Neutrino Event Generators (Technical)

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NuSTEC Summer School 2024
CERN

Science and Technology Facilities Council

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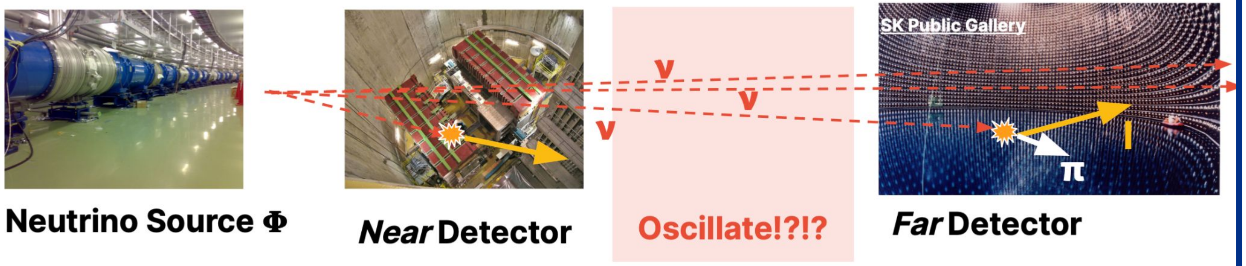
THE ROYAL SOCIETY

The Problem

Anatomy of an Oscillation Experiment



$$N(E_{\mathrm{obs}}) = \int dE_\nu \, \Phi(E_\nu) \cdot P_{osc}(E_\nu) \cdot \sigma(E_\nu) \cdot \mathrm{D}$$

1. Find or make a source of neutrinos
2. **Constrain model uncertainties before oscillation with *Near* Detector**
3. Predict the expected rate with a flux/cross-section/detector model
4. Look in your detector/box... **See appearance/disappearance?**

We need to predict what we will see in an experiment to be able to make inferences through models of processes of interest.

It ultimately amounts to solving the integral above.

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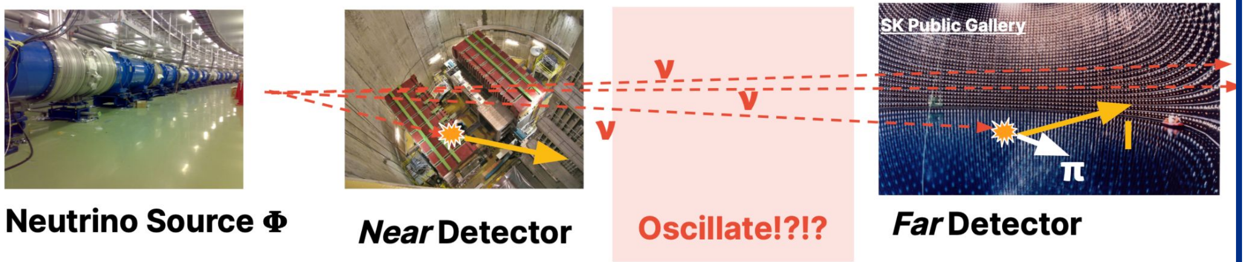

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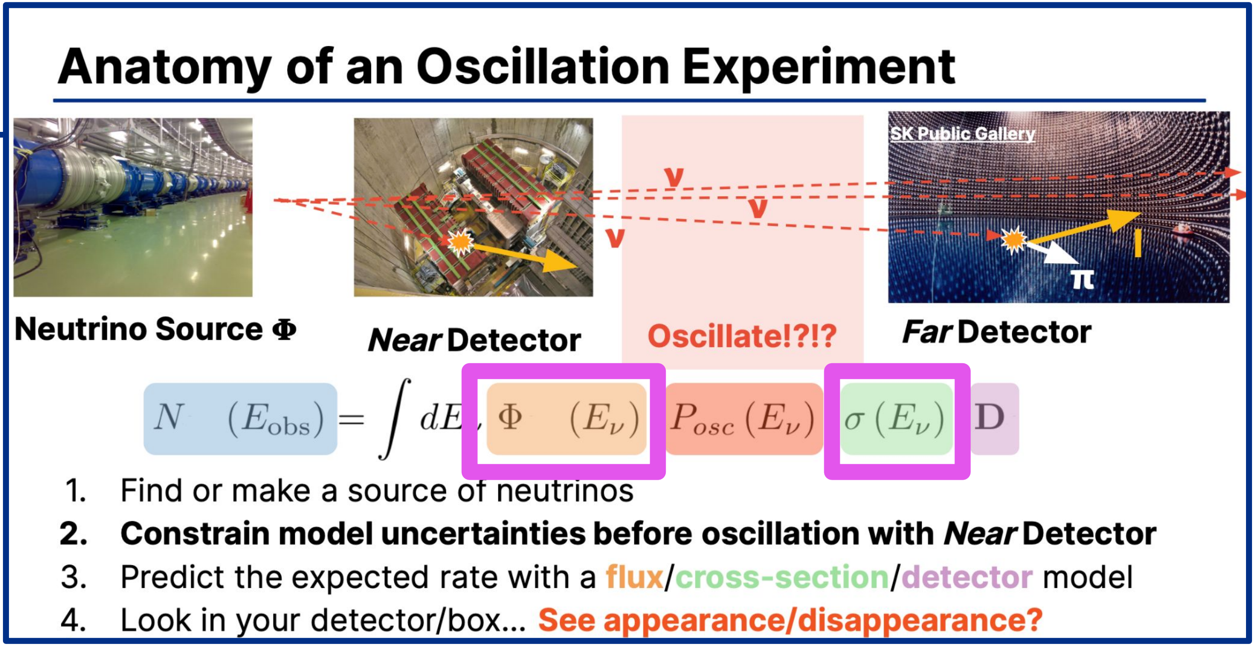
The Problem



We're going to focus on these parts of the integrand.

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The Plan

- We have heard, starting with Aaron about all the amazing and sophisticated models of neutrino–nucleon and neutrino–nucleus reactions.

CCQE Cross section

Full cross section obtained from square $|\mathcal{M}_{\mathrm{CCQE}}|^2$...

+(−) for (anti)neutrino

$$\frac{d\sigma^{CCQE}}{dQ^2}(E_\nu, Q^2) \propto \frac{1}{E_\nu^2}\left(A(Q^2) \mp \left(\frac{s-u}{M_N^2}\right) B(Q^2) + \left(\frac{s-u}{M_N^2}\right)^2 C(Q^2)\right)$$

$$s - u = 4M_N E_\nu - Q^2 - m_\ell^2$$

$$A(Q^2) = \left(\frac{m_\ell^2}{M_N^2} + 4\eta\right)\left[(1+\eta)F_A^2 - (1-\eta)(F_1^2 + \eta F_2^2) + 4\eta F_1 F_2 - \frac{m_\ell^2}{4M_N^2}\left((F_1+F_2)^2 + (F_A + 2F_P)^2 - 4(1+\eta)F_P^2\right)\right]$$

$$B(Q^2) = 4\eta F_A (F_1 + F_2) \qquad C(Q^2) = \frac{1}{4}\left(F_A^2 + F_1^2 + \eta F_2^2\right) \qquad \eta \equiv \frac{Q^2}{4M_N^2}$$

[Llewellyn Smith, 1972]
see also[Rev.Mod.Phys. 84 (2012)]

We will explore this in more detail...

[Aaron's talk from Monday]

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1. Start with a differential cross section in some interaction kinematics, Enu, Q^2
2. Apply any kinematic constraints
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4. Sample from the cross section

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Learning Outcomes

- An intuitive understanding of the technical steps between writing down a theoretical model and making stochastic predictions of what an experiment might expect to see were that model to be a good description of nature.
- How rejection sampling and MC integration works.
- That while modern generators are quite complicated, the fundamentals are understandable and implementable in not-too-many lines of code.
- That `numpy` is utter magic.

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Structure Of This Course

This course will be mostly hands on, with the pace somewhat dictated by you.

I will start with a short primer on the core python techniques that we will use. This is not a python course.

I will walk through a Jupyter notebook, stopping occasionally to allow you to answer short questions by poking and prodding the toy generator that we will build together.

There will be some longer homework options that we may or may not get to.

Structure Of This Course

Ask questions!

Stop me!

Ask me to slow down or try explaining something again!

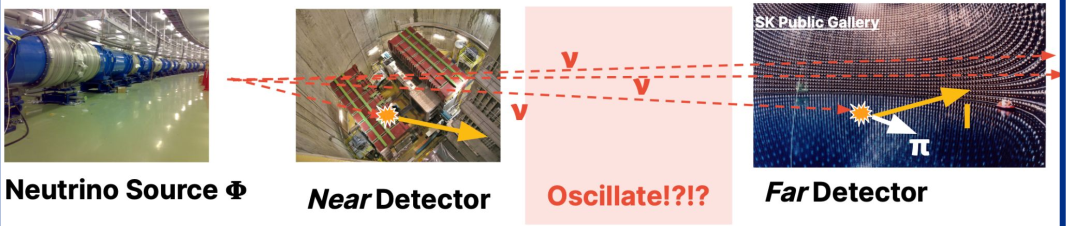
There are no stupid questions: I genuinely lost 2 full days doing some relatively simple algebra in making this course.

I would rather we covered ¼ of the material and some of you built up new foundational understanding of MC techniques than we get to the end of the notebook.

Lecture 2

Why MCEGs

Anatomy of an Oscillation Experiment



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A key point that I didn't highlight yesterday, but has come up today is that many of the analyses techniques that we apply:

Selections, complex projections, etc..

Cannot be analytically included in this integral. We need MCEGs to 'realise' events, so that we can perform the same analysis on simulation as data. We can't always just integrate the model.

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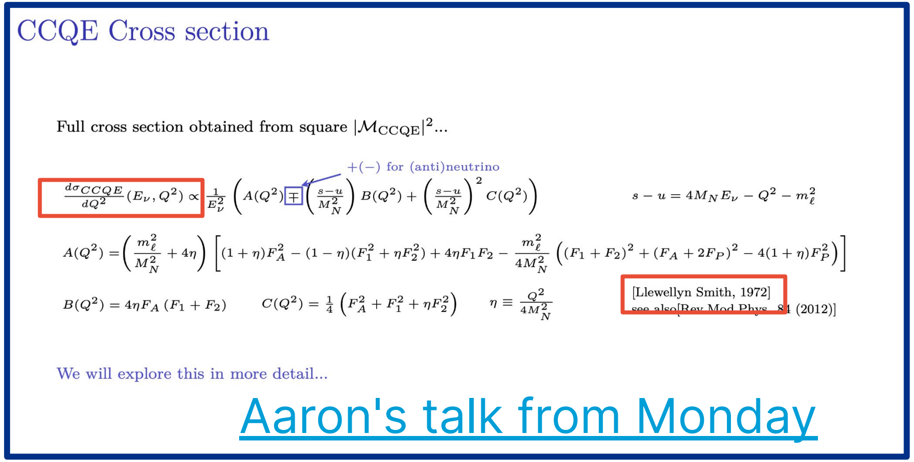

Aaron's talk from Monday

Other Important Bits and Bobs

- We have been building a 'single target' generator. It fires neutrinos at a single nucleon or nucleus and samples the cross section.
  - Real experiments are not infinite volumes of a pure target material
    - Compound rather than elemental targets
    - Interactions with detector elements can form important background
    - Interactions with the rock/shielding around detectors can send muons through the detector
  - Choosing where to place an interaction in a detector geometry is a modest extension of the MC techniques that we've learned here. Things get more complicated, but the fundamentals are essentially the same
- Systematic uncertainty tools and techniques:
  - How we use 'free' parameters in our models to produce uncertainties in our analyses?
  - How to use different model options to produce uncertainties in our analyses?
  - How do we down-select model options and tune free parameters?

Feedback

I would really appreciate some feedback on this course.

Talking/working time okay?

Amount of material covered okay?

Coding experience required/supported?

Please fill this out if you have 2 minutes:
https://forms.gle/ypWTjbuv4FhA349R7

Your Continued Journey To Generator Devhood

- This was just the beginning: We tried to work through a few of the foundational building blocks of MC simulations.
- Working with generators is a lot of fun and, I think, quite rewarding.
- The models and code you write become central moving parts of frontier physics in our field!
  - Unfortunately, we are a bit short on event generator developers (and model builders, who are the real smarty pants).
- If any of this piqued your interest, please get in touch with me or one of the other generator authors and get involved!