Deep inelastic scattering from nucleons and nuclei

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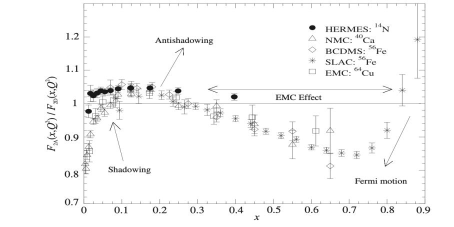
Outline

1. The study of (anti)neutrino reactions from the nuclear targets has been emphasized as almost all the present generation (anti)neutrino experiments use moderate to heavy nuclear targets like $^{12}C, ^{16}O, ^{40}Ar, ^{56}Fe, ^{208}Pb$, where the interactions take place with the nucleons that are bound inside the nucleus.
2. Various experiments like MINERνA, NOvA, T2K, etc., are being performed in the few GeV energy region where the contribution to the scattering cross section comes from all the possible channels, viz., quasielastic, inelastic, and deep inelastic scattering processes.
3. The precision with which the basic neutrino-nucleon cross sections in nuclear targets are known is still not better than 20-30%.
4. Neutrino oscillation experiments measure events that are a convolution of
(i) energy-dependent neutrino flux and
(ii) energy-dependent cross section.
5. In the Deep Underground Neutrino Experiment (DUNE), it is expected that more than 30% of the events would come from the DIS region

NME is broadly divided into four parts

M. Sajjad Athar and S. K. Singh, The Physics of Neutrino Interactions (CUP, 2020)



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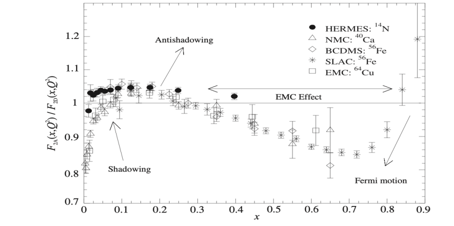
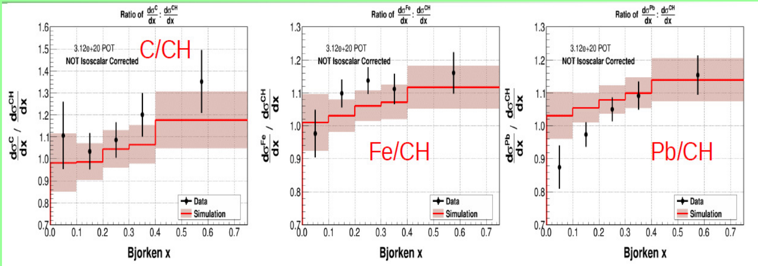


MINERνA: PRD93 071101(2016)



Phenomenological Efforts

Phenomenological group	data types used
EKS98	l+A DIS, p+A DY
HKM	l+A DIS
HKN04	l+A DIS, p+A DY
nDS	l+A DIS, p+A DY
EKPS	l+A DIS, p+A DY
HKN07	l+A DIS, p+A DY
EPS08	l+A DIS, p+A DY, $h^{\pm}, \pi^0, \pi^{\pm}$ in d+Au
EPS09	l+A DIS, p+A DY, π^0 in d+Au
nCTEQ	l+A DIS, p+A DY
nCTEQ	l+A and ν+A DIS, p+A DY
DSSZ	l+A and ν+A DIS, p+A DY, $\pi^0, \pi^{\pm}$ in d+Au

Paukkunen and Salgado:JHEP2010: "find no apparent disagreement with the nuclear effects in neutrino DIS and those in charged lepton DIS."

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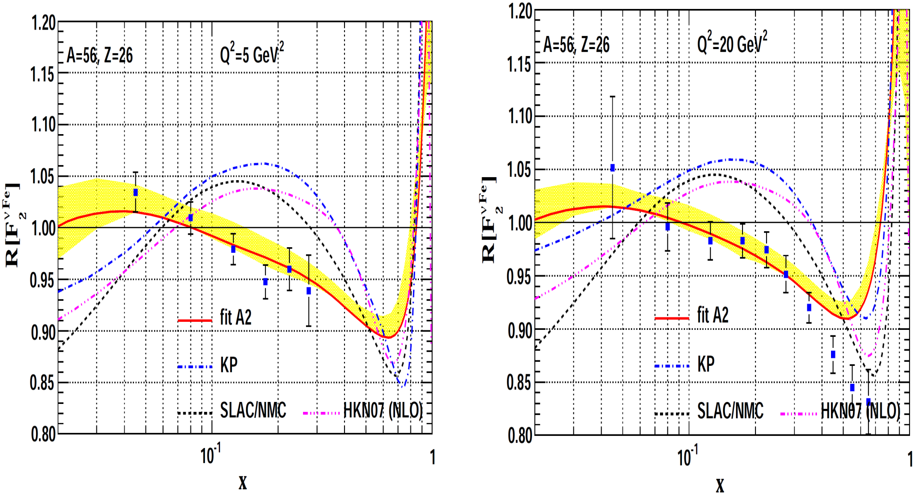
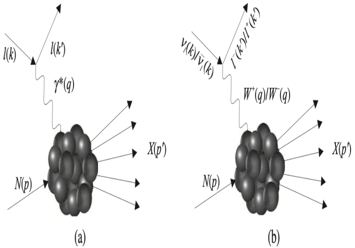


J G Morfin J. of Physics: Conf. Ser. 408 (2013) 012054;
Kovarik et al. Phys.Rev.Lett. 106 (2011) 122301

If we look inside the nucleus



We have incorporated the following NME in the present calculation

1. Fermi motion
2. Pauli blocking
3. Nucleon correlations
4. Pion and rho meson cloud contributions
5. Shadowing and antishadowing

Nucleon binding

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- This affects the free particle kinematics and the peak of the energy distribution is shifted in the energy distribution of the nucleus around the peak corresponding to $\Delta E = \frac{-q^2}{2M}$

Fermi motion

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- In a shell model picture, the nucleons move in a central mean field described by a potential $V(r)$; the motion is described nonrelativistically by a Hamiltonian given by $H = \frac{-q^2}{2M} + V(r)$

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- This momentum distribution is called the spectral function of the nucleon $S(\vec{p}, E)$. In the simplest case of the Fermi gas model, it is given by:
  $S(\vec{p}, E) \propto \theta(p_F - p)\delta(E - \sqrt{(|\vec{p}|^2 + M^2 + \epsilon)})$
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- The cross sections from a nucleon of a given momentum $\vec{p}$ is then convoluted with the spectral function $S(\vec{p}, E)$.

Pauli blocking

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- Similarly, in a Fermi gas picture of the nuclei, all the nuclear states in the Fermi sea are filled up to the momentum p_F
- In any nuclear reaction, the nucleons from a certain filled state are excited to a higher unoccupied state depending upon the energy transfer, creating a hole in the previously occupied state. This is called the creation of a particle-hole(1p-1h) state in the in the Fermi sea.

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- All the nuclear states up to a certain momentum in the phase space are inaccessible for occupation after scattering.This is called Pauli blocking and leads to the reduction in the cross section which could be substantial in certain kinematical regions especially in the region of low momentum transfers.

Approach to add Nuclear effects

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- Fermi motion, binding energy and nucleon correlations through spectral function and is calculated using Lehmann's representation for the relativistic nucleon propagator.
- Nuclear many body theory is used to calculate it for an interacting Fermi sea in nuclear matter. A local density approximation is then applied to translate these results to finite nuclei.

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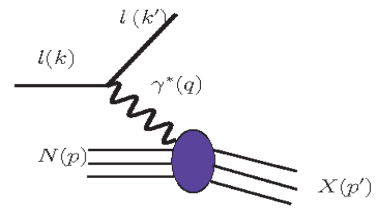
- This leads to an increase in the interaction probability of virtual mediating quanta with the meson cloud. The effect of meson cloud is more pronounced in heavier nuclear targets and dominate in the intermediate region of x($0.2<\mathrm{x}<0.6$).
- The shadowing suppression at small x occurs due to coherent multiple scattering of quark-anti quark pair coming from the virtual boson with destructive interference of the amplitudes and is incorporated following the works of Kulagin and Petti. Phys. Rev. D **76**, 094033(2007).

$l^\pm - N$ *scattering*

$$l^\pm(k) + N(p) \to l^\pm(k') + X(p'),$$



$l^\pm$-N DCX:

$$\frac{d^2\sigma^N}{d\Omega' dE'} = \frac{\alpha^2}{q^4} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L^{\alpha\beta}\, W^N_{\alpha\beta}$$

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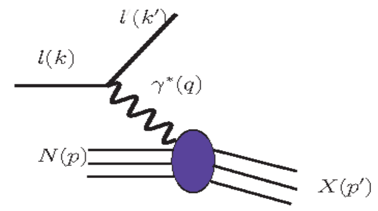

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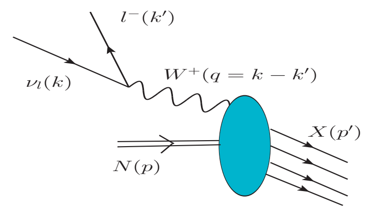
$\nu(\bar{\nu}) - N$ *scattering*

$$\nu_l(\bar{\nu}_l)(k) + N(p) \rightarrow l^{\pm}(k') + X(p'),$$



ν-N DCX:

$$\frac{d^2\sigma^N}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} \left(\frac{m_W^2}{q^2 - m_W^2} \right)^2 L^{\alpha\beta} W^N_{\alpha\beta}$$

$l^{\pm} - N$ *scattering*

Leptonic Tensor

$$L^{\alpha\beta} = 2(k^{\alpha} k'^{\beta} + k^{\beta} k'^{\alpha} - k.k' g^{\alpha\beta})$$

Hadronic tensor

$$\begin{aligned} W^{N}_{\alpha\beta} &= \left(\frac{q_{\alpha} q_{\beta}}{q^2} - g_{\alpha\beta}\right) W_{1N} + \frac{1}{M^2} \\ &\times \left(p_{\alpha} - \frac{p.q}{q^2} q_{\alpha}\right)\left(p_{\beta} - \frac{p.q}{q^2} q_{\beta}\right) W_{2N} \end{aligned}$$

$$MW_{1N}(\nu, Q^2) = F_1^N(x, Q^2)$$

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$$\begin{aligned} F_2^{ep}(x) &= x\left[\frac{4}{9}(u(x) + \bar{u}(x)) + \frac{1}{9}(d(x) + \bar{d}(x)) \right. \\ &\left. + \frac{1}{9}(s(x) + \bar{s}(x)) + \frac{4}{9}(c(x) + \bar{c}(x))\right] \end{aligned}$$

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$\nu(\bar\nu) - N$ *scattering*

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$$L^{\alpha\beta} = k^\alpha k'^\beta + k^\beta k'^\alpha - k.k' g^{\alpha\beta} \pm i\epsilon^{\alpha\beta\rho\sigma} k_\rho k'_\sigma$$

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$$\begin{aligned} W^N_{\alpha\beta} &= \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W^{\nu(\bar\nu)}_{1N} + \frac{1}{M^2} \\ &\times \left(p_\alpha - \frac{p.q}{q^2} q_\alpha\right)\left(p_\beta - \frac{p.q}{q^2} q_\beta\right) W^{\nu(\bar\nu)}_{2N} \\ &- \frac{i}{2M^2} \epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W^{\nu(\bar\nu)}_{3N} \end{aligned}$$

$$\begin{aligned} MW_{1N}(\nu, Q^2) &= F_1^N(x, Q^2) \\ \nu W_{2N}(\nu, Q^2) &= F_2^N(x, Q^2) \\ \nu W_{3N}(\nu, Q^2) &= F_3^N(x, Q^2) \\ F_2^{\nu p} &= 2x[d(x) + s(x) + \bar{u}(x) + \bar{c}(x)], \\ xF_3^{\bar\nu p} &= 2x[u(x) + c(x) - \bar{d}(x) - \bar{s}(x)] \end{aligned}$$

$l^{\pm} - N$ *scattering*

The differential cross section:

$$\frac{d^2\sigma^l}{dxdy} = \frac{8M_N E_l \pi\alpha^2}{Q^4}\left\{xy^2 F_{1N}(x,Q^2) + \left(1 - y - \frac{xyM_N}{2E_l}\right) F_{2N}(x,Q^2)\right\}.$$

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$\nu(\bar{\nu}) - N$ *scattering*

The differential cross section:

$$\begin{aligned}\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx\ dy} = & \frac{G_F^2 M E_\nu}{\pi(1+Q^2/M_W^2)^2}\left(\left[y^2 x + \frac{m_l^2 y}{2E_\nu M}\right] F_{1N}(x,Q^2)\right. \\ & + \left[(1 - \frac{m_l^2}{4E_\nu^2}) - (1 + \frac{Mx}{2E_\nu})y\right] F_{2N}(x,Q^2) \\ & \left.\pm \left[xy(1 - \frac{y}{2}) - \frac{m_l^2 y}{4E_\nu M}\right] F_{3N}(x,Q^2)\right)\end{aligned}$$

The cross section for an element of volume dV in the nucleus is related to the probability per unit time (Γ) of the lepton interacting with the nucleons:

$$d\sigma = \Gamma dt dS = \Gamma \frac{dt}{dl} dS dl = \Gamma \frac{1}{v} dV = \Gamma \frac{E_l}{\mid \mathbf{k} \mid} dV = \Gamma \frac{E_l}{\mid \mathbf{k} \mid} d^3 r,$$

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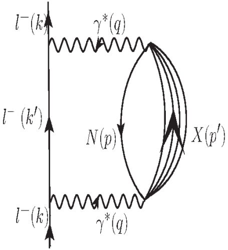
Γ is also related to imaginary part of lepton self energy:

$$-\frac{\Gamma}{2} = \frac{m_l}{E_l(\mathbf{k})} Im\Sigma$$

$$d\sigma = \frac{-2m}{E_l(\mathbf{k})} Im\Sigma(k) \frac{E_l(\mathbf{k})}{|\mathbf{k}|} d^3r,$$



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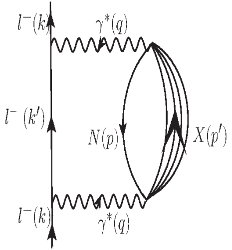


Lepton self energy $\Sigma(k)$:

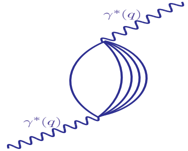
$$-i\Sigma(k) = \int \frac{d^4q}{(2\pi)^4} \bar{u}_l(\mathbf{k})\, ie\gamma^\mu\, i\frac{\not{k}' + m}{k'^2 - m^2 + i\epsilon} ie\gamma^\nu u_l(\mathbf{k}) \frac{-ig_{\mu\rho}}{q^2} (-i)\, \Pi^{\rho\sigma}(q) \frac{-ig_{\sigma\nu}}{q^2}$$

Imaginary part of lepton self energy:

$$Im\Sigma(k) = e^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_l} \theta(q^0)\, Im(\Pi^{\alpha\beta}) \frac{1}{q^4} \frac{1}{2m} L_{\alpha\beta}$$

In this expression, $\Pi^{\alpha\beta}$ is the photon self-energy, which is written in terms of the nucleon (G_l) and meson (D_j) propagators.

photon self-energy $\Pi^{\alpha\beta}(q)$ in the nuclear medium:



$$\Pi^{\alpha\beta}(q) = e^2 \int \frac{d^4p}{(2\pi)^4} G(p) \sum_X \sum_{s_p, s_l} \prod_{i=1}^{N} \int \frac{d^4p'_i}{(2\pi)^4} \prod_l G_l(p'_l) \prod_j D_j(p'_j)$$

$$< X|J^\mu|H >< X|J^\nu|H >^* (2\pi)^4 \, \delta^4(q + p - \sum_{i=1}^{N} p'_i)$$

where s_p is the spin of the nucleon, s_l is the spin of the fermions in X, $< X|J^\mu|H >$ is the hadronic current for the initial state nucleon to the final state hadrons, index l, j are respectively, stands for the fermions and for the bosons in the final hadronic state X, and $\delta^4(q + p - \sum_{i=1}^{N} p'_i)$ ensures the conservation of four momentum at the vertex.

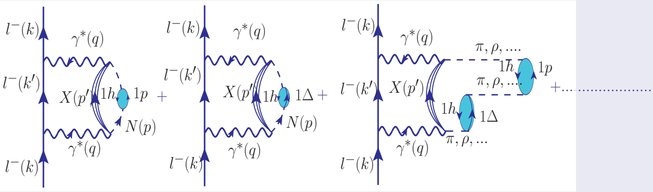
The nucleon propagator $G(p)$ inside the nuclear medium provides information about the propagation of the nucleon from the initial state to the final state or vice versa.

Relativistic Dirac propagator $G^0(p_0, \mathbf{p})$ for a free nucleon:

$$G^0(p_0, \mathbf{p}) = \frac{M}{E(\mathbf{p})} \left\{ \frac{\sum_r u_r(p)\bar{u}_r(p)}{p^0 - E(\mathbf{p}) + i\epsilon} + \frac{\sum_r v_r(-p)\bar{v}_r(-p)}{p^0 + E(\mathbf{p}) - i\epsilon} \right\}$$

- Only the positive energy contributions are retained as the negative energy contributions are suppressed. In the interacting Fermi sea, the relativistic nucleon propagator is then written in terms of the nucleon self-energy $\sum^N(p^0, p)$.
- In the nuclear many body technique, the quantity that contains all the information on single nucleon properties in the nuclear medium is the nucleon self-energy $\sum^N(p^0, p)$.
- For an interacting Fermi sea, the relativistic nucleon propagator is written in terms of the nucleon self-energy and in nuclear matter, the interaction is taken into account through Dyson series expansion.

The nucleon propagator in the interacting Fermi sea:



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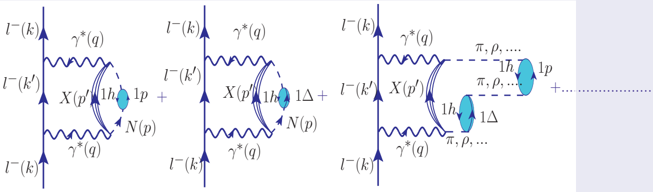

$$G(p_0, \mathbf{p}) = \frac{M}{E(\mathbf{p})} \frac{\sum_r u_r(p)\bar{u}_r(p)}{\left(p^0 - E(\mathbf{p}) + \mathbf{i}\epsilon\right)} + \left(\frac{M}{E(\mathbf{p})}\right)^2 \frac{1}{\left(p^0 - E(\mathbf{p}) + \mathbf{i}\epsilon\right)} \sum \frac{\sum_r u_r(p)\bar{u}_r(p)}{\left(p^0 - E(\mathbf{p}) + \mathbf{i}\epsilon\right)} + ..$$

$$= \frac{M}{E(\mathbf{p})} \frac{\sum_r u_r(p)\bar{u}_r(p)}{\left(p^0 - E(\mathbf{p}) + \mathbf{i}\epsilon \frac{\mathbf{M}}{\mathbf{E(p)}} \sum\right)}$$

Relativistic nucleon propagator in the nuclear medium:

$$G(p^0,\mathbf{p}) = \frac{M}{E(\mathbf{p})}\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p})\left[\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\mathbf{p})}{p^0-\omega-i\epsilon} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega,\mathbf{p})}{p^0-\omega+i\epsilon}\right]$$

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for $p^0 \leq \mu$

$$S_h(p^0,\mathbf{p}) = \frac{1}{\pi}\frac{\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p})}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma(p^0,\mathbf{p}))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p}))^2}$$

for $p^0 > \mu$

$$S_p(p^0,\mathbf{p}) = -\frac{1}{\pi}\frac{\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p})}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma(p^0,\mathbf{p}))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p}))^2}$$

P.Fernandez de Cordoba and E. Oset, PRC 46, 1697(1992)

Spectral function is normalized to mass number 'A':

$$4\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, \mathbf{p}, \rho(r))\, d\omega = A\,,$$

where $\rho(r)$ is the baryon density for the nucleus.

Kinetic energy $< T >$:

$$< T > = \frac{4}{A}\int d^3r \int \frac{d^3p}{(2\pi)^3}(E(\mathbf{p}) - M) \int_{-\infty}^{\mu} S_h(p^0, \mathbf{p}, \rho(r))\, dp^0\,,$$

$$< E > = \frac{4}{A}\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(p^0, \mathbf{p}, \rho(r))\, p^0 dp^0\,,$$

and the binding energy per nucleon:

$$|E_A| = -\frac{1}{2}(< E - M > + \frac{A-2}{A-1} < T >)$$

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Lepton self energy $\Sigma(k)$ in the nuclear medium:

$$Im\Sigma(k) = e^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_l} \theta(q^0)\, Im(\Pi^{\alpha\beta}) \frac{1}{q^4} \frac{1}{2m} L_{\alpha\beta}$$

Scattering cross section: $d\sigma = -\frac{2m_\nu}{|\mathbf{k}|} \,\mathrm{Im}\,\Sigma\, d^3r\,.$

Differential scattering cross section for $l^{\pm} - A$ interaction:

$$\frac{d^2\sigma}{d\Omega' dE'} = -\frac{\alpha}{(q)^4} \frac{|\mathbf{k}'|}{|\mathbf{k}|} \frac{1}{(2\pi)^2} L_{\alpha\beta} \int d^3r \mathrm{Im}\Pi^{\alpha\beta}(q)\,.$$

$l^{\pm}$-N DCX: $\frac{d^2\sigma^N}{d\Omega' dE'} = \frac{\alpha^2}{q^4} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L^{\alpha\beta}\, W^N_{\alpha\beta}$

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$$W^A_{\alpha\beta} = -\int d^3r \mathrm{Im}\Pi_{\alpha\beta}(q)$$

Nuclear hadronic tensor:

It is written as a convolution of nucleonic hadronic tensor with the hole spectral function

$$W^A_{\alpha\beta} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \frac{M}{E(\mathbf{p})} S_h(p^0, \mathbf{p}, \rho(r)) W^N_{\alpha\beta}(p,q)$$

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$$W^A_{\mu\nu} = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) W^A_1(\nu, Q^2) + \frac{W^A_2(\nu, Q^2)}{M^2_A}\left(p_\mu - \frac{p.q}{q^2} q_\mu\right)\left(p_\nu - \frac{p.q}{q^2} q_\nu\right)$$

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$$W^N_{\mu\nu} = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) W_{1N}(\nu, Q^2) + \frac{W_{2N}(\nu, Q^2)}{M^2}\left(p_\mu - \frac{p.q}{q^2} q_\mu\right)\left(p_\nu - \frac{p.q}{q^2} q_\nu\right)$$

Taking the xx component

$$W_{xx}^N = \left(\frac{q_x q_x}{q^2} - g_{xx}\right) W_1^N + \frac{1}{M^2}\left(p_x - \frac{p.q}{q^2}\, q_x\right)\left(p_x - \frac{p.q}{q^2}\, q_x\right) W_2^N$$

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Choosing **q** along the z-axis

$$W_{xx}^N(\nu_N, Q^2) = W_1^N(\nu_N, Q^2) + \frac{1}{M^2} p_x^2 W_2^N(\nu_N, Q^2)$$

Similarly taking xx component of nuclear hadronic tensor

$$W^A_{xx}(\nu_A, Q^2) = W^A_1(\nu_A, Q^2) = \frac{F^A_1(x_A)}{AM}$$

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$F_1(x) = M\ W_1(\nu, Q^2), \quad F_2(x) = \nu\ W_2(\nu, Q^2)$

$$\frac{F_1^A(x_A)}{AM} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(\mathbf{r})) \times \left[\frac{F_1^N(x_N)}{M} + \frac{1}{M^2}{p_x}^2\frac{F_2^N(x_N)}{\nu}\right]$$

$$F_2(x) = \nu W_2(\nu, Q^2)$$

$F_2^A(x_A)$ *in nuclear medium*

$$F_2^A(x_A) = 2\sum_{p,n}\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h^{p,n}(p^0, \mathbf{p}, \rho_{p,n}(\mathbf{r}))F_2^N(x_N)C$$

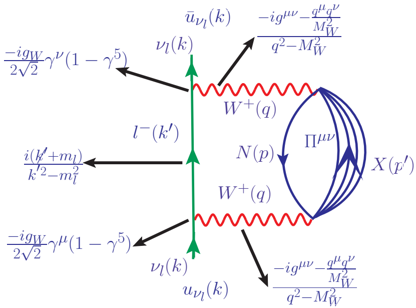
$$C = \left[\frac{Q^2}{q_z^2}\left(\frac{p^2 - p_z^2}{2M^2}\right) + \frac{(p.q)^2}{M^2\nu^2}\left(\frac{p_z\ Q^2}{p.qq_z} + 1\right)^2 \frac{q_0 M}{p_0\ q_0 - p_z\ q_z}\right]$$

Formalism: $\nu_l/\bar{\nu}_l - A$ scattering

- To calculate the scattering cross section for a neutrino interacting with a target nucleon in the nuclear medium, we write

$$d\sigma_A = -2\frac{m_\nu}{|\,\mathbf{k}\,|} Im\Sigma(k) d^3r.$$

- The neutrino self energy $\Sigma(k)$:



$$\Sigma(k) = \frac{iG_F}{\sqrt{2}} \int \frac{d^4q}{(2\pi)^4} \frac{4L^{WI}_{\mu\nu}}{m_l} \frac{1}{(k'^2 - m_l^2 + i\epsilon)} \left(\frac{M_W}{q^2 - M_W^2}\right)^2 \Pi^{\mu\nu}(q)\,,$$

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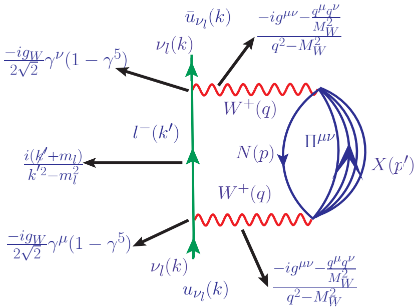

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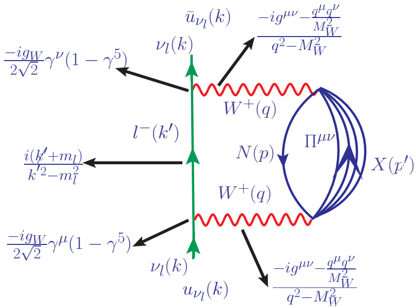

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Weak Nuclear Structure Function

$$F_1^A(x_A) = 4AM \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(\mathbf{r})) \left[\frac{F_1^N(x_N)}{M} + \frac{1}{M^2} {p_x}^2 \frac{F_2^N(x_N)}{\nu} \right]$$

$$F_2^A(x_A) = 2 \sum_{p,n} \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h^{p,n}(p^0, \mathbf{p}, \rho_{p,n}(\mathbf{r})) F_2^N(x_N) C$$

$$C = \left[\frac{Q^2}{q_z^2} \left(\frac{p^2 - p_z^2}{2M^2} \right) + \frac{(p.q)^2}{M^2 \nu^2} \left(\frac{p_z\, Q^2}{p.q q_z} + 1 \right)^2 \frac{q_0 M}{p_0\, q_0 - p_z\, q_z} \right]$$

$$F_3^A(x_A, Q^2) = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(\mathbf{r})) \frac{p^0\gamma - p_z}{(p^0 - p_z\gamma)\gamma} F_3^N(x_N)$$

π and ρ mesons contributions

"Significant at low and mid x"

1. There are virtual mesons associated with each nucleon bound inside the nucleus.
2. These meson clouds get strengthened by the strong attractive nature of nucleon-nucleon interactions.
3. This leads to an increase in the interaction probability of virtual photons with the meson cloud.
4. The effect of meson cloud is more pronounced in heavier nuclear targets and dominate in the intermediate region of x(0.2<x<0.6).

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For meson cloud contribution

$$2\pi \frac{M}{E(\mathbf{p})} S_h(p_0, \mathbf{p}) W_N^{\alpha\beta}(p,q) \rightarrow 2 Im D(p) \theta(p_0) W_\pi^{\alpha\beta}(p,q)$$

Pion propagator in the nuclear medium

$$D(p) = [{p_0}^2 - \mathbf{p}^2 - m_\pi^2 - \Pi_\pi(p_0, \mathbf{p})]^{-1}$$

$$\Pi_\pi = \frac{f^2/m_\pi^2 F^2(p) \mathbf{p}^2 \Pi^*}{1 - f^2/m_\pi^2 V_L' \Pi^*}$$

πNN form factor $F(p) = (\Lambda^2 - m_\pi^2)/(\Lambda^2 + \mathbf{p}^2)$

$F_{1,\pi}^{A}(x_A)$:

$$F_{1,\pi}^{A}(x_\pi) = -6AM\int d^3r\int \frac{d^4p}{(2\pi)^4}\theta(p_0)\ \delta ImD(p)\,2m_\pi\ \times \left[\frac{F_{1\pi}(x_\pi)}{m_\pi} + \frac{|\mathbf{p}|^2 - p_z^2}{2(p_0\ q_0 - p_z q_z)}\frac{F_{2\pi}(x_\pi)}{m_\pi}\right]$$

$F_{2,\pi}^{A}(x_A)$:

$$F_{2,\pi}^{A}(x_\pi) = -6\int d^3r\int \frac{d^4p}{(2\pi)^4}\,\theta(p_0)\ \delta ImD(p)\,2m_\pi\ \frac{m_\pi}{p_0 - p_z\ \gamma}C_1F_{2\pi}(x_\pi)$$

$$C_1 = \frac{Q^2}{q_z^2}\left(\frac{|\mathbf{p}|^2 - p_z^2}{2m_\pi^2}\right) + \frac{(p_0 - p_z\ \gamma)^2}{m_\pi^2}\left(\frac{p_z\ Q^2}{(p_0 - p_z\ \gamma)q_0q_z} + 1\right)^2$$

$F^A_{1,\rho}(x_A)$:

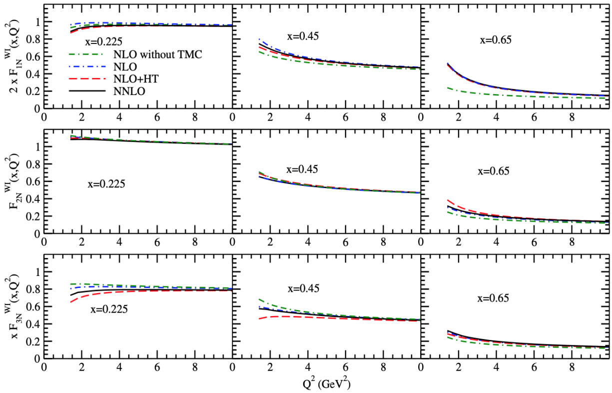
$$F^A_{1,\rho}(x_\rho) = -12AM \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\ \delta Im D_\rho(p)\, 2m_\rho \times \left[\frac{F_{1\rho}(x_\rho)}{m_\rho} + \frac{|\mathbf{p}|^2 - p_z^2}{2(p_0\ q_0 - p_z\ q_z)} \frac{F_{2\rho}(x_\rho)}{m_\rho} \right]$$

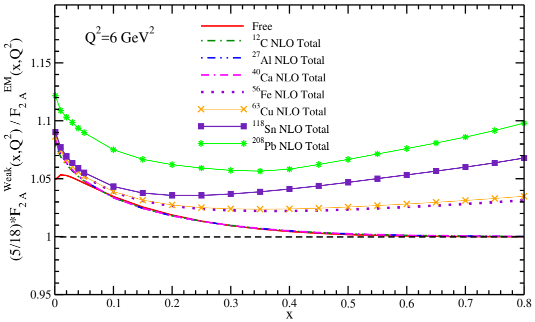
$F^A_{2,\rho}(x_A)$:

$$F^A_{2,\rho}(x_\rho) = -12 \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\ \delta Im D_\rho(p)\, 2m_\rho\ \frac{m_\rho}{p_0 - p_z\ \gamma} C_2 F_{2\rho}(x_\rho)$$

$$C_2 = \frac{Q^2}{q_z^2} \left(\frac{|\mathbf{p}|^2 - p_z^2}{2m_\rho^2} \right) + \frac{(p_0 - p_z\ \gamma)^2}{m_\rho^2} \left(\frac{p_z\ Q^2}{(p_0 - p_z\ \gamma) q_0 q_z} + 1 \right)^2$$

WEAK STRUCTURE FUNCTIONS IN ν_l-N AND ν_l-A … PHYS. REV. D **101,** 033001 (2020)




Q^2=6 GeV^2
Free
12C NLO Total
27Al NLO Total
40Ca NLO Total
56Fe NLO Total
63Cu NLO Total
118Sn NLO Total
208Pb NLO Total
(5/18)*F_2A^Weak(x,Q^2) / F_2A^EM(x,Q^2)
x
0.95
1
1.05
1.1
1.15
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

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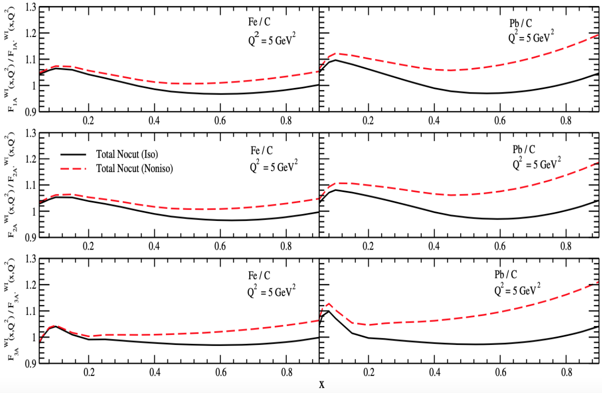
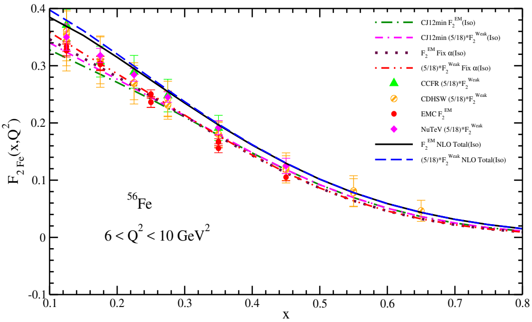


Nuclear Medium Effects, Left: are different in EM and Weak interactions. Right: show A dependence



Haider *et al.*, Nuc Phys A, 955, 2016, 58


Thank
you!