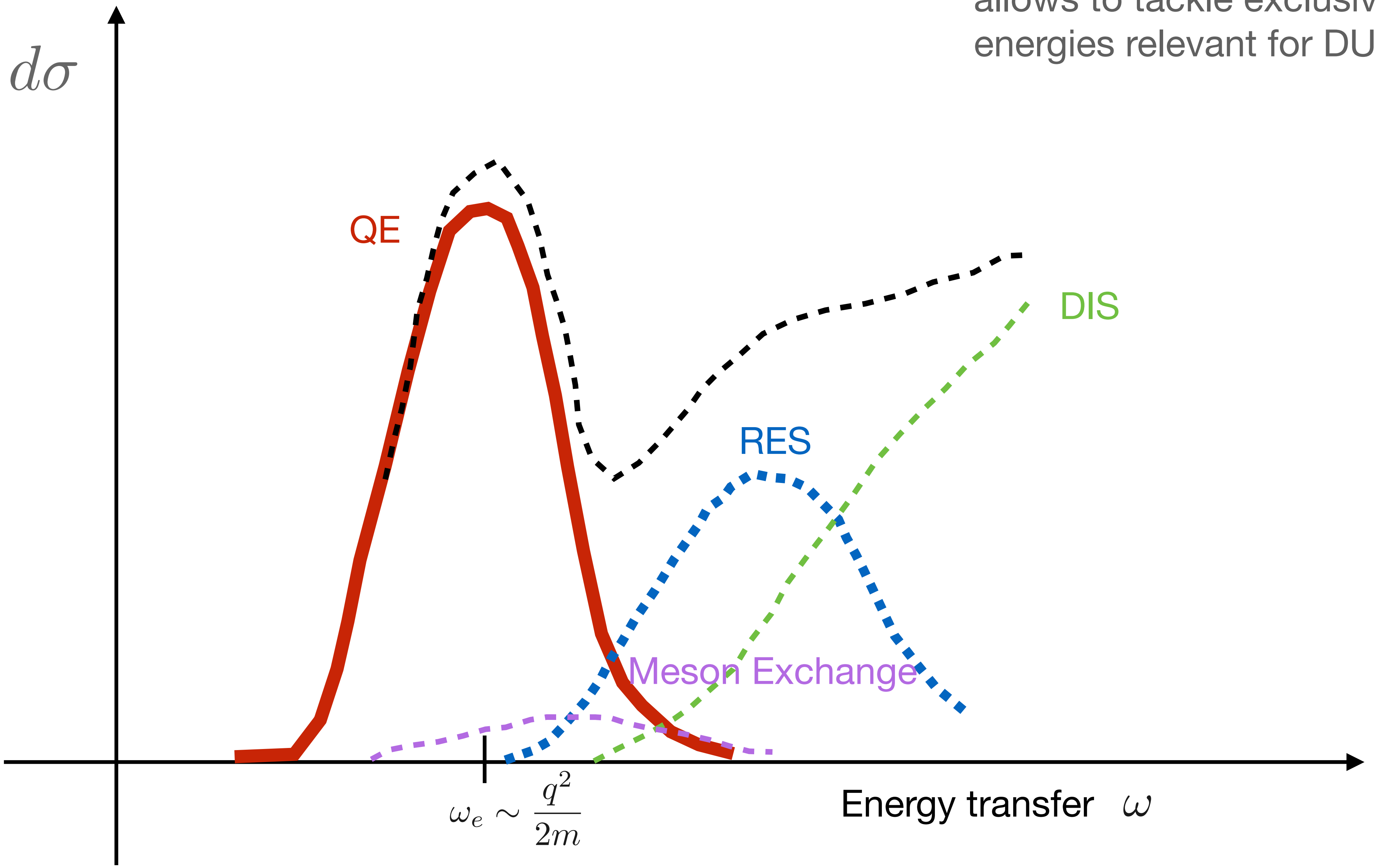
Factorization approach + spectral function : QE

More spectral function

Factorization approach + spectral function : MEC + interference (+pion)

Scaling properties

Factorization Based Approaches



Factorization of the hadronic final states: allows to tackle exclusive channels + higher energies relevant for DUNE

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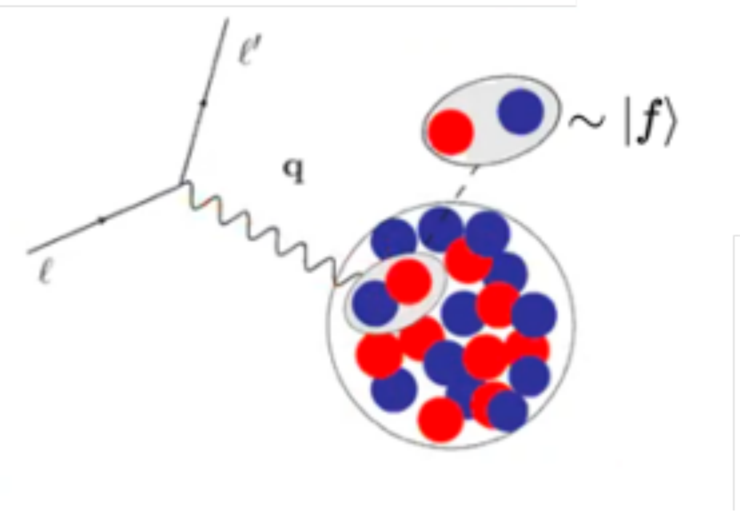
Short-Time Approximation

NR, **Frontiers in Phys. 8 (2020) 116**

- Based on Factorization
- Retains two-body physics
- Response functions are given by the scattering from pairs of fully interacting nucleons that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities



The sum over all final states is replaced by a two nucleon propagator

$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

The STA restricts the propagation to two active nucleons and allows to compute density functions of the CoM and relative momentum of the pair

$$R^{\rm STA}(q,\omega) \sim \int \delta(\omega + E_0 - E_f) de\, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

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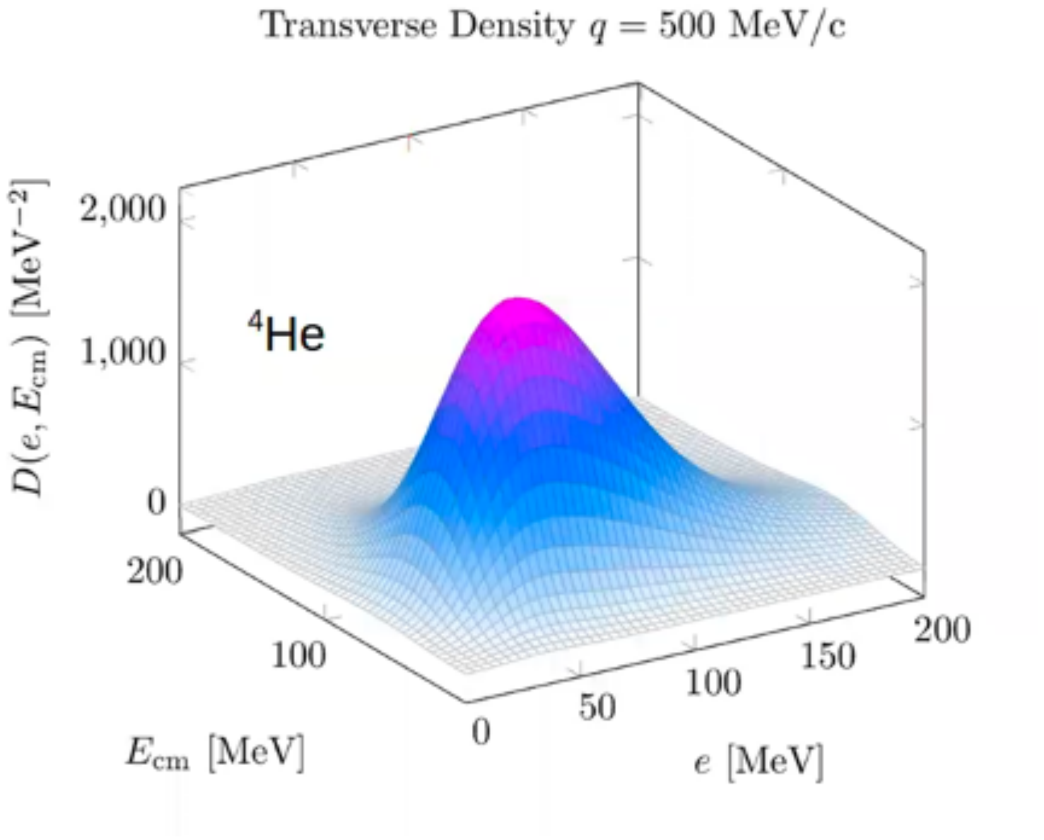
Short-Time Approximation

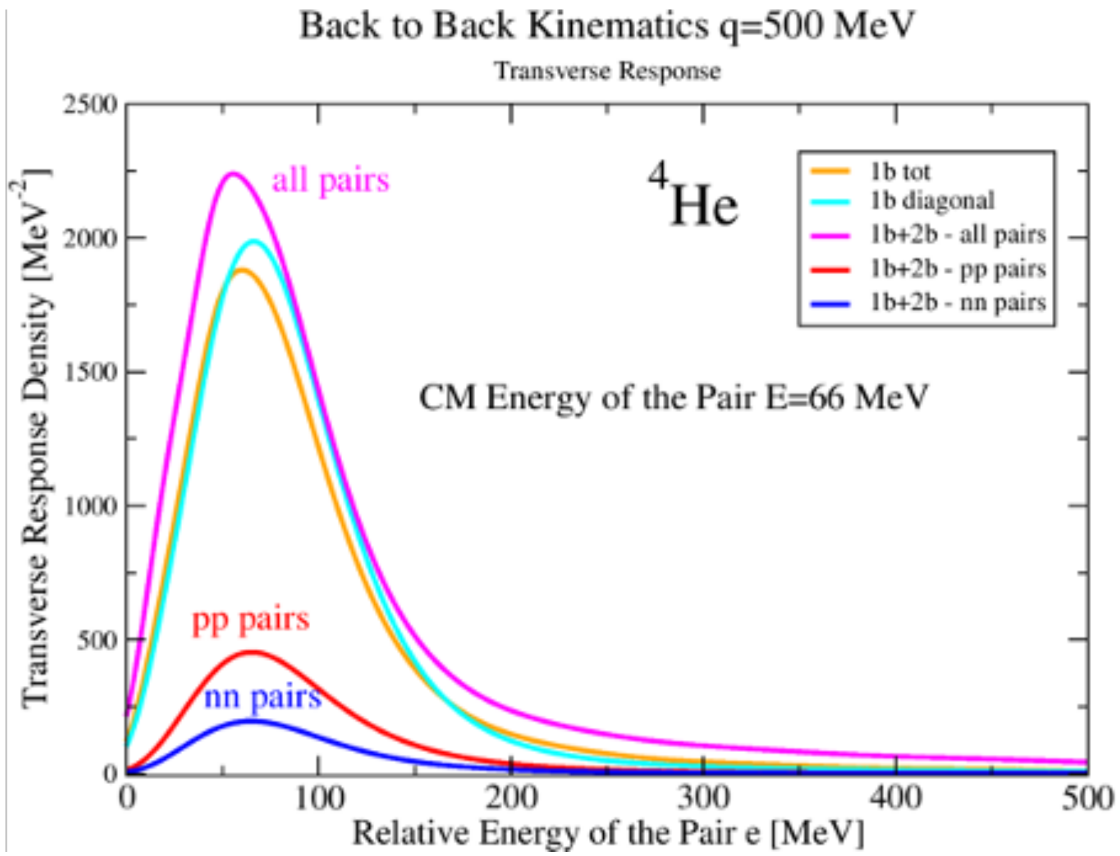


$$R^{\rm STA}(q,\omega) \sim \int \delta(\omega + E_0 - E_f) de\, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

Electron scattering from 4He:

- Response density as a function of (E,e)
- Give access to particular kinematics for the struck nucleon pair

Pastore et al. PRC101(2020)044612



- pp pairs
- nn pairs
- 1 body tot
- all pairs tot

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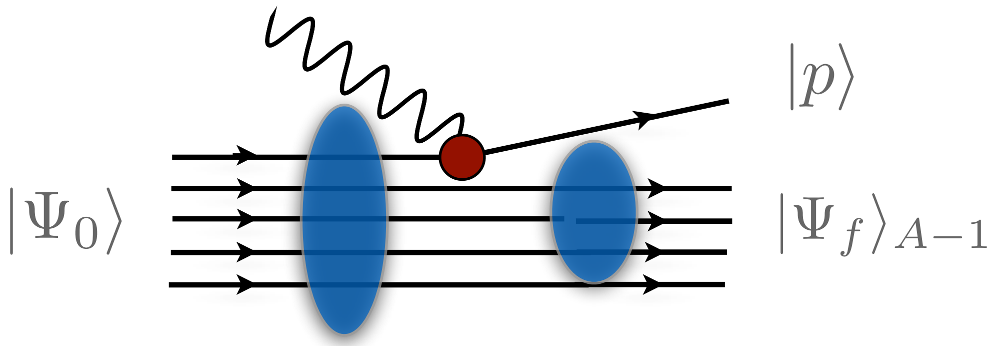
Spectral function approach

The general expression for the hadronic tensor reads

$$R^{\mu\nu}(\mathbf{q},\omega) = \sum_f \langle\Psi_0|j^{\mu\dagger}|\Psi_f\rangle\langle\Psi_f|j^\nu|\Psi_0\rangle\delta(E_0+\omega-E_f)$$



At large momentum transfer, the scattering reduces to the sum of individual terms

$$J_\alpha = \sum_i j_\alpha^i \qquad\qquad |\Psi_f\rangle \rightarrow |p\rangle \otimes |\Psi_f\rangle_{A-1}$$

We employ the factorization ansatz and insert a single- nucleon completeness relation

$$\langle\Psi_f|j^\mu|\Psi_0\rangle \rightarrow \sum_k \left[{}_{A-1}\langle\Psi_f| \otimes \langle k|\right]|\Psi_0\rangle\langle p|\sum_i j_i^\mu|k\rangle$$

The incoherent contribution of the one-body response reads

$$R^{\mu\nu}(\mathbf{q},\omega) = \sum_{p,k,f}\sum_i \langle k|j_i^{\mu\dagger}|p\rangle\langle p|j_i^\nu|k\rangle\Big[\langle\Psi_0||\Psi_f\rangle_{A-1}\otimes|k\rangle\Big]^2$$
$$\times\,\delta(\omega - e(\mathbf{p}) - E_f^{A-1} + E_0^A)$$

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Spectral function approach

We can rewrite the delta using the identity:

$$\delta(\omega - e(\mathbf{p}) - E_f^{A-1} + E_0) = \int dE\delta(\omega + E - e(\mathbf{p}))\delta(E + E_f^{A-1} - E_0)$$

The response tensor is given by

$$R^{\mu\nu}(\mathbf{q},\omega) = \int d^3k dE\, P_h(\mathbf{k},E) \frac{m_N^2}{e(\mathbf{k})e(\mathbf{k}+\mathbf{q})} \sum_i \langle k\,|\,j_i^{\mu\dagger}\,|\,p\rangle\langle p\,|\,j_i^{\nu}\,|\,k\rangle\delta(\omega + E - e(\mathbf{k}+\mathbf{q}))$$

Spectral Function

Implicit covariant normalization of the four-spinors

The hole spectral function reads

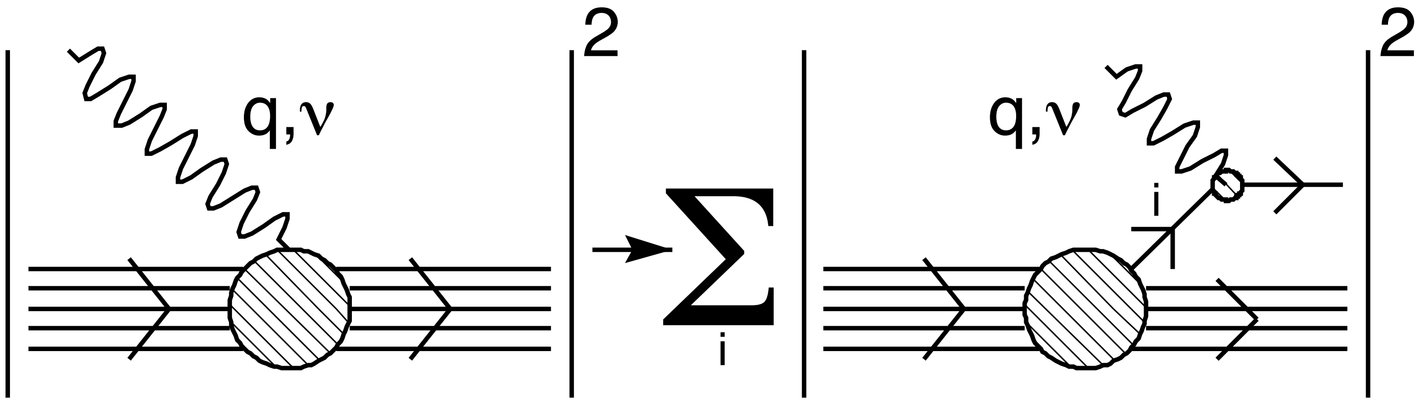
$$P_h(\mathbf{k},E) = \sum_f \left| \langle\Psi_0\,|\,\left[|\,k\rangle \otimes |\,\Psi_f^{A-1}\rangle\right]\right|^2 \times \delta(E + E_f^{A-1} - E_0)$$

The proton spectral function $P_h(\mathbf{k},E)$ describes the **probability distribution** of removing a proton of momentum **k** from the target nucleus, leaving the residual system with excitation energy E

Spectral function approach

The response tensor is given by

$$R^{\mu\nu}(\mathbf{q},\omega) = \int d^3k dE\, P_h(\mathbf{k},E) \frac{m_N^2}{e(\mathbf{k})e(\mathbf{k}+\mathbf{q})} \sum_i \langle k \,|\, j_i^{\mu\dagger} \,|\, p \rangle \langle p \,|\, j_i^{\nu} \,|\, k \rangle \delta(\omega + E - e(\mathbf{k}+\mathbf{q}))$$



The nuclear cross section is replaced by the incoherent sum of cross sections describing scattering off individual nucleons, the recoiling (A – 1)-nucleon system acting as a spectator.

The scattering takes place on a bound nucleon, how do we introduce this effect?

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Off shellness effects

The nucleon tensor can be rewritten as

$$r_N^{\mu\nu} = \sum_i \langle k \,|\, j_i^{\mu\dagger} \,|\, p \rangle \langle p \,|\, j_i^{\nu} \,|\, k \rangle \delta(\tilde{\omega} + e(\mathbf{k}) - e(\mathbf{k}+\mathbf{q}))$$

Where: $\tilde{\omega} = e(\mathbf{k}+\mathbf{q}) - e(\mathbf{k}) = E_0 + \omega - E_f^{A-1} - e(\mathbf{k}) = \omega - E + m - e(\mathbf{k})$

We can identify the nucleon tensor as describing the scattering off a **free nucleon** at four momentum

$$q = (\omega, \mathbf{q}) \rightarrow \tilde{q} = (\tilde{\omega}, \mathbf{q})$$

The quantity $\delta\omega = \omega - \tilde{\omega}$ is the amount of energy going into the recoiling spectator system

Let's rewrite the explicit expression of the nucleon tensor in the case of quasi elastic scattering:

$$r_N^{\mu\nu} = w_1^N\Big(- g^{\mu\nu} + \frac{\tilde{q}^\mu \tilde{q}^\nu}{\tilde{q}^2}\Big) + \frac{w_2^N}{m^2}\Big(k^\mu - \frac{k\tilde{q}}{\tilde{q}^2}\tilde{q}^\mu\Big)\Big(k^\nu - \frac{k\tilde{q}}{\tilde{q}^2}\tilde{q}^\nu\Big)$$

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Off shellness effects

While the replacement of $\omega \to \tilde{\omega}$ is reasonable on physics grounds, it poses a considerable conceptual problem in that it leads to a violation of current conservation, which requires

$$q_\mu r_N^{\mu\nu} = 0$$

A possibility is to adopt a convention from de Forest, where the different components of the tensor are defined taking z along the **q** directions

$$\tilde{r}_N^{\mu\nu} = r_N^{\mu\nu}(\tilde{q})$$

For μ and/or $\nu = 0$

$$\tilde{r}_N^{3\nu} = \frac{\omega}{|\mathbf{q}|} r_N^{0\nu}(\tilde{q})$$

Different procedures can be used to restore gauge invariance. However, this only affects the longitudinal response. As a consequence, they are expected to become less and less important as the momentum transfer increases.

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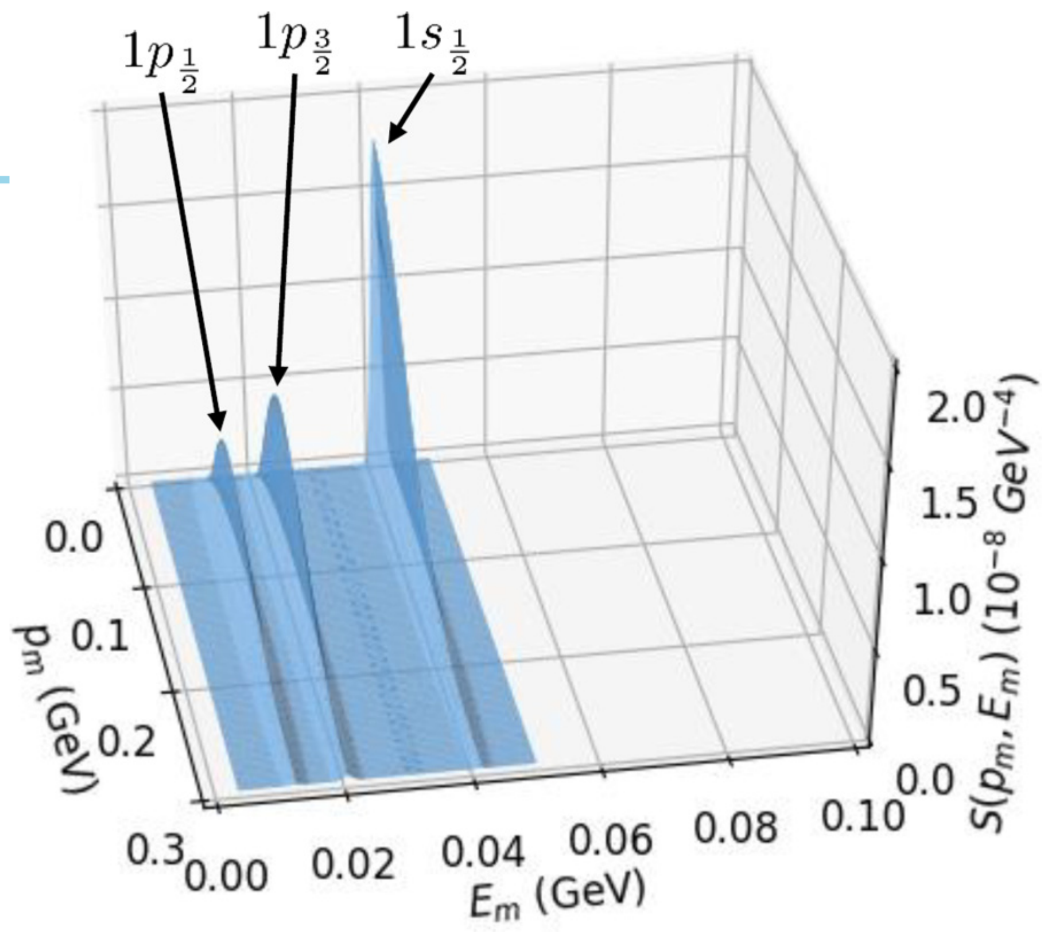
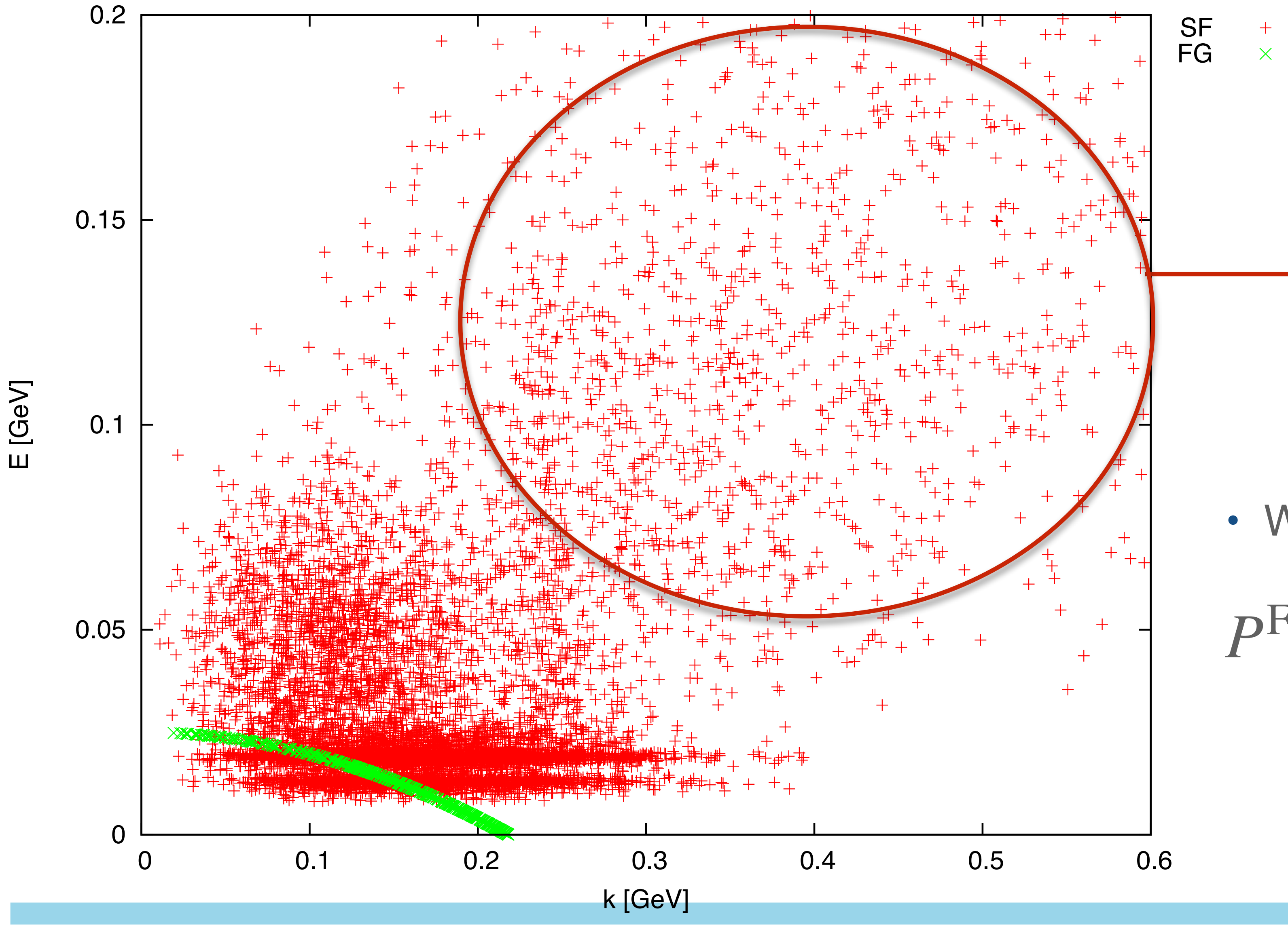
Spectral function approach

- Single-nucleon spectral function:

$$P_h(\mathbf{k}, E) = P_{\rm MF}(\mathbf{k}, E) + P_{\rm corr}(\mathbf{k}, E)$$





High energy and momentum correlated pairs

- Within the Global Fermi gas, one can write

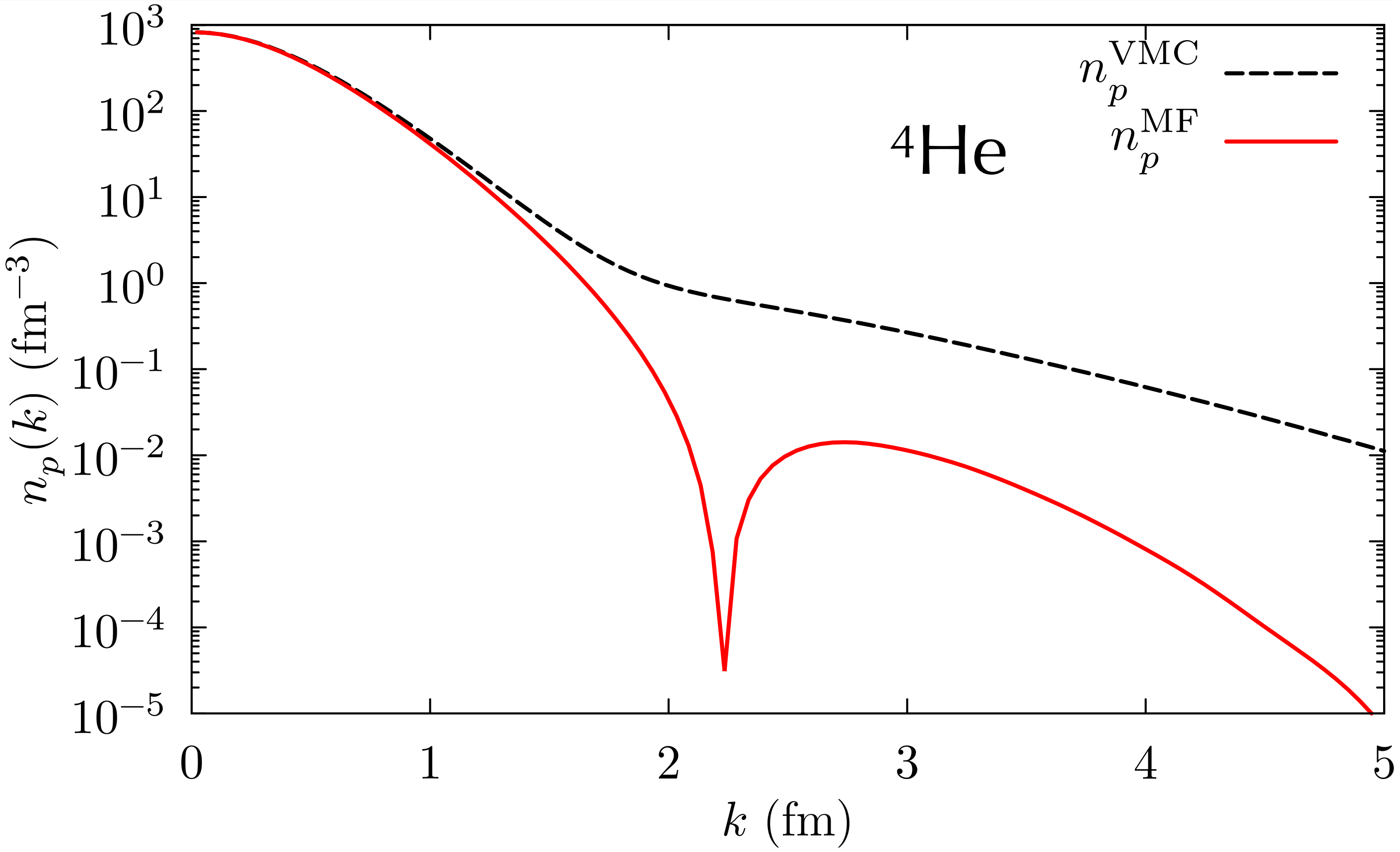
$$P^{\rm FG}(\mathbf{k}, E) = \delta(E - \epsilon_B)\theta(k_F - |\mathbf{k}|)$$

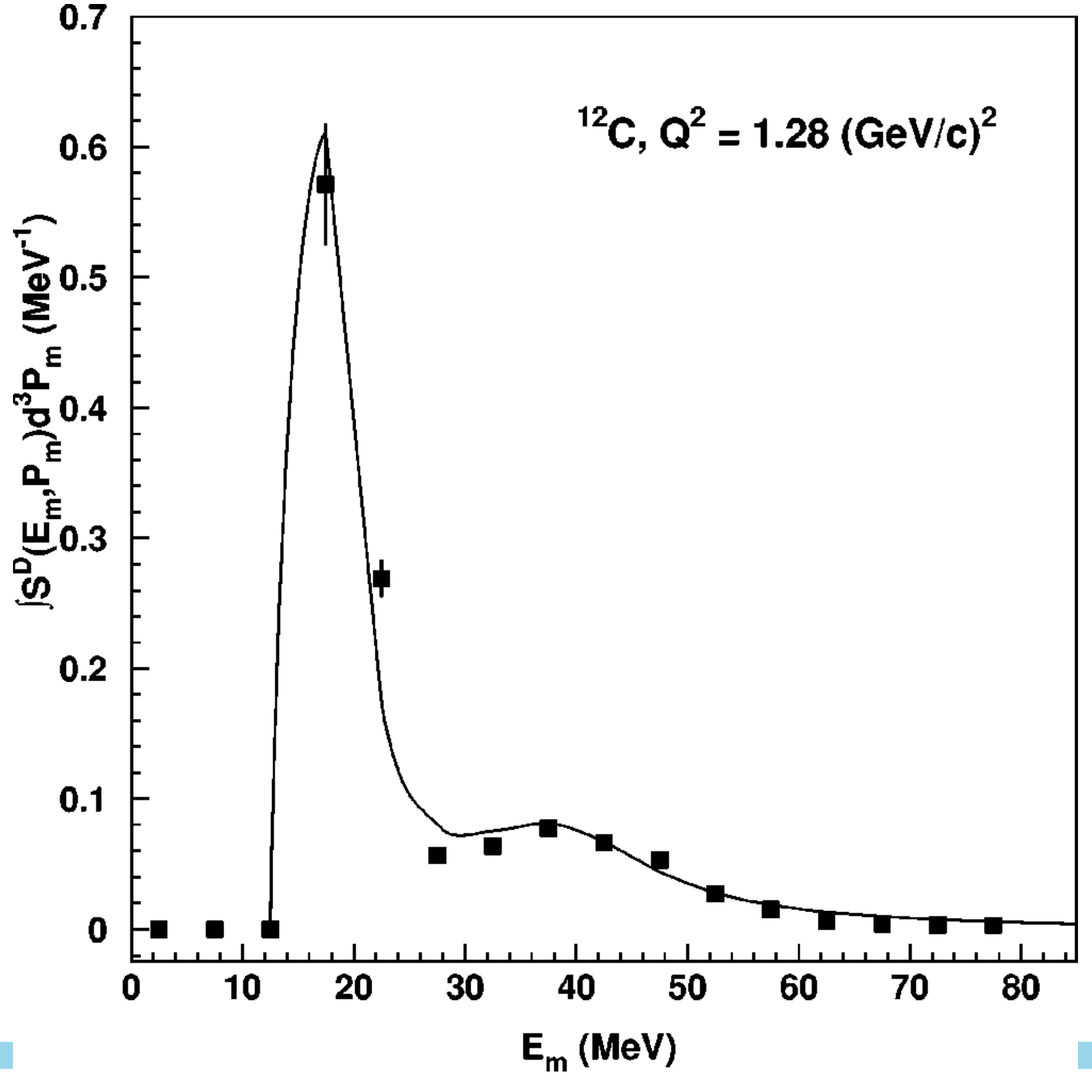
Spectral function approach

From the Spectral function we can obtain

- Single-nucleon momentum distribution:

$$n(k) = \int dE P_h(\mathbf{k}, E)$$





- Missing energy distribution

$$S(E) = \int d^3k P_h(\mathbf{k}, E)$$

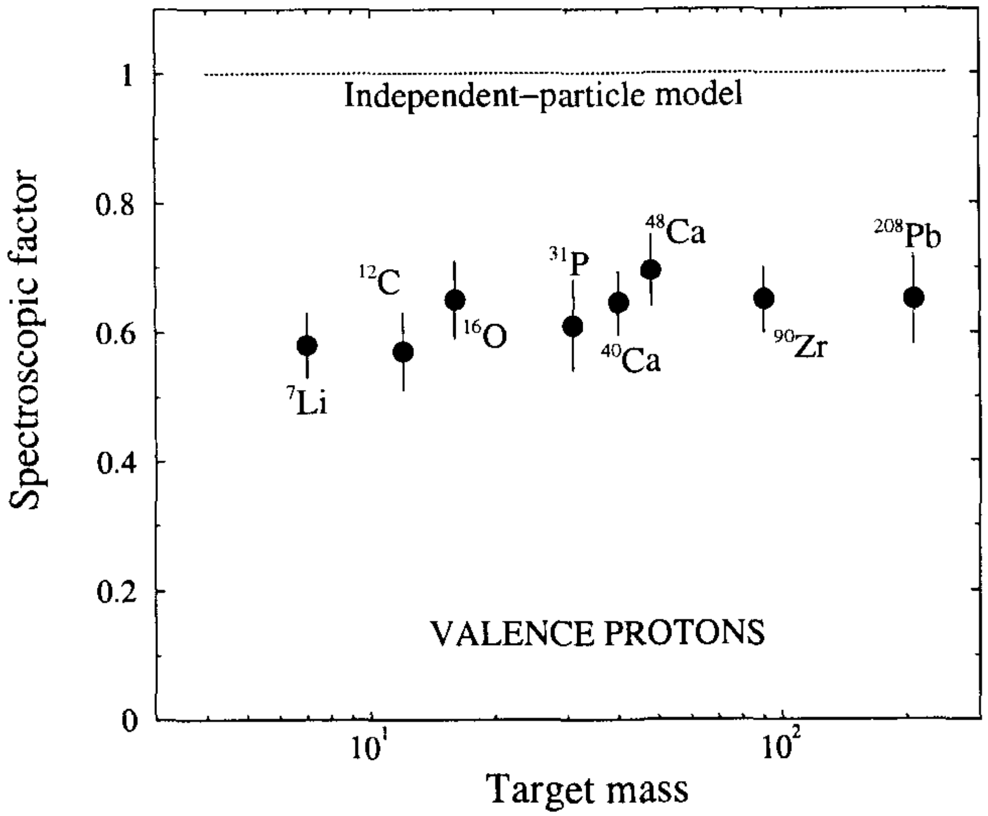
D. Dutta, et al., Phys. Rev. C 68, 064603 (2003).

Correlated Basis Function Approach

- ^{16}O Spectral Function obtained within CBF and using the Local Density Approximation

$$P_{LDA}(\mathbf{k}, E) = P_{MF}(\mathbf{k}, E) + P_{corr}(\mathbf{k}, E)$$

$$\boxed{\sum_n Z_n |\phi_n(\mathbf{k})|^2 F_n(E - E_n)}$$



O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989).

O. Benhar, A. Fabrocini, S. Fantoni, and I. Sick, Nucl. Phys. A579, 493 (1994)

Noemi Rocco, nrocco@fnal.gov

Correlated Basis Function Approach

- ^{16}O Spectral Function obtained within CBF and using the Local Density Approximation

$$P_{LDA}(\mathbf{k},E) = P_{MF}(\mathbf{k},E) + P_{corr}(\mathbf{k},E) \longrightarrow \boxed{\int d^3r P^{NM}_{corr}(\mathbf{k},E;\rho=\rho_A(r))}$$

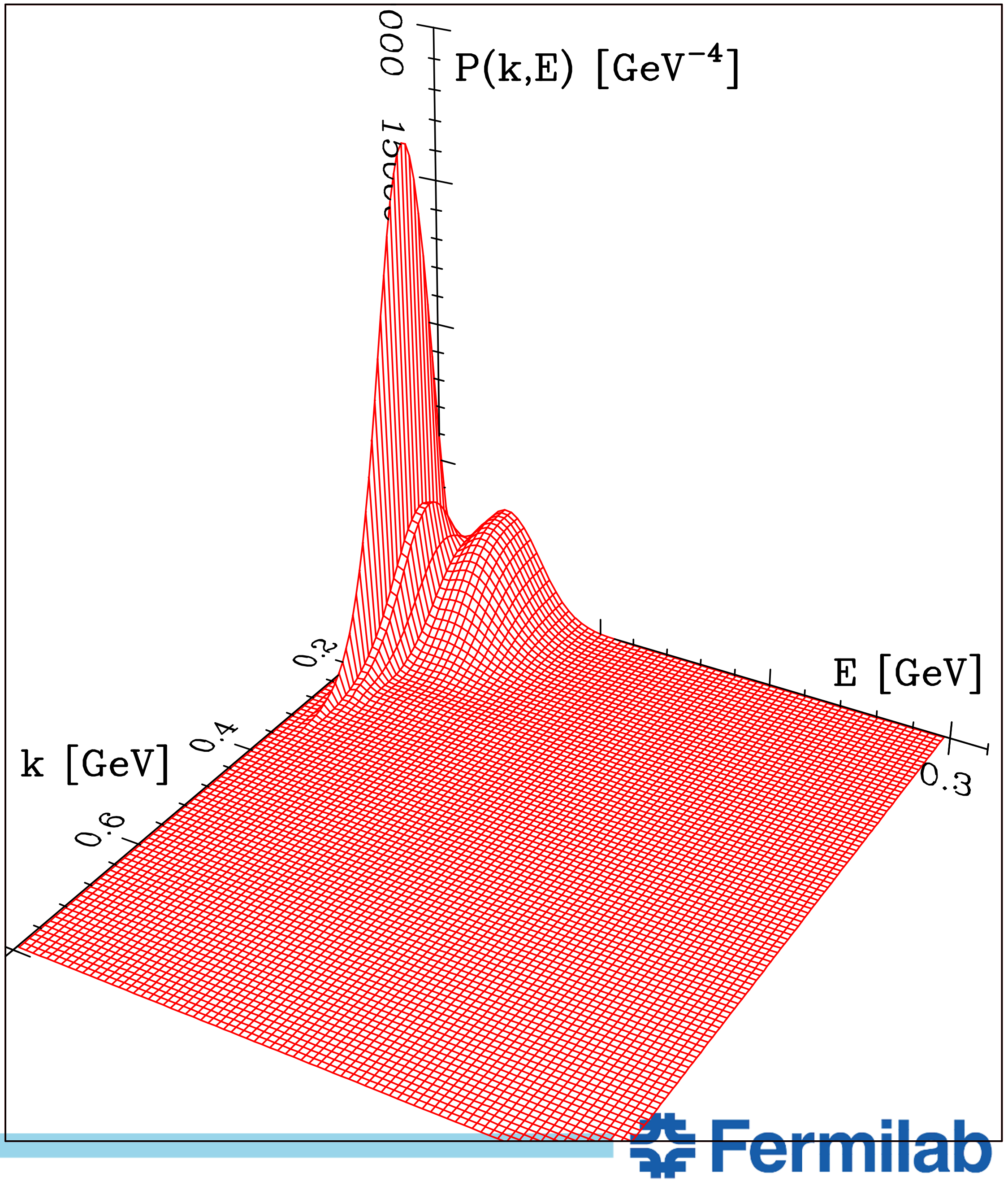
- The one-body Spectral function of nuclear matter:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} \underset{\substack{\downarrow \\ \text{Argonne v18}}}{v_{ij}} + \sum_{i<j<k} \underset{\substack{\downarrow \\ \text{UIX, IL7}}}{V_{ijk}} + \dots$$

- The Correlated Basis Function approach accounts for correlations induced by the nuclear interactions

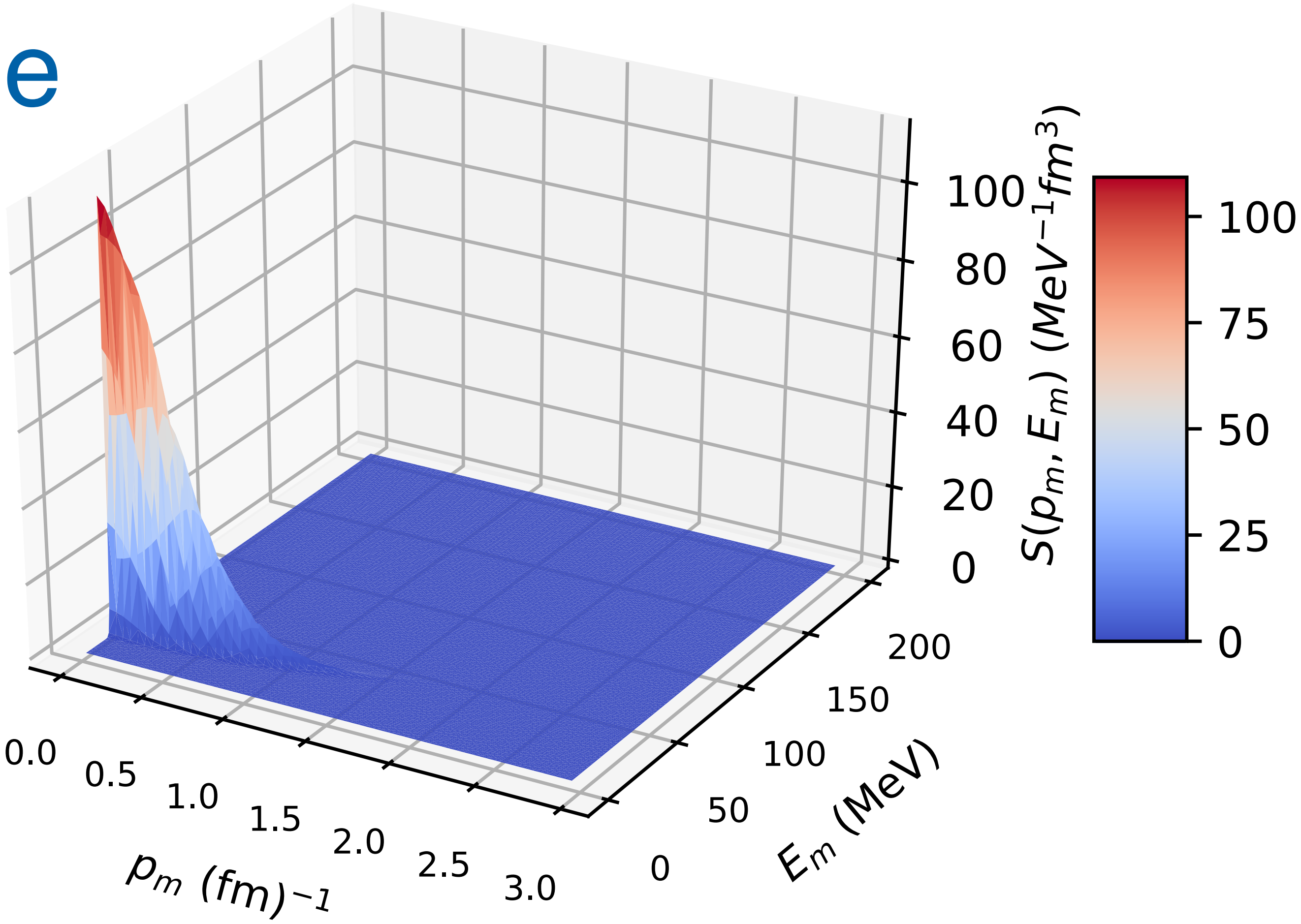
$$\Phi_n(x_1 \dots x_A) \longrightarrow \mathcal{F}\,\Phi_n(x_1 \dots x_A)$$

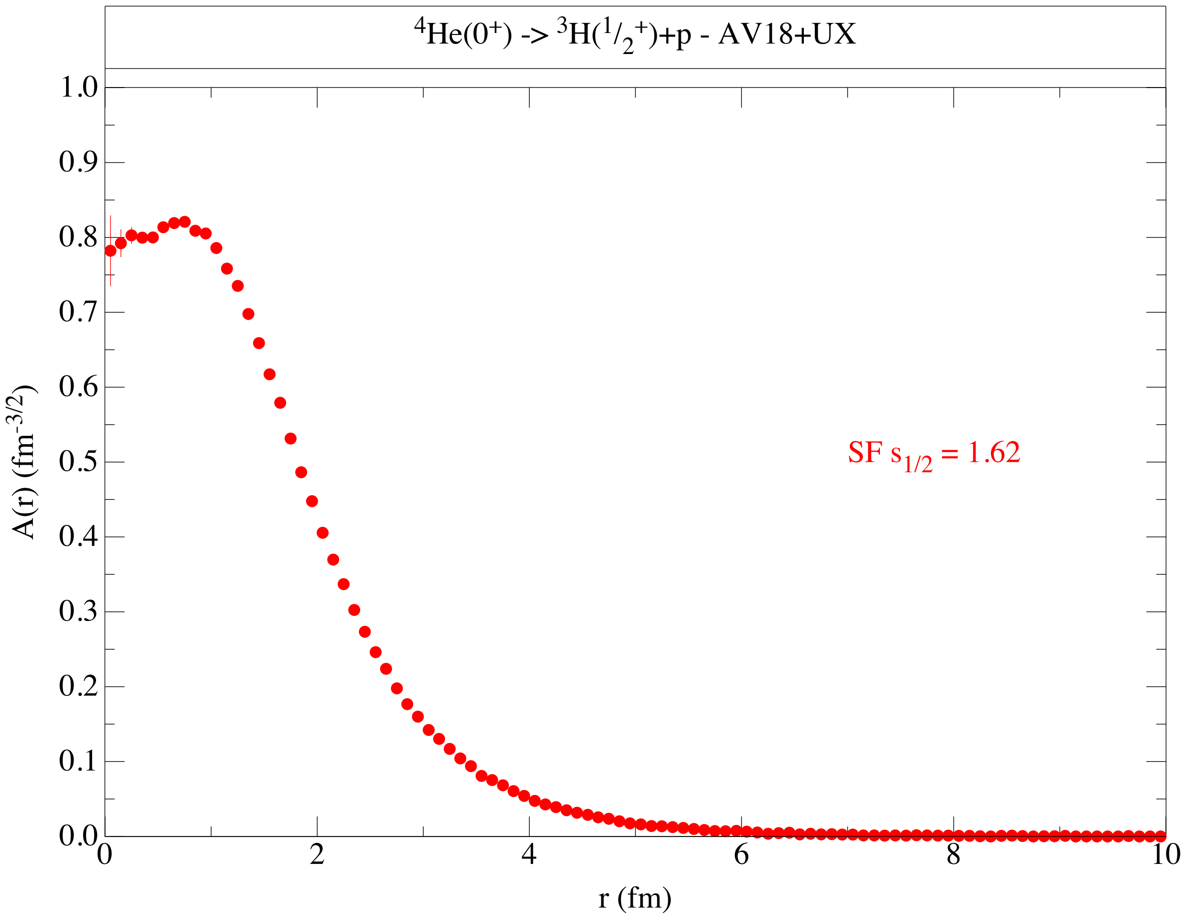
O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989).



QMC Spectral Function of ^{4}He

$$P_{\tau_k}(\mathbf{k},E) = \sum_n |\langle\Psi_0^A|[|k\rangle\,|\Psi_n^{A-1}\rangle]|^2$$
$$\times\,\delta(E+E_0^A-E_n^{A-1})$$





$$P_p^{\rm MF}(\mathbf{k},E) = \boxed{n_p^{\rm MF}(\mathbf{k})}\,\delta\Big(E - B_{^4{\rm He}} + B_{^3{\rm H}} - \frac{k^2}{2m_{^3{\rm H}}}\Big)$$

$$\downarrow$$

$$|\langle\Psi_0^{^4{\rm He}}|[|k\rangle \otimes |\Psi_0^{^3{\rm H}}\rangle]|^2$$

- The single-nucleon overlap has been computed within VMC (center of mass motion fully accounted for)

QMC Spectral Function of ^{4}He

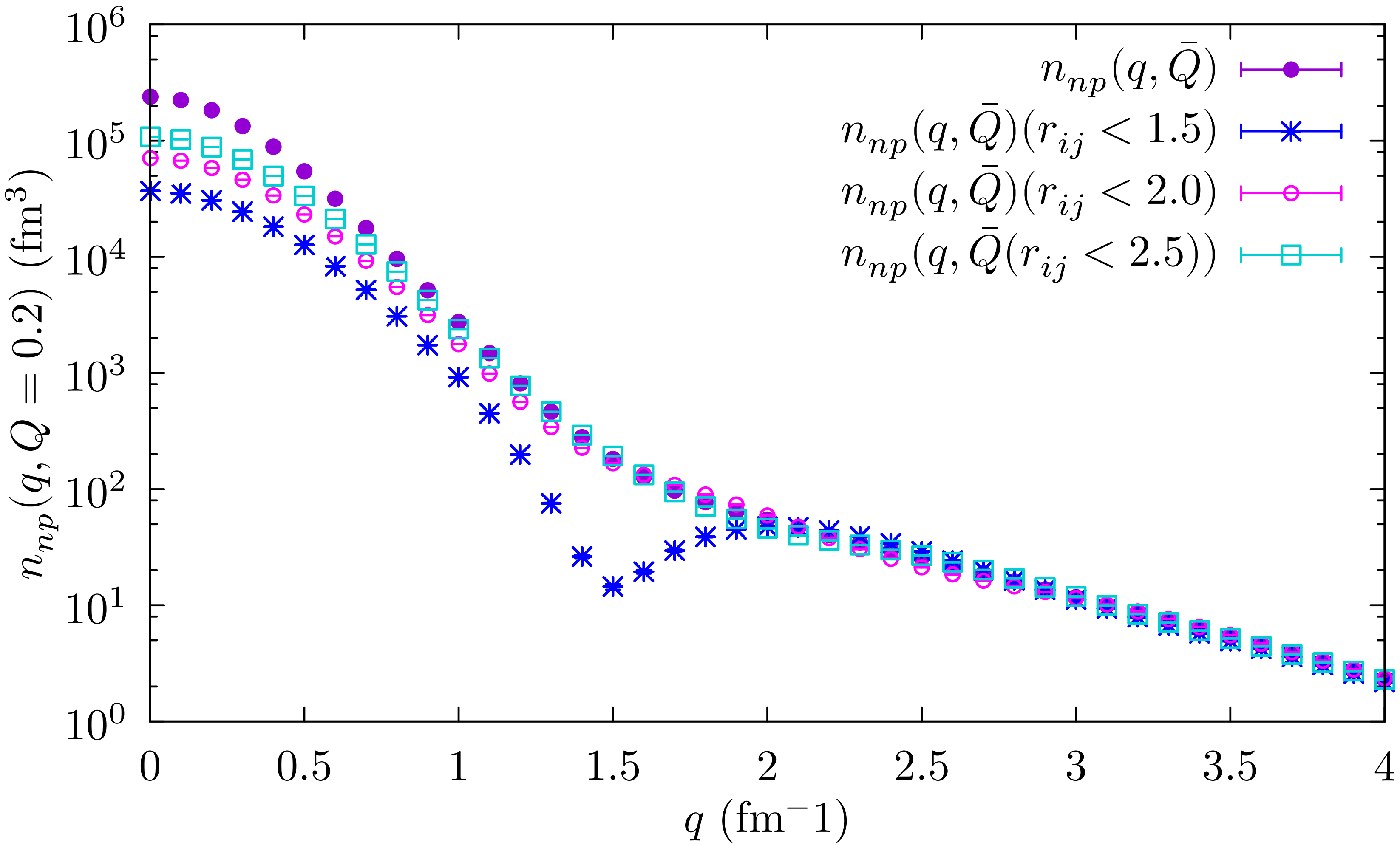
$$P_p^{\rm corr}(\mathbf{k},E)=\sum_n\int\frac{d^3k'}{(2\pi)^3}|\langle\Psi_0^A|[|k\rangle\,|k'\rangle\,|\Psi_n^{A-2}\rangle]|^2\delta(E+E_0^A-e(\mathbf{k}')-E_n^{A-2})$$

Using QMC techniques

$$\sum_{\tau_{k'}=p,n} n_{p,\tau_{k'}}(\mathbf{k},\mathbf{k}')\delta\Big(E-B_A \quad -e(\mathbf{k}')+B_{A-2}-\frac{(\mathbf{k}+\mathbf{k}')^2}{2m_{A-2}}\Big)$$

Only SRC pairs should be considered: $|\Psi_0^{A-1}\rangle$ and $|k'\rangle|\psi_n^{A-2}\rangle$ be orthogonalized

One can introduce **cuts** on the **relative distance** between the particles in the two-body momentum distribution



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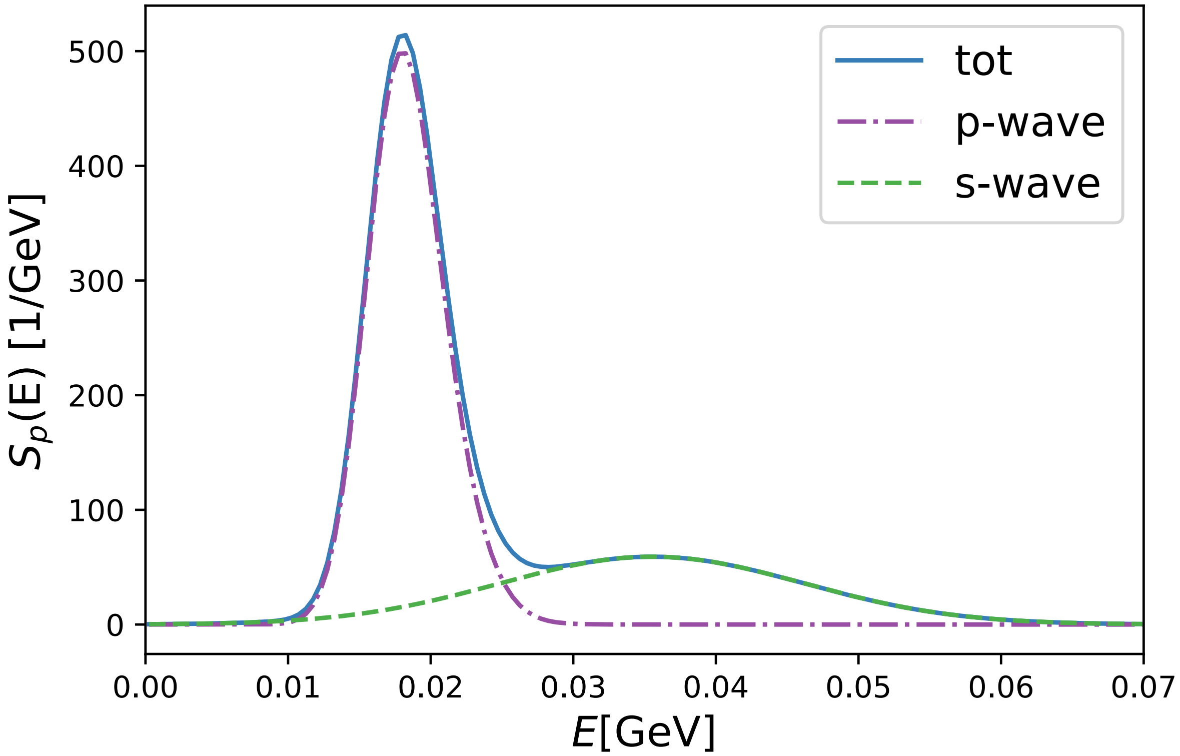
QMC Spectral Function of ^{12}C

- The p-shell contribution has been obtained by FT the radial overlaps:

$$^{12}\mathrm{C}(0^+) \rightarrow ^{11}\mathrm{B}(3/2^-) + p$$
$$^{12}\mathrm{C}(0^+) \rightarrow ^{11}\mathrm{B}(1/2^-) + p$$
$$^{12}\mathrm{C}(0^+) \rightarrow ^{11}\mathrm{B}(3/2^-)^* + p\,.$$

R. Crespo, et al, Phys.Lett.B 803 (2020) 135355

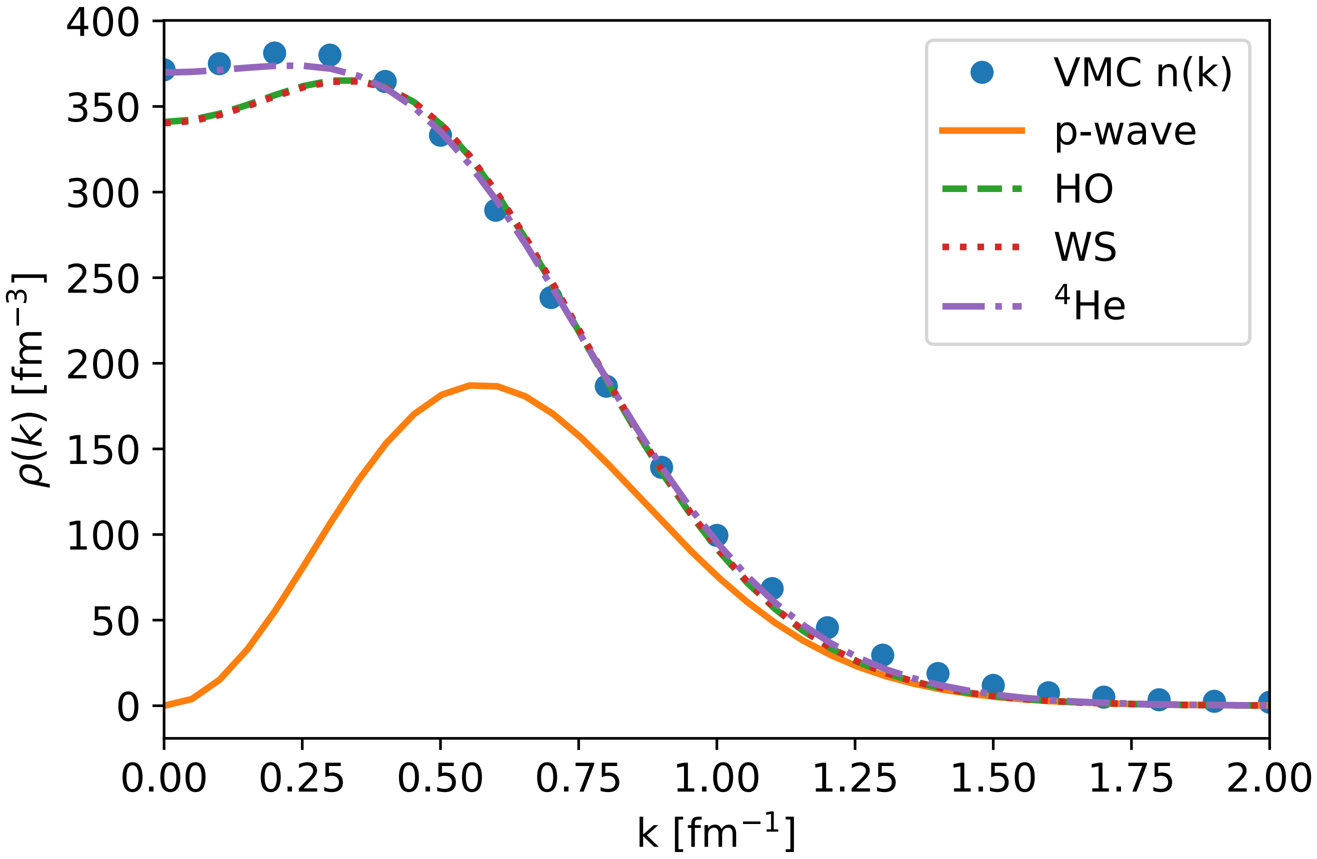


- The quenching of the spectroscopic factors automatically emerges from the VMC calculations

Computing the s-shell contribution is non trivial within VMC. We explored different alternatives:

- Quenched Harmonic Oscillator
- Quenched Wood Saxon
- VMC overlap associated for the $^4\mathrm{He}(0^+) \rightarrow \, ^3\mathrm{H}(1/2^+) + p$ transition

Korover, et al, CLAS collaboration submitted (2021)



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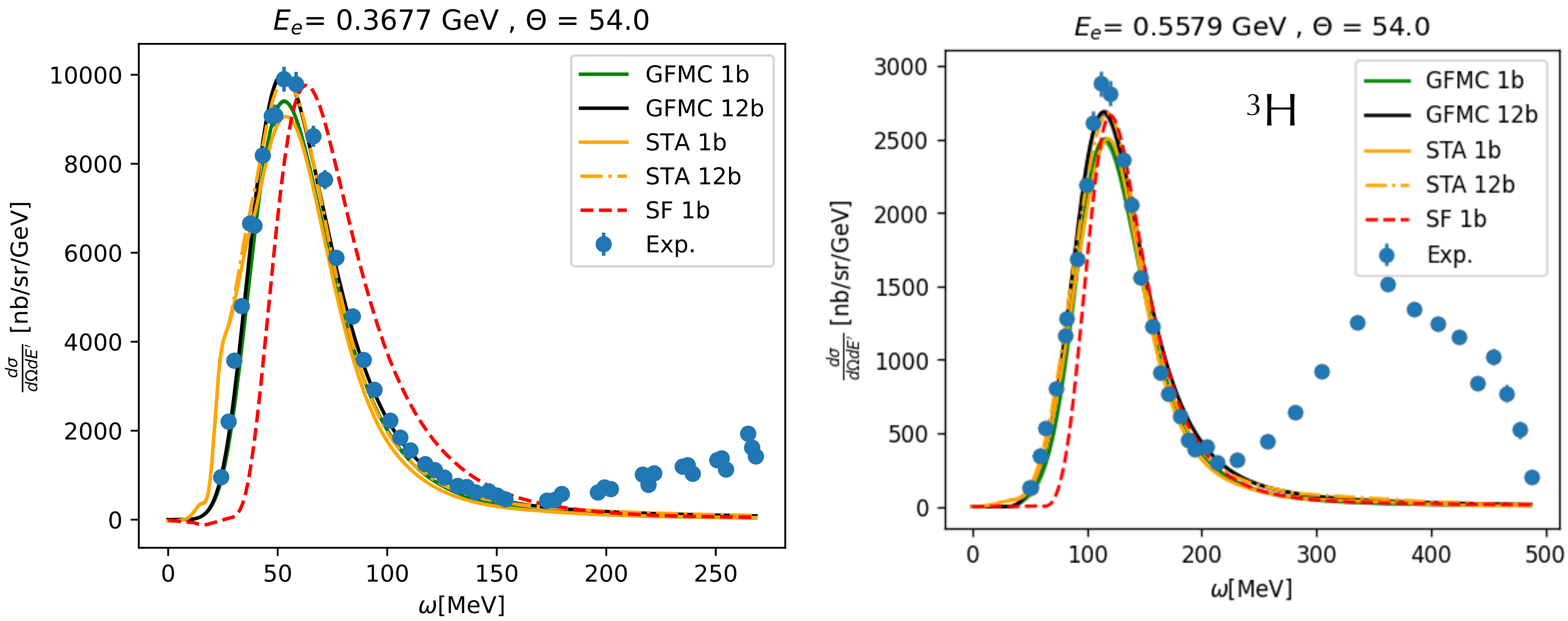
Comparing different many-body methods

- e -^{3}H: inclusive cross section

L. Andreoli, NR, et al, PRC 105 (2022) 1, 014002



- Comparisons among GFMC, SF, and STA approaches: first step to precisely **quantify the uncertainties i**nherent to the factorization of the final state.

- Gauge the role of **relativistic effects** in the energy region relevant for neutrino experiments.

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Two point Green’s function

- The nuclear matrix element can be rewritten in terms of the transition amplitude

$$[\langle\psi_f^{A-1}|\otimes\langle k|]|\psi_0^A\rangle=\sum_\alpha \mathcal{Y}_{f,\alpha}\tilde{\Phi}_\alpha(\mathbf{k})=\sum_\alpha\tilde{\Phi}_\alpha(\mathbf{k})\langle\psi_f^{A-1}|a_\alpha|\psi_0^A\rangle\,,$$

- The Spectral Function gives the probability distribution of removing a nucleon with momentum **k**, leaving the spectator system with an excitation energy E

$$P_h(\mathbf{k},E)=\sum_f|\langle\psi_0^A|[|\mathbf{k}\rangle\otimes|\psi_f^{A-1}\rangle]|^2\delta(E+E_f^{A-1}-E_0^A)$$
$$=\frac{1}{\pi}\sum_{\alpha\beta}\tilde{\Phi}^*_\beta(\mathbf{k})\tilde{\Phi}_\alpha(\mathbf{k})\mathrm{Im}\langle\psi_0^A|a_\beta^\dagger\frac{1}{E+(H-E_0^A)-i\epsilon}a_\alpha|\psi_0^A\rangle\,.$$

- The two points Green’s Function describes nucleon propagation in the nuclear medium

$$G_{h,\alpha\beta}(E)=\langle\psi_0^A|a_\beta^\dagger\frac{1}{E+(H-E_0^A)-i\epsilon}a_\alpha|\psi_0^A\rangle$$

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Self Consistent Green's Function

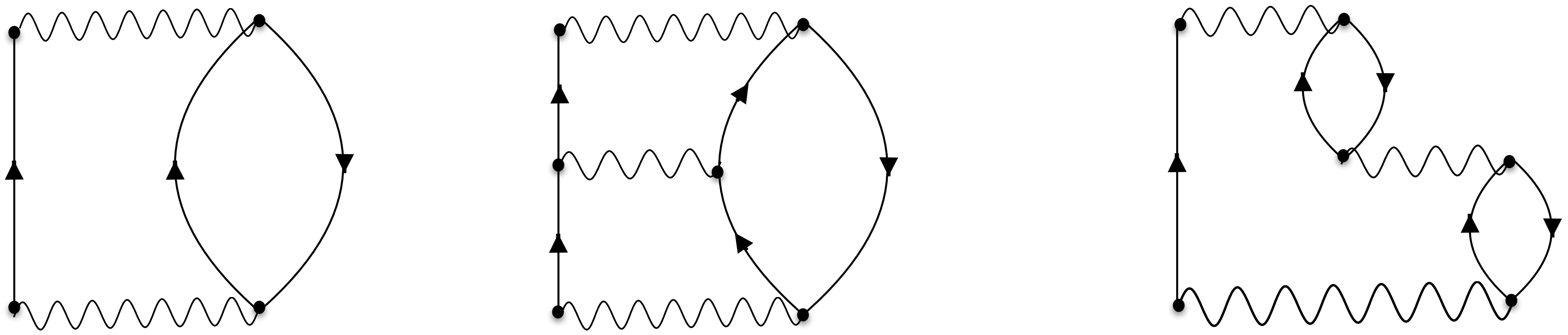
- The one-body Green's function is completely determined by solving the Dyson equation

$$G_{\alpha\beta}(E) = \boxed{G^0_{\alpha\beta}(E)} + \sum_{\gamma\delta} G^0_{\alpha\gamma}\Sigma^*_{\gamma\delta}(E)G_{\delta\beta}(E)$$

initial reference state, HF

- $\Sigma^* = \Sigma^*[G(E)]$, an iterative procedure is required to solve the Dyson equation self-consistently

- The self-energy is systematically calculated in a non-perturbative fashion within the Algebraic Diagrammatic Construction (ADC). The saturating chiral interaction at NNLO (NNLO$_{\mathrm{sat}}$) is used.



2nd and 3rd order diagrams with 2h1p (and 2p1h) intermediate configurations

✤ V. Somà et al, Phys.Rev. C87 (2013) no.1, 011303 : generalization of this formalism within Gorkov theory allows to describe open-shell nuclei such as Ar^{40}, Ti^{48} …

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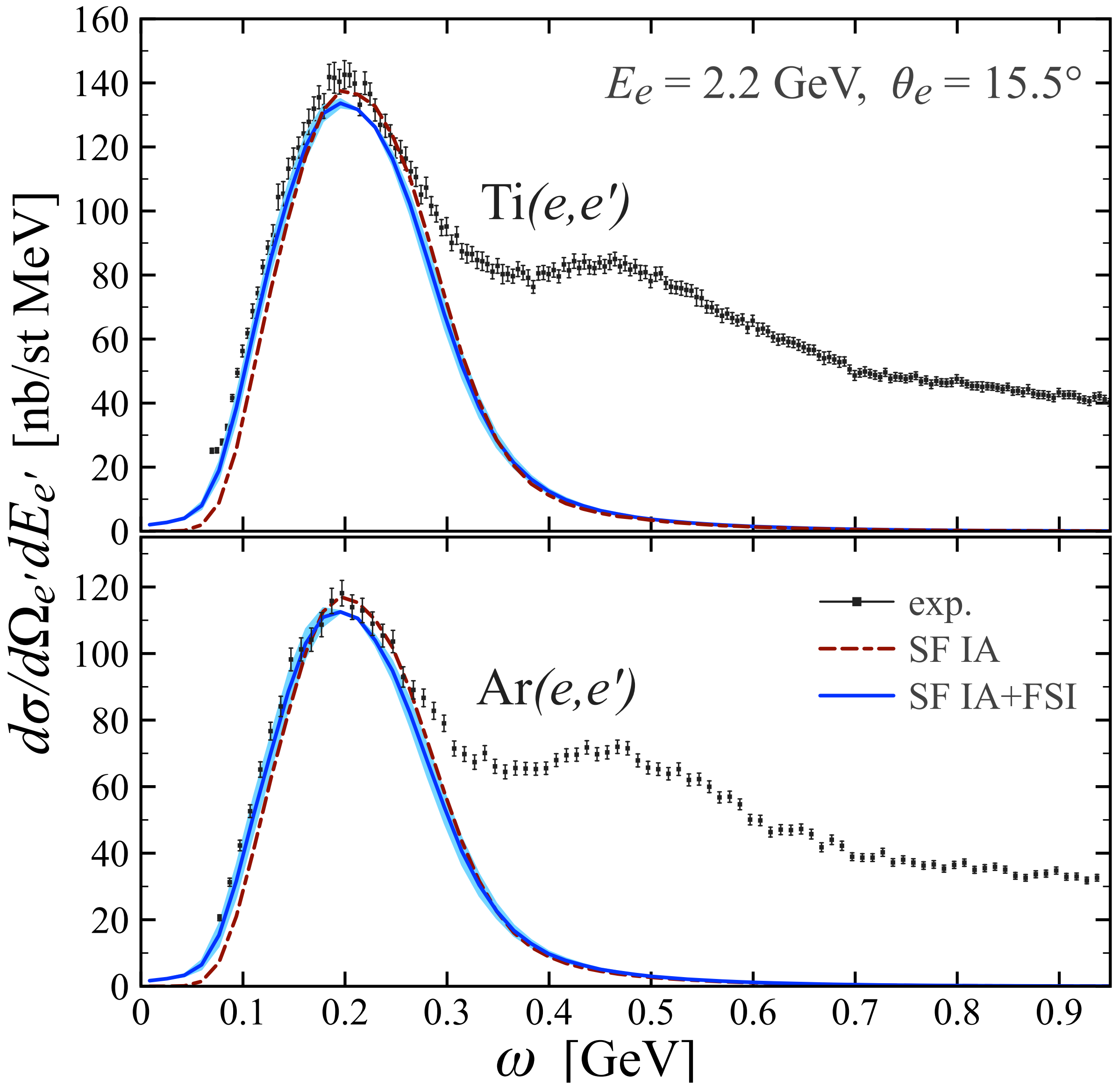
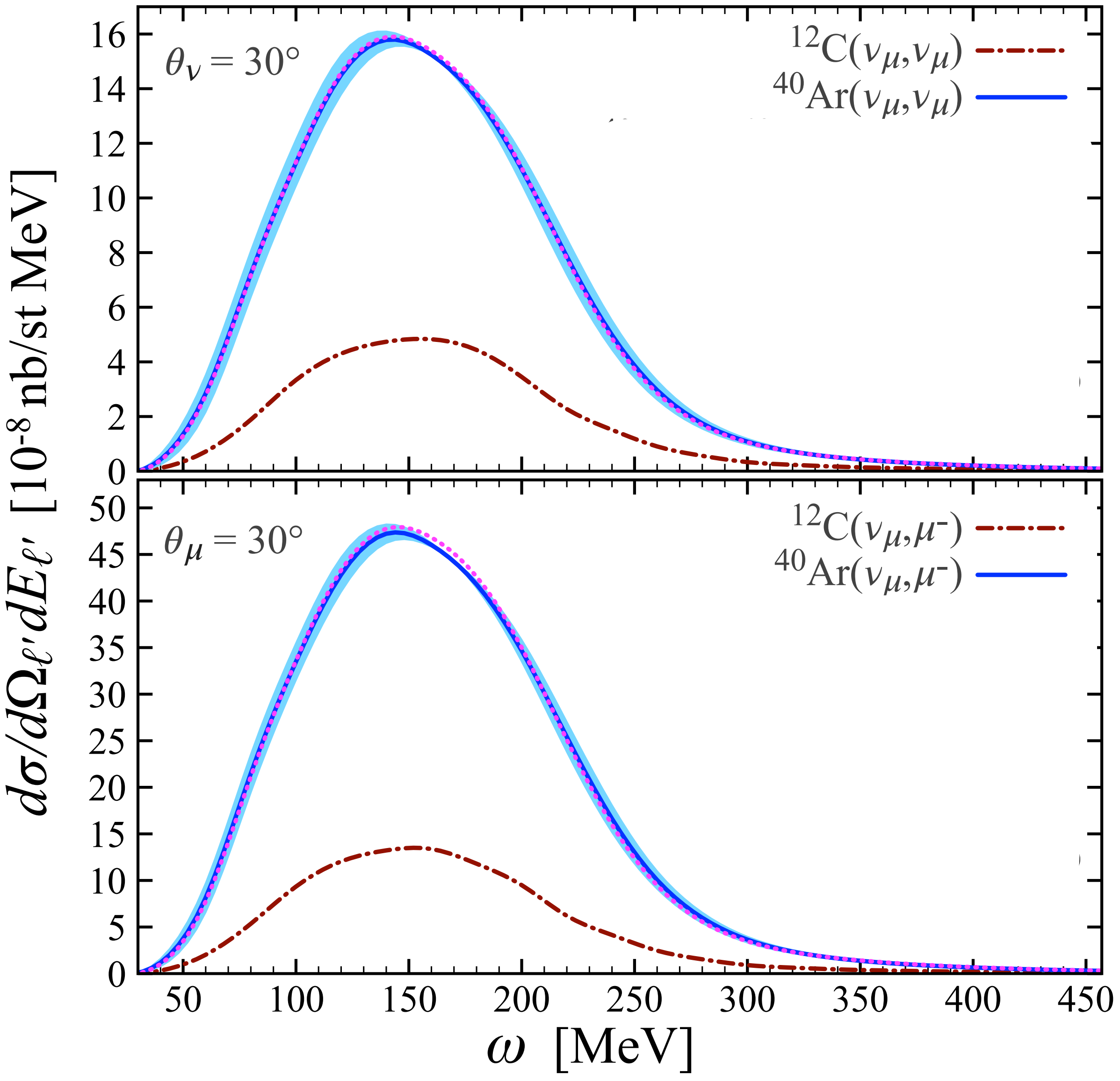
Self Consistent Green’s Function

^{40}Ar(e,e’) and ^{48}Ti(e,e’) cross sections

✐ C. Barbieri, NR, and V. Somà, PRC 100, no.6, 062501 (2019)



Charge current and neutral current ν_μ scattering on ^{12}C and Ar for $E\nu_\mu$ =1 GeV



The band comes from a **first estimate of the uncertainty on the spectral function** calculation obtained by varying the model-space and the harmonic oscillator frequency

Coupled-Cluster theory

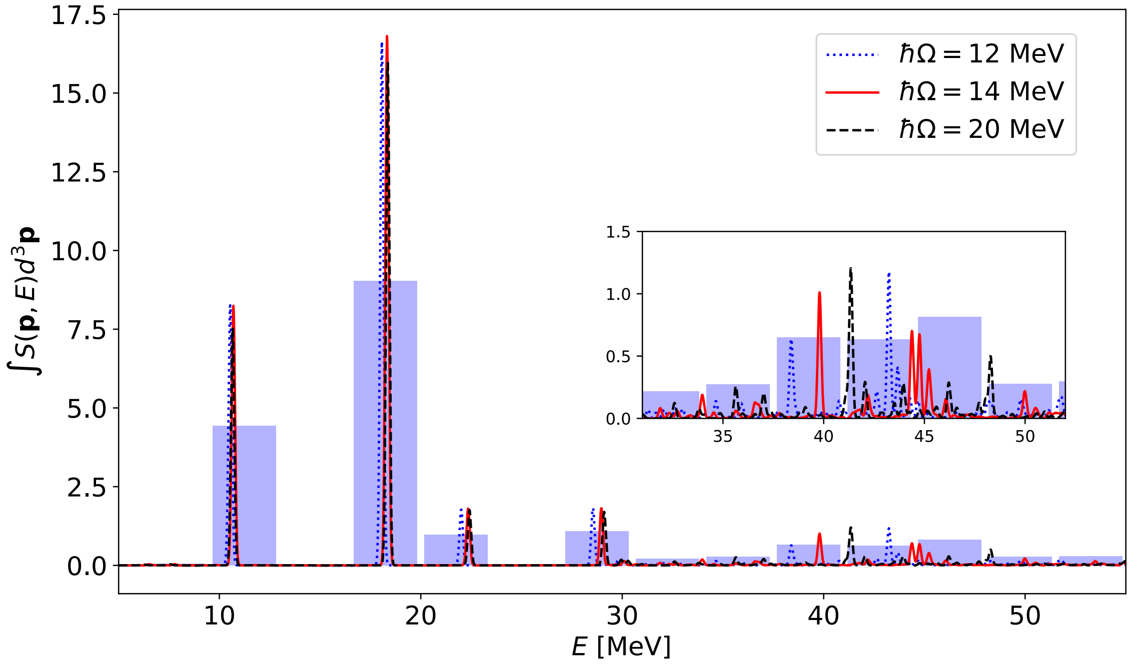
$$P_h(\mathbf{k},E) = \sum_f \Bigg| \langle \Psi_0 | \Big[| k \rangle \otimes | \Psi_f^{A-1} \rangle \Big]^2 \times \delta(E + E_f^{A-1} - E_0)$$

The sum over final A-1 states is comprised of **bound states** and **continuous spectrum**

This can be calculated considering its integral transform: regularizing the delta conservation with a kernel K_Λ

$$P_\Lambda(\mathbf{k},E) = \int K_\Lambda(E,E')P_h(\mathbf{k},E')dE'$$

Technically it is done by expanding the kernel into Chebyshev polynomials + building a histogram



More information:

Integral transform using Chebyshev polynomials:

A. Roggero Phys.Rev.A 102 (2020) 2, 022409

J.E. Sobczyk, A. Roggero Phys.Rev.E 105 (2022) 055310

Calculation of spectral function for ^{4}He & ^{16}O:

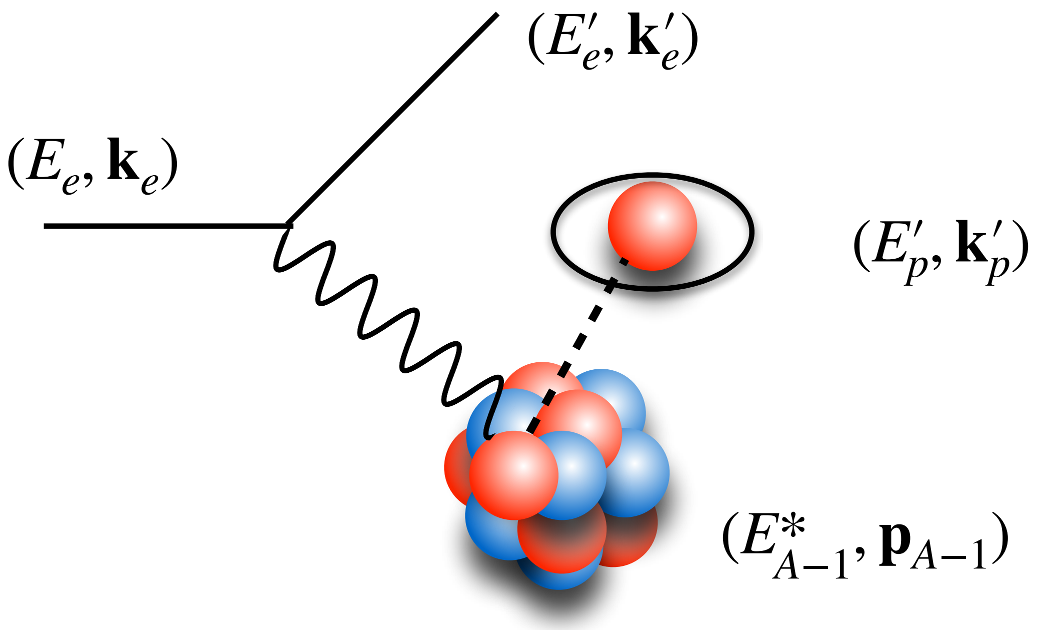
J.E. Sobczyk, S. Bacca, G. Hagen, T. Papenbrock Phys.Rev.C 106 (2022) 3, 034310

J.E. Sobczyk , S. Bacca, Phys.Rev.C 109 (2024) 044314

Extraction of ^{40}Ar spectral function - E12-14-01 exp

Jiang et al., PRD 105, 112002 (2022)

Jiang et al., PRD 107, 012005 (2023)



$$E_e + M_A = E_e' + E_p' + E^*_{A-1}$$

$$\mathbf{k}_e + 0 = \mathbf{k}_e' + \mathbf{p}' + \mathbf{p}_{A-1}$$

Without Final State interactions, one can connect: $-\mathbf{p}_{A-1} = \mathbf{p}_m$ and $E^*_{A-1} = M_A - M + E_m$

and rewrite the expressions:

$$E_e + M - E_m = E_e' + E_p'$$

$$\mathbf{k}_e + \mathbf{p}_m = \mathbf{k}_e' + \mathbf{p}'$$

Extraction of ^{40}Ar - Mean Field contribution

$$P_{\rm MF}(p_m, E_m) = \sum_\alpha S_\alpha |\phi_\alpha(p_m)|^2 f_\alpha(E_m)$$

Spectroscopic factors   Missing energy distributions

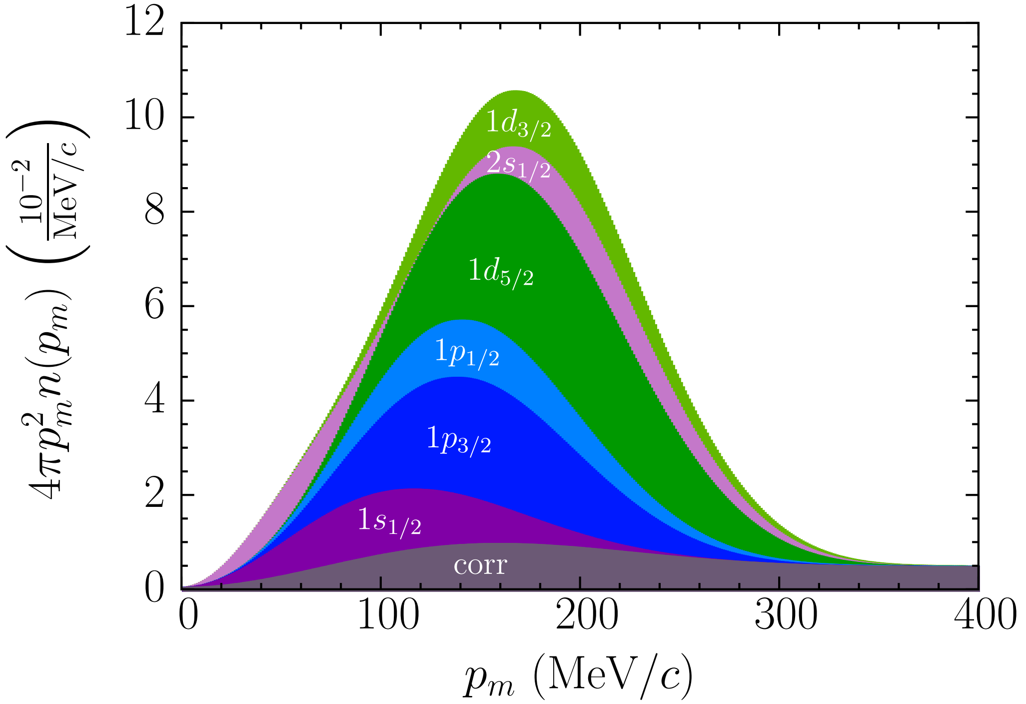
$$f_\alpha(E_m) = \frac{1}{\sqrt{2\pi}\sigma_\alpha} \exp\left[-\left(\frac{E_m - E_\alpha}{\sqrt{2}\sigma_\alpha}\right)^2\right]$$

Starting from shell model predictions; assume $S_\alpha = 0.8 N_\alpha$

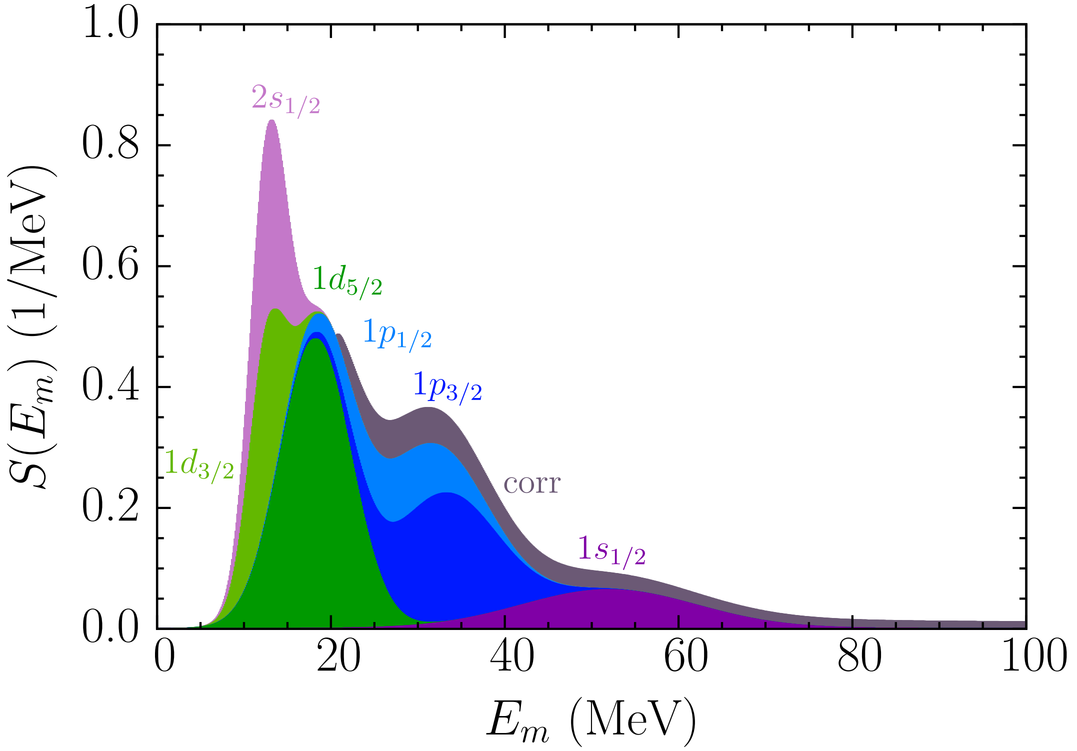
α	N_α	S_α	E_α (MeV)	σ_α (MeV)
$1d_{3/2}$	2	1.6	12.53	2
$2s_{1/2}$	2	1.6	12.93	2
$1d_{5/2}$	6	4.8	18.23	4
$1p_{1/2}$	2	1.6	28.0	6
$1p_{3/2}$	4	3.2	33.0	6
$1s_{1/2}$	2	1.6	52.0	10
Corr.	…	3.6	20.60	…

Jiang et al., PRD 105, 112002 (2022)

Momentum distribution of protons



Missing en distribution of protons

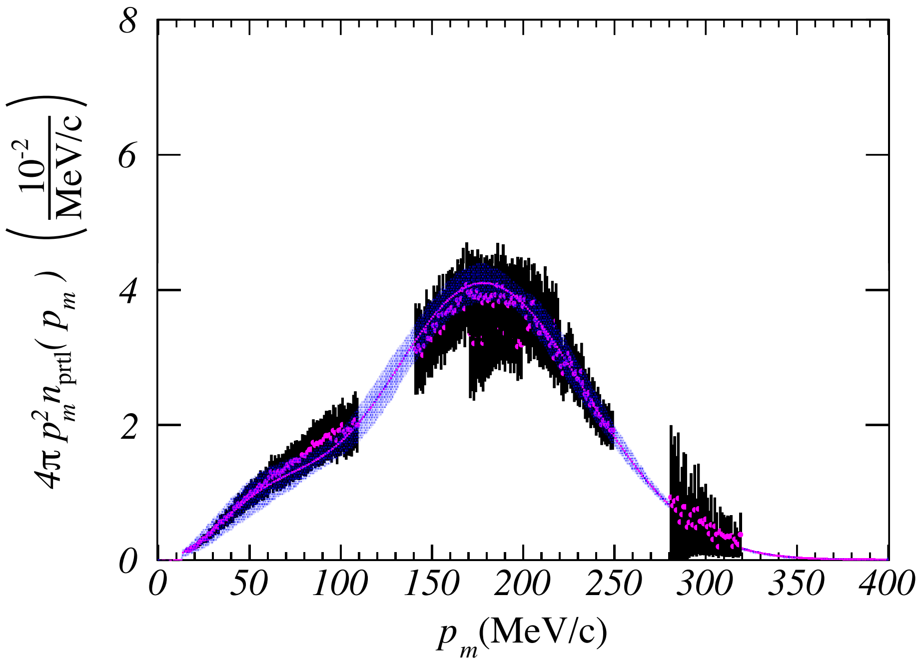


Extraction of ^{40}Ar proton spectral function

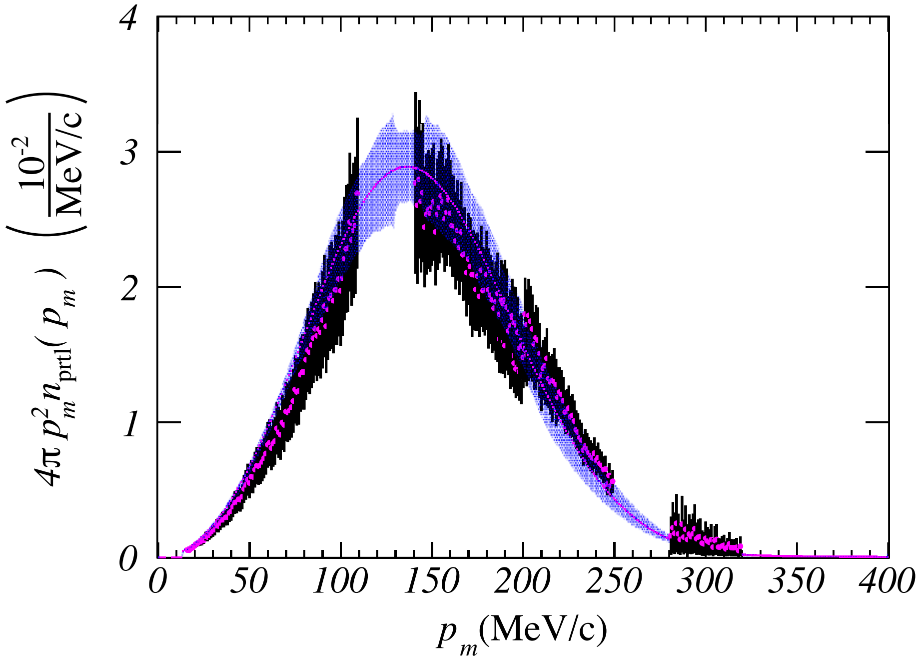
- Find the parameters of the p_m distribution (i.e., spectroscopic factors) from the fits to the reduced cross sections as a function of p_m
- Find the parameters of the Em distribution (i.e., spectroscopic factors, peak positions, distribution widths) from the fits to the reduced cross sections as a function of Em

		All priors	w/o p_m	w/o corr.
α	N_α	S_α		
$1d_{3/2}$	2	0.89 ± 0.11	1.42 ± 0.20	0.95 ± 0.11
$2s_{1/2}$	2	1.72 ± 0.15	1.22 ± 0.12	1.80 ± 0.16
$1d_{5/2}$	6	3.52 ± 0.26	3.83 ± 0.30	3.89 ± 0.30
$1p_{1/2}$	2	1.53 ± 0.21	2.01 ± 0.22	1.83 ± 0.21
$1p_{3/2}$	4	3.07 ± 0.05	2.23 ± 0.12	3.12 ± 0.05
$1s_{1/2}$	2	2.51 ± 0.05	2.05 ± 0.23	2.52 ± 0.05
Corr.	0	3.77 ± 0.28	3.85 ± 0.25	Excluded
$\sum_\alpha S_\alpha$		17.02 ± 0.48	16.61 ± 0.57	14.12 ± 0.42
d.o.f		206	231	232
χ^2/d.o.f.		1.9	1.4	2.0

Jiang et al., PRD 105, 112002 (2022)



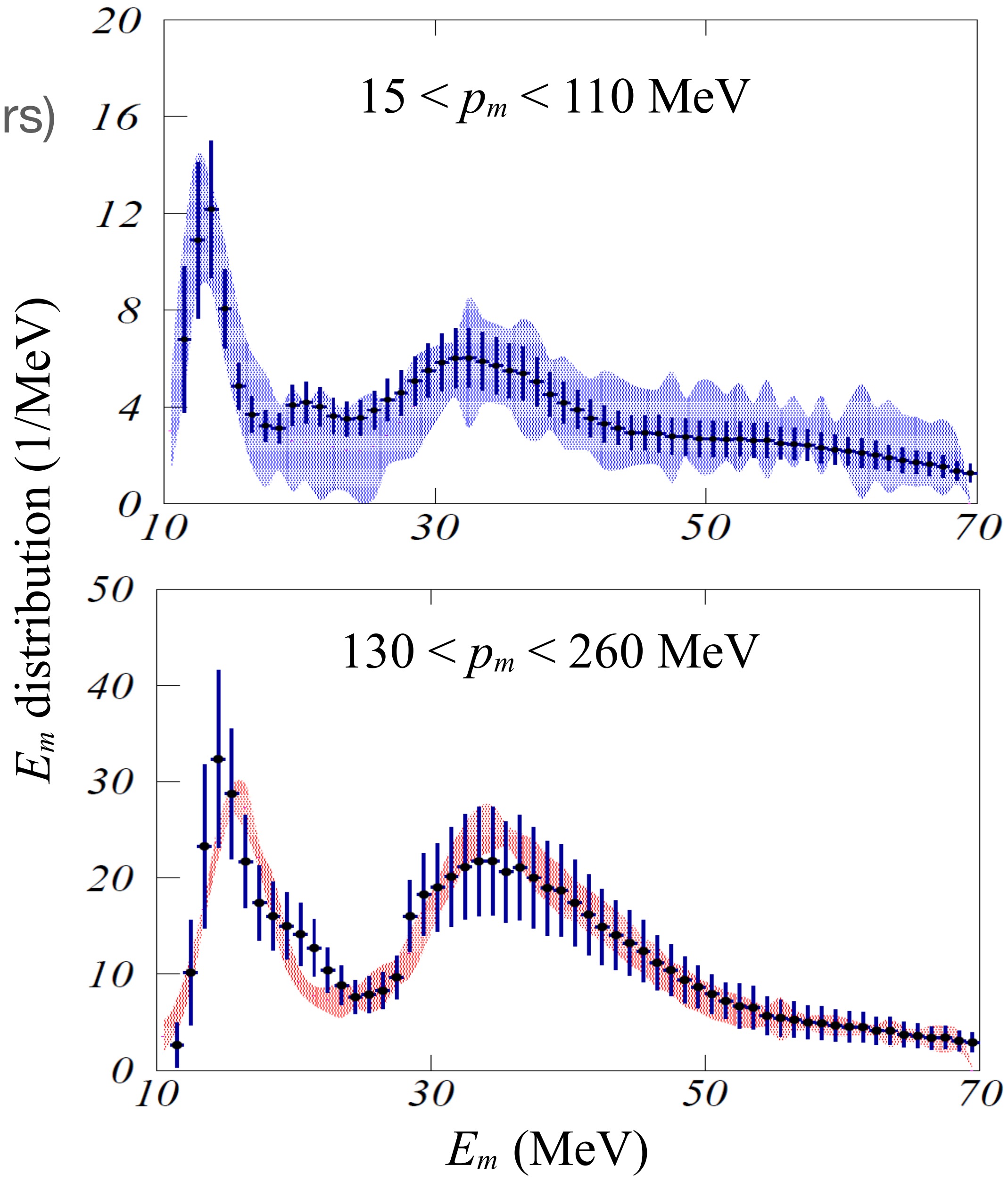
(a) $0 < E_m < 30$ MeV



(b) $30 < E_m < 54$ MeV

Extraction of 40Ar - MF

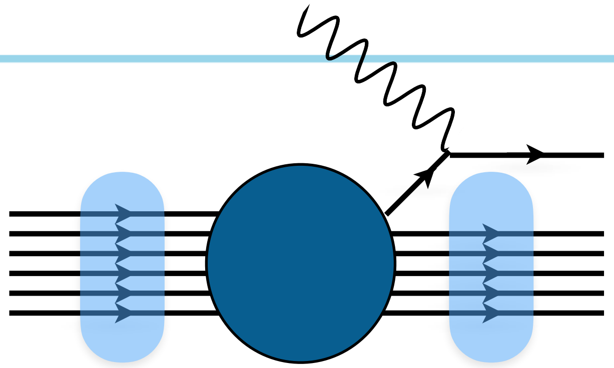
- Find the parameters of the p_m distribution (i.e., spectroscopic factors) from the fits to the reduced cross sections as a function of p_m
- Find the parameters of the Em distribution (i.e., spectroscopic factors, peak positions, distribution widths) from the fits to the reduced cross sections as a function of Em



	E_α (MeV)		σ_α (MeV)	
α	w/ priors	w/o priors	w/ priors	w/o priors
$1d_{3/2}$	12.53 ± 0.02	10.90 ± 0.12	1.9 ± 0.4	1.6 ± 0.4
$2s_{1/2}$	12.92 ± 0.02	12.57 ± 0.38	3.8 ± 0.8	3.0 ± 1.8
$1d_{5/2}$	18.23 ± 0.02	17.77 ± 0.80	9.2 ± 0.9	9.6 ± 1.3
$1p_{1/2}$	28.8 ± 0.7	28.7 ± 0.7	12.1 ± 1.0	12.0 ± 3.6
$1p_{3/2}$	33.0 ± 0.3	33.0 ± 0.3	9.3 ± 0.5	9.3 ± 0.5
$1s_{1/2}$	53.4 ± 1.1	53.4 ± 1.0	28.3 ± 2.2	28.1 ± 2.3
corr.	24.1 ± 2.7	24.1 ± 1.7	—	—

Jiang et al., PRD 105, 112002 (2022)

Phenomenology of Final State Interactions



- In the kinematical region in which the interactions between the struck particle and the spectator system can not be neglected, the IA results have to be modified to include the effect of final state interactions (FSI).

$$d\sigma_{FSI} = \int d\omega' f_{\mathbf{q}}(\omega - \omega') d\tilde{\sigma}_{IA} \quad , \quad \tilde{e}(\mathbf{p}) = e(\mathbf{p}) + \underbrace{\mathcal{U}(t_{kin}(\mathbf{p}))}_{\text{Optical Potential}}$$

Let's start discussing the effect of the energy shift. This is connected to the real part of an optical potential extracted from proton-nucleus scattering. (The ones used here are from Cooper et al using Dirac Phenomenology)

This optical potential depends on $t_{\text{kin}}(\mathbf{p})$ and **r**. How do we obtain something that only depends on p?

$$E'_{\text{tot}}(\mathbf{p}, r) = \sqrt{(m_N + U_S(\mathbf{p}, r))^2 + \mathbf{p}^2} + U_V(\mathbf{p}, r))$$

We get:

$$\mathcal{U}(t_{\text{tkin}}(\mathbf{p})) = \int d^3 r \rho(r) \Re(E'_{\text{tot}}) - E(\mathbf{p})$$

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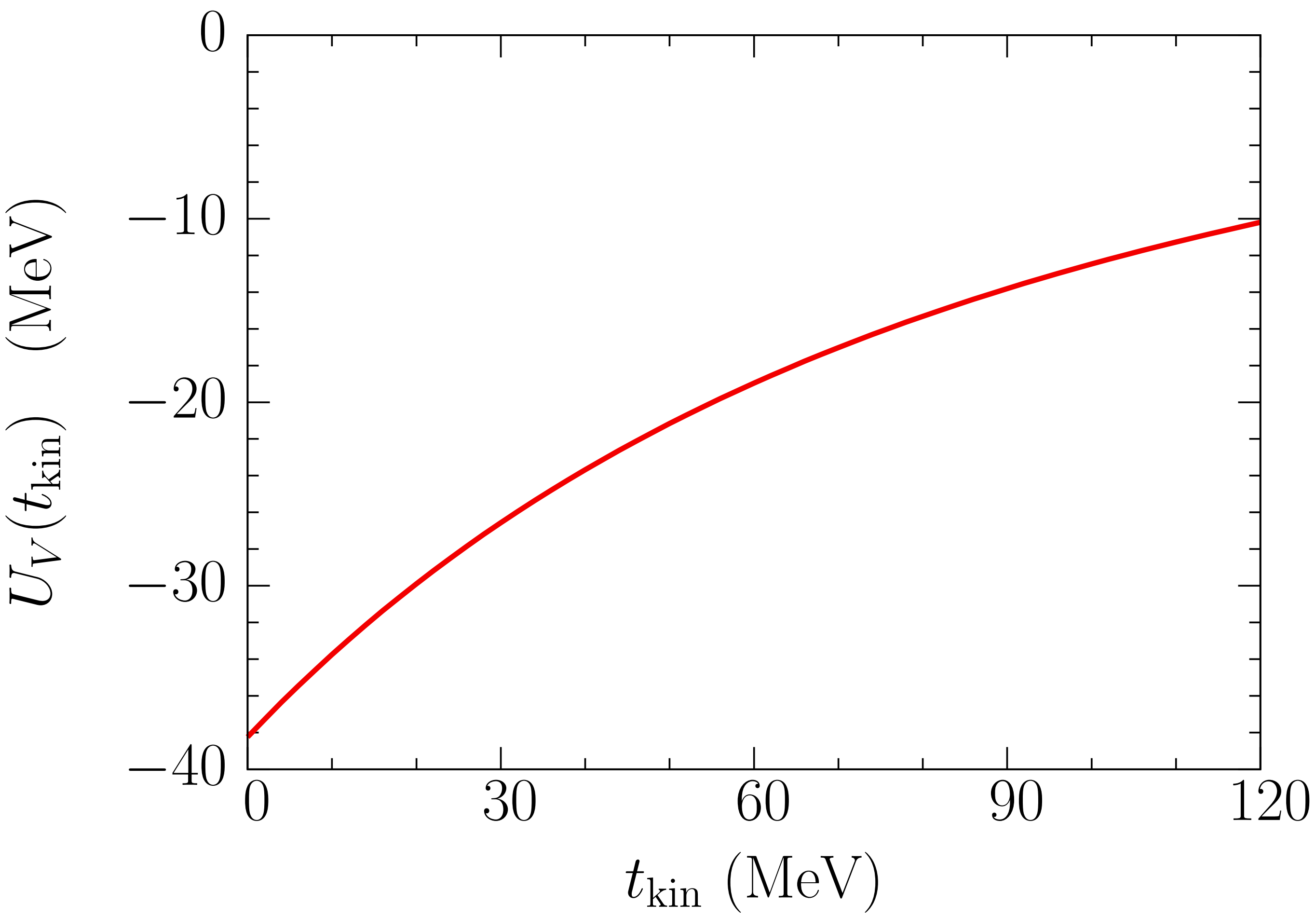
Phenomenology of Final State Interactions



$$d\sigma_{FSI} = \int d\omega' f_{\mathbf{q}}(\omega - \omega') d\tilde{\sigma}_{IA} \quad , \quad \tilde{e}(\mathbf{p}) = e(\mathbf{p}) + \mathcal{U}(t_{kin}(\mathbf{p}))$$

Optical Potential

It clearly appears that in the low-tkin region, particularly relevant to QE scattering, interactions with the spectator system lead to a sizable modification to the struck protons's spectrum.

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Phenomenology of Final State Interactions

- The theoretical approach to calculate the folding function consists on a generalization of Glauber theory of high energy proton-nucleus scattering

$$f_{\mathbf{q}}(\omega) = \delta(\omega)\underbrace{\sqrt{T_{\mathbf{q}}}}_{\text{Nuclear Transparency}} + \int \frac{dt}{2\pi} e^{i\omega t} \left[\underbrace{\bar{U}^{FSI}_{\mathbf{q}}(t)}_{\text{Glauber Factor}} - \sqrt{T_{\mathbf{q}}}\,\right]$$

A.Ankowski et al,Phys. Rev. D91, 033005 (2015)

O.Benhar, Phys. Rev. C87, 024606 (2013)

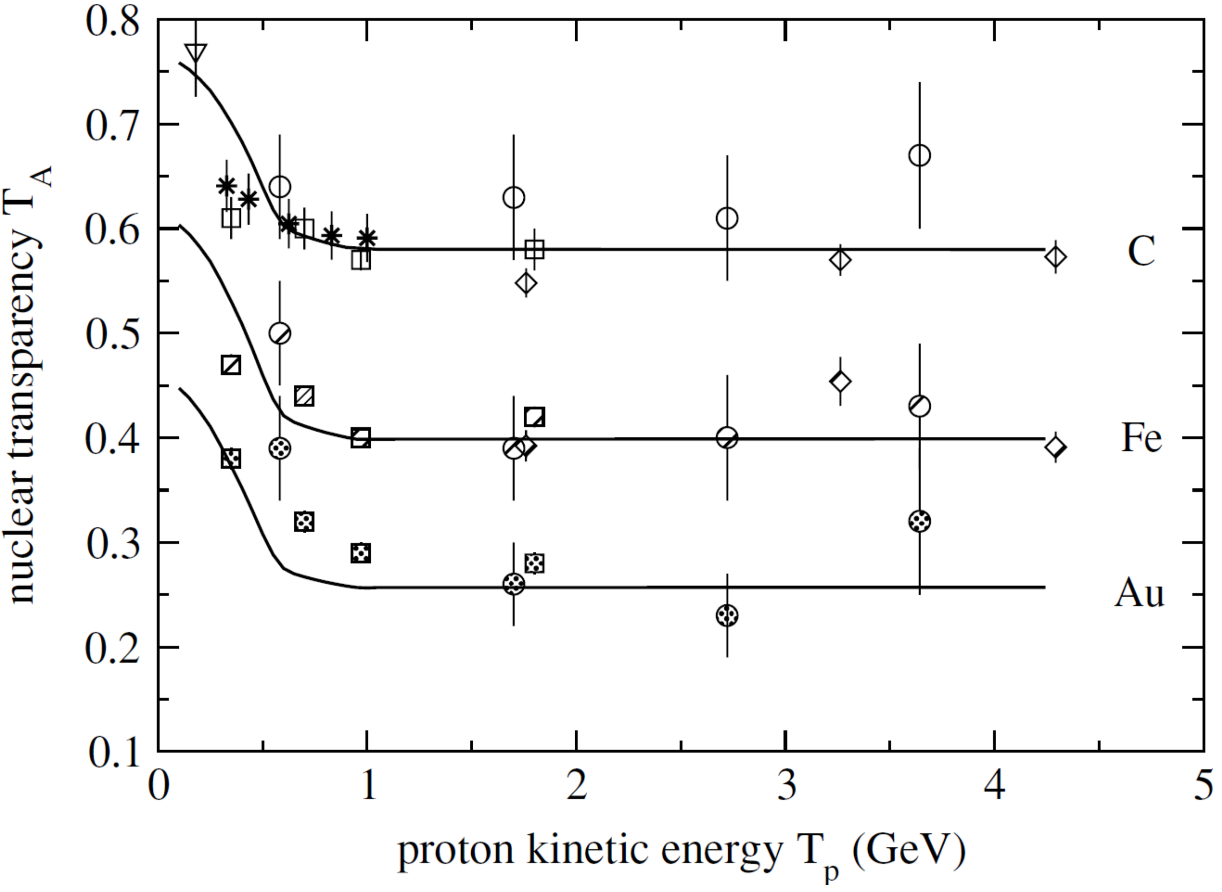
In the limit of a transparent nucleus: $T_{\mathbf{q}} \to 1$, the we recover the IA case.

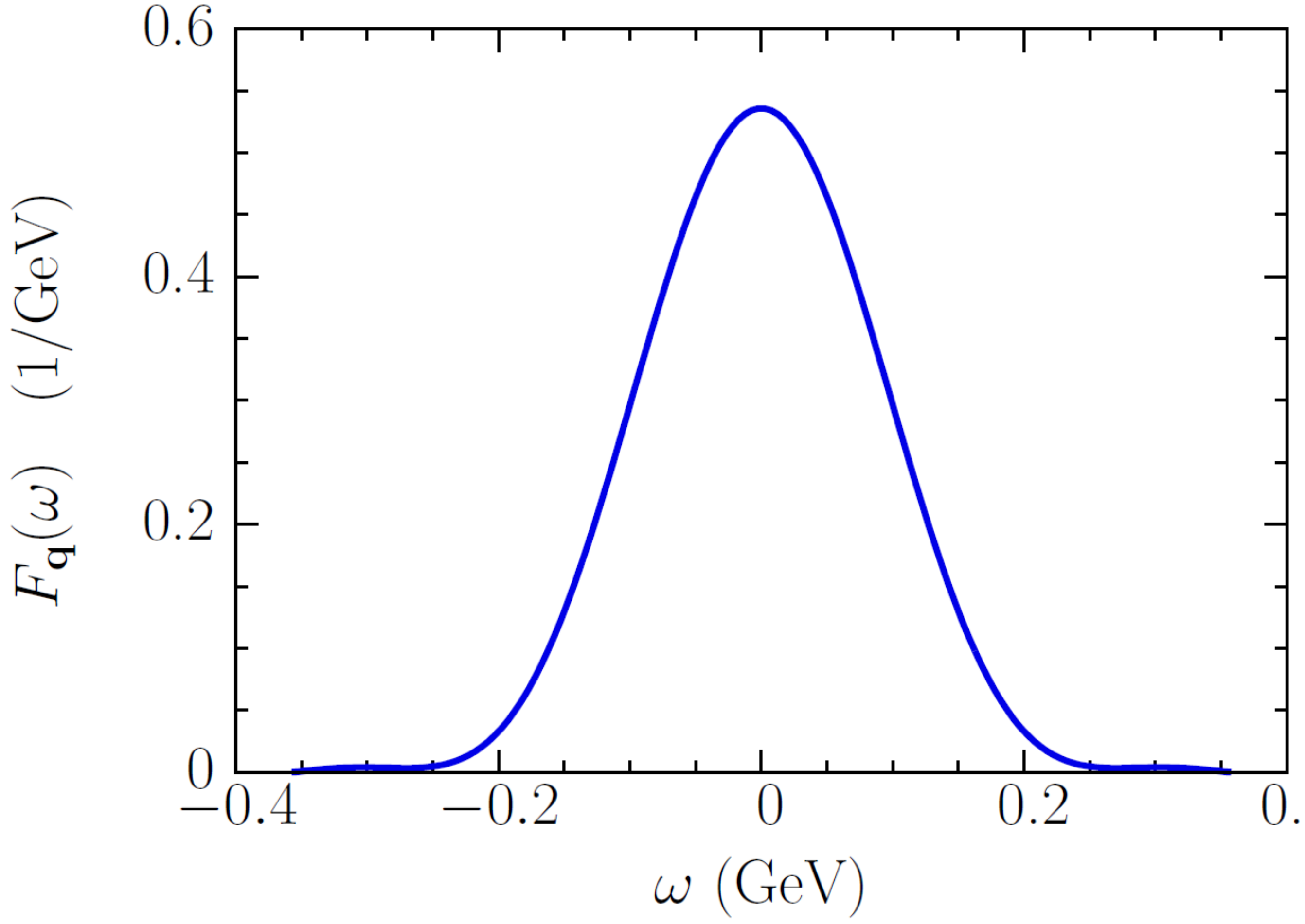
Qualitatively, it can be shown that the Glauber factor, is connected to the imaginary part of an optical potential which moves strength from single-particle excitations to more complex final states

$$F_{\mathbf{q}}(\omega) = \frac{1}{\pi} \frac{U_W}{U_W^2 + \omega^2}$$

Phenomenology of Final State Interactions

Rohe et al, PRC 72 054602 (2005)



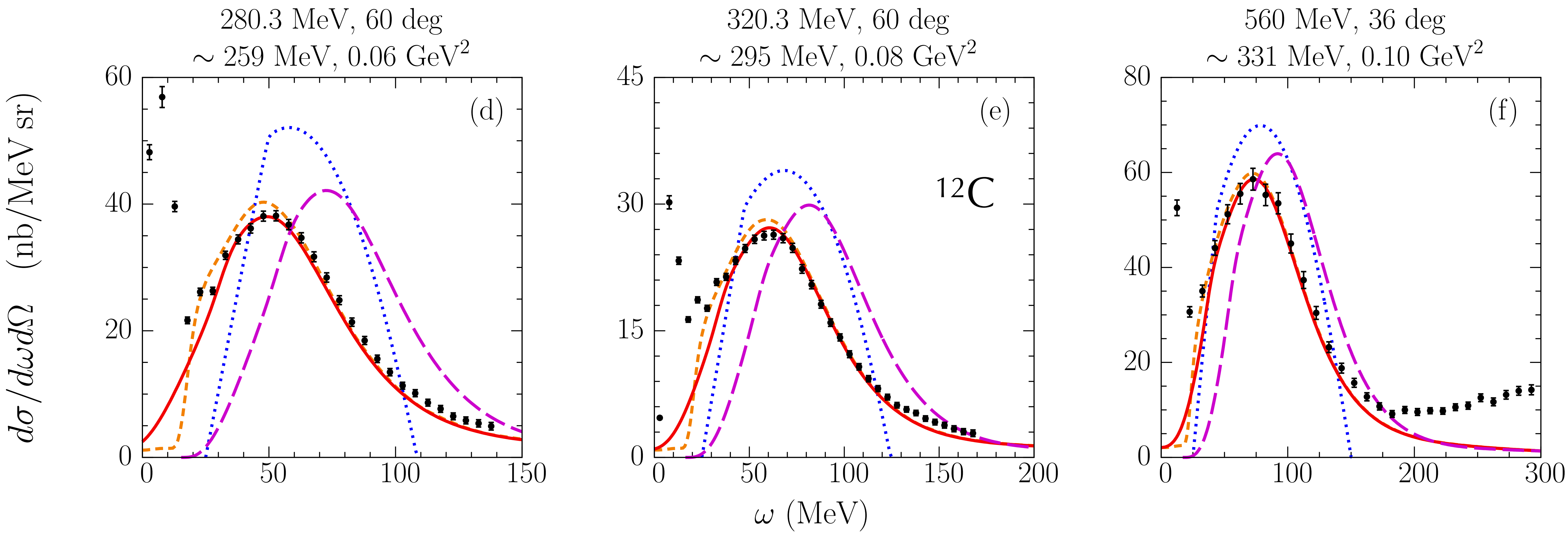


O. Benhar, PRC 87, 024606 (2013)

The folding function can be obtained from more refined calculations based on **eikonal approximation.** The main elements of the calculation are the measured **NN scattering cross sections**, and the **radial distribution function g(r)**: the probability of finding two nucleons separated by a distance r in the nuclear ground state.

Phenomenology of Final State Interactions

A. Ankowski, O. Benhar & M. Sakuda, PRD 91, 033005 (2015)



SF w/ FSI,
LDA treatment
of Pauli blocking

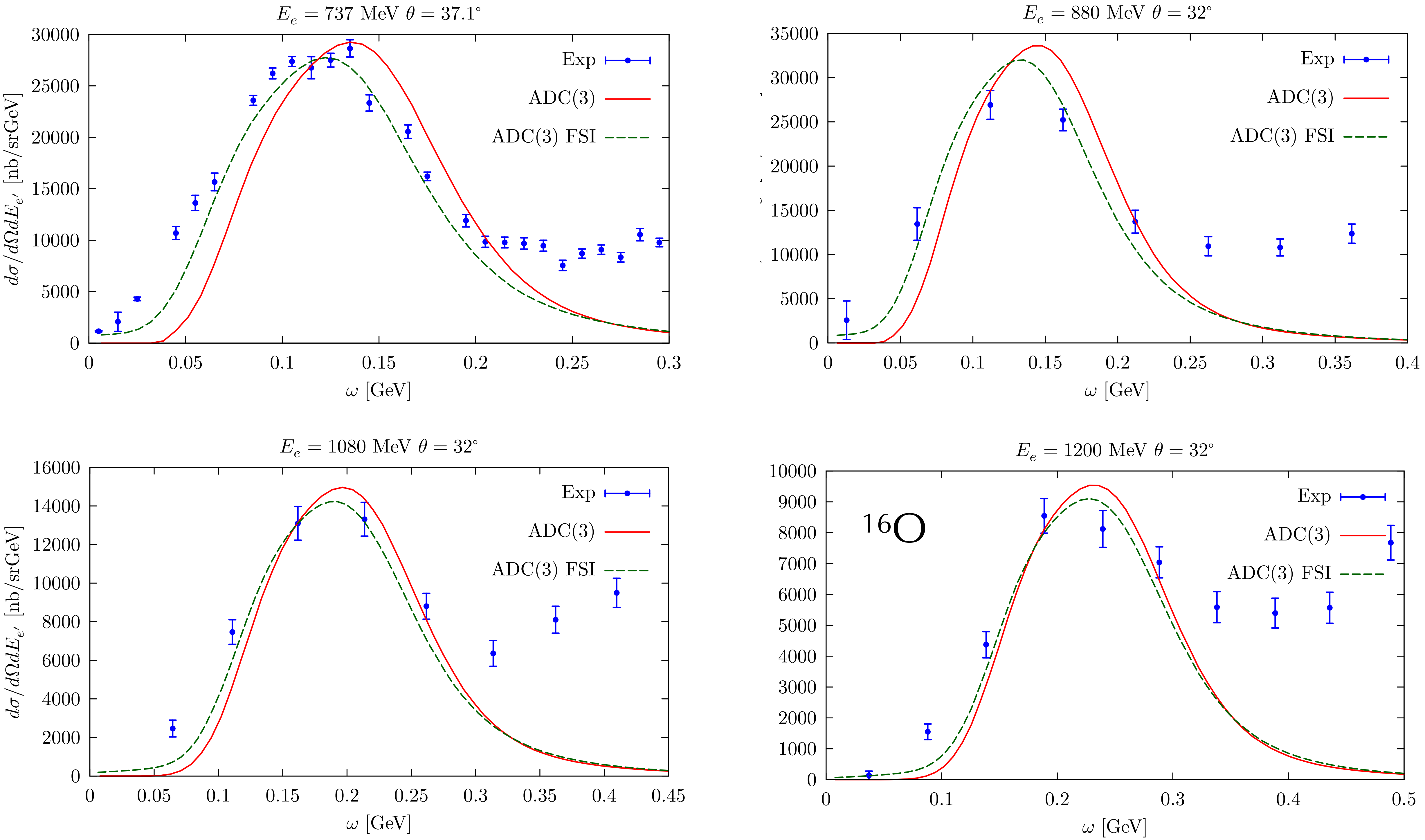
SF w/ FSI,
step function

RFG

SF w/o FSI

^{16}O(e,e’) cross sections within the SCGF approach

NR, C. Barbieri, Phys.Rev. C98 (2018) 025501



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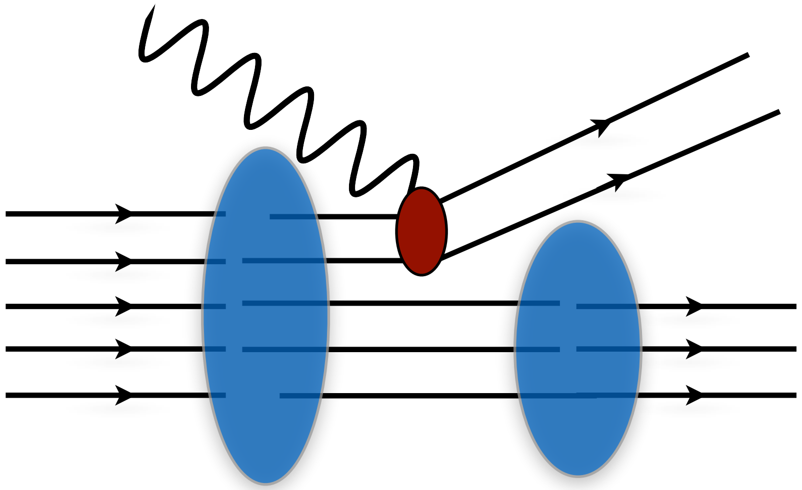
Two-body currents

$$\boxed{|f\rangle \rightarrow |pp'\rangle_a \otimes |f_{A-2}\rangle}$$



$$R^{\mu\nu}_{2b}(\mathbf{q},\omega) = \frac{V}{4}\int dE\, d^3k d^3k' d^3p d^3p' P_{2b}(\mathbf{k},\mathbf{k}',E) 2\langle kk'|j^{\mu}_{2b}|pp'\rangle_a \langle pp'|j^{\nu}_{2b}{}^{\dagger}|kk'\rangle$$
$$\times\, \delta(\omega - E + 2m - e(\mathbf{p}) - e(\mathbf{p}'))\delta^{(3)}(\mathbf{p}+\mathbf{q}-\mathbf{p}')$$

The normalization volume for the nuclear wave functions $V = \rho/A$ with $\rho = 3\pi^2 k_F^3/2$ depends on the Fermi momentum of the nucleus

Factor 1/4 accounts for the fact that we sum over indistinguishable pairs of particles, while the factor 2 stems from the equality of the product of the direct terms and the product of the two exchange terms after interchange of indices

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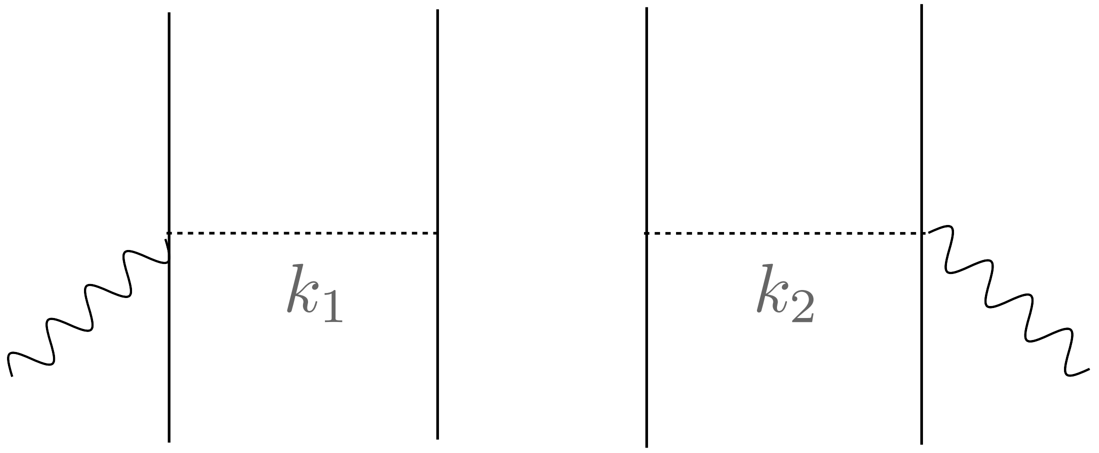
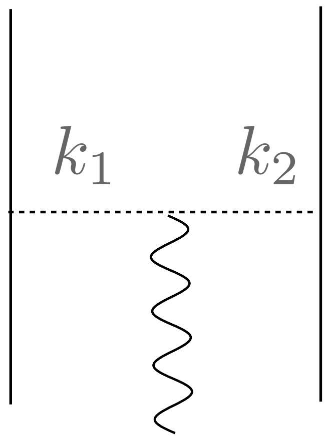
Two-body currents - pion contribution e.m. case



Seagull



Pion in flight

$$(J^\mu_{\rm sea}) = (I_V)_z \frac{f^2_{\pi NN}}{m^2_\pi} F^V_1(q) F^2_{\pi NN}(k_1) \Pi(k_1)_{(1)} \big(\gamma_5 \gamma^\mu\big)_{(2)} - (1 \leftrightarrow 2)$$

Isospin structure: $(I_V)_z = (\tau^{(1)} \times \tau^{(2)})_z$

$$J^\mu_{\rm pif} = \frac{f^2_{\pi NN}}{m^2_\pi} F^V_1(q) F_{\pi NN}(k_1) F_{\pi NN}(k_2) \Pi(k_1)_{(1)} \Pi(k_2)_{(2)} (k^\mu_1 - k^\mu_2)$$

Pion propagation and absorption: $\Pi(k) = \dfrac{\gamma_5 \not{k}}{k^2 - m^2_\pi}$

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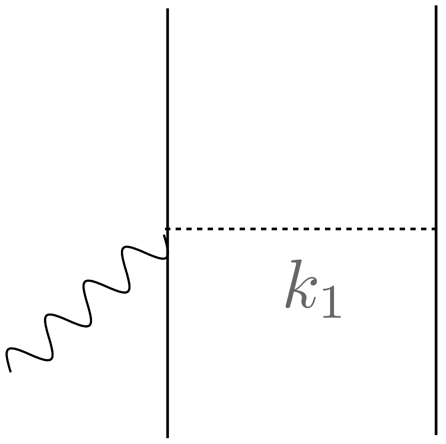
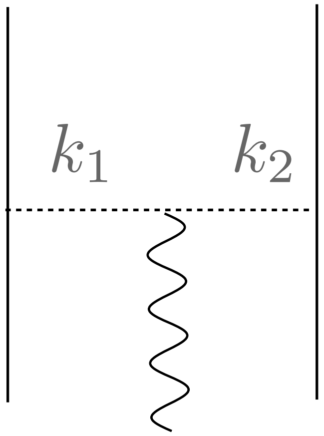
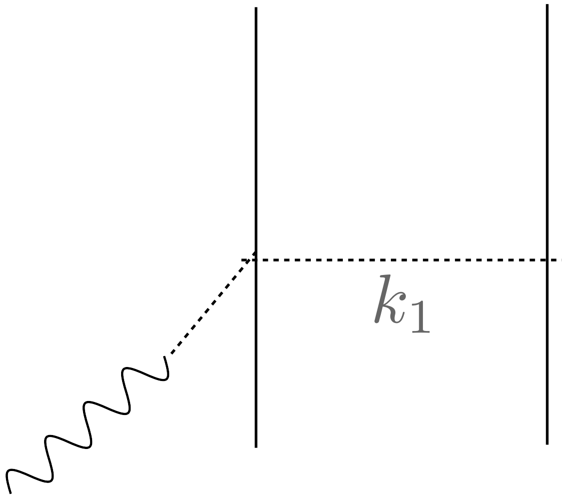
Two-body currents - pion contribution CC case



Seagull: A+V (1↔2)



Pion in flight: V



Pion-pole: A (1↔2)

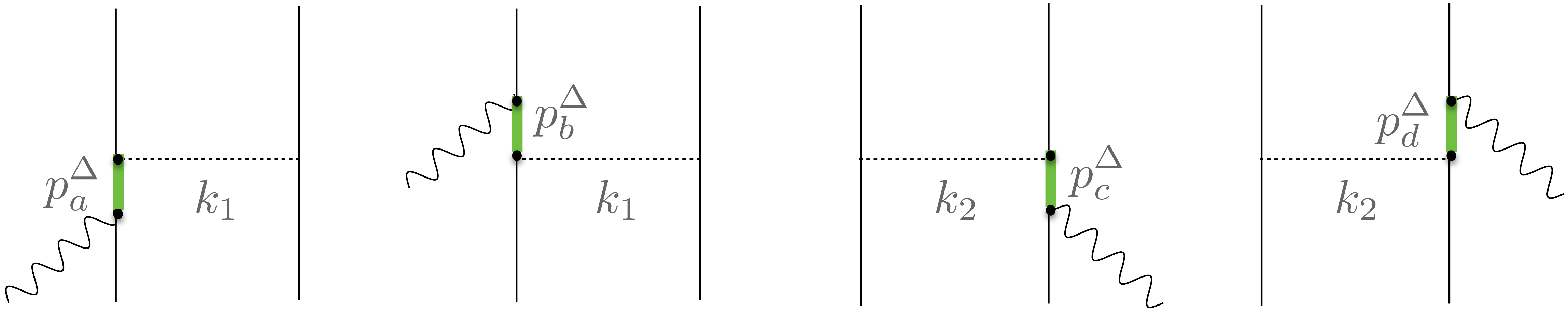
$$\left(J^\mu_{\rm sea}\right)_A = \frac{f^2_{\pi NN}}{m^2_\pi}\frac{1}{g_A} F_\rho(k_2) F^2_{\pi NN}(k_1)\Pi(k_1)_{(1)}(\gamma^\mu)_{(2)} \quad - (1 \leftrightarrow 2)$$

$$\left(J^\mu_{\rm pole}\right)_A = \frac{f^2_{\pi NN}}{m^2_\pi}\frac{1}{g_A} F_\rho(k_1) F^2_{\pi NN}(k_2)\Pi(k_2)_{(2)} \left(\frac{q^\mu \not{q}}{q^2 - m^2_\pi}\right)_{(1)} - (1 \leftrightarrow 2)$$

A form factor to account for the ρ dominance of the πNN coupling has been inserted

$$F_\rho(k) = \frac{1}{k^2 - m^2_\rho}, \quad m_\rho = 775.8 \text{ MeV}.$$

Two-body currents - Delta contribution - e.m. case



$$j^\mu_\Delta = \frac{3}{2}\frac{f_{\pi NN} f^*}{m_\pi^2}\Big\{\Pi(k_2)_{(2)}\Big[\Big(-\frac{2}{3}\tau^{(2)} + \frac{I_V}{3}\Big)_z F_{\pi NN}(k_2) F_{\pi N\Delta}(k_2)(J_a^\mu)_{(1)} - \Big(\frac{2}{3}\tau^{(2)} + \frac{I_V}{3}\Big)_z F_{\pi NN}(k_2) F_{\pi N\Delta}(k_2)(J_b^\mu)_{(1)}\Big] + (1 \leftrightarrow 2)\Big\}$$

where

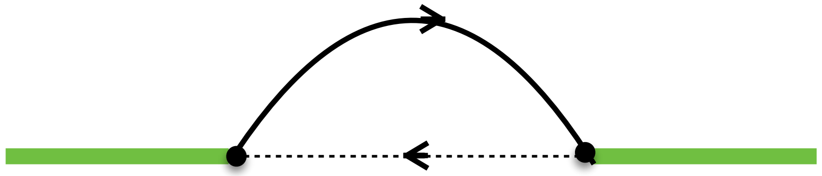
Rarita Schwinger propagator

$$(J_a^\mu)_V = (k_1)^\alpha G_{\alpha\beta}(p_\Delta)\Big[\frac{C_3^V}{m_N}\big(g^{\beta\mu}\not{q} - q^\beta\gamma^\mu\big) + \frac{C_4^V}{m_N^2}\big(g^{\beta\mu} q\cdot p_\Delta - q^\beta p_\Delta^\mu\big) + \frac{C_5^V}{m_N^2}\big(g^{\beta\mu} q\cdot k - q^\beta k^\mu + C_6^V g^{\beta\mu}\big)\Big]\gamma_5$$

Two-body currents - Delta contribution

Rarita-Schwinger propagator

$$G^{\alpha\beta}(p_\Delta) = \frac{P^{\alpha\beta}(p_\Delta)}{p_\Delta^2 - M_\Delta^2}$$



The spin 3/2 projection operator reads

$$P^{\alpha\beta}(p_\Delta) = (\not{p}_\Delta + M_\Delta)\Big[g^\alpha\beta - \frac{1}{3}\gamma^\alpha\gamma^\beta - \frac{2}{3}\frac{p_\Delta^\alpha p_\Delta^\beta}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\alpha\gamma^\beta - p_\Delta^\beta\gamma^\alpha}{M_\Delta}\Big] .$$

To account for the resonant behavior of the Δ: $M_\Delta \to M_\Delta - i\Gamma(p_\Delta)/2$

$$\Gamma(p_\Delta) = -2\mathrm{Im}\Sigma_{\pi N}(s) = \frac{(4f_{\pi N\Delta})^2}{12\pi m_\pi^2}\frac{|\mathbf{d}|^3}{\sqrt{s}}(m_N + E_d)R(\mathbf{r}^2)$$

d is the decay three-momentum in the πN center of mass frame

In medium effects of the Δ $\Gamma_\Delta(p_\Delta) \to \Gamma_\Delta(p_\Delta) - 2\mathrm{Im}[U_\Delta(p_\Delta, \rho = \rho_0)]$

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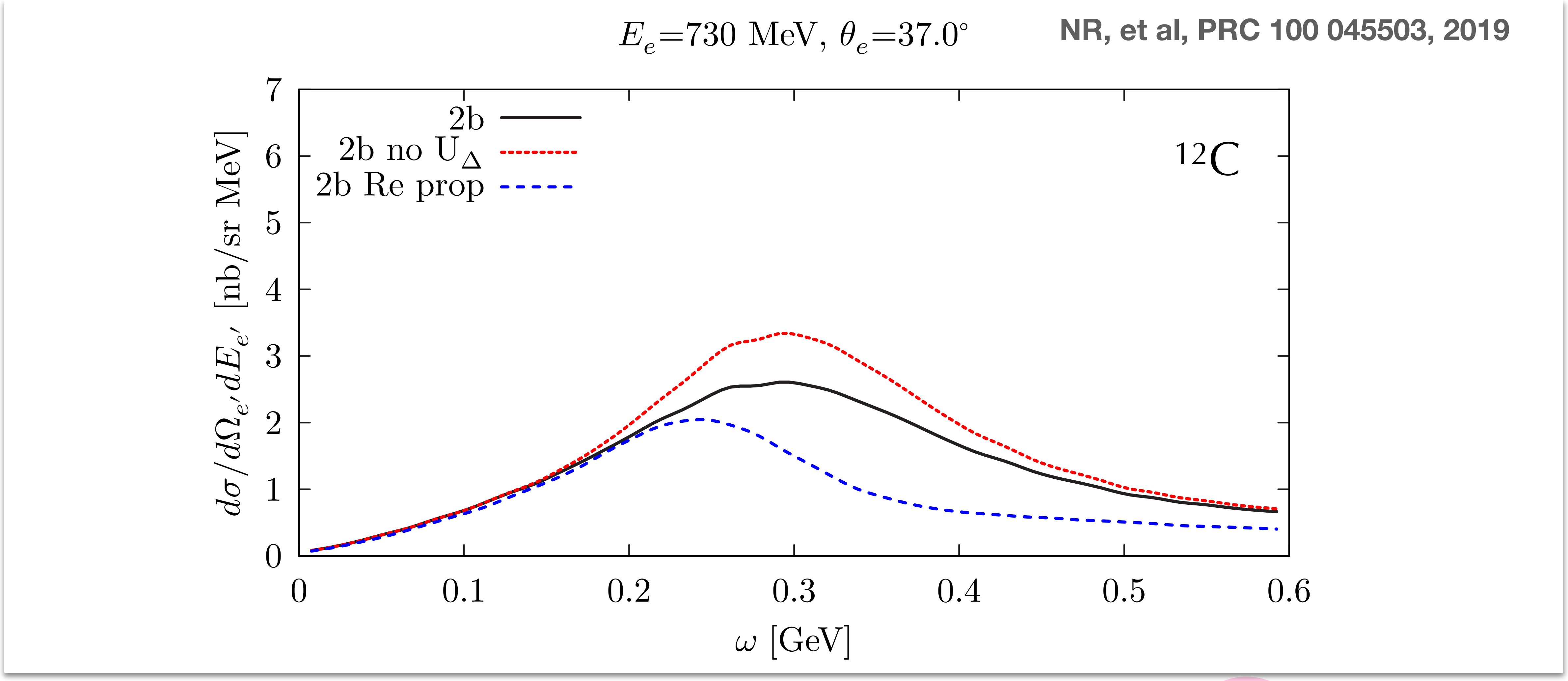
Two-body currents - Delta contribution



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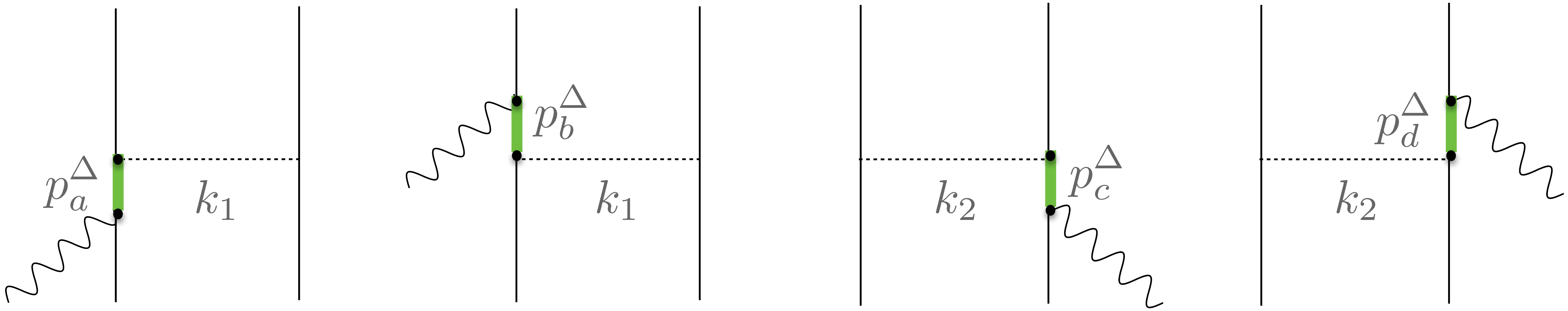
Two-body currents - Delta contribution - c.c. case



$$(j_a^\mu)_A = (k'_\pi)^\alpha G_{\alpha\beta}(p_\Delta)\Big[\frac{C_3^A}{m_N}\Big(g^{\beta\mu}\not{q} - q^\beta\gamma^\mu\Big) + \frac{C_4^A}{m_N^2}\Big(g^{\beta\mu} q\cdot p_\Delta - q^\beta p_\Delta^\mu\Big)$$

$$+ C_5^A g^{\beta\mu} + \frac{C_6^A}{m_N^2} q^\mu q^\alpha\Big]$$

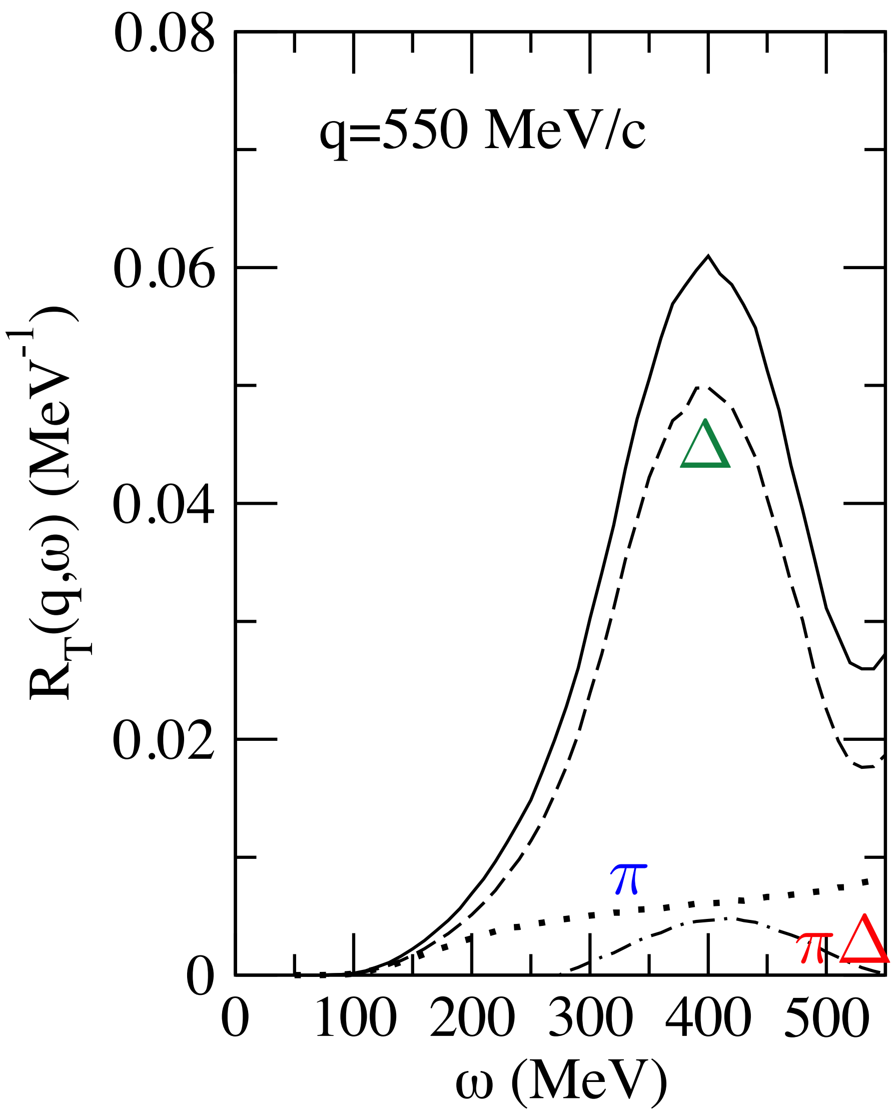
with $C_5^A = \frac{1.2}{\big(1 - q^2/M_{A\Delta}^2\big)^2}\,\frac{1}{1 - q^2/\big(3M_{A\Delta}^2\big)}$

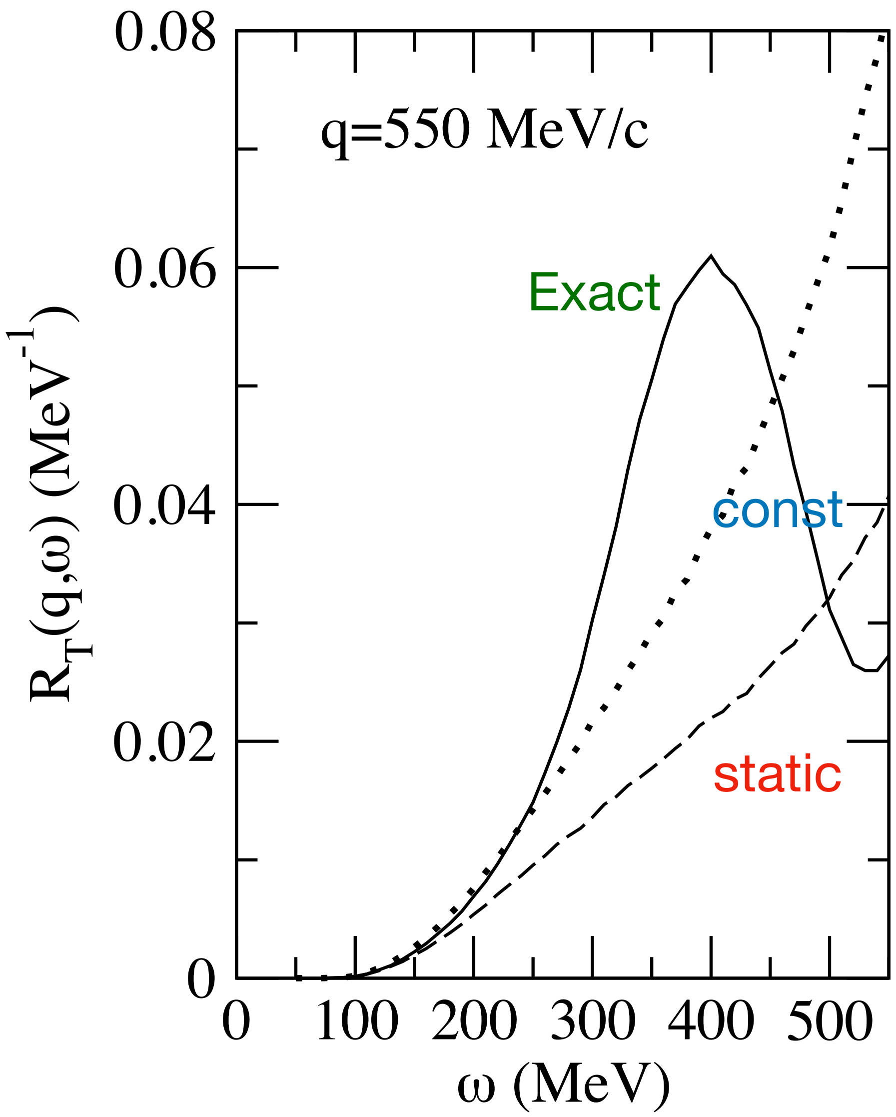
C_5^A and C_6^A are connected by PCAC. However, C_6^A is neglected in the cross section since it's $\propto m_\ell$.

Since there are no other theoretical constraints for C_3^A(q2), C_4^A(q2) and C_5^A(q2)/C_5^A(0), they have to be fitted to neutrino scattering data. The available information comes mainly from two bubble chamber experiments, ANL and BNL.

Two-body currents - Delta contribution

A. De Pace, et al, Nucl.Phys.A 726 (2003) 303-326





RGFG — electromagnetic response of A=56 k_F= 1.3 fm^{-1}

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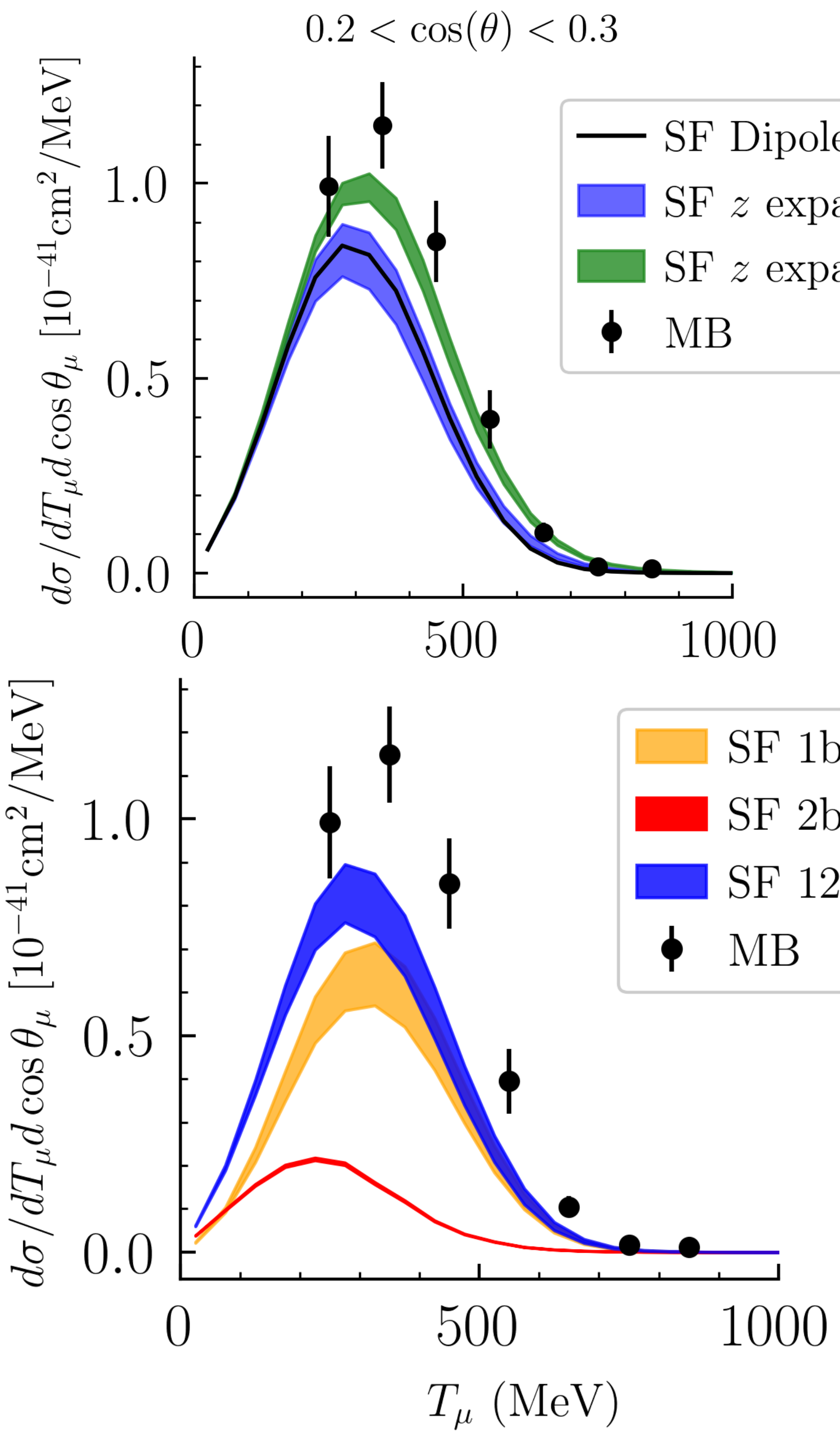
Axial Form Factors Uncertainty needs



D.Simons, N. Steinberg et al, 2210.02455

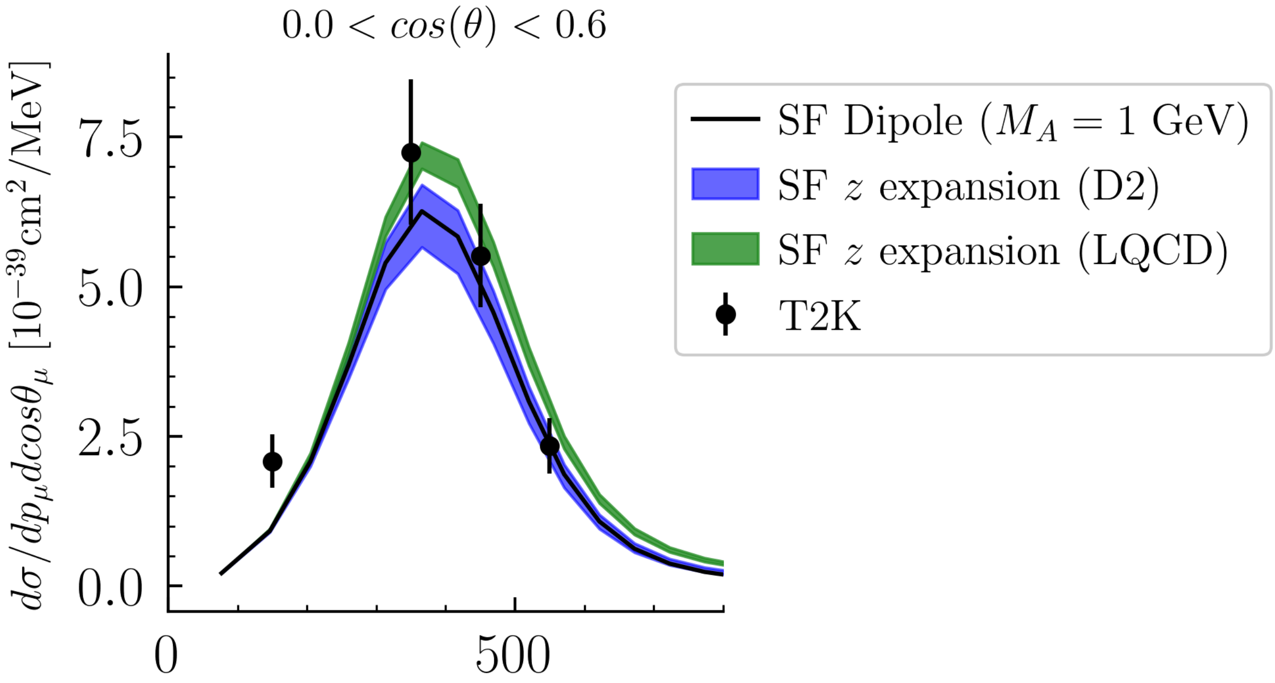
✳ Axial form factor dependence:

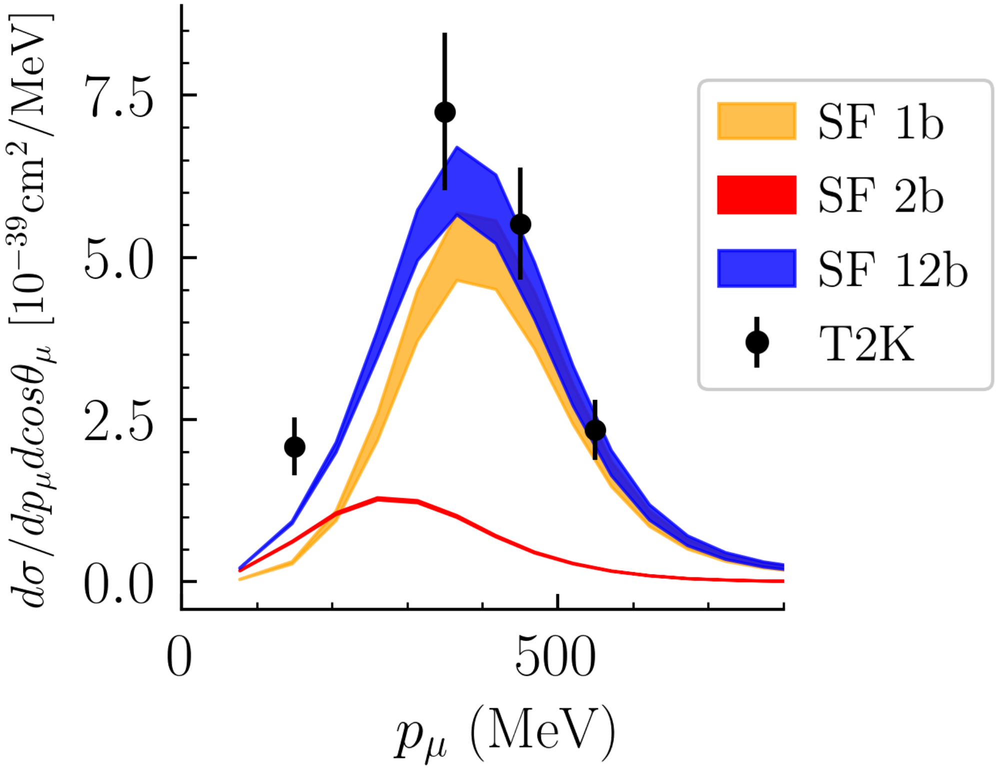
MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$
SF Difference in $d\sigma_{\text{peak}}$ (%)	16.3

✳ Many-body method dependence:

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	22.8

Axial Form Factors Uncertainty needs





D.Simons, N. Steinberg et al, 2210.02455

- ✳ Axial form factor dependence:

T2K	$0.0 < \cos\theta_\mu < 0.6$
SF difference in $d\sigma_{\text{peak}}$ (%)	15.3

- ✳ Many-body method dependence:

T2K	$0.0 < \cos\theta_\mu < 0.6$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	13.4

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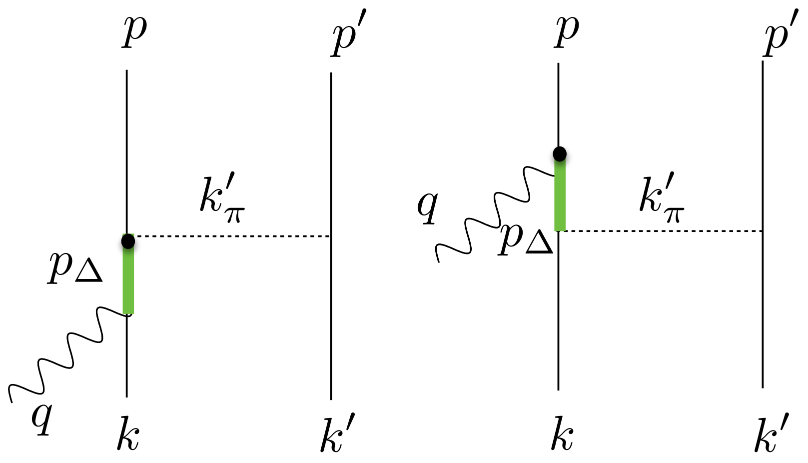
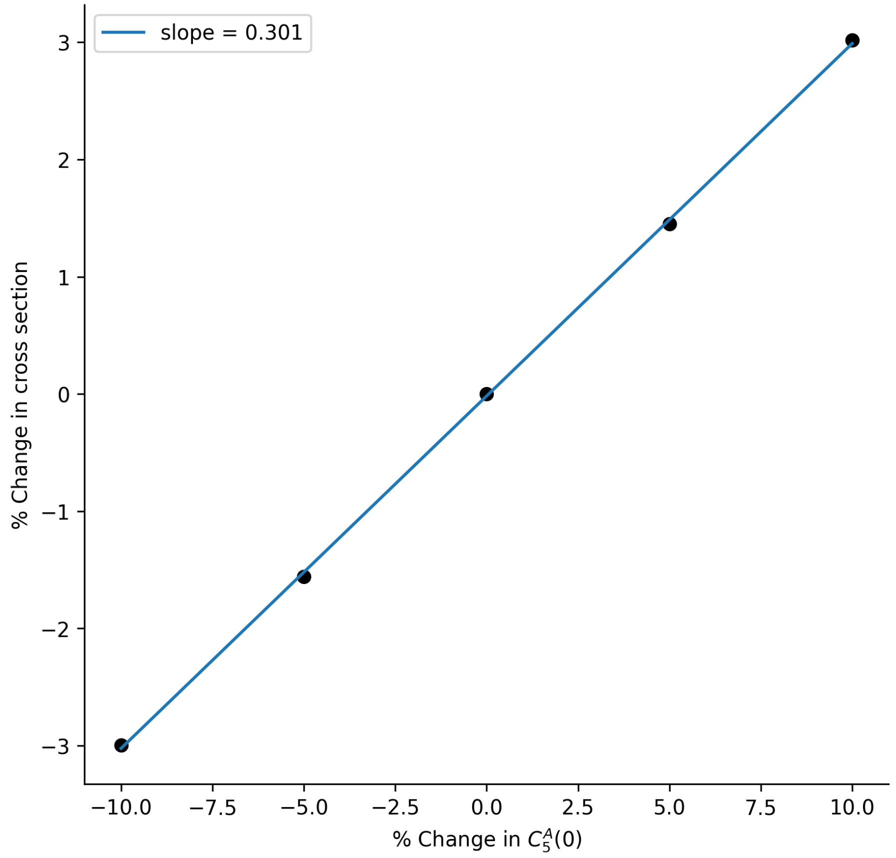
Resonance Uncertainty needs

The largest contributions to two-body currents arise from resonant $N \rightarrow \Delta$ transitions yielding pion production





D.Simons, N. Steinberg et al, 2210.02455

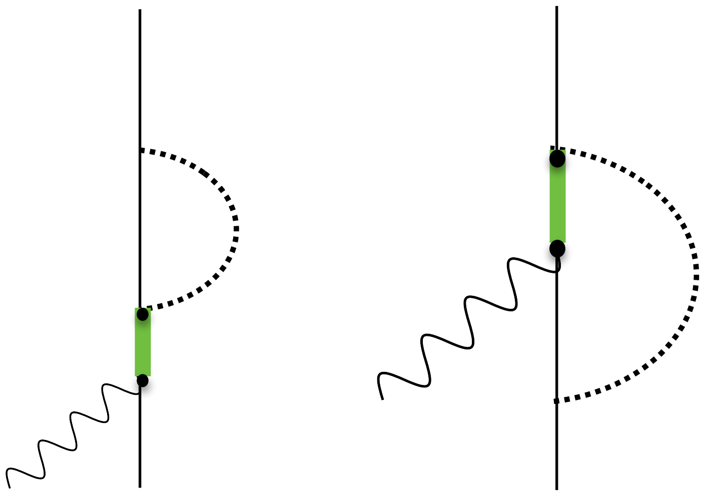
The normalization of the dominant $N \rightarrow \Delta$ transition form factor needs be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics

State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions)

Hernandez et al, PRD 81 (2010)

Further constraints on $N \rightarrow \Delta$ transition relevant for two-body currents and π production will be necessary to achieve few-percent cross-section precision

Including the one- and two-body interference



We recently included interference effects between one- and two-body currents yielding single nucleon knock-out

$$R^{\mu\nu}_{12\mathrm{b}} = \sum_f \langle 0|j^{\mu\dagger}_{2b}|f\rangle\langle f|j^{\nu}_{1b}|0\rangle\delta(\omega + E_0 - E_f)\,.$$

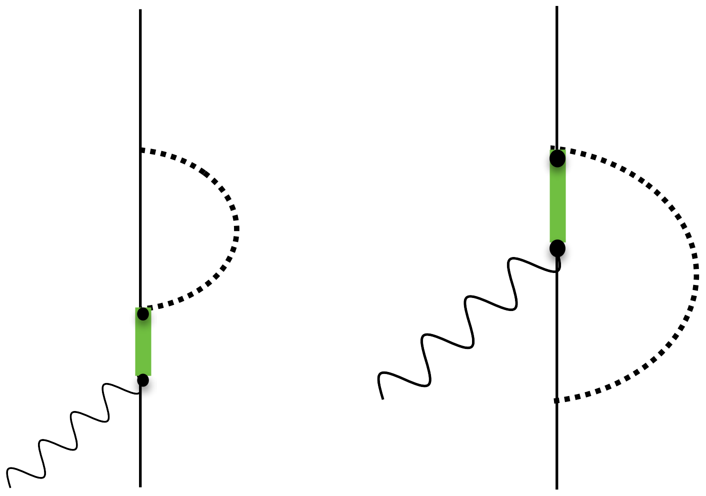
Within the SF formalism, we write this contribution as

$$R^{\mu\nu}_{12b} = -V\sum_{\eta_p,\eta_h,\eta_k}\int\frac{d^3h}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}dE\Big[\tilde{P}_{\mathrm{MF}}(\mathbf{h},E)$$
$$\times\,\tilde{n}_{\mathrm{MF}}(\mathbf{k})\delta(\mathbf{q}-\mathbf{h}+\mathbf{p})\theta(\mathbf{p}-k_F)\langle\eta_p|j_1|\eta_h\rangle$$
$$\times\,\langle\eta_p\eta_k|j_{12}|\eta_h\eta_k\rangle\delta(\tilde{\omega}-e(\mathbf{p})+e(\mathbf{h}))\Big]$$

We are retaining only the MF contribution, because we are considering states with one-nucleon emission only.

In the electromagnetic case, only the exchange contribution is present (spin sum and zero pi momentum leads to a negligible contribution for the direct)

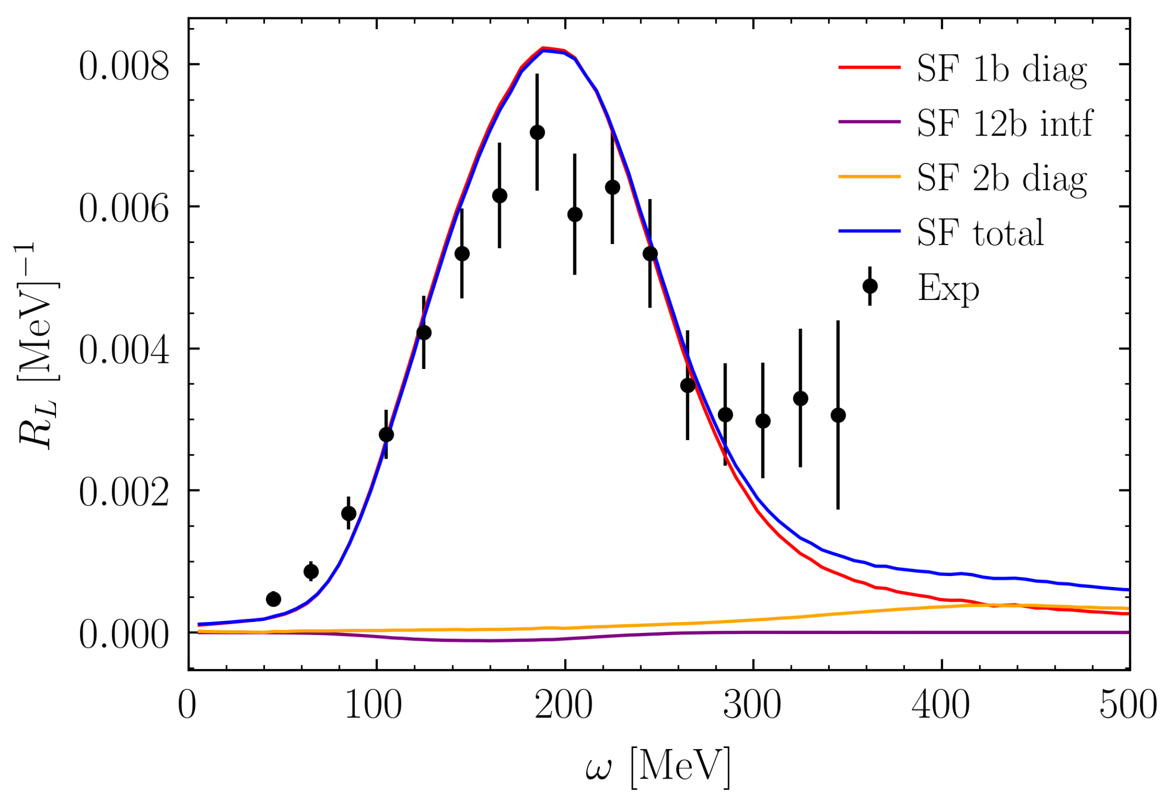
Including the one- and two-body interference

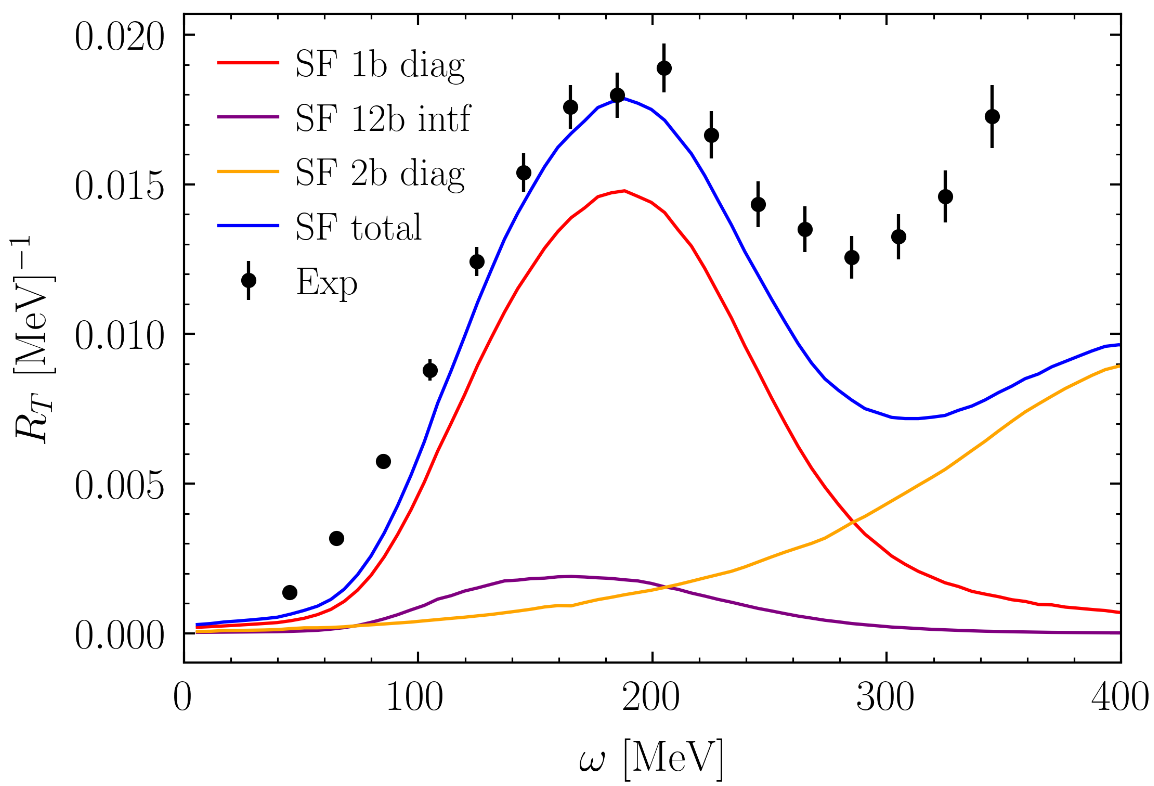


We fist consider the results obtained for the electromagnetic responses of 12C for a fixed value of q=570 MeV

Observe a small quenching in the longitudinal channel and an enhancement in the q.e. peak in the transverse → agreement with the GFMC

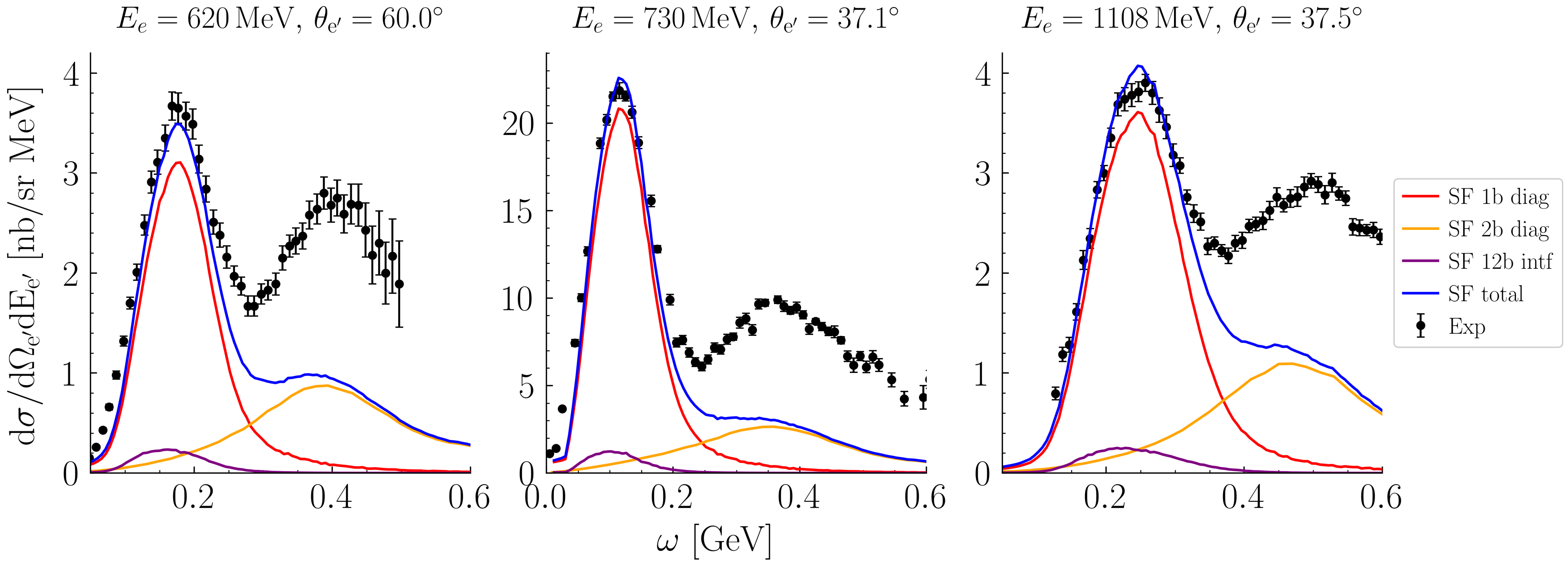
N. Steinberg, NR, A. Lovato, arXiv: 2312.12545

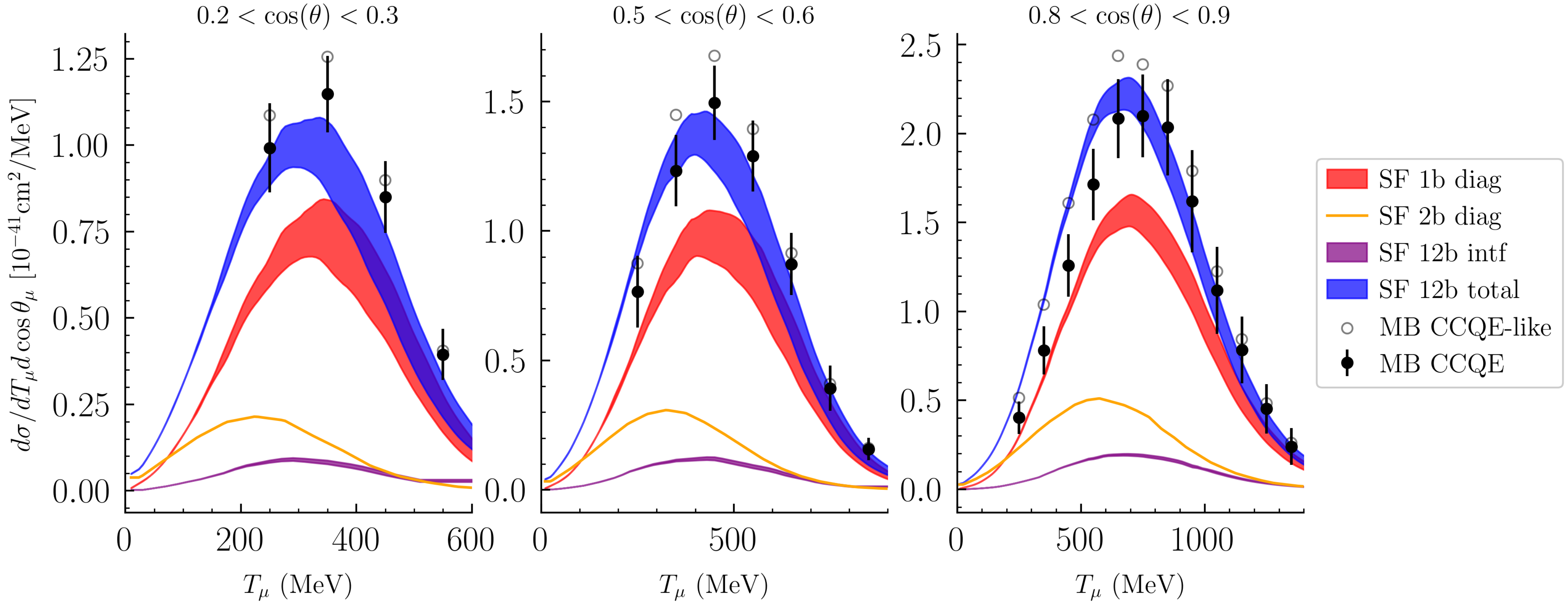




Including the one- and two-body interference

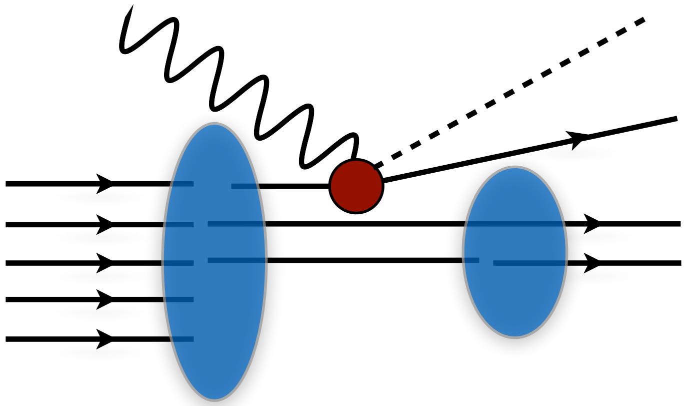
N. Steinberg, NR, A. Lovato, arXiv: 2312.12545





Pion production within extended factorization

$$\boxed{|f\rangle \to |p_\pi p\rangle \otimes |f_{A-1}\rangle}$$



Production of real π in the final state:

$$R^{\mu\nu}_{1b1\pi}(\mathbf{q},\omega) = \int \frac{d^3k}{(2\pi)^3} dE P_h(\mathbf{k},E) \frac{d^3p_\pi}{(2\pi)^3} \sum_I \langle k|j_I^{\mu\,\dagger}|p_\pi p\rangle\langle p_\pi p|j_I^\nu|k\rangle\Big|_{\mathbf{p}=\mathbf{k}+\mathbf{q}-\mathbf{p}_\pi}$$
$$\times\, \delta(\omega - E + m_N - e(\mathbf{k}+\mathbf{q}-\mathbf{p}_\pi) - e_\pi(\mathbf{p}_\pi))\,,$$

- Pion production elementary amplitudes currently derived within the **Dynamic Couple Chanel approach**;

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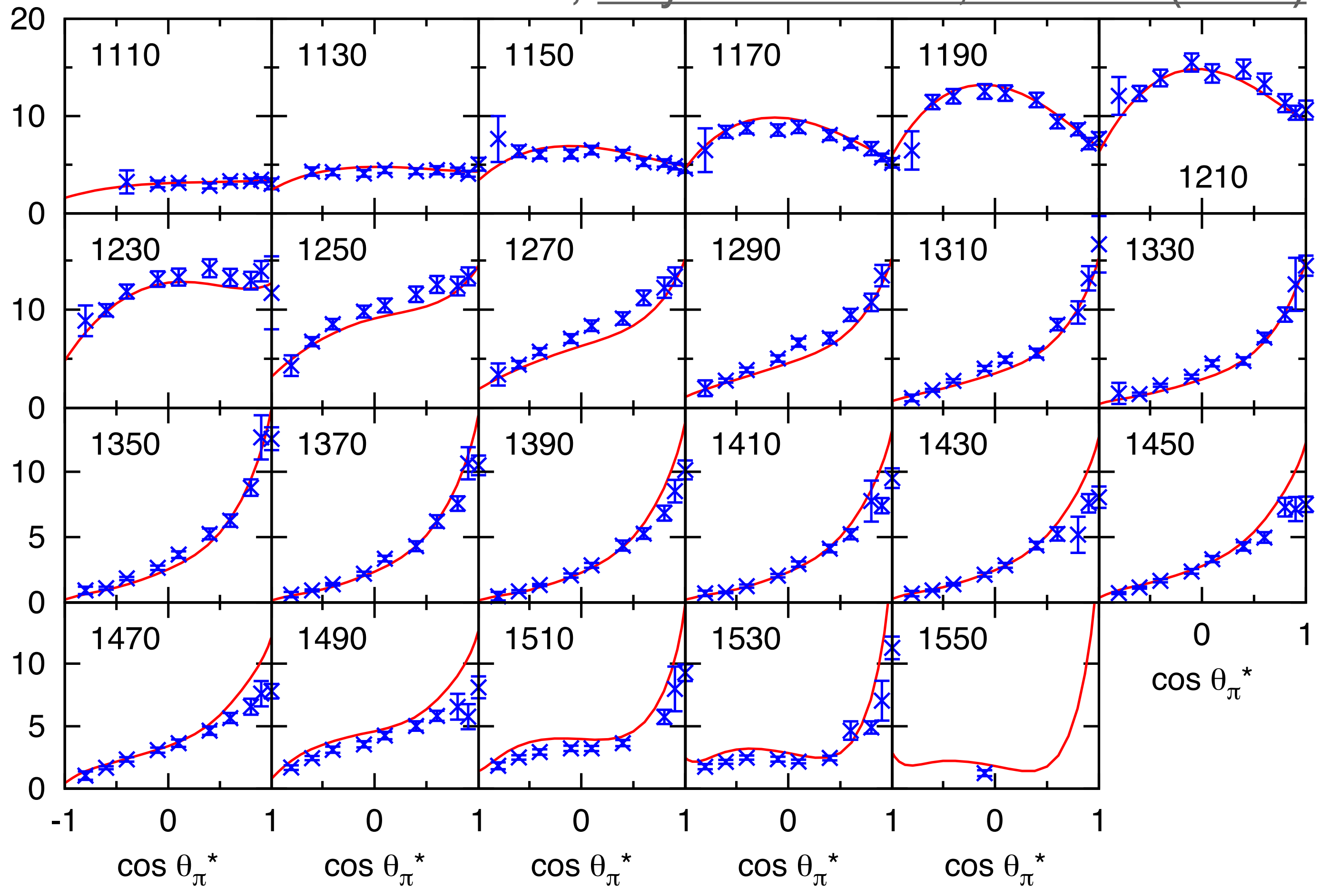
Pion production mechanisms

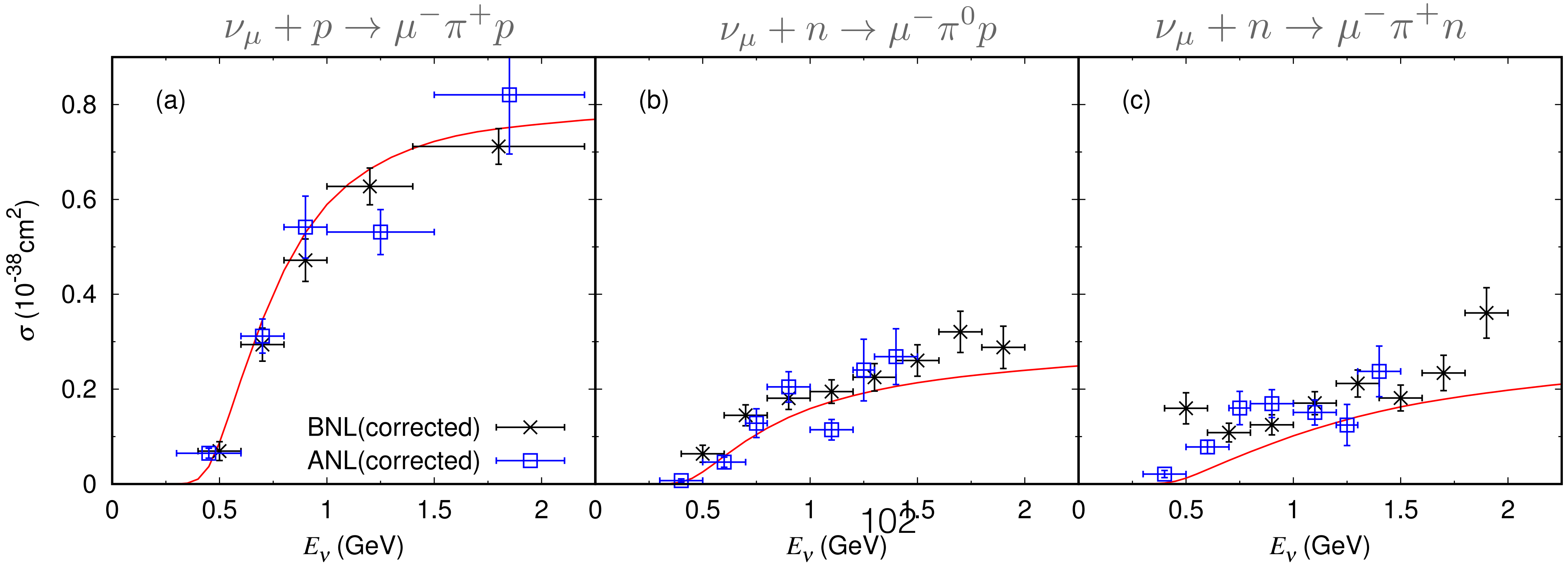
S.X. Nakamura et al, Phys. Rev. D 92, 074024 (2015)



- To fix the Q^2-dependence of the vector form factors: analyze data for electron-induced reactions off the proton and neutron targets

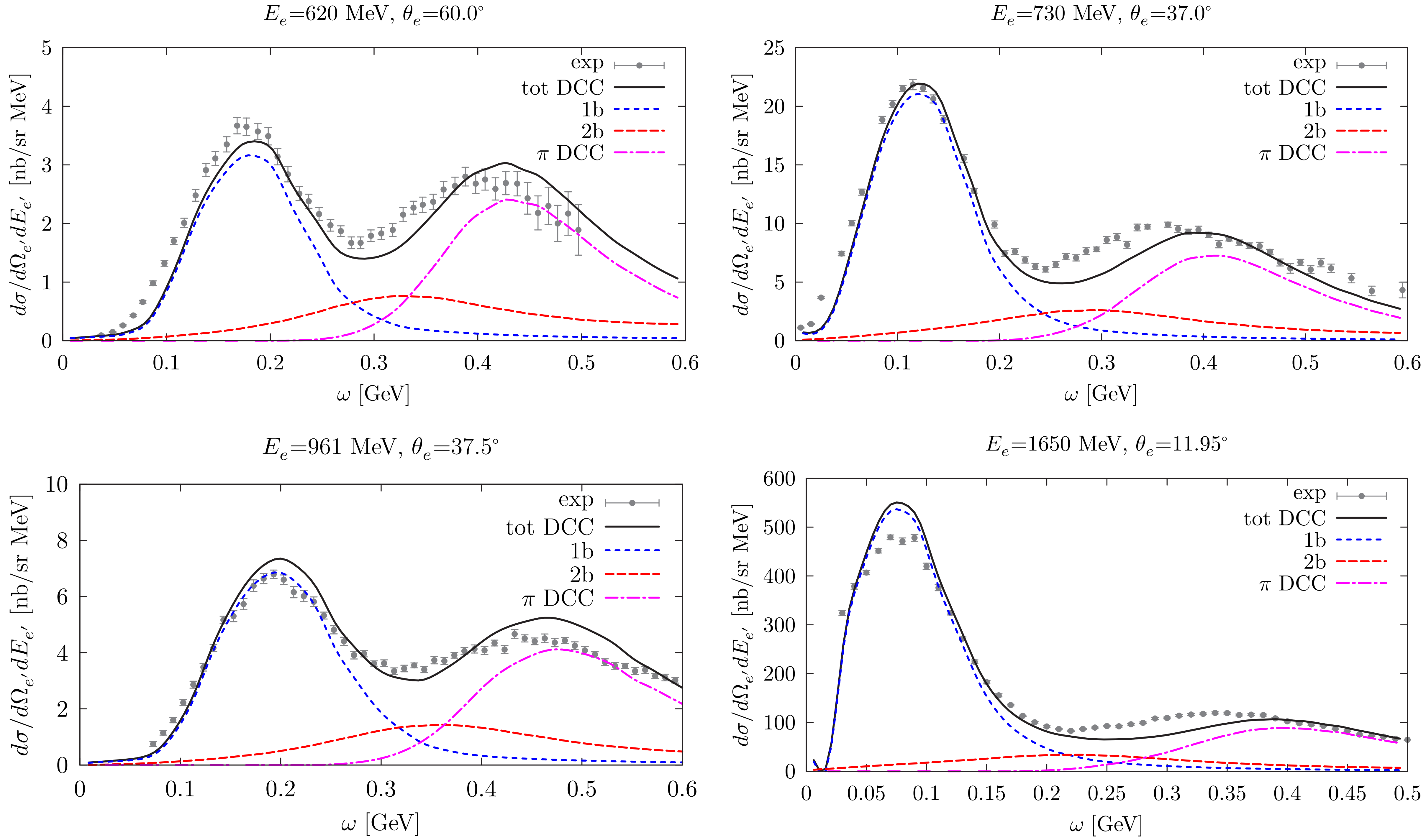
- Virtual photon cross section at Q^2=0.4 GeV2 for $p(e,e'\pi^+)n$

- For the axial-current matrix element at Q^2=0 the PCAC relation is used.



Spectral function approach

NR, et al, PRC 100 045503, 2019



Electron scattering results

Scaling in the Fermi gas model

- Scaling of the first kind: the nuclear electromagnetic responses divided by an appropriate function describing the single-nucleon physics no longer depend on two variables ω and **q**, but only upon $\psi(\mathbf{q},\omega)$

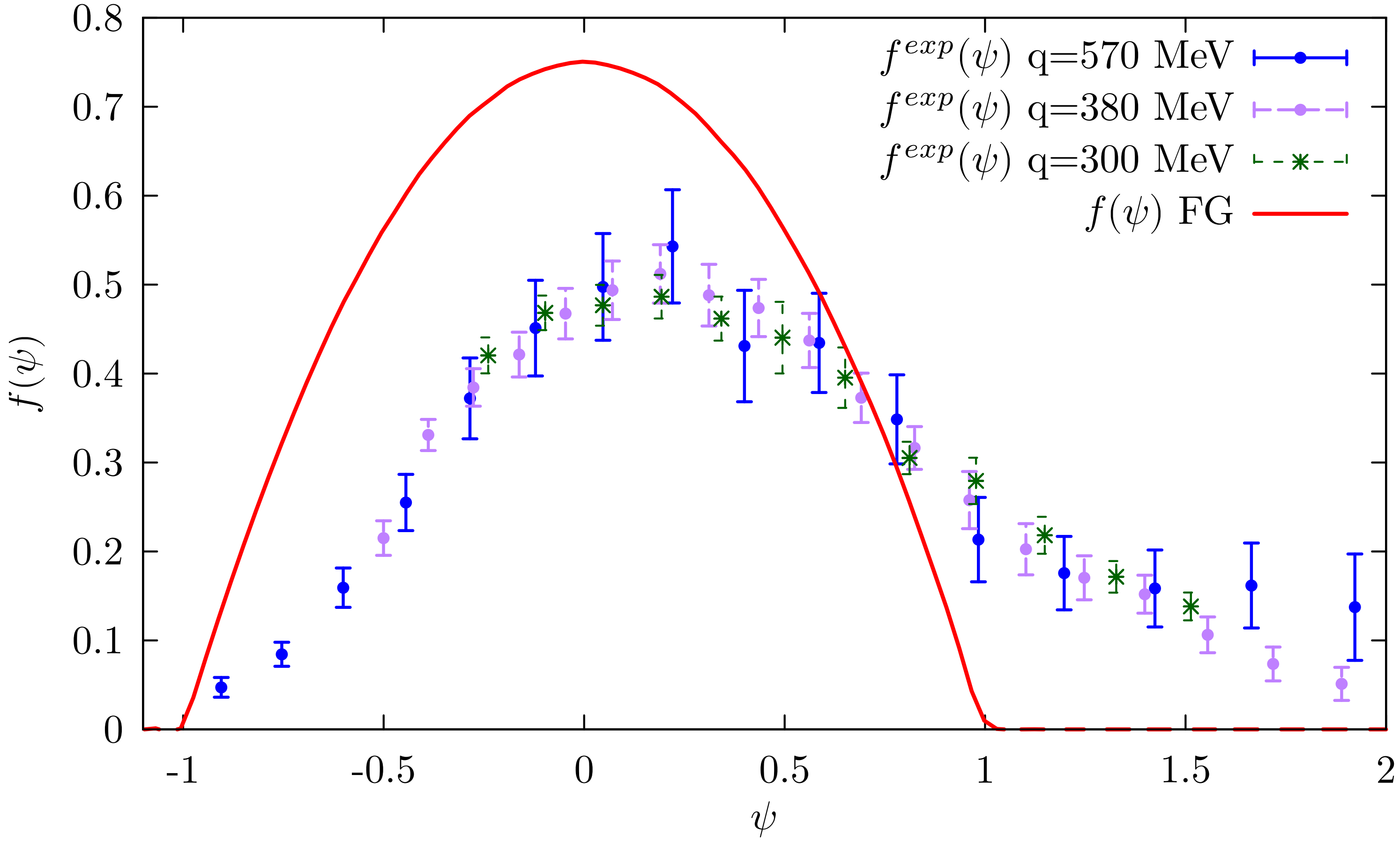
$$f_{L,T}(\mathbf{q},\omega) = p_F \frac{R_{L,T}}{G_{L,T}} = \frac{\text{nuclear response function}}{\text{elem single nucleon function}}$$

- For sufficiently large values of **q**

$$f(\mathbf{q},\omega) \rightarrow f(\psi)$$

- Within the Relativistic Fermi gas scaling of the first kind is exactly fulfilled

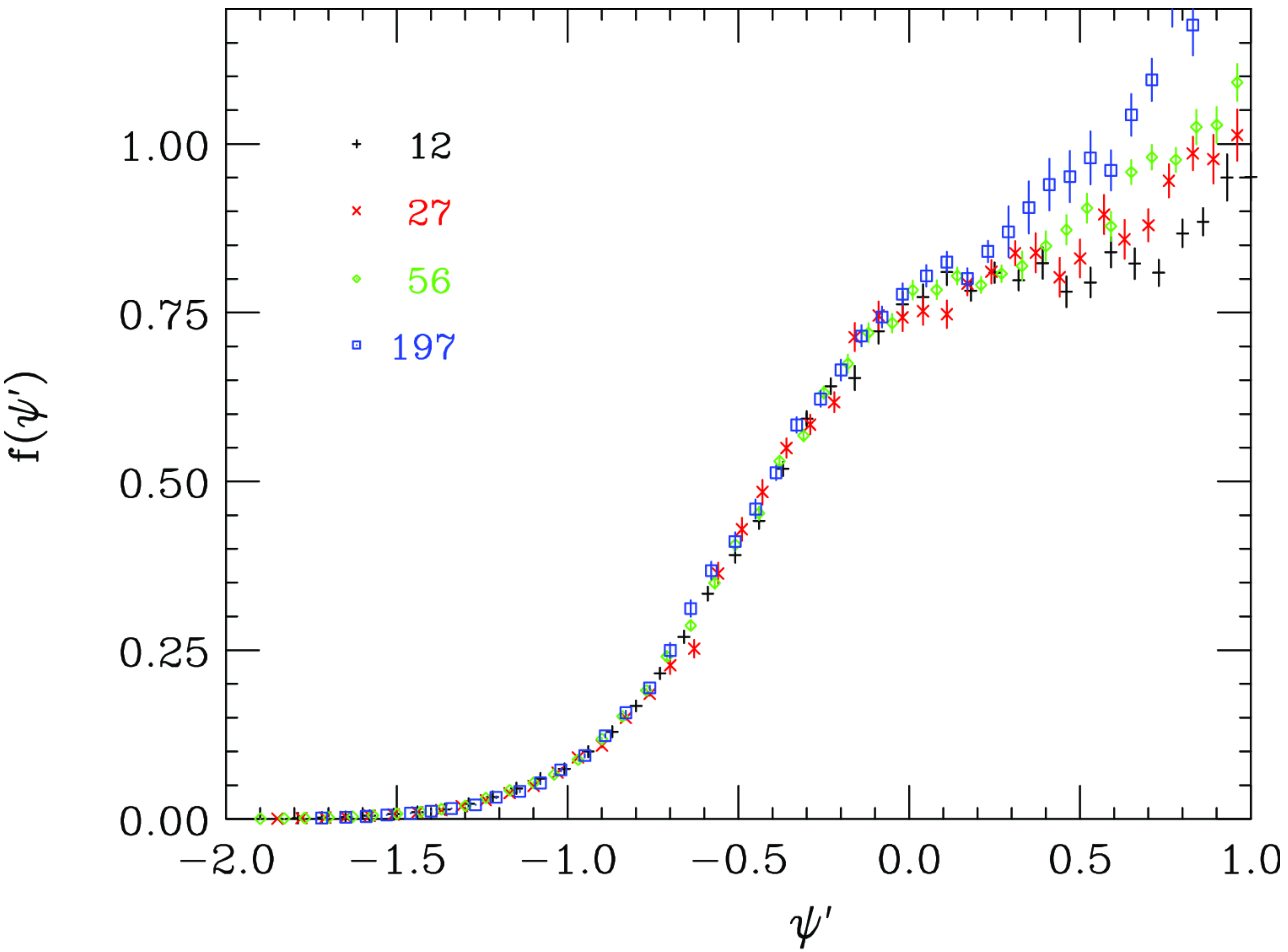
$$f_{\rm RFG} = \frac{3}{4}(1-\psi^2)$$



- Scaling function of ^{12}C, obtained dividing the R_L by the single nucleon functions for different **q**

Scaling in the Fermi gas model

- Scaling of the second kind: independence of the nuclear species A



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Scaling in the Fermi gas model

T. W. Donnelly and Ingo Sick PRC 60, 065502, 1999

Let's specify how the different variables are defined:

$$\lambda = \omega/2m \ ,$$

$$\kappa = |\mathbf{q}|/2m \ ,$$

$$\eta_F = p_F/m \ .$$

$$\xi_F = E_F/m - 1$$

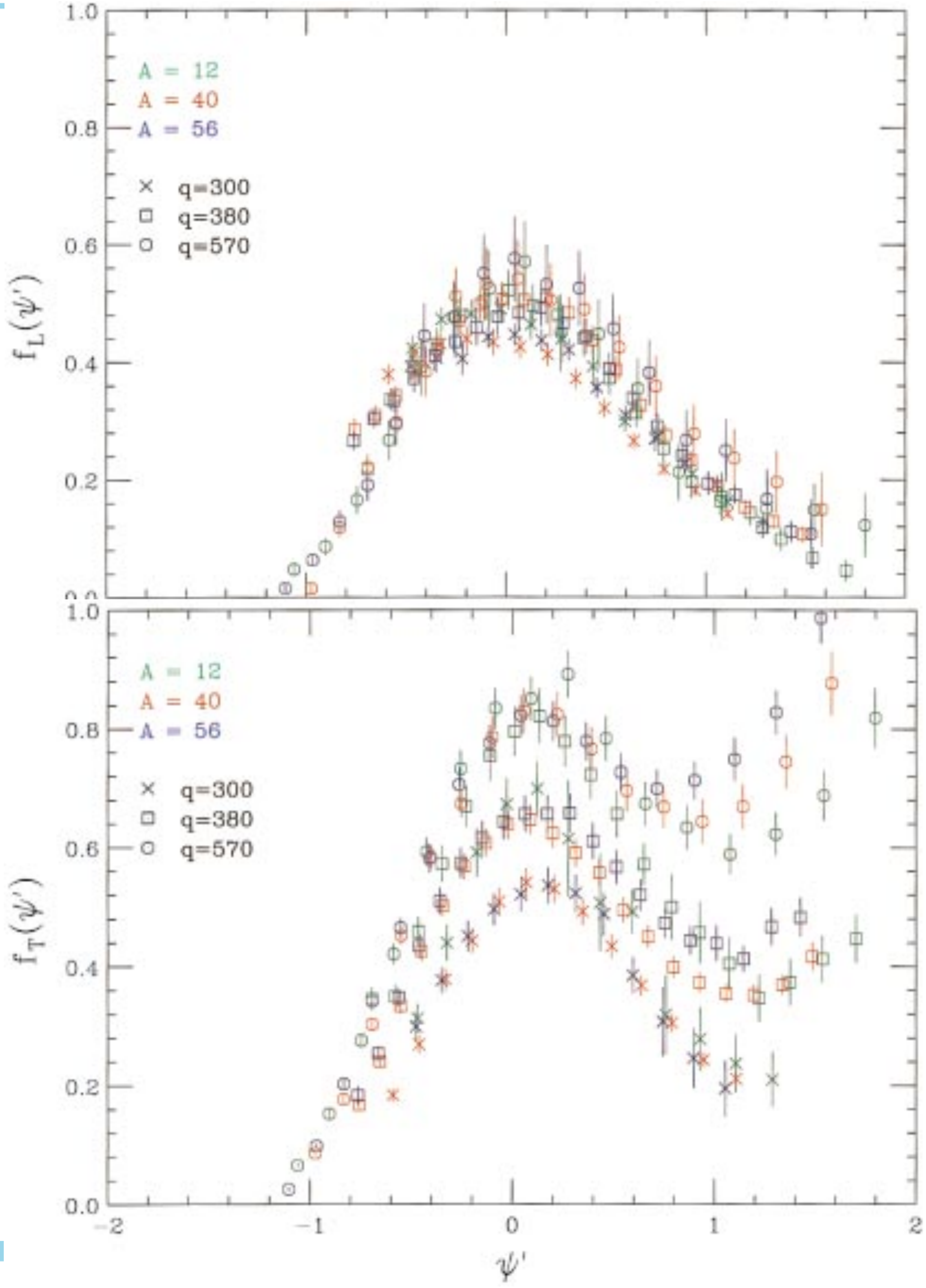
Scaling variable:

$$\psi = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\sqrt{(1+\lambda)\tau + \kappa\sqrt{\tau(1+\tau)}}}$$

Scaling functions:

$$f_L(\psi) = p_F \times \frac{R_L}{G_L} \ ,$$

$$f_T(\psi) = p_F \times \frac{R_T}{G_T} \ ,$$



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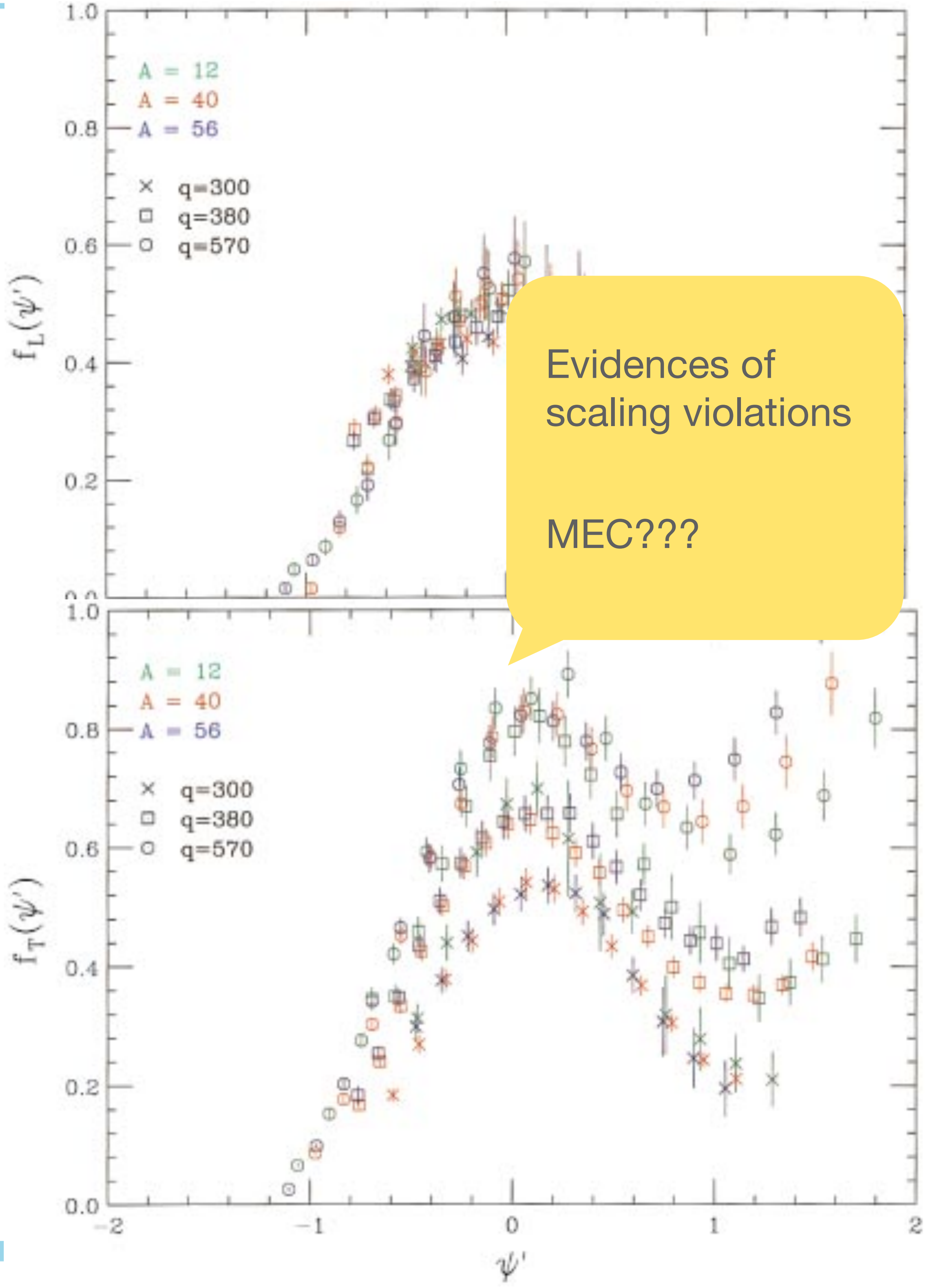


Evidences of scaling violations

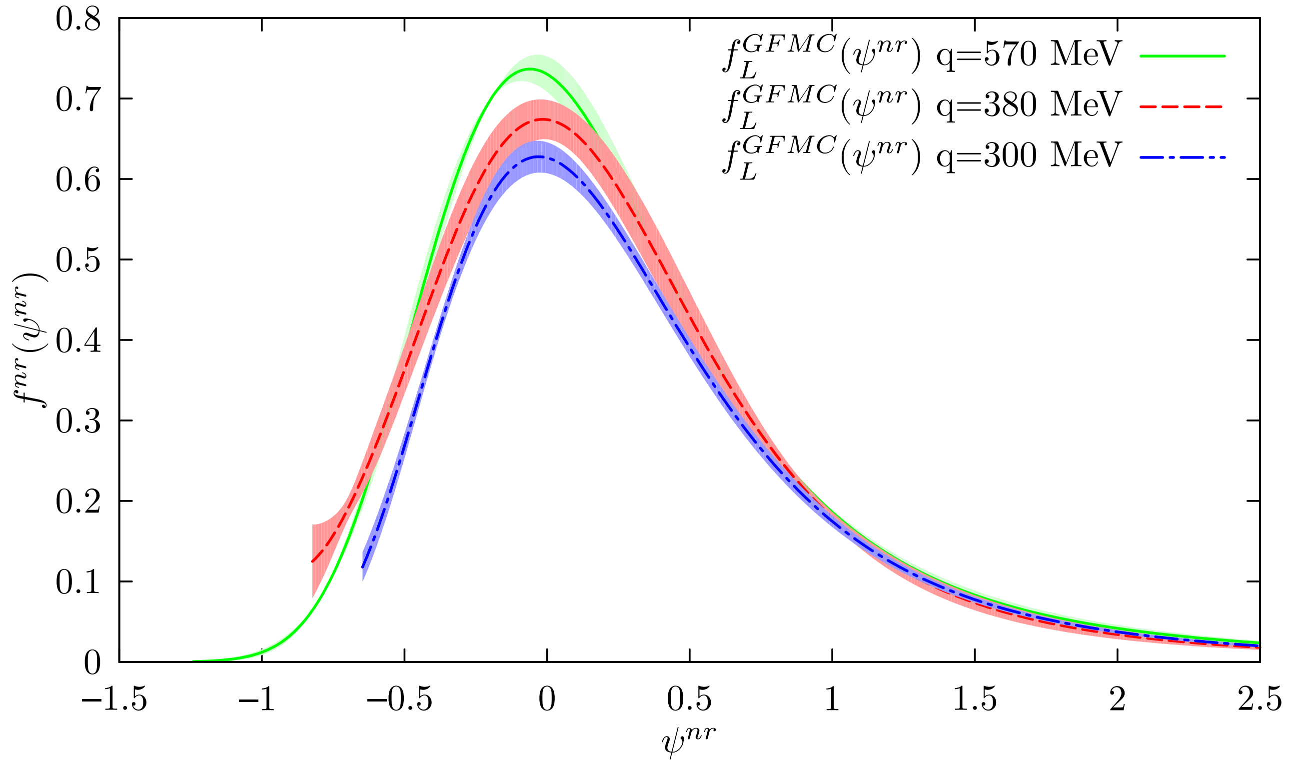
MEC???

Scaling in the GFMC approach

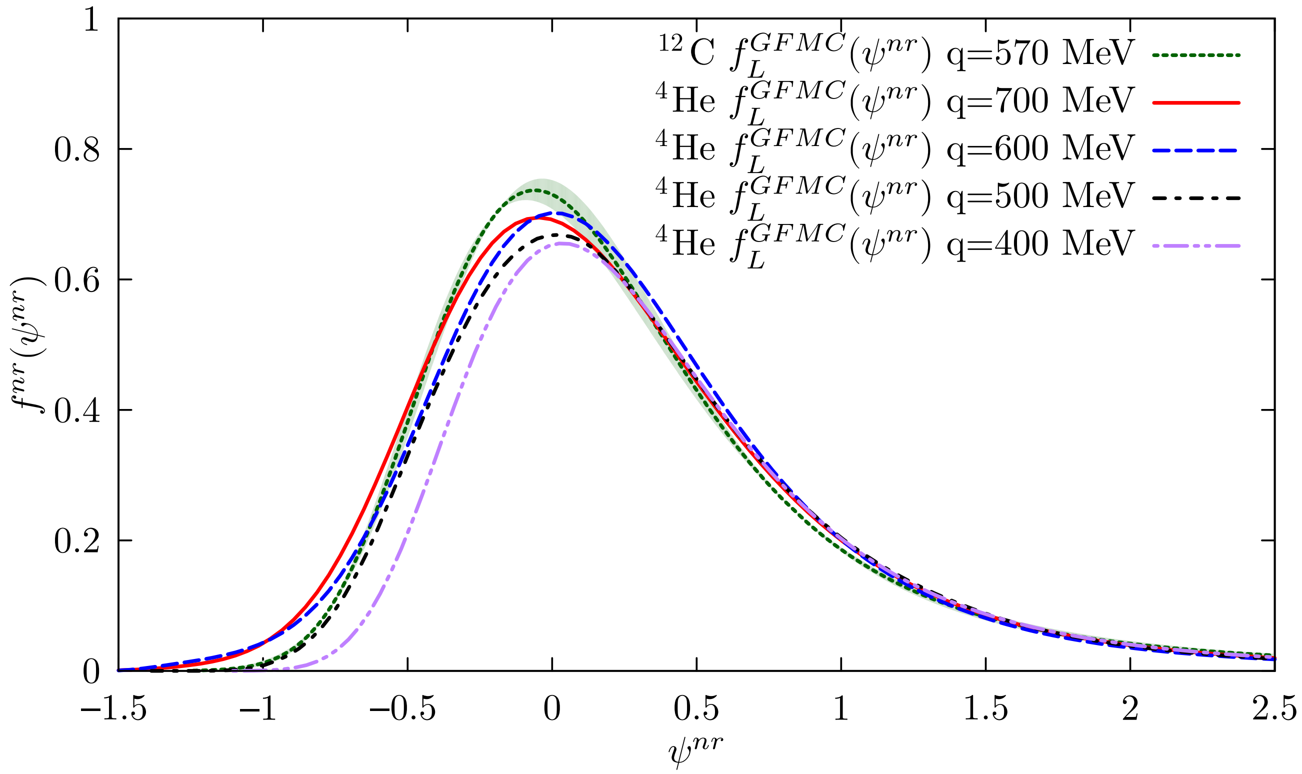
- In the relativistic case, the single nucleon function reads:

$$G_L = \frac{\mathcal{N}}{2\kappa}\left\{\frac{\kappa^2}{\tau}\left[G_E^2(\tau) + W_2(\tau)\Delta\right]\right\} \qquad G_T = \frac{\mathcal{N}}{2\kappa}\left\{2\tau G_M^2(\tau) + W_2(\tau)\Delta\right\}$$

- Same calculation can be carried out in the non-relativistic case and study scaling properties of the longitudinal response for the GFMC results

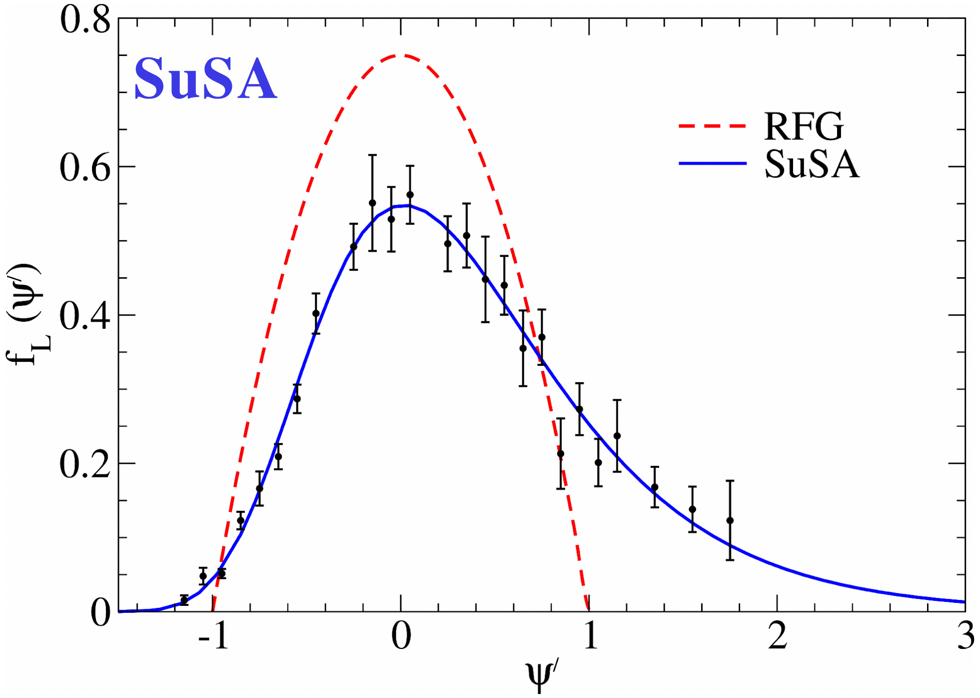


PHYSICAL REVIEW C 96, 015504 (2017)



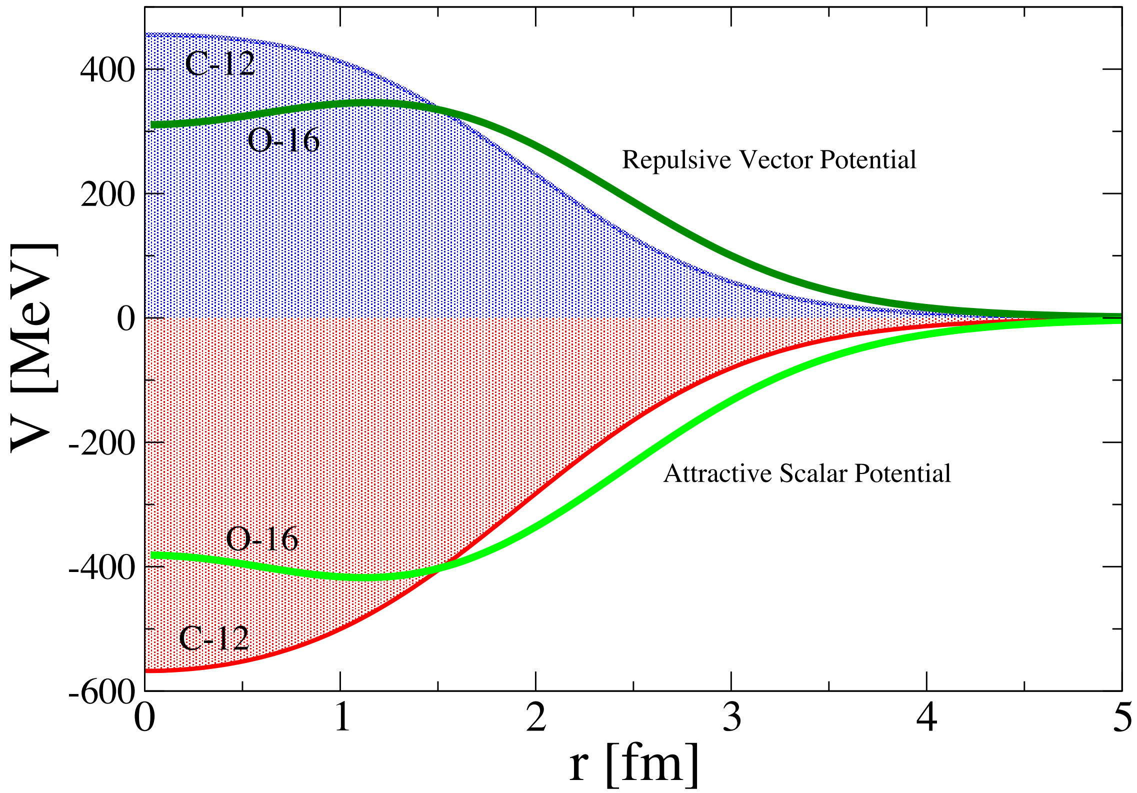
Super-Scaling (SuSA) model

Amaro *et al.*, PRC71 (2005)



Basic idea is to use the scaling function extracted from longitudinal (e,e') data to predict ν-scattering cross sections

Gonzalez-Jimenez *et al.*, PRC90(2014)



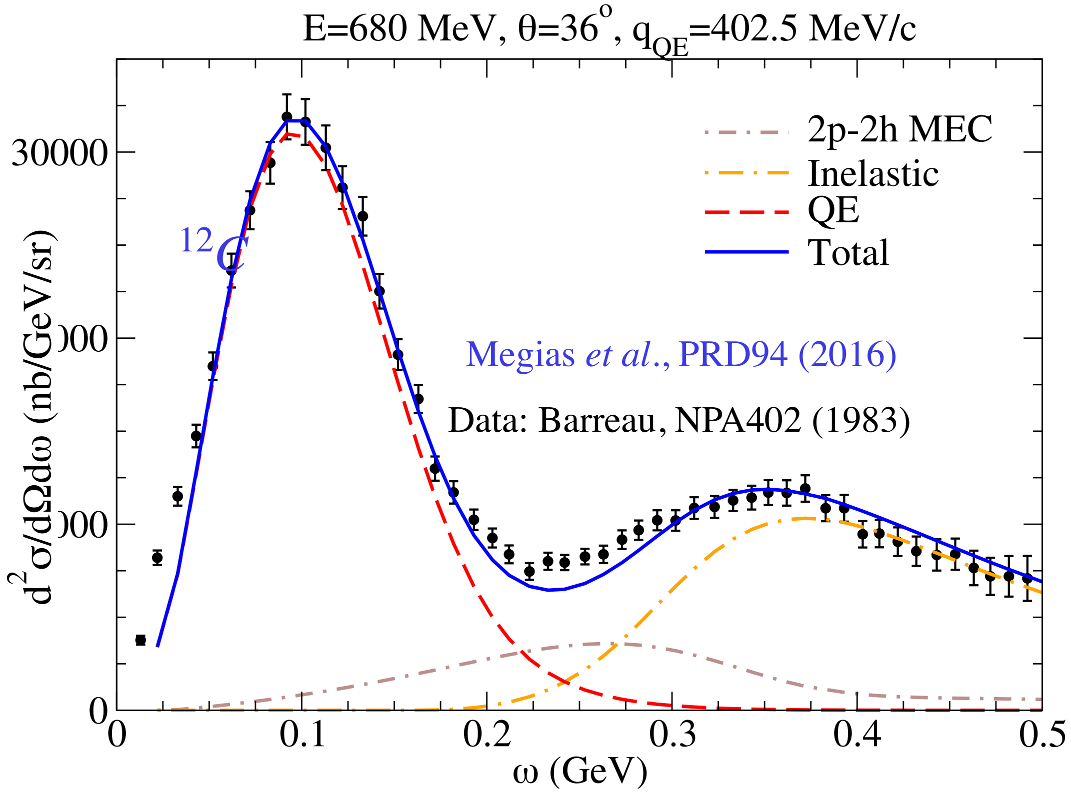
The nucleon wave functions are solutions of the Dirac equation with phenomenological relativistic scalar and vector mean field potentials

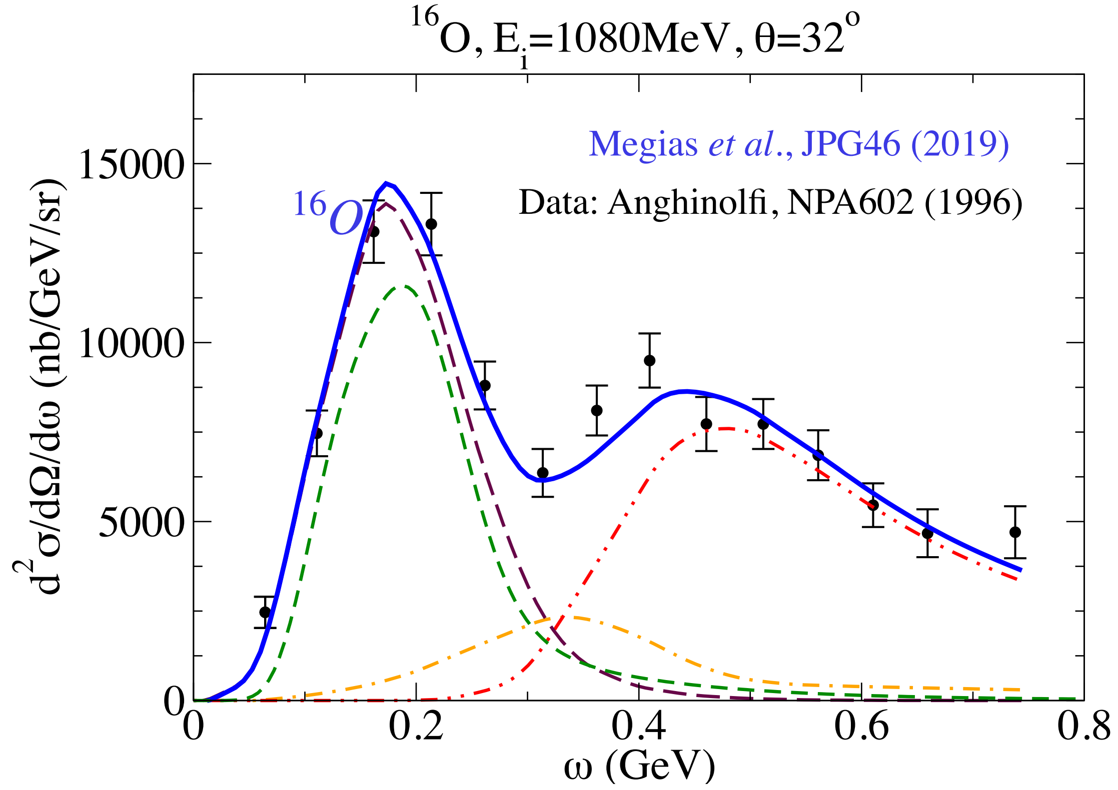
$$(i\gamma^\mu\partial_\mu - M - S + V)\,\psi(\vec{r},t) = 0$$

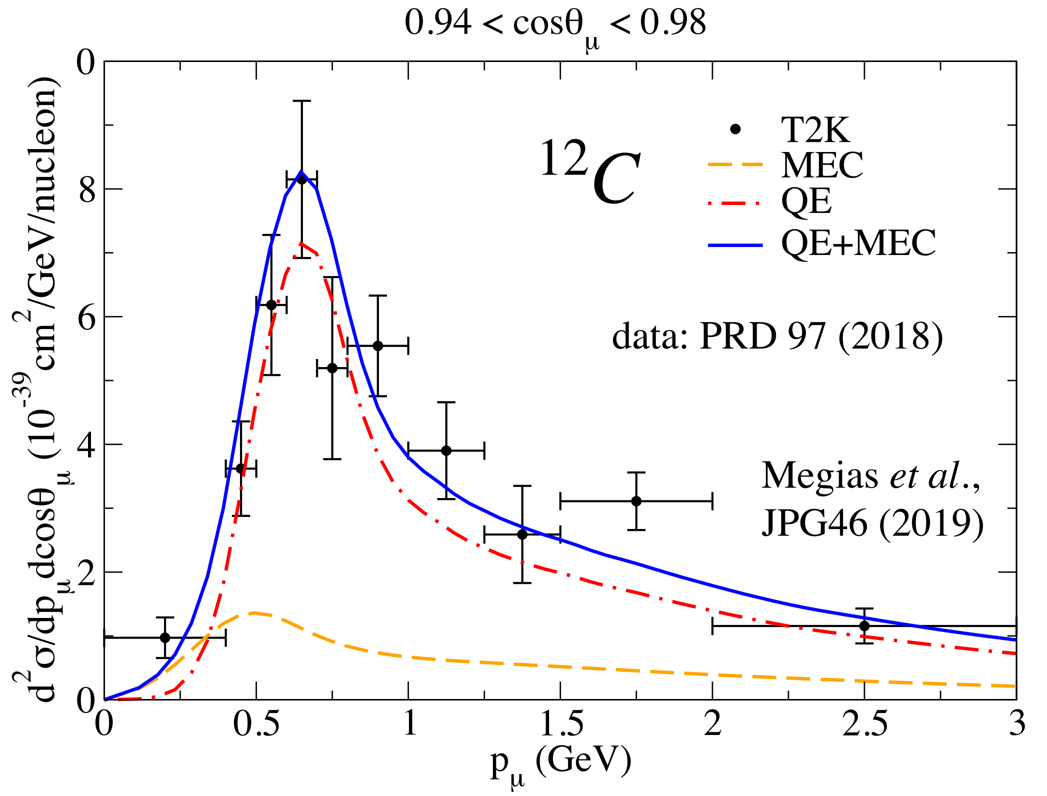
SuSAv2 : uses scaling functions extracted from Relativistic Mean Field calculations.

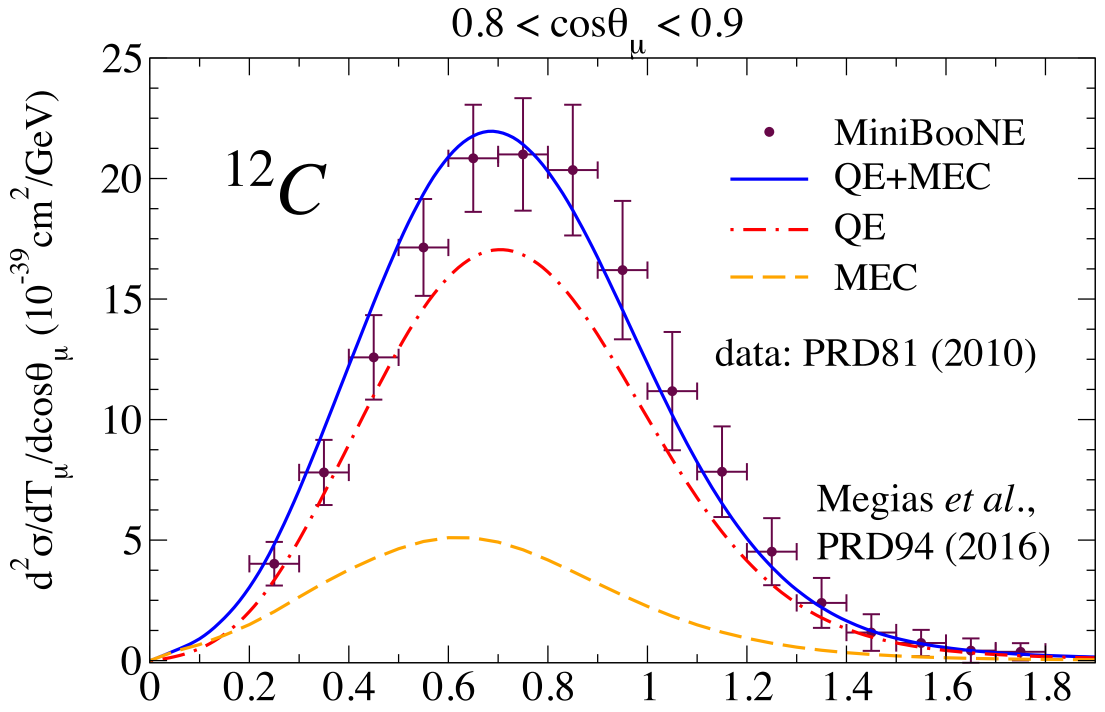
$f_T > f_L$ in agreement with L/T separated *(e,e')* data

SuSAv2 predictions





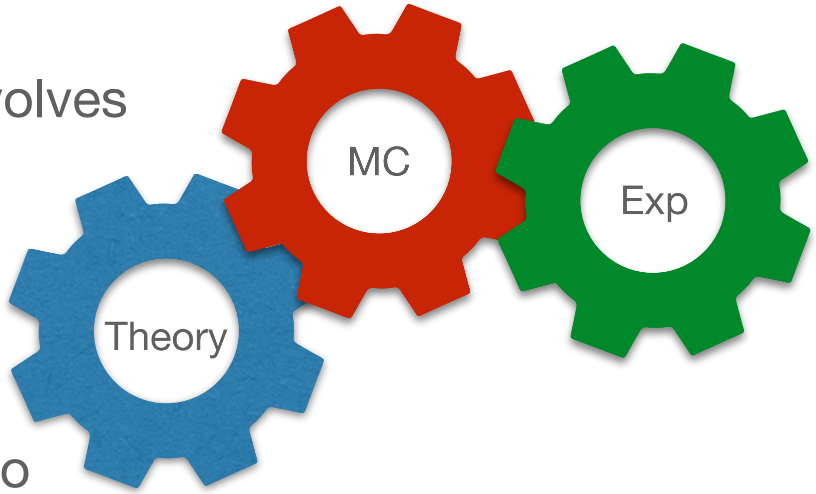




Conclusions

- Understanding neutrino-nucleus interactions is crucial for neutrino oscillation experiments

- To do this we rely on cross-sections: large important topic that involves



- What did we learn today about the Theory ?

  ✤ Electron and Neutrino scattering are connected, we need to start from the electromagnetic case first and then extend to the electroweak sector

  ✤ Nuclei are complicated objects, we need a good model to describe them

  ✤ There are different reactions mechanisms at play: we need to account for all of them. Factorization scheme + realistic spectral function is a possible way to do it

- The study of neutrino properties has the unique feature of connecting: Lattice-QCD, Nuclear, and Particle Physics

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Thank you for your attention!