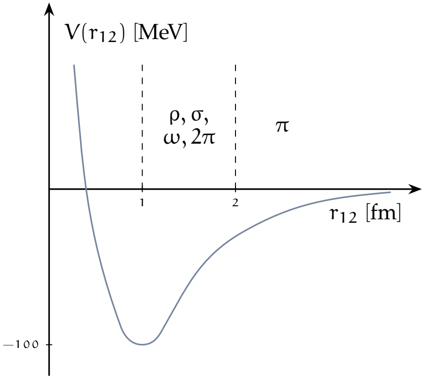
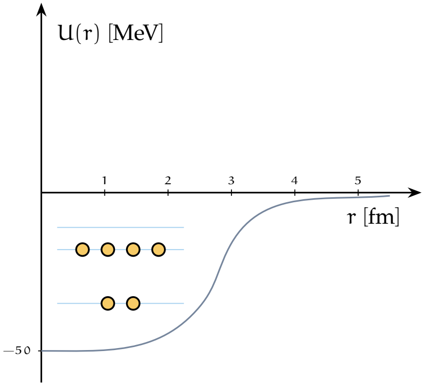
One- and two-nucleon knock-out in neutrino-nucleus scattering: Nuclear mean-field approaches

Kajetan Niewczas

Mean-field nuclear picture

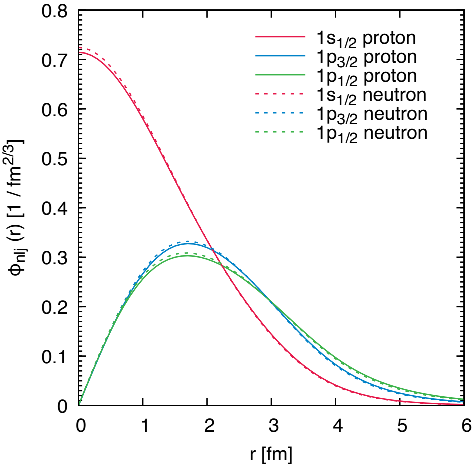


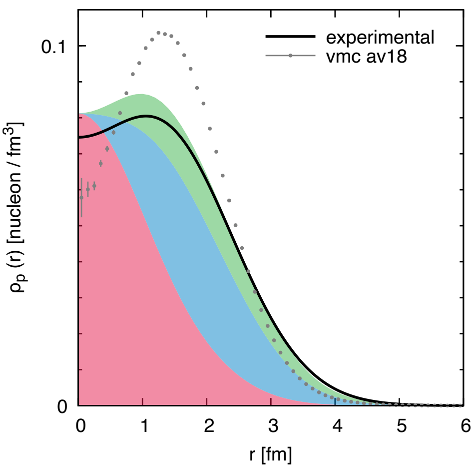
$\rightarrow$



$\rightarrow$ we use a realistic **nucleon-nucleon potential** to derive the **central nuclear potential**

Oxygen wave functions





http://discovery.phys.virginia.edu/research/groups/ncd/index.html
https://www.phy.anl.gov/theory/research/density/norfolk.html

Outline

Lecture 1. *the general framework of the nuclear mean-field model*

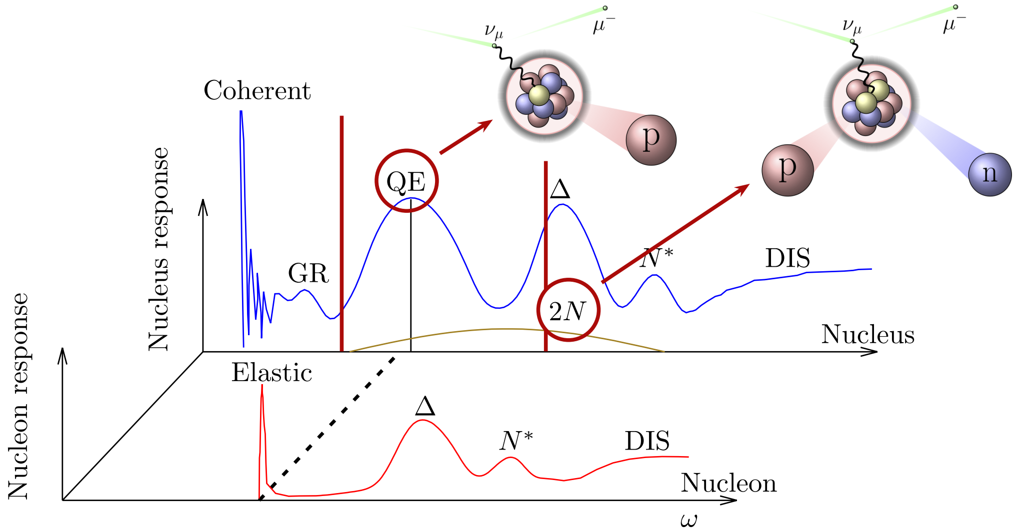
Lecture 2. *one- and two-nucleon knock-out in lepton-nucleus scattering*

(1) Kinematics and scattering cross section

(2) Distorted-wave calculations

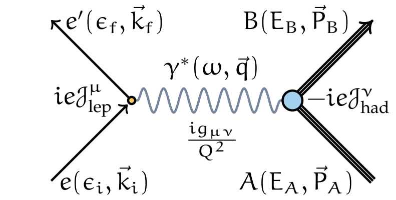
(3) Corrections and additional dynamics

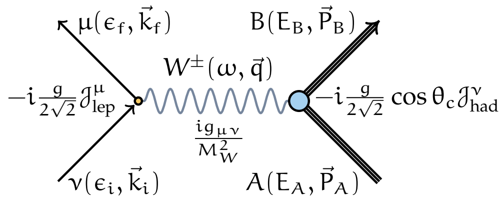
Nuclear response in the quasielastic and Δ regions



Kinematics and scattering cross section

Independent variables in a scattering problem





Counting independent variables:

- ○ 4 x 4-vectors $\rightarrow$ + 16 numbers
- ○ 4-mom. conservation $\rightarrow$ - 4 numbers
- ○ 4 x on-shell particles $\rightarrow$ - 4 numbers
- ○ target rest-frame $\rightarrow$ - 3 numbers
- ○ fixed projectile direction $\rightarrow$ - 2 numbers
- ○ fixed incoming energy $\rightarrow$ - 1 number
- for 2-to-2 scattering: **2 independent variables**

Note, the cross section does
not depend on the global ϕ rotation!

Independent variables in a scattering problem

Unknown particle 4-vectors	Variables
Initial lepton	4
Target nucleus	4
Final lepton	4
Remnant nucleus	4
Outgoing hadrons	$4N$
	$16 + 4N$

Physical effects	Variables
Particles on-shell	$-(3 + N)$
4-momentum conservation	-4
Target rest-frame	-3
Fixed projectile direction	-2
Fixed incoming energy	-1
	$-13 - N$
	$3 + 3N$

Counting the number of independent variables describing lepton-nucleus interactions while detecting N hadronic particles in the process, summing over the spin of the outgoing lepton, and leaving the remnant nucleus undetected.

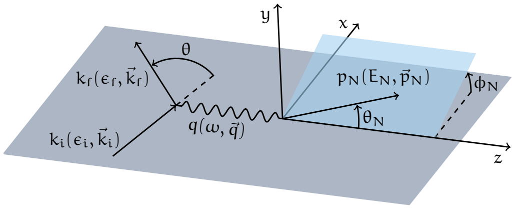
Scattering cross sections

Target	Process	Properties	Example formula
Free nucleon	(Quasi)elastic	$N = 0$, all particles on-shell	$\frac{d\sigma}{dQ^2}$
	Inelastic	$N = 0$, excited hadronic system	$\frac{d^2\sigma}{dQ^2 dW}$
	SPP	$N = 1$, all particles on-shell	$\frac{d^4\sigma}{dQ^2 dW d\Omega_\pi}$
Nucleus	Inclusive	$N = 0$, all hadrons integrated	$\frac{d^2\sigma}{d\Omega'}$
	1p1h	$N = 1$, detected one nucleon	$\frac{d^5\sigma}{dE' d\cos\theta' dT_{N'} d\Omega_{N'}}$
	2p2h	$N = 2$, detected two nucleons	$\frac{d^8\sigma}{dE' d\cos\theta' dT_{N'} d\Omega_{N'} dT_{N''} d\Omega_{N''}}$
	SPP	$N = 2$, detected nucleon and π	$\frac{d^8\sigma}{dE' d\Omega' dE_\pi d\Omega_\pi d\Omega_{N'}}$

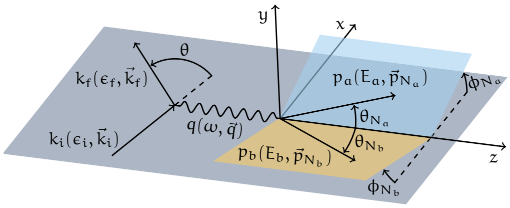
The dimensionality of cross section formulas for the most basic lepton scattering scenarios, off the free nucleon or on the nucleus.

Kinematics

One-nucleon knock-out (1p1h)



Two-nucleon knock-out (2p2h)



Inclusive cross section

Electron scattering

$$\frac{d\sigma^\gamma}{d\epsilon_f d\Omega_f} = 4\pi\sigma^{\text{Mott}}[\mathcal{V}_L^e \mathcal{W}_L + \mathcal{V}_T^e \mathcal{W}_T]$$

Neutrino scattering

$$\frac{d\sigma^W}{d\epsilon_f d\Omega_f} = 4\pi\sigma^W \zeta[\mathcal{V}_{CC}\mathcal{W}_{CC} + \mathcal{V}_{CL}\mathcal{W}_{CL} + \mathcal{V}_{LL}\mathcal{W}_{LL} + \mathcal{V}_T\mathcal{W}_T + h\mathcal{V}_{T'}\mathcal{W}_{T'}]$$

$\mathcal{V}_x$ - leptonic factors; $\mathcal{W}_x$ - hadronic responses; L/T - longitudinal/transverse relative to $\vec{q}$

Hadronic responses

In the **Born approximation** (1 boson), we have 16 terms coming from:

$$\begin{aligned}\frac{d\sigma}{dE' d\cos\theta' dT_{N'} d\Omega_{N'}} &\propto L_{\mu\nu}W^{\mu\nu} \\ &\propto [v_{CC}W_{CC} + v_{CL}W_{CL} + v_{LL}W_{LL} + v_T W_T + v_{TT}W_{TT} + v_{TC}W_{TC} + v_{TL}W_{TL} + v_{\overline{TT}}W_{\overline{TT}} \\ &+ v_{\overline{TC}}W_{\overline{TC}} + v_{\overline{TL}}W_{\overline{TL}} + h\left(v_{T'}W_{T'} + v_{TC'}W_{TC'} + v_{TL'}W_{TL'} + v_{\overline{CL}'}W_{\overline{CL}'} + v_{\overline{TC}'}W_{\overline{TC}'} + v_{\overline{TL}'}W_{\overline{TL}'}\right)]\end{aligned} \tag{1}$$

For **unpolarized processes**:

$$\begin{aligned}\frac{d\sigma}{dE' d\cos\theta' dT_{N'} d\Omega_{N'}} \propto [&v_{CC}W_{CC} + v_{CL}W_{CL} + v_{LL}W_{LL} + v_T W_T + v_{TT}W_{TT} \\ &+ v_{TC}W_{TC} + v_{TL}W_{TL} + h\left(v_{T'}W_{T'} + v_{TC'}W_{TC'} + v_{TL'}W_{TL'}\right)]\end{aligned} \tag{2}$$

Integrating out the **nucleon solid angle**:

$$\int d\Omega_{N'} \frac{d\sigma}{dE' d\cos\theta' dT_{N'} d\Omega_{N'}} \propto [v_{CC}W_{CC} + v_{CL}W_{CL} + v_{LL}W_{LL} + v_T W_T + h v_{T'}W_{T'}] \tag{3}$$

Using **conserved vector current**, $J_3(q) = (\omega/|\mathbf{q}|)J_0(q)$, and $h = 0$:

$$\frac{d\sigma}{dE' d\cos\theta'} \propto [v_L W_L + v_T W_T] \tag{4}$$

One variable mysteriously disappeared?

In the mean-field framework, we know the exact **energy states**:

$$\frac{d^5\sigma}{dE' d\cos\theta' dT_{N'} d\Omega_{N'}} \to \sum_{h'} \frac{d^4\sigma}{dE' d\cos\theta' d\Omega_{N'}} \tag{5}$$

where we used $\omega + E_{h'} = T_{N'}$

But for the **two-nucleon knock-out** case:

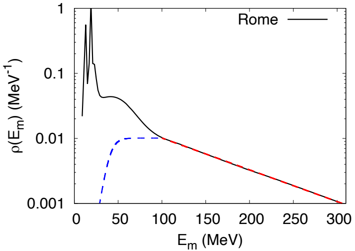
$$\frac{d^8\sigma}{dE' d\cos\theta' dT_{N'} d\Omega_{N'} dT_{N''} d\Omega_{N''}} \to \sum_{h',h''} \frac{d^7\sigma}{dE' d\cos\theta' d\Omega_{N'} dT_{N''} d\Omega_{N''}} \tag{6}$$

because $\omega + E_{h'} + E_{h''} = T_{N'} + T_{N''}$

In general, we can introduce a function $\rho(E_m)$:

- in a **pure shell model** $\rho(E_m)$ is $\sum_h \delta(E_m - E_h)$
- **phenomenological profiles** for $\rho(E_m)$

R. González-Jiménez et al., Phys.Rev. C 105 (2022), 025502



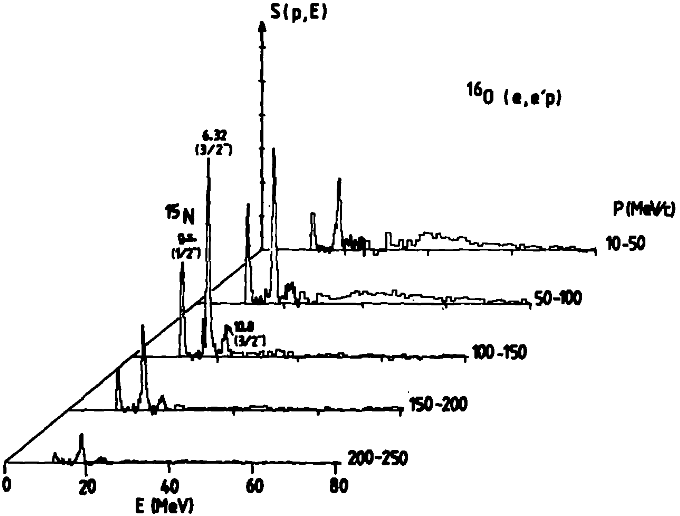
Distorted-wave calculations

Nuclear mean-field model

→ Nucleons exhibit discrete energy states characteristic of the **mean-field potential** picture

→ The redistribution of shell strength is caused by the **nucleon-nucleon correlations**

→ Residual nuclei can be excited above the **two-nucleon knock-out** threshold



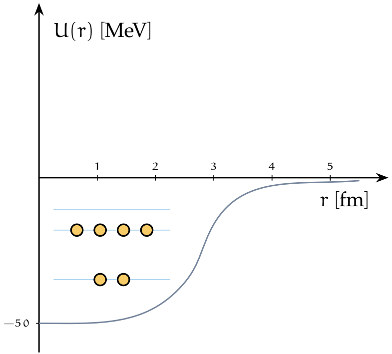
J. Mougey, Nucl.Phys. A 335 (1980) 35

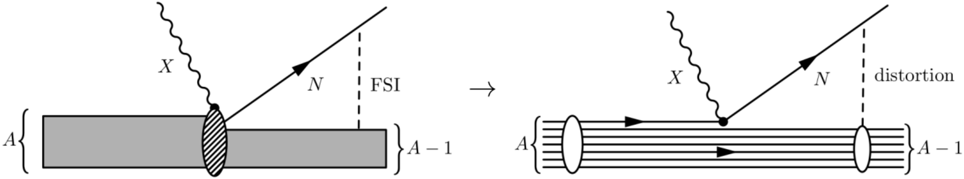
Our nuclear framework

→ Nucleons are solutions to the Schrödinger equation in a **mean-field potential**

→ We calculate single-particle states with the **Hartree-Fock** procedure and SkE2 NN force

→ We describe outgoing nucleons as **continuum states** of the nuclear potential





Impulse approximation

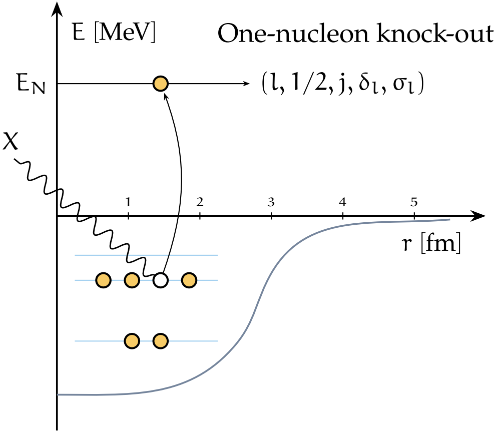
→ We evaluate the following **hadronic transition** currents

$$\mathcal{J}(\vec{r})^{\text{had}}_{\nu} = \langle\, \Psi_f \,|\, \hat{\mathcal{J}}(\vec{r})^{\text{had}}_{\nu} \,|\, \Psi_i \,\rangle$$

→ The nuclear many-body current is a sum of **one-body operators**

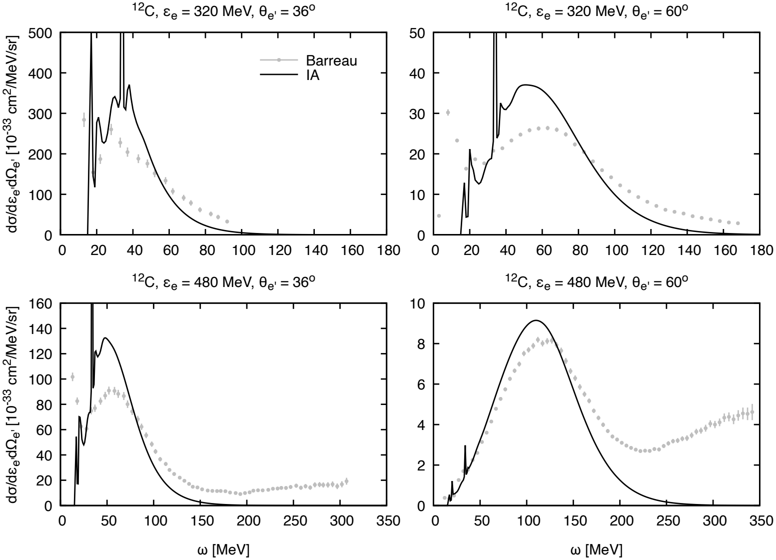
$$\hat{\mathcal{J}}(\vec{r})^{\text{had}}_{\nu} \simeq \hat{\mathcal{J}}(\vec{r})^{\text{IA}}_{\nu} = \sum_{j=1}^{A} \hat{\mathcal{J}}(\vec{r}_j)^{[1]}_{\nu} \delta^{(3)}(\vec{r} - \vec{r}_j)$$

→ We control numerical precision using a **multipole decomposition**

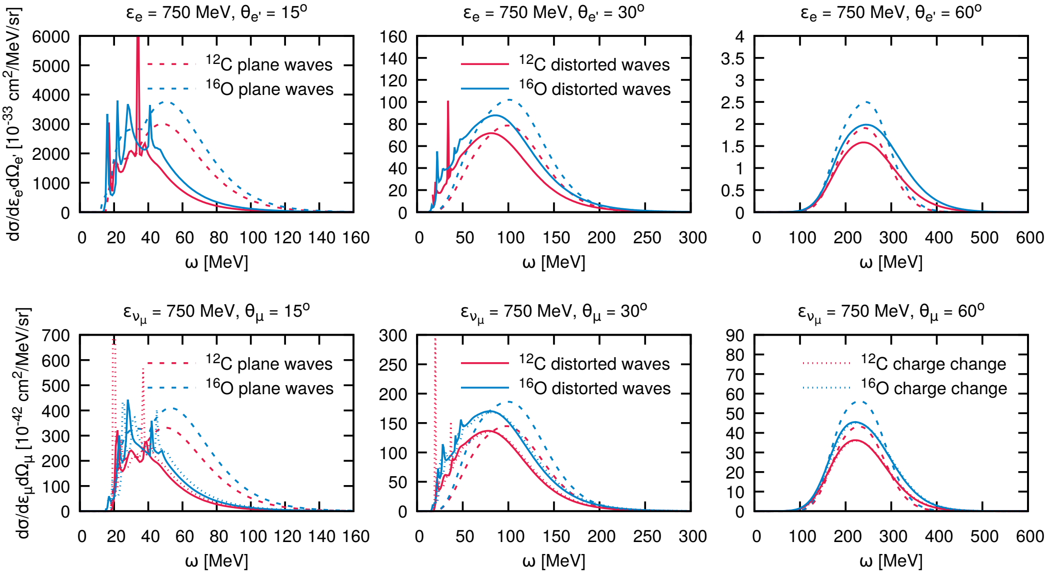


→ Comparing to **inclusive electron scattering data** allows for benchmarking of the model

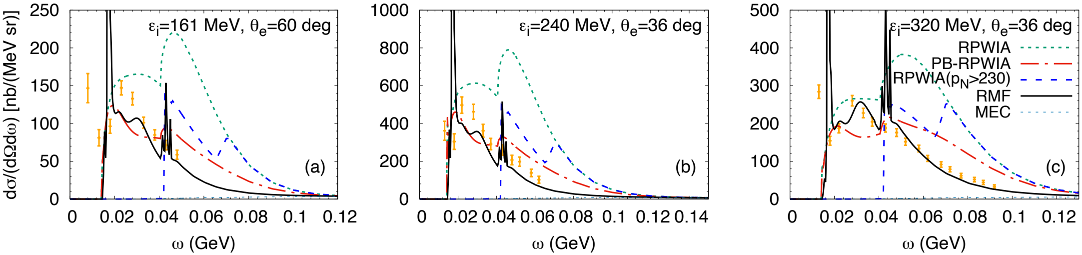
Impulse approximation: electron scattering



Impulse approximation: distorted waves

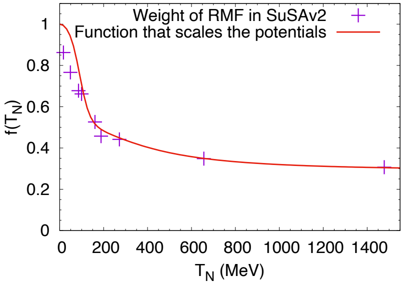


Impulse approximation: distorted waves



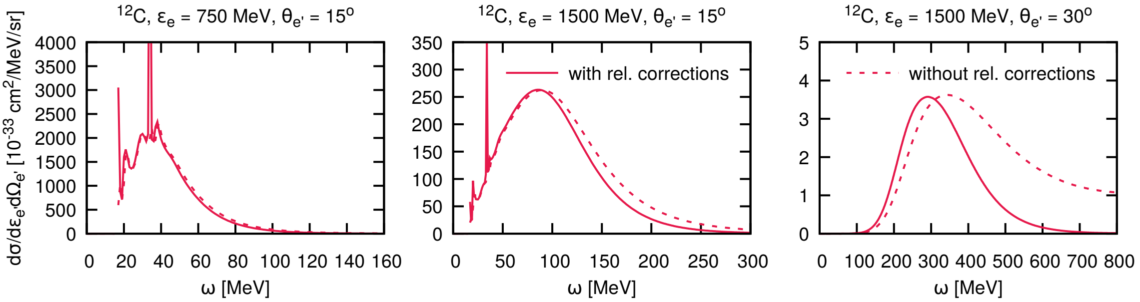
We can try different **phenomenological approaches**:

- hard **Pauli blocking** cuts
- trying to **orthogonalize wave functions**
- **momentum dependence** using plane waves



R. González-Jiménez et al., Phys.Rev. C 100 (2019), 045501

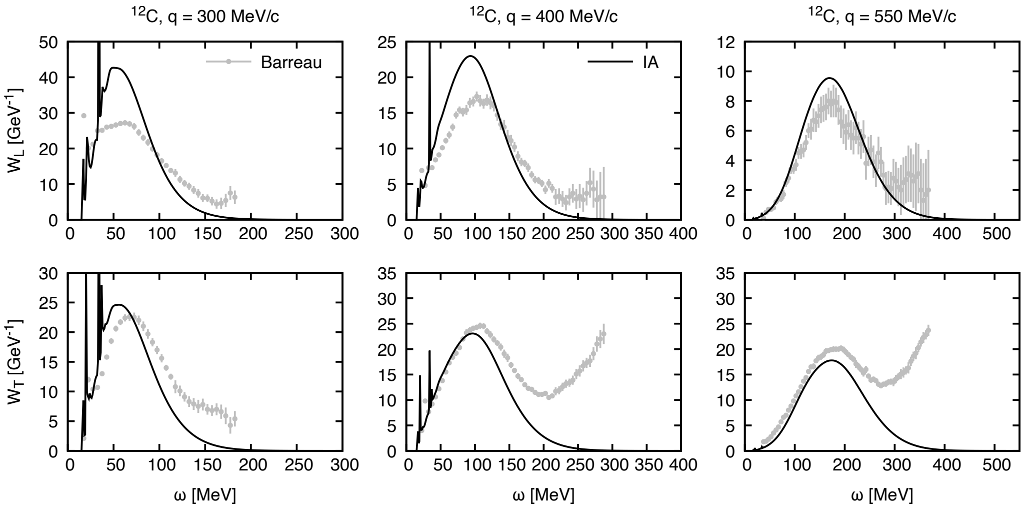
Impulse approximation: relativistic corrections



Fixing the **relativistic** position of the **quasielastic peak**

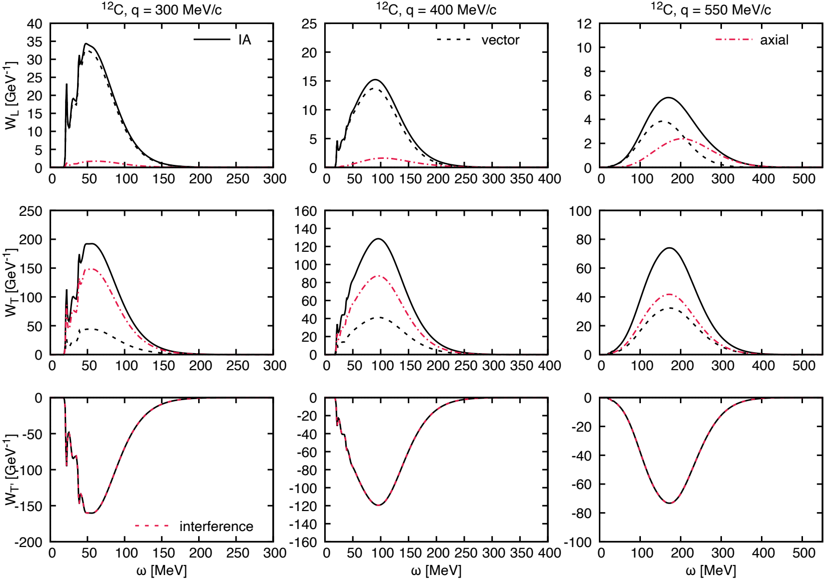
$$\omega \to \omega \left(1 + \frac{\omega}{2M_N}\right), \quad \text{then} \quad \omega_{QE} = \frac{|\vec{q}|^2}{2M_N} \to \frac{Q^2}{2M_N}$$

Impulse approximation: electron scattering



→ Calculation using **one-body currents** is fairly accurate

Impulse approximation: neutrino scattering

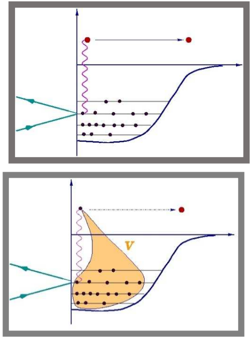


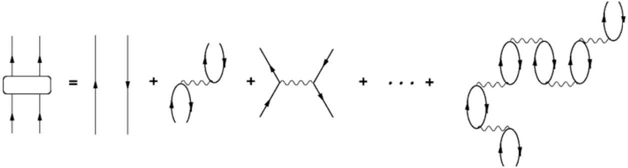
Corrections and additional dynamics

Continuum random-phase approximation

CRPA

- Green's function approach
- Skyrme SkE2 residual interaction
- self-consistent calculations

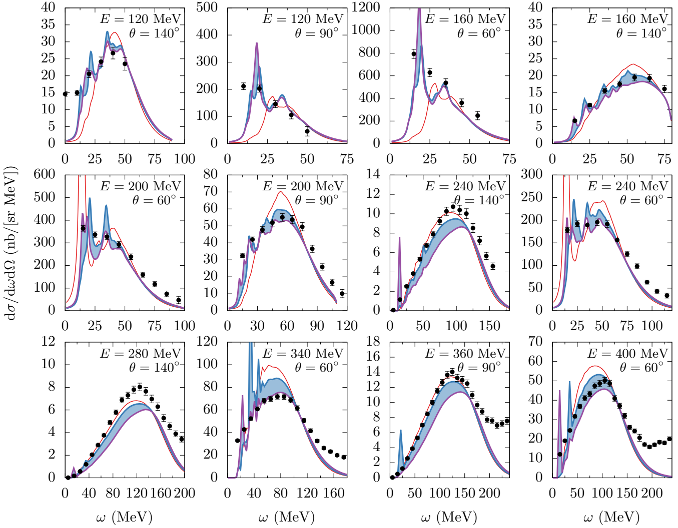




$$|\Psi_{RPA}\rangle = \sum_c \left\{ X_{(\Psi,C)} \left|ph^{-1}\right\rangle - Y_{(\Psi,C)} \left|hp^{-1}\right\rangle \right\}$$

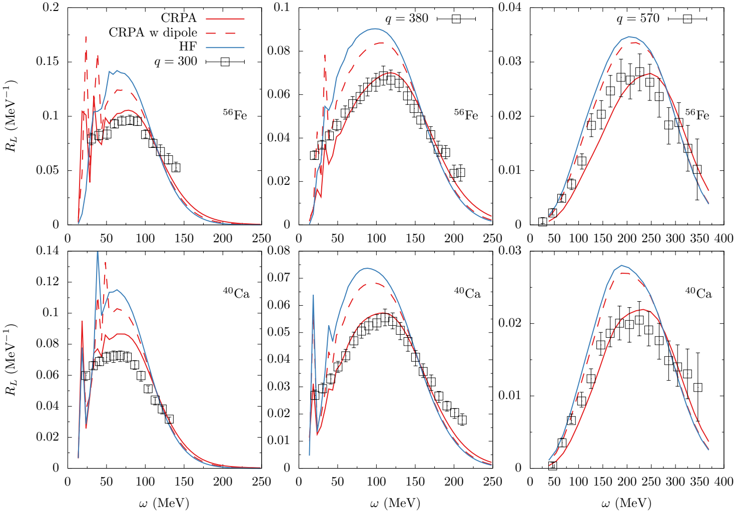
N. Jachowicz, NuSTEC School 2017

Continuum random-phase approximation



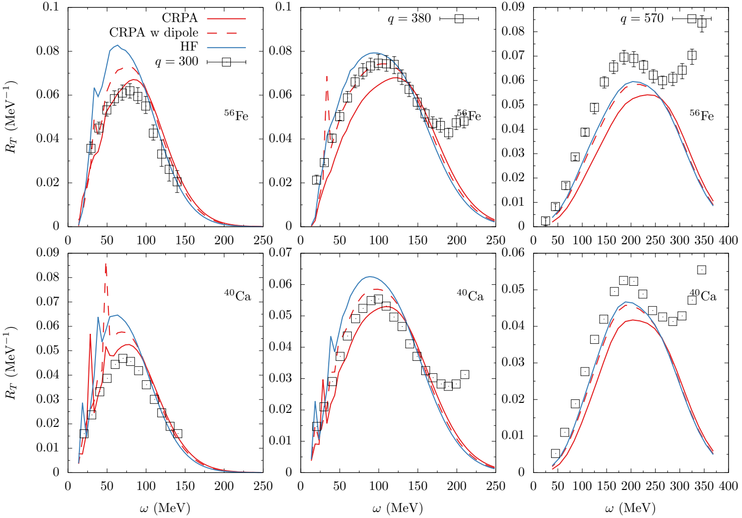
N. Jachowicz, A. Nikolakopoulos, Eur.Phys.J.ST 230 (2021), 4339-4356

Continuum random-phase approximation



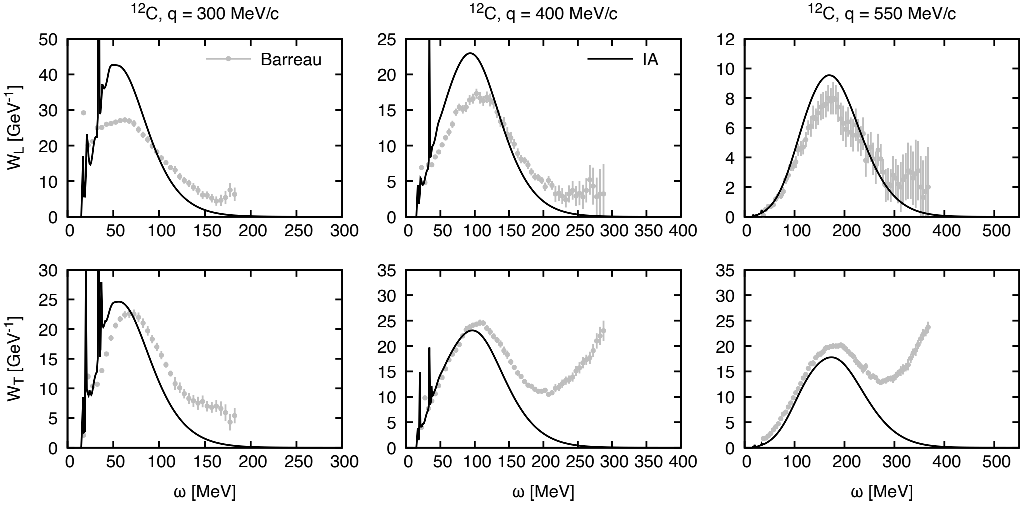
N. Jachowicz, A. Nikolakopoulos, Eur.Phys.J.ST 230 (2021), 4339-4356

Continuum random-phase approximation



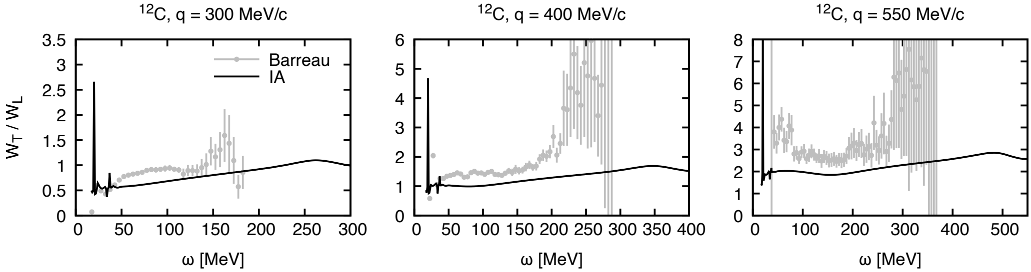
N. Jachowicz, A. Nikolakopoulos, Eur.Phys.J.ST 230 (2021), 4339-4356

Impulse approximation: electron scattering



→ Calculation using **one-body currents** is fairly accurate

Impulse approximation: electron scattering



→ Overestimation of the longitudinal and the underestimation of the transverse responses

Short-range correlations

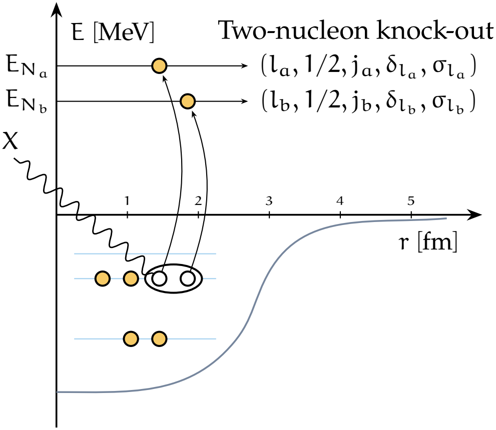
→ Nucleons with strongly **overlapping wave functions** for a short period of time

$$\hat{\mathcal{J}}^{\text{eff}}_{\nu} \simeq \sum_{i=1}^{A} \hat{\mathcal{J}}^{[1]}_{\nu}(i) + \sum_{i<j}^{A} \hat{\mathcal{J}}^{[1],\text{SRC}}_{\nu}(i,j)$$

with

$$\hat{\mathcal{J}}^{[1],\text{SRC}}_{\nu}(i,j) = \left[\hat{\mathcal{J}}^{[1]}_{\nu}(i) + \hat{\mathcal{J}}^{[1]}_{\nu}(j)\right] \hat{l}(i,j)$$

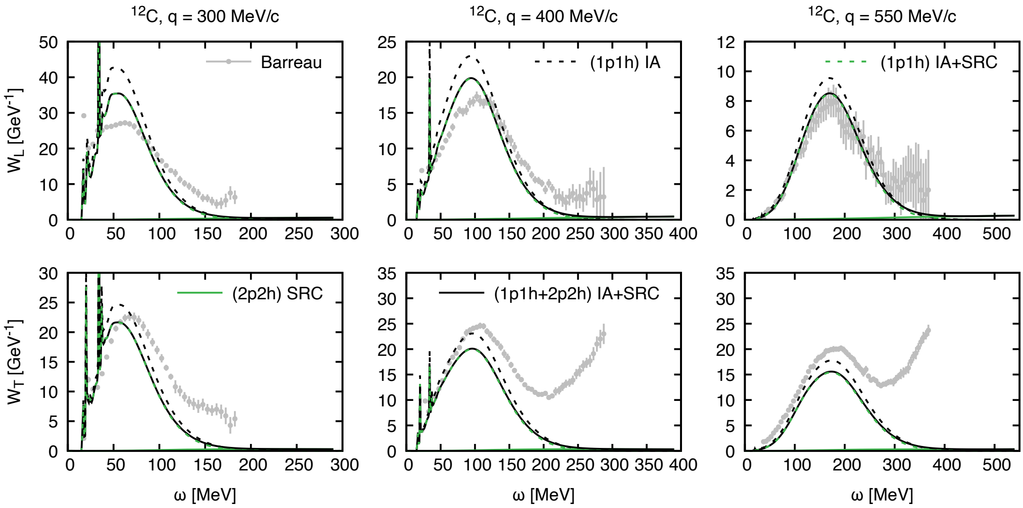
→ The correlation operator $\hat{l}(i,j)$ includes **central**, **tensor**, and **spin-isospin correlations**



→ First corrections to the **independent-particle model** picture for 1p1h

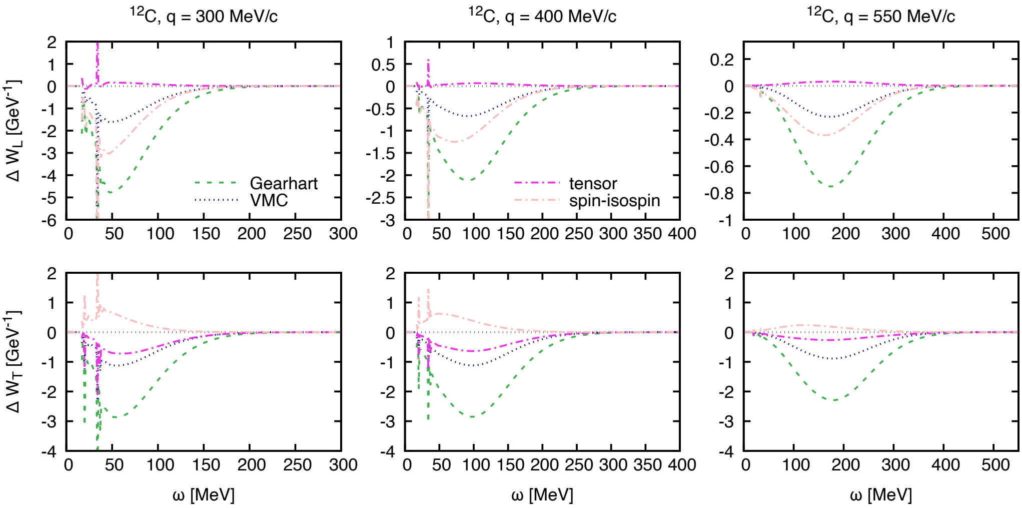
→ **Two-body currents** also leading to **two-nucleon knock-out** reactions

Short-range correlations: electron scattering



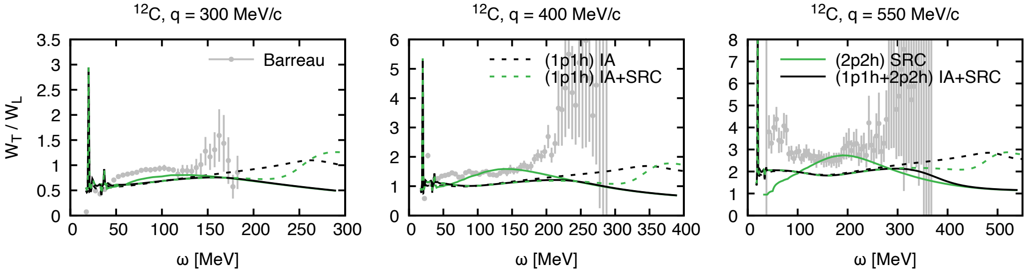
→ Significant **reduction of the 1p1h strength** and a minor 2p2h contribution

Short-range correlations: electron scattering



→ Interplay between different correlation effects

Short-range correlations: electron scattering

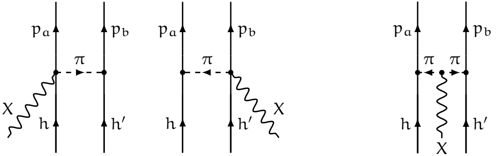


→ Including correlation effects does not fix the ratio

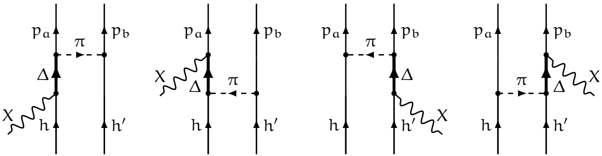
Meson-exchange currents

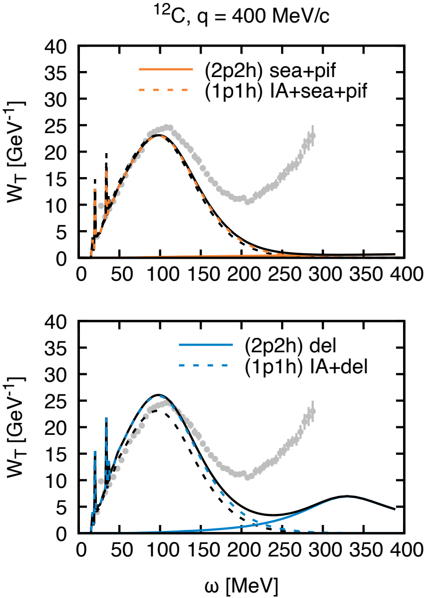
Explicit **two-body currents** contributing to both **1p1h** and **2p2h** final-states:

→ **Seagull** currents → **Pion-in-flight** current

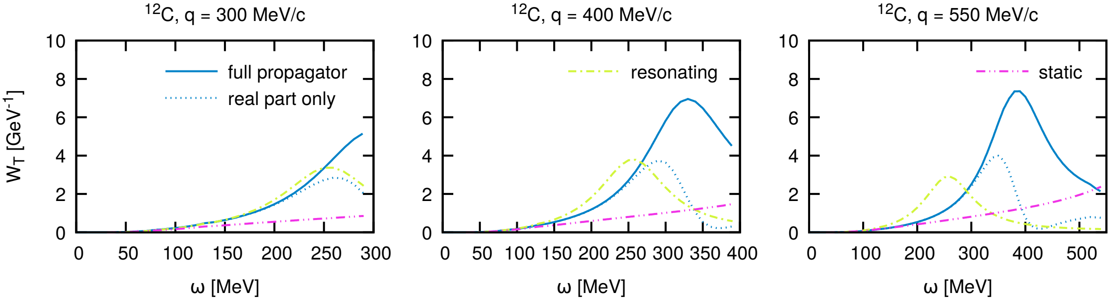


→ **Δ-isobar** degrees of freedom





Delta currents



Full propagators

$$G_\Delta^{res} = \frac{2M_\Delta}{M_\Delta^2 - s - iM_\Delta\Gamma_\Delta^{res} + 2M_\Delta V_\Delta}$$

$$G_\Delta^{nres} = \frac{2M_\Delta}{M_\Delta^2 - u}$$

Static approximation

$$G_\Delta^{res} = \frac{1}{M_\Delta - M_N}$$

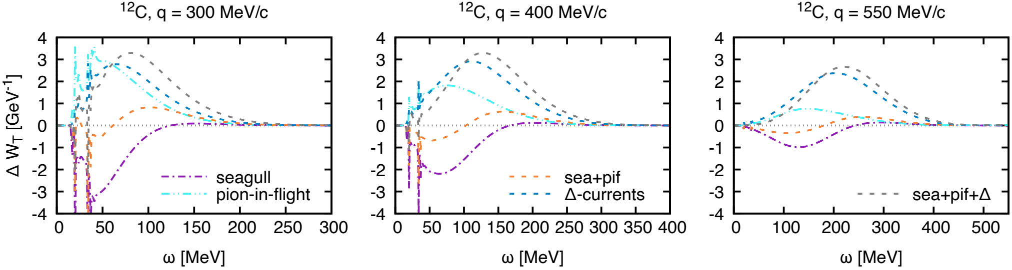
$$G_\Delta^{nres} = \frac{1}{M_\Delta - M_N}$$

Resonating approximation

$$G_\Delta^{res} + G_\Delta^{nres} = \frac{1}{M_\Delta - M_N - \omega - \frac{i}{2}\Gamma_\Delta^{res}}$$

$$G_\Delta^{res} - G_\Delta^{nres} = 0 \qquad + \frac{1}{M_\Delta - M_N + \omega}$$

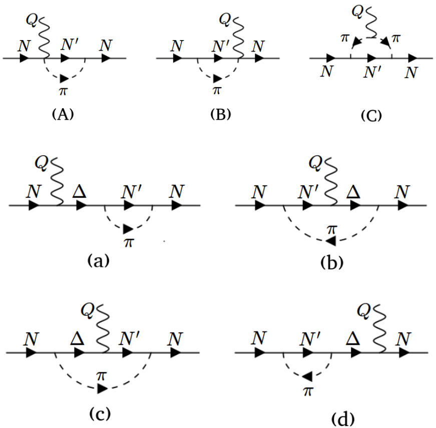
Meson-exchange currents: electron scattering



→ Meson-exchange currents **enhance the transverse response**

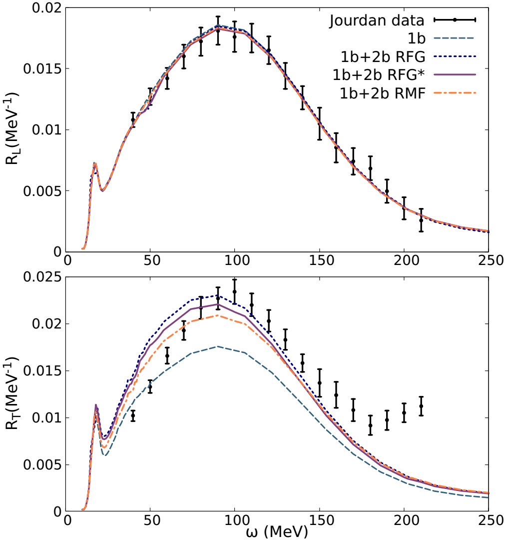
Meson-exchange currents in RMF

Meson-exchange currents derived from **ChPT**:

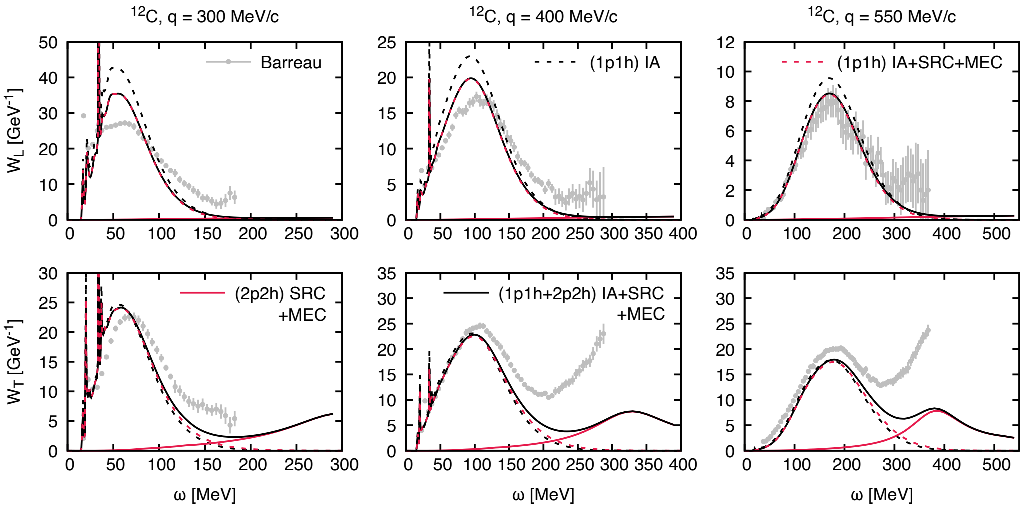


→ plus **spectroscopic factors**

T. Franco-Munoz et al., Phys.Rev. C 108 (2023), 064608

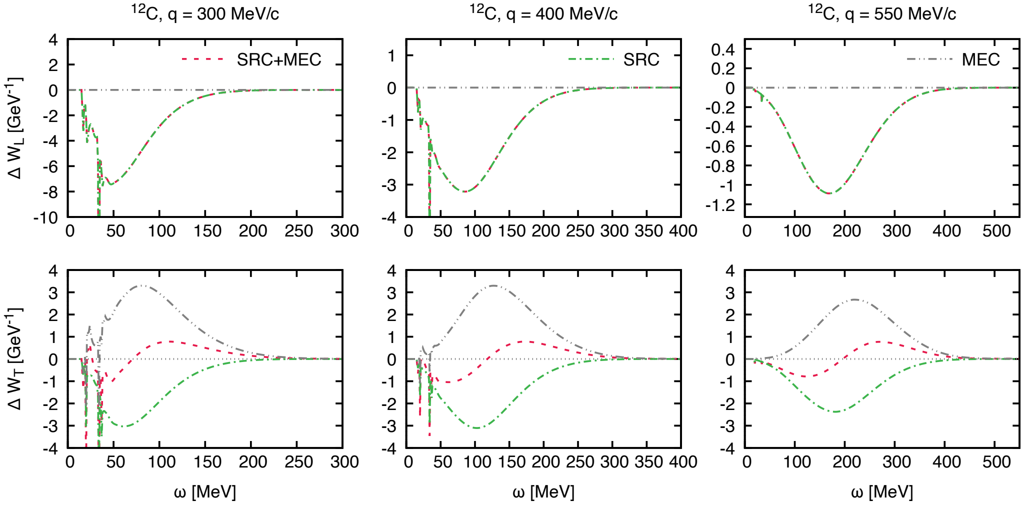


Consistent modeling of two-body currents: electron scattering



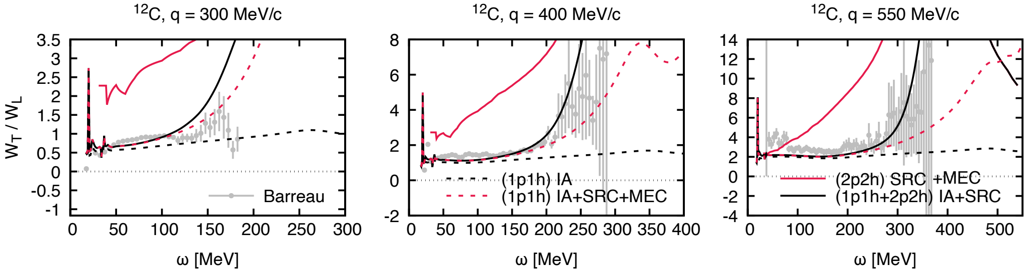
→ **Coherent sum of SRC and MEC** enhances our predictions

Consistent modeling of two-body currents: electron scattering



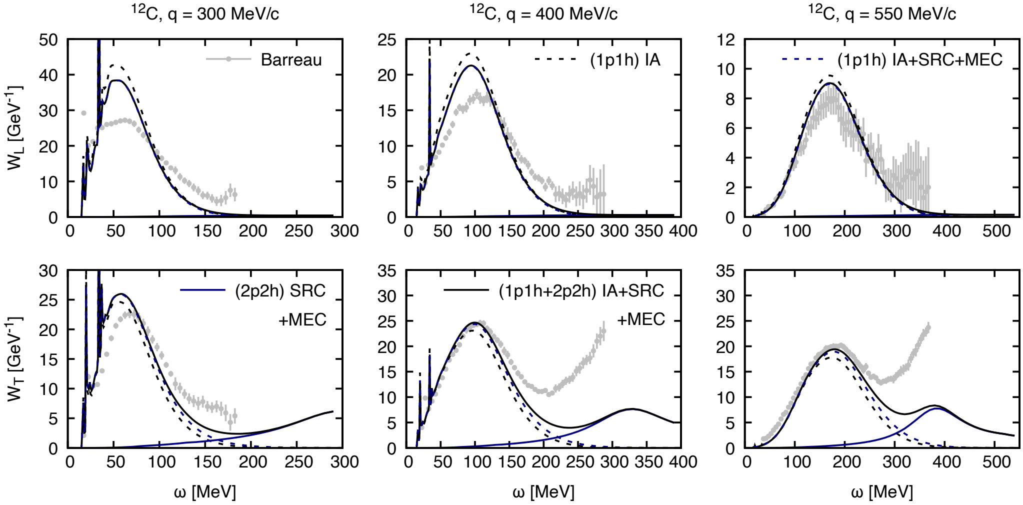
→ Interplay between SRC and MEC effects in the transverse response

Consistent modeling of two-body currents: electron scattering



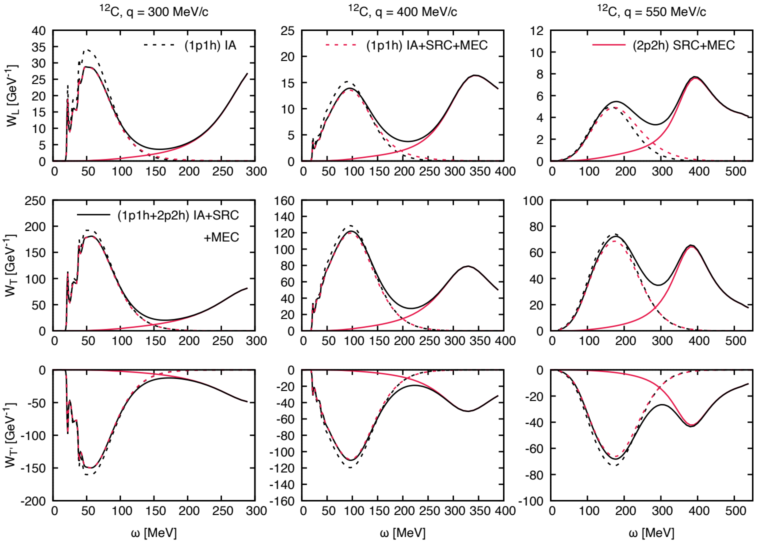
→ Meson-exchange currents are neccessary to fix the ratio

Consistent modeling of two-body currents: electron scattering



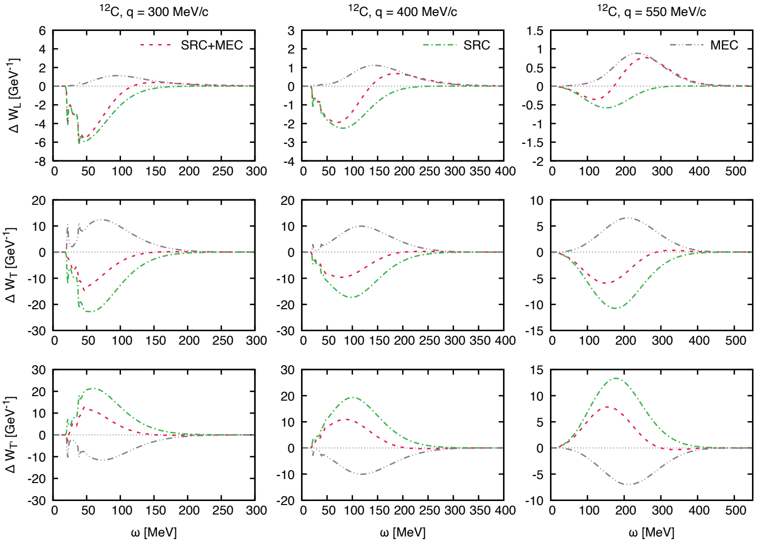
→ **Softer correlations** enhance the comparison for larger momentum transfer

Consistent modeling of two-body currents: neutrino scattering



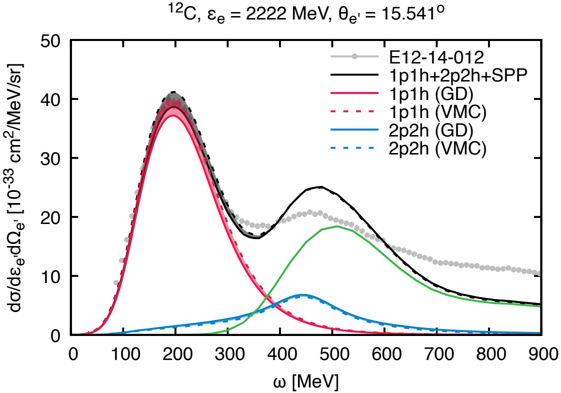
→ **Pronounced Δ peaks** for both longitudinal and transverse responses

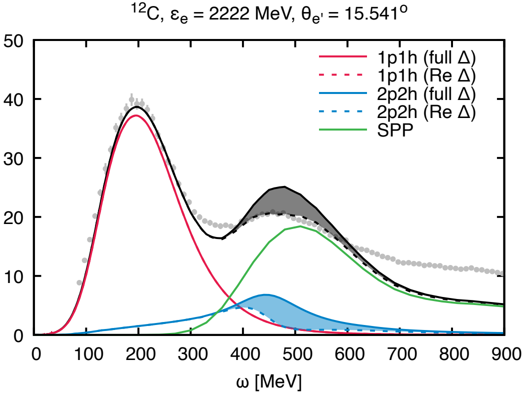
Consistent modeling of two-body currents: neutrino scattering



→ The enhancement appears only in the longitudinal response

JLab Hall A data

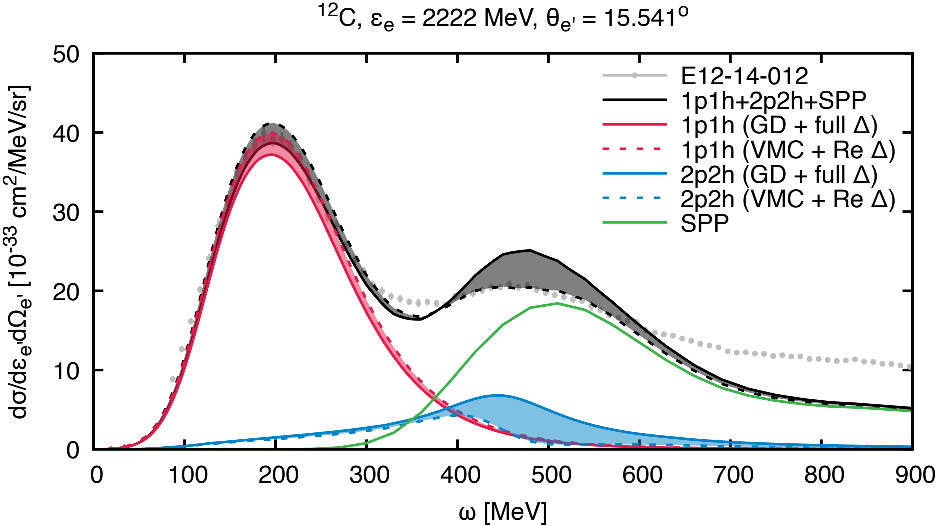




→ The choice of the different **central correlation functions** modifies the **QE peak strength** (GD–stronger, VMC–weaker)

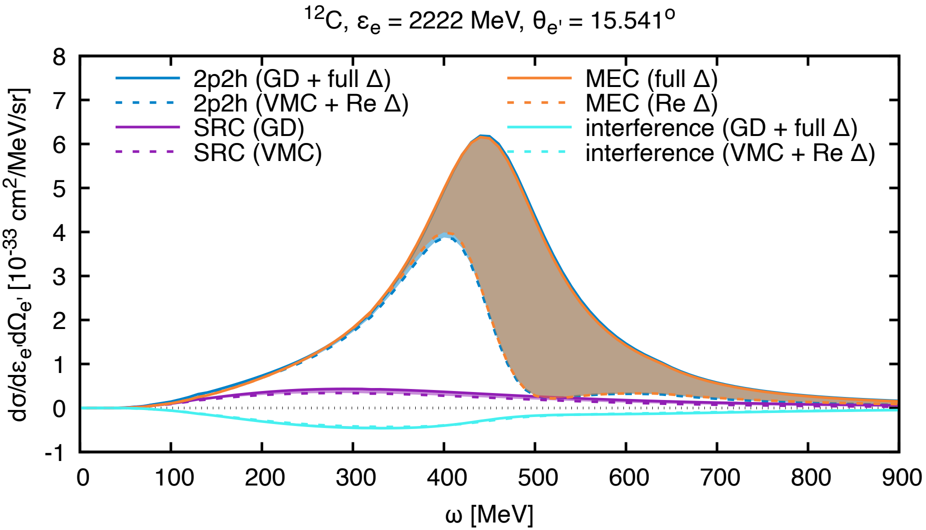
→ Modifying the Δ-propagator governs the **overlap between MEC and SPP** around the Δ peak (Re Δ–only the real part)

JLab Hall A data



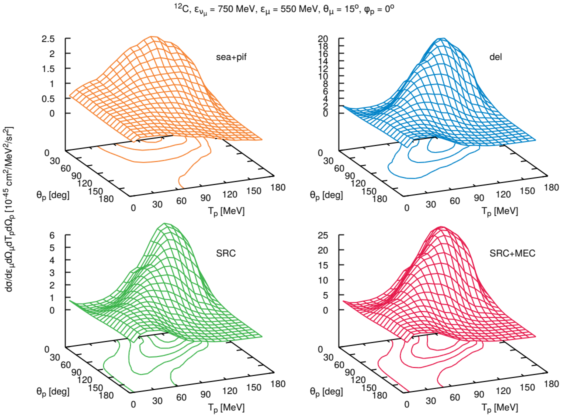
→ Combining variation in given d.f. provides **flexibility in describing QE and Δ peaks**

JLab Hall A data



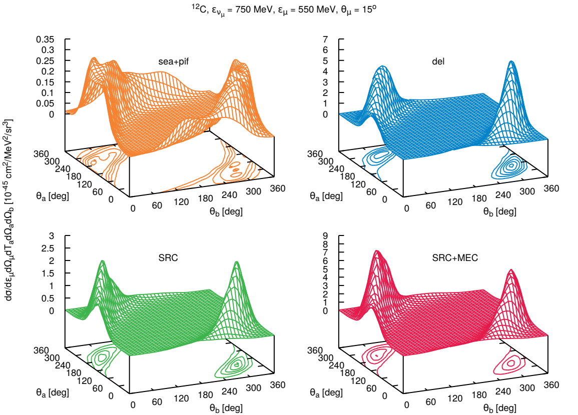
→ **Interferences** are vital in correct interpretation of scattering cross sections

Going more exclusive... in neutrino scattering



Semi-inclusive two-nucleon knock-out

Exclusive two-nucleon knock-out



Summary I

- One-nucleon knock-out:
  - → **factorized models**: PWIA, many MC approaches
  - → **unfactorized models**: DWIA, RPWIA, RMF, ...
  - → some **correlations included**, some added
  - → proper treatment of **Pauli blocking** requires an angular momentum base
- Two-nucleon knock-out:
  - → many models are based on **(local) Fermi gas**: Valencia, SuSAv2, ...
  - → some include **correlation currents**, some phenomenological SRCs
  - → many models provide too much strength and modify the Δ propagator
  - → **nobody really knows how to do it right** ...

Summary II

- **Nucleons in a central potential** is a natural approach to nuclear physics
- Mean-field framework allows for **realistic distorted-wave calculations**
- We are capable of performing certain **advanced corrections to this model**

  - **In-medium properties** and other dependencies are still largely unknown
  - **Neglecting double-counting and interferences** leads to "Frankenmodels"
  - There is a **long way to implement these models in MCs** in their full complexity

Problem session

Are you interested in nuclear models, modeling neutrino interactions, or Monte Carlo generators?

→ we are meeting to solve problems from yesterday's lecture together–**Tuesday after classes**!