Connections to BSM

NuSTEC Summer School 2024

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Homework Problems

Question 1. *BSM Particle Decay Widths*

Consider the three long-lived-particle scenarios discussed in the lecture: (a) dark photon $U(1)$, (b) dark Higgs boson, (c) heavy neutral lepton.

(a) Dark Photon $U(1)$ Assuming the "vector-portal" Lagrangian,

$$\mathcal{L} \supset -\frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} - \frac{\varepsilon}{2}F^{\mu\nu}F'_{\mu\nu} + \frac{m_{A'}^2}{2}A'_\mu A'^\mu, \tag{1}$$

this generates a dark photon A' that couples to the SM electromagnetic current with strength $e\varepsilon$. Determine the partial decay width of an unpolarized A'

$$\Gamma\left(A' \to e^+e^-\right).$$

Recall that for a two-body decay of an unpolarized particle the total width is

$$\Gamma = \frac{|\mathcal{M}|^2}{8\pi}\frac{|\boldsymbol{p}_1|}{M^2}, \tag{2}$$

where $\mathcal{M}$ is the matrix-element of the decay, $\boldsymbol{p}_1$ is the three-momentum of one of the outgoing particles in the decaying particle's rest frame, and M is the decaying particle's mass.

Assuming this is the only decay channel of A', determine its rest-frame lifetime with $m_{A'} = 100$ MeV and $\varepsilon = 10^{-6}$. For such a dark photon with energy $E_{A'} = 1$ GeV, what is the lab-frame decay-length of this particle?

(b) Dark Higgs Boson A dark Higgs boson ϕ couples to SM particles via some small mixing angle θ between ϕ and the SM Higgs boson. This implies that its couplings to SM fermions are proportional to $y_f \sin\theta$, where $y_f \propto m_f/v$ with $v = 246$ GeV being the Higgs' vacuum expectation value.

Determine the relative size of the partial widths $\Gamma(\phi \to \ell^+\ell^-)$. As you vary the mass ϕ and more channels (including those into pions, etc.) become accessible for the ϕ decay, what do you expect to occur to the branching ratio of ϕ into different final states? How does this vary from the $U(1)$ dark photon model above (a)?

(c) Heavy Neutral Lepton (HNL) Similar to the dark Higgs boson, a heavy neutral lepton N may mix with the SM neutrino(s) via mixing angles commonly expressed as $|U_{\alpha N}|^2$, where $\alpha = e,\ \mu,\ \tau$ indicate the different charged-lepton flavors with which the HNL may mix. Different nonzero $|U_{\alpha N}|^2$ will allow for different production and decay modes of the N in accelerator neutrino environments. For only $|U_{\mu N}|^2 \neq 0$, determine some of the allowed production modes of N and its decay channels for $m_N \in [10\text{ MeV}, 1\text{ GeV}]$.

Assuming only one nonzero mixing angle $|U_{\mu N}|^2$, one of the decay channels that will occur is $N \to 3\nu$. Determine the parametric dependence (i.e. do not worry about overall dimensionless prefactors) of the partial width $\Gamma\left(N \to 3\nu\right)$. *Hint: take $m_N \ll M_Z$ – this decay is mediated by an off-shell Z-boson. Treat this as a four-fermion interaction with an effective operator proportional to the Fermi constant, $G_F = 1.166 \times 10^{-5}$ GeV^{-2}.*

Question 2. *Dark Matter in a Neutrino Beam*

Consider the vector-portal dark matter scenario where a dark photon A' is kinetically mixed with the standard model via some mixing ε. Also consider a dark matter particle χ (fermion) or ϕ (scalar) that interacts with the dark photon with some dark version of QED with a "dark fine-structure constant" $\alpha_D = g_D^2/4\pi$, where g_D is the "dark" gauge coupling of this new $U(1)$. Further let us assume that $m_{A'} > 2m_{\text{DM}}$ so that the A' may decay on-shell into dark matter pairs.

In a proton beam-dump facility, the predominant detection mechanism of this dark matter goes as follows:

1. The dark photon is produced in a (rare) decay of a standard model meson or via some other rare interaction through the small coupling of A' with the electromagnetic current $\propto \varepsilon$.

2. The dark photon decays on-shell to dark matter pairs, and some fraction of the dark matter flux is travelling toward the neutrino detector.

3. The dark matter particle scatters with electrons/nuclei in the detector via the dark photon.

For this scenario, determine the parametric dependence of the signal rate on the parameters ε and α_D. Compare this dependence with other strategies that can search for this model, e.g., as in searches for visibly-decaying dark photons in an e^+e^- machine (arXiv:1406.2980). Can you construct other mechanisms for discovery? What are the relative advantages of the different searches (think about the parameter space, experimental luminosities, etc.)?

Question 3. *Long-lived Particles from Decay-at-Rest Production*

Consider an experimental setup where a large quantity of K^+ are decaying at rest, for instance in a neutrino beamline's absorber. Imagine a detector near this decay point $\mathcal{O}$(tens) of meters away that has volume $\sim$(few m)^3. Let us imagine that some rare decay of the kaon into a new long-lived particle X exists $K^+ \to \pi^+ X$, where $m_X < m_K - m_\pi$. In terms of the number of kaons decaying and any relevant detector geometry, determine (a) the flux Φ_X of X particles passing through the detector and (b) the energy distribution of those X particles.

Assuming that the X particle can decay $X \to e^+e^-$ with some partial width $\Gamma(X \to e^+e^-)$, determine the probability of the X particle decaying inside the detector volume (you may assume that the lab-frame lifetime of the X is much larger than the distance between its production and the detector).