Imperial College
London

Inelastic scattering; meson production
Lecture 3

Minoo Kabirnezhad

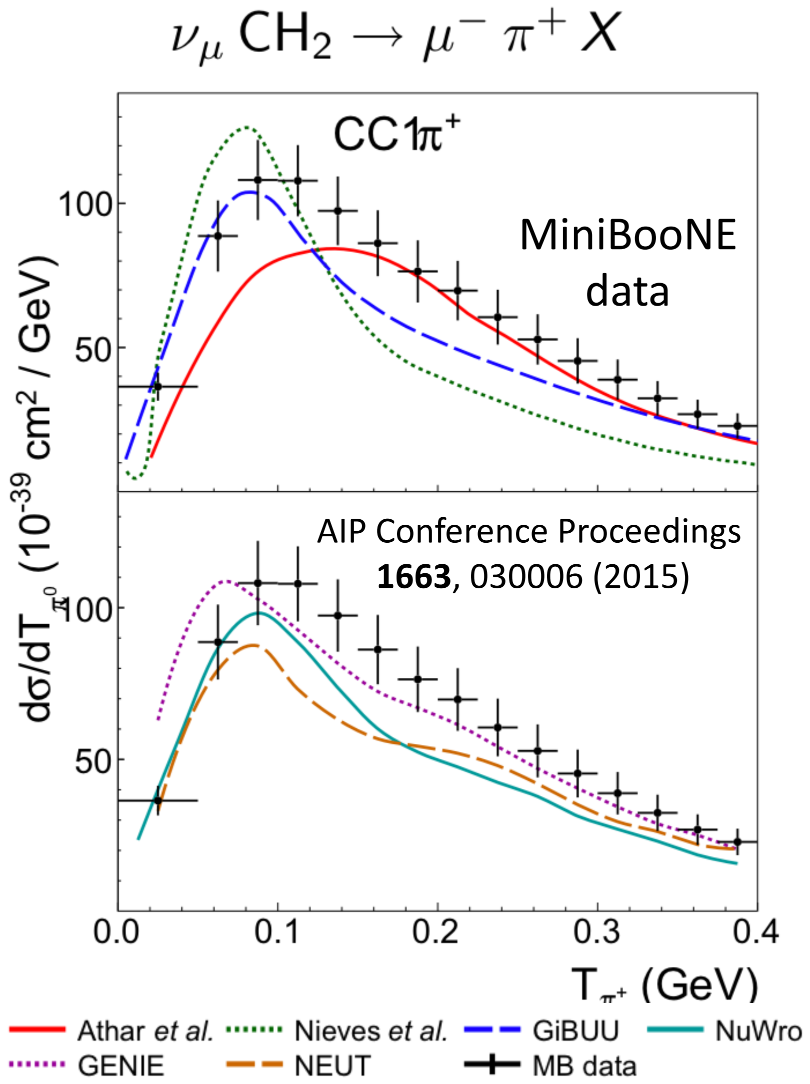
m.kabirnezhad@imperial.ac.uk

NuSTEC 2024

June 10, 2024

Who am I and why pion production?

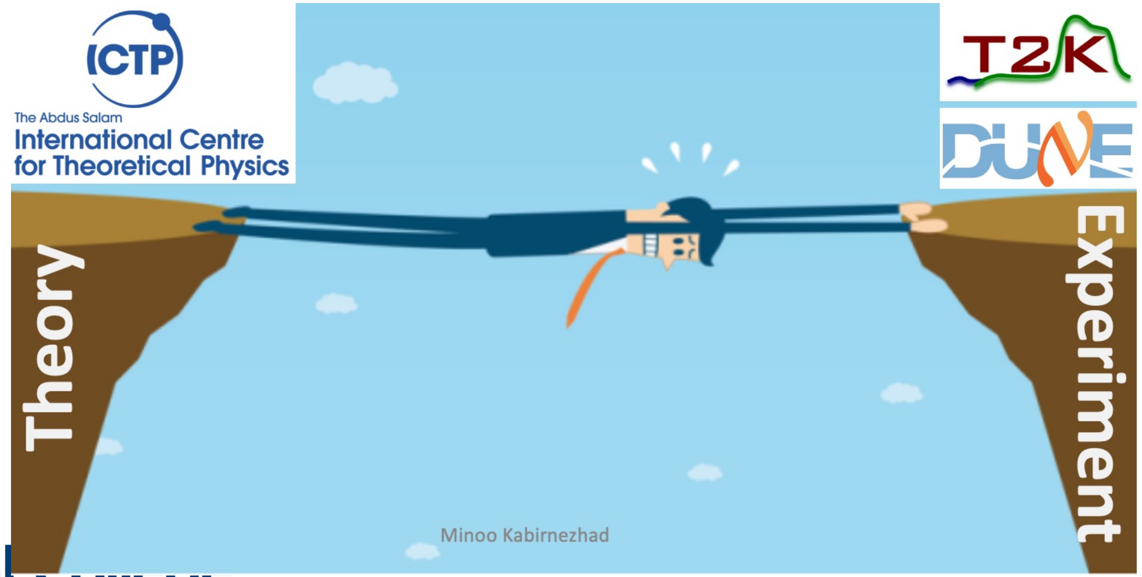
- Studied at the International Centre for Theoretical Physics (ICTP).
- Joined the T2K experiment for my PhD project.
- Identified discrepancies between model predictions for pion production, event generators, and experimental data.
- Noticed a gap in understanding between theorists and experimentalists regarding the requirements for neutrino experiments.



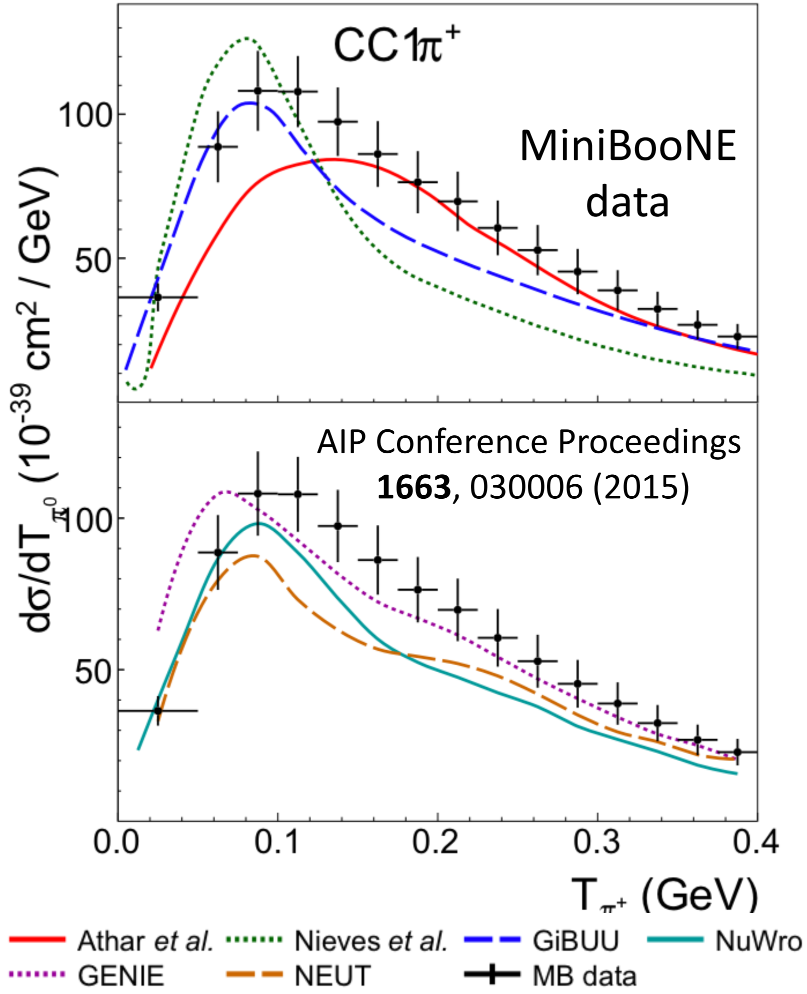
Imperial College London

Who am I and why pion production?

- **Goal**: Develop innovative approaches for building models that maximize our ability to extract and interpret key physics measurements.



$\nu_\mu \, CH_2 \rightarrow \mu^- \, \pi^+ \, X$



Rein-Sehgal model (1981)

1. D. Rein and L. M. Sehgal, Annals Phys. 133 (1981) 79.

- Describes the resonant production based on a quark model proposed by Feynrnan-Kislinger-Raundal (FKR)
- FKR Hamiltonian is given by the four-dimensional harmonic oscillator:

$$\mathcal{H} = 3\left(p_a^2 + p_b^2 + p_c^2\right) + \frac{1}{36}\Omega^2[(u_a - u_b)^2 + (u_b - u_c)^2 + (u_c - u_a)^2 + \text{const.}$$

Where p_a is 4-momentum operator of quark a and $p_{a\mu} = i(\frac{\partial}{\partial u_a^u})$.

Parameter Ω is determined from the Regge slope of baryon trajectories

Rein-Sehgal model (1981)

- The quark model proposed by Feynrnan-Kislinger-Raundal (FKR) is given by the four-dimensional harmonic oscillator Hamiltonian:

$$\mathcal{H} = 3\left(p_a^2 + p_b^2 + p_c^2\right) + \frac{1}{36}\Omega^2[(u_a - u_b)^2 + (u_b - u_c)^2 + (u_c - u_a)^2 + \text{const.}$$

- Hamiltonian can be written in terms of annihilation and creation operators.

$$\mathcal{H} = (p_a + p_b + p_c)^2 - \Omega\left(a_\mu^* a^\mu + b_\mu^* b^\mu\right) + \text{const.}$$

$$\left[a_\mu, a_\nu^*\right] = \left[b_\mu, b_\nu^*\right] = -g_{\mu\nu}$$

Rein-Sehgal model (1981)

- Hadron current can be described by creation operator a (transition from the ground state, the nucleon) spin operators (1, σ) and isospin operators (1, $\boldsymbol{\tau}$) . For charged current:

$$F_{\pm}^{V,A} \propto \tau^{+}(R^{V,A}\sigma_{\pm} + R^{V,A}\sigma_{\mp})e^{-a_z}$$

$$\left(F_0^{\lambda}\right)^V \propto \tau^{+} S e^{-a_z}, \qquad \left(F_0^{\lambda}\right)^A \propto \tau^{+}(C\sigma_z + B\sigma.a)e^{-a_z}$$

- $\epsilon^{\mu} J_{\mu}^{V,A} = C_L e_L^{\alpha} J_{\alpha} + C_R e_R^{\alpha} J_{\alpha} + \mathrm{C}_{\lambda} \mathrm{e}_{\lambda}^{\alpha} \mathrm{J}_{\alpha}$

$$\epsilon^{\mu} J_{\mu}^{V,A} = C_L F_{-} + C_R F_{+} + \mathrm{C}_{\lambda} \mathrm{F}_0^{\lambda}$$

Rein-Sehgal (1981) vs Berger-Sehgal (2007)

- In Rein-Sehgal $m_l = 0$ in CC ν-interactions. Therefore, lepton current is left-handed ($\lambda = -$).
- In Berger-Sehgal $m_l \neq 0$ in CC ν-interactions. Therefore, $\lambda = \mp$.

$$\epsilon^{\mu} J_{\mu}^{V,A} = C_L F_- + C_R F_+ + \mathrm{C}_{\lambda} \mathrm{F}_0^{\lambda}$$

- Therefore in the BS (RS) model we have 16 (12) helicity amplitudes because polarization of the gauge boson change $4 \rightarrow 3$.
- If we ignore lepton mass, the gauge boson polarisation in CC neutrino will be like gauge photon in electron interaction.

Rein (Berger)- Seghal model

- It is based on a quark model which is not what we expect for non-perturbative region.
- It is not a full kinematic model. The helicity amplitudes are **not** a function of pion angles! $d\,\sigma/dW\;dQ^2$
- Pion angles are described by density matrix. NEUT and GENIE **only** implemented the Δ resonance.
- It does not cover non-resonant interaction. It define an ad-hoc term based on spin ½ resonances. This is implemented in NEUT.
- It uses simple (dipole) form-factors for all resonances.

Why do we still use RS (BS) model?

- It is the default model used in both **NEUT** and **GENIE**.
- It is based on **helicity amplitudes** therefore it is not computationally expensive; suitable for event generators
- It describes all resonances in the resonance region (W<2 GeV).
- The model offers flexibility, allowing the dipole form factor to be adjusted. This enables the variation of parameters to evaluate systematic uncertainties.

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

Why do we still use RS (BS) model?

- It use relativistic Breit–Wigner parameterization instead of Decay amplitudes amplitudes in the FKR model.

$$\langle N\pi, \lambda_2|\varepsilon_\lambda^\alpha F_\alpha|N, \lambda_1\rangle = \langle N\pi, \lambda_2|R\lambda_R\rangle\langle R\lambda_R|\varepsilon_\lambda^\alpha F_\alpha|N\lambda_1\rangle$$

- Decay amplitudes:

$$\langle N\pi, \lambda_2|R, \lambda_R\rangle \propto \frac{-1}{W - M_R + i\Gamma_R/2},$$

$$\Gamma_R \propto \Gamma_0 (\text{Resonance width})$$

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

Breit-Wigner amplitudes

- Broad states like resonances in the final state, can be replaced by a spectral function of the particle. In the resonance rest frame:

$$\delta(p_R^2 - M_R^2) = \frac{1}{2M_R}\delta(W - M_R)$$
$$= \frac{1}{2M_R}\frac{1}{\pi}\lim_{\varepsilon\to 0}\frac{\varepsilon}{W^2+\varepsilon^2}.$$

$$\delta(p_R^2 - M_R^2) \to \mathscr{A}(p_R) = \frac{1}{2M_R}\frac{1}{\pi}\frac{\Gamma_R/2}{(W-M_R)^2+\Gamma_R^2/4}$$

Why do we still use RS (BS) model?

- It is based on **helicity amplitudes** therefore it is not computationally expensive; suitable for event generators
- It describes all resonances in the resonance region (W<2 GeV).
- It use relativistic Breit–Wigner parameterization instead of Decay amplitudes amplitudes in the FKR model.
- It is the default model used in both **NEUT** and **GENIE**.

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

Rein (Berger)- Sehgal model in **NEUT**

- It is not a full kinematic model. The helicity amplitudes are **not** a function of pion angles! $d\sigma/dW\,dQ^2$
- Pion angles are described by density matrix. NEUT **only** implemented the Δ resonance.
- It does not cover non-resonant interaction. It define an ad-hoc term based on spin ½ resonances. It **includes** interference between resonances. No interference effect.
- Two options for resonance form-factors: dipole (RS) and Graczyk-Sobczyk form-factor.

Rein (Berger)- Sehgal model in **GENIE**

- It is not a full kinematic model. The helicity amplitudes are **not** a function of pion angles! $d\sigma/dW\,dQ^2$
- Pion angles are described by density matrix. GENIE **only** implemented the Δ resonance.
- It does not cover non-resonant interaction as described here, but it is simulated by DIS contribution (GENIE v3). No interference effect.
- It **doesn't include** interference between resonances.
- Two options for resonance form-factors: dipole (RS) and Graczyk-Sobczyk form-factor.

Improving Rein (Berger)- Seghal model

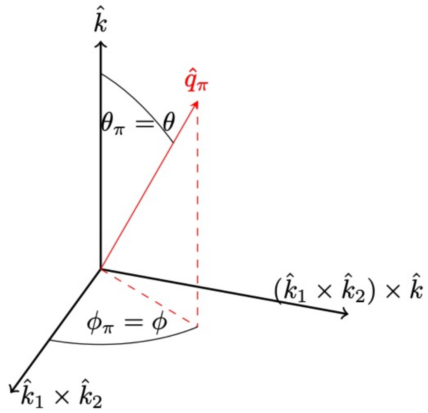
- It is not a full kinematic model. The helicity amplitudes are **not** a function of pion angles! $d\,\sigma/dW\;dQ^2$
- Pion angles are described by density matrix. NEUT and GENIE **only** implemented the Δ resonance.
- It does not cover non-resonant interaction

- The nonresonant interaction was added to the BS model in the First version of MK model 2018
- The output of the MK model is full kinematic $d\,\sigma/dW\;dQ^2 d\Omega_\pi$

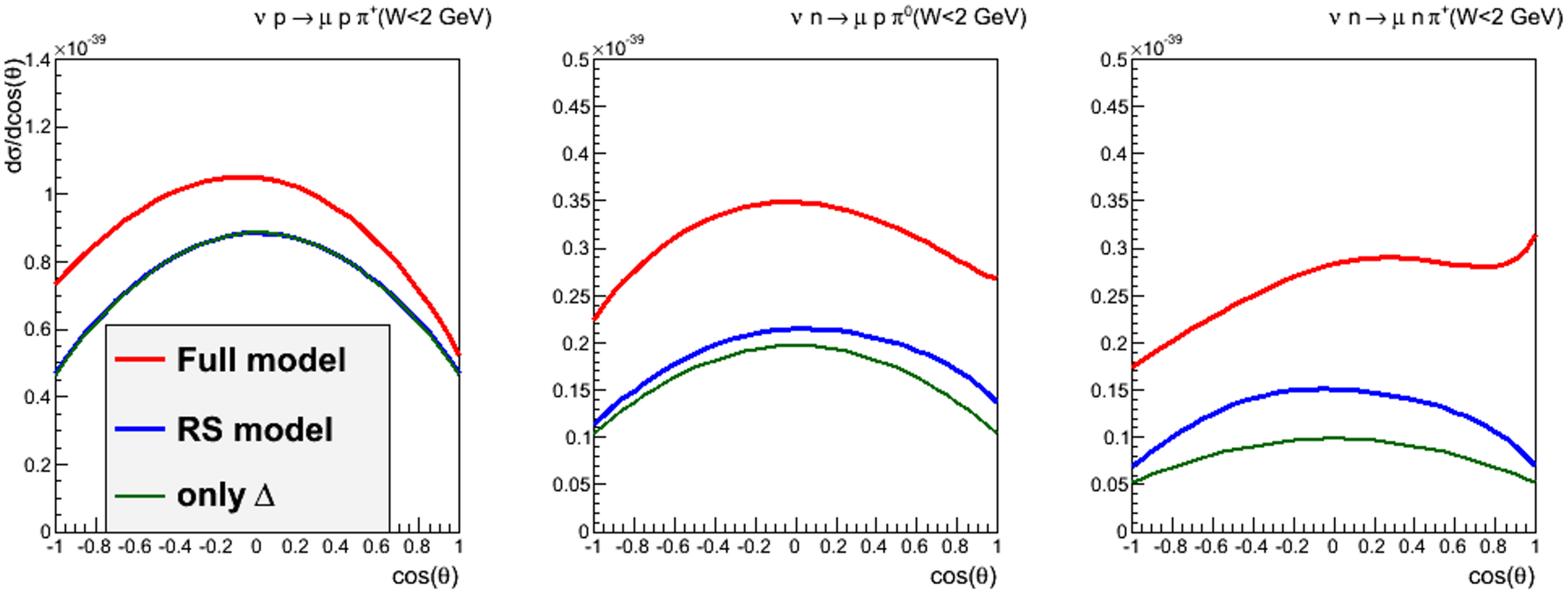
Effects we are expecting to see

1. pion angles
2. nonresonant and interference

Pion angle modifications



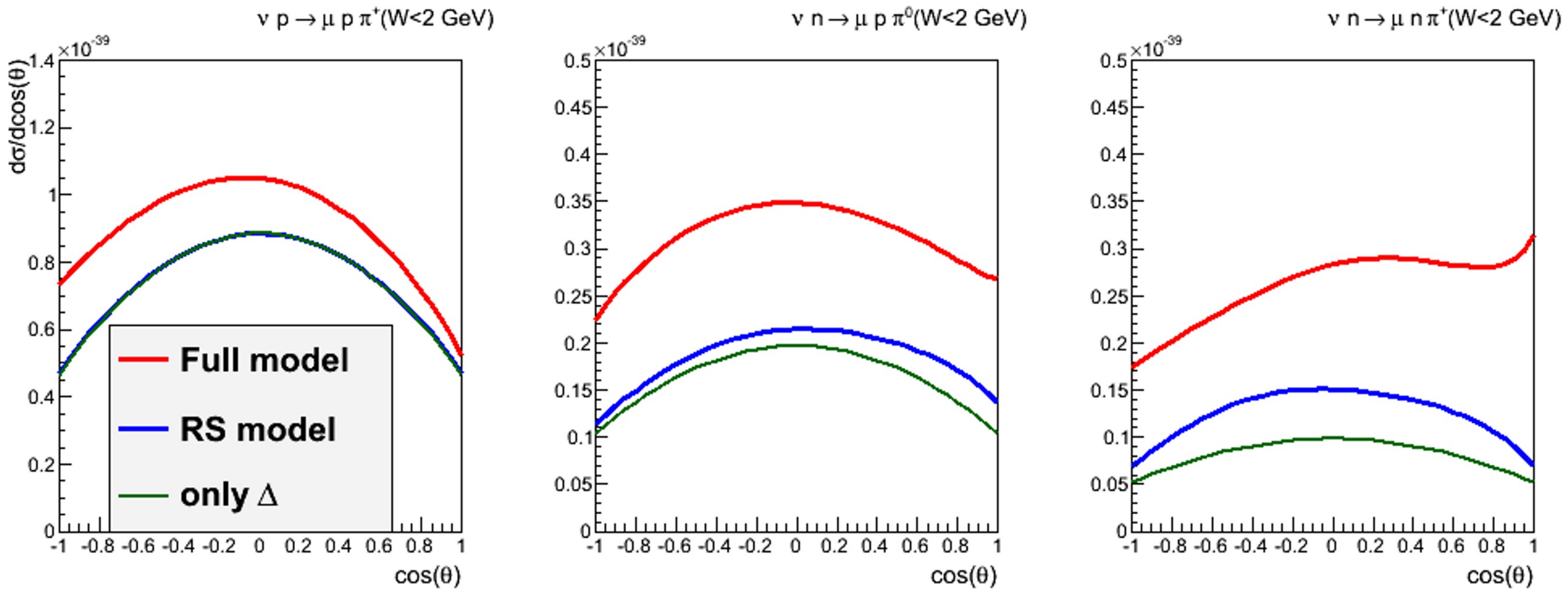
- Pion polar angular distribution in Adler frame



For T2K energy

Pion angle modifications

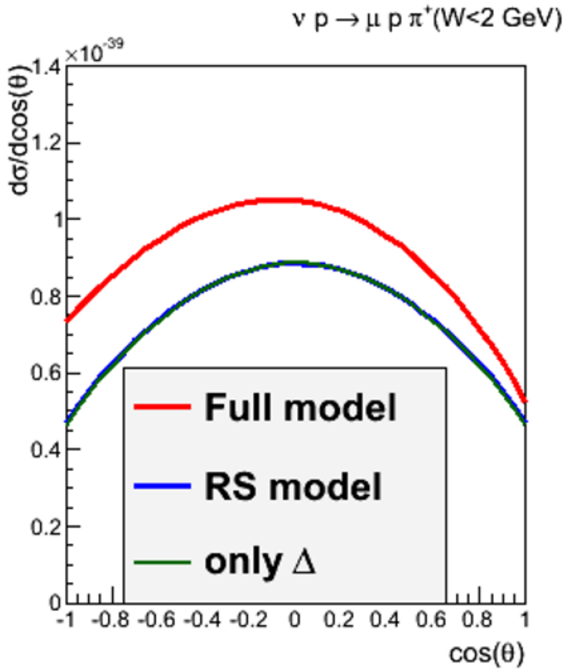
- “Only Δ” has a symmetric distribution.
- Any deviation from the symmetric distribution comes from the interference effects.

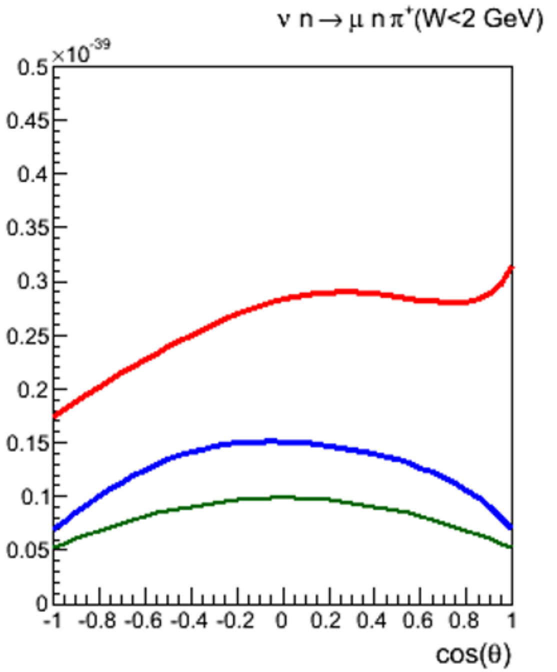


Imperial College London

Pion angle modifications

- Isospin symmetry allows only resonances with isospin 3/2 in $\nu_\mu p \rightarrow \mu p \pi^+$ channel and mainly Δ resonance. Therefore the shape of angular momentum is not changing much.
- According to isopspin symmetry, isospin ½ resonances have dominant contribution in the $\nu_\mu n \rightarrow \mu n \pi^+$ channel.
- This is for T2K energy (E_ν~0.6 GeV). This is more significant for higher energy like DUNE!

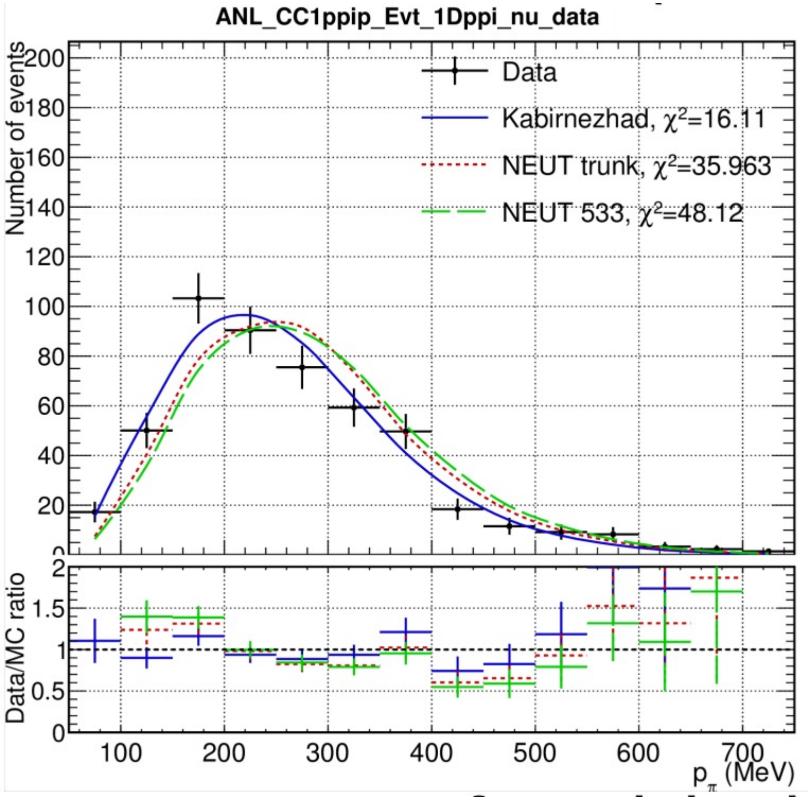


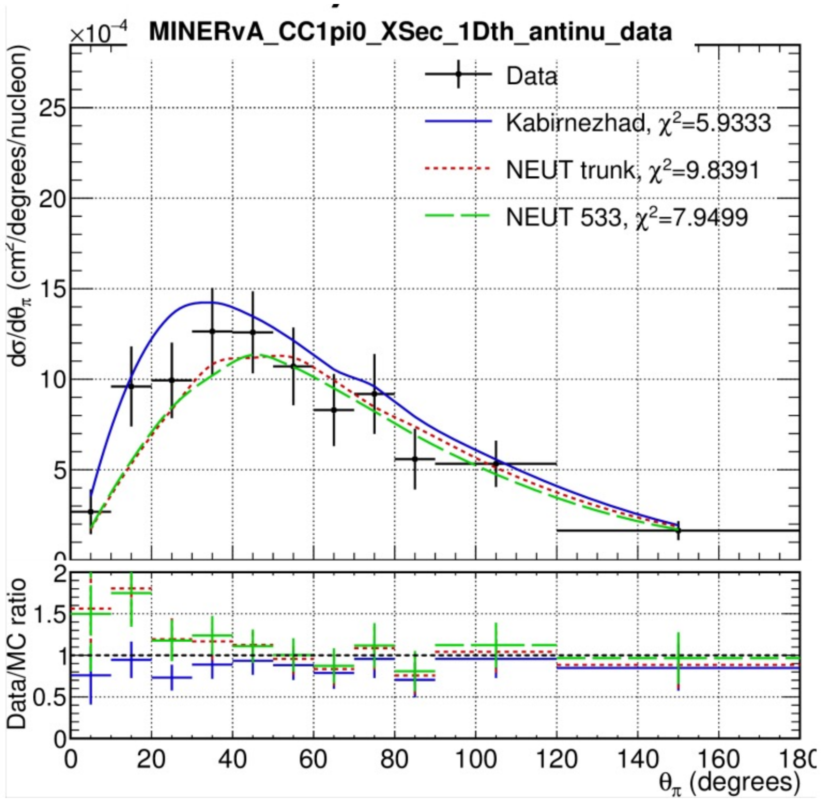


Imperial College London

Improving pion angle and momentum (2018)

Better predictions at low p_π (compare to RS model in NEUT) is due to the non-resonant contributions and the interference effects.



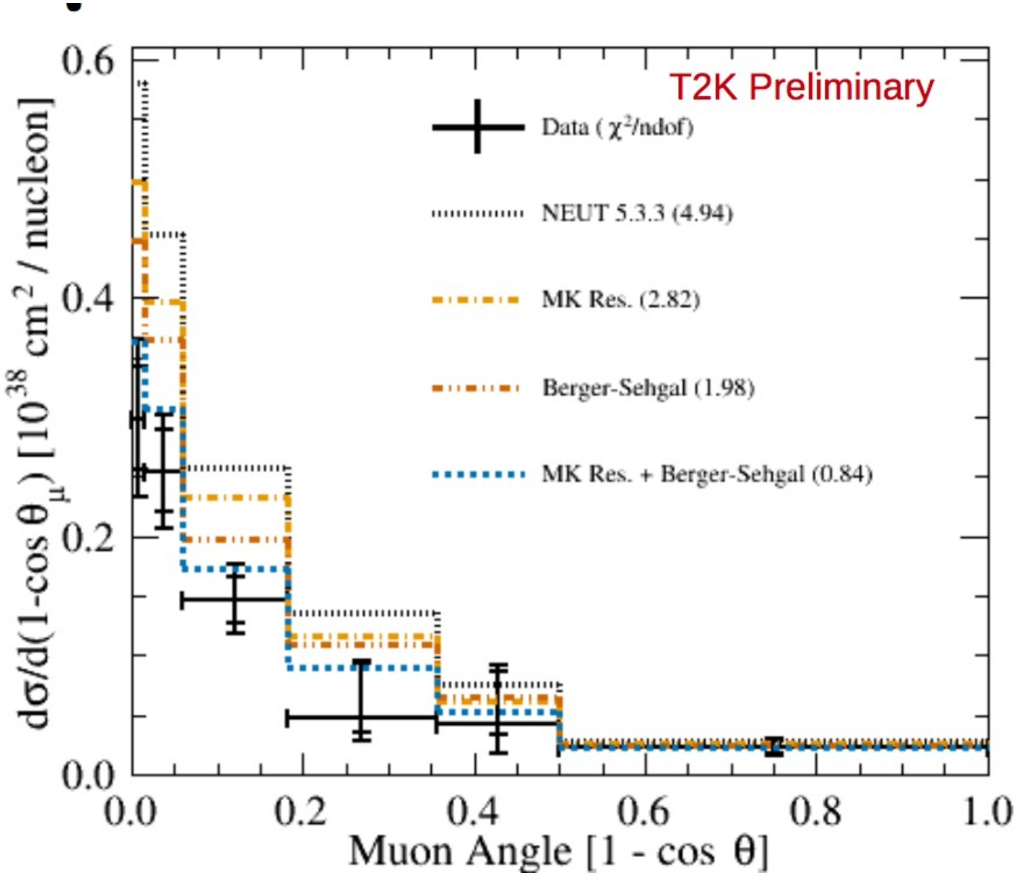


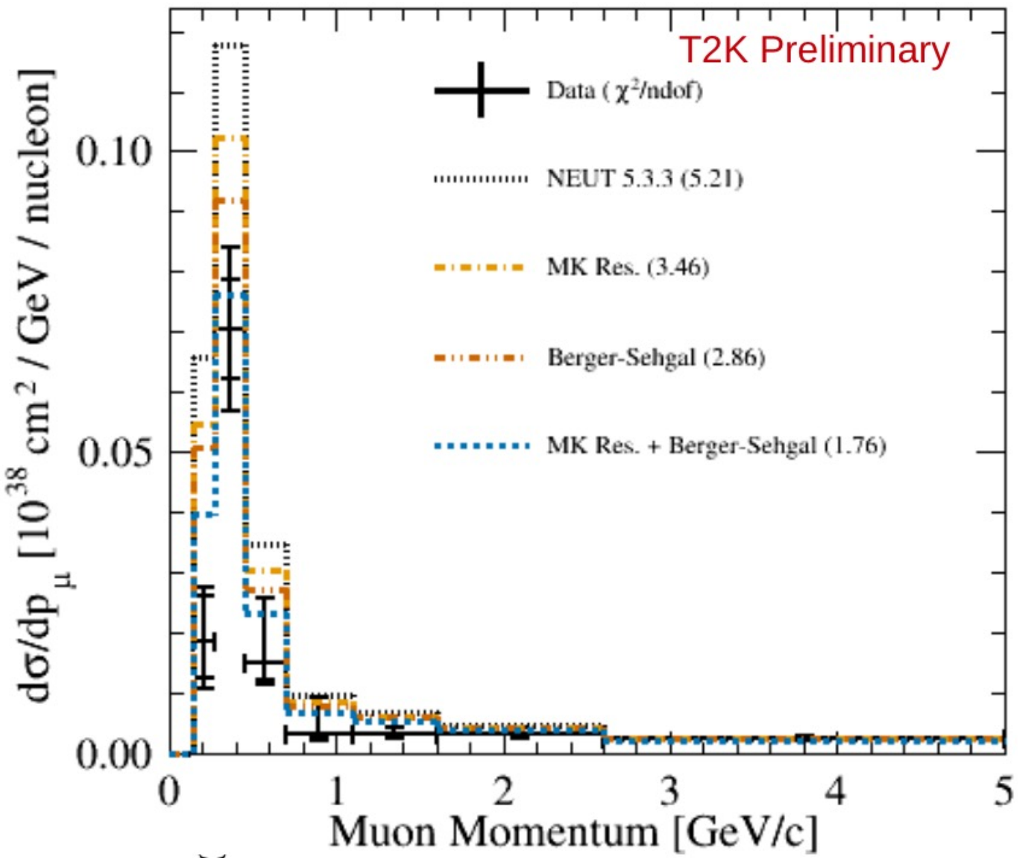
From Clarence Talk at NuInt 2017

Imperial College London

Improving muon angle and momentum (2018)

T2K CC1π^+ data

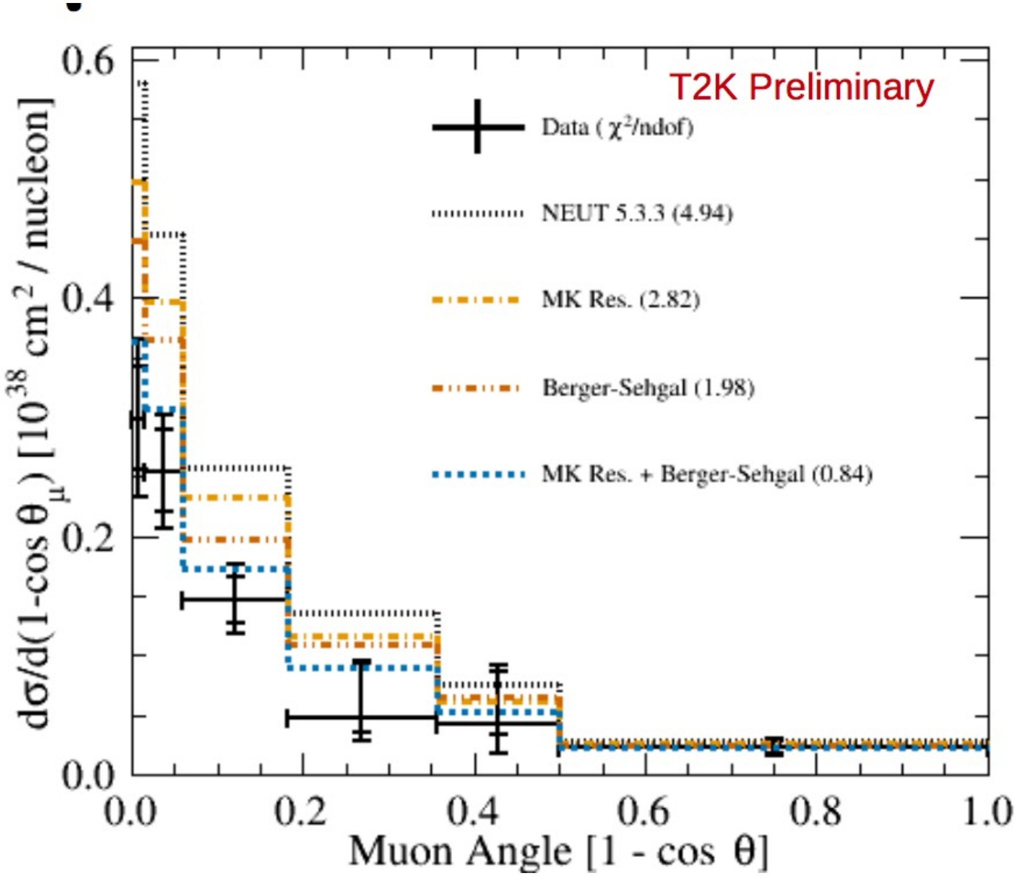


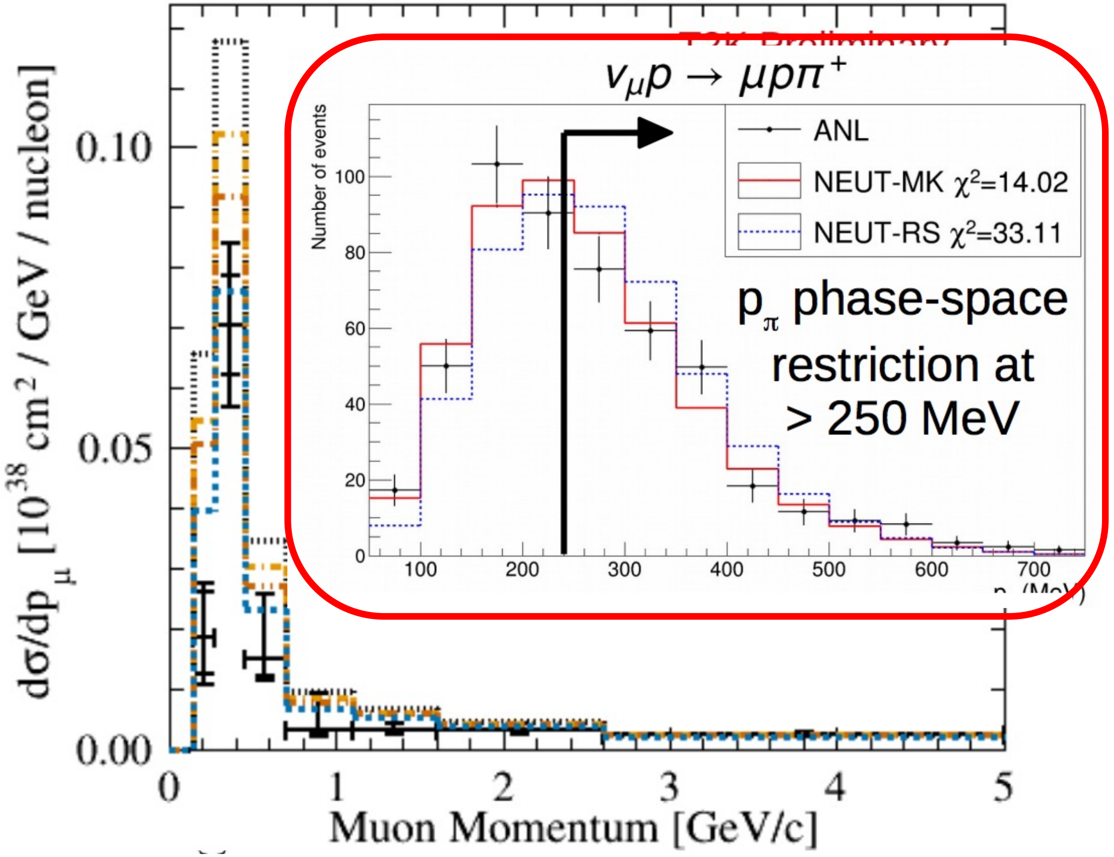


From Dan Cherdack talk at NuInt 18

Improving muon angle and momentum (2018)

- This reduction is due to the better prediction for pion momentum.

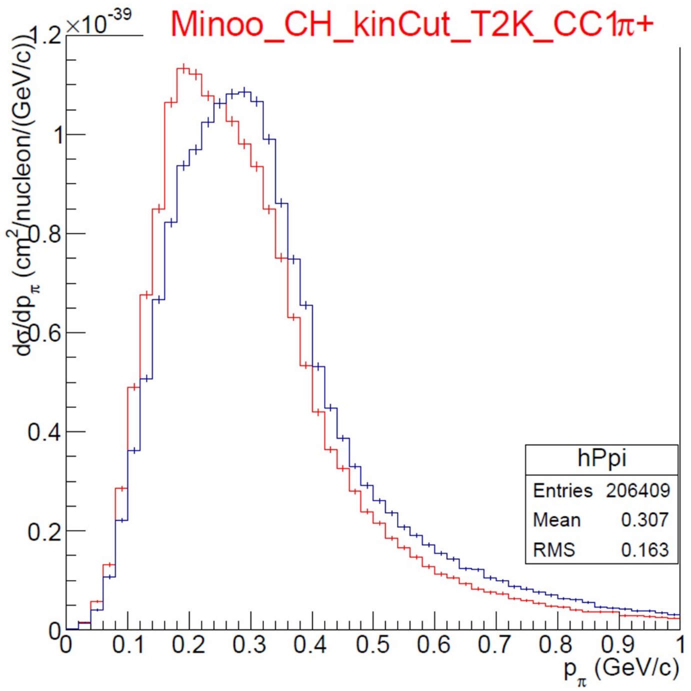




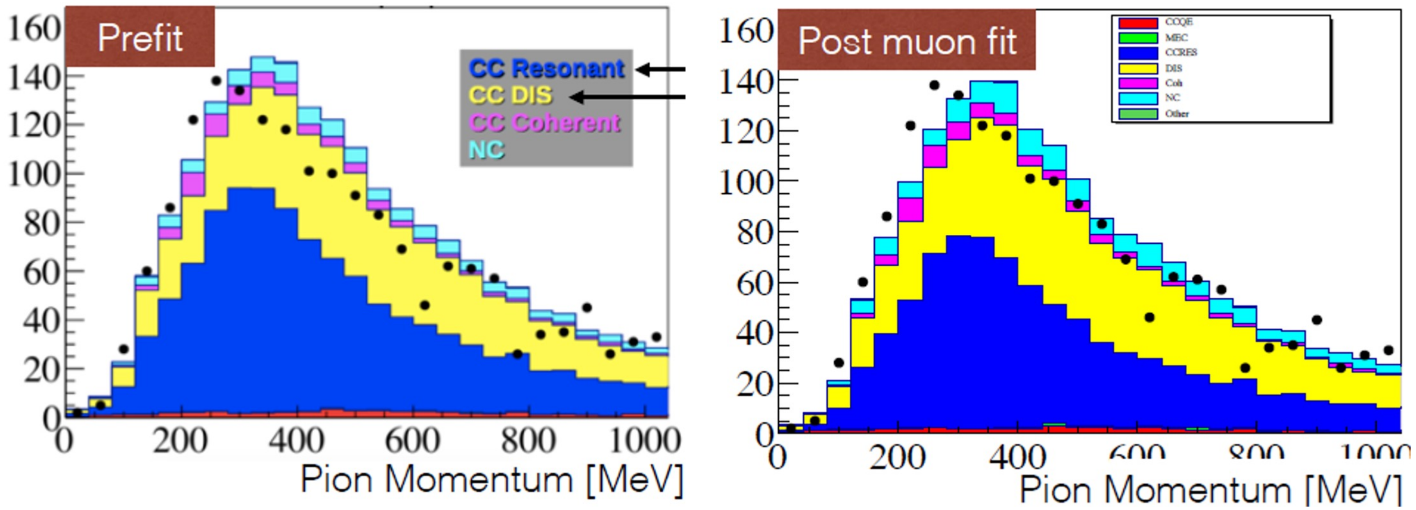
From [Dan Cherdack talk at NuInt 18]

Improving T2K analysis

- T2K also noticed this low pion momentum proble

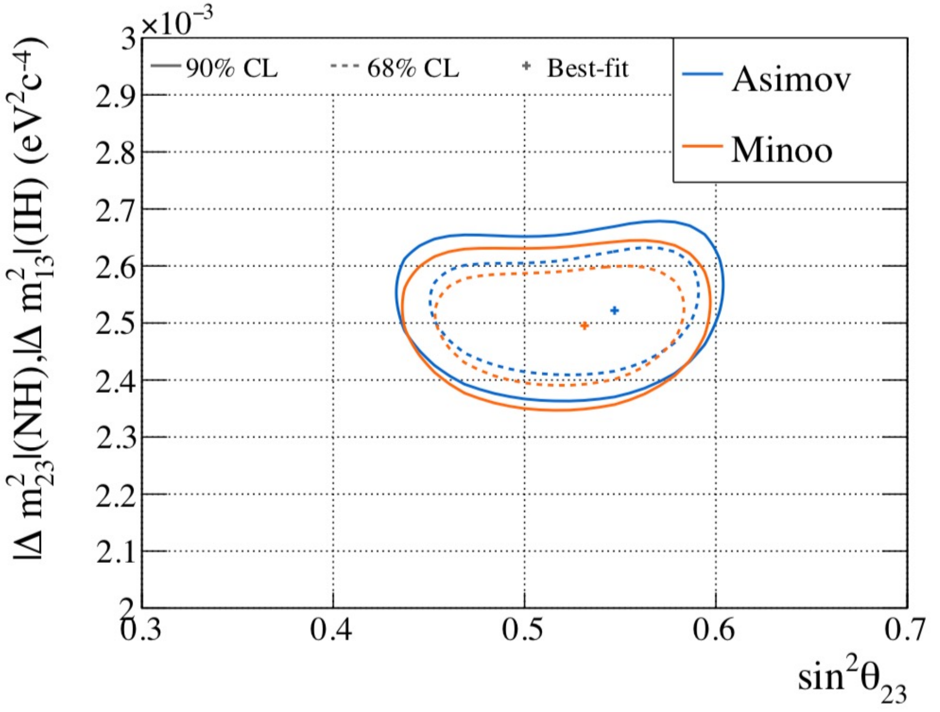


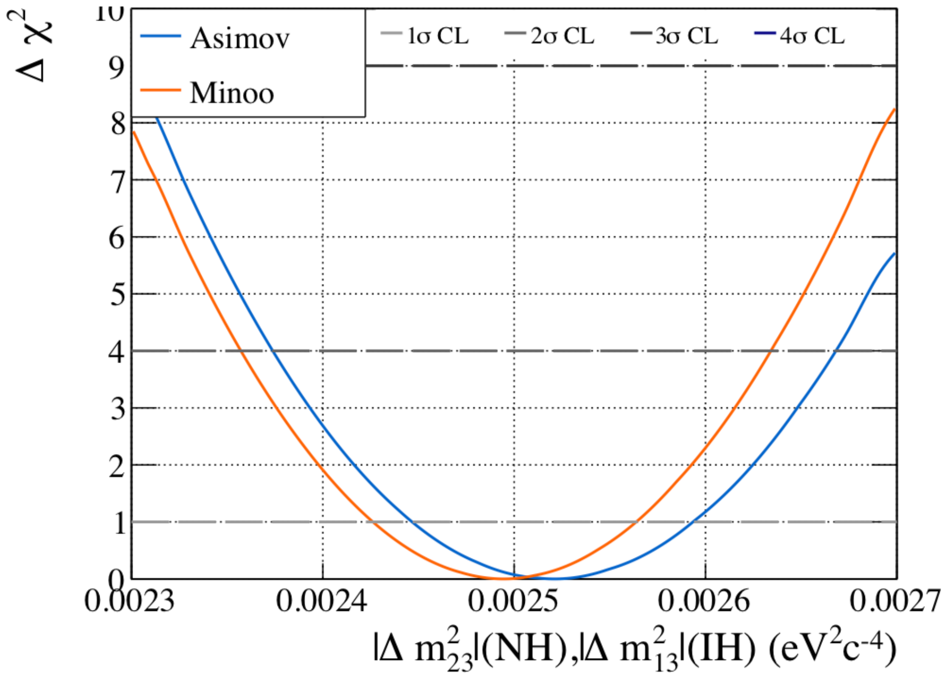
T2K prefit and post muon fit (2017)



MK-model and T2K analysis

- MK-model has been used as pseudodata to see if it causes issue in our current analysis (underestimated systematics).
- Although bias is small with respect to the uncertainty right now, clearly mis-modelling can produce **biased** oscillation parameters, and this will be a serious problem for next generation experiments.

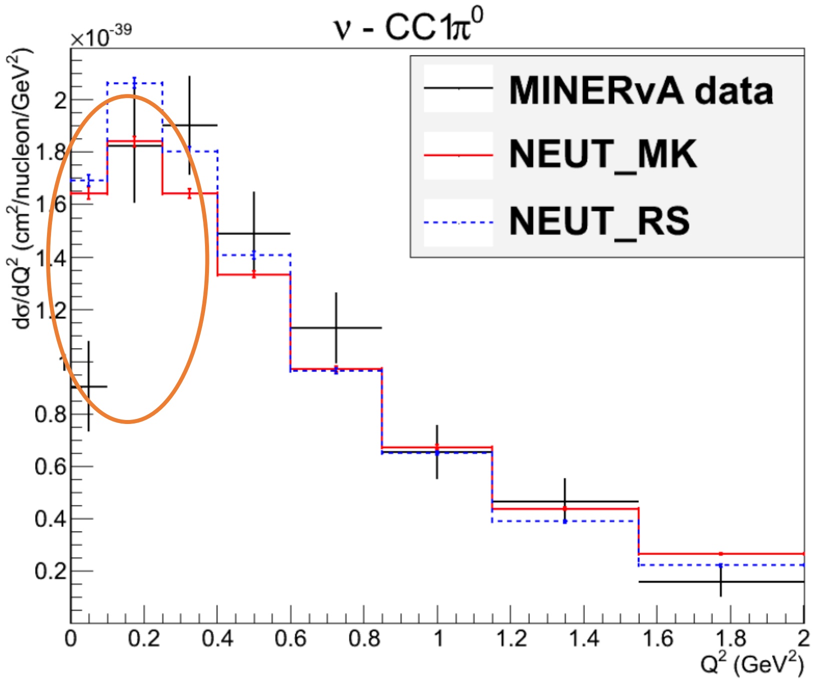


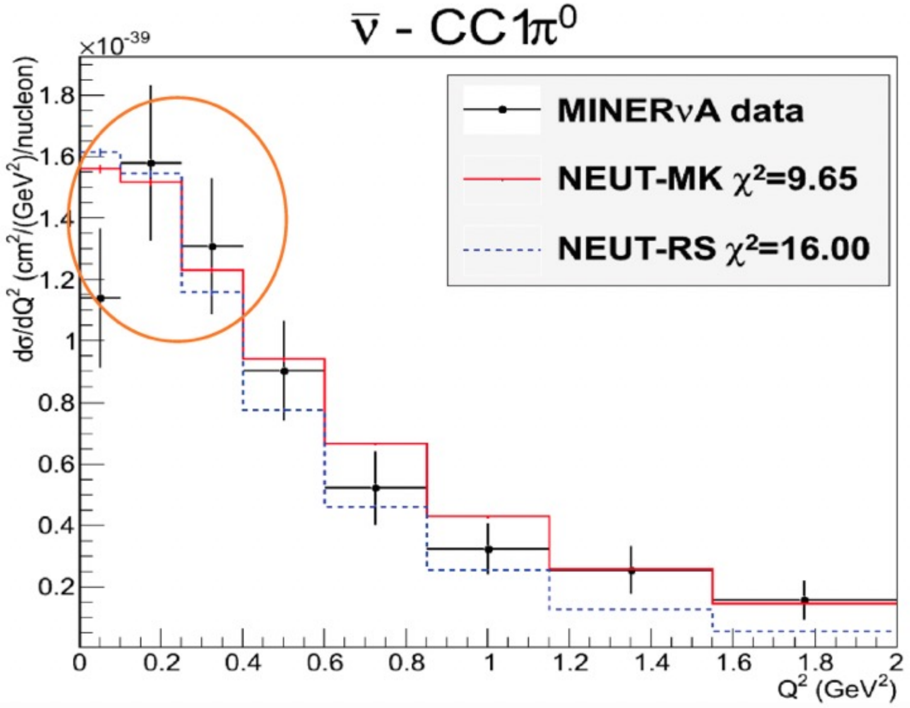


Imperial College London

What MK model couldn't improve

- Low Q^2 predictions from axial current.





Review of other resonance model

Adler model (1968)

- describes weak single-pion production in the first resonance region (only Δ resonance).
- Using linear sigma model for non-resonant interaction i.e three born diagrams (lecture 2)
- cross section is calculated from helicity amplitudes and multipoles.
- Multipoles are helicity amplitudes for definite angular momentum.

Lalakulich-Paschos (LP) model (2005)

- describes the resonances in the first and the second resonance region (4 resonances) using the Rarita-Schwinger (Lecture 2) and used Breit-Wigner amplitudes instead of Δ propagator.
- The relativistic Breit–Wigner parameterization represents a dressed propagator for an isolated resonance.
- They used MAID helicity amplitudes results to parameterise the resonances vector form factors.

Reminder: Resonance production (spin 3/2)

- $J^{\mu}_{3/2} = \bar{\psi}_{\nu}(p')\Gamma^{\nu\mu}_{3/2}u(p)$, $\bar{\psi}_{\nu}$ is Rarita-Schwinger spinor for S=3/2 resonances and $\Gamma^{\nu\mu}_{3/2}$ is the weak $WNR_{3/2}$vertex
  - For positive parity: $\Gamma^{\nu\mu}_{3/2+} = \left(\mathcal{V}^{\nu\mu}_{3/2} - \mathcal{A}^{\nu\mu}_{3/2}\right)\gamma^5$
  - For negative parity: $\Gamma^{\mu}_{3/2-} = \left(\mathcal{V}^{\mu}_{3/2} - \mathcal{A}^{\mu}_{3/2}\right)\mathbb{I}$

- $\mathcal{V}^{\nu\mu}_{3/2} = \frac{\mathcal{C}^V_3}{M}(g^{\nu\mu}\not{q} - q^{\nu}\gamma^{\mu}) + \frac{\mathcal{C}^V_4}{M^2}(g^{\nu\mu}q.p' - q^{\nu}p'^{\mu}) + \frac{\mathcal{C}^V_5}{M^2}(g^{\nu\mu}q.p - q^{\nu}p^{\mu}) + g^{\nu\mu}\mathcal{C}^V_6$
- $-\mathcal{A}^{\mu}_{3/2} = [\frac{\mathcal{C}^A_3}{M}(g^{\nu\mu}\not{q} - q^{\nu}\gamma^{\mu}) + \frac{\mathcal{C}^A_4}{M^2}(g^{\nu\mu}q.p' - q^{\nu}p'^{\mu}) + \mathcal{C}^A_5 g^{\nu\mu} + \frac{\mathcal{C}^V_6}{M^2}q^{\nu}q^{\mu}]\gamma^5$

Graczyk-Sobczyk form factor

- Equate the helicity amplitudes from the RS model with those in the LP model using the Rarita-Schwinger formalism.
- Partially solve the equations to extract a new form factor for the RS model (only for the Δ resonance), incorporating information from the LP model.

$$G_V^{RS,new}(W,Q^2) = \frac{1}{2}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left(1+\frac{Q^2}{4W^2}\right)^{-\frac{N}{2}}\sqrt{3(G_3(W,Q^2))^2+(G_1(W,Q^2))^2}$$

$$G_3(W,Q^2) = \frac{1}{2\sqrt{3}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}+\frac{C_3^V}{M}(W+M)\right],$$

$$G_1(W,Q^2) = -\frac{1}{2\sqrt{3}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}-C_3^V\frac{(M+W)M+Q^2}{MW}\right]$$

Hernandez et al. (HNV) model (2007)

- Introduced non-resonant mechanism from the non-linear chiral Lagrangian (lecture 2).
- describes the resonances in the first and the second resonance region (4 resonances) using the Rarita-Schwinger (Lecture 2) and the Δ propagator.
- Used form factors from Lalakulich-Paschos fit for resonant interaction.
- The model is partially unitarized by imposing Watson theorem

E. Hernandez, J. Nieves and M. Valverde, Phys. Rev. D **76** (2007) 033005

Hybrid model (2017)

R. González-Jiménez, *et al*
[Phys. Rev. D **95** (2017)]

- Using HNV model (previous slide)
- Extends the validity of the non-resonant model by using a Regge approach. (Lecture 2)
- The model combined low-W (ChPT) and high-W (ReChi) models in a phenomenological way, into a hybrid model.

Dynamic Couple Channel (DCC) model (2015)

- Solving a coupled channel equation for the Δ(1232) and higher resonances.
- The model includes resonant and non-resonant amplitudes, respecting the unitarity relation.
- Combine analysis to determine vector form-factors. All parameters (~ 440 + 406) for vector form factor and others for resonance mass and phases) are fixed (determined) in the analysis.
- Dipole axial form factor with $m_A = 1.02$ satisfying PCAC.
- The systematic uncertainties are not evaluated.

Nakamura et al. Phys. Rev. D 92, 074024 (2015)

MK model

M. Kabirnezhad
Phys. Rev. D **97** (2018)
Phys. Rev. D **102** (2020)
Phys.Rev.C **107** (2023)

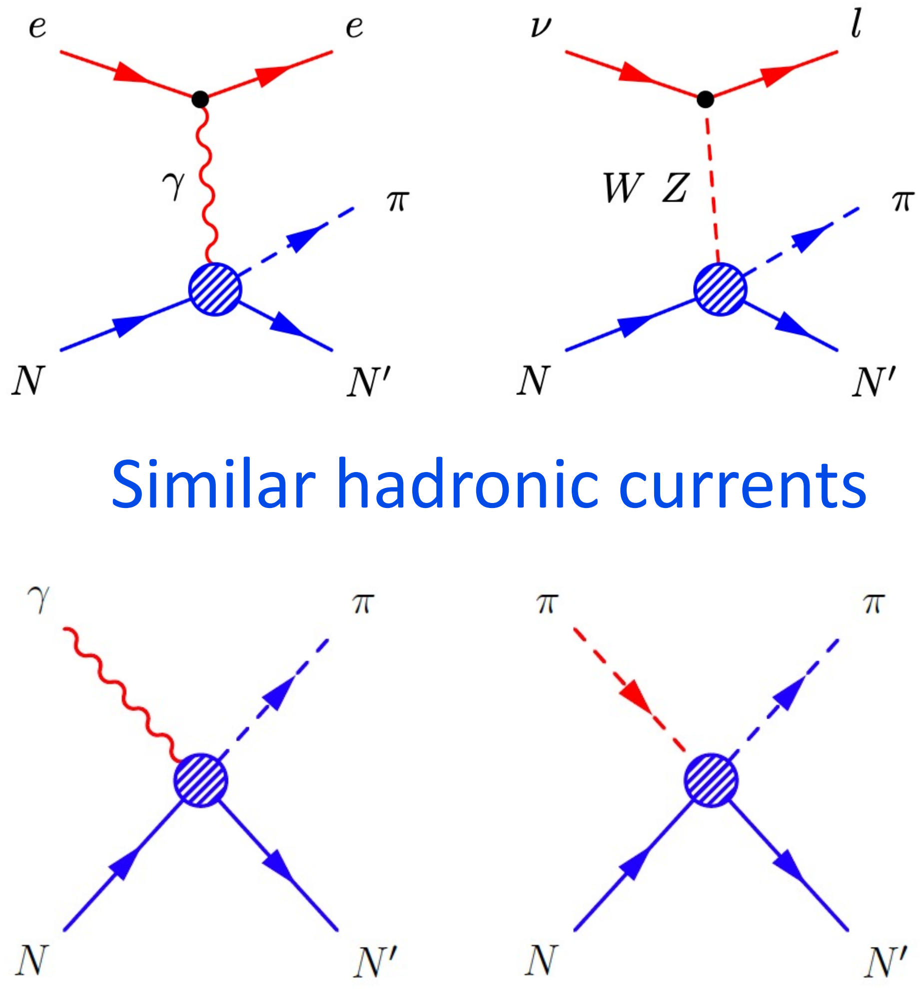
The MK model comprehensively describes single-pion production in interactions involving **photons, electrons, and neutrinos** with nucleons.

- Meson Dominance (MD) form factor: Maintains **unitarity** and integrates **QCD principles** for both resonant and non-resonant interactions. (Lecture 2)
- **CVC and PCAC** fulfilment: Ensures model consistency at low Q^2.
- Q^2 evolution: Utilises QCD calculations and **quark-hadron duality**.
- W evolution: Applies **Regge trajectory** and the Hybrid model.

R. González-Jiménez, *et al*
Phys. Rev. D **95** (2017)

How to define form factors in weak interaction

- Resonance phase space spans both perturbative and non-perturbative regimes, posing modelling challenges.
- Phenomenological models in this region must account for numerous processes and parameters.
- A unified model is essential for interpreting all interactions and maximising data utilisation.



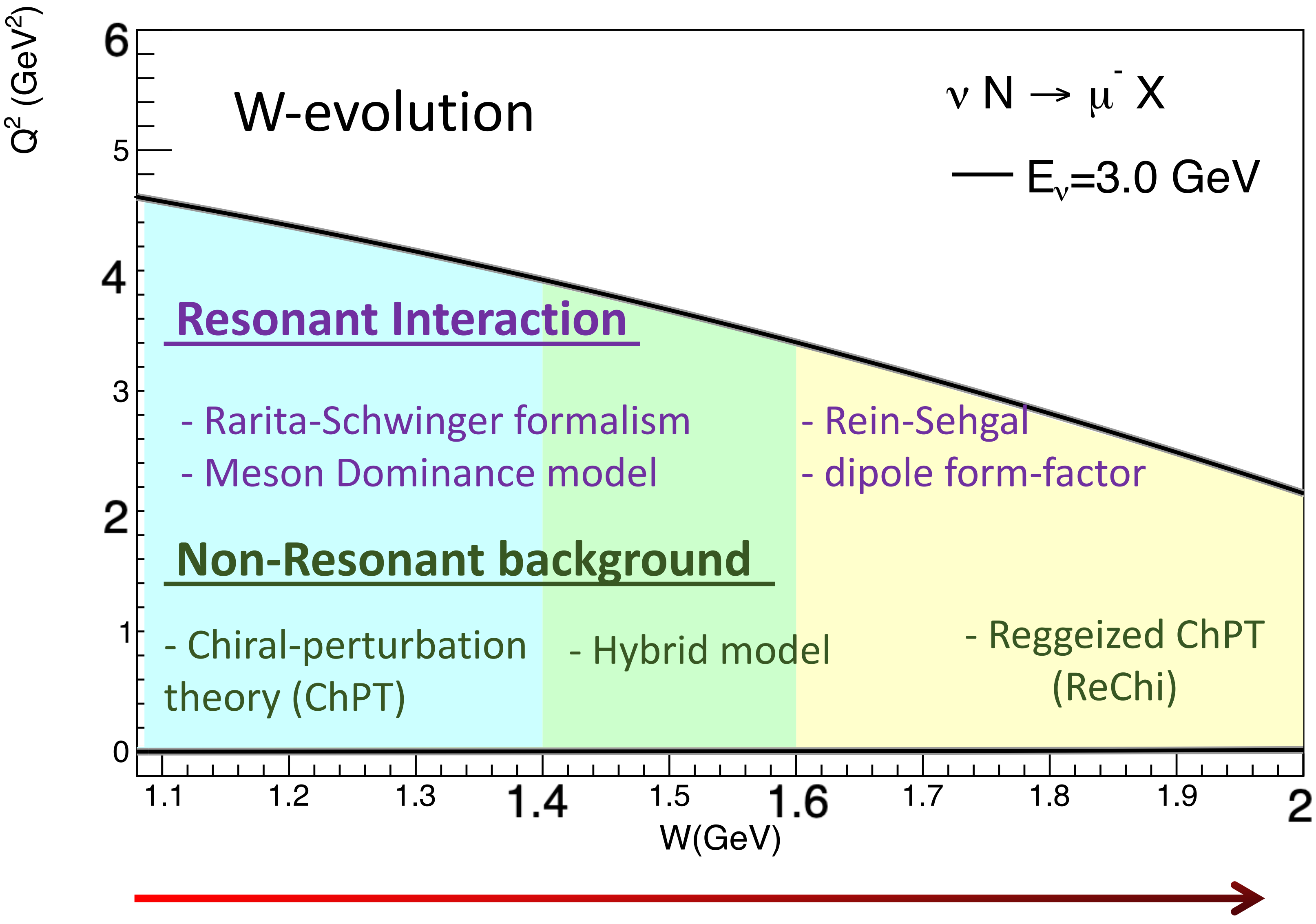
MK model

Resonant interaction

- Several resonances contribute at different invariant mass (W)

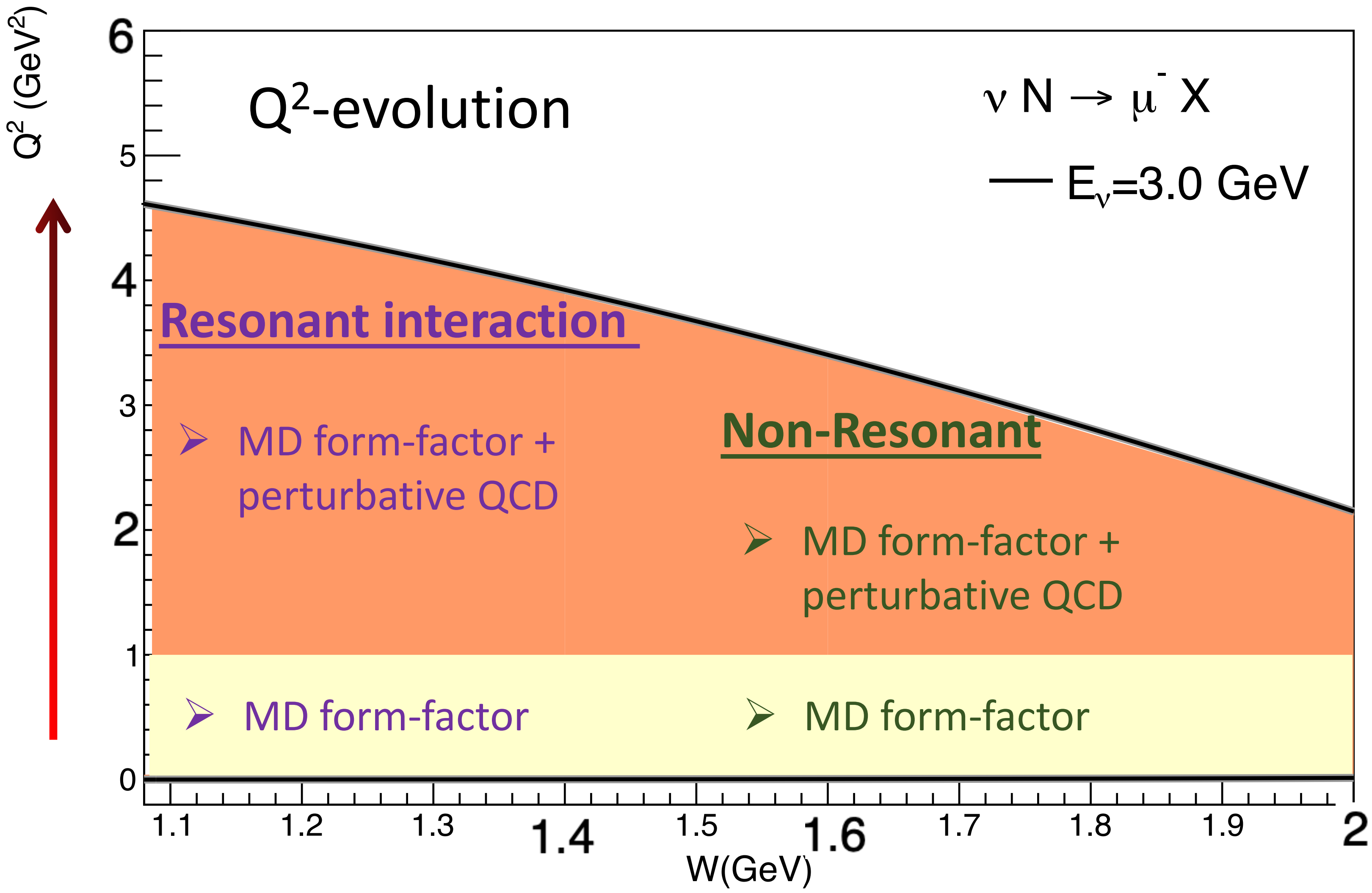
Non-resonant bkg

- Chiral perturbation at low W < 1.4 GeV
- Regge trajectory at high W
- Hybrid model



MK model

- Meson Dominance (MD) model describes form-factors in non-perturbative domain
- It can reproduce Q^2-evolution of form-factors to asymptotically join QCD expectations

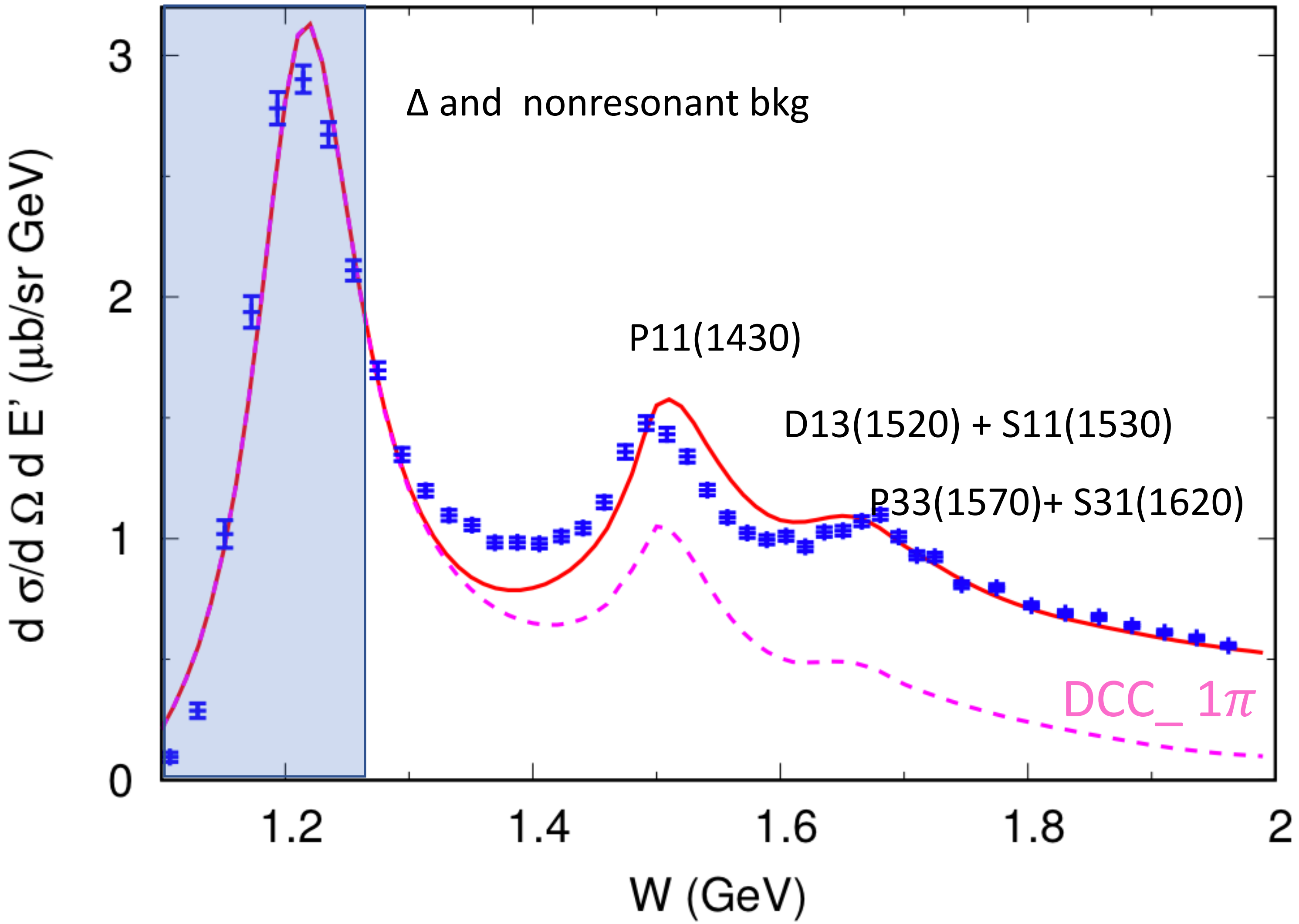


Valid kinematic region region for MK model

Data used in the Joint analysis

# data point	Photon, electron, pion, Neutrino Channels	Q^2 Range $(GeV/C)^2$	W Range GeV	Form Factors	
≈ 9800	$\gamma\, p \to n + \pi^+, \ \gamma p \to p + \pi^0$	0	1.08 – 2.0	Proton	Vector
≈ 31000	$ep \to en + \pi^+, \ ep \to ep + \pi^0$	0.16 – 6.0	1.08 – 2.0		
≈ 2500	$\gamma n \to p + \pi^-$	0	1.08 – 2.0	Neutron	
≈ 700	NEW $en \to ep + \pi^-$	0.4 – 1.0	1.08 – 1.8		
≈ 400	$\pi^+ p \to p + \pi^+, \ \pi^- p \to p + \pi^-$	0	1.08 – 2.0	Axial-Vector	
<100	$\nu N \to l^- N + \pi \ , \bar{\nu} N \to l^+ N + \pi$	$Q^2>0$ Integrated	1.08 – 2.0 Integrated		

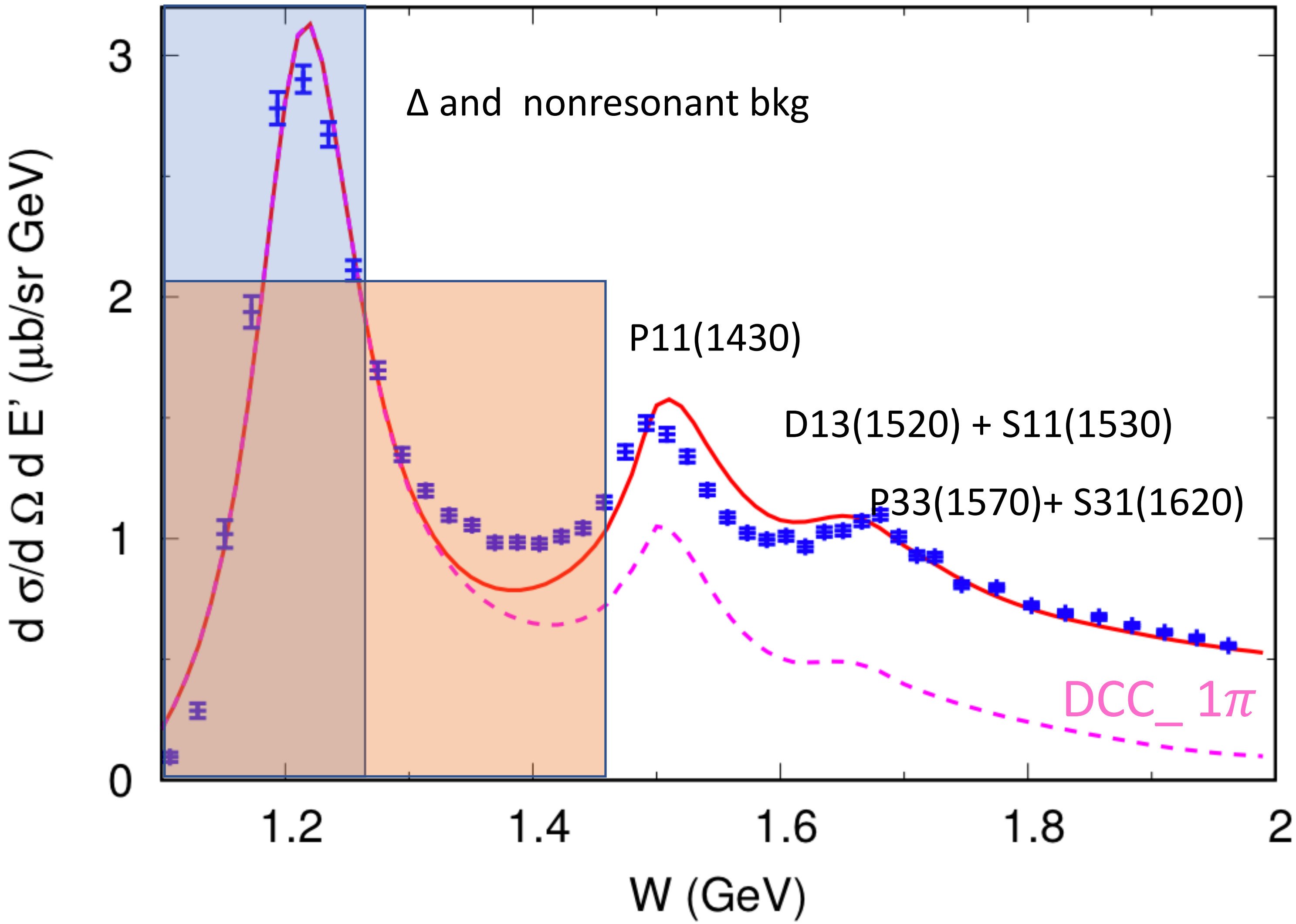
Analysis of electron-induced exclusive data



- Select Data in 1.08<W<1.28 GeV region to choose the best Δ and bkg form factors.

- Throw random starting point for minuit.

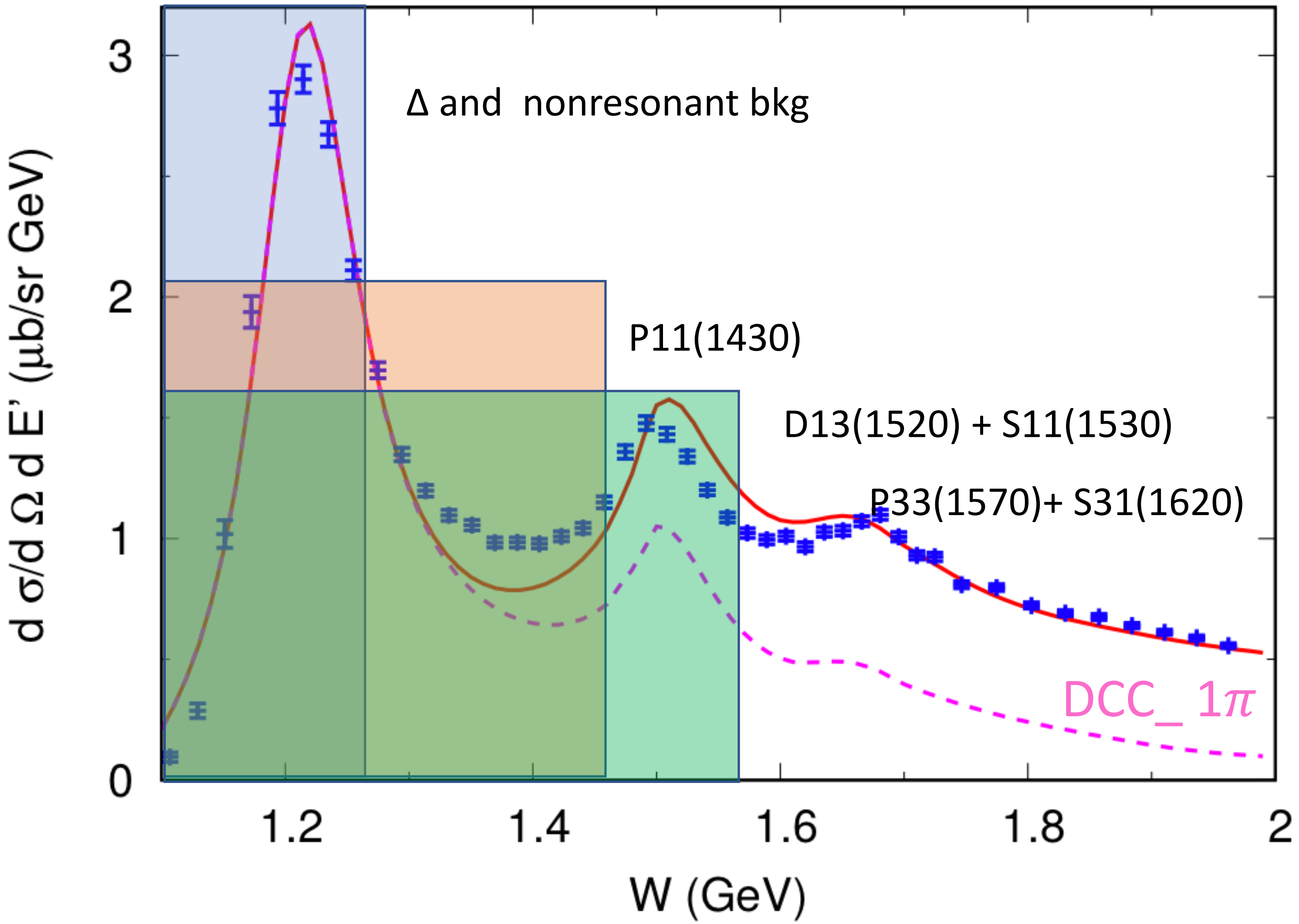
Analysis of electron-induced exclusive data



- Add data in 1.28<W<1440 MeV to choose the best P11(1430) resonance's form factor and best bkg_cut.

- Throw random starting point for minuit.

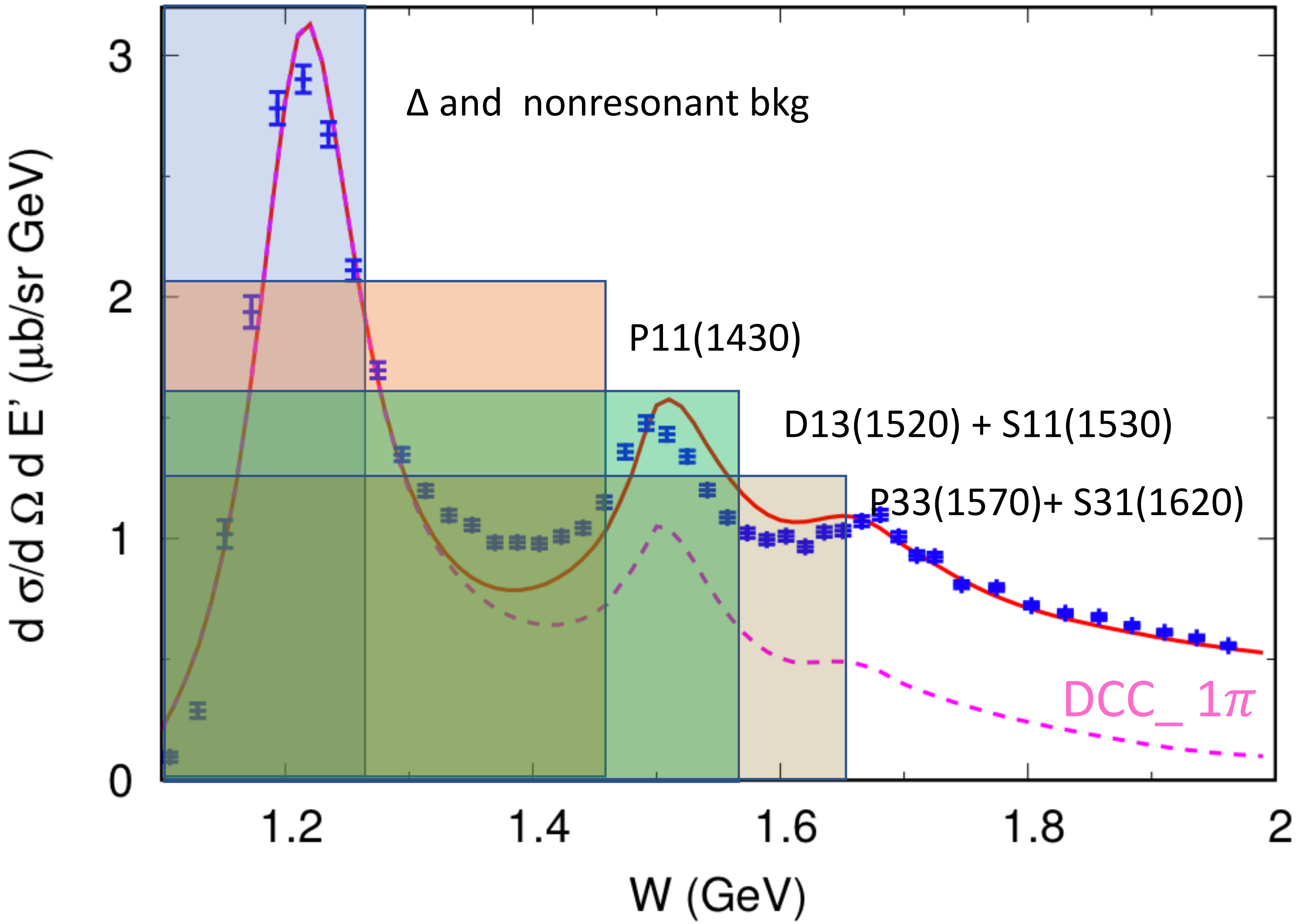
Analysis of electron-induced exclusive data



- Add data in 1440<W<1540 MeV region to choose D13(1520) + S11(1530) form factorsand the best bkg cut.

- Throw random starting point for minuit.

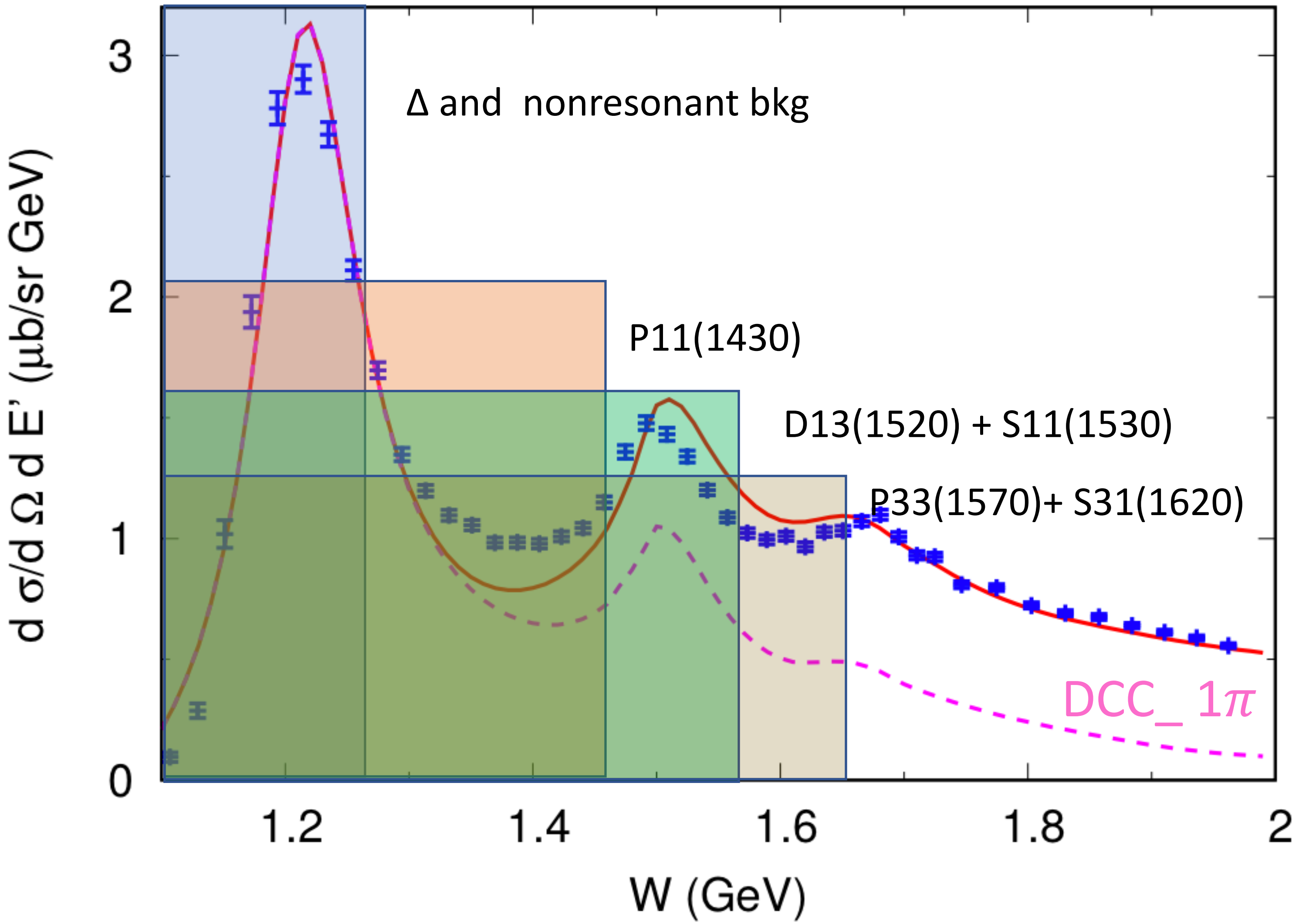
Analysis of electron-induced exclusive data



- Add data in 1540<W<1640 MeV region to P33(1570)+ S31(1620) form factor.

- Throw random starting point for minuit.

Analysis of electron-induced exclusive data

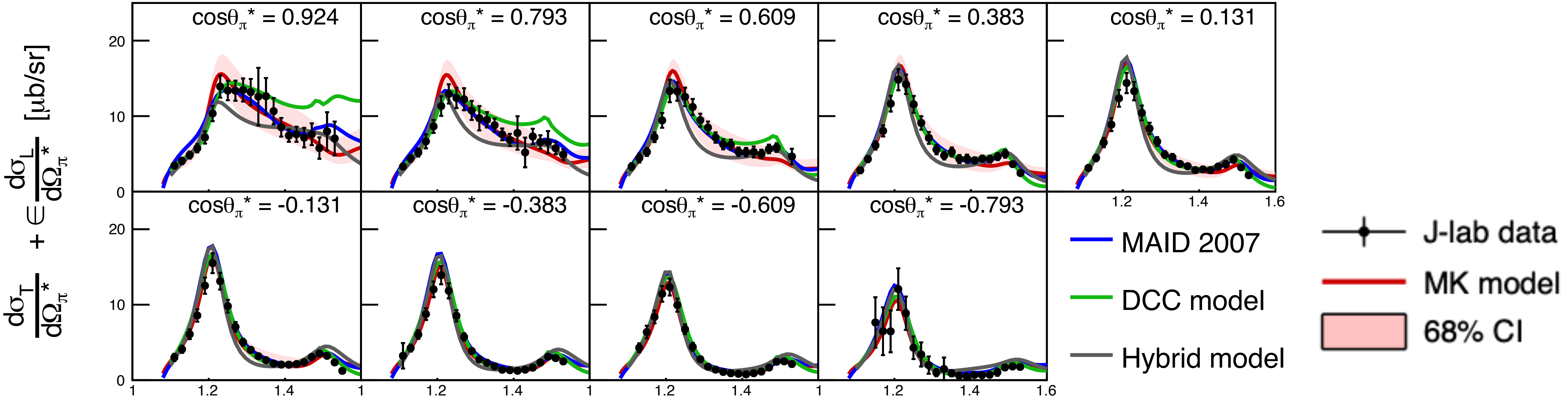


- In the final step all the parameters in the form-factors and the phases between these resonances and the nonresonant helicity amplitudes were fit.

Data/models comparison at low Q^2

M. Kabirnezhad
Phys.Rev.C 107 (2023)

$ep \rightarrow en + \pi^+$



E= 1.515 GeV

Q^2=0.4 GeV^2

1.1<W<1.41 GeV

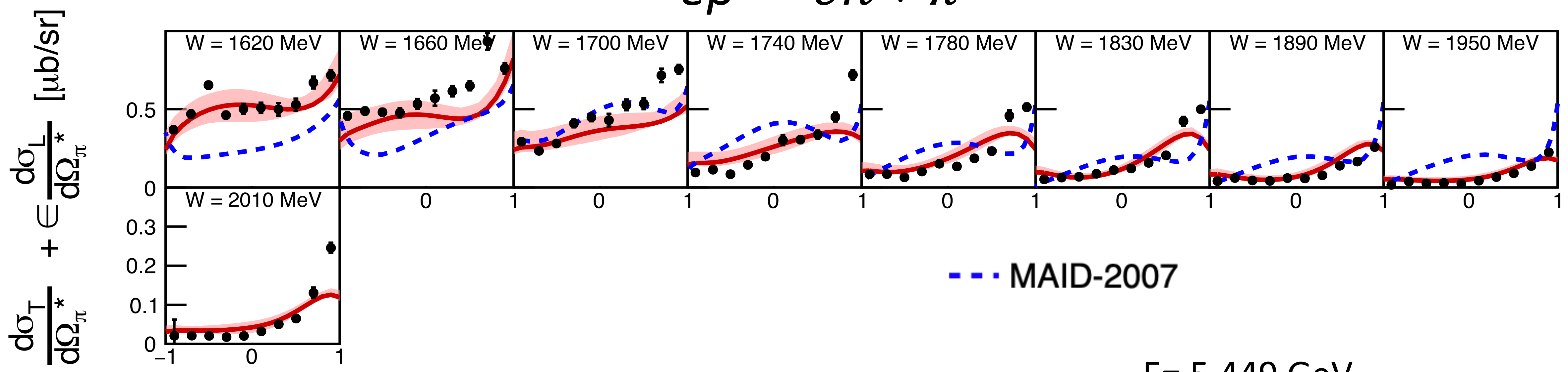
Imperial College London

Data/models comparison at high Q^2

M. Kabirnezhad
Phys.Rev.C 107 (2023)

- Only MAID model provide prediction for high Q^2

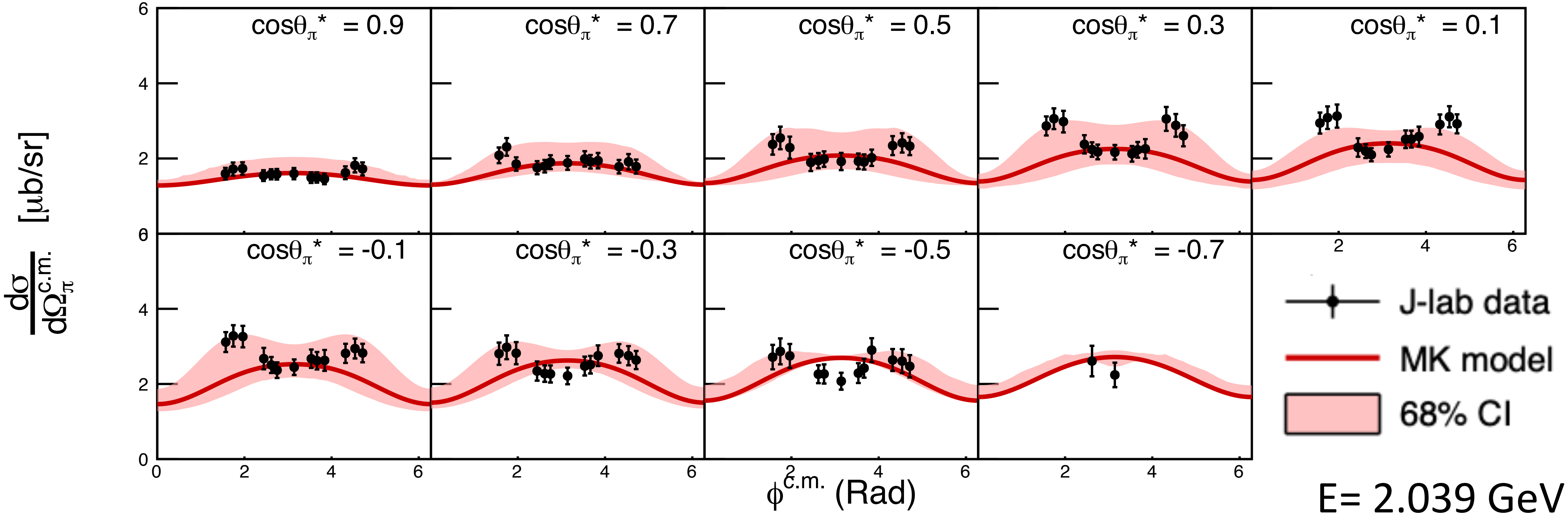
$$ep \rightarrow en + \pi^+$$



E= 5.449 GeV
Q^2=2.6 GeV^2
1.62 <W< 2.01 GeV
Third resonance region

MK model prediction after a joint-fit

$$e\,n \rightarrow e\,p + \pi^-$$



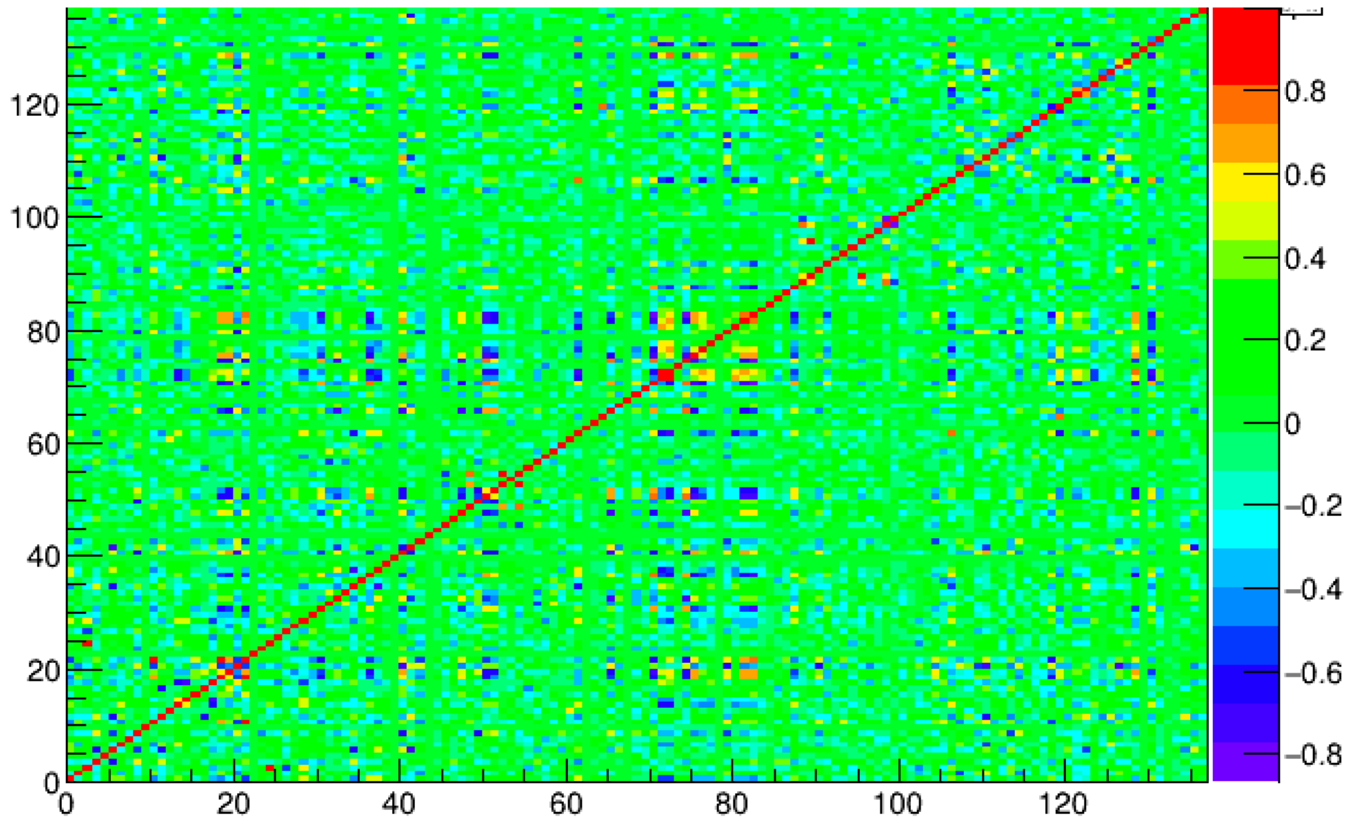
E= 2.039 GeV

Q^2=0.7 GeV^2

W=1.13 GeV

Systematic Uncertainties

- Systematic uncertainties are assessed by employing the covariance (correlation) matrix.
- They can be used to evaluate systematic uncertainties in neutrino measurements.
- 105+ 33 parameters for vector and axial form factor



Preliminary

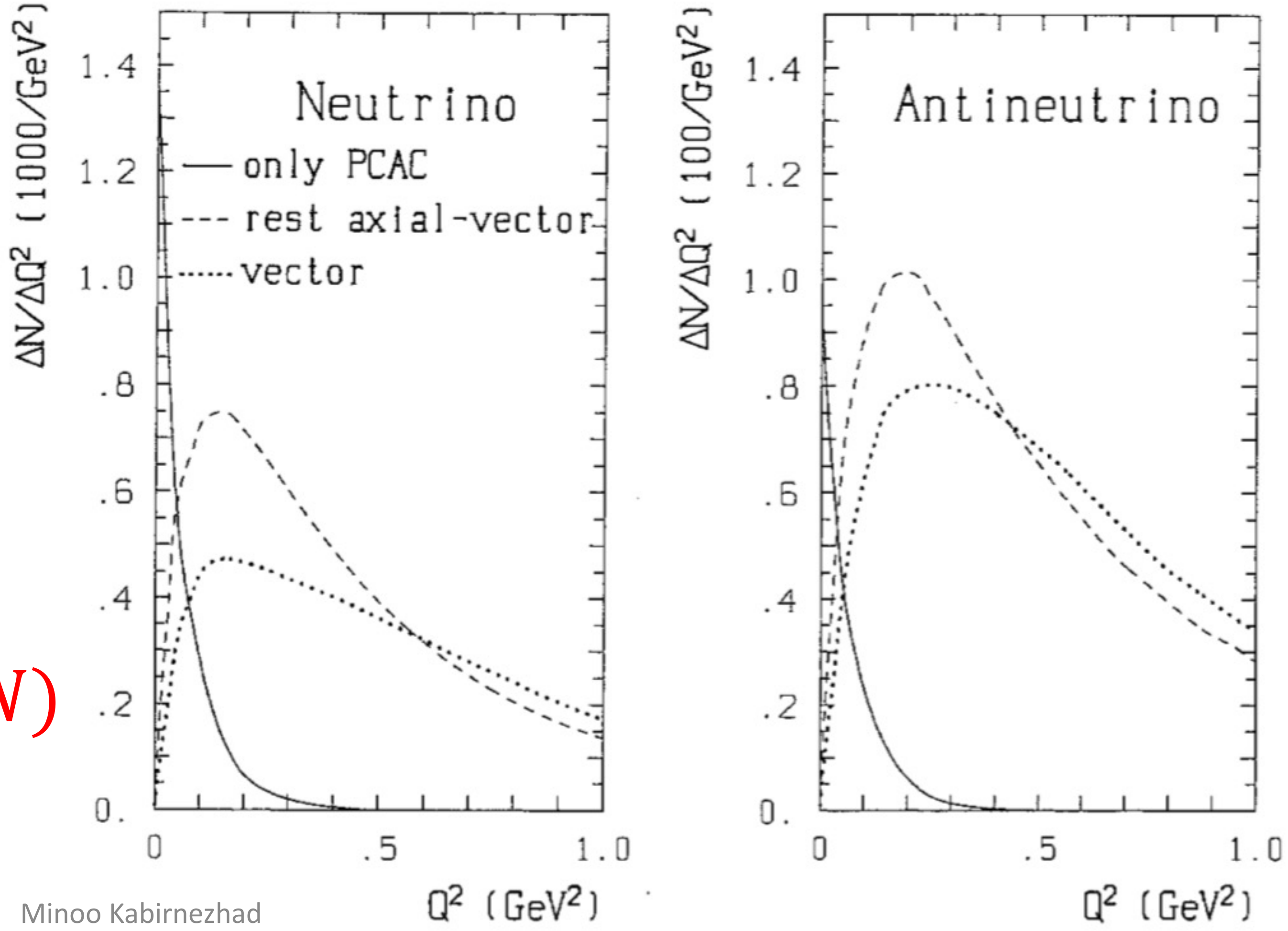
Axial current

Axial vs vector currents

$$\frac{d\sigma}{dQ^2} = \left(\frac{d\sigma}{dQ^2}\right)^{V} + \left(\frac{d\sigma}{dQ^2}\right)^{A}$$

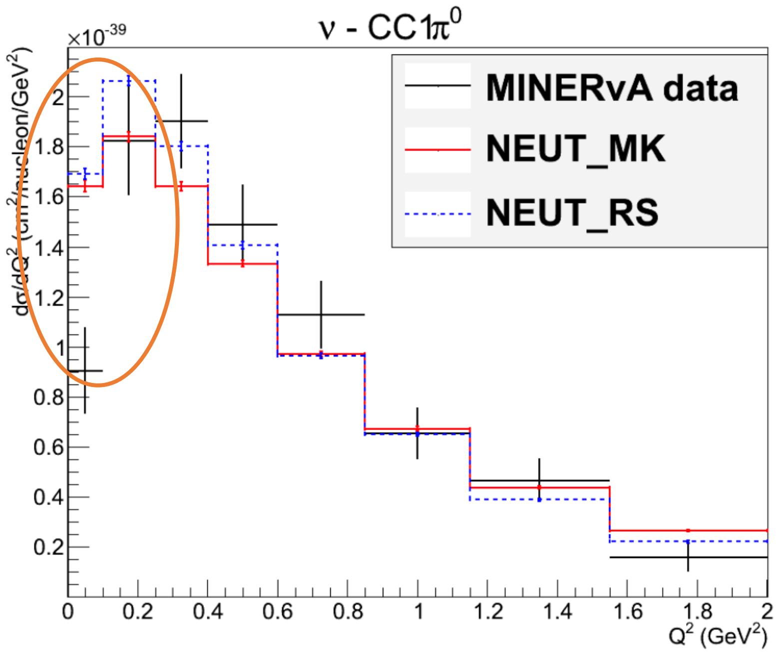
$$\left(\frac{d\sigma}{dQ^2}\right)^{VT} + \left(\frac{d\sigma}{dQ^2}\right)^{VL} \qquad \left(\frac{d\sigma}{dQ^2}\right)^{AT} + \left(\frac{d\sigma}{dQ^2}\right)^{AL}$$

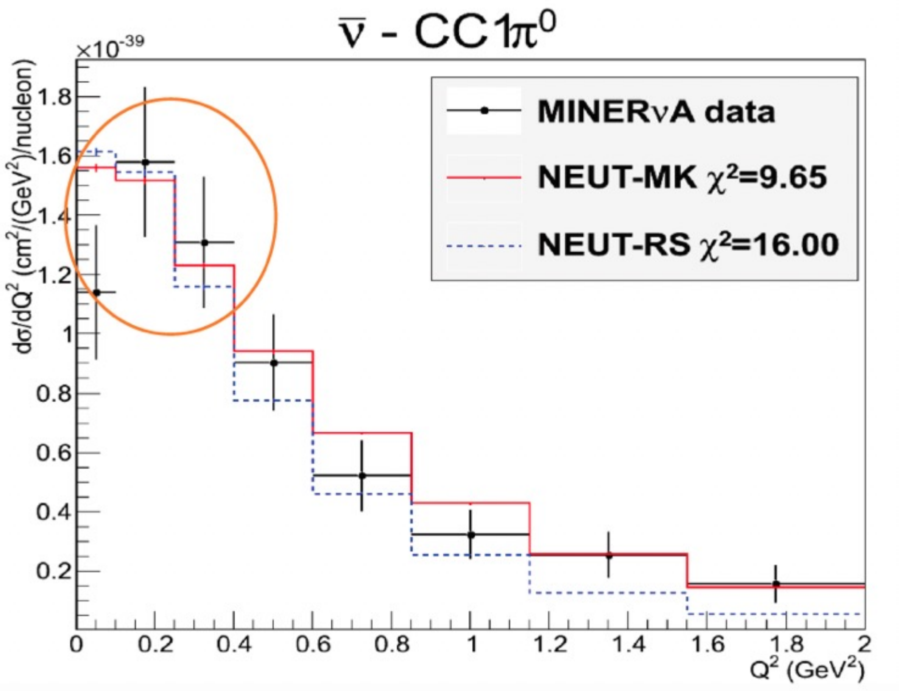
$$\left(\frac{d\sigma}{dQ^2\, dW}\right)^{AL} \Big|_{Q^2=0} \propto \sigma(\pi N \to \pi N)$$



What MK model couldn't improve

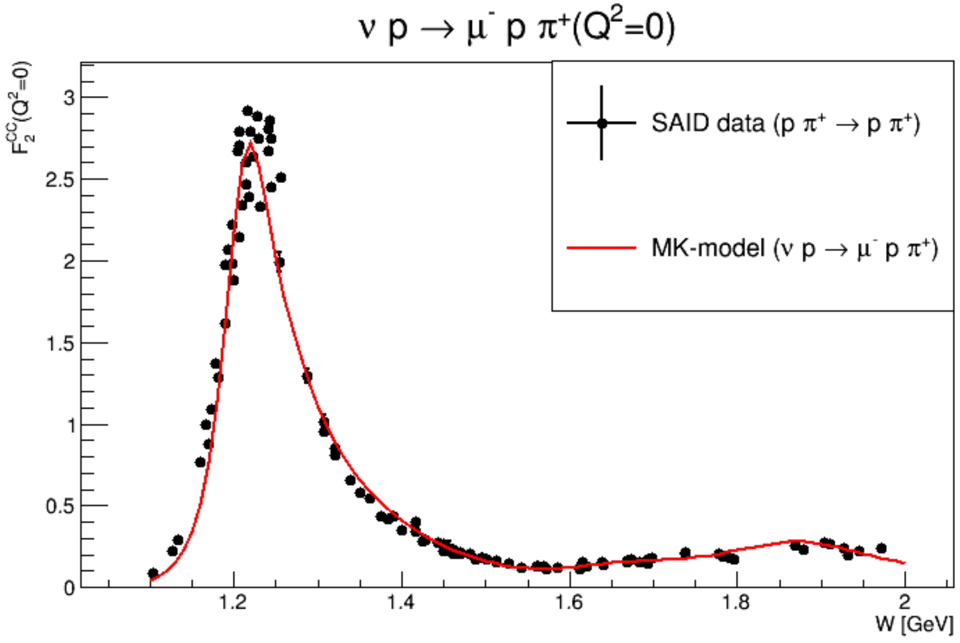
- The PCAC relation allows us to utilize pion scattering data at Q2=0Q^2 = 0Q2=0. At low Q2Q^2Q2 (<0.2 GeV), the axial current predominates due to the conservation of the vector current.

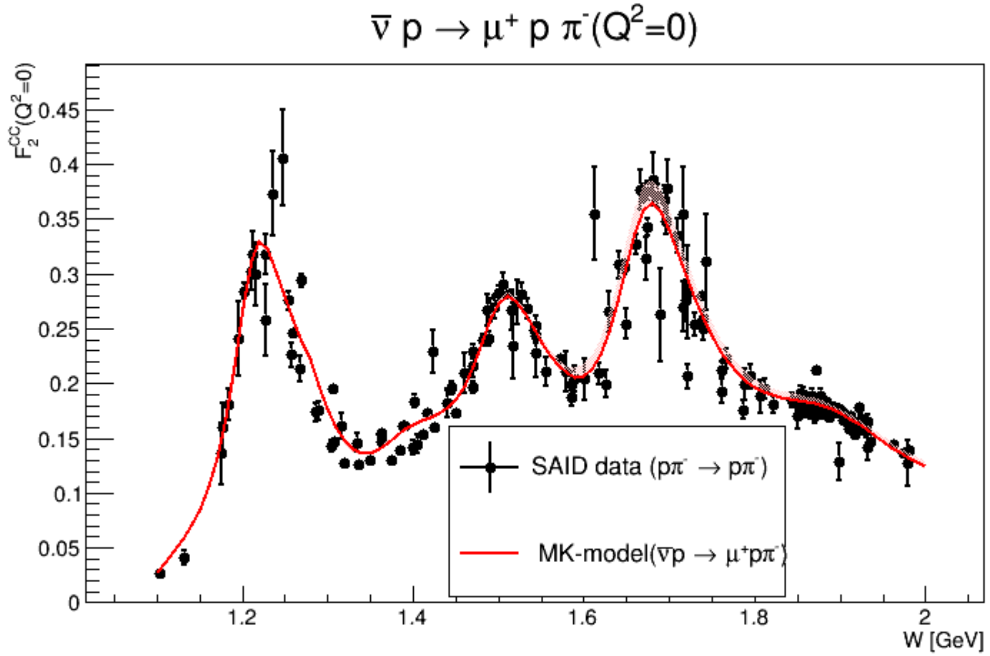




Improving the axial current using PCAC

- Using pion elastic scattering data on hydrogen to fit the axial form-factors at Q^2=0.





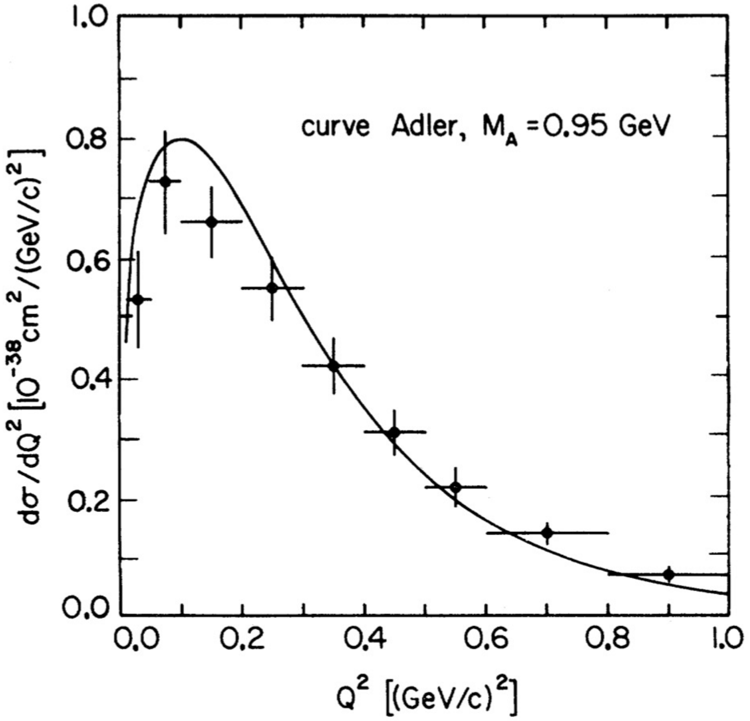
Neutrino data: ANL & BNL

- In the ANL experiment, data were initially taken with a hydrogen fill of the bubble chamber, and then data were taken with a deuterium fill for the remainder of the experiment.
- Event rates are only available as a **combination** of both hydrogen (30%) and deuterium fills of the detector.
- In the BNL experiments results are **separated** into hydrogen and deuterium measurements.
- There is no measurement for single pion production on hydrogen.

Available cross-section data: ANL

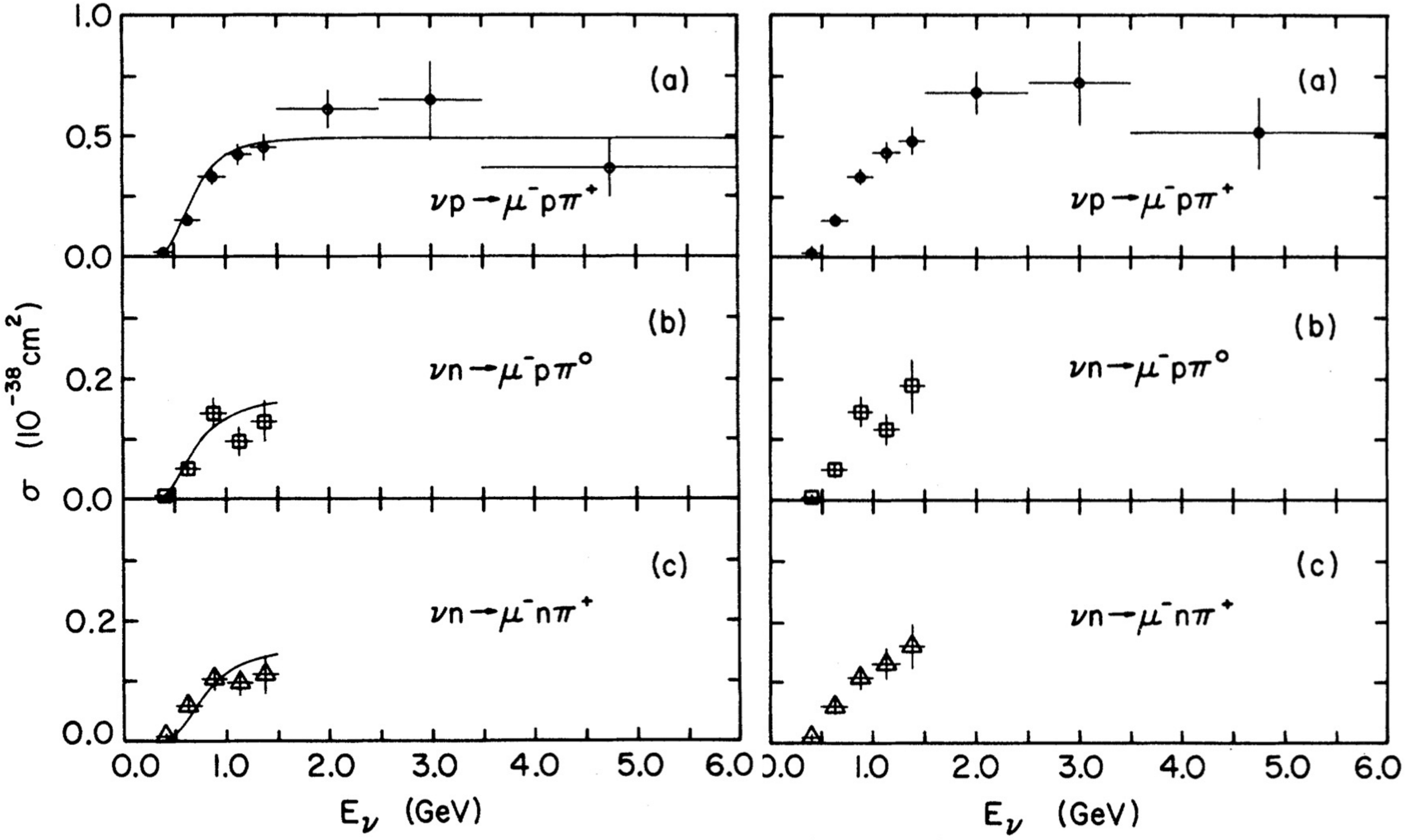
- Deuterium target

W<1.4 GeV



W<1.4 GeV

W<1.6 GeV



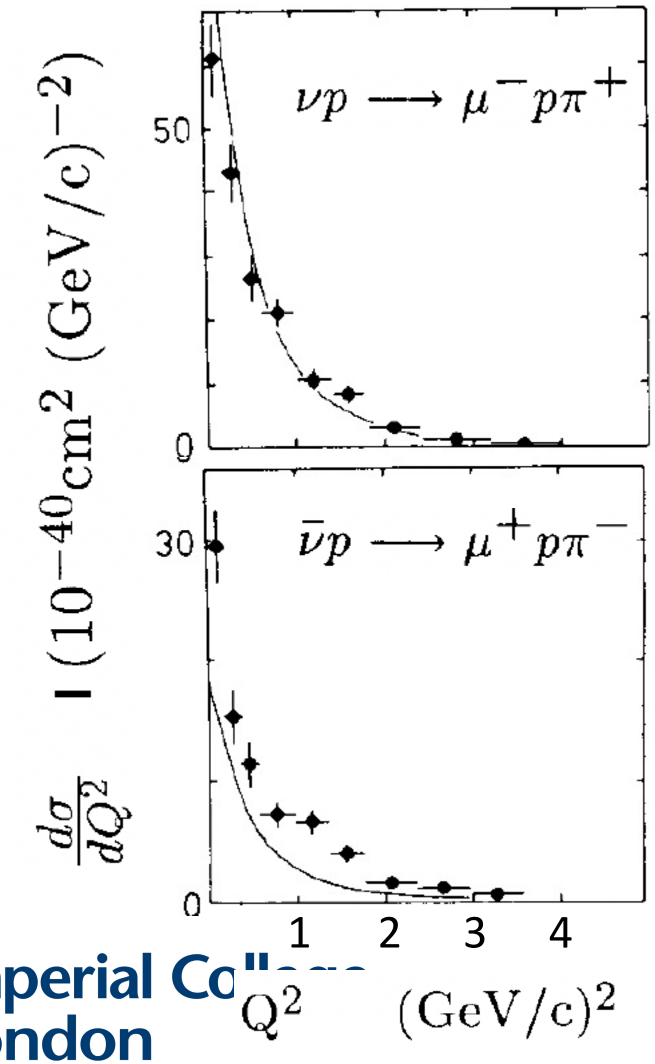
Neutrino data Used in the joint analysis

1. ANL & BNL measurements on Deuterium or mixed with Hydrogen targets ($E_\nu \approx 1$ GeV)
   - Employing the ratio of channels in the joint fit

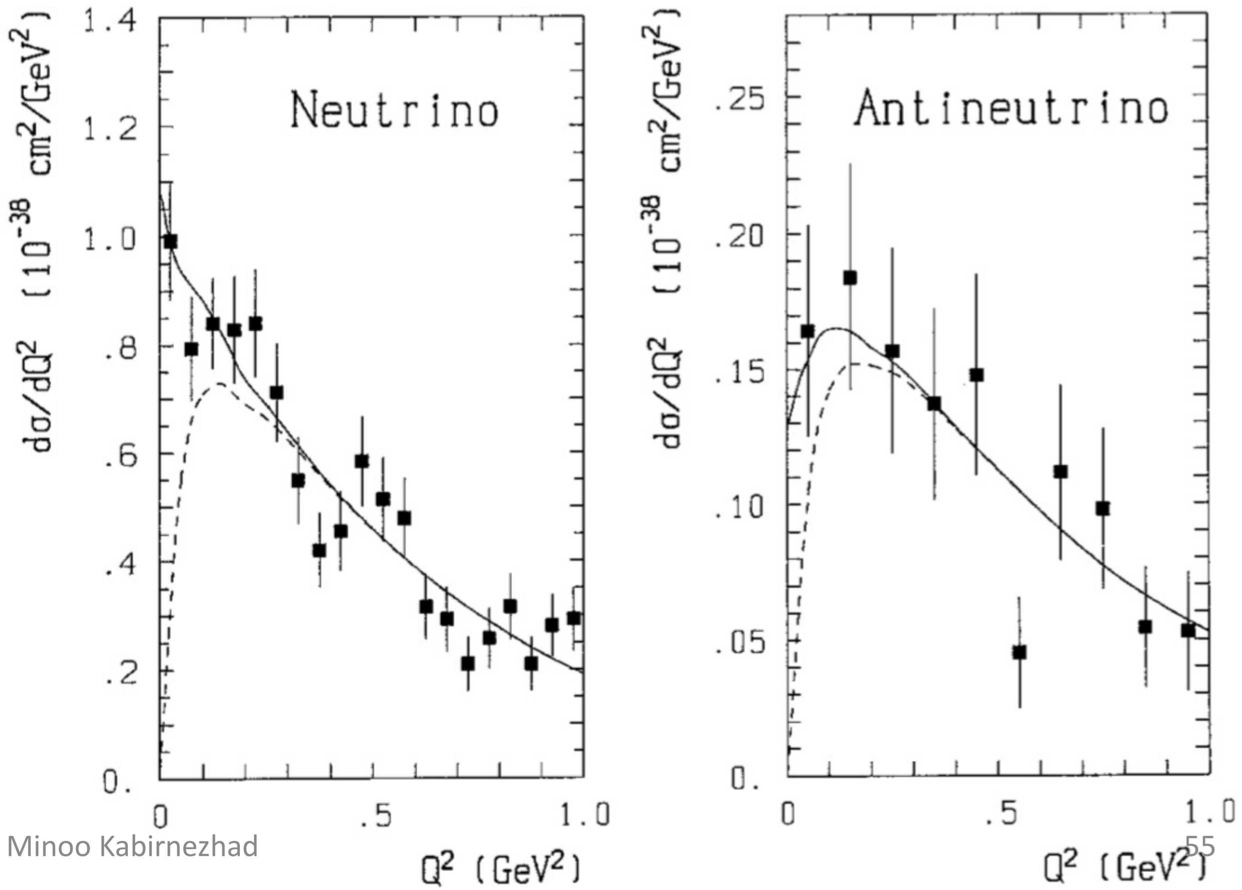
$$R = \frac{\sigma[\nu p(n_s) \rightarrow \mu p \pi^+(n_s)]}{\sigma[\nu n(p_s) \rightarrow \mu n \pi^+(p_s)]}$$

2. BEBC measurements on Hydrogen & Deuterium ($E_\nu \approx 20$ GeV)
   - Employing data on hydrogen and utilising the ratio of channels on deuterium in the fit
   - Despite high energy beam, cross-section is measured at low Q^2 and W

BEBC measurements (D_2 vs LH_2), W<1.4 GeV



Liquid hydrogen



Defining Vector + axial form factor in a joint fit

Highlight : NC channels

- Data from NC channels are exceptionally rare.
- The model successfully describes both electromagnetic and weak interactions simultaneously, and ensures rigorous control over the NC vector current.

- $V^{\mu}_{1,2,3}$ form a vector in isospin space (isovector current).
- V^{μ}_{Y} is hypercharge current

$$J^{\mu}{}_{EM} = \frac{1}{2} V^{\mu}_{Y} + V^{\mu}_{3}$$

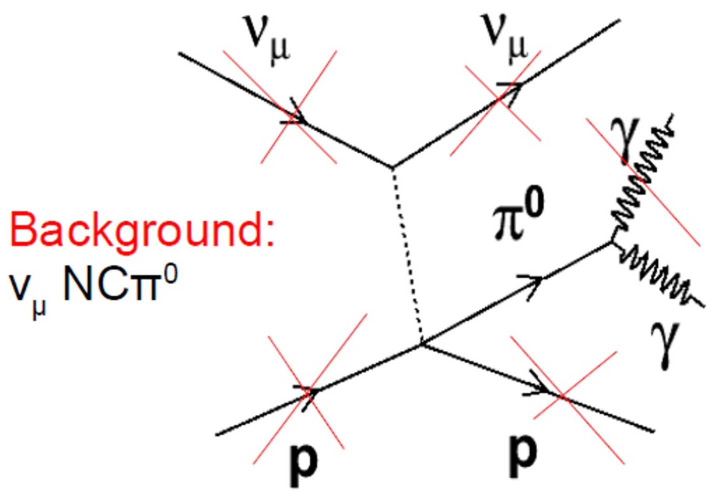
$$J^{\mu}{}_{CC} = V^{\mu}_{1} + iV^{\mu}_{2}$$

$$J^{\mu}{}_{NC} = (1 - 2\sin^2\theta_W)V^{\mu}_{3} - \sin^2\theta_W V^{\mu}_{Y}$$

Highlight : NC channels

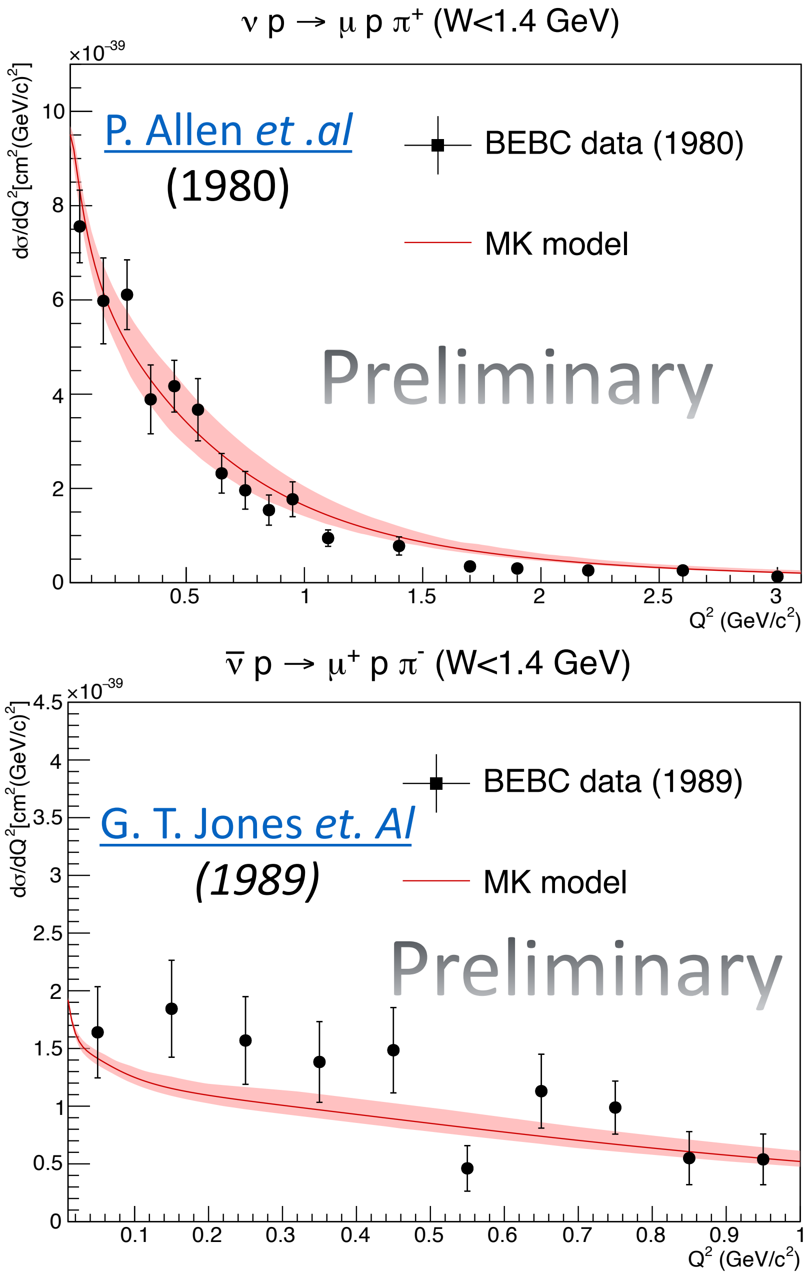
- Data from NC channels are exceptionally rare.
- The model successfully describes both electromagnetic and weak interactions simultaneously, and ensures rigorous control over the NC vector current.

- Event generators can not be tuned with very few bubble chamber data
- $\nu_\mu + p \rightarrow \nu_\mu p \pi^0$ is the main background for electron



neutrino vs anti-neutrino

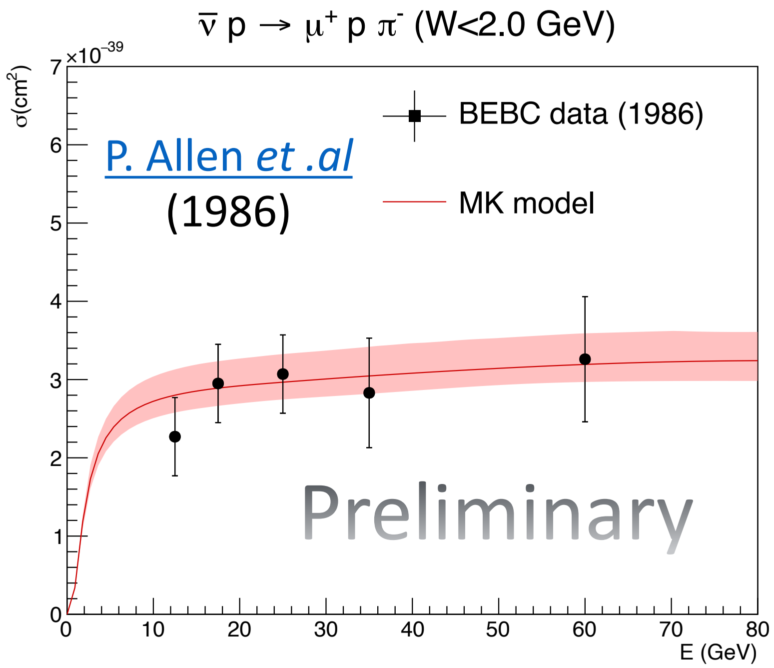
- By employing an advanced model for the form factors and incorporating data from both neutrino and anti-neutrino interactions, we can ensure a reliable prediction for both types of interactions.
- This is extremely important for measuring CP-violation!

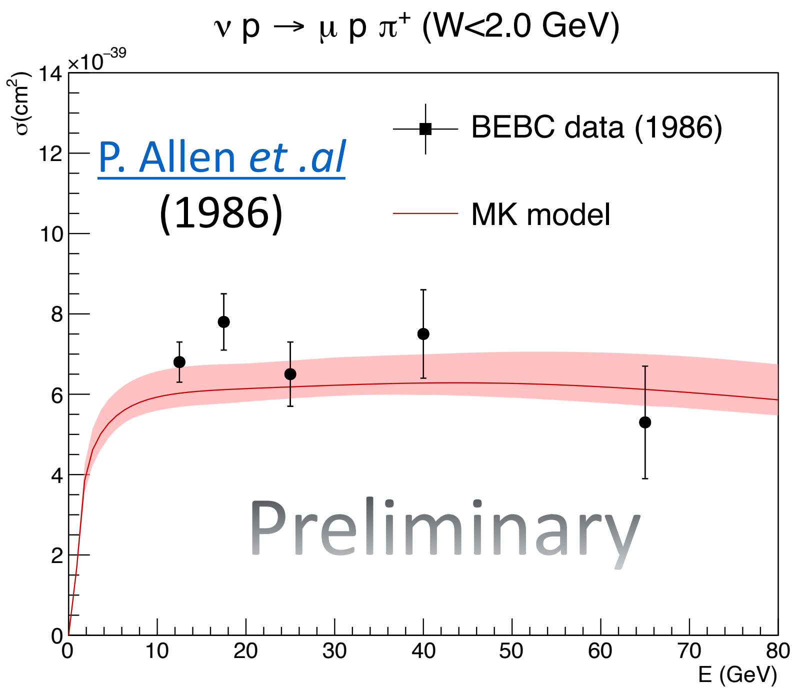


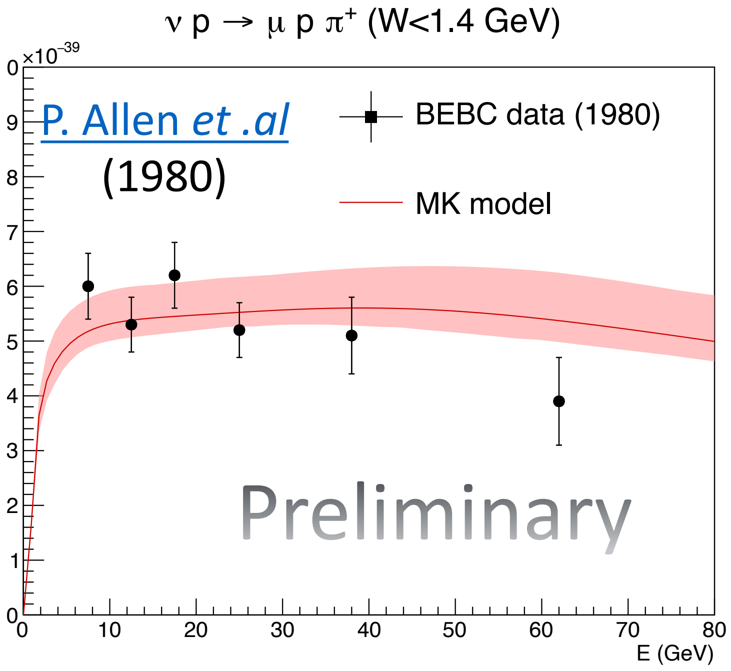
Imperial College London

neutrino vs anti-neutrino)

- Integrated cross section







Imperial College London

Backup

Axial current parametrisation

- Due to the scarcity of neutrino data, parameters must be minimised.
- The chiral symmetry of strong interactions dictates that the coupling constants of left and right fields must be equal, resulting in identical asymptotic behavior for vector and axial form factors.
- Chiral symmetry reduces the number of parameters in the axial form factors.

Highlight 3: Low Q^2 region

- The model is designed to address the low Q^2 region, where existing models struggle to predict empirical data:

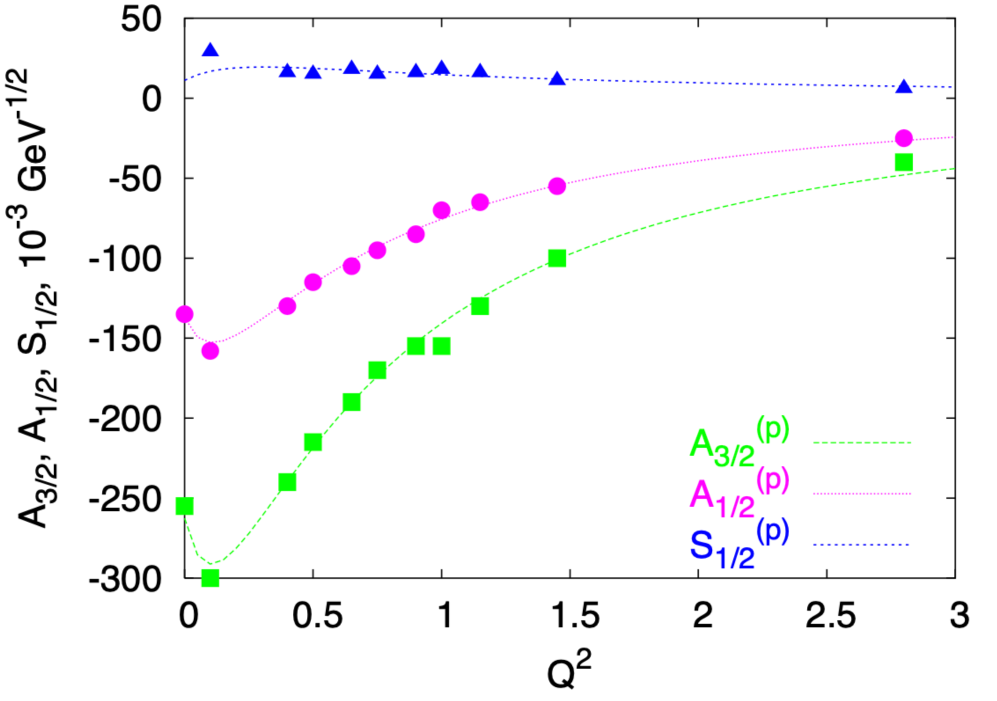
  1. For the vector current, it incorporates Conserved Vector Current (CVC) principles and photon scattering data.
  2. For the axial current, it utilises Partially Conserved Axial Current (PCAC) principles and pion scattering data.

$$\left.\frac{\mathrm{d}\sigma}{dQdW}\right|_{Q^2=0} \propto \sigma(\pi N \rightarrow \pi N)$$

Existing form-factors determination

- Helicity amplitudes for spin 3/2

$$A_{3/2} \propto \left\langle R, +\frac{3}{2} \middle| J_{em}.\,\epsilon^{(R)} \middle| N, +\frac{1}{2} \right\rangle$$
$$A_{1/2} \propto \left\langle R, +\frac{1}{2} \middle| J_{em}.\,\epsilon^{(R)} \middle| N, -\frac{1}{2} \right\rangle$$
$$S_{1/2} \propto \left\langle R, +\frac{1}{2} \middle| J_{em}.\,\epsilon^{(S)} \middle| N, +\frac{1}{2} \right\rangle$$



MAID "data": Helicity amplitudes for $P_{33}(1232)$ resonance

Existing form-factors determination

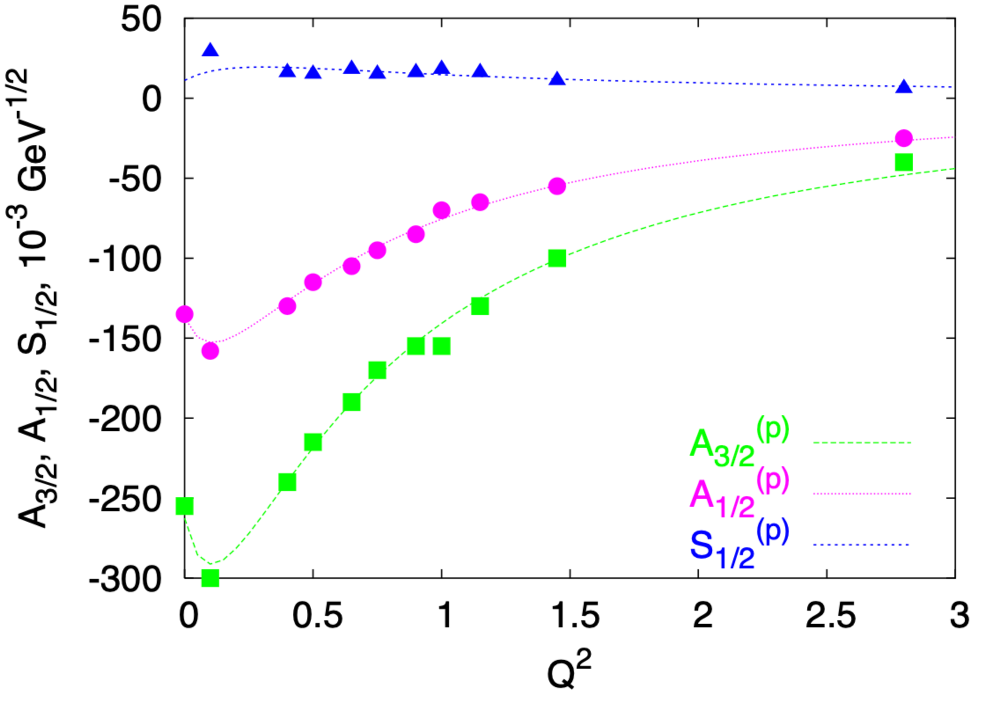
- Some of neutrino models, utilized the helicity amplitudes determined in the MAID analysis to extract form factors.

$$C_3^{(p)} = \frac{2.13/D_V}{1 + Q^2/4M_V^2},$$

$$C_4^{(p)} = \frac{-1.51/D_V}{1 + Q^2/4M_V^2},$$

$$C_5^{(p)} = \frac{0.48/D_V}{1 + Q^2/0.776M_V^2}.$$

$$D_V = \left(1 + \frac{Q^2}{M_V^2}\right)^2, \qquad M_V = 0.84\ GeV$$



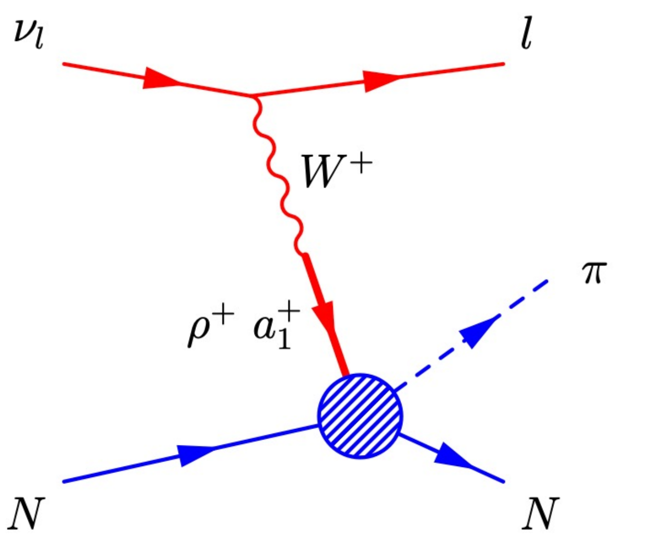
Fitted model to MAID analysis for $P_{33}(1232)$ resonance.
From Lalakulich et. al. (2006)

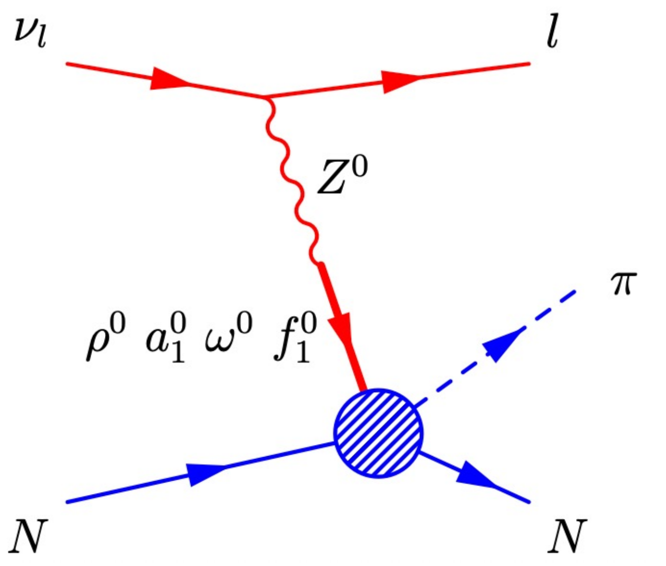
Meson Dominance (MD) model

- The MD model is rooted in the effective Lagrangian of quantum field theory.

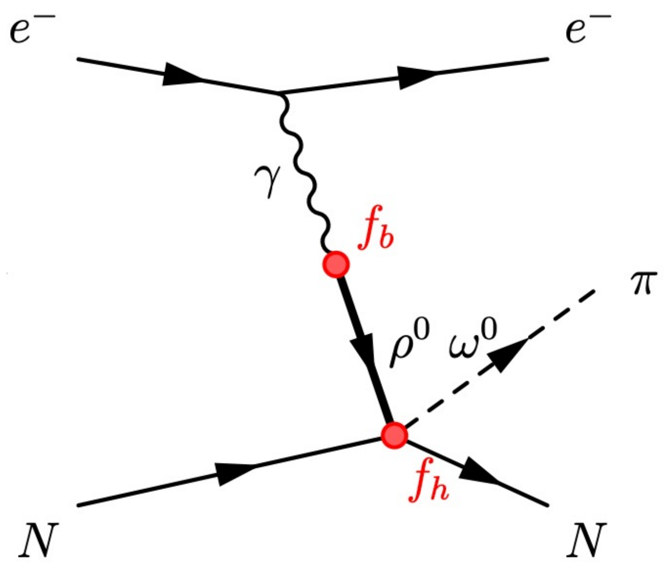
1. J. J. Sakurai, Annals Phys.11, 1 (1960)
2. M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961)

- It establishes connections between vector and axial currents and corresponding meson fields with analogous quantum properties.
- This framework explains the interaction between neutrinos and nucleons through meson exchange.





Meson Dominance (MD) model



- MD form factors can be expressed in terms of the meson masses and the coupling strengths, summing over all possible mesons:

$$\mathrm{F_N}(Q^2) = \sum_{j=1}^{n} \frac{m_j^2}{m_j^2 - Q^2} \left(\frac{f_h}{f_b}\right)$$

- Although they do not inherently comply to the unitarity condition (analytic model) or accurately predict behaviour at high Q^2, they can be **imposed**!

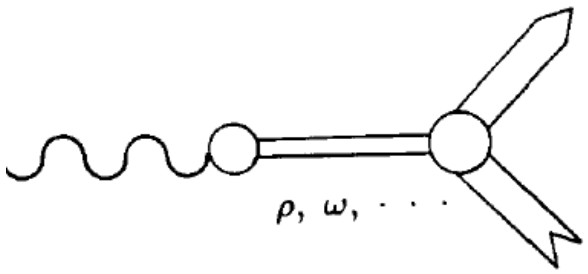
k	ρ -group	$m_{(\rho)k}$[GeV]	ω -group	$m_{(\omega)k}$[GeV]
1	$\rho(770)$	0.77526	$\omega(782)$	0.78265
2	$\rho(1450)$	1.465	$\omega(1420)$	1.410
3	$\rho(1700)$	1.720	$\omega(1650)$	1.670
4	$\rho(1900)$	1.885	$\omega(1960)$	1.960
5	$\rho(2150)$	2.150	$\omega(2205)$	2.205
k	a_1 -group	$m_{(a_1)k}$[GeV]	f_1 -group	$m_{(f_1)k}$[GeV]
1	$a_1(1260)$	1.230	$f_1(1285)$	1.2819
2	$a_1(1420)$	1.411	$f_1(1420)$	1.4263
3	$a_1(1640)$	1.655	$f_1(1510)$	1.518
4	$a_1(2095)$	2.096	$f_1(1970)$	1.1971

C. Adamuscin *et al*. Eur. Phys. J. C 28, 115 (2003)

Meson Dominance (MD) model

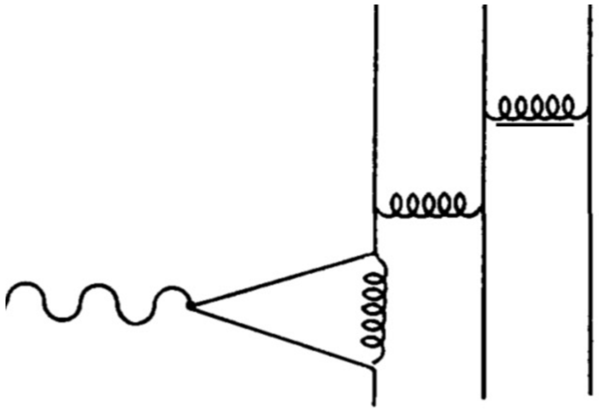
Representation of electron scattering:

Non-perturbative (low Q^2)



- Vector mesons propagate between the virtual photon and the nucleon

Perturbative (high Q^2)



- schematic quark model of VMD model

Asymptotic behaviour of form factor

- At large Q2, resonance form factors must align with the perturbative QCD constraints.
- For spin 3/2 resonance:

G. Vereshkov and N. Volchanskiy (PRD 2007)

$$F_\alpha(Q^2) \cong \left(\frac{4M_N^2}{Q^2}\right)^{p_\alpha} \frac{f_\alpha}{\ln^{n_\alpha}\left({Q^2}/{\Lambda_{QCD}^2}\right)}, \qquad (\alpha = 1-3)$$

$$p_1 = 3, p_2 = p_3 = 4,$$
$$n_3 > n_1 > n_2, \qquad n_1 \cong 3$$

MD form factors used in the model

- For spin 3/2 resonance:

$$F_\alpha(Q^2) = \frac{f_\alpha}{L_\alpha(Q^2)} \sum_{k=1}^{K} \frac{a_{\alpha k}\, m_k^2}{m_k^2 + Q^2}, \qquad (\alpha = 1 - 3)$$

$$\mathrm{L}_\alpha(\mathrm{Q}^2) = \left[1 + \mathrm{g}_\alpha \ln\left(1 + \frac{\mathrm{Q}^2}{\Lambda_{\mathrm{QCD}}^2}\right) + \mathrm{h}_\alpha \ln^2\left(1 + \frac{\mathrm{Q}^2}{\Lambda_{\mathrm{QCD}}^2}\right)\right]^{\mathrm{n}_\alpha}$$

$\mathrm{n}_1 = 3, \mathrm{n}_2 = 2, \mathrm{n}_3 = 4$

$\Lambda_{\mathrm{QCD}} \in [0.19 - 0.24]$ GeV

- $a_{\alpha k}$ and $b_{\beta k}$ are constrained by unitarity conditions that also satisfy asymptotic QCD requirements.

Nonresonant pion production (linear σ-model)

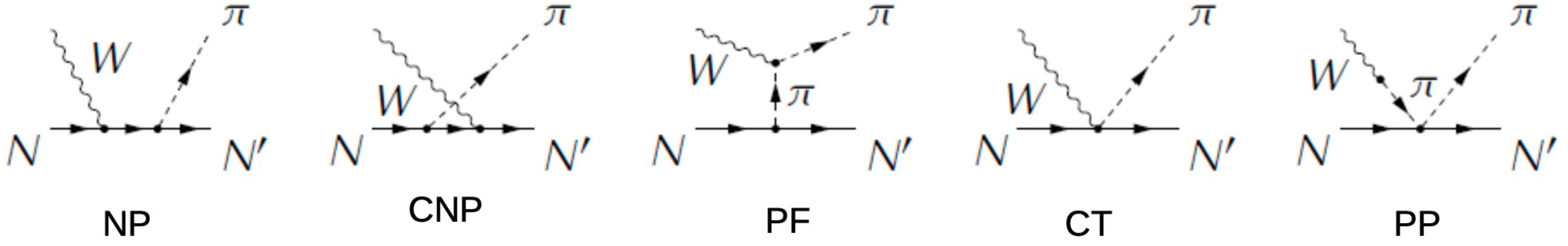
- Is based on $\mathrm{SU}(2)\times\mathrm{SU}(2)$ chiral symmetry, consistent with the symmetries of QCD.
- The ingredients of the model are Nucleon and pion fields plus an scalar σ field.
- The Lagrangian is linear with pion field.
- Three possible (Born) diagrams is the result of the linera σ model.
- There is no experimental evidence for σ particle

Nonresonant pion production (**non**-linear σ-model)

- Is based on $SU(2)\times SU(2)$ chiral symmetry, consistent with the symmetries of QCD.
- The ingredients of the model are Nucleon and pion fields.
- The Lagrangian is **not** linear with pion field.
- Five possible (Born) diagrams is the result of the **non**-linear σ model.
- Low energy Chiral Perturbative Theory (ChPT) is valid at low energy.

E. Hernandez, J. Nieves and M. Valverde, Phys. Rev. D **76** (2007) 033005

Nonresonant pion production



$$\mathcal{M}^{CC}_{NP} = \frac{g_A}{\sqrt{2} f_\pi} \cos\theta_C \frac{1}{s - M} \bar{u}(p_2) \; \not{q}\, \gamma_5 (\not{p}_1 + \not{k} + M) \epsilon^\mu \Gamma^{CC}_\mu \, u(p_1)$$

$$\mathcal{M}^{CC}_{CNP} = \frac{g_A}{\sqrt{2} f_\pi} \cos\theta_C \frac{1}{u - M} \bar{u}(p_2) \, \epsilon^\mu \Gamma^{CC}_\mu (\not{p}_2 - \not{k} + M) \; \not{q}\gamma_5 \, u(p_1)$$

$$\mathcal{M}^{CC}_{PF} = \frac{g_A}{\sqrt{2} f_\pi} \cos\theta_C \frac{1}{t - m_\pi^2} F_{PF}(k^2) \bar{u}(p_2) \; \gamma_5 \; [2q\epsilon - k\epsilon] \; u(p_1)$$

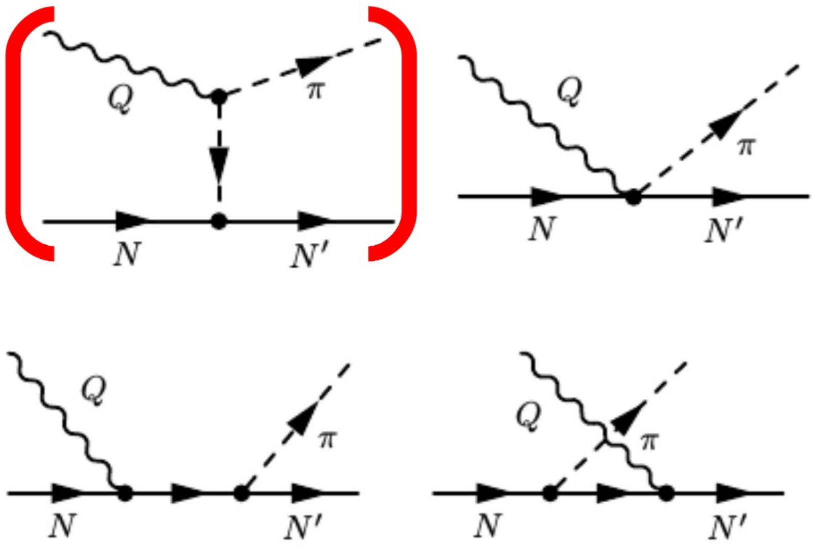
$$\mathcal{M}^{CC}_{CT} = \frac{1}{\sqrt{2} f_\pi} \cos\theta_C \; \bar{u}(p_2) \; \epsilon^\mu \gamma_\mu \; [g_A F^V_{CT}(k^2) \; \gamma_5 - F_\rho((k-q)^2)] \; u(p_1)$$

$$\mathcal{M}^{CC}_{PP} = \frac{1}{\sqrt{2} f_\pi} \cos\theta_C \; \bar{u}(p_2) \; \frac{\epsilon k}{k^2 - m_\pi^2} \; \not{k} \; u(p_1)$$

Hybrid Model for nonresonant pion production

R. González-Jiménez, *et al*
[Phys. Rev. D **95** (2017)]

- Use ChPT model at low energy (W).
- Use Regge formalism at high energy (W). Regge Theory provides the **high energy (s→∞) behavior** of the amplitude:



$$\frac{1}{t - m_\pi^2}$$

The pion propagator is replace by the Regge trajectory of the pion family

$$\mathcal{P}_\pi(t, s) = -\alpha'_\pi \varphi_\pi(t) \Gamma[-\alpha_\pi(t)] (\alpha'_\pi s)^{\alpha_\pi(t)}$$

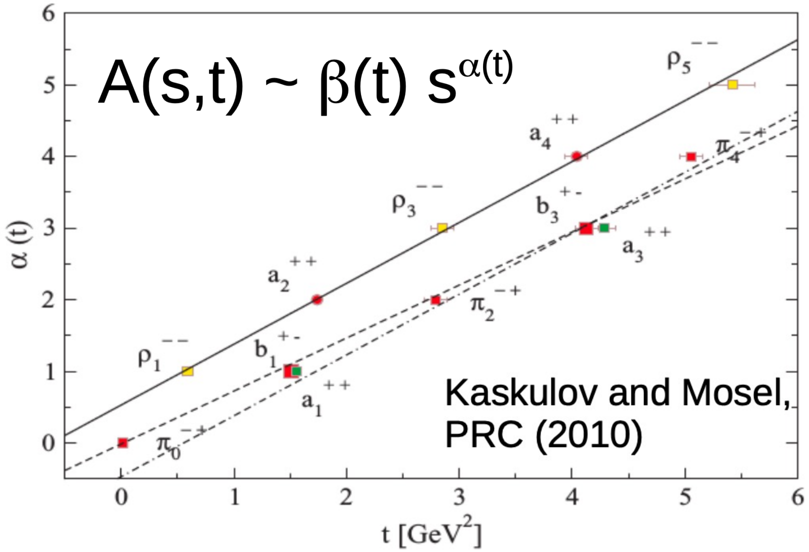
From Raúl González Jiménez
Presentation

Hybrid Model for nonresonant pion production

- Use ChPT model at low energy (W).
- Use Regge formalism at high energy (W).

R. González-Jiménez, *et al*
Phys. Rev. D **95** (2017)

α(t): Families or Regge trajectories



$$\frac{1}{t - m_\pi^2}$$

The pion propagator is replace by the Regge trajectory of the pion family

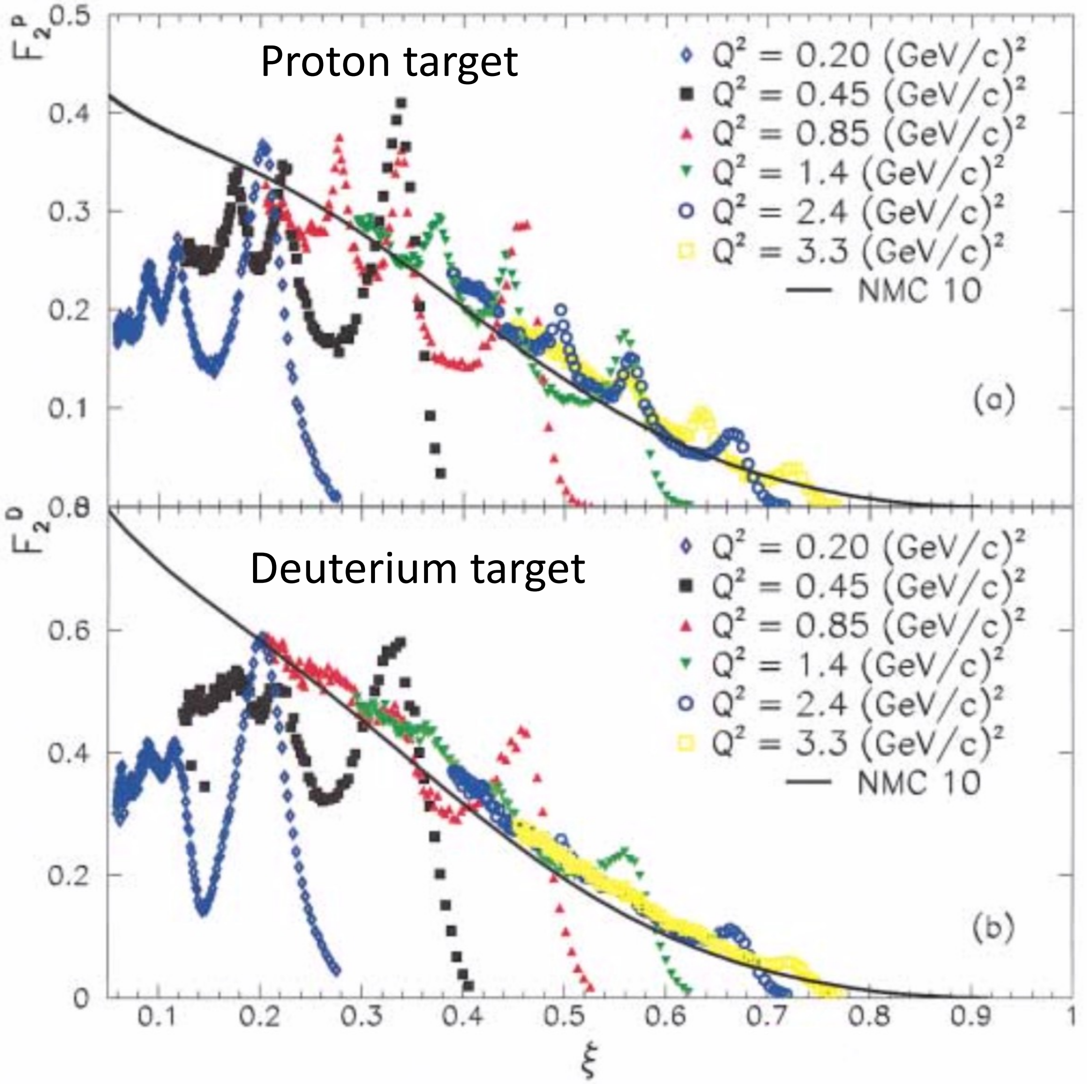
$$\mathcal{P}_\pi(t,s) = -\alpha'_\pi \varphi_\pi(t)\Gamma[-\alpha_\pi(t)](\alpha'_\pi s)^{\alpha_\pi(t)}$$

From Raúl González Jiménez
Presentation

From I. Niculescu *et al.*

Quark–hadron duality

- It was observed about 50 years ago.
- The resonances oscillate around an average scaling curve.
- Scaling behaviour would imply that the nucleon target appears as a collection of point-like constituents when probed at very high energies in DIS.
- Establishes a relationship between the quark–gluon description, and the hadronic description.



$$\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$$

$$x = Q^2/2M\nu$$

Imperial College London