Deep inelastic scattering from nucleons and nuclei

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Table of contents

Introduction:Definition of cross section for scattering or reactions

Event= $\sigma \times \phi(E) \times N_T \times t$
σ =scattering cross section
$\phi(E)$ =flux
$N_T =$ number of target particles
$t =$ time of interaction

1. The name 'cross section'is derived from the study of collision processes in classical mechanics, in which a particle collides with a fixed spherical target with radius r, lying in the interaction region of total volume V and area A.
2. The probability P that the incoming particle collides with the sphere of radius r is given by: $P = \frac{\pi r^2}{A}$
πr^2 is cross section of area relevant for the collision process and is generally represented by $\sigma = PA$.

1. The cross section σ is effectively an area in which the incident and target particle interact for the scattering to take place.
2. This concept of cross section σ is generalized to any collision process in which a beam of particles scatter with a fixed target or two beams of particles from opposite directions collide with each other.

Consider a beam of particles of density ρ, that is, number of particles per unit volume $(= \frac{V}{n})$ moving with velocity v. In time t, this beam, passing through an effective area of interaction A, will have $n = \rho v t A$ particles.

$$A = \frac{n}{\rho v t}, \sigma = \frac{P/t}{1/At}$$

Now we can write the σ for the present case

$$\sigma = \frac{|<f|s|i>|^2 V}{Tv}$$

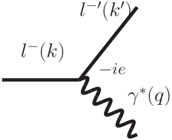
$$<f|s|i> = \delta_{fi} + 2\pi^4 \delta^4(P_f - P_i)\Pi_i \frac{1}{\sqrt{2VE_i}} \Pi_f \frac{1}{\sqrt{2VE_f}} M_{fi}$$

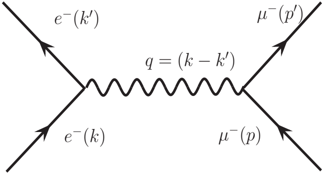
where P_f and P_i are the sum of 4-momenta of all the particles in final and initial state, respectively,
Using the integral representation of $\delta^4(P)$ function, Obtain the cross section expression as:

$$\sigma = (2\pi)^4 \delta^4(P_f - P_i) \frac{V^2}{v} \Pi_i (\frac{1}{2E_i V}) \Pi_f (\frac{1}{2E_f V}) |M_{fi}|^2$$

In quantum mechanics, the momentum is not fixed, but always lies in between a range of $\vec{p}$ and $\vec{p} + d\vec{p}$.
Multiply for the cross section, by the density of states $\rho(p)dp$,
This is given by the density of states in phase space
$\rho(p)dp = \frac{V d\vec{p}}{2\pi^3}$
Therefore, we get expression for scattering cross section as

$$\sigma = \frac{1}{4\sqrt{(p_1.p_2)^2 - m_1^2 m_2^2}} \int \Pi_f \frac{d\vec{p}_f}{(2\pi)^3 2E_f} (2\pi)^4 \delta^4 (P_f - P_i) |M_{fi}|^2$$



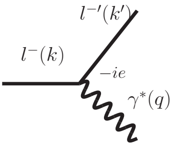


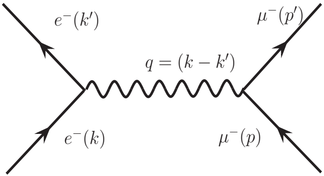
The interaction is described by the Lagrangian

$$\mathcal{L}_I = -ie\bar{\psi}(k')\gamma^\mu A_\mu \psi(k)$$

The transition amplitude M is given as:

$-iM =$ current at vertex 1 $\times$ propagator $\times$ current at vertex 2





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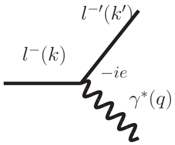

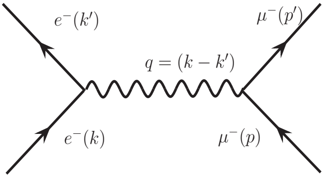

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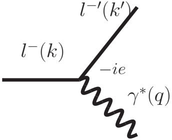

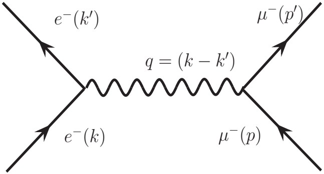

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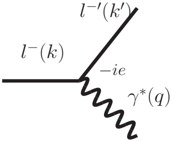

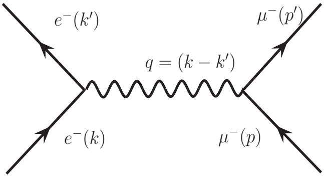

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$$\Rightarrow |M|^2 = \frac{e^4}{q^4} \mid \bar{u}(k')\gamma^\mu u(k) \mid^2 \ \mid \bar{u}(p')\gamma_\mu p(k) \mid^2$$

In the case of unpolarized e^- and μ^-, we average over the initial spins and sum over the final spins. Using the standard projection operators for spin 1/2 spinors , we may write:

$$\bar{\sum}\sum \mid M \mid^2 = \frac{e^4}{q^4}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot Tr\left[(\not{k}'+m_e)\gamma^\mu(\not{k}+m_e)\gamma^\nu\right]$$
$$Tr\left[(\not{p}'+m_\mu)\gamma_\mu(\not{p}+m_\mu)\gamma_\nu\right]$$

where the symbol $\bar{\sum}$ is the average taken over initial spins which gives a factor of 1/2 each for e^- and μ^- $\sum$ is the sum over the final spin states. m_e and $m\mu$ are the masses of electron and muon, respectively.

$$\bar{\sum}\sum \mid M \mid^2 = \frac{e^4}{q^4} L_e^{\mu\nu} L_{\mu\nu}^{muon},$$

We have now

$$L_e^{\mu\nu} = Tr\left[(\not{k}' + m_e)\gamma^\mu(\not{k} + m_e)\gamma^\nu\right]$$

$$L^{muon}_{\mu\nu} = Tr\left[(\not{p}' + m_\mu)\gamma_\mu(\not{p} + m_\mu)\gamma_\nu\right]$$

Problem:Using trace calculations obtain the below expressions for electron(Eq:1) and muon(Eq:2) leptonic tenors and contract them for final expression for the matrix element squared(Eq:3).

$$L_e^{\mu\nu} = \frac{1}{2}4[k^\mu k'^\nu + k'^\mu k^\nu - (k \cdot k' - m^2)g^{\mu\nu}] \tag{1}$$

$$L^{muon}_{\mu\nu} = \frac{1}{2}4[p_\mu p'_\nu + p'_\mu p_\nu - (p \cdot p' - m^2)g_{\mu\nu}] \tag{2}$$

$$\Rightarrow \bar{\sum}\sum |T|^2 = \frac{4e^4}{q^4}\left\{2k \cdot pk' \cdot p' + 2k \cdot p'k' \cdot p - 2(p \cdot p' - m_\mu^2)k \cdot k' \right. \tag{3}$$
$$\left. -2(k \cdot k' - m_e^2)p \cdot p' + 4(k \cdot k' - m_e^2)(p \cdot p' - m_\mu^2)\right\}$$

If q is the four momentum transfer, $q = k - k' = p' - p$, $q^2 = -2k.k' = -2EE'(1 - cos\theta)$ then the above equation in the limit m=0 becomes:

$$\bar{\sum}\sum \mid M \mid^2 = \left[\frac{8e^4}{q^4}\left\{2k'\cdot p\ k\cdot p + (-q^2/2)k\cdot p + (q^2/2)k'\cdot p - M^2 k\cdot k'\right\}\right]$$

In the lab frame when the target particle is at rest

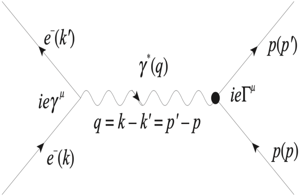
$$2p.kp.k' = 2M^2EE' \tag{5}$$

$$\Rightarrow \bar{\sum}\sum \mid M \mid^2 = \left[\frac{8e^4}{q^4}\left\{2m_\mu^2 E_e E'_e - \frac{q^2}{2}m_\mu(E_e - E'_e) + \frac{m_\mu^2 q^2}{2}\right\}\right] \tag{6}$$

$$\Rightarrow \bar{\sum}\sum \mid M \mid^2 = \left\{\frac{8e^4}{q^4}2m_\mu^2 E_e E'_e\left[cos^2\frac{\theta}{2} - \frac{q^2}{2m_\mu^2}sin^2\frac{\theta}{2}\right]\right\} \tag{7}$$

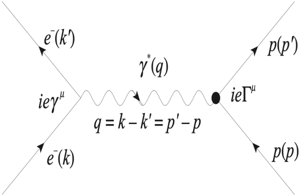
$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\ \pi^2 m_\mu E_e} \bar{\sum} \sum \mid M \mid^2 \frac{|\mathbf{k}'|^3}{(E_e + m_\mu)|\mathbf{k}'|^2 - k \cdot k' E'} \qquad (8)$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_e^2 sin^4(\frac{\theta}{2})} \frac{E_e'}{E_e} \left\{ cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} sin^2 \frac{\theta}{2} \right\} \qquad (9)$$



Invariant amplitude is written as

$$-iM = \bar{u}(k')ie\gamma^\mu u(k)$$



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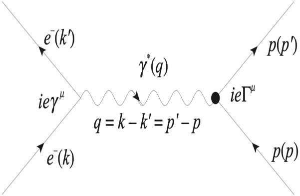

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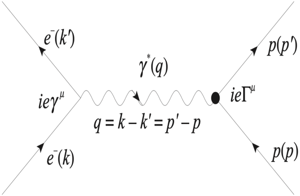

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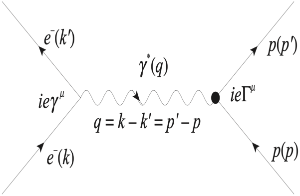
Γ^{ν} is written in terms of p, p', q and γ-matrices:

$$\begin{aligned}\Gamma^{\mu} &= A(Q^2)\gamma^{\mu} + B(Q^2)(p'-p)^{\mu} + C(Q^2)(p'+p)^{\mu} \\ &+ D(Q^2)i\sigma^{\mu\nu}(p'-p)_{\nu} + E(Q^2)i\sigma^{\mu\nu}(p'+p)_{\nu}\end{aligned}$$



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Problem: Obtain the below expression for Γ^μ

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2M}q_\nu F_2(q^2)$$

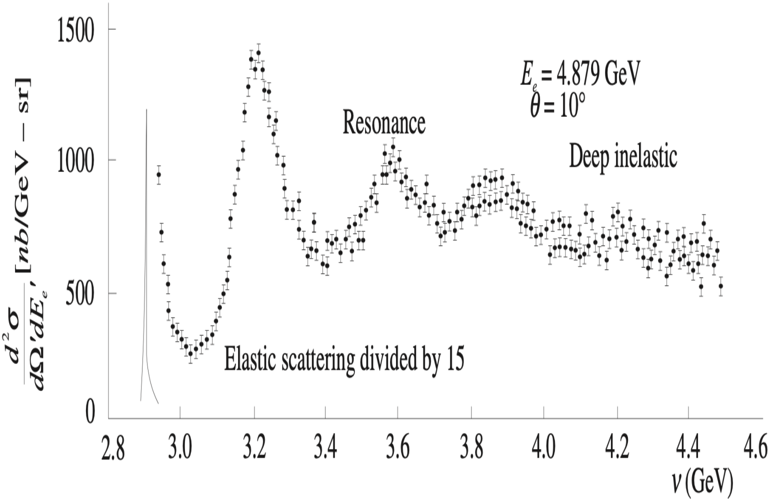
Using Gordan Decomposition Γ^μ may be given as

$$\Gamma^\mu = (F_1(q^2) + F_2(q^2))\gamma^\mu - \frac{F_2(q^2)}{2M}(p + p\prime)^\mu$$

What is Deep Inelastic Scattering?

Lepton-nucleon scattering was born in 1956, when Hofstadter et al. performed the first elastic $e - p$ scattering experiment and found a finite radius of the proton.
About 10 years later it became 'deep inelastic, when experiments at 20 times higher energies, at SLAC, established the partonic nature of the proton.

Scattering of 4.879 GeV electrons from protons at rest.



About the previous slide Fig.

1. Elastic electron scattering from the proton target, the peak is observed at low energy transfer or equivalently when the invariant mass $W \approx M$.
2. The structure of the proton is described in terms of the electric and magnetic Sachs form factors; the size of the proton and its magnetic moment may also be understood with the help of these form factors.
3. The Q^2 dependence of the form factors implies that the elastic cross section decreases with the increase in four-momentum transfer squared.
4. With increase in energy transfer, one observes inelastic scattering which results in the production of one pion, multipions, etc. for which $W > M$.

What happens when Q^2 becomes very large?

At high Q^2, the proton breaks up into a jet of hadrons and the final state is now a multiparticle state with large invariant mass.

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General process for the deep inelastic scattering is
$l(k) + N(p) \longrightarrow l'(k') + X(p'), \quad l, l' = e^{\pm}, \mu^{\pm}, \nu_l, \bar{\nu_l}, \; N = n, p$

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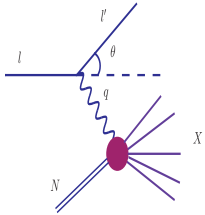
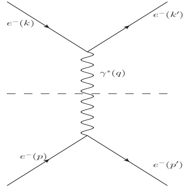
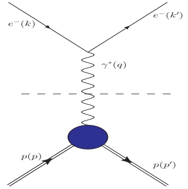
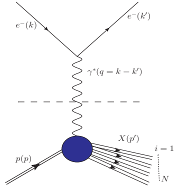
Kinematics(Nucleon in the rest frame)

$Q^2=-q^2=-(k-k')^2=4EE'\sin^2\frac{\theta}{2}$
$M^2=p^2$
$\nu=p.q=M(E-E')$
$x=\frac{Q^2}{2M\nu}=\frac{Q^2}{2p.q}=\frac{Q^2}{2MEy}$
$y=\frac{p.q}{p.k}=1-\frac{E'}{E}$
$W^2=M^2+2p.q-Q^2$





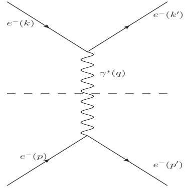


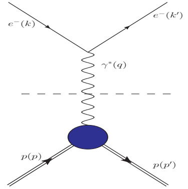


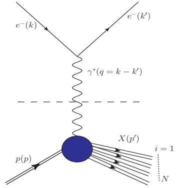
For the two body exclusive process $1 + 2 \to 3 + 4 + ... + n$ the differential cross section is

$$\sigma = \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - ...) \times$$

$$\Pi_{j=3}^{n} 2\pi \quad \delta(p_j^2 - m_j^2) \quad \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$







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Dynamics of the scattering is contained in

$$|\mathcal{M}|^2 \propto L_{\mu\nu} W^{\mu\nu}$$

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We write a general parameterization of the hadronic tensor

$$\begin{aligned} W^{\mu\nu} &= -g_{\mu\nu}\ W_1 + \frac{p_\mu p_\nu}{M^2}\ W_2 - i\epsilon_{\mu\nu\lambda\sigma}\frac{p^\lambda q^\sigma}{2M^2}\ W_3 + \frac{q_\mu q_\nu}{M^2}\ W_4 \\ &\quad + \frac{(p_\mu q_\nu + p_\nu q_\mu)}{2M^2}\ W_5 + \frac{i(p_\mu q_\nu - p_\nu q_\mu)}{2M^2}\ W_6 \end{aligned}$$

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By contraction of hadronic tensor with $L_{\mu\nu}$

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Applying CVC: $q_\mu W^{\mu\nu} = 0$

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\begin{aligned}
W_4 &= \frac{-2p\cdot q}{q^2} W_2 \\
W_5 &= \frac{M^2}{q^2} W_1 + \left(\frac{p\cdot q}{q^2}\right)^2 W_2
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W_1 and W_2 can be the functions of any two Lorentz-invariant scalars: q^2, $\nu = \frac{p.q}{M}$, $x = \frac{-q^2}{2p.q}$, $y = \frac{p.q}{p.k}$

$$L_{\mu\nu}W^{\mu\nu} = 4W_1(k.k') + \frac{2W_2}{M^2}[2(p.k)(p.k') - M^2(k.k')]$$

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$$k.k' = 2EE'\sin\frac{\theta}{2}, \qquad p.k = EM, \qquad p.k' = E'M$$

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$L_{\mu\nu}W^{\mu\nu}$ is obtained as:

$$L_{\mu\nu}W^{\mu\nu} = 4EE'\left[\cos^2\frac{\theta}{2}W_2(\nu,q^2) + \sin^2\frac{\theta}{2}2W_1(\nu,q^2)\right]$$

The differential scattering cross section in the energy and angle of the scattered electron may be written as:

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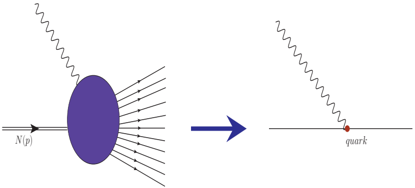
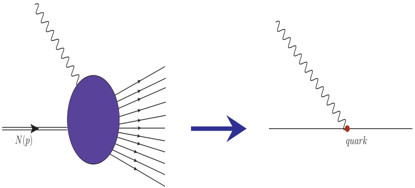

N(p)
quark



Proton structure functions:

$$2mW_1^{point}(\nu, Q^2) = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\nu W_2^{point}(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

Structure function is independent of ν and Q^2 and depends on the ratio $\frac{Q^2}{2m\nu}$.

In contrast one may observe the behaviour of elastic scattering by setting $G_E = G_M \equiv G$

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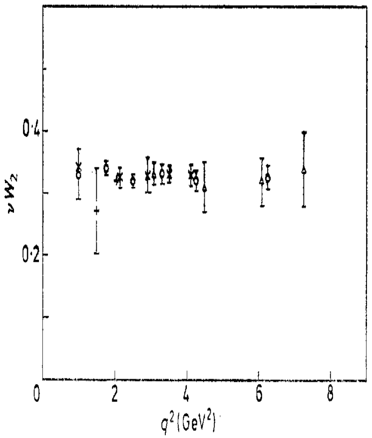
The structure functions contain a form factor $G^2(Q^2)$ and so cannot be the functions of a single dimensionless variable.



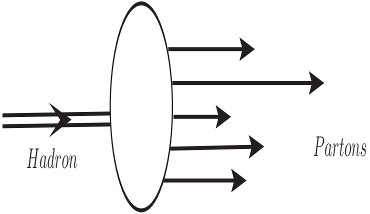
The naive parton model predicts that structure functions are independent of Q^2. This scale invariance is Bjorken scaling.

$$MW_1(\nu, Q^2) \longrightarrow F_1(\omega)$$
$$\nu W_2(\nu, Q^2) \longrightarrow F_2(\omega)$$

$\omega = \frac{2M\nu}{Q^2}$

Basic assumptions of parton model

1. A rapidly moving hadron appears as a jet of partons all of which travel in more or less same direction as the parent hadron



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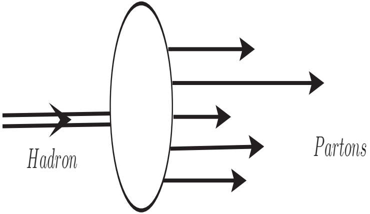
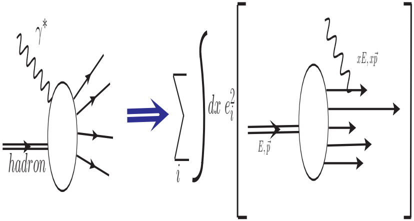
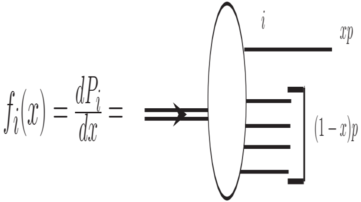


2. The rule for calculating reaction rate for hadron: the reaction rate for the basic process with free partons is calculated and summed incoherently over the contributions of partons in the hadron.



3 The three momentum of the hadron is shared out among the partons. One defines the parton momentum distribution



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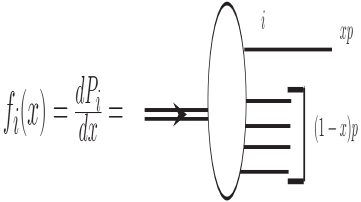

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4. They carry a different fraction x of the hadron's momentum and energy. All the fractions x add up to 1

$$\sum_{i'} \int dx \; x \; f_{i'}(x) = 1$$

	Hadron	Parton
Energy	E	x E
Momentum	p_L	xp_L
	$p_T = 0$	$p_T = 0$
Mass	M	m=xM

The dimensionless structure functions are given by:

$$F_1(\omega) = \frac{Q^2}{4m\nu x}\delta\left(1 - \frac{Q^2}{2m\nu}\right), \quad F_2(\omega) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

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For a proton, we may write F_1 and F_2 as

$$F_2(\omega) = \sum_i \int dx \; e_i^2 \; f_i(x) \; x \; \delta\left(x - \frac{1}{\omega}\right), \quad F_1(\omega) = \frac{\omega}{2}F_2(\omega)$$

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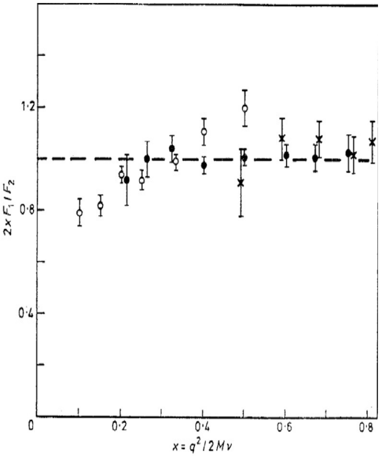
Structure functions are obtained as:

$$\nu W_2(\nu, Q^2) \longrightarrow F_2(x) = \sum e_i^2\ x\ f_i(x)$$
$$M W_1(\nu, Q^2) \longrightarrow F_1(x) = \frac{1}{2x}F_2(x)$$

$F_{1,2}$ corresponds to the total momentum fraction carried by all the quarks and antiquarks in the nucleon, weighted by the squares of the quark charges.



It is observed that $F_1(x)$ and $F_2(x)$ are not independent:

Callan-Gross relation for spin $\frac{1}{2}$ constituents: $F_1(x) = \frac{1}{2x} F_2(x)$

In the 'naive' parton model:

$$
\begin{aligned}
u_v(x) &= 2d_v(x) \\
s_v(x) &= \bar{u}_v(x) = \bar{d}_v(x) = \bar{s}_v(x) = 0 \\
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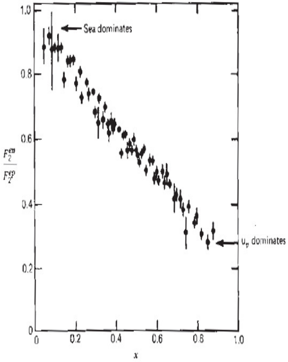

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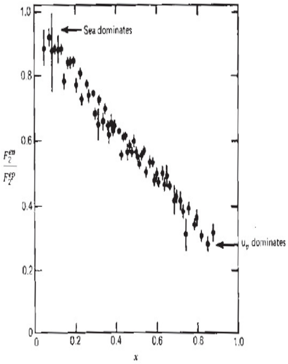

Valence quarks are dominant at large x and sea quarks at low x.

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The structure function scales i.e. $F(x, Q^2) \longrightarrow F(x)$ *in the asymptotic (Bjorken) limit:* $Q^2 \longrightarrow \infty$
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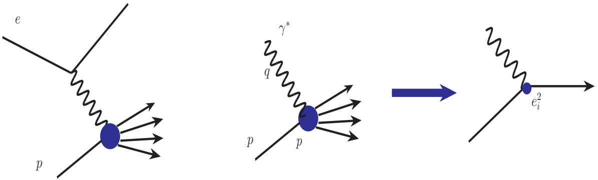

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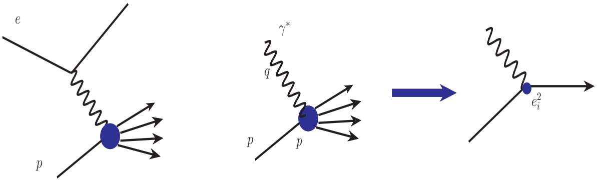
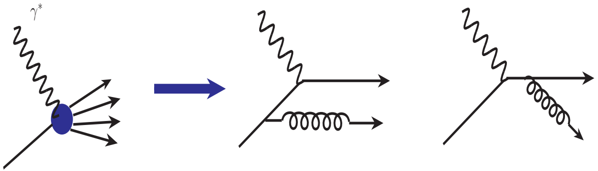


QCD extends the naive quark parton model by allowing interactions between the partons via the exchange of gluons.

Two possibilities may arise:

- Quark can radiate a gluon before or after being struck by the virtual photon i.e. $\gamma^* q \longrightarrow qg$.



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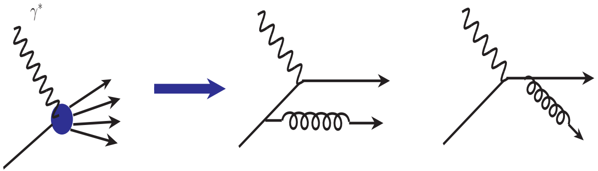
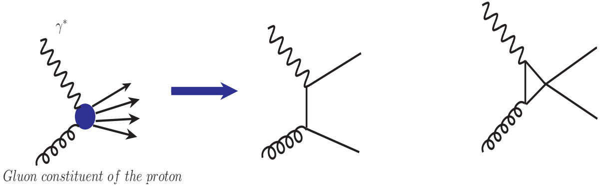


- Gluon constituents can contribute to DIS via $\gamma^* g \longrightarrow q\bar{q}$



Effect of gluons dynamics on structure functions:

Violation of scaling property of structure functions.
Outgoing quark will no longer be collinear with the virtual photon($p_T \neq 0$).

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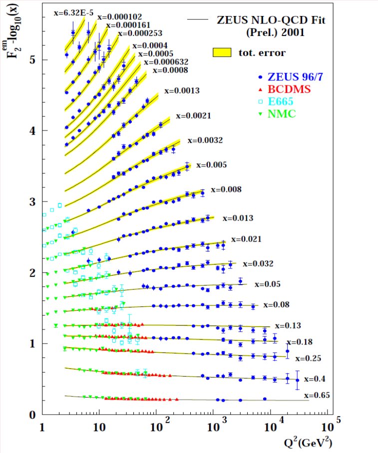
Considering $\gamma^* q \longrightarrow qg$, F_2 modifies to:

$$\frac{F_2(x)}{x} = \sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y) \left[\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{y}\right) log\left(\frac{Q^2}{\mu^2}\right)\right]$$

which introduces a logarithmic Q^2 dependence due to the gluon emission, violating the scaling behavior.

Logarithmic variation of Q^2 in F_2



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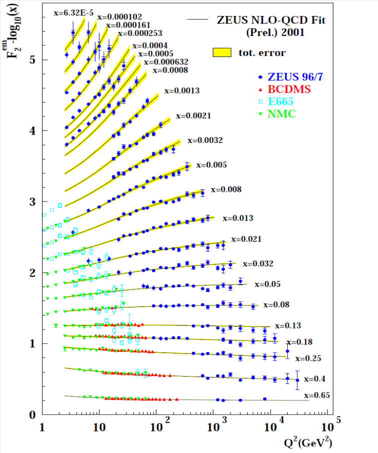

At large x, $F_2^{e.m.}$ decreases with Q^2, at small x, $F_2^{e.m.}$ increases with Q^2.

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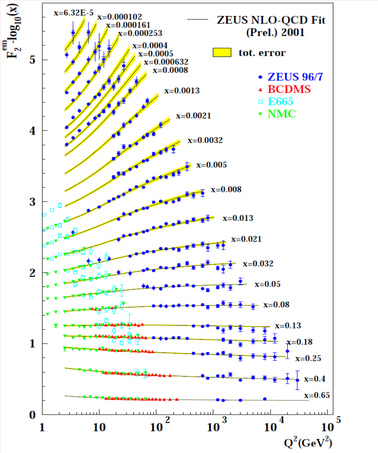


At large x, $F_2^{e.m.}$ decreases with Q^2, at small x, $F_2^{e.m.}$ increases with Q^2.

At Larger Q^2

- The probability of finding a quark at small x increases.
- The probability of finding a quark at large x decreases since high momentum quarks lose momentum by radiating gluons.

Re-expressing $\frac{F_2(x)}{x}$ as:

$$\frac{F_2(x)}{x} = \sum_q e_q^2 \int_x^1 \frac{dy}{y} \left[q(y) + \Delta q(y, Q^2)\right] \delta\left(1 - \frac{x}{y}\right)$$

where

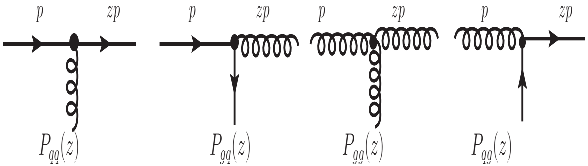
$$\Delta q(y, Q^2) = \left[\frac{\alpha_s}{2\pi}\left(\frac{x}{y}\right) log\left(\frac{Q^2}{\mu^2}\right)\right] \int_x^1 \frac{dy}{y} q(y) P_{qq}(\frac{x}{y})$$

$$\frac{dq(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q(y, Q^2) P_{qq}(\frac{x}{y})$$

This is 'Altarelli-Parisi Evolution equation'.

Splitting Functions P_{ab}

1. The effect of all interactions is described by splitting functions P_{ab}.
2. P_{ab} represents the probability that a parton of type a radiates a quark or gluon and becomes a parton of type b carrying fraction $\frac{x}{z}$of the momentum of parton a.
3. Splitting functions have been calculated from perturbative QCD.



Q^2 *evolution of parton densities:*

$$\frac{dq(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[q(y,Q^2)P_{qq}(\frac{x}{y}) + g(y,Q^2)P_{qg}(\frac{x}{y})\right]$$

Similarly, Q^2 *evolution of gluon densities:*

$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[q(y,Q^2)P_{gq}(\frac{x}{y}) + g(y,Q^2)P_{qg}(\frac{x}{y})\right]$$

In general the splitting function can be expressed as a power series in α_s: $P_{ab} = P_{ab}^{LO} + \alpha_s P_{ab}^{NLO} + \alpha_s^2 P_{ab}^{NNLO} + ...$

Proton contains both quarks and gluons, so coupled DGLAP:

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$$\frac{d}{d\log Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

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place arrow

$$(P_{ab} \otimes q)(x, Q^2) = \int_x^1 \frac{dy}{y} q(y, Q^2) P_{ab}(\frac{x}{y})$$

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$$(P_{ab} \otimes q)(x, Q^2) = \int_x^1 \frac{dy}{y} q(y, Q^2) P_{ab}(\frac{x}{y})$$

which allows us to write

$$\frac{dq(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}(P_{ab} \otimes q)(x, Q^2)$$

Differential scattering cross section for the reaction $ep \longrightarrow eX$ in terms of dimensionless structure functions:

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$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2 sin^4(\frac{\theta}{2})}\left[F_2(x)\cos^2\frac{\theta}{2} + 2F_1(x)\sin^2\frac{\theta}{2}\right]$$

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Differential scattering cross section may be expressed as:

$$\frac{d\sigma}{dxdy} = \frac{8\pi\alpha^2 ME}{Q^4}\left[\left(1-y-\frac{Mxy}{2E}\right)F_2(x) + \frac{y^2}{2}2xF_1(x)\right]$$

Electromagnetic structure functions

$$
\begin{aligned}
F_2^{ep} &= \frac{4\ x}{9}\left(u_v + u_s + \bar{u}_s + c + \bar{c} + ...\right) + \frac{x}{9}\left(d_v + d_s + \bar{d}_s + s + \bar{s} + ...\right) \\
F_2^{en} &= \frac{x}{9}\left(u_v + u_s + \bar{u}_s + s + \bar{s} + ...\right) + \frac{4\ x}{9}\left(d_v + d_s + \bar{d}_s + c + \bar{c} + ...\right)
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Electromagnetic structure functions

$$F_2^{ep} = \frac{4\,x}{9}\left(u_v + u_s + \bar{u}_s + c + \bar{c} + ...\right) + \frac{x}{9}\left(d_v + d_s + \bar{d}_s + s + \bar{s} + ...\right)$$
$$F_2^{en} = \frac{x}{9}\left(u_v + u_s + \bar{u}_s + s + \bar{s} + ...\right) + \frac{4\,x}{9}\left(d_v + d_s + \bar{d}_s + c + \bar{c} + ...\right)$$

where $v \equiv$ valence quark and $s \equiv$ sea quark

These distribution functions are generally determined by using global QCD analysis with the inputs from various sets of experimental data specially obtained from DIS experiments revealing proton structure.

1. *The fraction of nucleon momentum carried by struck quark is defined to be x, the Bjorken scaling variable.*
2. *The physical meaning of x cannot hold when the scattering occurs at very large x and low $Q^2 \sim M_N^2$.*
3. *The mass of the quark at very large x is effectively same as the nucleon mass. Therefore, the nucleon mass cannot be ignored at low Q^2.*

These effects are important to determine the valence quark distribution at high x and therefore x is replaced by Nachtmann scaling variable($x \to \xi$).

$$\xi = \frac{2x}{1+\sqrt{1+4\mu x^2}}, \qquad \mu = \frac{M^2}{Q^2}$$

At large values of Q^2, $\xi \longrightarrow x$. However, for Q^2 less than a few times the target mass of ~ 1 GeV, ξ can deviate significantly from x, especially at large x values.

1. *The fraction of nucleon momentum carried by struck quark is defined to be x, the Bjorken scaling variable.*
2. *The physical meaning of x cannot hold when the scattering occurs at very large x and low $Q^2 \sim M_N^2$.*
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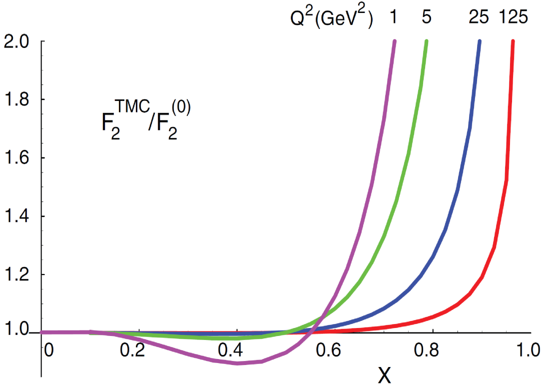
At large values of Q^2, $\xi \longrightarrow x$. However, for Q^2 less than a few times the target mass of $\sim$ 1 GeV, ξ can deviate significantly from x, especially at large x values.

$$F_2^{TMC}(x,Q^2) \approx \frac{x^2}{\xi^2\gamma^3} F_2(\xi) \left(1 + \frac{6\mu x\xi}{\gamma}(1-\xi)^2\right) \gamma = \sqrt{1+\frac{4M^2x^2}{Q^2}}$$

Schienbein et al. JPG 35 (2008) 053101



Dynamical Higher Twist Effect (Significant at High x and low Q^2)

Higher twist corrections include multiparton correlation in the nucleon

1. At low Q^2 Lepton/Neutrino-nucleon scattering can involve multiple partons.
2. Compared to the leading twist diagrams (pQCD) the higher twist diagrams are suppressed by powers of $\frac{1}{Q^2}$, so they are important at low Q^2.
3. This is also the region where the strong coupling constant is large and pQCD is invalid

$$F_2(x,Q^2) = F^{LT+TMC}(x,Q^2)\left(1 + \frac{D_2(x,Q^2)}{Q^2}\right),$$

For Details: E. Stein et al. Nucl. Phys. B 536, 318 (1998)
M. Dasgupta et al. Phys. Lett. B 382, 273

Formalism: $\nu_l/\bar{\nu}_l - N$ *scattering*

The basic reaction for the (anti)neutrino induced charged current deep inelastic scattering process on a free nucleon target is given by

$$\nu_l(k)/\bar{\nu}_l(k) + N(p) \;\; \rightarrow \;\; l^-(k')/l^+(k') + X(p'), \qquad\qquad (l = e, \nu, \tau)$$

The general expression for the double differential scattering cross section (DCX):

$$\frac{d^2\sigma}{dxdy} = \frac{yM_N}{\pi} \; \frac{E}{E'} \; \frac{|\mathbf{k}'|}{|\mathbf{k}|} \; \frac{G_F^2}{2} \; \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 \; L_{\mu\nu} \; W_N^{\mu\nu} \; ,$$

Leptonic tensor:

$$L_{\mu\nu} = 8(k_\mu k'_\nu + k_\nu k'_\mu - k.k' g_{\mu\nu} \pm i\epsilon_{\mu\nu\rho\sigma} k^\rho k'^\sigma)$$

Hadronic tensor:

$$\begin{aligned} W_N^{\mu\nu} &= -g^{\mu\nu}\, W_{1N}(\nu,Q^2) + W_{2N}(\nu,Q^2)\frac{p^\mu p^\nu}{M_N^2} \\ &\quad - \frac{i}{M_N^2}\epsilon^{\mu\nu\rho\sigma} p_\rho q_\sigma W_{3N}(\nu,Q^2) + \frac{W_{4N}(\nu,Q^2)}{M_N^2} q^\mu q^\nu \\ &\quad + \frac{W_{5N}(\nu,Q^2)}{M_N^2}(p^\mu q^\nu + q^\mu p^\nu) + \frac{i}{M_N^2}(p^\mu q^\nu - q^\mu p^\nu) W_{6N}(\nu,Q^2)\,. \end{aligned}$$

$W_{iN}(\nu,Q^2)$ $(i = 1-6)$ are the weak nucleon structure functions

In the limit of $Q^2 \to \infty$, $\nu \to \infty$, $x \to$ finite and $W_{iN}(\nu, Q^2)$ $(i = 1-5)$ are written in terms of the dimensionless nucleon structure functions as:

$$F_{1N}(x) = W_{1N}(\nu, Q^2)$$

$$F_{2N}(x) = \frac{Q^2}{2xM_N^2} W_{2N}(\nu, Q^2)$$

$$F_{3N}(x) = \frac{Q^2}{xM_N^2} W_{3N}(\nu, Q^2)$$

$$F_{4N}(x) = \frac{Q^2}{2M_N^2} W_{4N}(\nu, Q^2)$$

Formalism: $\nu_l/\bar{\nu}_l - N$ scattering

The differential scattering cross section is given by

$$\begin{aligned}\frac{d^2\sigma}{dxdy} &= \frac{G_F^2 M_N E_\nu}{\pi(1+\frac{Q^2}{M_W^2})^2}\Bigg\{\left[y^2 x + \frac{m_l^2 y}{2E_\nu M_N}\right]F_{1N}(x,Q^2) \\ &+\left[\left(1-\frac{m_l^2}{4E_\nu^2}\right)-\left(1+\frac{M_N x}{2E_\nu}\right)y\right]F_{2N}(x,Q^2) \\ &\pm\left[xy\left(1-\frac{y}{2}\right)-\frac{m_l^2 y}{4E_\nu M_N}\right]F_{3N}(x,Q^2) \\ &+\frac{m_l^2(m_l^2+Q^2)}{4E_\nu^2 M_N^2 x}F_{4N}(x,Q^2)-\frac{m_l^2}{E_\nu M_N}F_{5N}(x,Q^2)\Bigg\}.\end{aligned}$$

At the leading order

Callan-Gross relation: $F_2(x) = xF_1(x)$
Albright-Jarlskog relations: $F_4(x) = 0, \qquad F_2(x) = 2xF_5(x)$

For $\nu(\bar{\nu})$-proton scattering

$$\begin{aligned}
F_{2p}^{\nu}(x) &= 2x[d(x)+s(x)+\bar{u}(x)+\bar{c}(x)] \\
F_{2p}^{\bar{\nu}}(x) &= 2x[u(x)+c(x)+\bar{d}(x)+\bar{s}(x)] \\
xF_{3p}^{\nu}(x) &= 2x[d(x)+s(x)-\bar{u}(x)-\bar{c}(x)] \\
xF_{3p}^{\bar{\nu}}(x) &= 2x[u(x)+c(x)-\bar{d}(x)-\bar{s}(x)]
\end{aligned}$$

For $\nu(\bar{\nu})$-neutron scattering

$$\begin{aligned}
F_{2n}^{\nu}(x) &= 2x[u(x)+s(x)+\bar{d}(x)+\bar{c}(x)] \\
F_{2n}^{\bar{\nu}}(x) &= 2x[d(x)+c(x)+\bar{u}(x)+\bar{s}(x)] \\
xF_{3n}^{\nu}(x) &= 2x[u(x)+s(x)-\bar{d}(x)-\bar{c}(x)] \\
xF_{3n}^{\bar{\nu}}(x) &= 2x[d(x)+c(x)-\bar{u}(x)-\bar{s}(x)].
\end{aligned}$$

Problem: Obtain the relation between electromagnetic and weak structure functions

Books

- The Physics of Neutrino Interactions by M. Sajjad Athar S. K. Singh.
- Quarks and Leptons by Halzen & Martin

Deep inelastic scattering from nuclei in next lecture