Imperial College London

Inelastic scattering; meson production
Lecture 1

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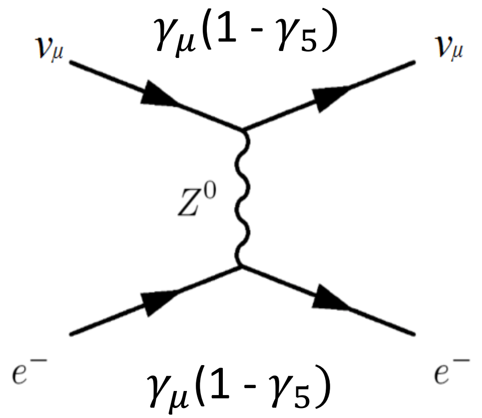
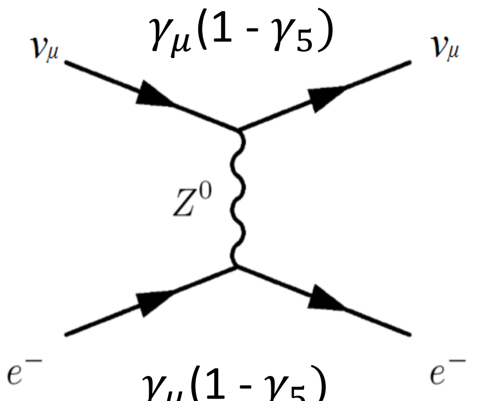
Neutrino interactions in the GeV regime

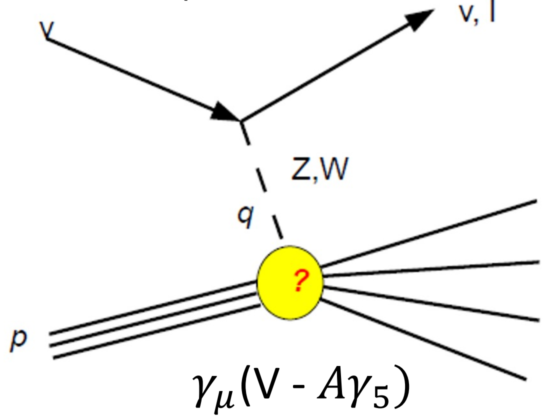
- Interaction of lepton and quarks
  - ➢Described by quantum field theory
  - ➢Targets in neutrino experiments are nuclear targets!



Neutrino (weak) interactions in the GeV regime

- Interaction with lepton and quarks
  - ➢Described by quantum field theory
- Interaction with free nucleon (Hydrogen)
  - ➢Elastic or quasielastic scattering
  - ➢Inelastic scattering
  - ➢Neutrino detectors in 60-80’s
- V and A are vector and axial-vector form-factors which shows how the scattering reduced from its value for a point-link nucleon.





Nucleons have structure

Bubble Chambers

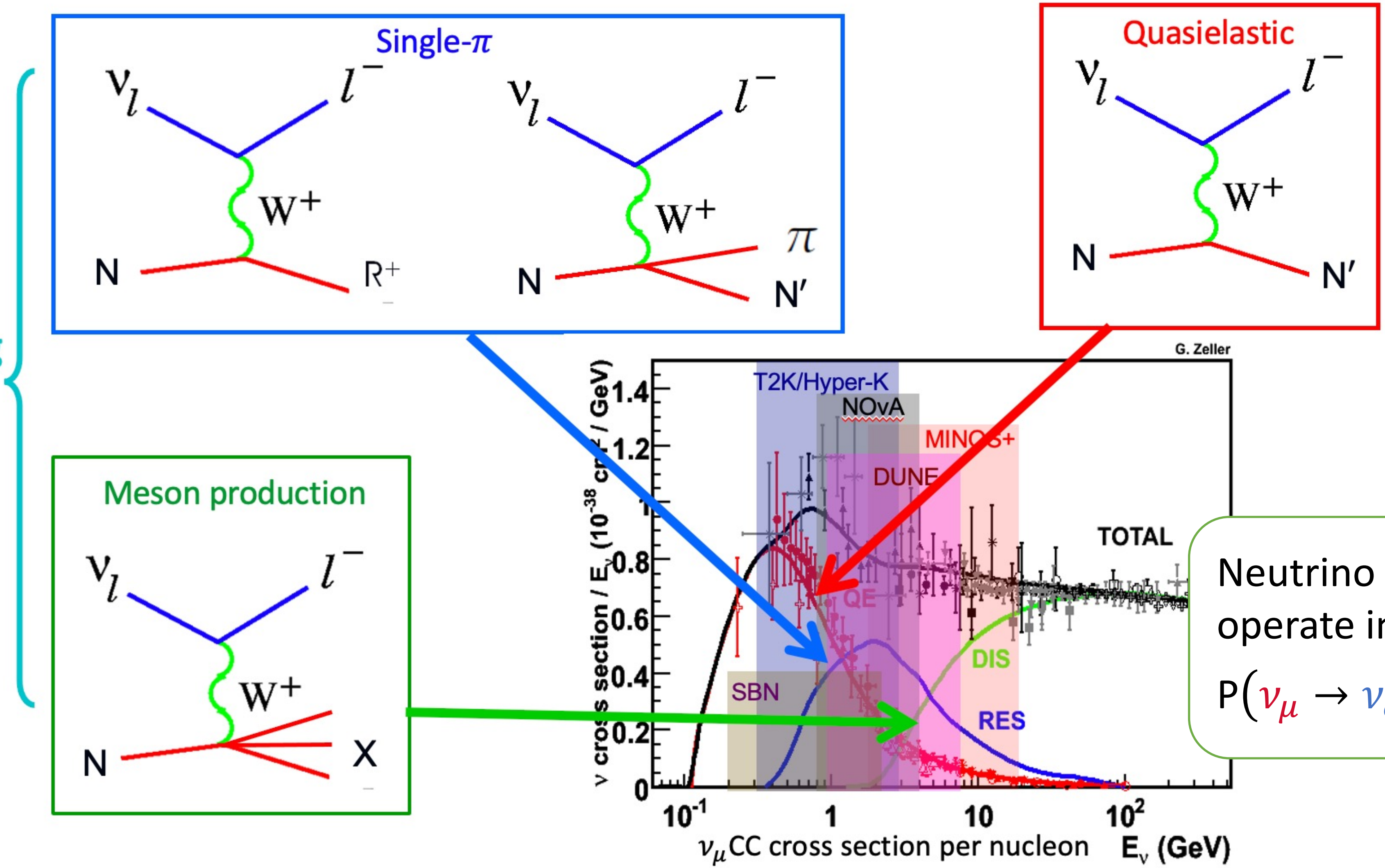
- Used for neutrino interaction search in the 60-80's.
- Now you can find them in the carparks of national labs.

The 15-foot Bubble Chamber FNAL

Big European Bubble Chamber (BEBC) CERN

ν-Nucleon interactions in the GeV regime

Inelastic scattering off a nucleon

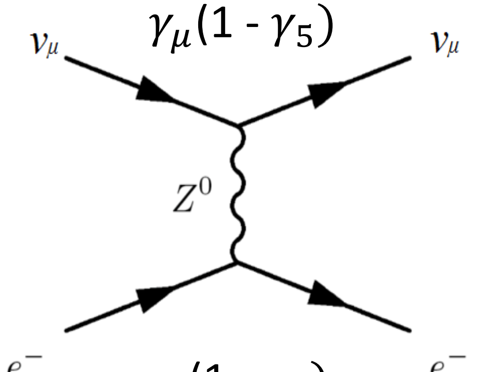


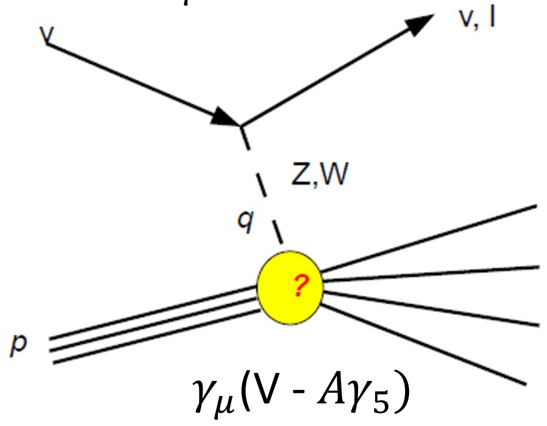
Neutrino experiments must operate in this region

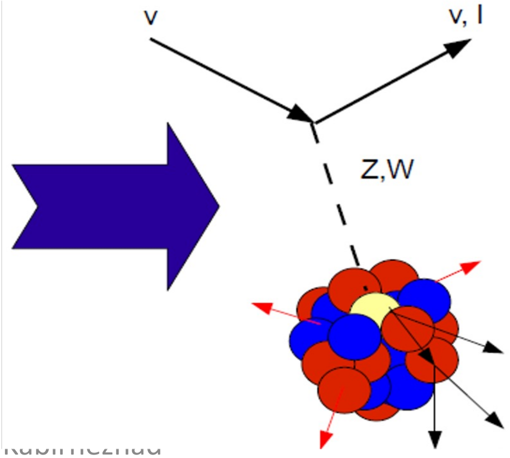
$P(\nu_\mu \to \nu_e) \propto \sin^2(\frac{\Delta m^2 L}{4E})$

Neutrino interactions in the GeV regime

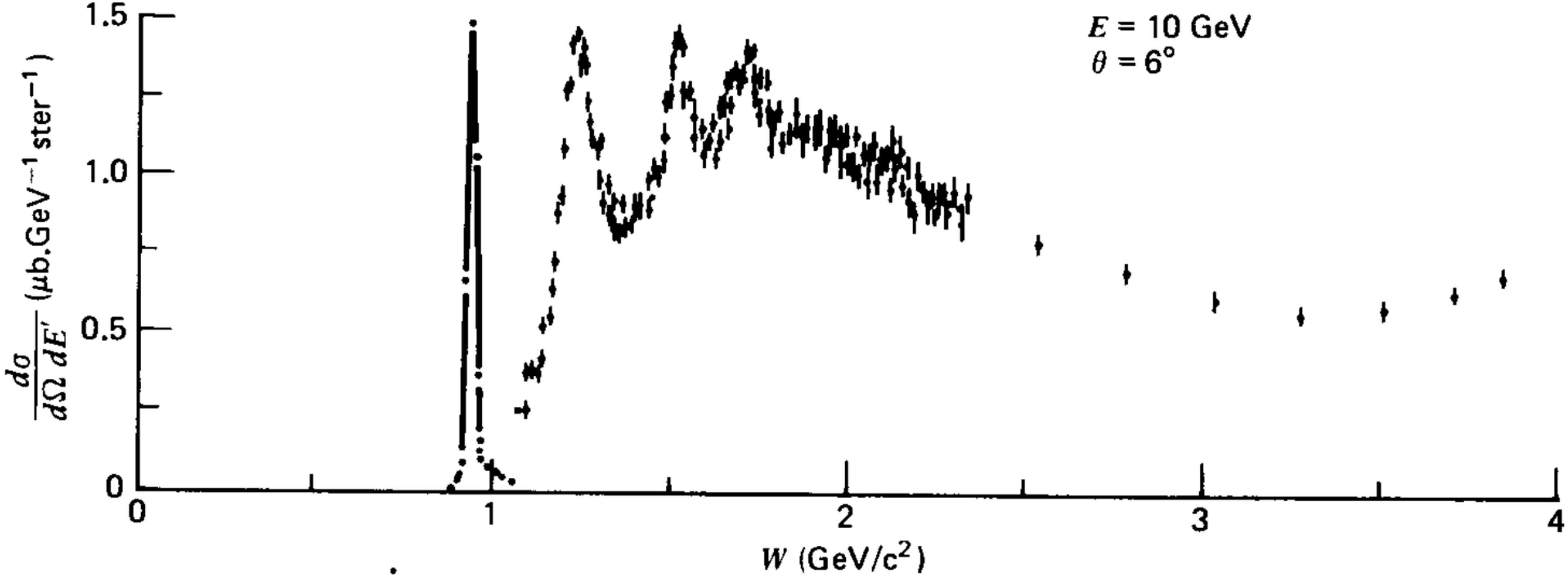
- Interaction with lepton and quarks
  - ➢Described by quantum field theory
- Interaction with free nucleon (Hydrogen)
  - ➢Elastic of quasielastic scattering
  - ➢Inelastic scattering
- Interaction with nucleus
  - ➢Modern neutrino experiments
  - ➢Targets are ^{12}C, ^{16}O or ^{40}Ar



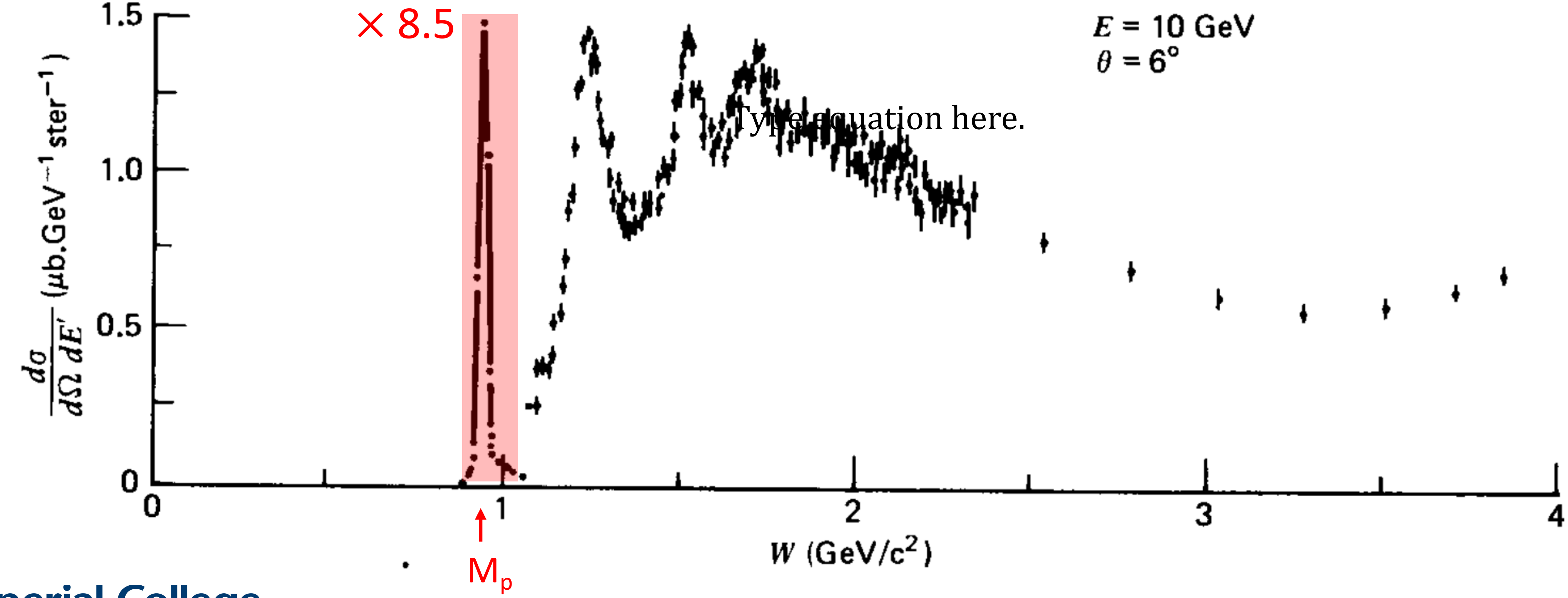




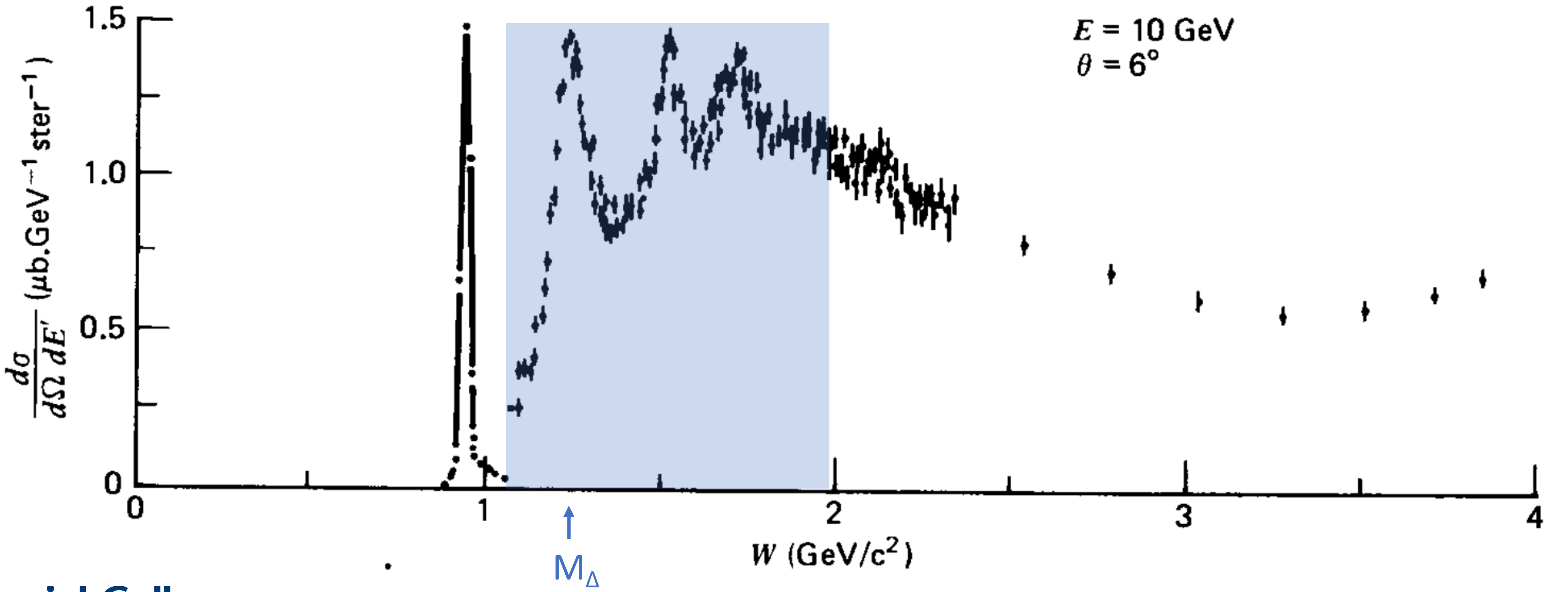
$ep \to eX$ cross section



Elastic peak ep → ep

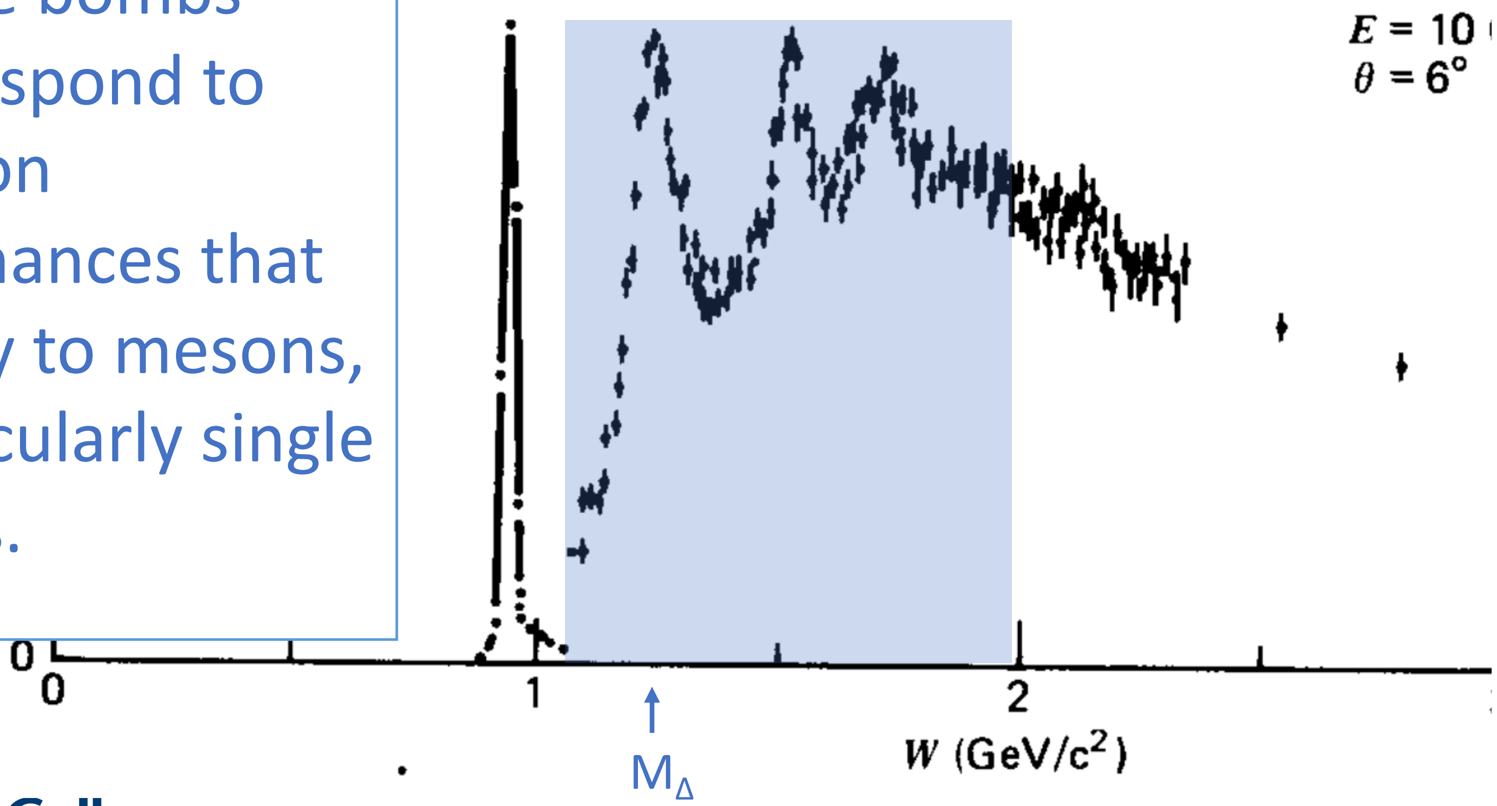


Proton excitations $ep \rightarrow epX$



Proton excitations ep → epX

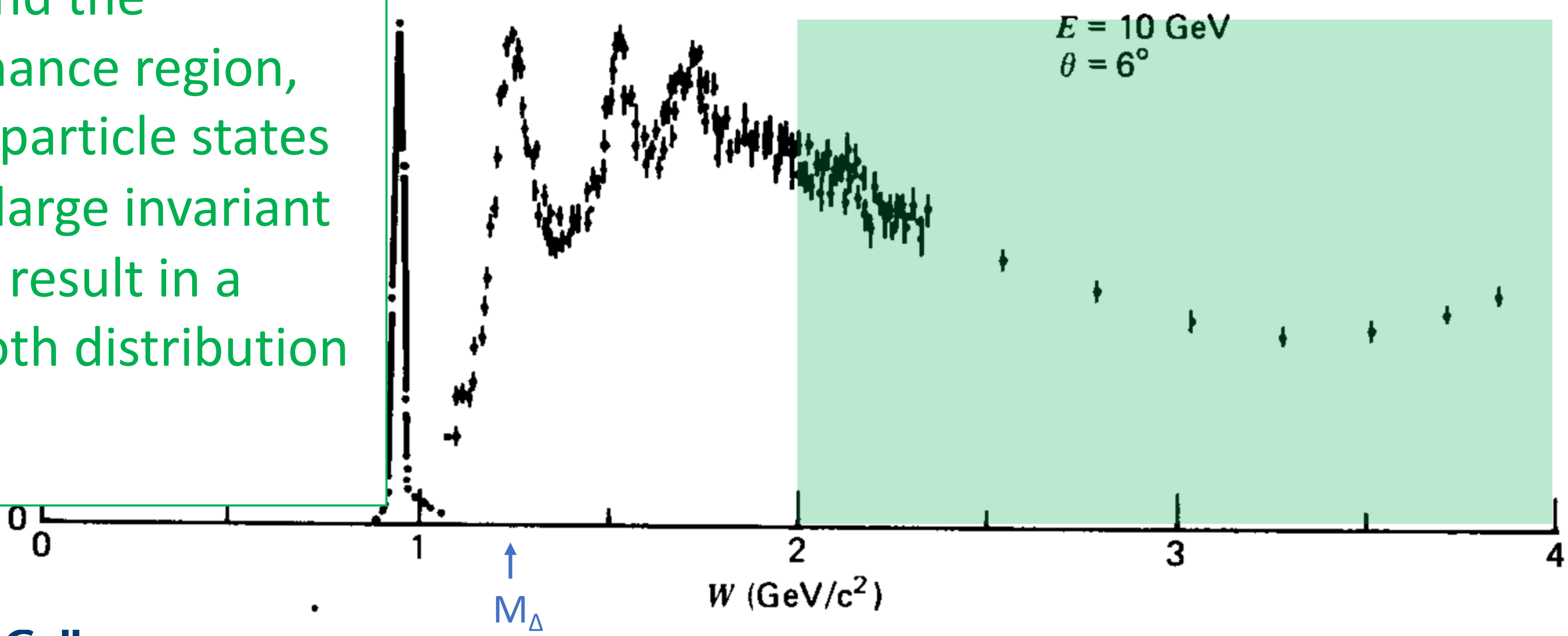
These bombs correspond to baryon resonances that decay to mesons, particularly single pions.



Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

Proton excitations $ep \rightarrow epX$

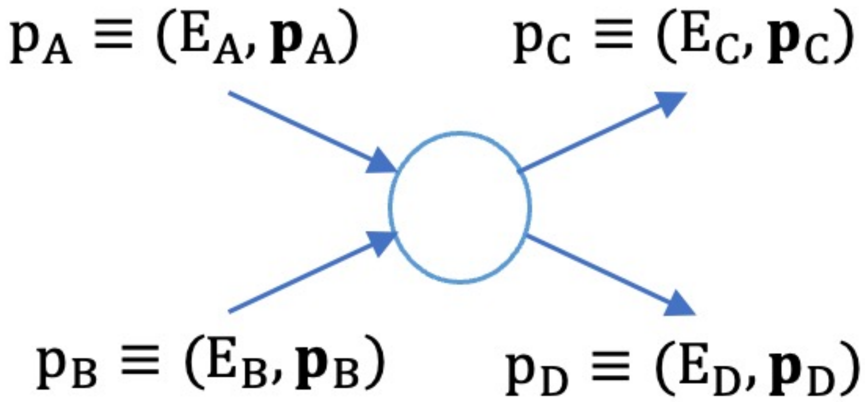
Beyond the resonance region, multiparticle states with large invariant mass result in a Smooth distribution in W



Differential cross section

- Differential cross section:

$$d\sigma = \frac{|\mathcal{M}|^2}{F} dQ$$

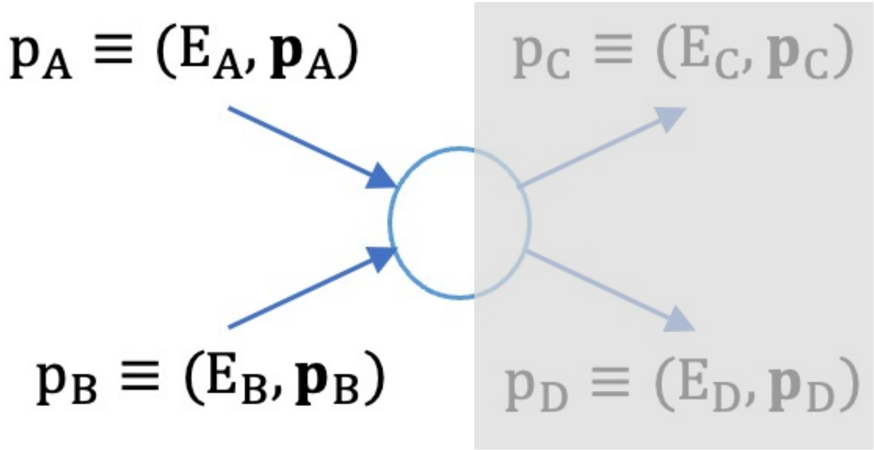


Differential cross section

- Differential cross section:

$$d\sigma = \frac{|\mathcal{M}|^2}{F} dQ$$

- F is Incident flux:

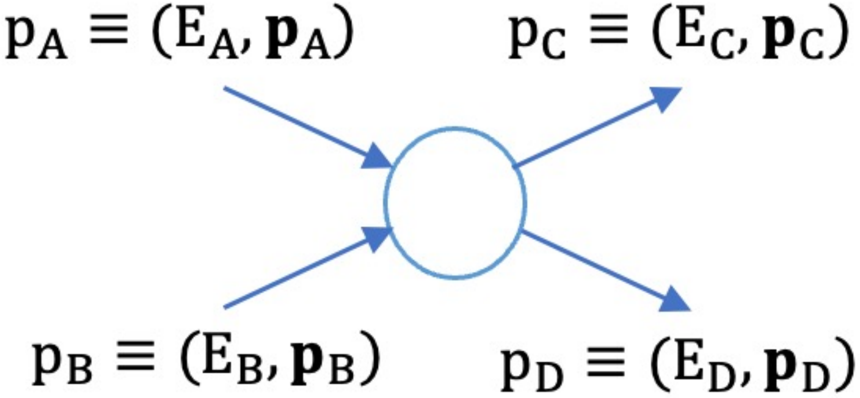


$$F = |\mathbf{v}_A - \mathbf{v}_B|.\, 2E_A.\, 2E_B$$
$$= 4(|\mathbf{p}_A|E_B + |\mathbf{p}_B|E_A)$$
$$= 4\left((p_A.\, p_B)^2 - m_A^2 m_B^2\right)^{1/2}$$ Lorentz invariant

Differential cross section

- Differential cross section:

$$d\sigma = \frac{|\mathcal{M}|^2}{F} dQ$$



- dQ is a Lorentz invariant phase space factor:

$$dQ = (2\pi)^4 \delta^4(p_C + p_D - p_A - p_B) \frac{d^3p_C}{(2\pi)^3 2E_C} \frac{d^3p_D}{(2\pi)^3 2E_D}$$

Phase space factor

$$d\sigma = \frac{|\mathcal{M}|^2}{F} dQ$$

- The general expression is:

$$dQ = (2\pi)^4 \delta^4 \left(\sum_f p_f - \sum_i p_i \right) \prod_f \frac{d^4 p_C}{(2\pi)^3} \delta(p_f^2 - m_f^2)$$

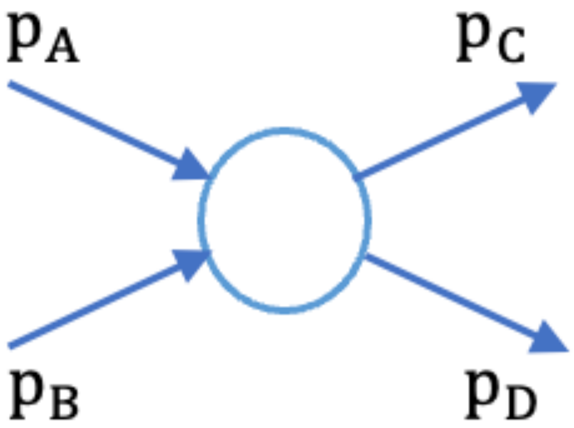
- For outgoing particles on their **mass sell** we can simplify the expression:

$$\frac{d^4 p_C}{(2\pi)^3} \delta\left(p_f^2 - m_f^2\right) = \frac{d^3 p_C}{(2\pi)^3 2p^0} = \frac{1}{(2\pi)^3} \frac{\sqrt{p0^2 - m^2}}{2} d\Omega dp^0$$

Counting degrees of freedom

Also see Raúl González Jiménez [Presentation]

- Elastic or quasilastic scattering:



$2\times$ 4-momentum	$\rightarrow$	8 variables
4-momentum conserv.	$\rightarrow$	$-$ 4 constraints
$2 \times (E^2 = M^2 + p^2)$	$\rightarrow$	-2 constaints
Independent variable left:		2

- You can choose your preferred (measurable) variables. Outgoing lepton kinemtic

Exercise 1:

In the center-of –mass frame for the process $AB \rightarrow CD$ show:

$$dQ = \frac{1}{4\pi^2}\frac{p_f}{4\sqrt{s}}d\Omega$$
$$F = 4p_i\sqrt{s}$$

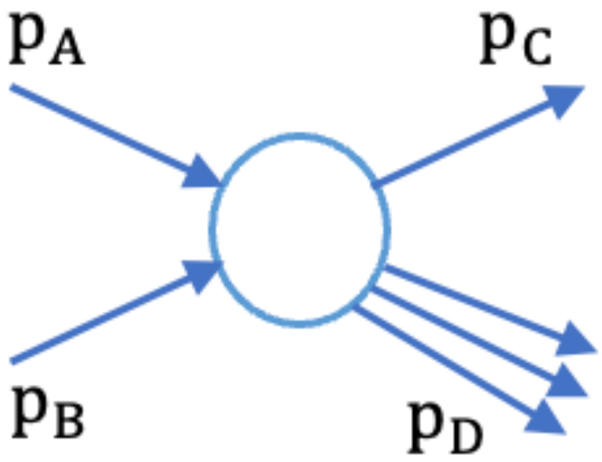
and therefore:

$$\frac{d\sigma}{d\Omega}\Big|_{cm} = \frac{1}{64\pi^2 s}\frac{P_f}{p_i}|\mathcal{M}|^2$$

where $d\Omega$ is the element of solid angle about $\mathbf{p}_C$, $s = (E_A + E_B)^2$,
$|\mathbf{p}_A| = |\mathbf{p}_B| = p_i$, $|\mathbf{p}_C| = |\mathbf{p}_D| = p_f$,

Counting degrees of freedom

- Inclusive scattering: we don't know about individual hadrons



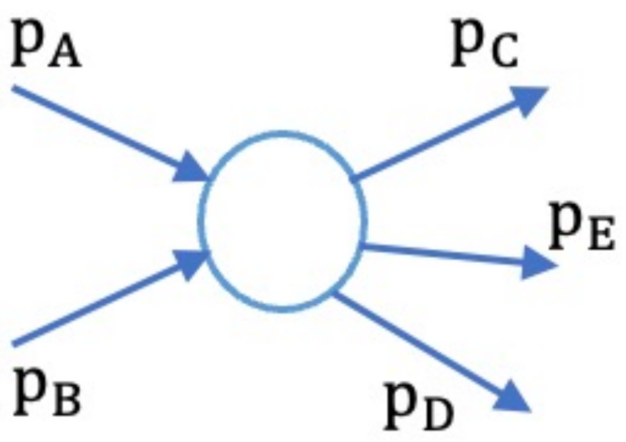
$2\times$ 4-momentum	$\rightarrow$	8 variables
4-momentum conserv.	$\rightarrow$	$-$ 4 constraints
$1 \times (E^2 = M^2 + p^2)$	$\rightarrow$	-1 constaints

Independent variable left: 3

- You can choose your preferred (measurable) variables. Outgoing lepton kinemtic

Counting degrees of freedom

- Single meson production on a free nucleon: $\nu_\mu p \rightarrow \mu p \pi^+$
- The event includes one lepton, one pion and one nucleon

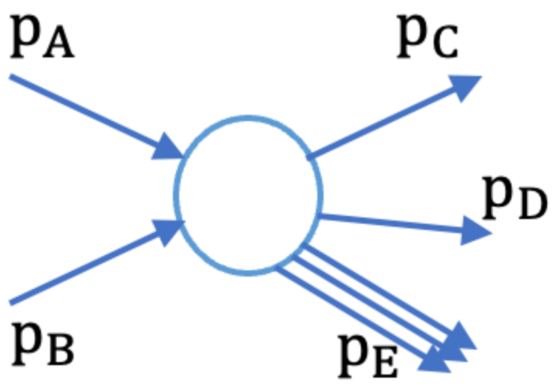


$3\times$ 4-momentum	$\rightarrow$	12 variables
4-momentum conserv.	$\rightarrow$	$-$ 4 constraints
$3 \times (\mathrm{E}^2 = \mathrm{M}^2 + \mathrm{p}^2)$	$\rightarrow$	-3 constaints

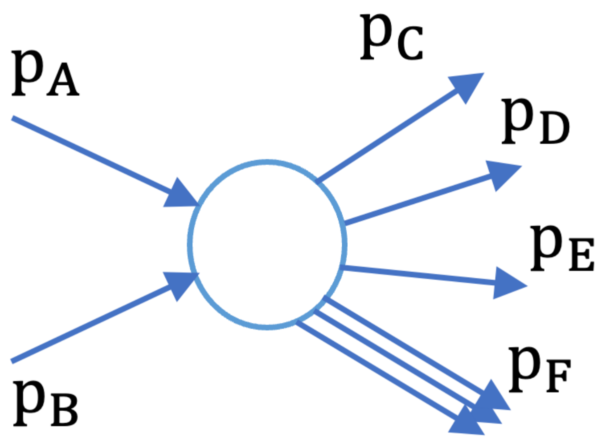
Independent variable left: 5

Exercise 2: Counting degrees of freedom

- one lepton and at least one hadron are detected. we don't know about the rest of hadron.



- one lepton and at least two hadron are detected. we don't know about the rest of hadron. Example 2-nucleon knockout or 1-pion production



Answer 2:

$3\times$ 4-momentum	$\rightarrow$	12 variables
4-momentum conserv.	$\rightarrow$	$-$ 4 constraints
$2\times (E^2 = M^2 + p^2)$	$\rightarrow$	$-$ 2 constaints

Independent variable left: 6

$4\times$ 4-momentum	$\rightarrow$	16 variables
4-momentum conserv.	$\rightarrow$	$-$ 4 constraints
$3\times (E^2 = M^2 + p^2)$	$\rightarrow$	$-$ 3 constaints

Independent variable left: 9

Differential cross section

- We may write the differential cross section in a symbolic form:

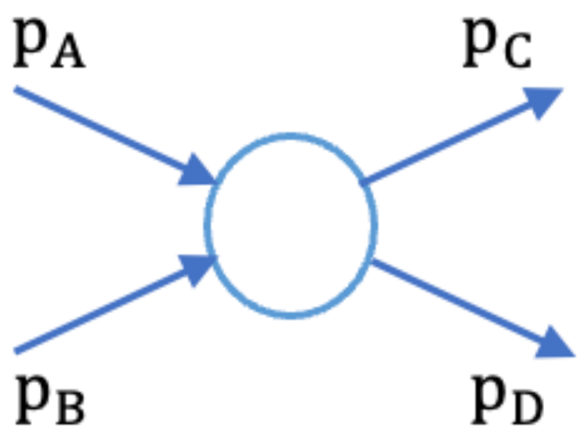
$$d\sigma = \frac{|\mathcal{M}|^2}{F} dQ$$

- F and dQ are only related to the kinematics and are not related to the dynamical models.
- F is related to the incident particles and not related to the interaction types, I.e. elastic and inelastic scatterings.
- dQ is related to the outgoing particles and related to the interaction types.

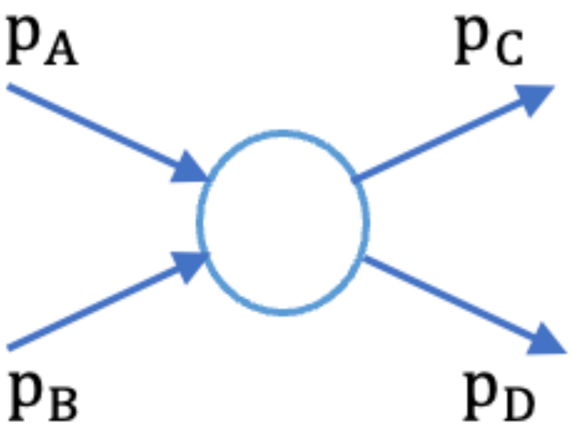
Invariant (Mandelstam) variable, s, t, u

- Possible invariant variables out of 4 four-momentum are: $p_A . p_B$, $p_A . p_C$, $p_A . p_D$
- Due to $p_A + p_B = p_C + p_D$, only two of them are independent.
- It is conventional to use the related Mandelstam variable:

$$s = (p_A + p_B)^2,$$
$$t = (p_A - p_C)^2,$$
$$u = (p_A - p_D)^2.$$



Exercise 3



1. Show that:

$$s + t + u = m_A^2 + m_B^2 + m_c^2 + m_D^2$$

Where m_i is the rest mass of particle i

2. If $e^- e^- \rightarrow e^- e^-$ is in the s channel process, $A + B \rightarrow C + D$, verify that

$$s = 4\left(k^2 + m^2\right)$$
$$t = -2k^2(1 - \cos\theta)$$
$$u = -2k^2(1 + \cos\theta)$$

Where θ is the center-of-mass scattering angle and $k = |\mathbf{k}_i| = |\mathbf{k}_f|$. $\mathbf{k}_i$ and $\mathbf{k}_f$ are the momentum of incident and scattered electron.

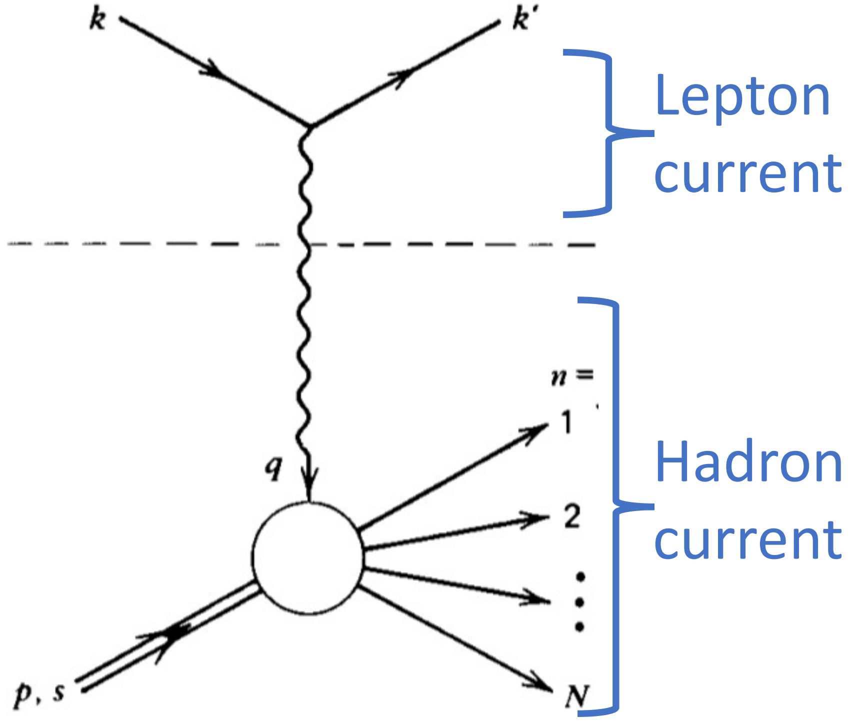
Invariant amplitude

$$d\sigma = \frac{|\mathcal{M}|^2}{F} dQ$$

- $|\mathcal{M}|^2 \propto L_{\mu\nu} H^{\mu\nu}$
- Lepton tenson: $L_{\mu\nu} = \sum\sum j_\mu^\dagger j_\nu$
- Hadron tensor: $H_{\mu\nu} = \sum\sum J_\mu^\dagger J_\nu$
- Lepton current: $j^\mu = \bar{u}(k')\gamma^\mu(1 - a\gamma^5)u(k)$

$$a = \begin{cases} 1, & \text{neutrino} \\ 0, & \text{EM} \\ -1, & \text{antineutrino} \end{cases}$$

- Hadron current $J^\mu = ?$



Lepton tensor

- Lepton current: $j^{\mu} = \bar{u}(k')\gamma^{\mu}(1 - a\gamma^{5})u(k)$ $\quad a = \begin{cases} 1, & \text{neutrino} \\ 0, & \text{EM} \\ -1, & \text{antineutrino} \end{cases}$
- Lepton tensor: $L_{\mu\nu} = \frac{1+|a|}{2} \sum_{s_i} \sum_{s_f} j_{\mu}^{\dagger} j_{\nu}$

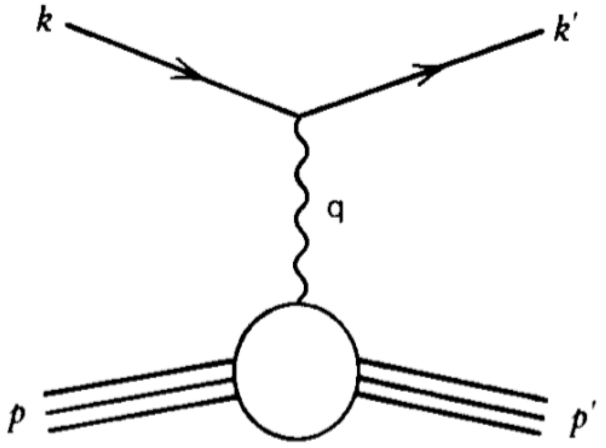
For NC neutrinos : $L_{\mu\nu} = 4(k_{\mu}k'_{\nu} + k_{\nu}k'_{\mu} - g_{\mu\nu}k.k' - i\epsilon_{\mu\nu\alpha\beta}k^{\alpha}k'^{\beta})$

- There is no averaging over (anti-)neutrino helicities since they are (right) left-handed
- Lepton tensor is known!

The Structure of nucleon- Form factors

- Experiments to study the interaction of quarks and gluons are performed with hadrons.
- We need to describe "wavefunctions" that describe nucleon in terms of its constituent quarks and gluons.
- Form-factors parametrize our ignorance of the detailed structure of nucleon represented by a blob.
- These form factors can be determines experimentally by measuring:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{point} |F(q)|^2$$



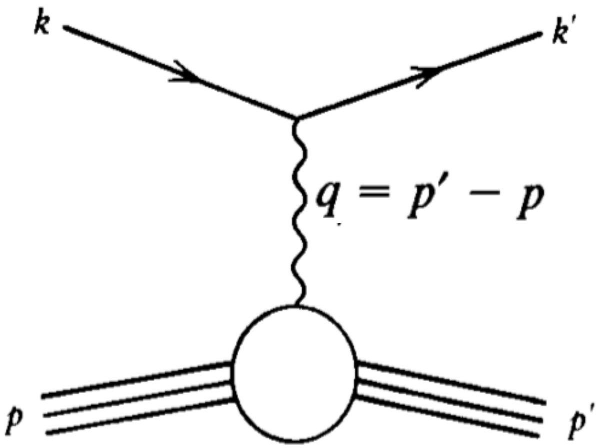
Lowest-order lepton-proton (quasi)elastic scattering

Quasielastic neutrino scattering

- Lepton and proton transition currents:

$$j_L^\mu = \bar{u}(k')\gamma^\mu(1-\gamma^5)u(k)$$

$$J^\mu = \bar{u}(p')\Gamma^\mu u(p) = \bar{u}(p')(\mathcal{V}^\mu - \mathcal{A}^\mu)u(p)$$



- The most general form of J^μ, consistence with Lorentz covariance, constructed from bilinear covariance, as well as p, p’ and q.

Hadron current for Quasielastic scattering

- Hadron current: $J^{\mu} = \bar{u}(p')(\mathcal{V}^{\mu} - \mathcal{A}^{\mu})u(p)$

$$\mathcal{V}^{\mu} = \gamma^{\mu}\mathcal{F}_1 + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}\mathcal{F}_2 + \frac{q^{\mu}}{M}\mathcal{F}_3$$

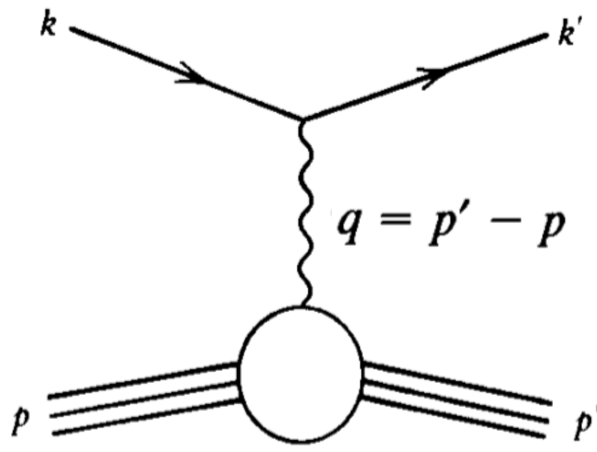
$$\mathcal{A}^{\mu} = \gamma^{\mu}\gamma_5\mathcal{G}_1 + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}\gamma_5\mathcal{G}_2 + \frac{q^{\mu}}{M}\gamma_5\mathcal{G}_3$$

- q^2 is the only independent scalar variable at the proton vertex
- Form factors are only a function of $Q^2 = -q^2$

$$\mathcal{F}_i = \mathcal{F}_i(Q^2), \quad \mathcal{G}_i = \mathcal{G}_i(Q^2)$$

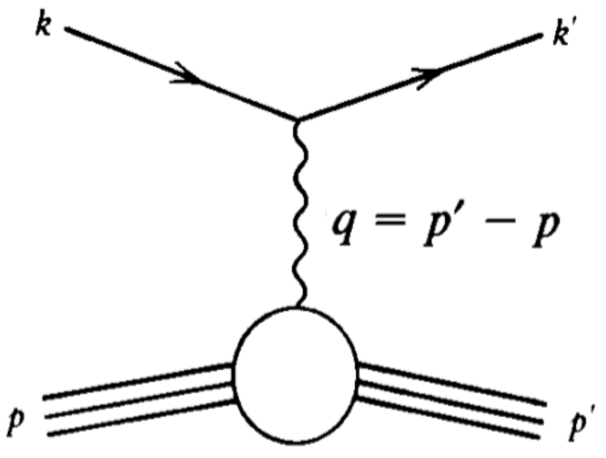
Exercise 4

- Show that p.q is not an independent scalar at the proton vertex by expressing it in terms of the variable q^2
- Hint: use the four-momentum conservation at the proton vertex.



Answer 4

- Hint: use the four-momentum conservation at the proton vertex.



$$(q + p)^2 = p'^2$$

$$q^2 + 2q.p + p^2 = p'^2$$
$$q^2 + 2p.q = 0$$

Hadron current for Quasielastic scattering

- Hadron current: $J^{\mu} = \bar{u}(p')(\mathcal{V}^{\mu} - \mathcal{A}^{\mu})u(p)$

$$\mathcal{V}^{\mu} = \gamma^{\mu}\mathcal{F}_1 + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}\mathcal{F}_2 + \frac{q^{\mu}}{M}\mathcal{F}_3$$

$$\mathcal{A}^{\mu} = \gamma^{\mu}\gamma_5\mathcal{G}_1 + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}\gamma_5\mathcal{G}_2 + \frac{q^{\mu}}{M}\gamma_5\mathcal{G}_3$$

- These bilinear covariant have certain definite properties under discrete transformation like C, P and T as well as the internal symmetries like the isospin and unitary symmetry.
- We can use symmetry properties to limit the number of independent form factors.

Symmetry properties

- Time reversal invariance holds → the form factors must be real
- G parity ($G = Ce^{i\pi I_Y}$) invariance → $\mathcal{F}_3(Q^2) = \mathcal{G}2(Q^2) = 0$
- Conserved vector current hypothesis: $\partial_\mu \mathcal{V}^\mu = 0 \rightarrow \mathcal{F}_3(Q^2) = 0$
- Partial conservation of axial current: $\lim_{m_\pi \rightarrow 0} \partial_\mu \mathcal{A}^\mu = 0$

Goldberger–Treiman relation: $\mathcal{G}_3(Q^2) = \frac{2M^2 \mathcal{G}_1(Q^2)}{m_\pi^2 + Q^2}$

- We end up having 3 independent real form factors:

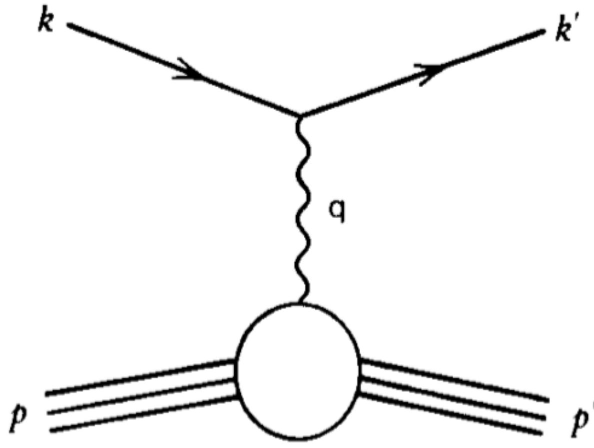
$$\mathcal{F}_1(Q^2), \mathcal{F}_2(Q^2), \mathcal{G}_1(Q^2)$$

Hadron current for Quasielastic scattering

- Hadron current: $J^{\mu}_{QE}(p, p') = \bar{u}(p')(\mathcal{V}^{\mu} - \mathcal{A}^{\mu})u(p)$

$$\mathcal{V}^{\mu} = \gamma^{\mu}F_1^V(Q^2) + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}F_2^V(Q^2)$$

$$\mathcal{A}^{\mu} = (\gamma^{\mu} + \frac{q^{\mu}}{M}\frac{2M^2}{m_{\pi}^2+Q^2})\gamma_5 G^A(Q^2)$$



- The Hadronic tensor for CCQE scattering, $\nu_l N \to l N'$, is determined by hadronic current.

$$H^{\mu\nu}_{QE} = \frac{1}{2}\sum_{s_i}\sum_{s_f} J^{\mu\dagger}_{QE}(p, p') J^{\nu}_{QE}(p, p')$$

Electromagnetic current

- EM current: $J_\mu^{em} = \bar{u}(p')V_\mu^{em}u(p)$

$$V_\mu^{em}(p,n) = \gamma_\mu F_1^{p,n}(Q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{(2M)}F_2^{p,n}(Q^2)$$

- Isospin property: define a Isospinor $u = \begin{pmatrix} u_p \\ u_n \end{pmatrix}$ under isospin transformation

$$\bar{u}_p V_\mu^{em} u_p = \bar{u}V_\mu^{em}\frac{\mathbb{I}+\tau_3}{2}u\,, \quad \bar{u}_n V_\mu^{em} u_n = \bar{u}V_\mu^{em}\frac{\mathbb{I}-\tau_3}{2}u$$

Implying the isoscalar and isovector current:

$$\bar{u}\mathbb{I}V_\mu^{em}u = \bar{u}_p V_\mu^{em}u_p + \bar{u}_n V_\mu^{em}u_n,$$
$$\bar{u}\tau_3 V_\mu^{em}u = \bar{u}_p V_\mu^{em}u_p - \bar{u}_n V_\mu^{em}u_n$$

Exercise 5: Isospin symmetry

- Show that charged weak current is purely isovector current.

Hint: show $\bar{\mathrm{u}}_{\mathrm{p}}\mathcal{V}_{\mu}^{CC}\mathrm{u}_{\mathrm{n}} = \bar{\mathrm{u}}\mathcal{V}_{\mu}^{\mathrm{CC}}\tau^{+}\mathrm{u}$ and $\bar{\mathrm{u}}_{\mathrm{n}}\mathcal{V}_{\mu}^{CC}\mathrm{u}_{\mathrm{p}} = \bar{\mathrm{u}}\mathcal{V}_{\mu}^{\mathrm{CC}}\tau^{-}\mathrm{u}$

Where $\tau^{\pm} = \frac{\tau_1 \pm i\tau_2}{2}$, are isospin raising and lowering operator.

Exercise 6: Isospin symmetry

- If we parametrize the isoscalar (with $F^S_{1,2}$ form factors) and isovector (with $F^V_{1,2}$ form factors) current

$$\bar{\mathrm{u}}\tau_3 \mathrm{V}^{\mathrm{em}}_{\mu}\mathrm{u} = \bar{\mathrm{u}}\left[\gamma^{\mu}\mathcal{F}_1(Q^2) + i\sigma_{\mu\nu}\frac{q^{\nu}}{(2M)}\mathcal{F}_2(Q^2)\right]\tau_3 \mathrm{u}$$

$$\bar{\mathrm{u}}\mathbb{I}\mathrm{V}^{\mathrm{em}}_{\mu}\mathrm{u} = \bar{\mathrm{u}}\left[\gamma^{\mu}\mathrm{F}^{\mathrm{S}}_1(Q^2) + i\sigma_{\mu\nu}\frac{q^{\nu}}{(2M)}F^S_2(Q^2)\right]\mathrm{u}$$

Show:

$$\mathrm{F}^{\mathrm{V}}_{1,2}(\mathrm{Q}^2) = \mathrm{F}^{\mathrm{p}}_{1,2}(\mathrm{Q}^2) - \mathrm{F}^{\mathrm{n}}_{1,2}(\mathrm{Q}^2), \quad \mathrm{F}^{\mathrm{S}}_{1,2}(\mathrm{Q}^2) = \mathrm{F}^{\mathrm{p}}_{1,2}(\mathrm{Q}^2) + \mathrm{F}^{\mathrm{n}}_{1,2}(\mathrm{Q}^2)$$

Exercise7: Isospin symmetry

Show the vector form factors for Neutral currents are:

$$\tilde{f}^{p}_{1,2}(Q^2) = (\frac{1}{2} - 2\sin^2\theta_W)\, F^{p}_{1,2}(Q^2) - \frac{1}{2} F^{n}_{1,2}(Q^2)$$

$$\tilde{f}^{n}_{1,2}(Q^2) = (\frac{1}{2} - 2\sin^2\theta_W)\, F^{n}_{1,2}(Q^2) - \frac{1}{2} F^{p}_{1,2}(Q^2)$$

where we ignored the strangeness vector form factor

Isospin relation for vector form factor

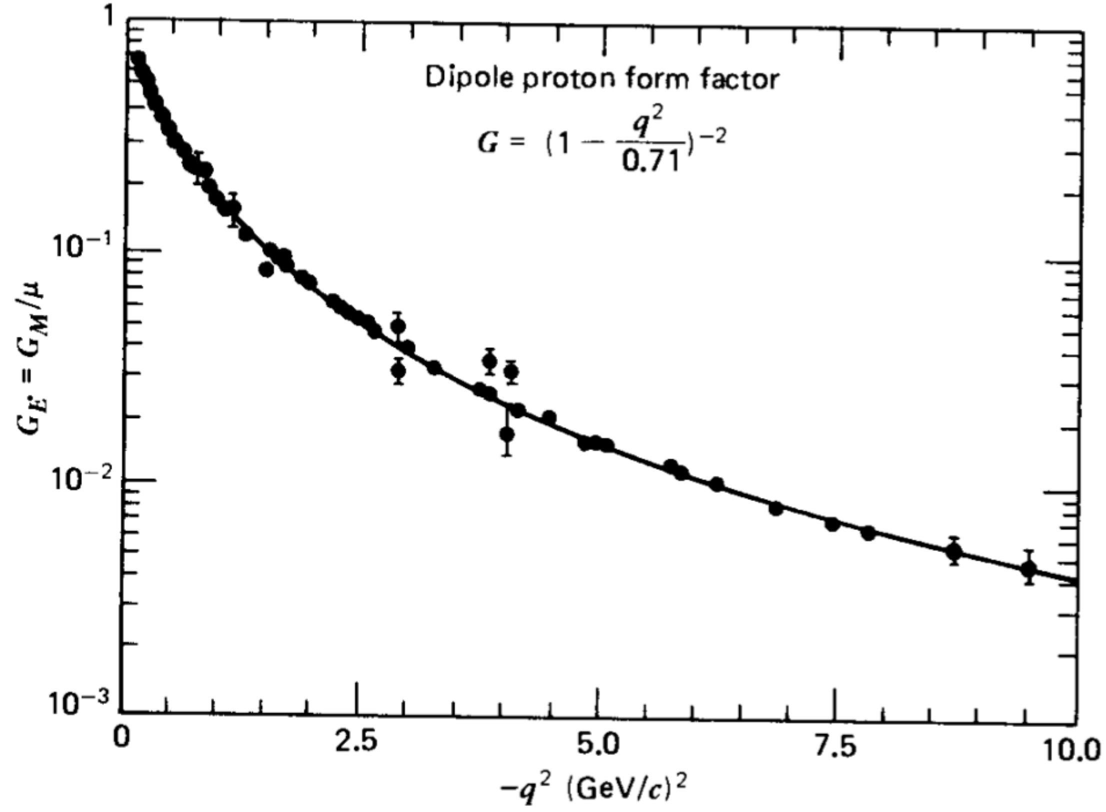
- If we know proton and neutron form factors, we know vector CC and NC form factors.

reaction	replace $\mathcal{F}_i$ in Eq. (4.7) with
$\ell^- p \to \ell^- p$	F_i^p
$\ell^- n \to \ell^- n$	F_i^n
$\nu n \to \ell^- p$	$F_i^V = F_i^p - F_i^n$
$\nu p \to \nu p$	$\tilde{F}_i^p = (\frac{1}{2} - 2\sin^2\theta_W) F_i^p - \frac{1}{2} F_i^n - \frac{1}{2} F_i^s$
$\nu n \to \nu n$	$\tilde{F}_i^n = (\frac{1}{2} - 2\sin^2\theta_W) F_i^n - \frac{1}{2} F_i^p - \frac{1}{2} F_i^s$

Dipole Form Factors

- Electromagnetic data show a dipole form for proton factor factors.
- Sachs form factors:

$$G_E(Q^2) = F_1^{em}(Q^2) - \frac{Q^2}{2M}\mathrm{F}_2^{\mathrm{em}}(\mathrm{Q}^2)$$
$$G_M(Q^2) = F_1^{em}(Q^2) + F_2^{em}(Q^2)$$



Charge distribution and Form facctor

- From a **static target** form factor is a Furier transform of the charge distribution

$$F(q^2) = \int \rho(x) e^{iq.x} d^3x$$

- If the charge distribution has an exponential form $\rho(r) = e^{-mr}$, the form factor has a dipole form:

$$F(q^2) = \left(1 - \frac{q^2}{m^2}\right)^{-2}$$

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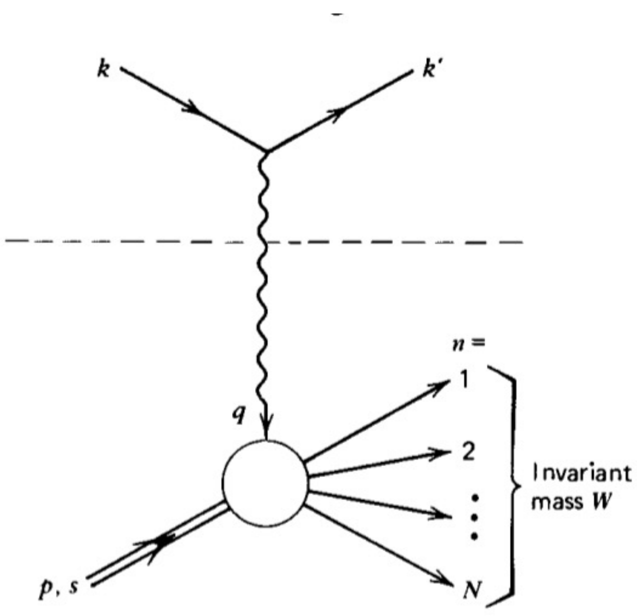
Inelastic scattering

- By Increasing the $Q^2= -q^2$ we can take a detailed look at the structure of nucleon.
- This can be done by requiring a large energy loss of the bombarding lepton.
- At very large transfer of energy, proton with break up and looses its identity.
- At modest Q^2 proton get excited to baryon resonances such as Δ resonance. The excited state promptly decay to a nucleon and mesons.



Inelastic scattering

- The main challenge now is that we have more than one hadron in the final states and it is not straightforward to calculate the hadron current. Therefore, J^{μ} has a more complex structure than Elastic scattering

Elastic

$$d\sigma \propto L^{e}_{\mu\nu}\, H^{\mu\nu}$$

Lepton tenson $L^{e}_{\mu\nu} = \sum\sum j^{\dagger}_{\mu}\, j_{\nu}$

Hadron tensor $H_{\mu\nu} = \sum\sum J^{\dagger}_{\mu}\, J_{\nu}$

Hadron current $J^{\mu} \propto \bar{u}\Gamma^{\mu}u$

Inelastic

$$d\sigma \propto L^{e}_{\mu\nu}\, W^{\mu\nu}$$

Hadron tensor $W_{\mu\nu} = \sum\sum {J_{\mu}}^{\dagger}\, J_{\nu}$

There are more than single hadron in the final state and the current is complicated.

Inelastic scattering

- The hadronic tensor W serves to parametrise our total ignorance of the form of inelastic hadron current.
- The most general form of the tensor W must be constructed from $g^{\mu\nu}$ and independent momenta p and q.

$$\mathcal{W}^{\mu\nu} = \frac{1}{2m_N} \sum \langle p|J^\mu(0)|\Delta\rangle\langle\Delta|J^\nu(0)|p\rangle\delta(W^2 - M_R^2)$$
$$= -\mathcal{W}_1 g^{\mu\nu} + \frac{\mathcal{W}_2}{m_N^2} p^\mu p^\nu - i\varepsilon^{\mu\nu\sigma\lambda} p_\sigma q_\lambda \frac{\mathcal{W}_3}{2m_N^2}$$
$$+ \frac{\mathcal{W}_4}{m_N^2} q^\mu q^\nu + \frac{\mathcal{W}_5}{m_N^2}(p^\mu q^\nu + q^\mu p^\nu) + i\frac{\mathcal{W}_6}{m_N^2}(p^\mu q^\nu - q^\mu p^\nu)$$

γ^μ is not included as we are parametrizing the cross section which is already summed and averaged over spins.

Inelastic scattering

- W_i s are function of the Lorentz scalar variables that can be constructed from four-momenta at the hadronic vertex.
- Unlike elastic scattering, there are two independent variables.

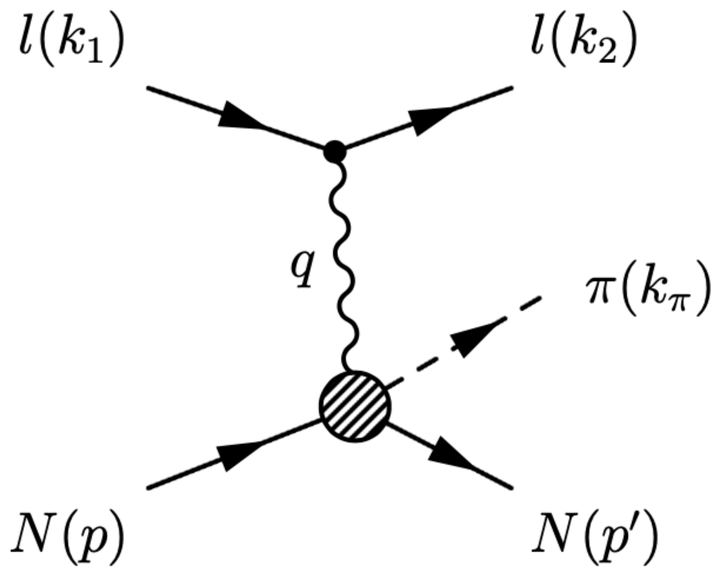
$$\mathcal{W}^{\mu\nu} = \frac{1}{2m_N} \sum \langle p|J^\mu(0)|\Delta\rangle\langle\Delta|J^\nu(0)|p\rangle \delta(W^2 - M_R^2)$$
$$= -\mathcal{W}_1 g^{\mu\nu} + \frac{\mathcal{W}_2}{m_N^2} p^\mu p^\nu - i\varepsilon^{\mu\nu\sigma\lambda} p_\sigma q_\lambda \frac{\mathcal{W}_3}{2m_N^2}$$
$$+ \frac{\mathcal{W}_4}{m_N^2} q^\mu q^\nu + \frac{\mathcal{W}_5}{m_N^2}(p^\mu q^\nu + q^\mu p^\nu) + i\frac{\mathcal{W}_6}{m_N^2}(p^\mu q^\nu - q^\mu p^\nu)$$

Exercise 8

Unlike elastic scattering, there are two independent scalar variables in inelastic scattering. Show that p.q is an independent variable in single pion production:

Hint: use four-momentum conservation relation:

$$q + p = p' + k_\pi$$



Answer 8

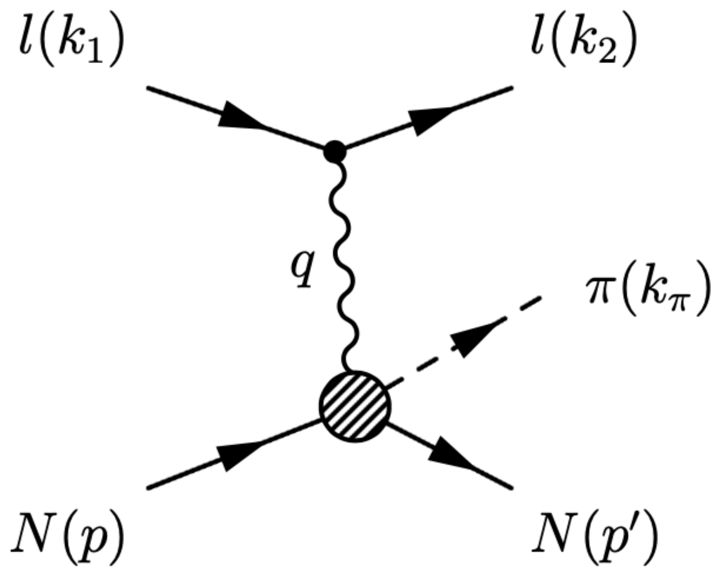
Four-momentum conservation:

$$(q+p)^2 = (p' + k_\pi)^2$$
$$q^2 + 2p.q + p^2 = (p' + k_\pi)^2$$

Introducing:

$$W^2 = (p' + k_\pi)^2 = (p+q)^2$$

Instead of the nucleon mass we have invariant hadrc mass (W) which had similar definition in the hadron rest frame: $W = p^0 + k_\pi^0$



Scalar variables for Hadron tensor in IE

- You can choose different sets of variables:
  1. $Q^2 = -q^2 = -(k_1 - k_2)^2, \quad \nu = \frac{p.q}{M} = q_L^0$ in Lab frame
  2. Lorentz invariant W, Q^2:

$$Q^2, \qquad W = p + q = p' + k_\pi$$

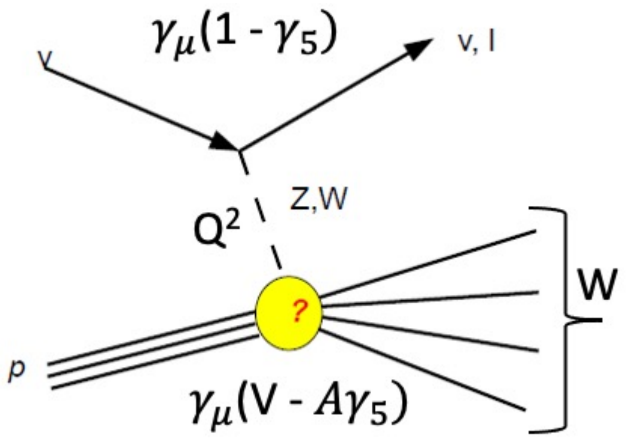
Where $W^2 = (p+q)^2 = M^2 + 2M\nu + q^2$

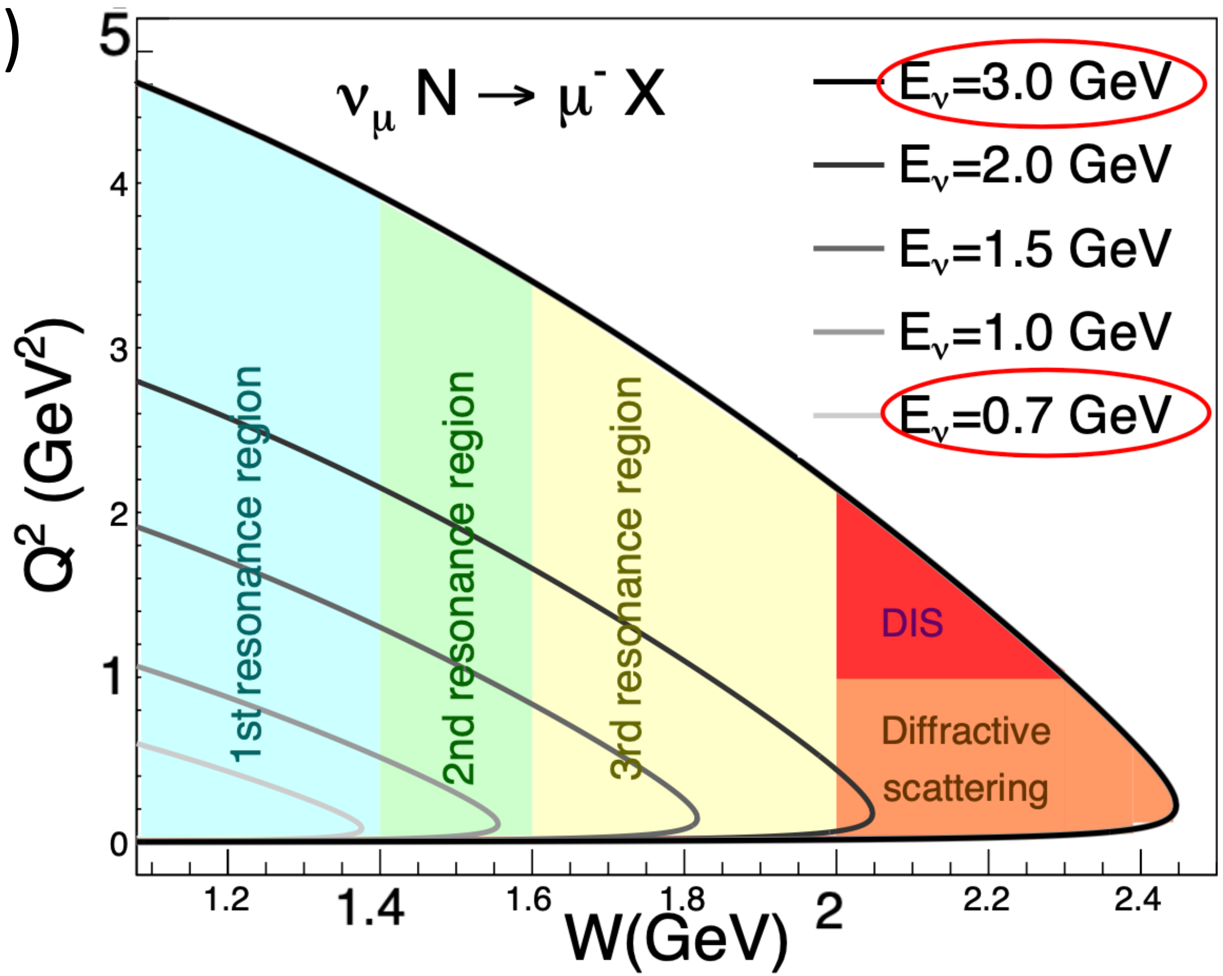
  3. dimensionless variables:

$$x = \frac{-q^2}{2p.q} = \frac{-q^2}{2M\nu}, \qquad y = \frac{p.q}{p.k_\pi}$$

Kinematic region for meson production

- Free nucleon (Hydrogen)





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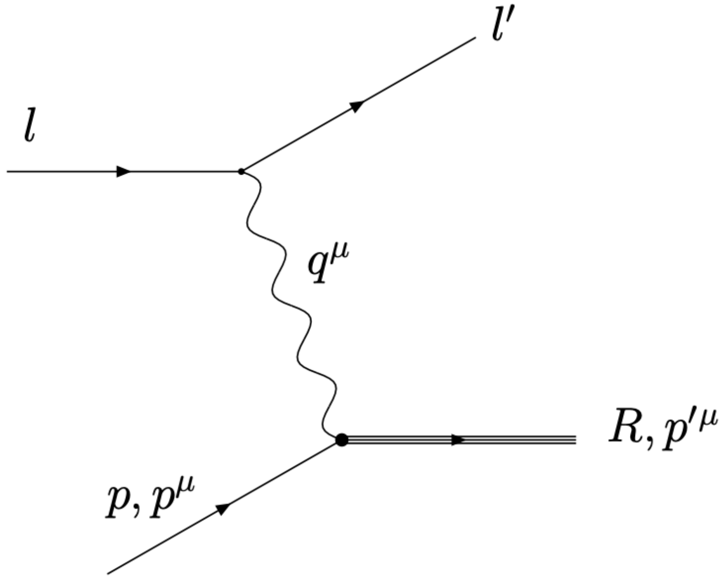
Resonance production

- $\nu_l N \rightarrow lR$
- Resonance mass: M_R
- Resonance width: $\Gamma 0$
- $R = l_{2I,2J}(M_R)$,

$$S: l = 0,$$
$$P: l = 1,$$
$$\text{etc}$$

- Parity: $P = -(-1)^l$



Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

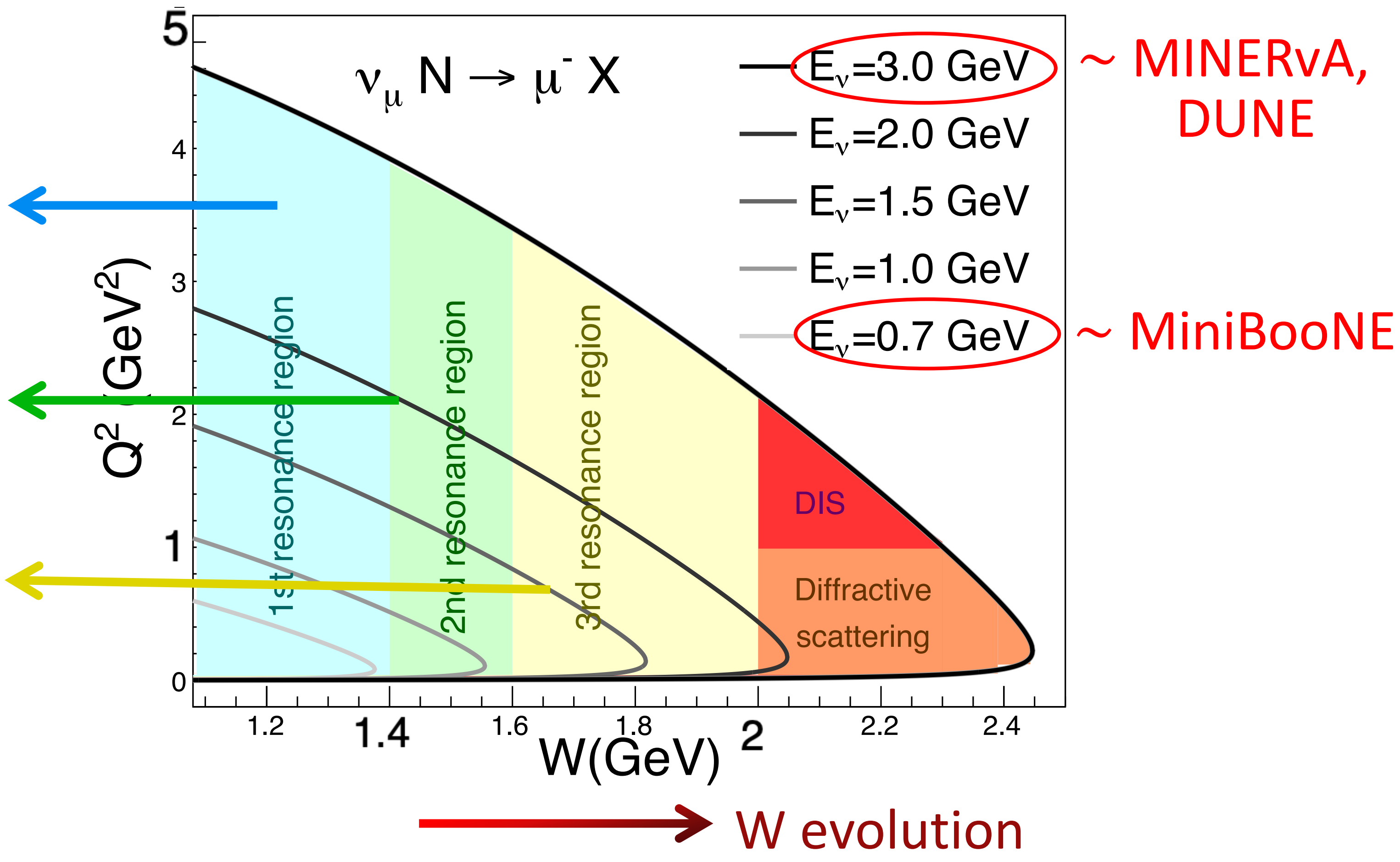
Meson production in ν-Nucleon interaction

Resonant interaction

- $\Delta(1232)$ resonance
- 1-π production

- $P_{11}(1440), D_{13}(1520)$ and $S_{11}(1535)$

- 2-π, η, etc. production
- About 13 resonances overlap



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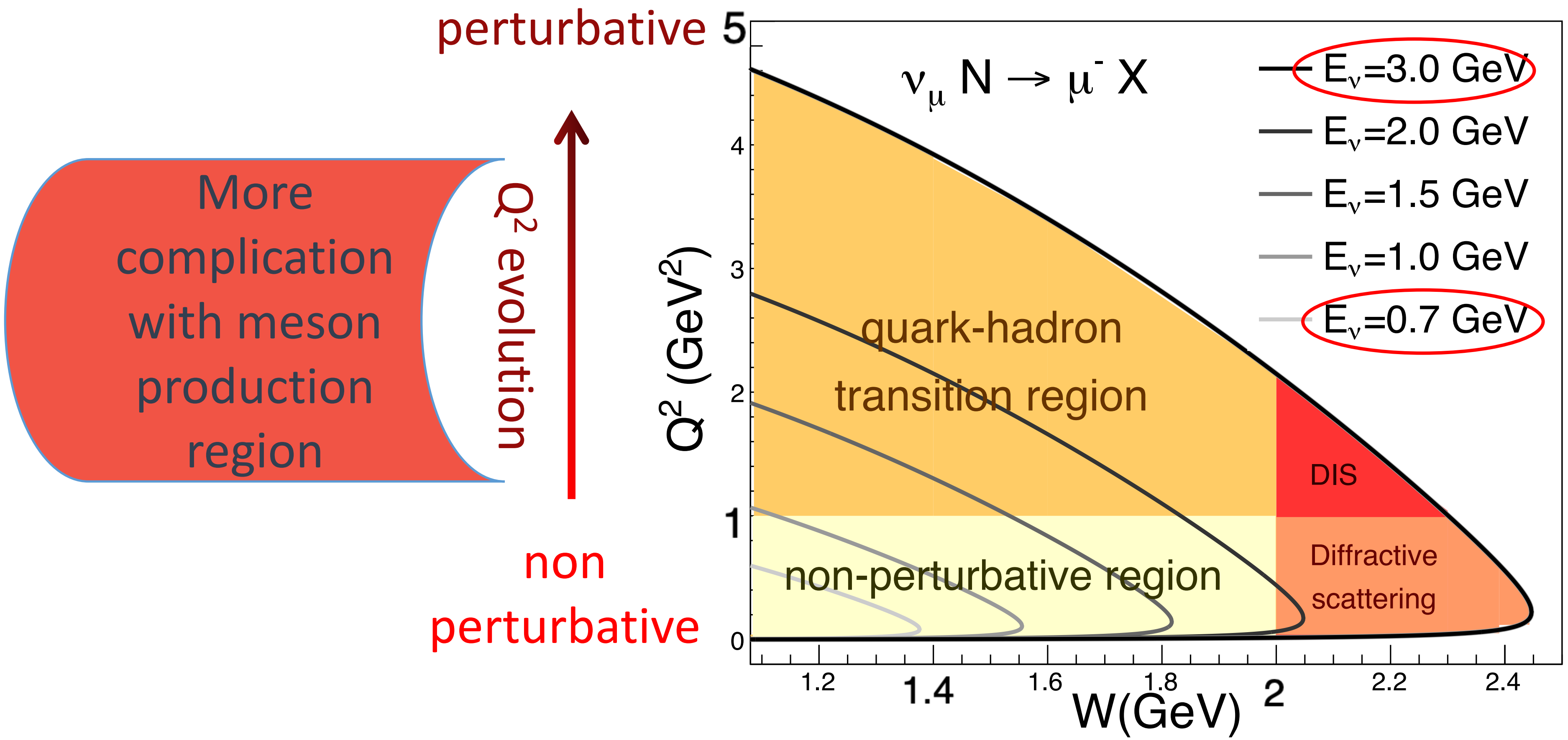
Meson production in ν-Nucleon interaction



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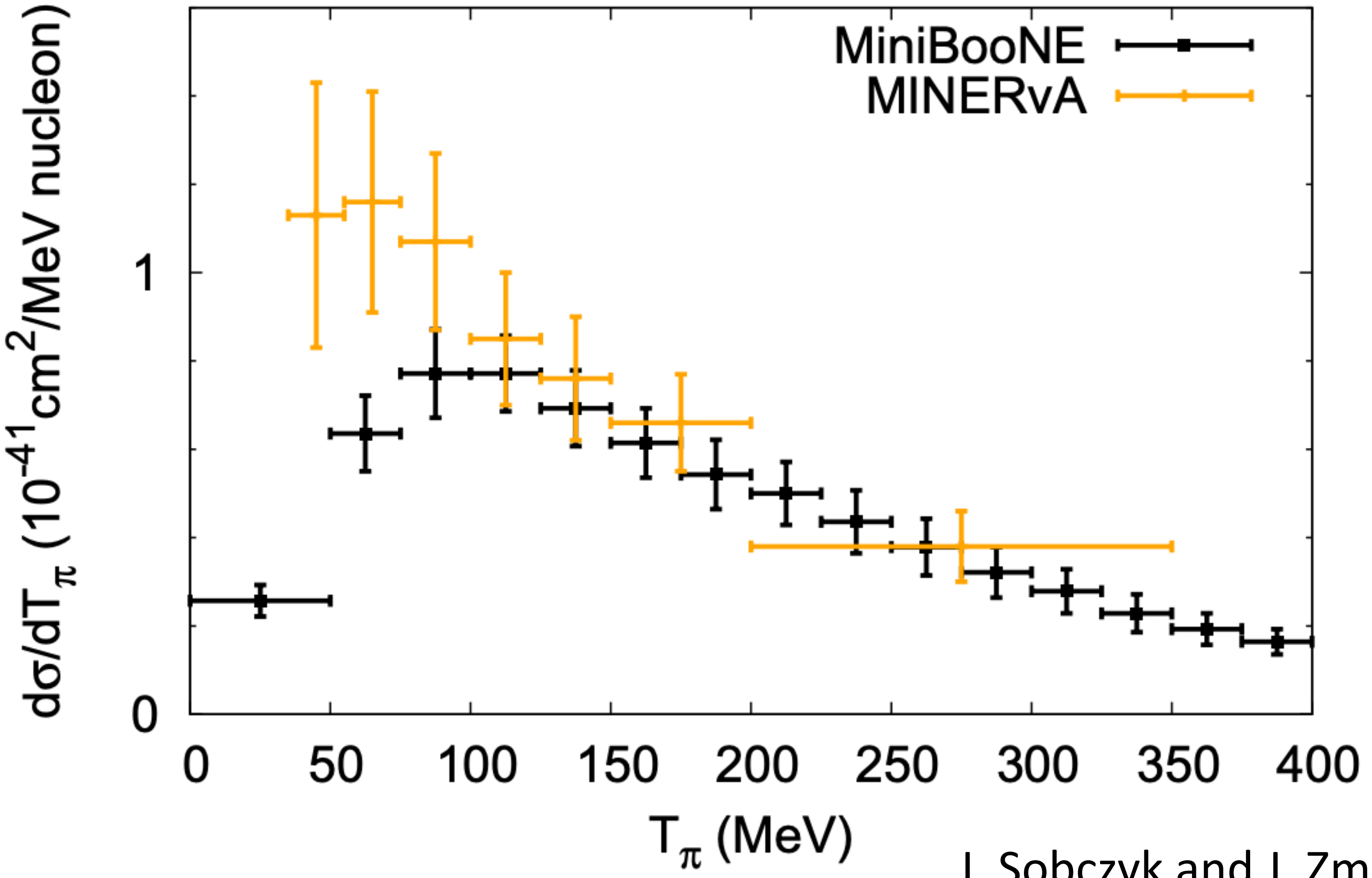
Non-perturbative region

Tensions between **MiniBooNE** and **MINERvA**

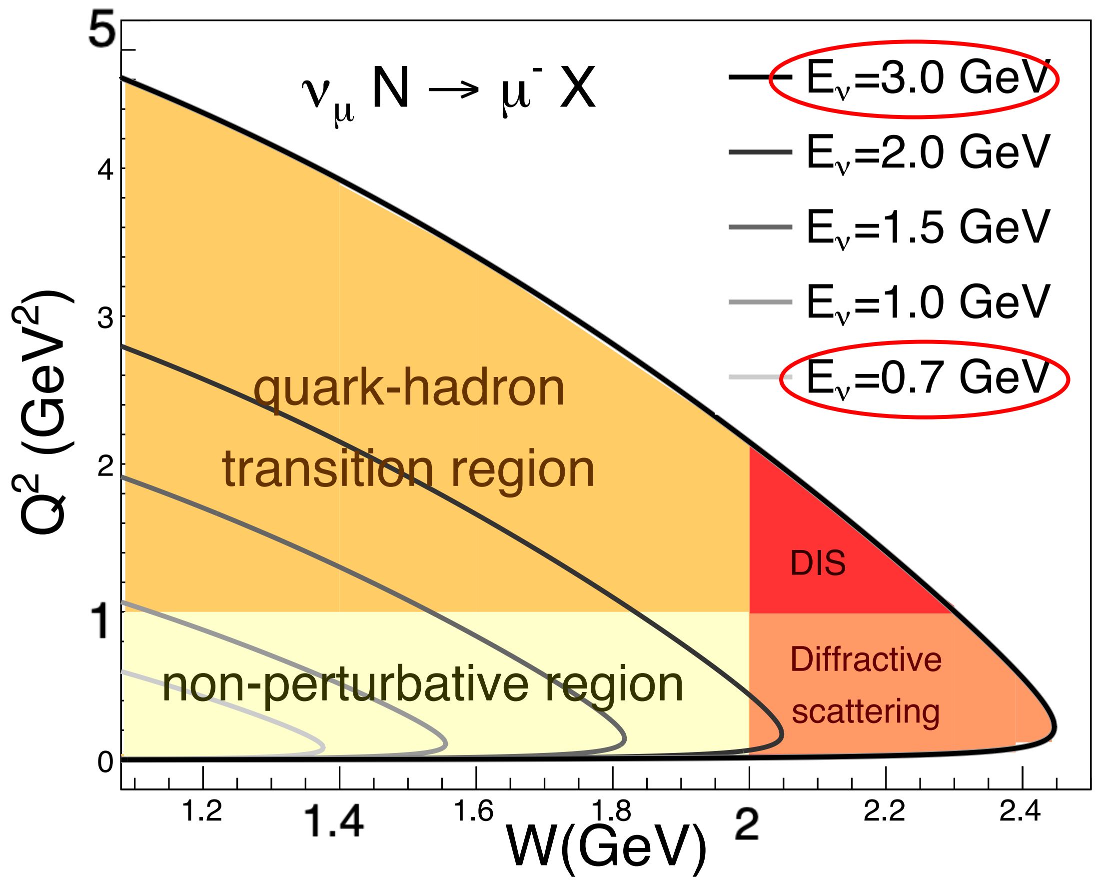
- Tensions between MiniBooNE and MINERvA for single pion production measurements on CH_2 and CH targets in the **first resonance region**.



J. Sobczyk and J. Zmuda
Phys. Rev. C **91** (2015)

Meson production in ν-Nucleon interaction

- Degree of freedom at E < 1 GeV (MiniBooNE) is **hadrons**
- Degree of freedom at E > 1 GeV (MINERvA) is a mixture of **hadrons** and **partons**



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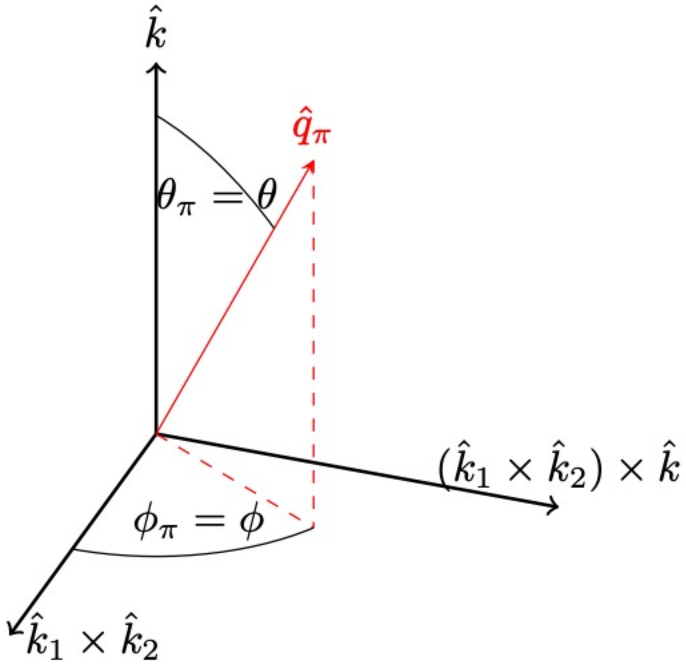
~ MiniBooNE

perturbative region

Resonance rest frame or Adler frame

- Coordinate frame in the Nπ center of mass system is the most suitable for calculating resonance contribution.
- The z-axis is along momentum transfer **q**
- $\mathbf{p} + \mathbf{q} = \mathbf{p}' + \mathbf{k_\pi} = \mathbf{0}$



Exercise 9

Show lepton energy transfer in terms of invariant variables W and q^2

$$q^0 = \frac{q^2+W^2-M^2}{2W}, \qquad q_L^0 = \frac{-q^2-M^2+W^2}{2M}$$

Hint: Use four-momentum conservation in the resonance rest frame and the lab frame.

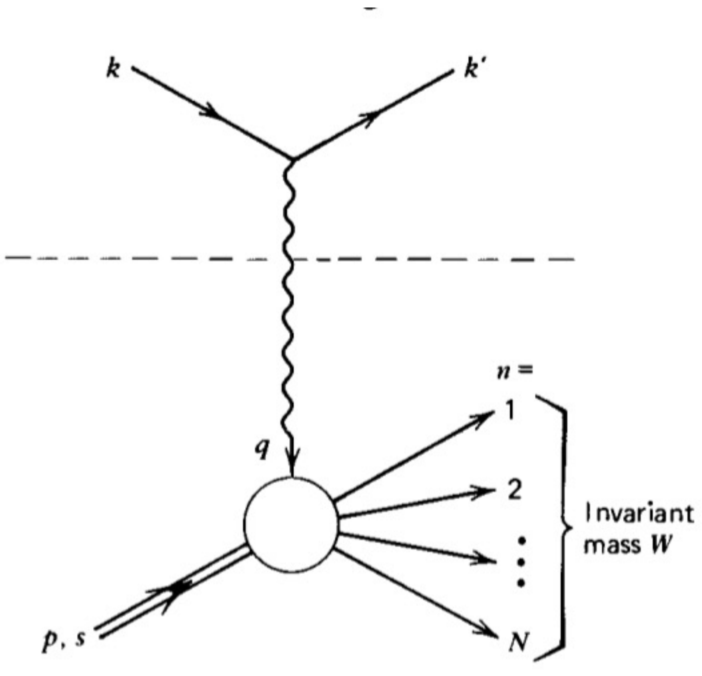
Exercise 10:

- Apart from the energy transfer, It is easier to calculate other kinematics in the hadron CM frame. Show pion energy and outgoing lepton energy in terms of invariant mass W:

$$q^0 = \frac{W^2 + m_\pi^2 - M_N^2}{2W}, \qquad p'^0 = \frac{W^2 - m_\pi^2 + M_N^2}{2W}$$

Inelastic scattering as virtual gauge boson with nucleon

- The important challenge is to know what happens below the dash line where a gauge boson interacts with a nucleon.
- The role of lepton current is that it is responsible for the presence of the virtual gauge boson.
- This is similar to the scattering a real photon with energy q^0 and (transverse) polarisation ϵ off nucleon target producing final-state particles.



Lowest order diagram for $lN \rightarrow lX$

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Polarization of gauge bosons

- For a virtual virtual gauge boson in lepton scattering, the polarizations include transverse and longitudinal and lepton current can be written in terms of 4 polarization: $\epsilon_\lambda^\alpha = C_{L_\lambda} e_L^\alpha + C_{R_\lambda} e_R^\alpha + C_\lambda e_\lambda^\alpha, \lambda = \pm$
- For electron scattering ($m_e = 0$), $\epsilon^\alpha q_\alpha = 0$ and we can have 3 independent polarization.
- For real photon on mass shell $q^2 = 0$, we can have two transverse polarization:

$$e_L^\alpha = \frac{1}{\sqrt{2}}(0 \quad 1 \quad -i \quad 0)$$

$$e_R^\alpha = \frac{1}{\sqrt{2}}(0 \quad -1 \quad -i \quad 0)$$

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Polarization of gauge bosons

- For a virtual virtual gauge boson in lepton scattering, the polarizations include transverse and longitudinal and lepton current can be written in terms of 4 polarization: $\epsilon_\lambda^\alpha = C_{L_\lambda} e_L^\alpha + C_{R_\lambda} e_R^\alpha + C_\lambda e_\lambda^\alpha,\ \lambda = \pm$

$$e_\lambda^\alpha = \frac{1}{\sqrt{|(\epsilon_\lambda^0)^2 - (\epsilon_\lambda^3)^2|}} \begin{pmatrix} \epsilon_\lambda^0 & 0 & 0 & \epsilon_\lambda^3 \end{pmatrix},$$

where $\epsilon_\lambda^{0,3}$ are components of lepton current

- Therefore the invariant amplitudes can be written as:

$$\mathcal{M}(\nu N \to \mu N \pi) = \frac{G_F}{\sqrt{2}} \cos\theta_C \epsilon^\alpha \langle N\pi | J_\alpha | N \rangle$$

Exercise 11:

- For charge current neutrino interaction: $\nu_\mu N \rightarrow \mu X$

where $\mathrm{m}_\mu \neq 0$, show :

$$\epsilon^\alpha q_\alpha \neq 0$$

- Hint: Use $\epsilon^\alpha = \bar{u}(k')\gamma^\alpha(1-\gamma_5)u(k)$ and $q^\alpha = k^\alpha - k'^\alpha$

Answer 11:

$$\epsilon^{\alpha} q_{\alpha} = \bar{\mathrm{u}}(\mathrm{k}')\gamma^{\alpha} q_{\alpha}(1-\gamma_5)\mathrm{u}(\mathrm{k})$$
$$\epsilon^{\alpha} q_{\alpha} = \bar{\mathrm{u}}(\mathrm{k}')\,(\not{k} - \not{k}')(1-\gamma_5)\mathrm{u}(\mathrm{k})$$
$$\epsilon^{\alpha} q_{\alpha} = \mathrm{m}_{\nu}\,\bar{\mathrm{u}}(\mathrm{k}')(1-\gamma_5)\mathrm{u}(\mathrm{k}) - \mathrm{m}_{l}\,\bar{\mathrm{u}}(\mathrm{k}')(1-\gamma_5)\mathrm{u}(\mathrm{k})$$

Exercise 12: Components of Lepton current

- Use lepton current in terms of 4×4 Dirac matrices and spinors:

$$\epsilon^{\alpha} = \bar{u}(k')\gamma^{\alpha}(1-\gamma_5)u(k)$$

And write it in terms of 2×2 Pauli matrices and spinors:

$$N_i = \sqrt{E_i + m_i}$$

$$\varepsilon_{\lambda}^{0} = N_1 N_2^* \chi^{\dagger}_{(s_2,\lambda)} \left(1 - \frac{\boldsymbol{\sigma}.\mathbf{k}_2}{k_{02}+m_l}\right)\left(1 - \frac{\boldsymbol{\sigma}.\mathbf{k}_1}{k_{01}}\right)\chi_{s_1}$$

$$\varepsilon_{\lambda}^{j} = N_1 N_2^* \chi^{\dagger}_{(s_2,\lambda)} \left(1 - \frac{\boldsymbol{\sigma}.\boldsymbol{k}_2}{k_{02}+m_l}\right)(-\sigma^j)\left(1 - \frac{\boldsymbol{\sigma}.\mathbf{k}_1}{k_{01}}\right)\chi_{s_1}$$

Where Pauli spinors can be calculated in your frame.

Exercise 12a: Components of Lepton current

- Use lepton current in terms of 4×4 Dirac matrices and spinors:

$$\epsilon^{\alpha} = \bar{u}(k')\gamma^{\alpha}(1-\gamma_5)u(k)$$

$$u_{\lambda}(k_i, s_i) = N_i \begin{pmatrix} \chi_{s_i,\lambda} \\ \frac{\boldsymbol{\sigma}.\mathbf{k}_i}{k_{0i}+m_i}\chi_{s_i,\lambda} \end{pmatrix}$$

$$N_i = \sqrt{E_i + m_i}$$

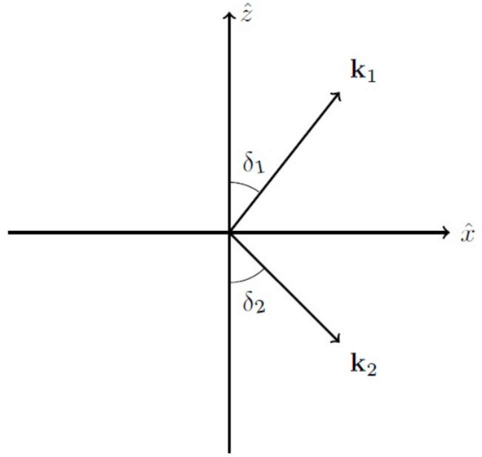
And write it in terms of 2×2 Pauli matrices and spinors:

$$\varepsilon^0_{\lambda} = N_1 N_2^* \, \chi^{\dagger}_{(s_2,\lambda)} \left(1 - \frac{\boldsymbol{\sigma}.\mathbf{k}_2}{k_{02}+m_l}\right)\left(1 - \frac{\boldsymbol{\sigma}.\mathbf{k}_1}{k_{01}}\right)\chi_{s_1}$$

$$\varepsilon^j_{\lambda} = N_1 N_2^* \, \chi^{\dagger}_{(s_2,\lambda)} \left(1 - \frac{\boldsymbol{\sigma}.\boldsymbol{k}_2}{k_{02}+m_l}\right)(-\sigma^j)\left(1 - \frac{\boldsymbol{\sigma}.\mathbf{k}_1}{k_{01}}\right)\chi_{s_1}$$

Exercise 12b: Components of Lepton current

- Use the pauli spinor for lepton in the Adler frame:



$$|\downarrow\rangle_\nu = \begin{pmatrix} -\sin\delta_1/2 \\ \cos\delta_1/2 \end{pmatrix}$$

$$|\downarrow\rangle_l = \begin{pmatrix} \cos\delta_2/2 \\ -\sin\delta_2/2 \end{pmatrix}, \qquad |\uparrow\rangle_l = \begin{pmatrix} \sin\delta_2/2 \\ \cos\delta_2/2 \end{pmatrix}$$

- And drive the lepton current elements:

Where $\delta = \pi - (\delta_1 + \delta_2)$

And $A_\pm = \sqrt{k_{01}(k_{02} \pm |\mathbf{k}_2|)}$

$$\begin{aligned} \varepsilon^0_\lambda &= 2\lambda A_\lambda \sqrt{1-\lambda cos\delta} \\ \varepsilon^1_\lambda &= 2\lambda A_\lambda \frac{k_{01}-\lambda|\mathbf{K_2}|}{|\mathbf{k}|}\sqrt{1+\lambda cos\delta} \\ \varepsilon^2_\lambda &= 2iA_\lambda \sqrt{1+\lambda cos\delta} \\ \varepsilon^3_\lambda &= 2\lambda A_\lambda \frac{k_{01}+\lambda|\mathbf{K_2}|}{|\mathbf{k}|}\sqrt{1-\lambda cos\delta}, \end{aligned}$$

Helicity amplitudes:

$$\mathcal{M}_{CC}(\nu N \to l_\lambda N'\pi) = \frac{G_F}{\sqrt{2}} \cos\theta_C \langle N'\pi | \varepsilon_\lambda^\rho J_\rho | N \rangle$$
$$= \frac{G_F}{\sqrt{2}} \cos\theta_C \langle N'\pi | C_{L_\lambda} e_L^\rho J_\rho + C_{R_\lambda} e_R^\rho J_\rho + C_\lambda e_\lambda^\rho J_\rho | N \rangle.$$

- Helicity amplitudes can be defined by knowing the helicity of incident and outgoing nucleons and gauge boson's polarization. 16 helicity amplitudes for each vector and axial.

$$\tilde{F}^{\lambda_k}_{\lambda_2,\lambda_1} = \langle N\pi | e^\rho_{\lambda_k} J^V_\rho | N \rangle$$

$$\tilde{G}^{\lambda_k}_{\lambda_2,\lambda_1} = \langle N\pi | e^\rho_{\lambda_k} J^A_\rho | N \rangle$$

Helicity amplitudes:

- Dirac equation allows us to have 16 independent Lorentz covariance
- Conservation of vector current reduce the number of O(V_i) to six.
- O(V) and O(A) are 4*4 matrices in terms of Dirac matrices and particle's 4-momenta.

4×4 matrices

$$J_V^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{6} V_i \, \bar{u}_N(p_2) O^{\lambda_k}(V_i) u_N(p_1)$$

Dirac Spinors

$$J_A^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{8} A_i \, \bar{u}_N(p_2) O^{\lambda_k}(A_i) u_N(p_1)$$

2×2 matrices

$$J_V^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{6} \mathscr{F}_i \, \chi_2^* \, \Sigma_i^{\lambda_k} \, \chi_1$$

Pauli spinors

$$J_A^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{8} \mathscr{G}_i \, \chi_2^* \, \Lambda_i^{\lambda_k} \, \chi_1$$

Cross section

$$\frac{d\sigma(\nu N \to lN\pi)}{dQ^2 dW d\Omega_\pi} = \frac{1}{(2\pi)^4}\frac{1}{(4ME_\nu)^2}\frac{|\mathbf{q}|}{4}|\mathscr{M}|^2,$$

$$\frac{d\sigma(\nu N \to lN\pi)}{dk^2 dW d\Omega_\pi} = \frac{G_F^2}{2}\frac{1}{(2\pi)^4}\frac{|\mathbf{q}|}{4}\frac{-k^2}{(k^L)^2}\sum_{\lambda_2,\lambda_1}\Bigg\{$$

$$\left|C_{L_-}(\tilde{F}^{e_L}_{\lambda_2\lambda_1}(\theta,\phi)-\tilde{G}^{e_L}_{\lambda_2\lambda_1}(\theta,\phi))+C_{R_-}(\tilde{F}^{e_R}_{\lambda_2\lambda_1}(\theta,\phi)-\tilde{G}^{e_R}_{\lambda_2\lambda_1}(\theta,\phi))+C_-(\tilde{F}^{e_-}_{\lambda_2\lambda_1}(\theta,\phi)-\tilde{G}^{e_-}_{\lambda_2\lambda_1}(\theta,\phi))\right|^2$$

$$+\left|C_{L_+}(\tilde{F}^{e_L}_{\lambda_2\lambda_1}(\theta,\phi)-\tilde{G}^{e_L}_{\lambda_2\lambda_1}(\theta,\phi))+C_{R_+}(\tilde{F}^{e_R}_{\lambda_2\lambda_1}(\theta,\phi)-\tilde{G}^{e_R}_{\lambda_2\lambda_1}(\theta,\phi))+C_+(\tilde{F}^{e_+}_{\lambda_2\lambda_1}(\theta,\phi)-\tilde{G}^{e_+}_{\lambda_2\lambda_1}(\theta,\phi))\right|^2\Bigg\}$$

Backup

Four-vector

- Four-vector: Any set of four quantities which transform like (ct, x) under Lorentz transformations:

$$ct' = \cosh\theta\, ct - \sinh\theta\, z,$$

$$z' = -\sinh\theta\, ct + \cosh\theta\, z,$$

$$\tanh\theta = v/c$$

V along z axis

- Notation:

$$(ct, \mathbf{x}) \equiv (x^0, x^1, x^2, x^3) \equiv x^\mu.$$

basic invariant is $c^2t^2 - \mathbf{x}^2$

$$\left(\frac{E}{c}, \mathbf{p}\right) \equiv (p^0, p^1, p^2, p^3) = p^\mu$$

basic invariant $(E^2/c^2) - \mathbf{p}^2$

Lorentz covariance

- A cornerstone of modern physics is that the fundamental laws have the same form in all Lorentz frames
- If an equation is a Lorentz covariance we must ensure that all unrepeated indices (upper and lower separately) balance on either side of the equation and all repeater indices appear once as and upper and once as lower.
- A relativistic theory a covariant copy on the non-relativistic perturbation theory

Lorentz invariant

- Not changing under Lorentz transformation:
    1. Scalar products of two four-vectors
- The rule for forming a Lorentz invariant is to make the upper indices (contravariant) balance with the lower indices (covariant)

Definition of a Free particle

- For a free particle we have $p^2 = m^2$.
- We say particle is on its mass shell

$$\frac{d\sigma}{d\Omega dE'} = \frac{G^2}{16\pi^2}\cos^2\theta_C \frac{E'}{E} L_{\mu\nu}\mathcal{W}^{\mu\nu}$$

$$L_{\mu\nu} = Tr[\gamma_\mu(1-\gamma_5)\ \not{k}\gamma_\nu\ \not{k}']$$

$$\mathcal{W}^{\mu\nu} = \frac{1}{2m_N}\sum \langle p|J^\mu(0)|\Delta\rangle\langle\Delta|J^\nu(0)|p\rangle\delta(W^2 - M_R^2)$$

$$\delta(W^2 - M_R^2) = \frac{M_R\Gamma_R}{\pi}\frac{1}{(W^2 - M_R^2)^2 + M_R^2\Gamma_R^2}$$

Resonance has an observable width,

$$\langle\Delta^{++}|J^\nu|p\rangle = \sqrt{3}\bar{\psi}_\lambda(p')d^{\lambda\nu}u(p)$$

$$d^{\lambda\nu} = g^{\lambda\nu}\left[\frac{C_3^V}{m_N}\ \not{q} + \frac{C_4^V}{m_N^2}(p'q) + \frac{C_5^V}{m_N^2}(pq) + C_6^V\right]\gamma_5 - q^\lambda\left[\frac{C_3^V}{m_N}\gamma^\nu + \frac{C_4^V}{m_N^2}p'^\nu + \frac{C_5^V}{m_N^2}p^\nu\right]\gamma_5$$

$$+g^{\lambda\nu}\left[\frac{C_3^A}{m_N}\ \not{q} + \frac{C_4^A}{m_N^2}(p'q)\right] - q^\lambda\left[\frac{C_3^A}{m_N}\gamma^\nu + \frac{C_4^A}{m_N^2}p'^\nu\right] + g^{\lambda\nu}C_5^A + q^\lambda q^\nu\frac{C_6^A}{m_N^2}.$$