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Royal Commission for the Exhibition of 1851

Inelastic scattering; meson production
Lecture 2

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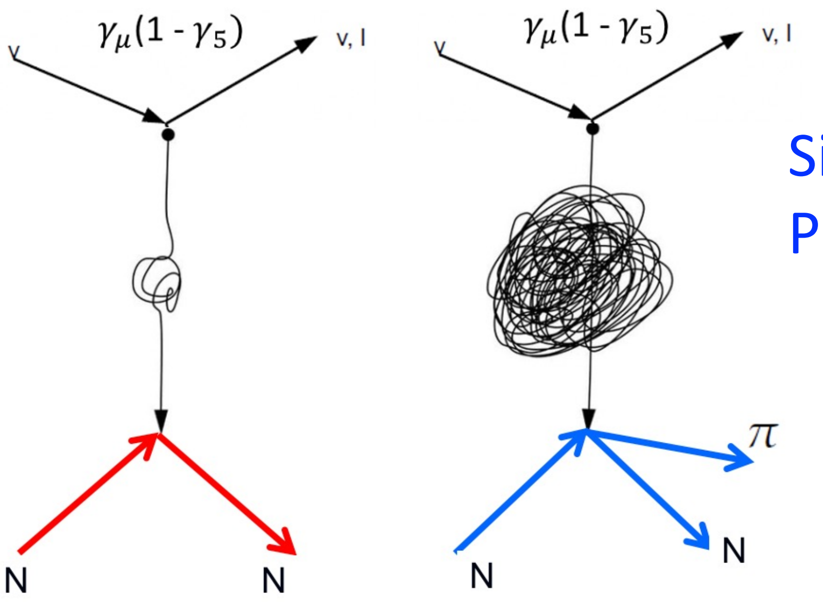
Reminder

- Modelling inelastic interactions are already difficult at nucleon level!

Elastic or Quasi-elastic

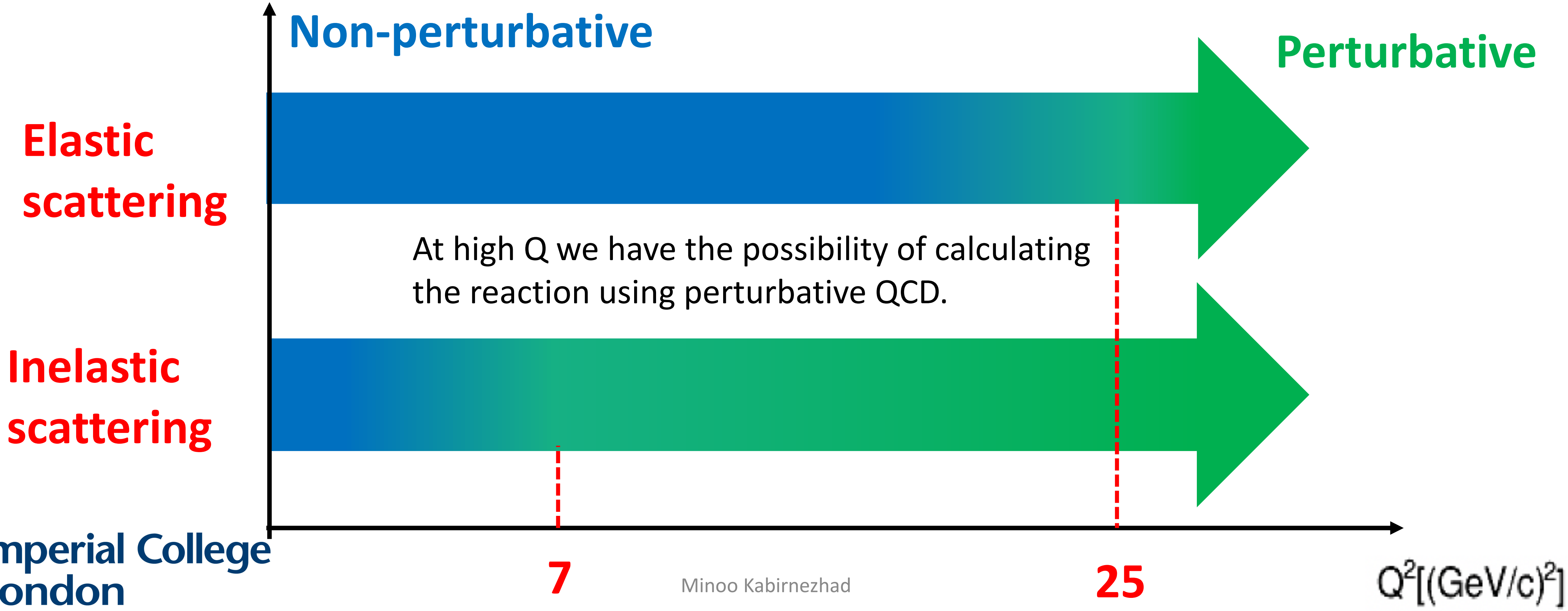


Single Pion Production

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

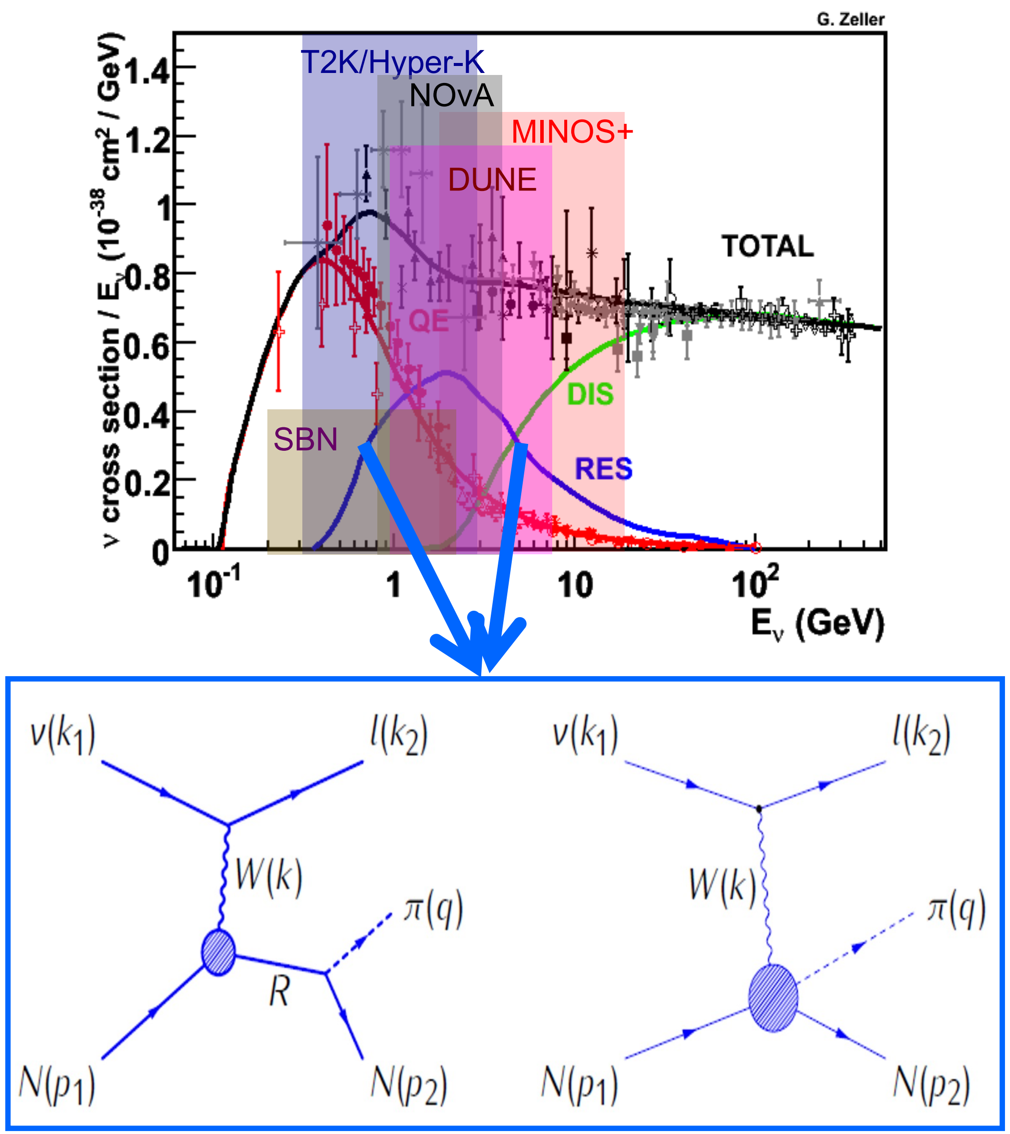
Reminder: Perturbative vs Non-perturbative

- the region of the perturbative QCD applicability is estimated differently in the **elastic** and **inelastic** scattering.



Single pion production

- The production of a single pion is the simplest IE process which starts at a threshold energy of $E_\nu \sim 280$ MeV for a CC $\nu_\mu N \rightarrow \mu N \pi$ channel.
- Single pion can be produced via decay of resonance excitations or non-resonant interactions.
- Their amplitudes must be added coherently.



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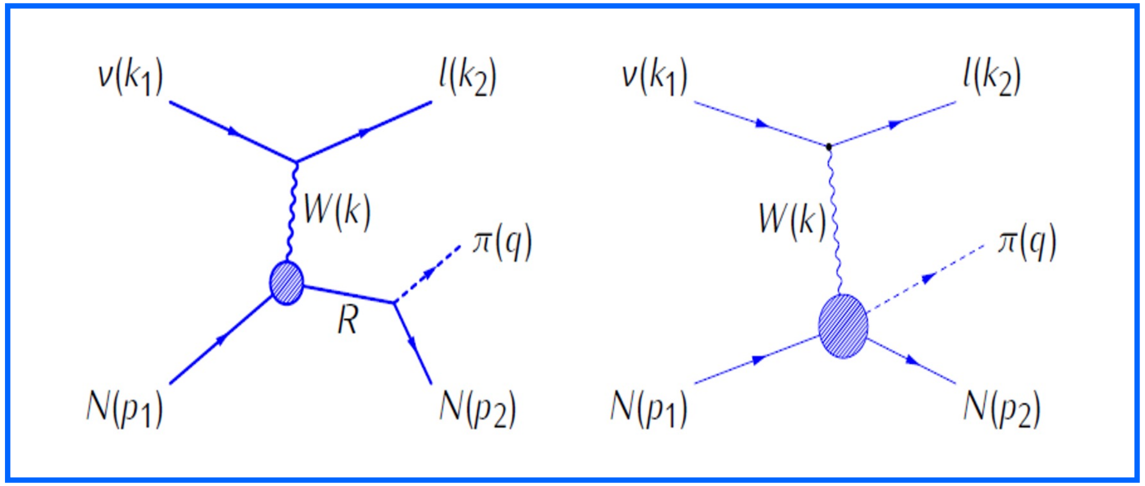
Single pion production channels

	ν	$\bar{\nu}$
CC	$\nu p \to \mu^- p \pi^+$	$\bar{\nu} n \to \mu^+ n \pi^-$
	$\nu n \to \mu^- p \pi^0$	$\bar{\nu} p \to \mu^+ n \pi^0$
	$\nu n \to \mu^- n \pi^+$	$\bar{\nu} p \to \mu^+ p \pi^-$
NC	$\nu p \to \nu p \pi^0$	$\bar{\nu} p \to \bar{\nu} p \pi^0$
	$\nu p \to \nu n \pi^+$	$\bar{\nu} p \to \bar{\nu} n \pi^+$
	$\nu n \to \nu n \pi^0$	$\bar{\nu} n \to \bar{\nu} n \pi^0$
	$\nu n \to \nu p \pi^-$	$\bar{\nu} n \to \bar{\nu} p \pi^-$

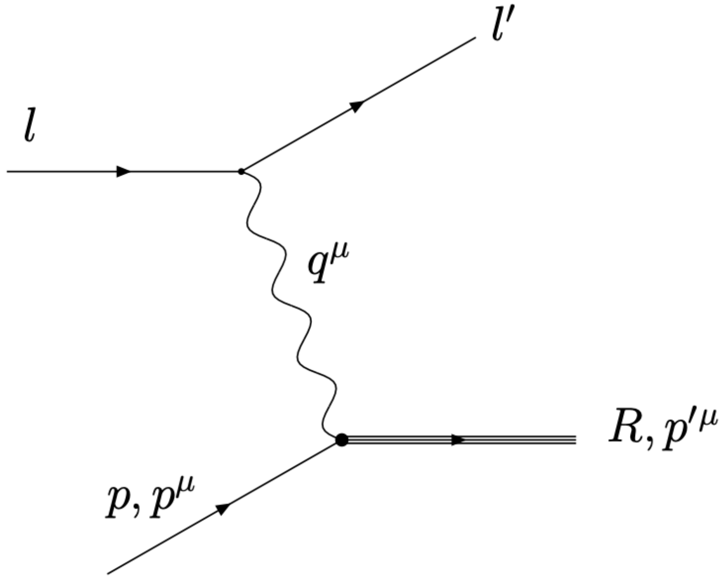


Resonance production

- $\nu_l N \rightarrow lR$
- Resonance mass: M_R
- Resonance width: $\Gamma 0$
- $R = l_{2I,2J}(M_R)$,

$$S \equiv l = 0,$$
$$P \equiv l = 1,$$
$$\text{etc}$$

- Parity: $P = -(-1)^l$



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Resonance production (spin ½)

- $J^{\mu}_{1/2} = \bar{u}(p')\Gamma^{\mu}_{1/2}u(p)$
  - For positive parity: $\Gamma^{\mu}_{1/2+} = \left(\mathcal{V}^{\mu}_{1/2} - \mathcal{A}^{\mu}_{1/2}\right)\mathbb{I}$
  - For negative parity: $\Gamma^{\mu}_{1/2-} = \left(\mathcal{V}^{\mu}_{1/2} - \mathcal{A}^{\mu}_{1/2}\right)\gamma^{5}$

- $\mathcal{V}^{\mu}_{1/2} = \frac{\mathcal{F}_1}{(2M)^2}(Q^2\gamma^{\mu} + ꝗq^{\mu}) + \frac{\mathcal{F}_2}{2M}i\sigma^{\mu\nu}q_{\nu}$
- $-\mathcal{A}^{\mu}_{1/2} = \mathcal{F}_A\gamma^{\mu}\gamma^{5} + \frac{\mathcal{F}_p}{M}q^{\mu}\gamma^{5}$

PCAC hypothesis for Spin 1/2

- $-\mathcal{A}^{\mu}_{1/2} = \mathcal{F}_A \gamma^{\mu}\gamma^5 + \frac{\mathcal{F}_p}{M} q^{\mu}\gamma^5$
- Goldberger-Treiman relation (PCAC): $\mathcal{F}_p(Q^2) = \frac{M(M+M_R)}{Q^2+m_\pi^2}\mathcal{F}_A(Q^2)$
- For the axial current, it utilises PCAC principles and pion scattering data.

$$\left.\frac{d\sigma}{dQdW}\right|_{Q^2=0} \propto \sigma(\pi N \rightarrow \pi N)$$

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Isospin relation for vector FF (spin ½)

- $\mathcal{V}^{\mu}_{1/2} = \frac{\mathcal{F}_1}{(2M)^2}(Q^2\gamma^{\mu} + \not{q}q^{\mu}) + \frac{\mathcal{F}_2}{2M} i\sigma^{\mu\nu}q_{\nu}$

	replace $\mathcal{F}_i$ in Eq. (5.5) with	
	for $I = 1/2$	for $I = 3/2$
$e^- p \to e^- R^+$	F_i^p	F_i^N
$e^- n \to e^- R^0$	F_i^n	F_i^N
$\nu p \to \ell^- R^{++}$	-	$\sqrt{3}F_i^V = -\sqrt{3}F_i^N$
$\nu n \to \ell^- R^+$	$F_i^V = F_i^p - F_i^n$	$F_i^V = -F_i^N$
$\nu p \to \nu R^+$	$\tilde{F}_i^p = (\frac{1}{2} - 2\sin^2\theta_W)F_i^p - \frac{1}{2}F_i^n - \frac{1}{2}F_i^s$	$\tilde{F}_i^N = (1 - 2\sin^2\theta_W)F_i^N$
$\nu n \to \nu R^0$	$\tilde{F}_i^n = (\frac{1}{2} - 2\sin^2\theta_W)F_i^n - \frac{1}{2}F_i^p - \frac{1}{2}F_i^s$	$\tilde{F}_i^N = (1 - 2\sin^2\theta_W)F_i^N$

From Tina Leitner thesis

Resonance production (spin 3/2)

- $J^{\mu}_{3/2} = \bar{\psi}_{\nu}(p')\Gamma^{\nu\mu}_{3/2}u(p)$, $\bar{\psi}_{\nu}$ is Rarita-Schwinger spinor for S=3/2 resonances and $\Gamma^{\nu\mu}_{3/2}$ is the weak $WNR_{3/2}$vertex
  - For positive parity: $\Gamma^{\nu\mu}_{3/2+} = \left(\mathcal{V}^{\nu\mu}_{3/2} - \mathcal{A}^{\nu\mu}_{3/2}\right)\gamma^5$
  - For negative parity: $\Gamma^{\mu}_{3/2-} = \left(\mathcal{V}^{\mu}_{3/2} - \mathcal{A}^{\mu}_{3/2}\right)\mathbb{1}$

- $\mathcal{V}^{\nu\mu}_{3/2} = \frac{\mathcal{C}^V_3}{M}(g^{\nu\mu}\not{q} - q^{\nu}\gamma^{\mu}) + \frac{\mathcal{C}^V_4}{M^2}(g^{\nu\mu}q.p' - q^{\nu}p'^{\mu}) + \frac{\mathcal{C}^V_5}{M^2}(g^{\nu\mu}q.p - q^{\nu}p^{\mu}) + g^{\nu\mu}\mathcal{C}^V_6$
- $-\mathcal{A}^{\mu}_{3/2} = [\frac{\mathcal{C}^A_3}{M}(g^{\nu\mu}\not{q} - q^{\nu}\gamma^{\mu}) + \frac{\mathcal{C}^A_4}{M^2}(g^{\nu\mu}q.p' - q^{\nu}p'^{\mu}) + \mathcal{C}^A_5 g^{\nu\mu} + \frac{\mathcal{C}^V_6}{M^2}q^{\nu}q^{\mu}]\gamma^5$

Isospin relation for vector FF (spin 3/2)

- $\mathcal{V}_{3/2}^{\nu\mu} = \frac{\mathcal{C}_3^V}{M}(g^{\nu\mu}\not{q} - q^\nu\gamma^\mu) + \frac{\mathcal{C}_4^V}{M^2}(g^{\nu\mu}q.p' - q^\nu p'^\mu) + \frac{\mathcal{C}_5^V}{M^2}(g^{\nu\mu}q.p - q^\nu p^\mu) + g^{\nu\mu}\mathcal{C}_6^V$

	replace C_i^V in Eq. (5.14) with	
	for $I = 1/2$	for $I = 3/2$
$e^- p \to e^- R^+$	C_i^p	C_i^N
$e^- n \to e^- R^0$	C_i^n	C_i^N
$\nu p \to \ell^- R^{++}$	-	$\sqrt{3}C_i^V = -\sqrt{3}C_i^N$
$\nu n \to \ell^- R^+$	$C_i^V = C_i^p - C_i^n$	$C_i^V = -C_i^N$
$\nu p \to \nu R^+$	${}_p\tilde{C}_i^V = (\frac{1}{2} - 2\sin^2\theta_W)C_i^p - \frac{1}{2}C_i^n - \frac{1}{2}C_i^{s_V}$	${}_N\tilde{C}_i^V = (1 - 2\sin^2\theta_W)C_i^N$
$\nu n \to \nu R^0$	${}_n\tilde{C}_i^V = (\frac{1}{2} - 2\sin^2\theta_W)C_i^n - \frac{1}{2}C_i^p - \frac{1}{2}C_i^{s_V}$	${}_N\tilde{C}_i^V = (1 - 2\sin^2\theta_W)C_i^N$

CVC and PCAC hypothesis for Spin 3/2

- $\mathcal{V}^{\nu\mu}_{3/2} = \frac{\mathcal{C}^V_3}{M}(g^{\nu\mu}\not{q} - q^\nu\gamma^\mu) + \frac{\mathcal{C}^V_4}{M^2}(g^{\nu\mu}q.p' - q^\nu p'^\mu) + \frac{\mathcal{C}^V_5}{M^2}(g^{\nu\mu}q.p - q^\nu p^\mu) + g^{\nu\mu}\mathcal{C}^V_6$
- $-\mathcal{A}^{\mu}_{3/2} = [\frac{\mathcal{C}^A_3}{M}(g^{\nu\mu}\not{q} - q^\nu\gamma^\mu) + \frac{\mathcal{C}^A_4}{M^2}(g^{\nu\mu}q.p' - q^\nu p'^\mu) + \mathcal{C}^A_5 g^{\nu\mu} + \frac{\mathcal{C}^V_6}{M^2}q^\nu q^\mu]\gamma^5$
- CVC: $\mathcal{C}^V_6 = 0$
- Goldberger-Treiman relation (PCAC): $\mathcal{C}^A_6(Q^2) = \frac{M^2}{Q^2+m_\pi^2}\mathcal{C}^A_5(Q^2)$
- For the axial current, it utilises PCAC principles and pion scattering data.

$$\frac{d\sigma}{dQdW}\Big|_{Q^2=0} \propto \sigma(\pi N \rightarrow \pi N)$$

Resonances with spin > 3/2

- Any formalism describing resonances with spin greater than 3/2 is highly complicated
- In most of the models, resonance with spin>3/2 either ignore or treated with spin 3/2 formalism.

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
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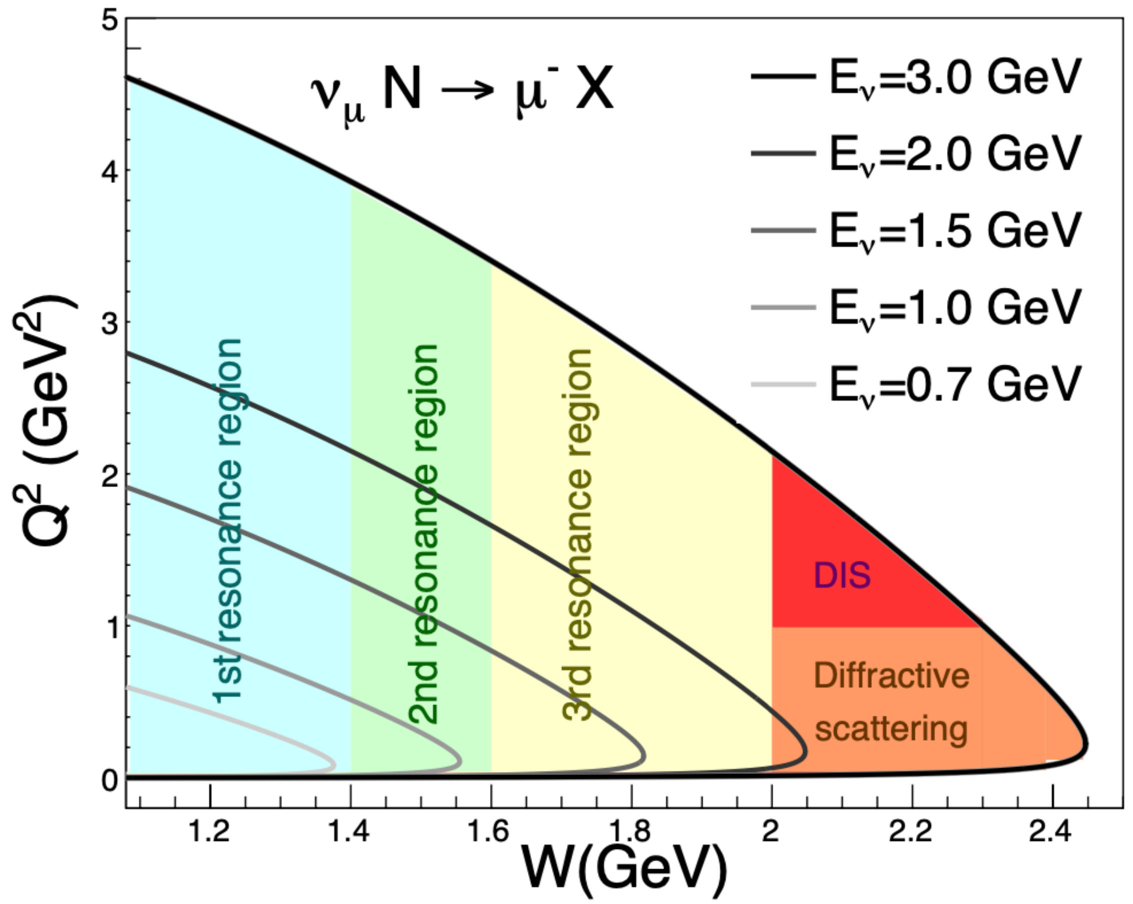
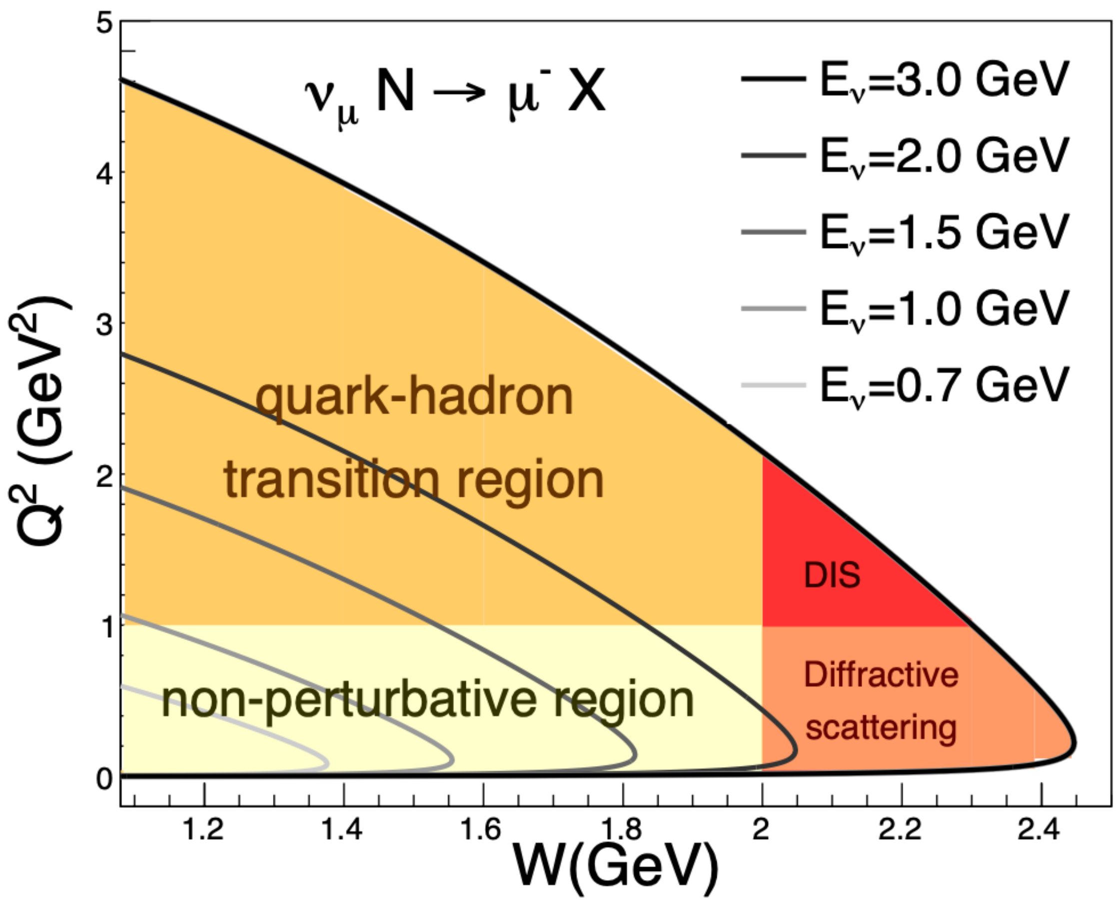
How to define form factors in weak interaction?

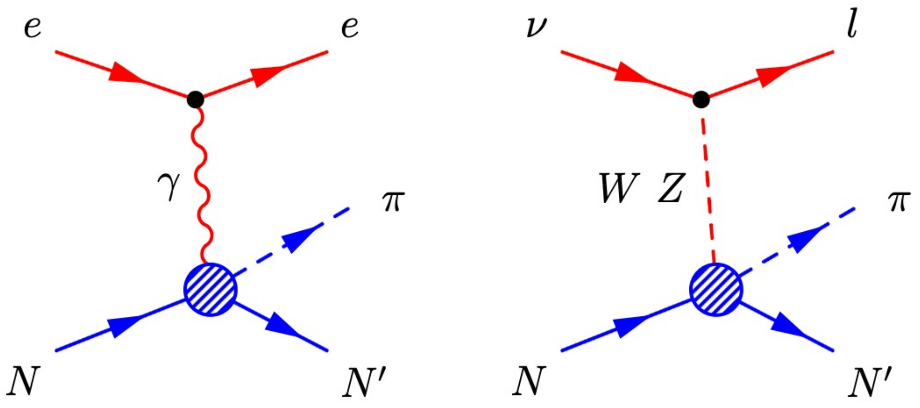
Reminder: so many processes are going on here and neutrino data is limited.



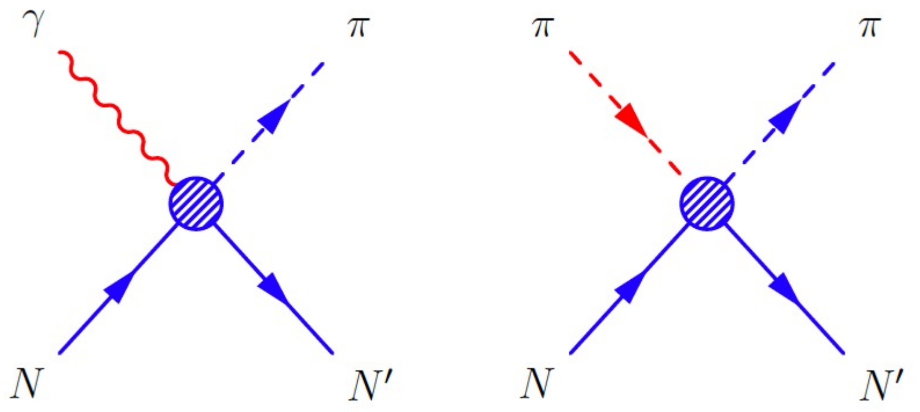


How to define form factors in weak interaction

- Resonance phase space spans both perturbative and non-perturbative regimes, posing modelling challenges.
- Phenomenological models in this region must account for numerous processes and parameters.
- A unified model is essential for interpreting all interactions and maximising data utilisation.



Similar hadronic currents



Existing data

# data point	Photon, electron, pion, Neutrino Channels	Q^2 Range $(GeV/C)^2$	W Range GeV	Form Factors	
≈ 9800	$\gamma\, p \to n + \pi^+\,,\ \ \gamma p \to p + \pi^0$	0	1.08 – 2.0	Proton	Vector
≈ 31000	$ep \to en + \pi^+,\ \ ep \to ep + \pi^0$	0.16 – 6.0	1.08 – 2.0		
≈ 2500	$\gamma n \to p + \pi^-$	0	1.08 – 2.0	Neutron	
≈ 700	NEW $en \to ep + \pi^-$	0.4 – 1.0	1.08 – 1.8		
≈ 400	$\pi^+ p \to p + \pi^+,\ \pi^- p \to p + \pi^-$	0	1.08 – 2.0	Axial-Vector	
<100	$\nu N \to l^- N + \pi\ ,\bar{\nu} N \to l^+ N + \pi$	$Q^2>0$ Integrated	1.08 – 2.0 Integrated		

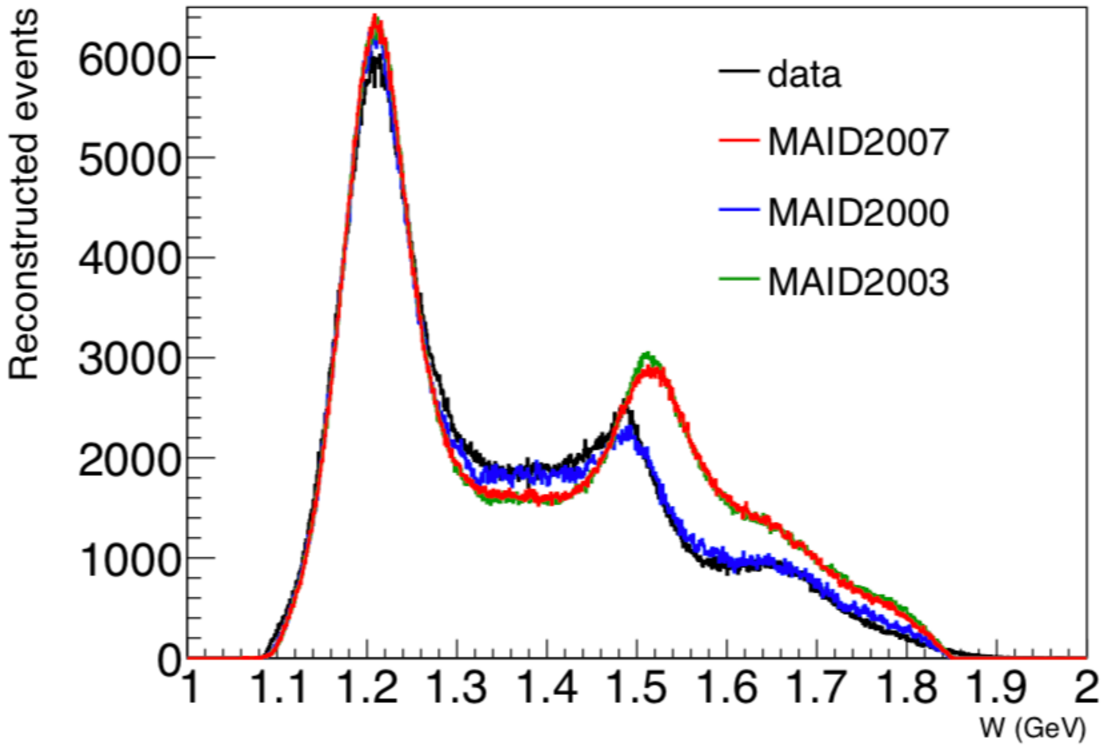
How to define **vector** form factors in weak interaction?

Existing form-factors determination

- The only way to obtain resonance form factors is through experimental data.
- MAID is a unitary isobar model for partial wave analysis on the world data of pion photo and electroproduction in the resonance region.
- The MAID group extracted **helicity amplitudes** for proton and neutron using the MAID model and all available data.
- These helicity amplitudes are used by other neutrino model to extract resonance for factors!

Electron-neutron scattering data

- A First-Time Endeavour!
- Utilisation of Data for Fitting Isospin ½ Resonances (Second Region)
- MAID2007, the latest version, is used by theorists to fit neutron form-factors.
- Neutron and proton form-factors are the same for the Δ resonance.

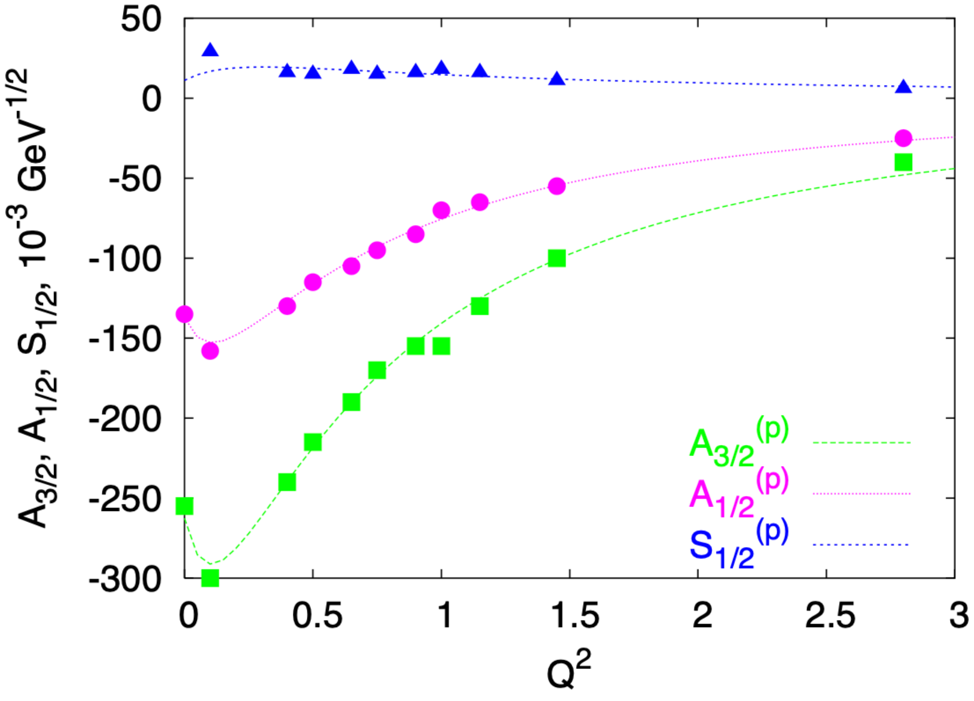


Y. Tian *et al.* [CLAS]
Phys. Rev. C **107** (2023)

Existing form-factors determination

- Helicity amplitudes for spin 3/2

$$A_{3/2} \propto \left\langle R, +\frac{3}{2} \middle| J_{em}.\,\epsilon^{(R)} \middle| N, +\frac{1}{2} \right\rangle$$
$$A_{1/2} \propto \left\langle R, +\frac{1}{2} \middle| J_{em}.\,\epsilon^{(R)} \middle| N, -\frac{1}{2} \right\rangle$$
$$S_{1/2} \propto \left\langle R, +\frac{1}{2} \middle| J_{em}.\,\epsilon^{(S)} \middle| N, +\frac{1}{2} \right\rangle$$



MAID "data": Helicity amplitudes for $P_{33}(1232)$ resonance

Existing form-factors determination

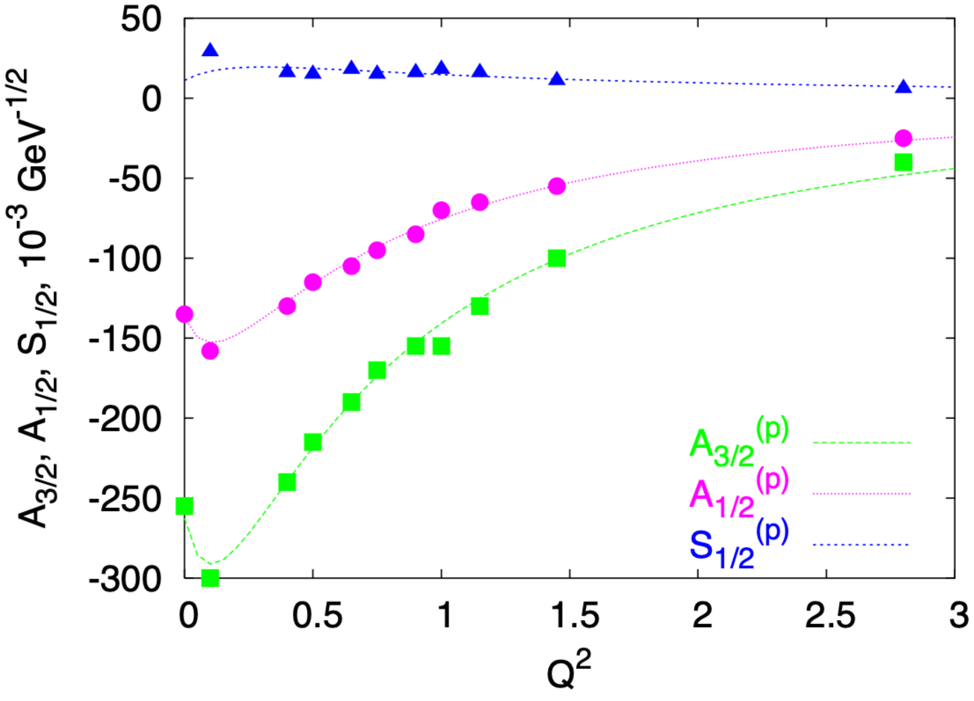
- Some of neutrino models, utilized the helicity amplitudes determined in the MAID analysis to extract form factors.

$$C_3^{(p)} = \frac{2.13/D_V}{1+Q^2/4M_V^2},$$

$$C_4^{(p)} = \frac{-1.51/D_V}{1+Q^2/4M_V^2},$$

$$C_5^{(p)} = \frac{0.48/D_V}{1+Q^2/0.776M_V^2}.$$

$$D_V = \left(1+\frac{Q^2}{M_V^2}\right)^2, \qquad M_V = 0.84\ GeV$$



Fitted model to MAID analysis for $P_{33}(1232)$ resonance.
From [Lalakulich et. al. (2006)]

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Existing form-factors determination

- Other models, use other model's form factors fitted to other model fitted to actual data!

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What can we do better

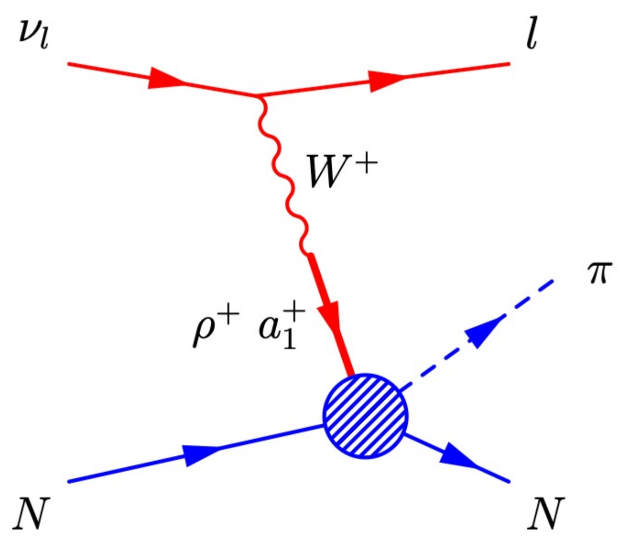
- We know dipole form factor is not the best form factor in a large kinematic region, maybe it is good enough for some resonances and only for low Q^2.
- We need to parametrise form factors and use all theoretical input such as perturbative QCD at high Q^2.
- Meson dominance form factor model allows the desired parametrisation.

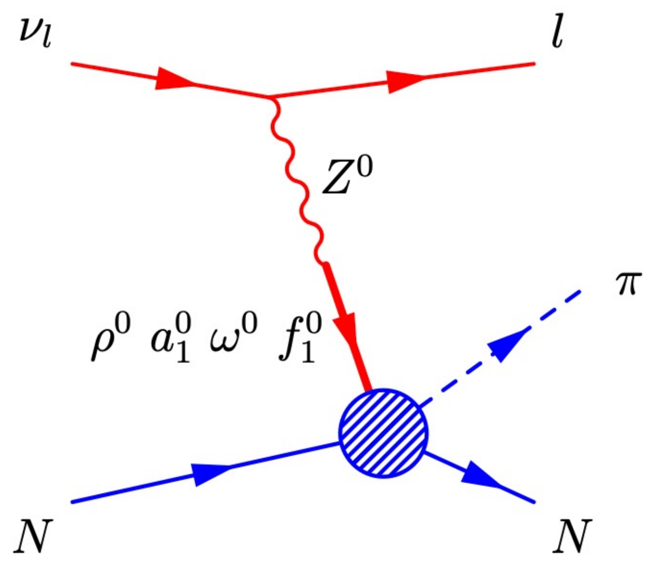
Meson Dominance (MD) model

- The MD model is rooted in the effective Lagrangian of quantum field theory.

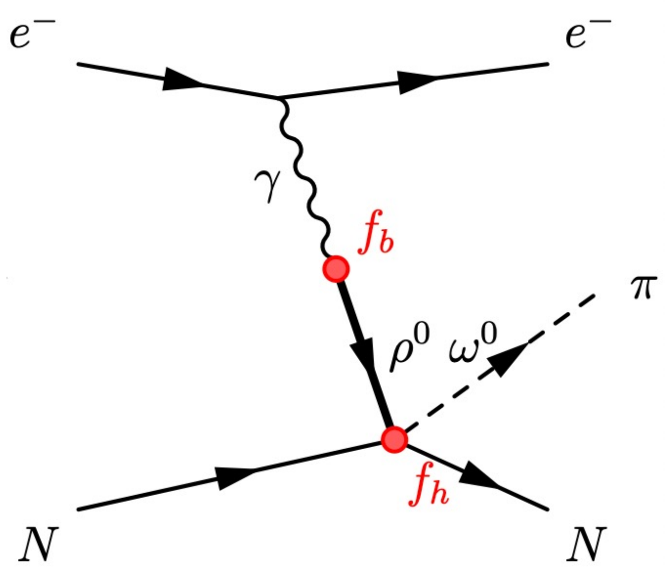
1. J. J. Sakurai, Annals Phys.11, 1 (1960)
2. M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961)

- It establishes connections between vector and axial currents and corresponding meson fields with analogous quantum properties.
- This framework explains the interaction between neutrinos and nucleons through meson exchange.





Meson Dominance (MD) model



- MD form factors can be expressed in terms of the meson masses and the coupling strengths, summing over all possible mesons:

$$\mathrm{F_N}(Q^2) = \sum_{j=1}^{n} \frac{m_j^2}{m_j^2 - Q^2} \left(\frac{f_h}{f_b}\right)$$

- Although they do not inherently comply to the unitarity condition (analytic model) or accurately predict behaviour at high Q^2, they can be **imposed**!

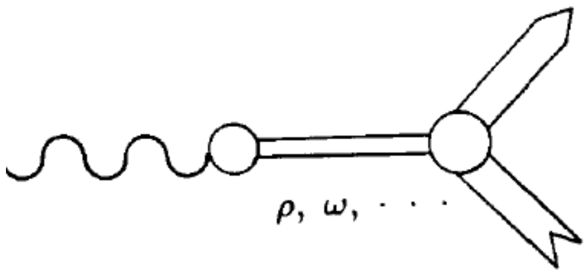
k	ρ -group	$m_{(\rho)k}$[GeV]	ω -group	$m_{(\omega)k}$[GeV]
1	$\rho(770)$	0.77526	$\omega(782)$	0.78265
2	$\rho(1450)$	1.465	$\omega(1420)$	1.410
3	$\rho(1700)$	1.720	$\omega(1650)$	1.670
4	$\rho(1900)$	1.885	$\omega(1960)$	1.960
5	$\rho(2150)$	2.150	$\omega(2205)$	2.205
k	a_1 -group	$m_{(a_1)k}$[GeV]	f_1 -group	$m_{(f_1)k}$[GeV]
1	$a_1(1260)$	1.230	$f_1(1285)$	1.2819
2	$a_1(1420)$	1.411	$f_1(1420)$	1.4263
3	$a_1(1640)$	1.655	$f_1(1510)$	1.518
4	$a_1(2095)$	2.096	$f_1(1970)$	1.1971

C. Adamuscin *et al*. Eur. Phys. J. C 28, 115 (2003)

Meson Dominance (MD) model

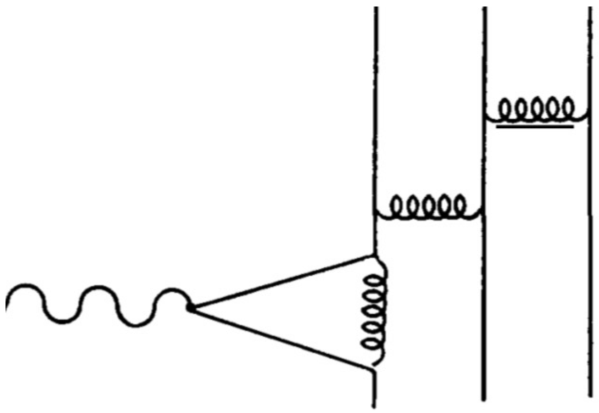
Representation of electron scattering:

Non-perturbative (low Q^2)



- Vector mesons propagate between the virtual photon and the nucleon

Perturbative (high Q^2)



- schematic quark model of VMD model

Asymptotic behaviour of form factor

- At large Q2, resonance form factors must align with the perturbative QCD constraints.
- For spin 3/2 resonance:

G. Vereshkov and N. Volchanskiy (PRD 2007)

$$F_\alpha(Q^2) \cong \left(\frac{4M_N^2}{Q^2}\right)^{p_\alpha} \frac{f_\alpha}{\ln^{n_\alpha}\left({Q^2}/{\Lambda_{QCD}^2}\right)}, \qquad (\alpha = 1-3)$$

$$p_1 = 3, p_2 = p_3 = 4,$$
$$n_3 > n_1 > n_2, \qquad n_1 \cong 3$$

MD form factors used in the model

- For spin 3/2 resonance:

$$F_\alpha(Q^2) = \frac{f_\alpha}{L_\alpha(Q^2)} \sum_{k=1}^{K} \frac{a_{\alpha k}\, m_k^2}{m_k^2 + Q^2}, \qquad (\alpha = 1 - 3)$$

$$\mathrm{L}_\alpha(\mathrm{Q}^2) = \left[1 + \mathrm{g}_\alpha \ln\left(1 + \frac{\mathrm{Q}^2}{\Lambda_{\mathrm{QCD}}^2}\right) + \mathrm{h}_\alpha \ln^2\left(1 + \frac{\mathrm{Q}^2}{\Lambda_{\mathrm{QCD}}^2}\right)\right]^{\mathrm{n}_\alpha}$$

$\mathrm{n}_1 = 3,\ \mathrm{n}_2 = 2,\ \mathrm{n}_3 = 4$

$\Lambda_{\mathrm{QCD}} \in [0.19 - 0.24]$ GeV

- $a_{\alpha k}$ and $b_{\beta k}$ are constrained by unitarity conditions that also satisfy asymptotic QCD requirements.

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Nonresonant pion production (linear σ-model)

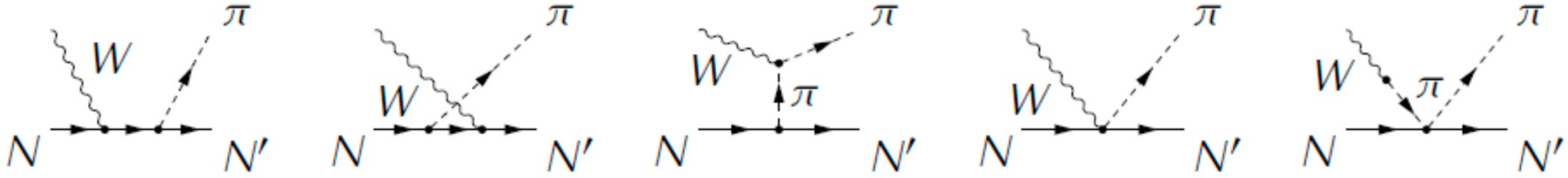
- Is based on $\mathrm{SU}(2)\times\mathrm{SU}(2)$ chiral symmetry, consistent with the symmetries of QCD.
- The ingredients of the model are Nucleon and pion fields plus an scalar σ field.
- The Lagrangian is linear with pion field.
- Three possible (Born) diagrams is the result of the linera σ model.
- There is no experimental evidence for σ particle

Nonresonant pion production (**non**-linear σ-model)

- Is based on $SU(2)\times SU(2)$ chiral symmetry, consistent with the symmetries of QCD.
- The ingredients of the model are Nucleon and pion fields.
- The Lagrangian is **not** linear with pion field.
- Five possible (Born) diagrams is the result of the **non**-linear σ model.
- Low energy Chiral Perturbative Theory (ChPT) is valid at low energy.

E. Hernandez, J. Nieves and M. Valverde, Phys. Rev. D **76** (2007) 033005

Nonresonant pion production



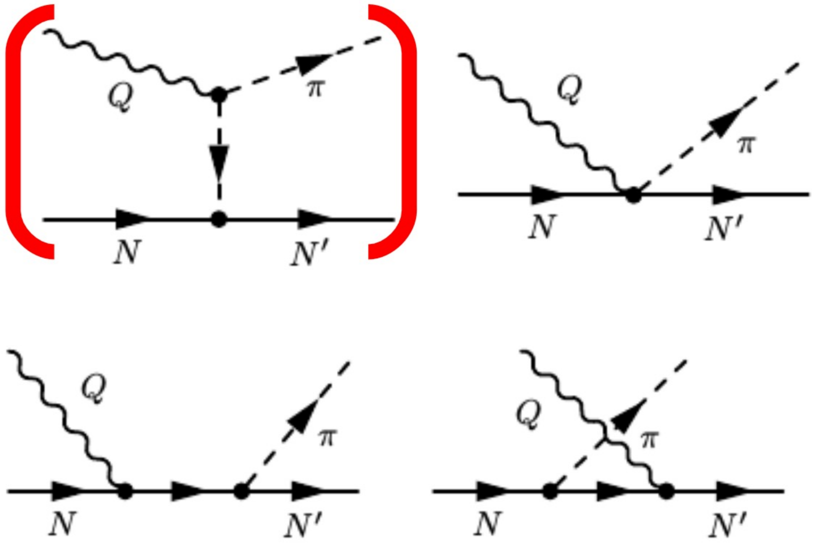
NP CNP PF CT PP

$$
\begin{aligned}
\mathcal{M}^{CC}_{NP} &= \frac{g_A}{\sqrt{2}f_\pi}\cos\theta_C\,\frac{1}{s-M}\,\bar{u}(p_2)\;\not{q}\,\gamma_5(\not{p}_1+\not{k}+M)\epsilon^\mu\Gamma^{CC}_\mu\,u(p_1)\\
\mathcal{M}^{CC}_{CNP} &= \frac{g_A}{\sqrt{2}f_\pi}\cos\theta_C\,\frac{1}{u-M}\,\bar{u}(p_2)\,\epsilon^\mu\Gamma^{CC}_\mu(\not{p}_2-\not{k}+M)\;\not{q}\gamma_5\,u(p_1)\\
\mathcal{M}^{CC}_{PF} &= \frac{g_A}{\sqrt{2}f_\pi}\cos\theta_C\,\frac{1}{t-m_\pi^2}\,F_{PF}(k^2)\bar{u}(p_2)\;\gamma_5\;[2q\epsilon-k\epsilon]\;u(p_1)\\
\mathcal{M}^{CC}_{CT} &= \frac{1}{\sqrt{2}f_\pi}\cos\theta_C\,\bar{u}(p_2)\;\epsilon^\mu\gamma_\mu\;[g_A F^V_{CT}(k^2)\;\gamma_5-F_\rho((k-q)^2)]\;u(p_1)\\
\mathcal{M}^{CC}_{PP} &= \frac{1}{\sqrt{2}f_\pi}\cos\theta_C\,\bar{u}(p_2)\,\frac{\epsilon k}{k^2-m_\pi^2}\;\not{k}\,u(p_1)
\end{aligned}
$$

Hybrid Model for nonresonant pion production

R. González-Jiménez, *et al*
[Phys. Rev. D **95** (2017)]

- Use ChPT model at low energy (W).
- Use Regge formalism at high energy (W). Regge Theory provides the **high energy (s→∞) behavior** of the amplitude:



$$\frac{1}{t - m_\pi^2}$$

The pion propagator is replace by the Regge trajectory of the pion family

$$\mathcal{P}_\pi(t,s) = -\alpha'_\pi \varphi_\pi(t)\Gamma[-\alpha_\pi(t)](\alpha'_\pi s)^{\alpha_\pi(t)}$$

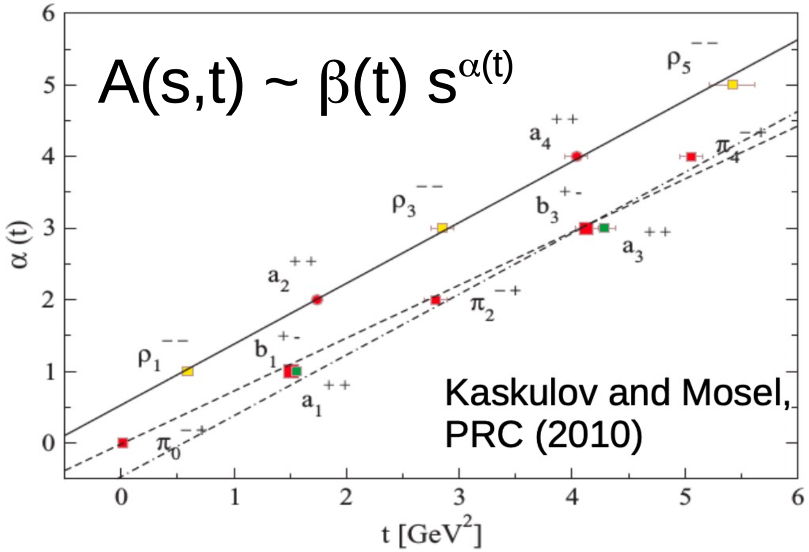
From Raúl González Jiménez
[Presentation]

Hybrid Model for nonresonant pion production

- Use ChPT model at low energy (W).
- Use Regge formalism at high energy (W).

R. González-Jiménez, *et al*
[Phys. Rev. D **95** (2017)]

$\alpha(t)$: Families or Regge trajectories



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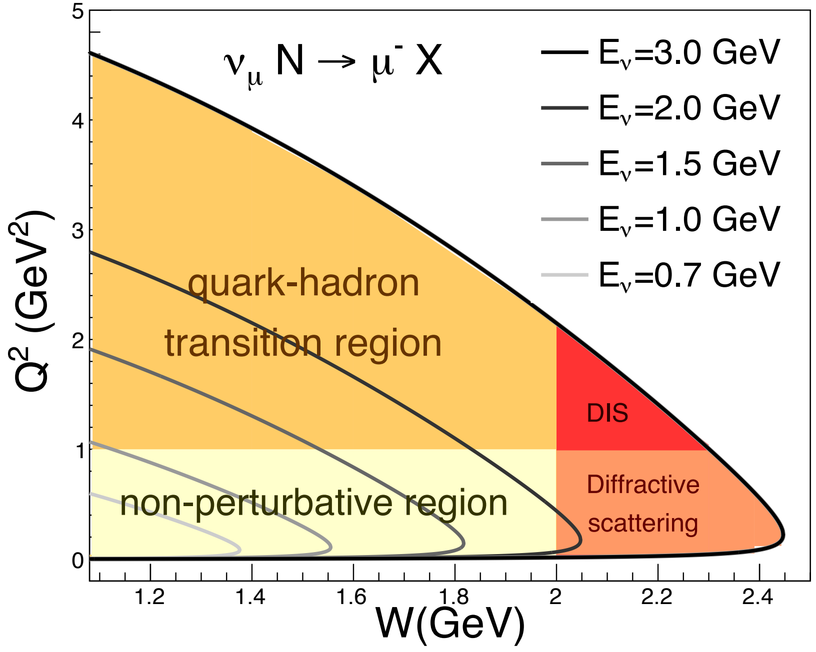
$$\mathcal{P}_\pi(t,s) = -\alpha'_\pi \varphi_\pi(t) \Gamma[-\alpha_\pi(t)](\alpha'_\pi s)^{\alpha_\pi(t)}$$

From Raúl González Jiménez
Presentation

Shallow Inelastic Scattering (SIS)



- Refers to the nonresonant meson production and non-perturbative multi-quark meson production.
- SIS is not a well-defined region. It refers to two different regions:
    1. Nonresonant meson production region.
    2. Transition region ($Q^2 > 1$ GeV); interactions occur through multi-quark processes until Q^2 increases sufficiently to enter the meson production regime via single quark perturbative QCD DIS scattering.

MK model

M. Kabirnezhad
Phys. Rev. D **97** (2018)
Phys. Rev. D **102** (2020)
Phys.Rev.C **107** (2023)

The MK model comprehensively describes single-pion production in interactions involving **photons, electrons, and neutrinos** with nucleons.

- Meson Dominance (MD) form factor: Maintains **unitarity** and integrates **QCD principles** for both resonant and non-resonant interactions.
- **CVC and PCAC** fulfilment: Ensures model consistency at low Q^2.
- Q^2 evolution: Utilises QCD calculations and **quark-hadron duality**.
- W evolution: Applies **Regge trajectory** and the Hybrid model.

R. González-Jiménez, *et al*
Phys. Rev. D **95** (2017)

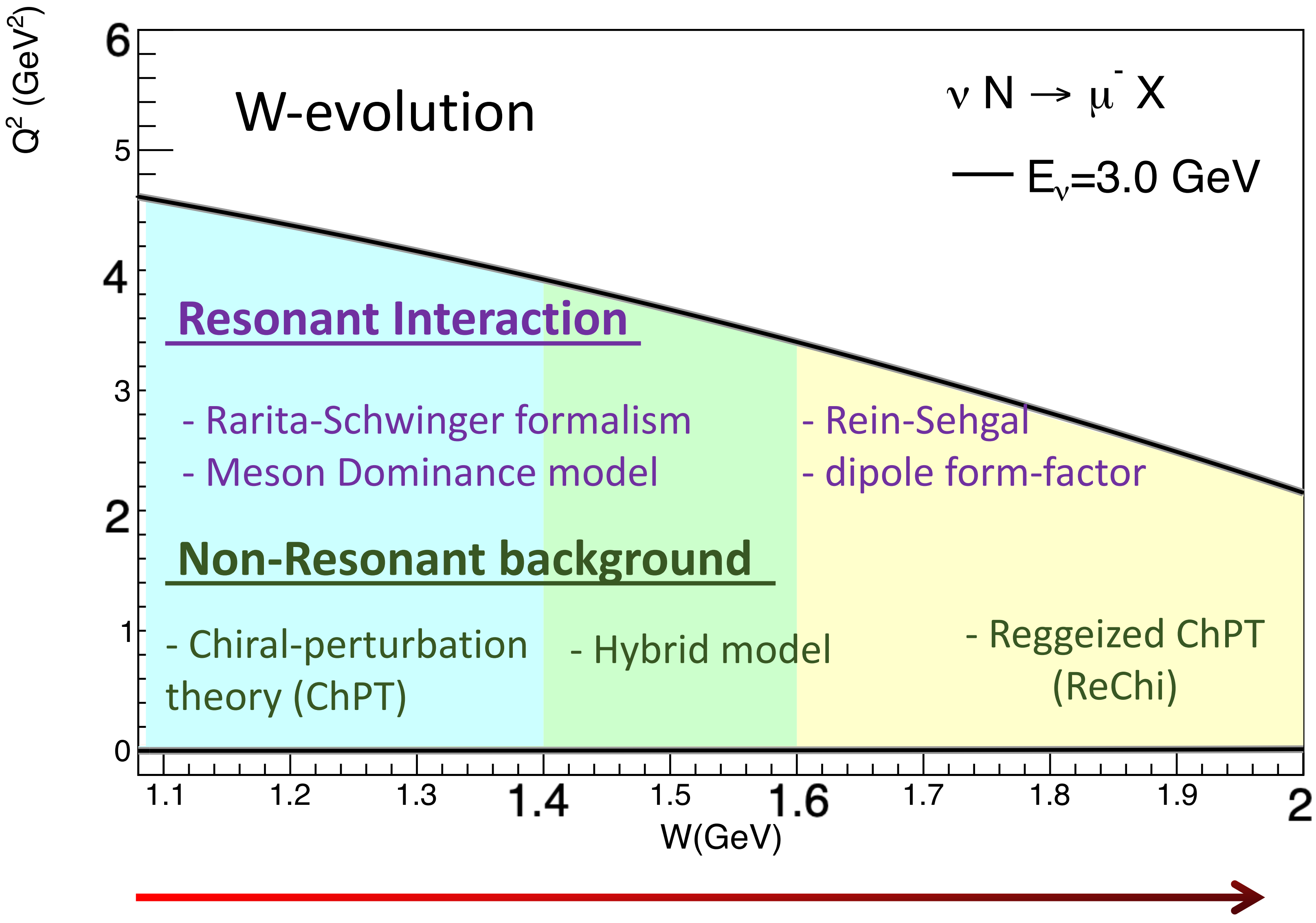
MK model

Resonant interaction

- Several resonances contribute at different invariant mass (W)

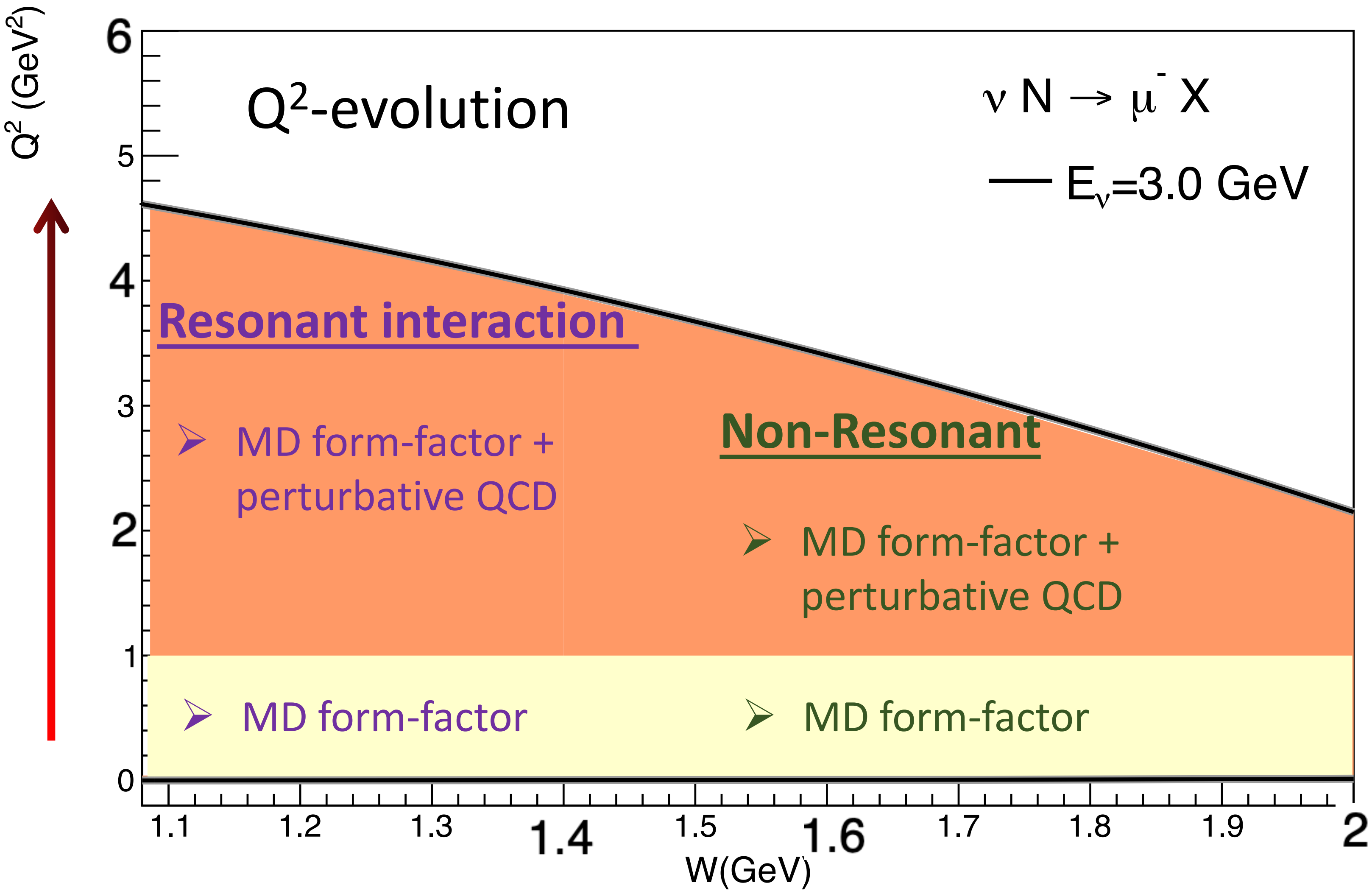
Non-resonant bkg

- Chiral perturbation at low W < 1.4 GeV
- Regge trajectory at high W
- Hybrid model



MK model

- Meson Dominance (MD) model describes form-factors in non-perturbative domain
- It can reproduce Q^2-evolution of form-factors to asymptotically join QCD expectations



Valid kinematic region region for MK model

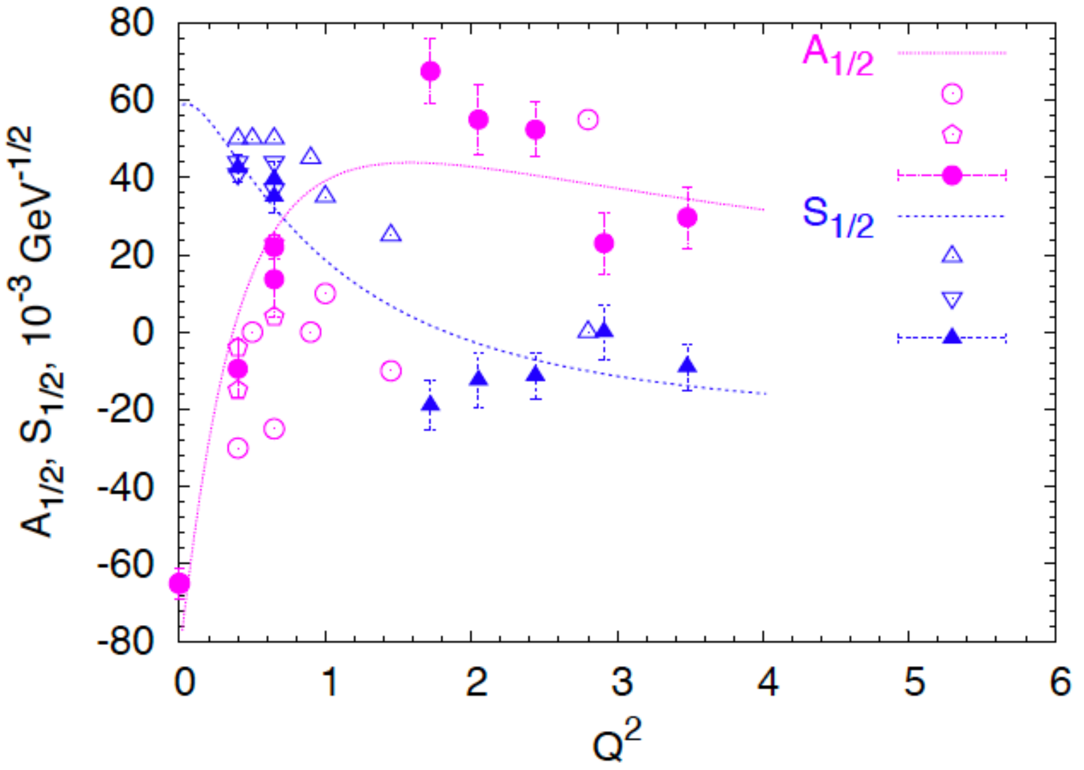
Backup

Lalakulich *et al*
Resonance $P_{11}(1440)$

$$A_{1/2}^{P_{11}} = \sqrt{N}\frac{\sqrt{2}q^z}{p'^0 + M_R}\left[\frac{g_1^{(em)}}{\mu^2}Q^2 + \frac{g_2^{(em)}}{\mu}(M_R + m_N)\right]$$

Helicity amplitudes

$$S_{1/2}^{P_{11}} = \sqrt{N}\frac{q_z^2}{p'^0 + M_R}\left[\frac{g_1^{(em)}}{\mu^2}(M_R + m_N) - \frac{g_2^{(em)}}{\mu}\right]$$



$$g_1^{(p)} = \frac{2.3/D_V}{1 + Q^2/4.3M_V^2},$$
$$g_2^{(p)} = \frac{-0.76}{D_V}\left[1 - 2.8\ln\left(1 + \frac{Q^2}{1\ \mathrm{GeV}^2}\right)\right]$$

These numbers can be different for varies models

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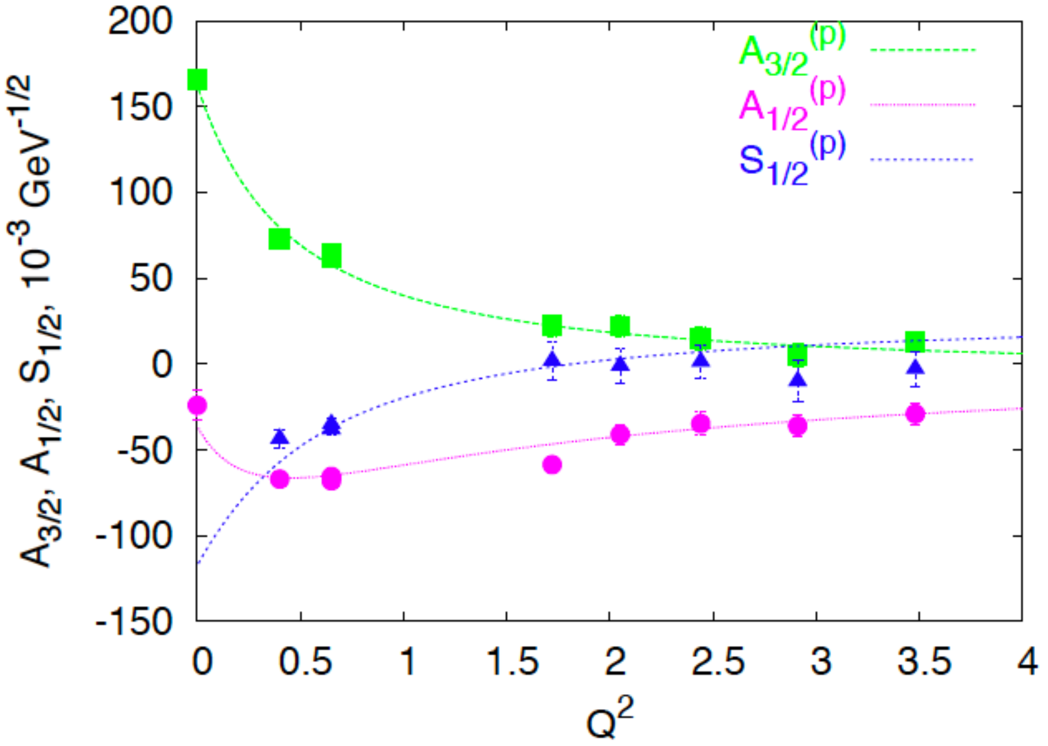
Lalakulich *et al*
Resonance $D_{13}(1520)$

$$A_{3/2}^{D_{13}} = \sqrt{N}\left[\frac{C_3^{(em)}}{m_N}(M_R - m_N) + \frac{C_4^{(em)}}{m_N^2} q\cdot p' + \frac{C_5^{(em)}}{m_N^2} q\cdot p\right]$$

$$A_{1/2}^{D_{13}} = \sqrt{\frac{N}{3}}\left[\frac{C_3^{(em)}}{m_N}\left(M_R - m_N - \frac{2m_N}{M_R}\frac{q_z^2}{p'^0 + M_R}\right) + \frac{C_4^{(em)}}{m_N^2} q\cdot p' + \frac{C_5^{(em)}}{m_N^2} q\cdot p\right] \quad \text{(IV.6)}$$

$$S_{1/2}^{D_{13}} = \sqrt{\frac{2N}{3}}\frac{q^z}{M_R}\left[\frac{C_3^{(em)}}{m_N}(-M_R) + \frac{C_4^{(em)}}{m_N^2}(Q^2 - 2m_N q^0 - m_N^2) - \frac{C_5^{(em)}}{m_N}(q^0 + m_N)\right] \quad \text{(IV.7)}$$



$$C_3^{(p)} = \frac{2.95/D_V}{1 + Q^2/8.9M_V^2},$$

$$C_4^{(p)} = \frac{-1.05/D_V}{1 + Q^2/8.9M_V^2},$$

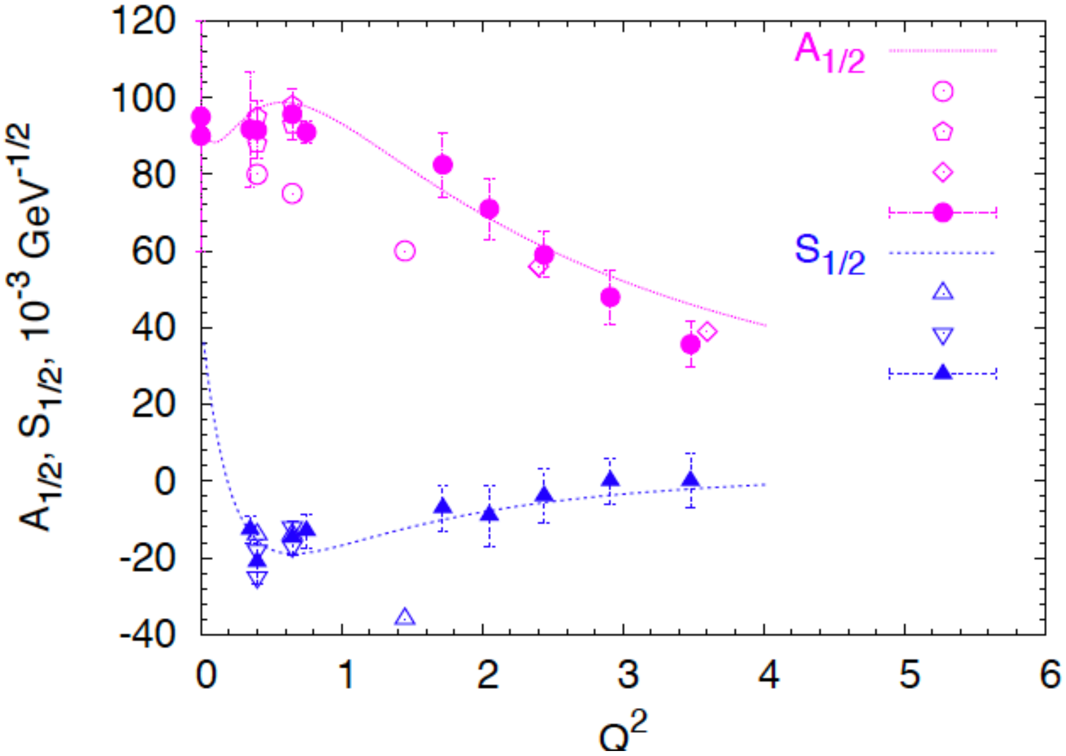
$$C_5^{(p)} = \frac{-0.48}{D_V}.$$

$$D_V = (1 + Q^2/M_V^2)^2$$

Lalakulich *et al*
Resonance $S_{11}(1535)$

$$A_{1/2}^{S_{11}} = \sqrt{2N}\left[\frac{g_1^{(em)}}{\mu^2}Q^2 + \frac{g_2^{(em)}}{\mu}(M_R - m_N)\right]$$

$$S_{1/2}^{S_{11}} = \sqrt{N}q_z\left[-\frac{g_1^{(em)}}{\mu^2}(M_R - m_N) + \frac{g_2^{(em)}}{\mu}\right]$$



$S_{11}(1535)$:

$$g_1^{(p)} = \frac{2.0/D_V}{1 + Q^2/1.2M_V^2}\left[1 + 7.2\ln\left(1 + \frac{Q^2}{1\ \mathrm{GeV}^2}\right)\right]$$

$$g_2^{(p)} = \frac{0.84}{D_V}\left[1 + 0.11\ln\left(1 + \frac{Q^2}{1\ \mathrm{GeV}^2}\right)\right].$$

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Four-vector

- Four-vector: Any set of four quantities which transform like (ct, x) under Lorentz transformations:

$$ct' = \cosh\theta\, ct - \sinh\theta\, z,$$

$$z' = -\sinh\theta\, ct + \cosh\theta\, z,$$

$$\tanh\theta = v/c$$

V along z axis

- Notation:

$$(ct, \mathbf{x}) \equiv (x^0, x^1, x^2, x^3) \equiv x^\mu.$$

basic invariant is $c^2t^2 - \mathbf{x}^2$

$$\left(\frac{E}{c}, \mathbf{p}\right) \equiv (p^0, p^1, p^2, p^3) = p^\mu$$

basic invariant $(E^2/c^2) - \mathbf{p}^2$

Lorentz covariance

- If an equation is a Lorentz covariance we must ensure that all unrepeated indices (upper and lower separately) balance on either side of the equation and all repeater indices appear once as and upper and once as lower.
- A relativistic theory a covariant copy on the non-relativistic perturbation. theory

A cornerstone of modern physics is that the fundamental laws have the same form in all Lorentz frames; that is, in reference frames which have a uniform relative velocity. The fundamental equations are said to be *Lorentz covariant*. Recall that the theory of special relativity is based on the premise that the velocity of light, c, is the same in all Lorentz frames. A Lorentz transformation relates the coordinates in two such frames. The basic invariant is $c^2t^2 - \mathbf{x}^2$.

Lorentz invariant

- Not changing under Lorentz transformation:
    1. Scalar products of two four-vectors
- The rule for forming a Lorentz invariant is to make the upper indices (contravariant) balance with the lower indices (covariant)

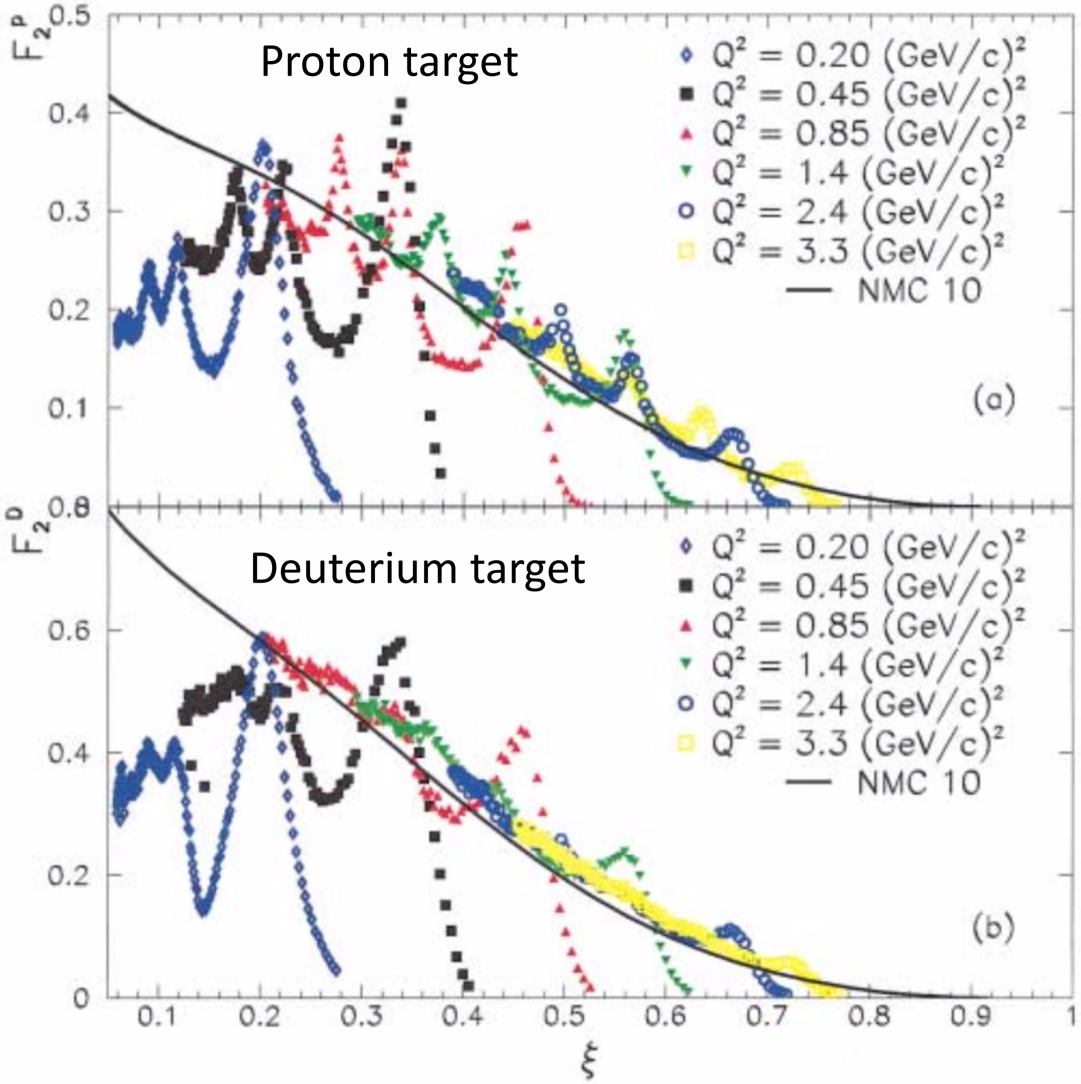
Definition of a Free particle

- For a free particle we have $p^2 = m^2$.
- We say particle is on its mass shell

From I. Niculescu *et al.*

Quark–hadron duality

- It was observed about 50 years ago.
- The resonances oscillate around an average scaling curve.
- Scaling behaviour would imply that the nucleon target appears as a collection of point-like constituents when probed at very high energies in DIS.
- Establishes a relationship between the quark–gluon description, and the hadronic description.



$$\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$$

$$x = Q^2/2M\nu$$

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