Lecture 3

Radiative Corrections

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- Neutrino experiments plan to push precision goals to % level
- QED radiative corrections are important for any SM observable at the % level

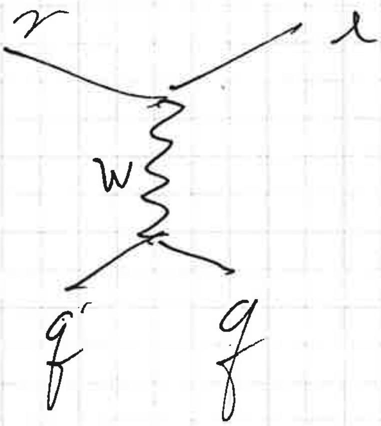
Goals of today

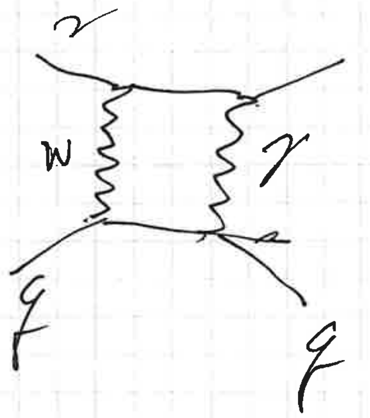
↳ How to think about radiative corrections

↳ Coulomb field as a simple example

↳ Running & Short distance cor.

Long vs. Short-Distance

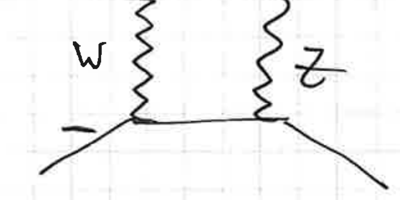




$$g^4 \int \frac{d^4L}{(2\pi)^4} \frac{1}{\not L} \frac{1}{\not L} \frac{1}{L^2-M_W^2} \frac{1}{L^2}$$

$$\sim \frac{d^4L}{L^6} \sim \frac{g^2}{M_W^2} \frac{g^2}{16\pi^2} \quad \#$$

Leads to a correction $\mathcal{O}(\alpha)$

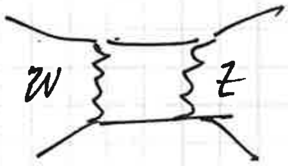


$$\sim \int \frac{d^4L}{(2\pi)^4} \frac{1}{\not L} \frac{1}{\not L} \frac{1}{L^2-M_W^2} \frac{1}{L^2-M_Z^2}$$

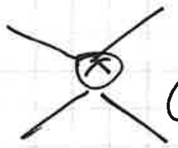
$$\sim \frac{g^2}{M_W^2} \frac{g^2}{16\pi^2} f\left(\frac{M_W^2}{M_Z^2}\right) \leftarrow \mathcal{O}(1)$$

- This may lead you to think the 4-Fermi theory is just an approximation at tree level

- This is not the case!



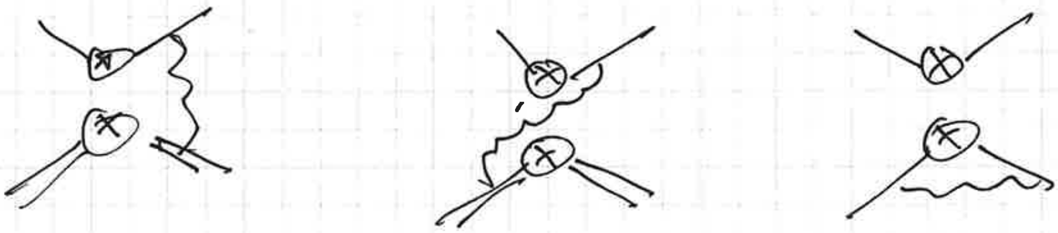
this diagram can be absorbed in shifted low-energy constant



$$C = \left(\frac{g^2}{M_W^2} + \frac{\alpha}{4\pi} \delta C^{(1)} \right)$$

We can calculate these corrections with quarks & gluons

Long Distance Corrections



- Need to use realistic physical states
- can not set "external scales" to zero
- ~~[illegible]~~

M_W

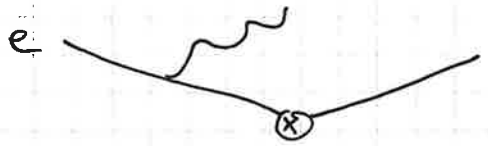
quarks & gluons

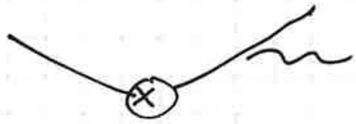
$\mu \sim 2\,\text{GeV}$ ⟸ Wilson coeff's here

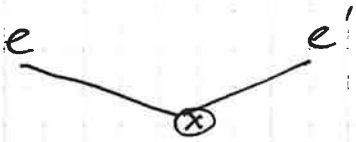
Long Distance Radiative corr here

Caveat: Need $s \ll M_W^2$

Simple Example $e \to e' \gamma$ Bkg potential







Tree level $= \quad \bar{u}\, \gamma_0\, \tilde{V}(\vec{Q})\, u$

$$q_\mu = (0, \vec{Q}) \qquad Q^2 = \vec{Q}^2$$

$$\bar{u}(p')\left[\not\epsilon \, \frac{\not p' + \not q + m_e}{(p'+q)^2 - m_e^2}\gamma_0 + \gamma_0 \frac{\not p - \not q + m_e}{(p-q)^2 - m_e^2} \not\epsilon \right] u(p)$$

Notice that

$$\not\epsilon\,(\not p' + M) = 2p'\cdot\epsilon + (-\not p' + M)\not\epsilon$$

$$\bar{u}\,(-\not p + M) = 0$$

$$\hookrightarrow \epsilon^\mu \bar{u}\left[\frac{2p'_\mu + \gamma_\mu \not q}{2p'\cdot q + \cancel{q^2}} + \frac{2p_\mu + \cancel{\gamma_\mu} \not q \epsilon_\mu}{2p\cdot q} \right] u$$

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Let's focus on the ~~soft~~ limit $q \ll P$

$$(\underbrace{\bar{u}\ \Gamma\ u}_{\text{Tree level}}) \otimes \left[e \left(\frac{P'\cdot\epsilon^*}{P'\cdot k} - \frac{P\cdot\epsilon^*}{P\cdot k} \right) \right]$$

$$\bar{u}\,\Gamma\,u \left[e \left(\frac{P'_\mu}{P'\cdot k} - \frac{P_\mu}{P\cdot k} \right) \right] (\epsilon^\mu)^*$$

notice the presence of <u>both</u> terms is necessary for gauge inv.

$$\epsilon_\mu \to \epsilon_\mu + \xi q_\mu$$

$$\sum_s \left| \frac{p'\cdot\epsilon_s}{p'\cdot k} - \frac{p\cdot\epsilon_s}{p\cdot k} \right|^2 = \frac{2p'\cdot p}{(p'\cdot k)(p\cdot k)} + \left(\begin{matrix}\text{lepton}\\ \text{mass suppressed}\end{matrix}\right)$$

Okay: $\frac{2p'\cdot p}{(p'\cdot k)(p\cdot k)}$

$|\vec{p}| \gg m_\ell \qquad p_\mu = E$

$$\simeq \cancel{\frac{2\,v'\cdot v}{(v'\cdot k)(v\cdot k)}} \qquad \frac{2p'\cdot p}{|p||p'|(1-\cos\theta)(1-\cos\theta')E_\gamma^2}$$

But not quite

$$\simeq \frac{2\;p'\cdot p}{|p||p'|\left(1-\cos\theta+\frac{1}{2}\frac{m_e^2}{p^2}\right)\left(1-\cos\theta'+\frac{1}{2}\frac{m_e^2}{p'^2}\right)|E_\gamma|^2}$$

$\lim_{\theta\to 0}$
$$\simeq \frac{2p'\cdot p}{\frac{1}{4}|p||p'|\left(\theta^2+m_e^2/p^2\right)\left(\theta'^2+m_e^2/p'^2\right)|E_\gamma|^2}$$

Recall $d\Phi = \frac{d^3p}{(2\pi)^3 2E_p}$

$$\int d\Phi \, |\mathcal{M}_{(1\gamma)}|^2 = \int \frac{d^3q}{(2\pi)^3 2E_q} \frac{p\cdot p'}{\cancel{4} E_q^2 |p||p'| (\theta^2+\delta^2)(\theta'^2+\delta'^2)}$$

$\hookrightarrow$

Two peaks $\theta = 0$ & $\theta' = 0$

Two <u>different kinds of</u> divs here

$\hookrightarrow$ Collinear $\int d\theta^2 \frac{1}{(\theta^2+\delta^2)} \sim \log(\delta)$

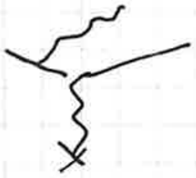
$$\int_{\Delta E}^{E} dE_\gamma \frac{E_\gamma^2}{E_\gamma^3} \sim \log\left(\frac{E}{\Delta E}\right)$$

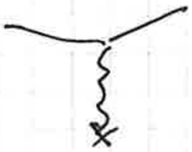
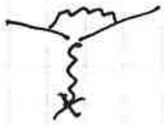
These logarithms overlap

$\left(\frac{\alpha}{4\pi}\right) \mathrm{Log}(\ ) \times \mathrm{Log}(\ ) \sim 20\%$ in some cases

"Large Logarithms"

Virtual vs. real corr. $\quad e \to e(n\gamma)$

 $+$  $= i\mathcal{M}_{(1\gamma)}$

 $+$  $+ (1+\delta Z_2)$  $= i\mathcal{M}_{(0\gamma)}$

- Both $i\mathcal{M}_{(1\gamma)}$ & $i\mathcal{M}_{(0\gamma)}$ have IR divergences
- $i\mathcal{M}_{(0\gamma)}$ has UV divergences, but these can be handled w/ Renormalization
- IR divergences cancel between

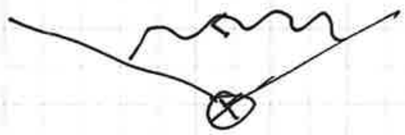
$$|\mathcal{M}_{0\gamma}|^2 + \int d\Phi_{1\gamma}\, |\mathcal{M}_{1\gamma}|^2 + \ldots$$

After integrating over photon phase space

Cancellation of ~~Real~~ Real & Virtual Corrections

↳ we just found out that the prob. to radiate a photon is **large**

↳ But to $\mathcal{O}\left(\frac{\alpha}{4\pi}\right)$ we need to include ~~Real radiation~~ Virtual corr as well



$$= \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2} \gamma_\mu \frac{\not p + \not q + m}{2p\cdot q + q^2} \Gamma \frac{\not p' + \not q + m}{2p'\cdot q + q^2} \gamma_\nu \ldots$$

This has the same singularity structure

$$\int \frac{d^4q}{q^4} \qquad \text{soft}$$

$$\int d^4q \quad \frac{1}{p\cdot q} \quad \frac{1}{p'\cdot q} \qquad \text{collinear}$$

Theorem: KLN sum over all degenerate final states & there are no singularities

$$\Rightarrow \quad d\sigma_{(0\gamma)} + \int d\sigma_{(1\gamma)}\, d\Phi_{1\gamma} \cdots$$

No large logs

(II)

Consider the flavour ratio

$$\frac{\sigma_{\nu_e A \to e X}}{\sigma_{\nu_\mu A \to \mu X}}$$

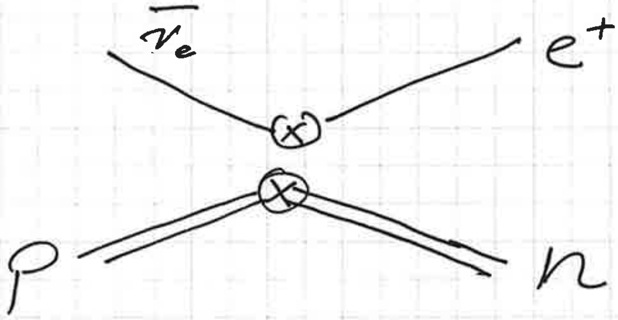
what controls the size of radiative corr to this quantity?

Let's take an Example

~~$\bar{\nu}_e \, n \to$~~

$$\bar{\nu}_e \; p \longrightarrow e^+ \, n \qquad (\text{I.B.D})$$

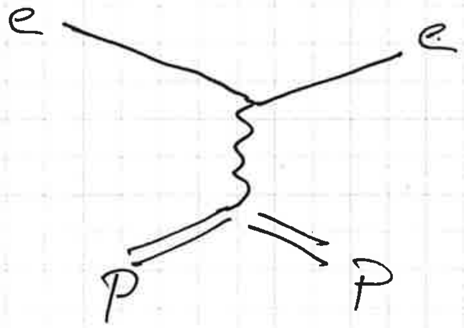
Tree-Level



$Q^2 \lesssim (5 \text{ MeV})^2$

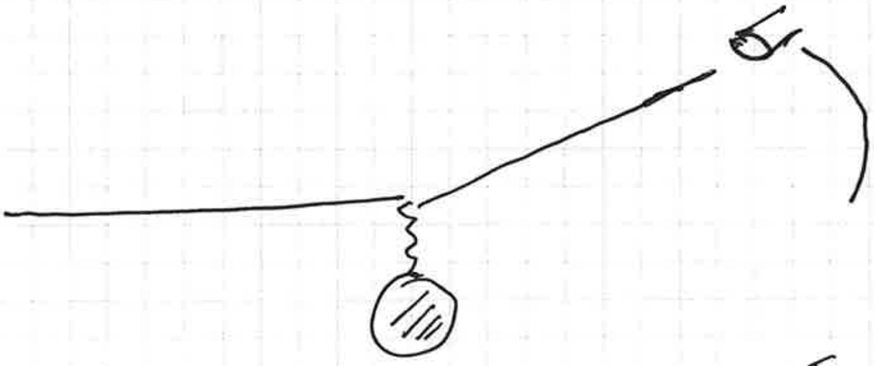
$$\langle n | J_\mu | p \rangle \simeq \bar{u}\left[C_V \gamma_\mu + C_A \gamma_\mu \gamma_5 \right] u$$

Compare w/ $\quad e\, p \to e\, p$



Leptonic System Preserves Charge

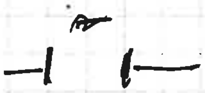
Neutrino Detector v.s. Spectrometer

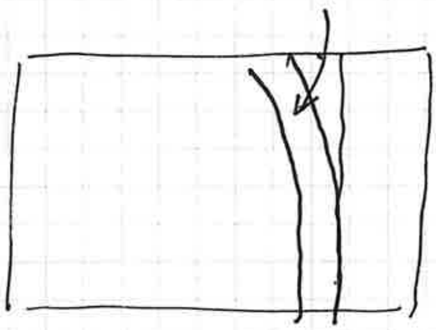


θ Fixed

E fixed

E.g. using B-field

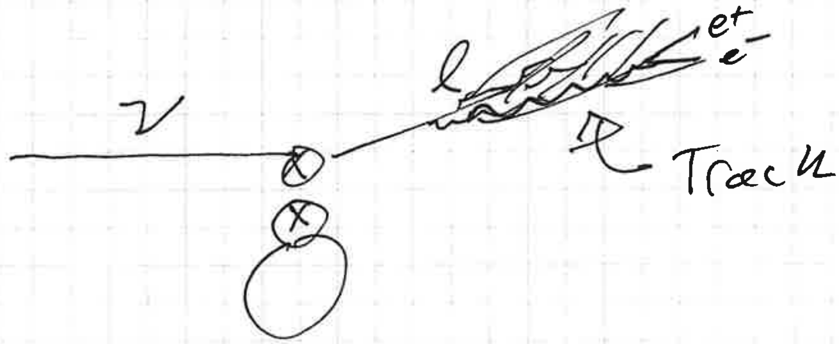




Any real radiation has a

<u>very large</u> effect

Neutrino Detector



↳ Fully inclusive

↳ Prompt radiation may be absorbed into shower

↳ KLN protects inclusive objects from logarithms

Factorization

↳ Soft photons factorize from hard processes

↳ Soft photons exponentiate

↳ $$S \sim \left(\frac{E}{\Delta E}\right)^{\nu}$$

$$\nu \sim \frac{\alpha}{2\pi} \log\left(\frac{1+\beta}{1-\beta}\right)$$

$$d\sigma_{E_\gamma \leq \Delta E} \quad S \cdot d\sigma^{(0)}$$

Basically the

Why do soft photons exponentiate?

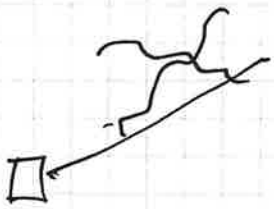
Exercise

- Show that

$$\frac{1}{p\cdot k_1}\,\frac{1}{p\cdot(k_1+k_2)} + \frac{1}{(p\cdot k_2)(p\cdot (k_1+k_2))}$$

$$= \frac{1}{p\cdot k_1}\,\frac{1}{p\cdot k_2}$$

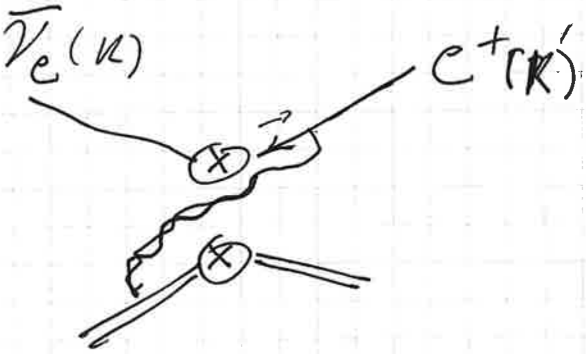
$+$



- Argue based on Bose statistics we have over counted & should include a factor of $\frac{1}{2}$!

- Read Peskin & Schroeder

what do radiative corr look like



$$= (-ie)\gamma_\mu$$

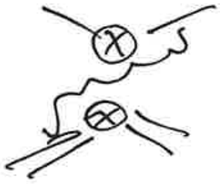
$$= v_\mu(+ie)$$

$$v_\mu = (1,0,0,0)$$

$$\int \frac{d^4L}{(2\pi)^4} \frac{-i}{L^2} (-ie)(+ie)\, \bar{\nu}\, \Gamma(1-\gamma_5)\gamma_\mu \frac{-(\not{k}+\not{L})+m_e}{2L\cdot p + L^2} \cancel{\not{v}}\, v$$

use on-shell Dirac eq'n

$$\sim \int \frac{d^4L}{(2\pi)^4} \frac{-i}{L^2} (-ie)(+ie)\, \bar{\nu}\,(1-\gamma_5)\gamma_\mu \frac{2E - \not{K}\not{v}}{2L\cdot p + L^2}\, v \ldots$$



Example of how to compute

P part of propagator

$$\int \frac{d^4 L}{(2\pi)^4} \frac{1}{L^2+i0} \, \frac{1}{2L\cdot P + L^2 + i0} \, \frac{1}{V\cdot L + i0}$$

IR div

$L \to 0$

UV finite

$$= \int_0^1 dx \int [dL] \frac{1}{(L^2+2x L\cdot P)^2} \, \frac{1}{V\cdot L}$$

$\lambda = 2\alpha$

$d\lambda = 2 d\alpha$

$$= \int_0^\infty d\lambda \int_0^1 dx \int [dL] \frac{4}{(\underbrace{L^2 + \lambda x L\cdot P + 2\lambda V\cdot L}_{\text{complete square}})^3}$$

* use dim reg

$$\int [dL] \frac{1}{(L^2+\Delta)^n} = i \, \frac{\Gamma(n-d/2)}{\Gamma(n)\,(4\pi)^{d/2}} (-1)^n \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}}$$

$d = 4 - 2\epsilon$

$$= (-i) \frac{\Gamma(3-2+\epsilon)}{\Gamma(3)\,(4\pi)^{2-\epsilon}} \int_0^1 dx \int_0^\infty d\lambda \, \frac{4}{(xP+\lambda V)^{2+\epsilon}}$$

$\lambda = x\alpha$

UV-finite

$$= (-i) \frac{\Gamma(1+\epsilon)}{\Gamma(3)(4\pi)^{2-\epsilon}} \int_0^1 dx \frac{4}{x^{1+\epsilon}} \int_0^\infty d\alpha \frac{1}{(p^2+2\alpha v\cdot p+\alpha^2)^{1+\epsilon}}$$

IR div

$$= (-i) \frac{\Gamma(1+\epsilon)}{\Gamma(3)(4\pi)^{2-\epsilon}} \frac{4}{\epsilon} \int_0^\infty d\alpha \frac{1}{(p^2+2\alpha v\cdot p+\alpha^2)^{1+\epsilon}}$$

$$= (-i) \frac{\Gamma(1+\epsilon)}{\Gamma(3)(4\pi)^{2-\epsilon}} \frac{4}{\epsilon} \left[\int_0^\infty d\alpha \frac{1}{(p^2+2\alpha v\cdot p+\alpha^2)} \right.$$

$$- \epsilon \int_0^\infty d\alpha \frac{\log(p^2+2\alpha v\cdot p+\alpha^2)}{(p^2+2\alpha v\cdot p+\alpha^2)}$$

$$\left. + \mathcal{O}(\epsilon^2) \right]$$

These can be written in terms of logarithms

Second Integral ↪

$$\int \frac{d^4L}{(2\pi)^4} \frac{\not{L}}{(V\cdot L)(2P\cdot L + L^2)(L^2)} = A\not{P} + B\not{V}$$

can find A & B using

$$\mathrm{Tr}(\not{V}\ldots)$$

$$\mathrm{Tr}(\not{P}\ldots)$$

will get integrals involving

$$\int \frac{d^4L}{(2\pi)} \frac{\cancel{(V\cdot L)}}{\cancel{(V\cdot L)}(2P\cdot L + L^2)(L^2)} \Leftarrow \text{easy!}$$

$$\int \frac{d^4L}{(2\pi)^4} \frac{2P\cdot L}{(V\cdot L)(2P\cdot L + L^2)(L^2)} = \int \frac{d^4L}{(2\pi)^4} \frac{(2P\cdot L + L^2) - L^2}{(V\cdot L)(2P\cdot L + L^2)(L^2)}$$

Exercises (I)

Consider two different observables

1) $\left(\frac{d\sigma}{dQ^2}\right)_{ep\to ep}$ elastic cross section for ep scattering
$E_e \approx 1\,\text{GeV}$
$Q^2 \simeq (1\,\text{GeV})^2$

2) Life time of the muon

τ_μ

- Is (1) subject to larger or smaller radiative corrections than (2) ?

- What are the "natural" sizes" of the radiative corrections

- What about a third process

$\frac{d\sigma}{dE_\mu}(\bar{\nu}_\mu p \to \mu^+ n)$?