Neutrino Event Generators (Technical)

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NuSTEC Summer School 2024
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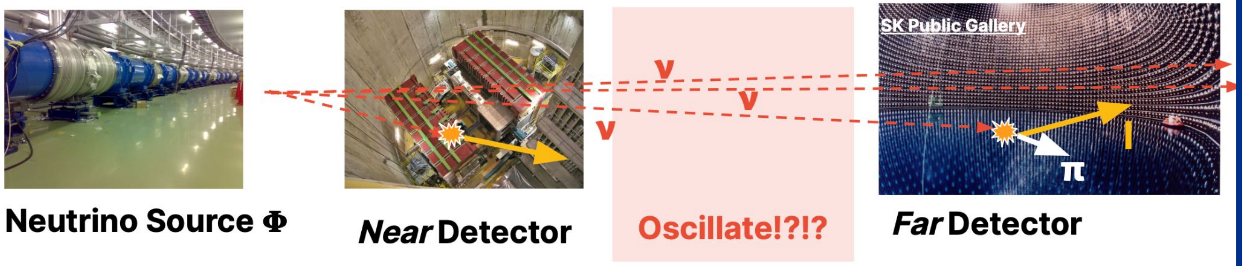
The Problem

Anatomy of an Oscillation Experiment



$$N(E_{\text{obs}}) = \int dE_\nu \, \Phi(E_\nu) \cdot P_{osc}(E_\nu) \cdot \sigma(E_\nu) \cdot \mathrm{D}$$

1. Find or make a source of neutrinos
2. **Constrain model uncertainties before oscillation with *Near* Detector**
3. Predict the expected rate with a **flux**/**cross-section**/**detector** model
4. Look in your detector/box... **See appearance/disappearance?**

We need to predict what we will see in an experiment to be able to make inferences through models of processes of interest.

It ultimately amounts to solving the integral above.

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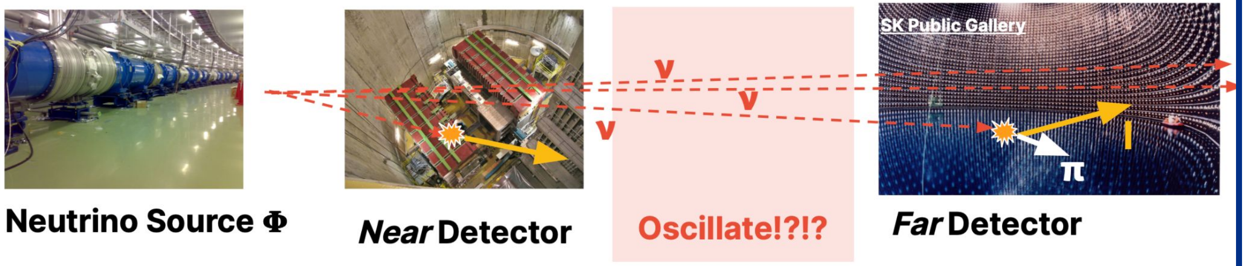

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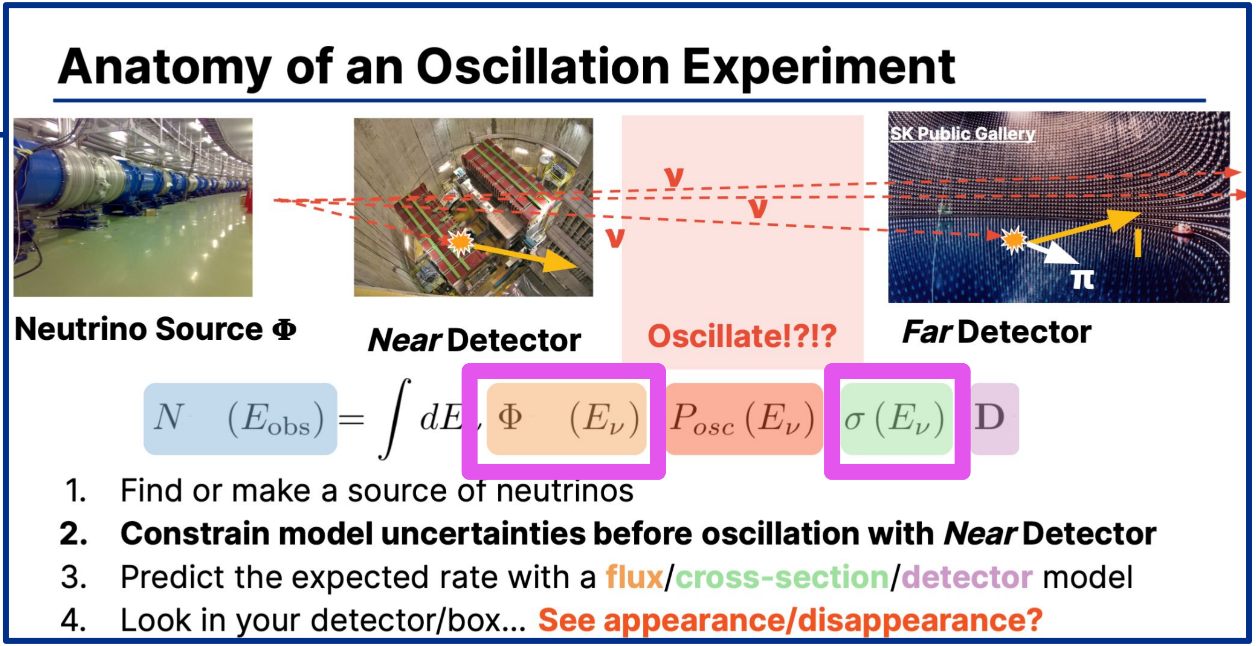

We're going to focus on these parts of the integrand.

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The Plan

- We have heard, starting with Aaron about all the amazing and sophisticated models of neutrino–nucleon and neutrino–nucleus reactions.

CCQE Cross section

Full cross section obtained from square $|\mathcal{M}_{\text{CCQE}}|^2$...

+(−) for (anti)neutrino

$$\frac{d\sigma^{CCQE}}{dQ^2}(E_\nu, Q^2) \propto \frac{1}{E_\nu^2}\left(A(Q^2) \boxed{\mp} \left(\frac{s-u}{M_N^2}\right) B(Q^2) + \left(\frac{s-u}{M_N^2}\right)^2 C(Q^2)\right) \qquad s - u = 4M_N E_\nu - Q^2 - m_\ell^2$$

$$A(Q^2) = \left(\frac{m_\ell^2}{M_N^2} + 4\eta\right)\left[(1+\eta)F_A^2 - (1-\eta)(F_1^2 + \eta F_2^2) + 4\eta F_1 F_2 - \frac{m_\ell^2}{4M_N^2}\left((F_1+F_2)^2 + (F_A + 2F_P)^2 - 4(1+\eta)F_P^2\right)\right]$$

$$B(Q^2) = 4\eta F_A (F_1 + F_2) \qquad C(Q^2) = \frac{1}{4}\left(F_A^2 + F_1^2 + \eta F_2^2\right) \qquad \eta \equiv \frac{Q^2}{4M_N^2}$$

[Llewellyn Smith, 1972]
see also[Rev.Mod.Phys. 84 (2012)]

We will explore this in more detail...

[Aaron's talk from Monday]

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1. Start with a differential cross section in some interaction kinematics, Enu, Q^2
2. Apply any kinematic constraints
3. Integrate to a cross section
4. Sample from the cross section

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Learning Outcomes

- An intuitive understanding of the technical steps between writing down a theoretical model and making stochastic predictions of what an experiment might expect to see were that model to be a good description of nature.
- How rejection sampling and MC integration works.
- That while modern generators are quite complicated, the fundamentals are understandable and implementable in not-too-many lines of code.
- That `numpy` is utter magic.

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Structure Of This Course

This course will be mostly hands on, with the pace somewhat dictated by you.

I will start with a short primer on the core python techniques that we will use. This is not a python course.

I will walk through a Jupyter notebook, stopping occasionally to allow you to answer short questions by poking and prodding the toy generator that we will build together.

There will be some longer homework options that we may or may not get to.

Structure Of This Course

Ask questions!

Stop me!

Ask me to slow down or try explaining something again!

There are no stupid questions: I genuinely lost 2 full days doing some relatively simple algebra in making this course.

I would rather we covered ¼ of the material and some of you built up new foundational understanding of MC techniques than we get to the end of the notebook.