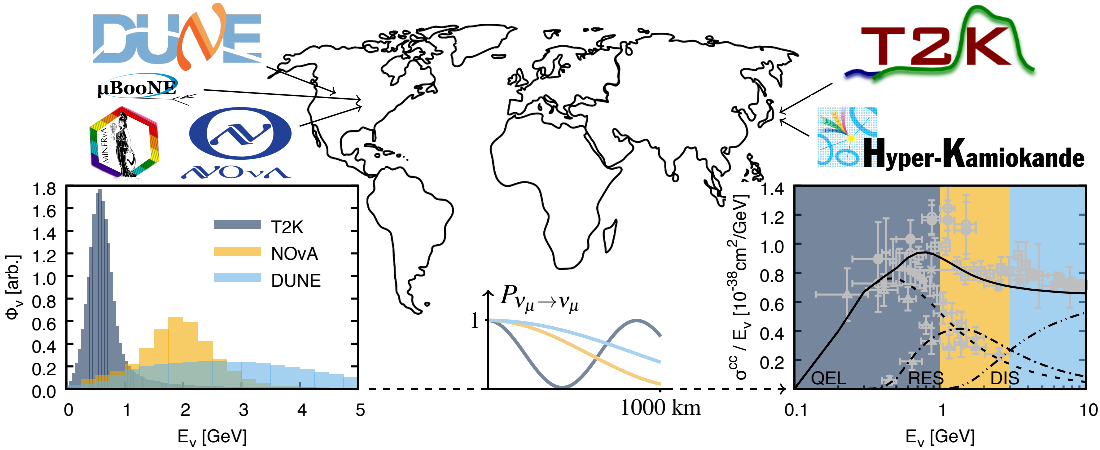
One- and two-nucleon knock-out in neutrino-nucleus scattering: Nuclear mean-field approaches

Kajetan Niewczas



$$P(\nu_\mu \to \nu_e) \simeq \sin^2(2\theta)\,\sin^2\left(1.27\frac{\Delta m^2 L}{E_\nu}\right)$$

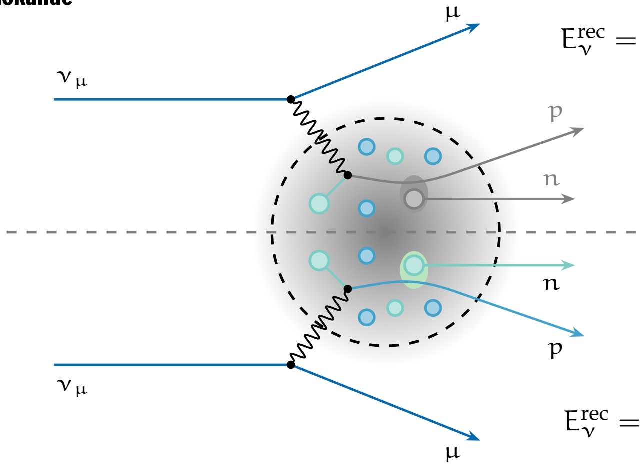
oscillation — amplitude — frequency

$$A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)}$$

asymmetry — oscillation ratio

Kinematical energy reconstruction

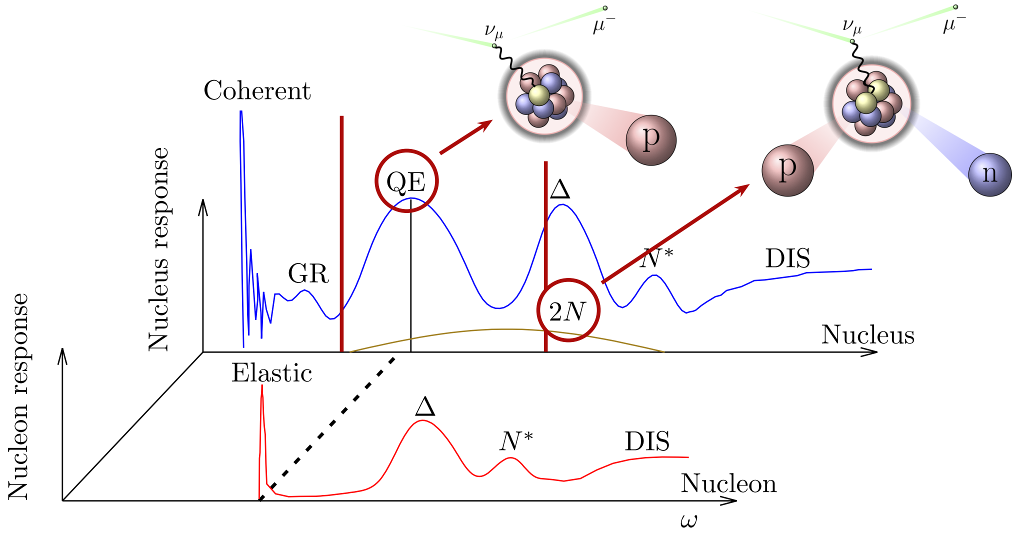
$$E_\nu^{rec} = \frac{2M_N E_\mu - m_\mu{}^2 + M_{N'}^2 - M_N^2}{2(M_N - E_\mu + p_\mu \cos\theta)}$$



$$E_\nu^{rec} = E_\mu - E_B + \sum\nolimits_{nucl.} T_i + \sum\nolimits_{mes.} E_j$$

Calorimetric energy reconstruction

Nuclear response in the quasielastic and Δ regions



Outline

Lecture 1. *the general framework of the nuclear mean-field model*

(1) Independent-particle model

(2) Nucleon in a central potential

(3) Mean-field nuclear potential

Lecture 2. *one- and two-nucleon knock-out in lepton-nucleus scattering*

Let's model a nucleus

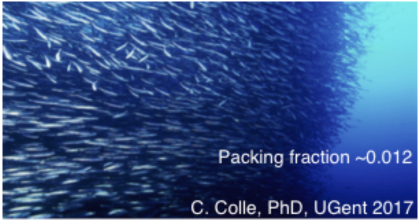
Basic property of the nucleus–**binding**

$$M(Z, N) = ZM_p + NM_n - B$$

Nuclear packing fraction:

- → for nuclei: $0.07 < \text{NFP} < 0.42$
- → for hard spheres: ≈ 0.74
- → for liquid argon: ≈ 0.032

Nucleus is like a **dense quantum liquid**



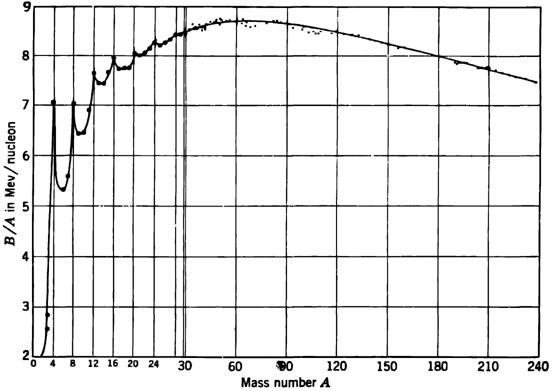


Fig. 3.1 Average binding energy B/A in Mev per nucleon for the naturally occurring nuclides (and Be^8), as a function of mass number A. Note the change of magnification in the A scale at $A = 30$. The Pauli four-shells in the lightest nuclei are evident. For $A \geq 16$, B/A is roughly constant; hence, to a first approximation, B is proportional to A.

R. Evans, The Atomic Nucleus (1955)

Liquid-drop model

Bethe-Weiszäcker **mass formula**

$$
\begin{aligned}
B = a_V A & \quad \rightarrow \quad \text{volume} \\
- a_S A^{2/3} & \quad \rightarrow \quad \text{surface} \\
- a_C \frac{Z^2}{A^{1/3}} & \quad \rightarrow \quad \text{Coulomb} \\
- a_A \frac{(N-Z)^2}{A} & \quad \rightarrow \quad \text{asymmetry} \\
\pm \Delta & \quad \rightarrow \quad \text{pairing}
\end{aligned}
$$

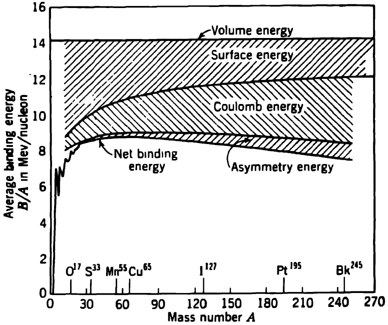
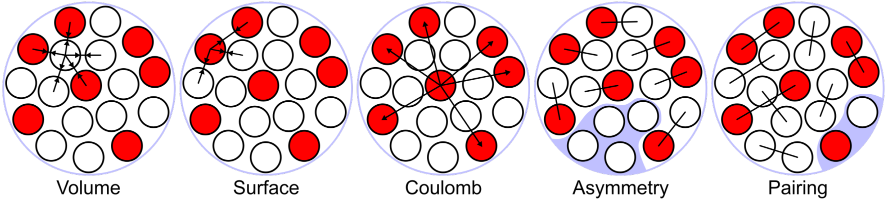


Fig. 3.5 Summary of the semiempirical liquid-drop-model treatment of the average binding-energy curve from Fig. 3.1 of Chap. 9. Note how the decrease in surface energy and the increase in coulomb energy conspire to produce the maximum observed in B/A at $A \sim 60$. For these curves, the constants used in the semiempirical mass formula are given in the last line of Table 3.3.



Independent-particle model

Basic assumptions

Elementary model of nuclear physics:

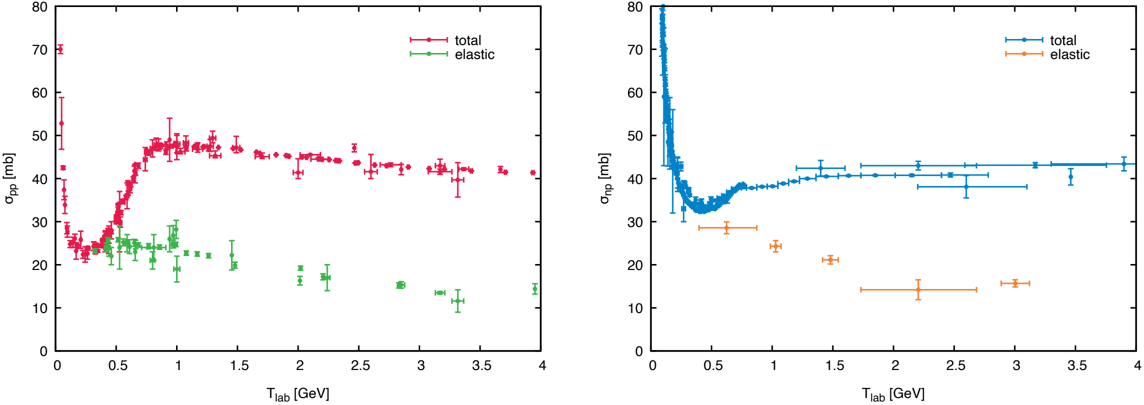
- **nonrelativistic**,
- **nucleons** are explicit degrees of freedom,
- described by the following Hamiltonian

$$\hat{H} = \sum_{i}^{A} \hat{T}_i + \sum_{i<j}^{A} \hat{V}_{ij} + \sum_{i<j<k}^{A} \hat{V}_{ijk} + \ldots$$

- **two-body potential** obtained from
  - → phenomenology,
  - → one-boson exchange models,
  - → using χEFT;
- **three-body potential** obtained from
  - → phenomenology,
  - → using χEFT;

Nucleon-nucleon interaction

K. A. Olive et al. (Particle Data Group), Chin.Phys. C 38 (2014), 090001



Two-body potentials (e.g. Argonne v_{18}) use **angular momentum** and **isospin** operators of the form

$$\{1, L \cdot S, \sigma_1 \cdot \sigma_2, S_{12}, L^2, (L \cdot S)^2, L^2 \sigma_1 \cdot \sigma_2\}, \quad \{1, \tau_1 \cdot \tau_2\}$$

Nucleon-nucleon interaction

Nuclear force:

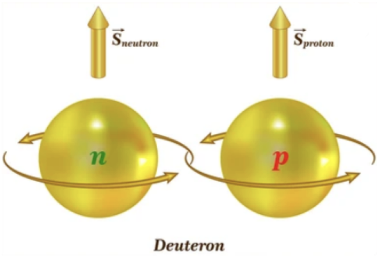
- Short range
- Repulsive core
- **Charge symmetry and independence**
- **Spin dependence**

Isospin symmetry:

- isospin $T = 1/2$
  - $\rightarrow$ neutron $T_z = -1/2$
  - $\rightarrow$ proton $T_z = +1/2$
  - $\rightarrow$ nucleus $T_z = 1/2(Z - N)$

Approximately **conserved in nuclei**

- nn, pp: $T = 1$
  - $\rightarrow$ must have $S = 0$
  - $\rightarrow$ marginally unbound
- np: $T = 0, 1$
  - $\rightarrow$ $S = 0$ is unbound
  - $\rightarrow$ **$S = 1$ is bound** with $B = 2.2$ MeV



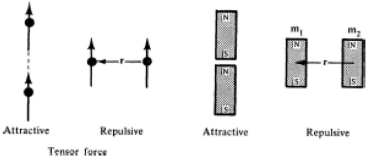


Figure 14.10: The tensor force in the deuteron is attractive in the cigar-shaped configuration and repulsive in the disk-shaped one. Two bar magnets provide a classical example of a tensor force.

Nucleon-nucleon interaction

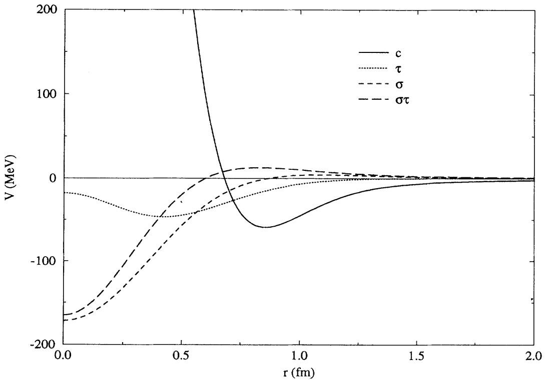


FIG. 6. Central, isospin, spin, and spin-isospin components of the potential. The central potential has a peak value of 2031 MeV at $r = 0$.

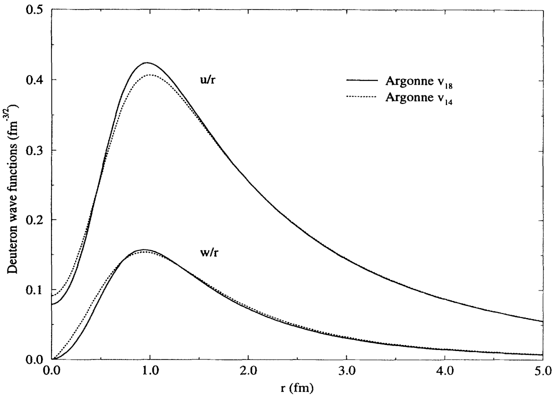


FIG. 11. The deuteron S- and D-wave function components divided by r.

R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys.Rev. C 51 (1995), 38–51

Independent-particle model

What do we know so far:

- nuclei are **made of nucleons**,
- **binding per nucleon** is relatively small ($\simeq 7.5$ MeV for ^{12}C),
- **distance between particles** larger than the nucleon radius ($\simeq 1-2$ fm),

Probability for a particle to propagate over a distance x with **no interactions** is

$$P(x) = \frac{1}{\lambda}\exp(-x/\lambda)$$

where $\lambda = (\rho\sigma)^{-1}$ is the **mean free path**, while ρ is **target density** and σ is **interaction cross section**

For nucleons inside nuclei:

$$\tilde{\lambda} \ll d < \lambda < R$$

where $\tilde{\lambda}$ is the de Broglie **wavelength**, d is the **distance** between targets, and R is the **nuclear radius**

$\rightarrow$ nucleus can be modeled as a **system of independent, quasifree nucleons**

Independent-particle model

General characteristics:

- **discrete energy levels** of a particle in a potential well

$$E_i = T_i - U(r_i) < 0,$$

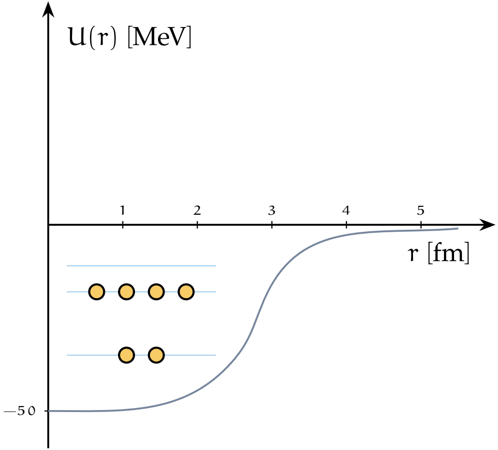
→ **nuclear binding**

$$B = \sum_{i}^{A} (T_i - U(r_i)),$$

→ **separation energy**

$$E_s = T_{max} - U(r),$$

- **Coulomb barrier** for protons



Fermi gas model

Let's assume a gas of nucleons:

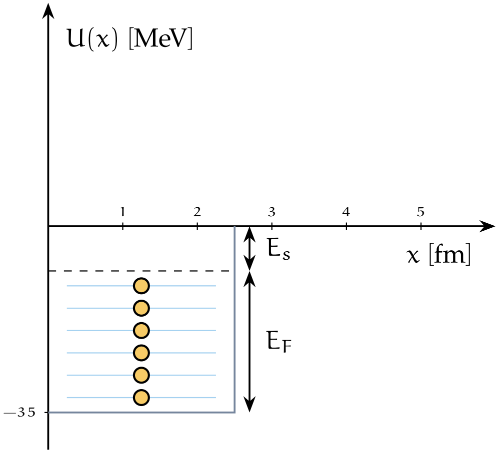
- nucleons are **fermions**,
  - → wave functions are **antisymmetric**

$$\psi(\dots, x_a, \dots, x_b, \dots) = -\psi(\dots, x_b, \dots, x_a, \dots)$$

- degeneracy pressure from **Pauli principle**,
- **no interactions** between nucleons,
- everything immersed in an **infinite potential well**

$$\frac{-\hbar^2}{2m}\nabla^2\psi(x, y, z) = E\psi(x, y, z)$$

→ stationary **Schrödinger equation**



Infinitely deep potential well

The **wave functions**:

$$\Psi_{n_x,n_y,n_z}(x,y,z) = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z) = \sin(k_x x)\sin(k_y y)\sin(k_z z) \quad (1)$$

The **energies**:

$$E_{n_x,n_y,n_z} = \frac{\hbar^2 p_x^2}{2m} + \frac{\hbar^2 p_y^2}{2m} + \frac{\hbar^2 p_z^2}{2m} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) \quad (2)$$

The **number of states** up to the Fermi momentum:

$$p_x^2 + p_y^2 + p_z^2 < p_F^2 \implies n_x^2 + n_y^2 + n_z^2 < \frac{p_F^2 L^2}{\pi^2\hbar^2} \quad (3)$$

We calculate the **number of occupied of states**:

$$n = 2\frac{1}{8}\frac{4}{3}\pi\left(\frac{p_F L}{\pi\hbar}\right)^3 = \frac{1}{3}\pi\left(\frac{p_F}{\pi\hbar}\right)^3 V = \frac{1}{3}\pi\left(\frac{p_F}{\pi\hbar}\right)^3\frac{4}{3}\pi r_A^3 \quad (4)$$

(we took only **$1/8$ of the total sphere** ($n_x > 0$, $n_y > 0$, $n_z > 0$), but with **2 spin states**)

Finally, using $r_A = r_0 A^{1/3}$, we obtain the **Fermi momenta for protons and neutrons**:

$$p_F = \frac{\hbar}{r_0}\sqrt[3]{\frac{9\pi}{4}}\sqrt[3]{\frac{Z}{A}} \quad \text{and} \quad p_F = \frac{\hbar}{r_0}\sqrt[3]{\frac{9\pi}{4}}\sqrt[3]{\frac{A-Z}{A}} \quad (5)$$

Fermi gas model... why not?

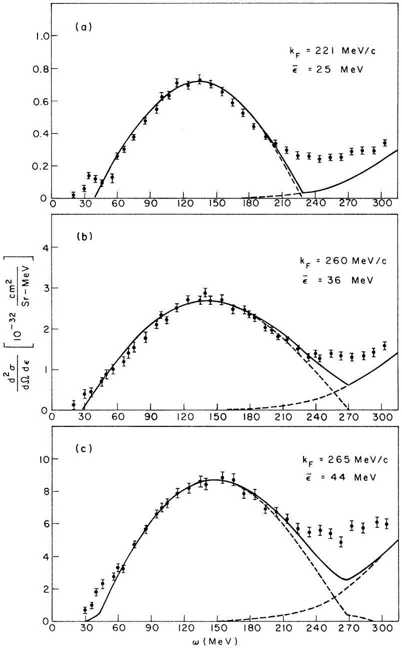
- **analytical model** for efficient computations,
- **nucleon transition** from below the Fermi sea to above

$$\theta(k_F - |\vec{k}|) \to \theta(|\vec{k} + \vec{q}| - k_F)$$

$$|\vec{k}|^2/2M - V \to (|\vec{k} + \vec{q}|^2 + M^2)^{1/2}$$

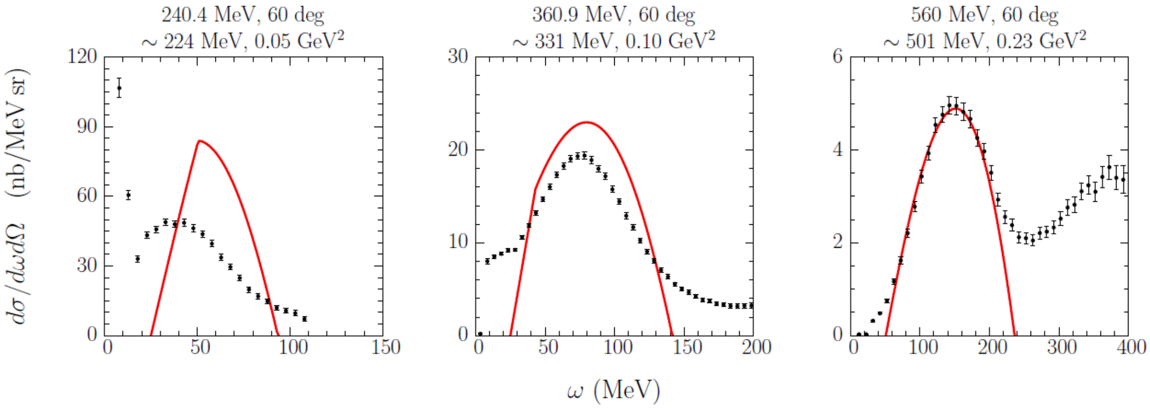
  → final nucleon is a plane wave;

- captures general features of **quasielastic peak**
  - → **Fermi momentum** controls the spread,
  - → **average interaction energy** controls the shift; $(\bar{\epsilon} = \langle E \rangle \ ? \ \bar{\epsilon} \neq V)$

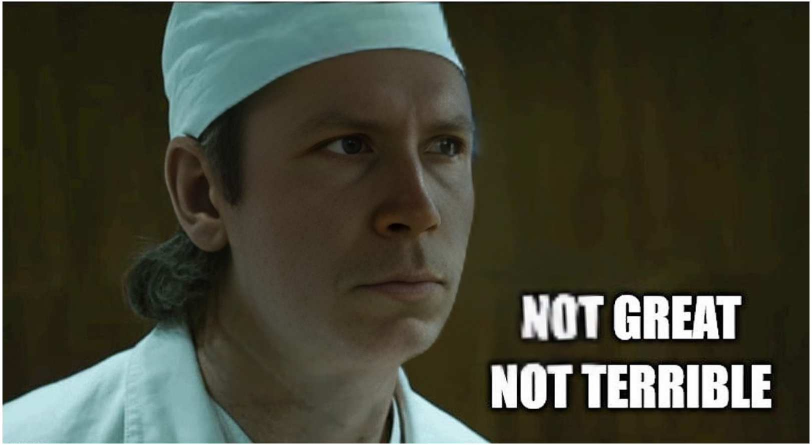


E. J. Moniz et al., Phys.Rev.Lett. 26 (1971) 445-448

...maybe better not

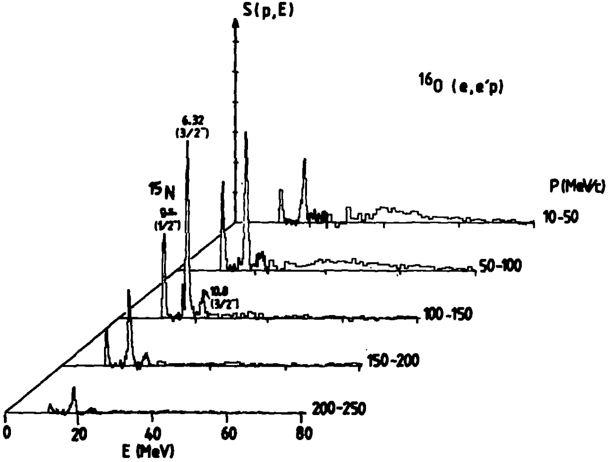


Artur Ankowski *R. Whitney et al., Phys.Rev. C 9 (1974), 2230*


NOT GREAT
NOT TERRIBLE

Fermi gas model

- **general assumptions are unclear**
  - → taken limits are inconsistent;
- fails to predict proper **energy levels**
  - → unreliable for exclusive processes;
- **lack of nucleon-nucleon interactions**
  - → overestimates the inclusive data;
- **local Fermi gas** is more robust
  - → but makes even less sense;



J. Mougey, Nucl.Phys. A 335 (1980) 35

Problem 1.

Let's take the ^{12}C nucleus with $k_F = 221$ MeV/c and $\bar{\epsilon} = 25$ MeV.

(1) What are the general properties of this Fermi gas (E_F,V,E_s)?

→ what is the average nucleon energy?

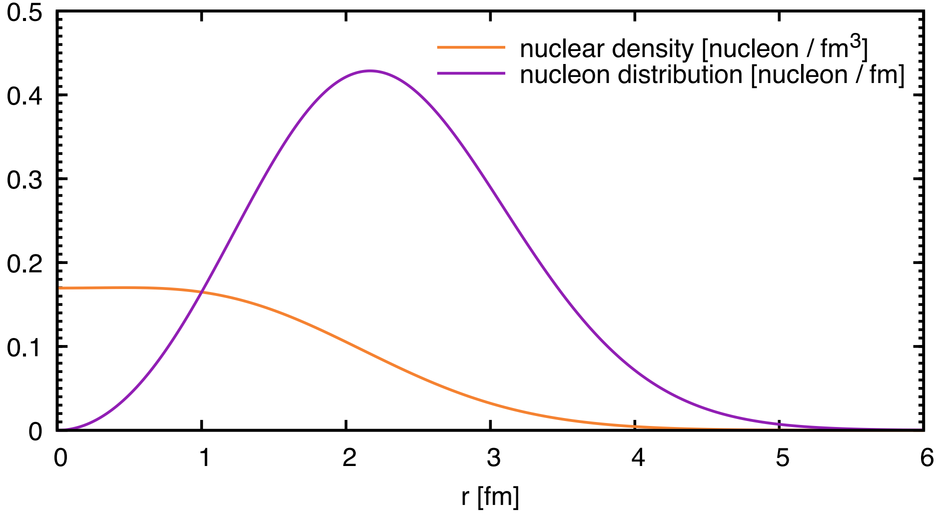
(2) How does the spectral function of the Fermi gas model look like?

→ what is the energy-momentum relation?

(3) How does the spectral function of a local Fermi gas looks like?

→ how can we parametrize k_F as a function of density $\rho(r)$?

Nuclear density and nucleon distribution for Carbon



http://discovery.phys.virginia.edu/research/groups/ncd/index.html

Nucleon in a central potential

Nucleon in a central potential

Let's consider a nucleon in a **central nuclear potential**

→ $V = V(r)$ only

→ angular momentum is conserved

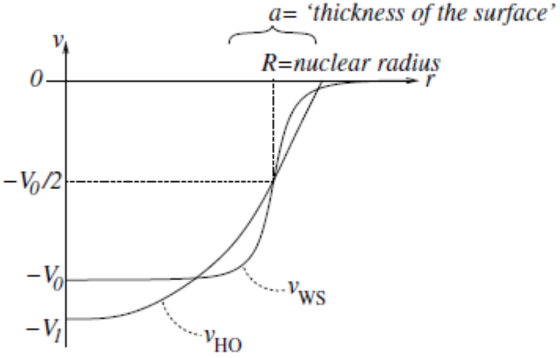
- harmonic oscillator

$$V_{HO}(r) = \frac{1}{2}m\omega^2 r^2 - V_1$$

- Woods-Saxon potential

$$V_{WS}(r) = -V_0 \frac{1}{1 + \exp((r - R)/a)}$$

with $V_0 \simeq 50$ MeV and $a \simeq 0.60$



Nucleon in a central potential

The Schrödinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi - V(r)\psi = E\psi \tag{1}$$

→ in **carthesian coordinates**:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{2}$$

→ in **spherical coordinates**:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial^2\phi}\right) \tag{3}$$

In spherical coordinates we **separate variables**:

$$\psi(r,\theta,\phi) = R(r)Y(\theta,\phi) \tag{4}$$

We obtain **two equations**:

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{2mr^2}{\hbar^2}[V(r)-E] = l(l+1) \tag{5}$$

$$\frac{1}{Y}\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right\} = -l(l+1) \tag{6}$$

Nucleon in a central potential

Let's consider the **angular part** and use $Y(\theta, \phi) = \Theta(\theta)\Phi(\phi)$:

$$\frac{1}{\Theta}\left[\sin\theta \frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right)\right] + l(l+1)\sin^2\theta = m^2 \tag{7}$$

$$\frac{1}{\Phi}\frac{d^2\Theta}{d\phi^2} = -m^2 \tag{8}$$

First we solve the latter:

$$\Phi(\phi) = e^{im\phi} \tag{9}$$

→ applying the condition $\Phi(\phi) = \Phi(2\pi + \phi)$ we must have $m = 0, \pm 1, \pm 2, ...$

Then, we solve the remaining:

$$\Theta(\theta) = AP_l^m(\cos\theta) \tag{10}$$

→ where $P_l^m(\cos\theta)$ are the associate Legendre polynomials, and $l = 0, 1, 2, ...$ for $m = -l, -l+1, ..., l-1, 2$

$Y(\theta, \phi) = \Theta(\theta)\Phi(\phi)$ are the **spherical harmonics**:

$$Y(\theta, \phi) = (-1)^m \sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}\, e^{im\phi} P_l^m(\cos\theta) \tag{11}$$

Nucleon in a central potential

The spherical harmonics are the angular solution to **any central potential problem**

The shape of the potential $V(r)$ **only affects the radial part** of the wave function

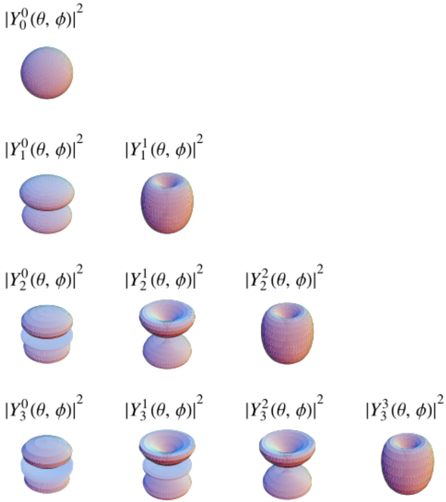
We have:

$$L^2 Y_{lm}(\theta, \phi) = l(l+1)\hbar^2 Y_{lm}(\theta, \phi) \quad (12)$$

$$L_z Y_{lm}(\theta, \phi) = m\hbar Y_{lm}(\theta, \phi) \quad (13)$$

Angular momentum is quantized:

- → the allowed values of l are $0, 1, 2, \ldots$
- → sometimes we use the letters $s, p, d, f, \ldots$
- → the allowed values of m are $0, \pm 1, \ldots, \pm l$
- → the eigenvalues of L^2,L_z are $l(l+1)\hbar^2$ and $m\hbar$



Nucleon in a central potential

Now, let us come back to the **radial part**:

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{2mr^2}{\hbar^2}[V(r) - E] = l(l+1) \tag{14}$$

We introduce $R(r) = u(r)/r$:

$$-\frac{\hbar}{2m}\frac{d^2u(r)}{dr^2} + \left(\frac{l(l+1)\hbar^2}{2mr^2} + V(r)\right)u(r) = Eu(r), \tag{15}$$

where

$$u(\infty) = 0, \quad u(0) = 0, \quad \int_0^\infty u^2(r)dr = 1 \tag{16}$$

E.g., for the **harmonic oscillator** of $U(r) = \frac{1}{2}m\omega^2r^2$:

$$u_{k,l}(r) = \left(\frac{m\omega}{\hbar}\right)^{l/2+1/2} e^{-\frac{m\omega}{2\hbar}} r^{l+1} L_k^{l+1/2}\left(\frac{m\omega}{\hbar}r^2\right) \tag{17}$$

with **energy levels**

$$E_{k,l} = \hbar\omega(2k + l + 3/2) = \hbar\omega(N + 3/2) \tag{18}$$

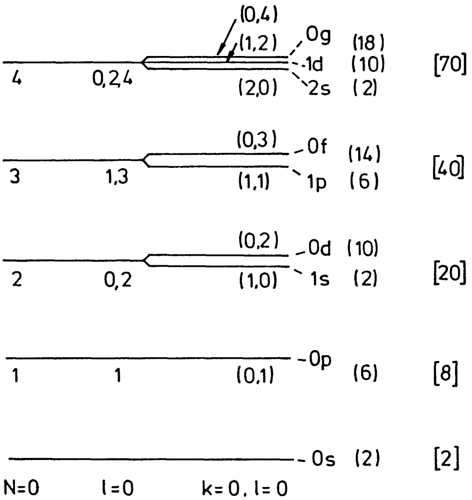
Nucleon in a central potential

Harmonic oscillator **energy spectrum is degenerate**

Energy levels are quantized:

- → major oscillator quantum number: $N = 0, 1, 2, ...$
- → orbital quantum number: $l = N, N-2, ..., 1, 0$
- → radial quantum number: $k = (N - l)/2$

Magic numbers appear in the spectrum



Spin-orbit coupling

So far we worked with the following **Hamiltonian**:

$$H_0 = \sum_i^A (T_i + U(r_i)) = \sum_i^A h_0(i), \quad E_0 = \sum_i^A \epsilon_{a_i} \tag{19}$$

Let's introduce a **spin-orbit term**:

$$h = h_0 + \zeta(r)\mathbf{l}\cdot\mathbf{s} \tag{20}$$

Nobel Prize in Physics 1963, E.P. Wigner, M. Goeppert Meyer, J.H.D. Jensen

So far, both **parallel and antiparallel orientations** have the same energies:

$$\langle nlj, m|\, h_0\, |nlj, m\rangle = \epsilon^{(0)}_{nlj}, \quad \langle \mathbf{r}, \sigma|nlj, m\rangle = \frac{u_{nl}(r)}{r}\left[\mathbf{Y}_l(\theta,\phi)\otimes\chi^{1/2}(\sigma)\right]^{(j)}_m \tag{21}$$

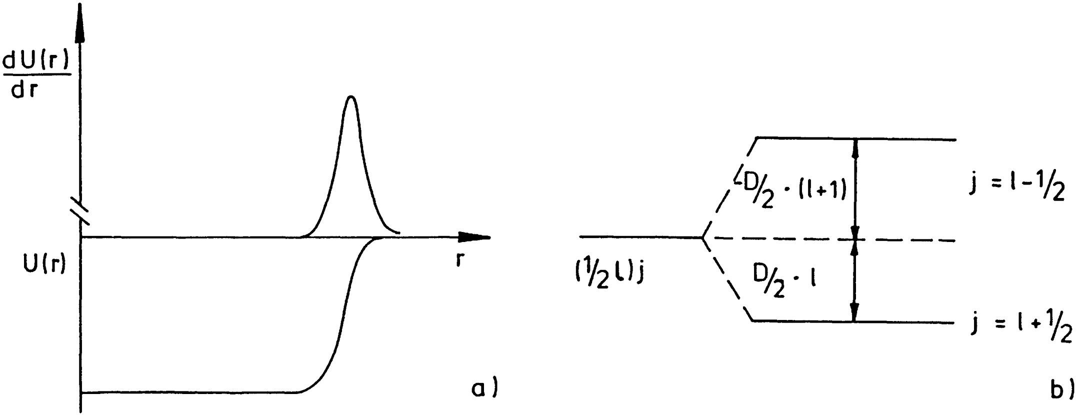
We can express the spin-orbit term as $\zeta(r)\frac{1}{2}(\mathbf{j}^2 - \mathbf{l}^2 - \mathbf{s}^2)$ and obtain $\epsilon_{nlj} = \epsilon^{(0)}_{nlj} + \Delta\epsilon_{nlj}$ with

$$\Delta\epsilon_{nlj} = \langle nlj, m|\, \zeta(r)\mathbf{l}\cdot\mathbf{s}\, |nlj, m\rangle, \quad \Delta\epsilon_{nlj} = \frac{D}{2}\left[j(j+1) - l(l+1) - \frac{3}{4}\right] \tag{22}$$

Finally, defining $D = \int u^2_{nl}(r)\zeta(r)dr$ and $\zeta(r) = V_{ls} r_0^2 \frac{1}{r}\frac{\partial U(r)}{\partial r}$, we get:

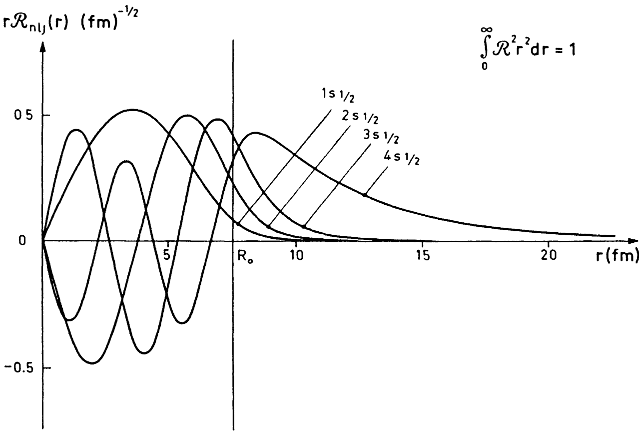
$$\Delta\epsilon_{nl\ j=l+1/2} = (D/2)\cdot l, \quad \Delta\epsilon_{nl\ j=l-1/2} = -(D/2)\cdot(l+1) \tag{23}$$

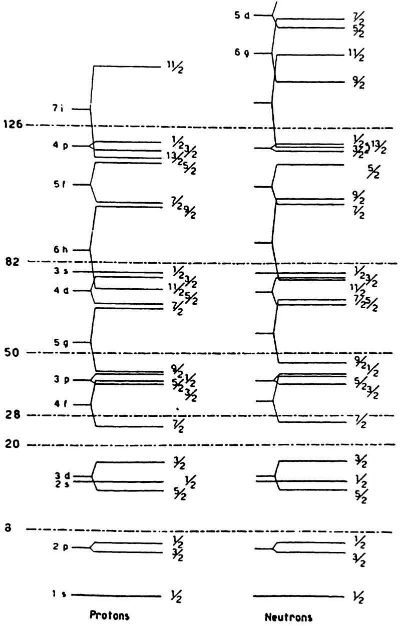
Spin-orbit coupling



K. Heyde, The Nuclear Shell Model (1990)

Woods-Saxon potential





→ correct **magic numbers**: 2, 8, 20, 28, 50, 82, 126, ...

Problem 2.

Let's take the nucleon in a spherical potential well:

$$\frac{dR(r)}{dr} + \frac{2}{r}\frac{dR(r)}{dr} + \left[\frac{2m}{\hbar}(E - V(r)) - \frac{l(l+1)}{r^2}\right] R(r) = 0$$

(1) What are the radial solutions to this problem?

→ *Handbook of Mathemathical Functions..., M. Abramowitz, I. A. Stegun, Eq. 10.1.1*

(2) What are the energy levels for nucleons?

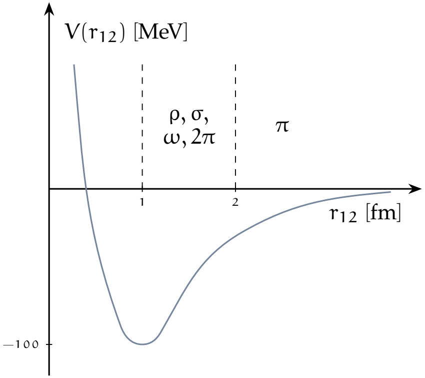
→ what are the roots of the solution and their relation to $k = \hbar p$?

(3) What is the average nucleon energy as confronted with a Fermi gas?

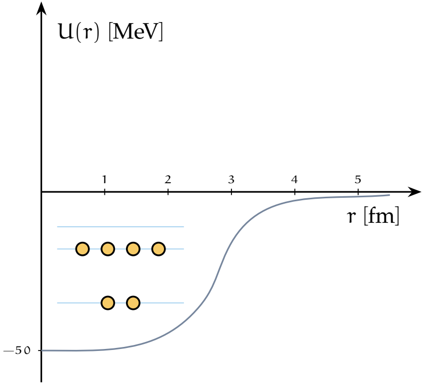
→ what is the depth of the potential using the same separation energy as before?

Mean-field nuclear potential

Mean-field nuclear picture



→



→ let's try to use a realistic **nucleon-nucleon potential** to derive the **central nuclear potential**

Mean-field potential

Single-particle **radial Schrödinger equation**:

$$(T + U(r))\,\phi_a(r) = \epsilon_a \phi_a(r) \tag{1}$$

Nuclear **Hamiltonian**:

$$H_0 = \sum_{i=1}^{A} (T_i + U(r_i)) = \sum_{i=1}^{A} h_0(i), \qquad E_0 = \sum_{i=1}^{A} \epsilon_{a_i}(r_i) \tag{2}$$

Nuclear wave function is a **Slater determinant**:

$$\Phi_{a_1,\dots,a_A}(r_1,\dots,r_A) = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_{a_1}(r_1) & \dots & \phi_{a_1}(r_A) \\ \vdots & \ddots & \vdots \\ \phi_{a_A}(r_1) & \dots & \phi_{a_A}(r_A) \end{vmatrix} \tag{3}$$

Let's restrict ourselves to two-body interactions only and evaluate the **mean-field potential**:

$$H = \sum_{i=1}^{A} T_i + \frac{1}{2} \sum_{i,j=1}^{A} V_{ij} \tag{4}$$

$$H = \sum_{i=1}^{A} (T_i + U(r_i)) + \left(\frac{1}{2} \sum_{i,j=1}^{A} V_{ij} - \sum_{i=1}^{A} U(r_i) \right) = H_0 + H_{res} = \sum_{i=1}^{A} h_0(i) + H_{res}, \tag{5}$$

Hartree-Fock methods

Let's consider a density in terms of the **occupied single-particle states**:

$$\rho(\mathbf{r}) = \sum_b \phi_b^*(\mathbf{r})\phi_b(\mathbf{r}) \tag{6}$$

The **Hartree potential** at a given point generated by the **two-body interaction**:

$$U_H(\mathbf{r}) = \sum_b \int \phi_b^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_b(\mathbf{r}')d\mathbf{r}' \tag{7}$$

The **Schrödinger equation** becomes:

$$\begin{aligned} -\frac{\hbar^2}{2m}\nabla^2\phi_i(\mathbf{r}) &+ \sum_b \int \phi_b^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_b(\mathbf{r}')d\mathbf{r}' \cdot \phi_i(\mathbf{r}) \\ &- \sum_b \int \phi_b^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_b(\mathbf{r})\phi_i(\mathbf{r}')d\mathbf{r}' = \epsilon_i\phi_i(\mathbf{r}) \end{aligned} \tag{8}$$

$$-\frac{\hbar^2}{2m}\nabla^2\phi_i(\mathbf{r}) + U_H(\mathbf{r})\phi_i(\mathbf{r}) - \int U_F(\mathbf{r},\mathbf{r}')\phi_i(\mathbf{r}')d\mathbf{r}' = \epsilon_i\phi_i(\mathbf{r}) \tag{9}$$

where the exchange term is driven by the **Fock potential**:

$$U_F(\mathbf{r}) = \sum_b \phi_b^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_b(\mathbf{r}) \tag{10}$$

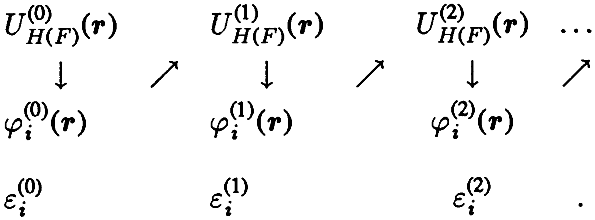
The iterative Hartree-Fock method

→ start with an **initial guess** for the **average field** or the **wave functions**

→ using the nucleon-nucleon potential $V(\mathbf{r}, \mathbf{r}')$ **solve the equation**

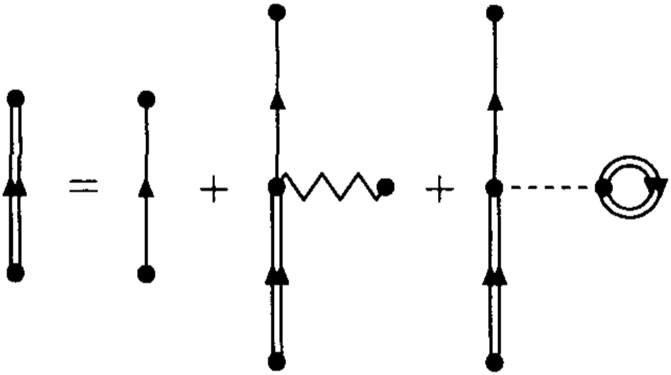
$$-\frac{\hbar^2}{2m}\nabla^2\phi_i(\mathbf{r}) + U_H(\mathbf{r})\phi_i(\mathbf{r}) - \int U_F(\mathbf{r}, \mathbf{r}')\phi_i(\mathbf{r}')d\mathbf{r}' = \epsilon_i\phi_i(\mathbf{r})$$

→ determine new values of $U_H(r)$, $U_F(\mathbf{r}, \mathbf{r}')$, $\phi_i(\mathbf{r})$, ϵ_i



→ at convergence: the **final field** $U_H(\mathbf{r})$, **wave function** $\phi_i(\mathbf{r})$, and **single-particle energy** ϵ_i

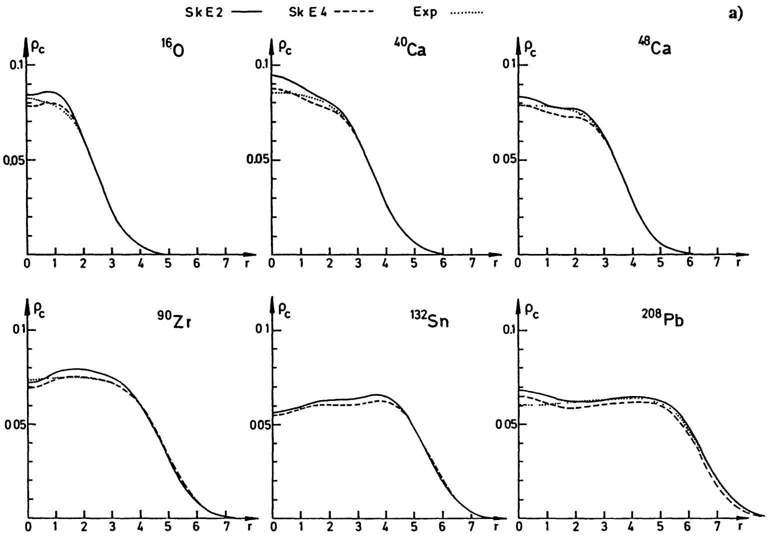
Nucleons in the mean-field potential



W. H. Dickhoff, D. Van Neck, Many-body Theory Exposed! (2005)

→ **nucleon lines are dressed** according to the Hartree-Fock procedure

Charge densities from the mean-field framework



K. Heyde, The Nuclear Shell Model (1990)

Charge densities from the mean-field framework

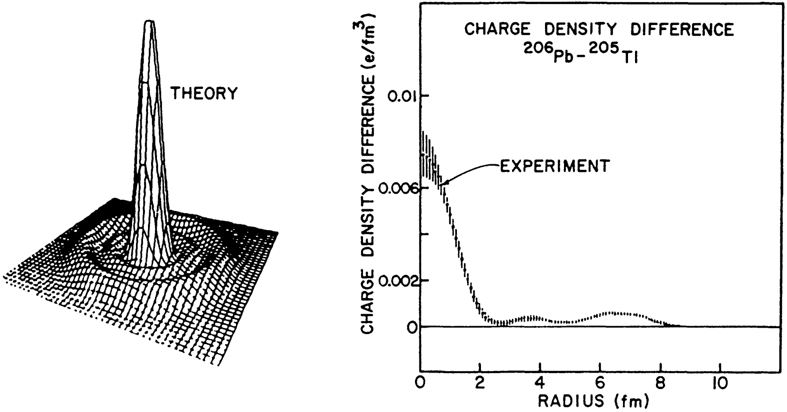


Fig. 3.18. The nuclear density distribution for the least bound proton in ^{206}Pb. The shell-model predicts the last ($3s_{1/2}$) proton in ^{206}Pb to have a sharp maximum at the centre, as shown at the left-hand side. On the right-hand side the nuclear charge density difference $\varrho_c\,(^{206}\mathrm{Pb}) - \varrho_c\,(^{205}\mathrm{Tl}) = \varphi^2_{3s_{1/2}}(r)$ is given [taken from (Frois 1983) and Doe 1983)]

K. Heyde, The Nuclear Shell Model (1990)

Relativistic mean-field

All of this can be also done in a **relativistic** framework:

- Schrödinger equation → **Dirac equation**,
- Wave functions → **Dirac spinors**,
- Spin-orbit term comes for free!

$$\left(\tilde{E}\gamma_0 - \vec{p}\cdot\vec{\gamma} - \tilde{M}\right)\psi = 0$$

$$\tilde{E} = E - V(r)$$
$$\tilde{M} = M - S(r)$$