Ultra-low emittance tuning using normal mode BPM calibration

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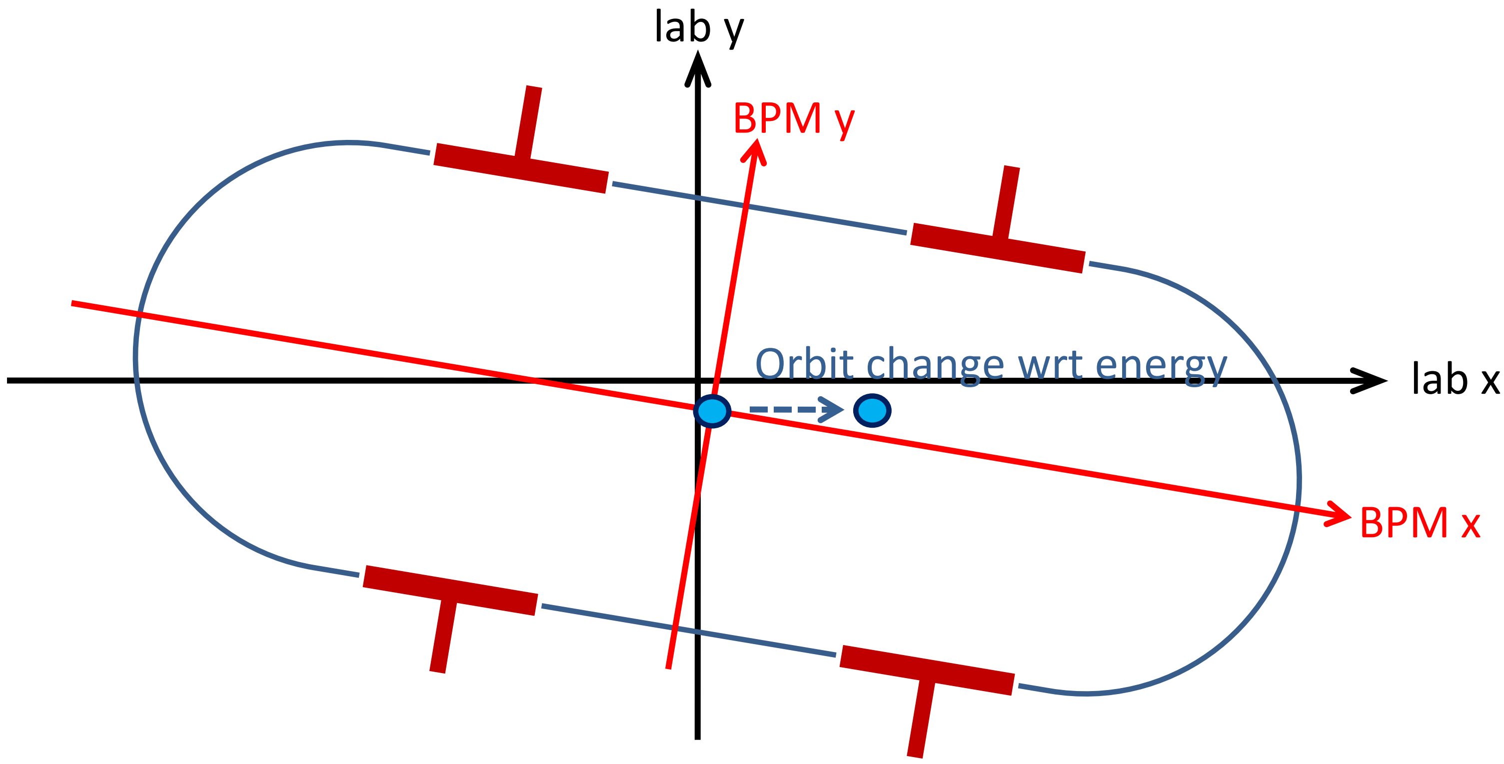
Introduction

- A number of storage rings have reported achieving vertical emittances of a few picometres.
- Achieving and maintaining emittances at this level requires precise correction of errors that lead to betatron coupling and vertical dispersion.
- The impact of **systematic BPM errors** is one of the major issues with ultra-low emittance tuning.
- I shall describe a technique for **calibrating the BPMs from normal mode beam motion**, which leads to a fast and straightforward procedure for ultra-low emittance tuning.

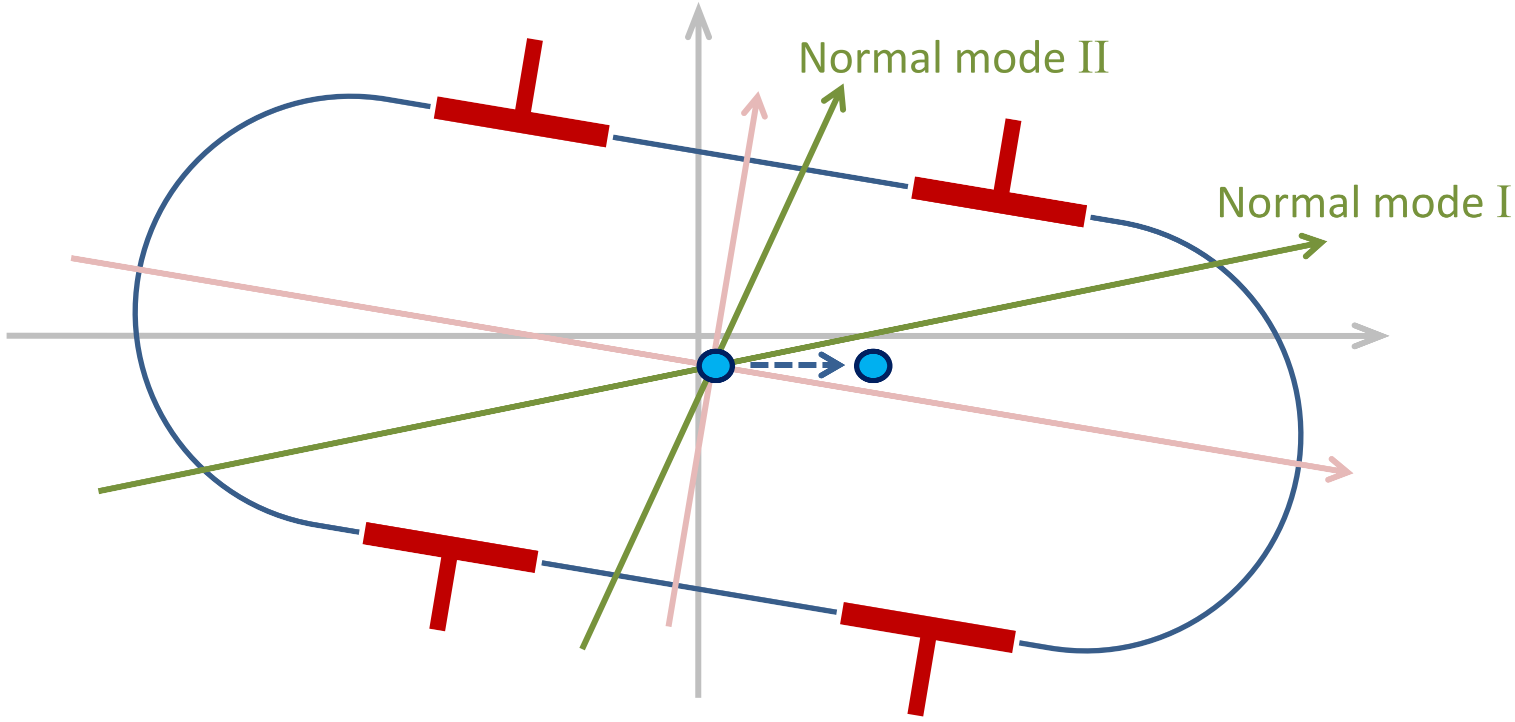
Coordinate Systems in a BPM

Alignment and amplifier gain errors in BPMs can lead to erroneous measurements of dispersion.



Coordinate Systems in a BPM

What really counts for emittance generation is the mode II dispersion in the dipoles…



Generation of Mode II Emittance

The normal mode (invariant) emittance is given by:

$$\varepsilon_{II} \approx C_q \gamma^2 \frac{I_{5,II}}{I_2}$$

where the synchrotron radiation integrals are:

$$I_2 = \oint \frac{ds}{\rho^2} \qquad\qquad I_{5,II} = \oint \frac{\mathcal{H}_{II}}{|\rho|^3} ds$$

and:

$$\mathcal{H}_{II} = \gamma_{II}\eta_{II}^2 + 2\alpha_{II}\eta_{II}\eta'_{II} + \beta_{II}\eta'^2_{II}$$

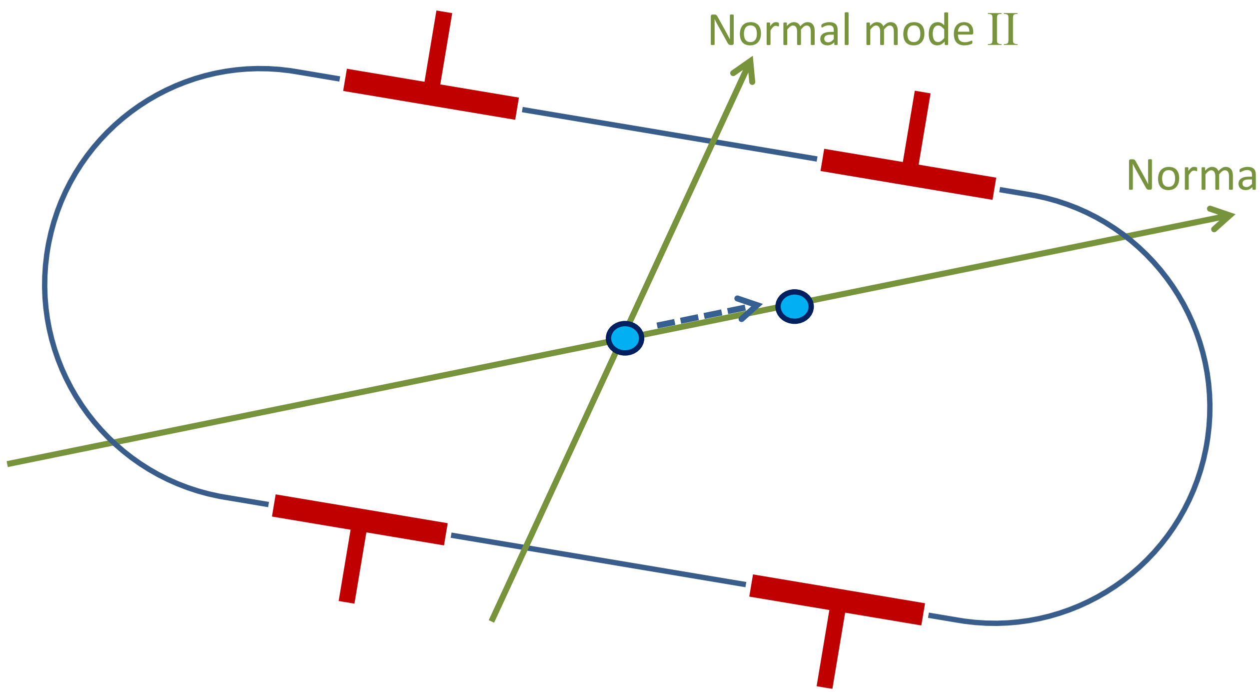
It is the normal mode dispersion that counts for the generation of the invariant emittance.

Correcting the mode II dispersion reduces simultaneously the contribution of betatron coupling and dispersion to the emittance.

Principle of Tuning Using the Normal Modes

To tune the ring for ultra-low invariant (mode II) emittance:

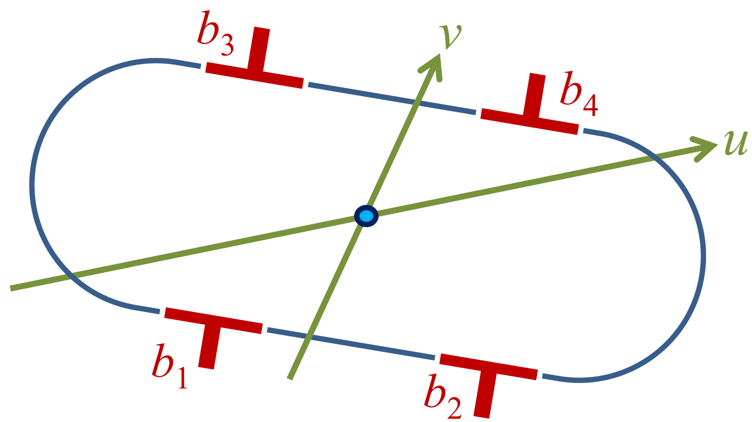
- calibrate the BPMs from observation of normal mode oscillations, to read the beam coordinates on the normal mode axes;
- use corrector elements (e.g. skew quadrupoles) to minimise the mode II dispersion in the dipoles.



Calibrating a BPM

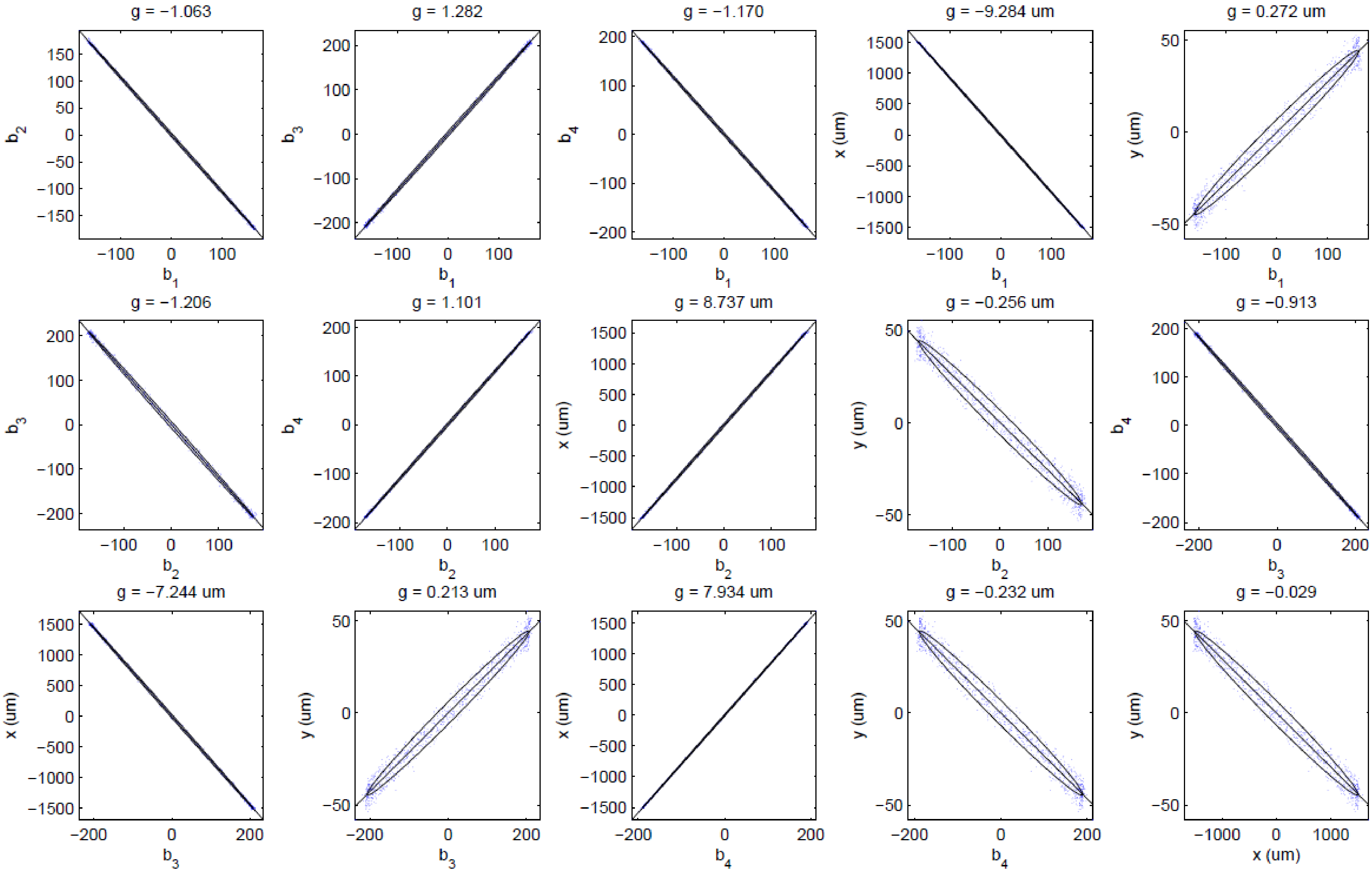
$$\begin{pmatrix}\Delta b_1\\ \Delta b_2\\ \Delta b_3\\ \Delta b_4\end{pmatrix}=\begin{pmatrix}\left(\frac{\partial b_1}{\partial u}\right)_v & \left(\frac{\partial b_1}{\partial v}\right)_u\\ \left(\frac{\partial b_2}{\partial u}\right)_v & \left(\frac{\partial b_2}{\partial v}\right)_u\\ \left(\frac{\partial b_3}{\partial u}\right)_v & \left(\frac{\partial b_3}{\partial v}\right)_u\\ \left(\frac{\partial b_4}{\partial u}\right)_v & \left(\frac{\partial b_4}{\partial v}\right)_u\end{pmatrix}\cdot\begin{pmatrix}\Delta u\\ \Delta v\end{pmatrix}$$

$$=\begin{pmatrix}1 & 1\\ \left(\frac{\partial b_2}{\partial b_1}\right)_v & \left(\frac{\partial b_2}{\partial b_1}\right)_u\\ \left(\frac{\partial b_3}{\partial b_1}\right)_v & \left(\frac{\partial b_3}{\partial b_1}\right)_u\\ \left(\frac{\partial b_4}{\partial b_1}\right)_v & \left(\frac{\partial b_4}{\partial b_1}\right)_u\end{pmatrix}\cdot\begin{pmatrix}\left(\frac{\partial b_1}{\partial u}\right)_v & 0\\ 0 & \left(\frac{\partial b_1}{\partial v}\right)_u\end{pmatrix}\cdot\begin{pmatrix}\Delta u\\ \Delta v\end{pmatrix}$$

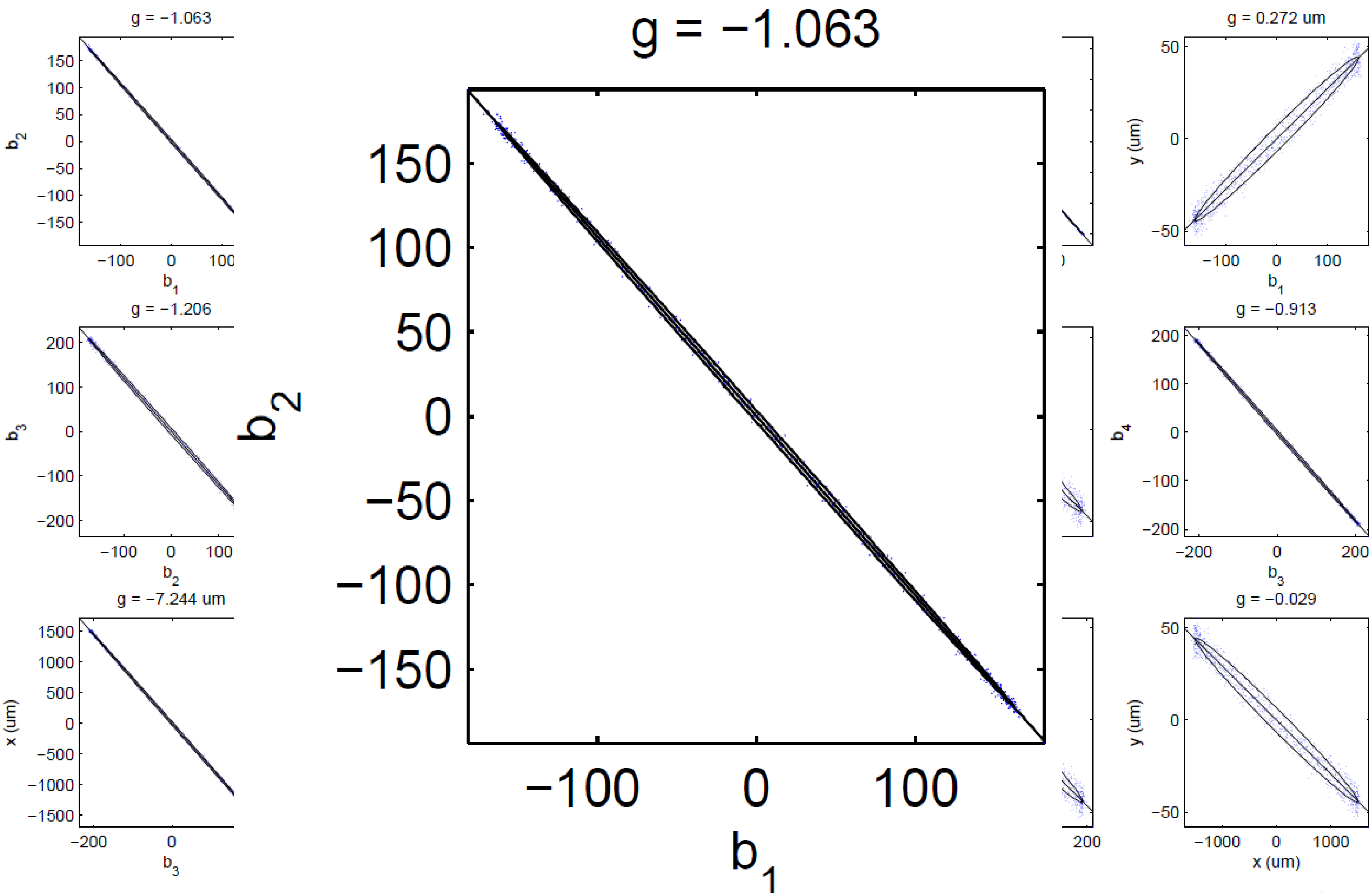


We can obtain the components of the calibration matrix from correlation plots of the button signals during normal mode beam excitation…

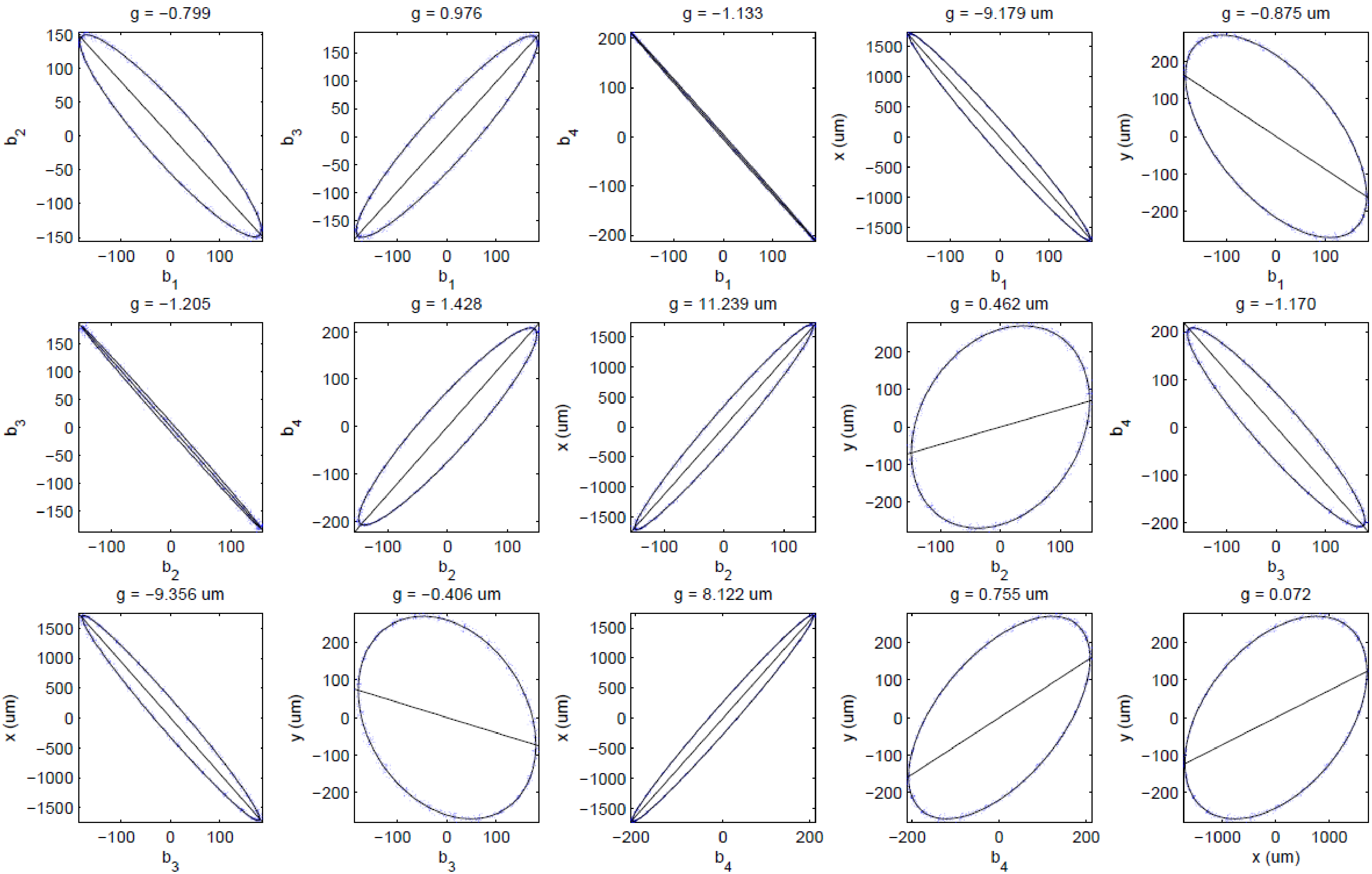
Typical Calibration Data from CesrTA



Typical Calibration Data from CesrTA

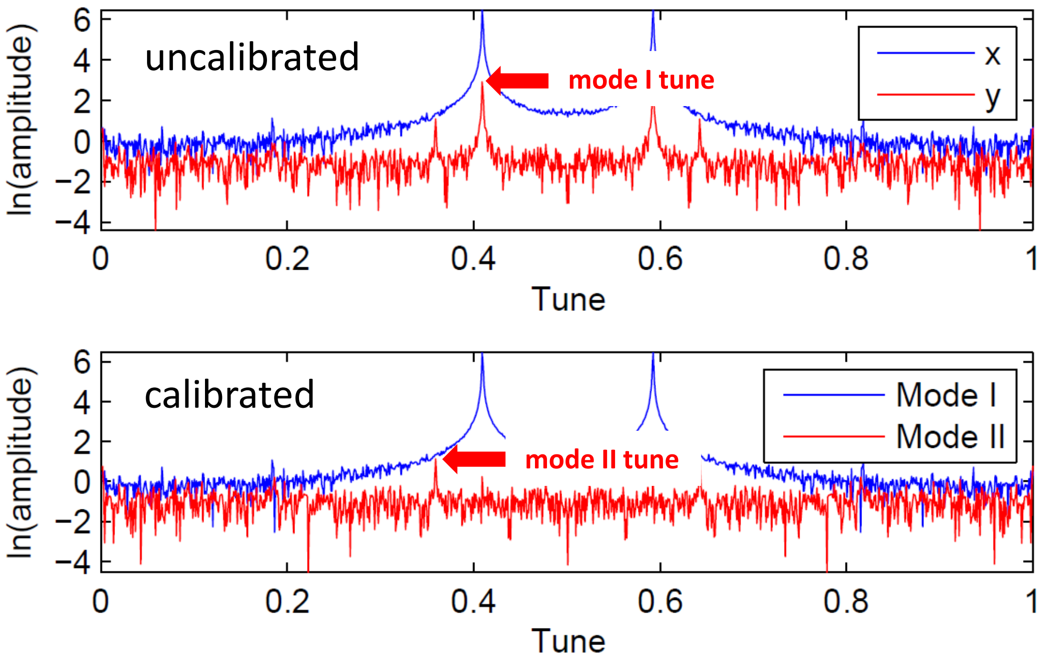


Calibration Data with Strong Coupling



Testing the Calibration

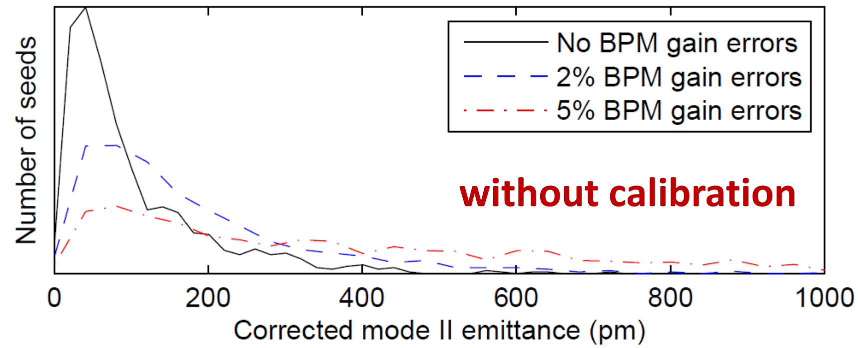
- The calibrated BPM returns the beam coordinates along the normal mode axes.
- If we excite the beam in a normal mode, and plot the Fourier spectrum of the turn-by-turn coordinates, we should see the mode I tune in only the mode I data, and the mode II tune in only the mode II data…



Fourier spectra of turn-by-turn coordinates from a single BPM, obtained during mode I beam excitation

Correction Simulations

- To test the correction technique, we simulated the calibration and correction procedure for 1000 seeds of machine errors.
- The correction procedure involved orbit correction using steering magnets, followed by vertical or mode II dispersion correction using skew quadrupoles.



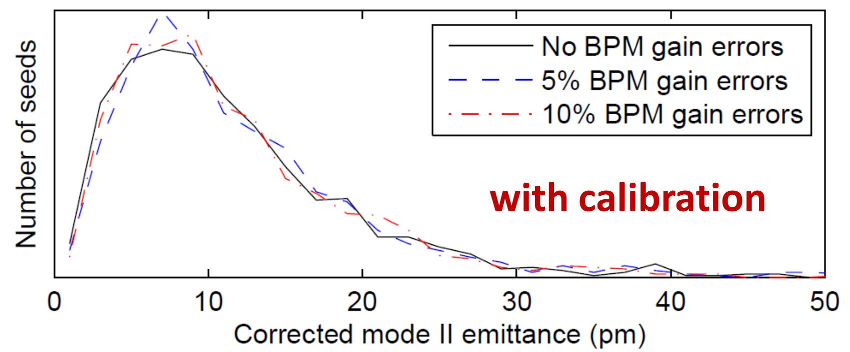


TABLE I. RMS values for error distributions.

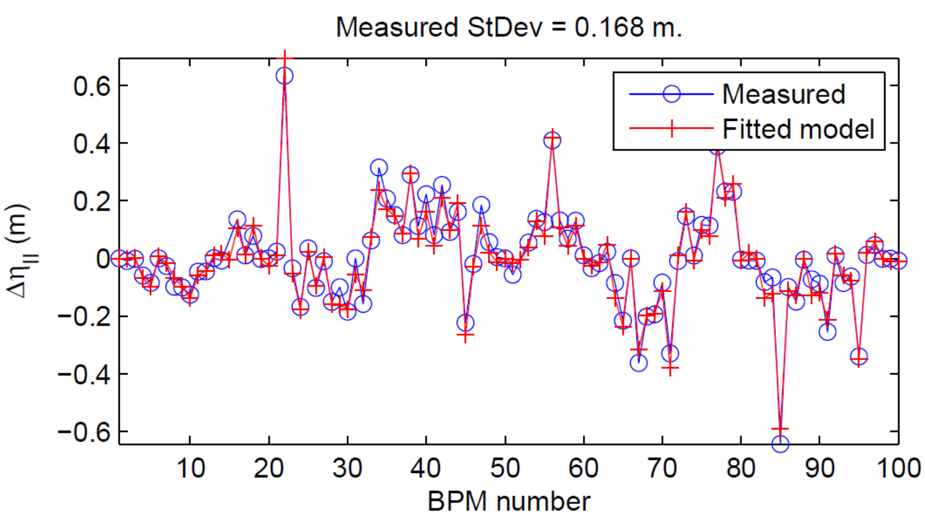
Error type	Distribution rms
Dipole tilt	300 μrad
Quadrupole tilt	300 μrad
Quadrupole vertical alignment	250 μm
Sextupole vertical alignment	250 μm
BPM vertical alignment	100 μm
BPM tilt	20 mrad
BPM gain	0 – 10%
Dispersion measurement resolution	12 mm

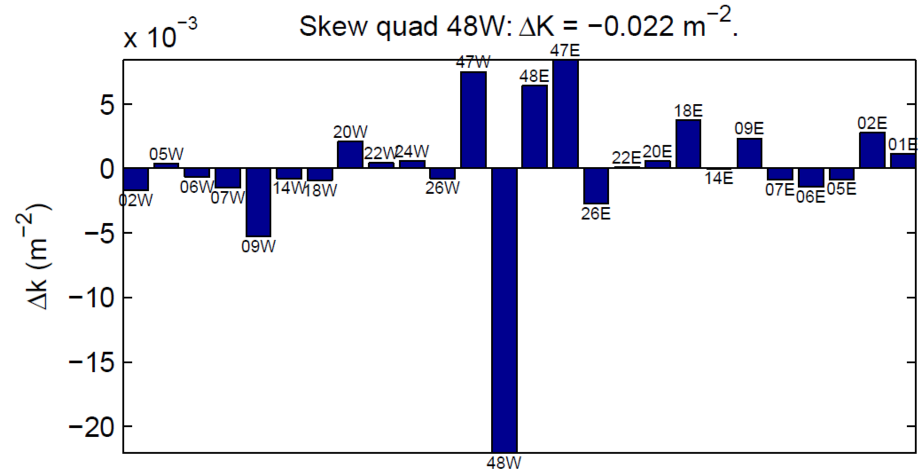
Note: nonlinear model used for BPM button response:

$$b_i = \frac{g_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}$$

Testing the Model Used for Correction

- The calibrated BPMs can be used to measure directly the mode II dispersion: this is model independent.
- The skew quad strengths needed to correct the mode II dispersion are calculated using a response matrix computed using a machine model.
- To test the model, we measured the change in mode II dispersion in response to a known change in strength of a single skew quadrupole; then we used the model to fit changes in *all* skew quads from the change in mode II dispersion.

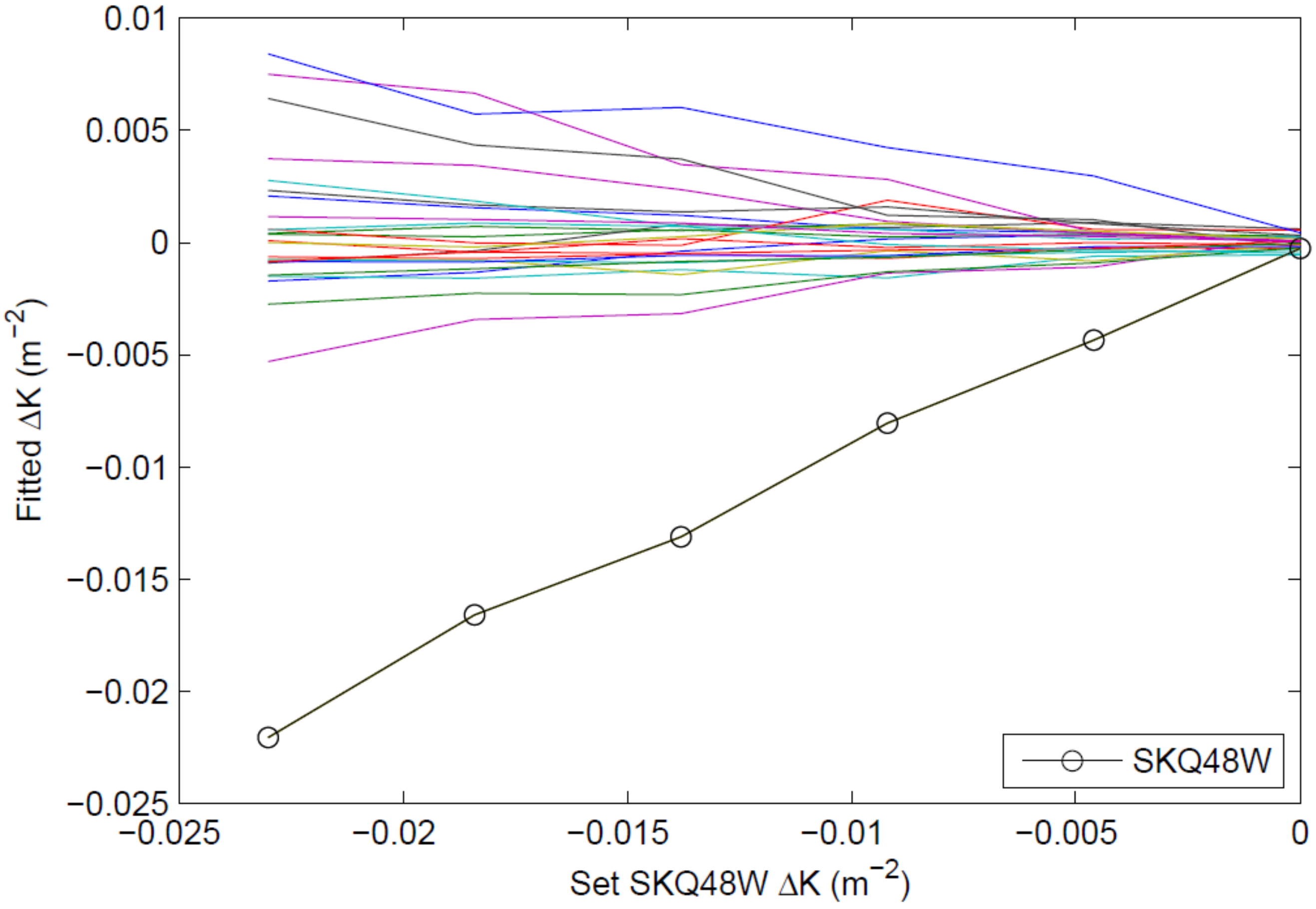




Note: Skew Quad 48W changed by $\Delta K = -0.023$ m^{-2}. This skew quad is located where $\eta_x = 0$.

Testing the Model Used for Correction

- The agreement between the model and the machine may be reasonable, but is far from perfect…



Results from Experimental Tests

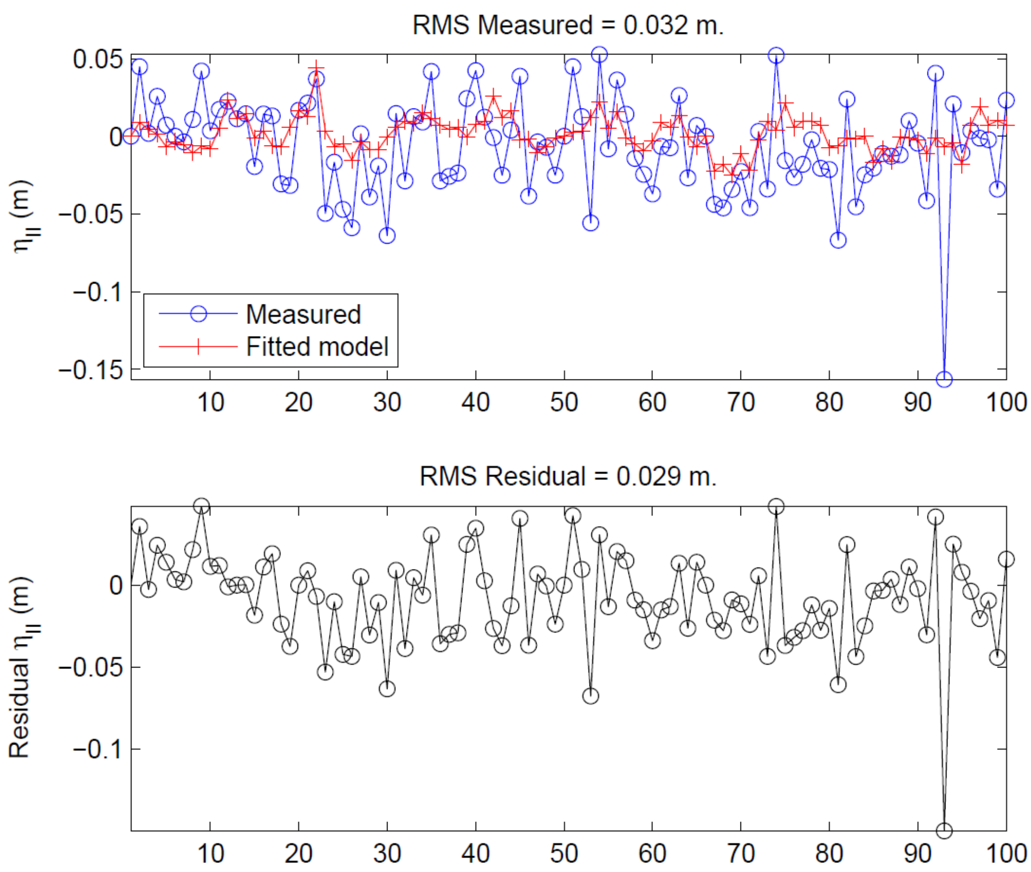
- Tuning tests were carried out, using a fast x-ray beam size monitor to estimate the vertical emittance.

Machine conditions	RMS η_{II} (mm)	RMS σ_y (μm)	η_y at bsm (mm)	ε_y (pm)
Following initial tuning (orbit, dispersion and coupling correction)	38	21	6	14
All skew quadrupoles turned off	32	27	3	24
After first correction of mode II dispersion	32	22	8	14
After further correction of mode II dispersion	31	28	13	21

- An increase in the emittance following a second iteration of the correction is often observed in simulation: it likely follows from the equal weighting given to the correction of the mode II dispersion at all BPMs.

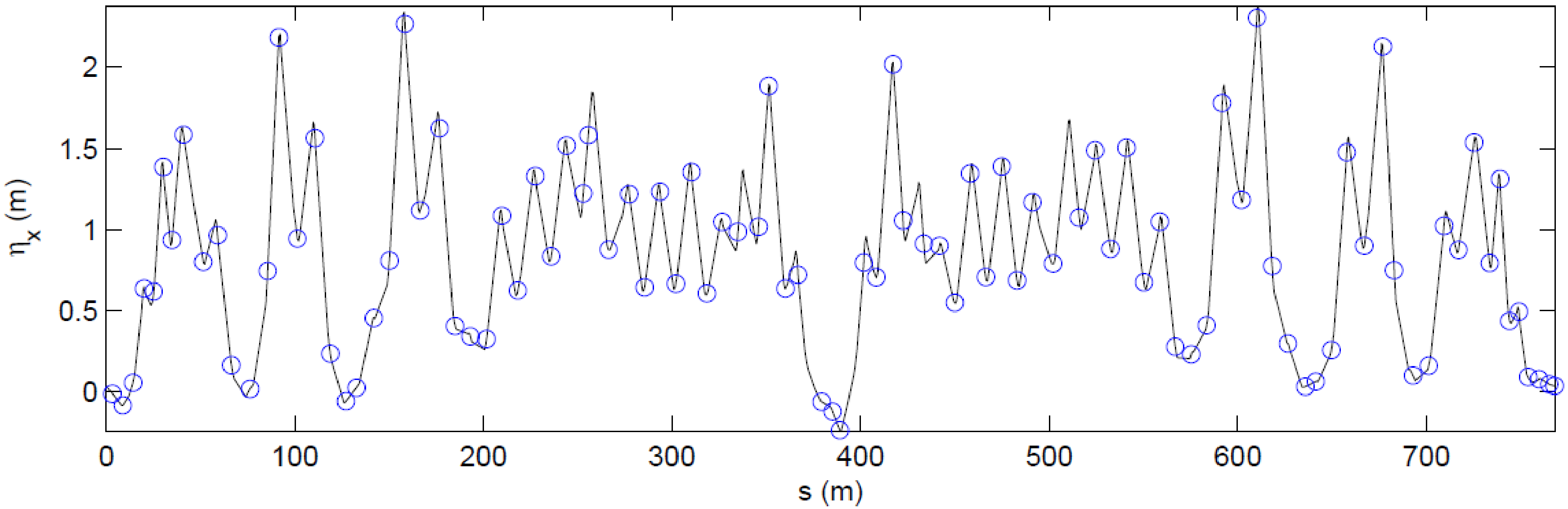
Results from Experimental Tests

- The measured mode II dispersion could not be well fitted using the available skew quadrupoles…
- …in this case, the effectiveness of the correction could be limited.



Potential Improvement: Weighting the BPMs

- Where the horizontal dispersion is large, a small amount of betatron coupling can lead to a large mode II dispersion…
- …but the mode II dispersion is only significant for the generation of emittance in the dipoles and insertion devices…
- … which is where the dispersion is smallest.
- Giving equal weight to all BPMs is not the right thing to do!



Horizontal dispersion in CesrTA design lattice. Circles indicate the positions of the BPMs.

Final Remarks

- Normal mode BPM calibration provides a fast and (at least in simulation) very effective technique for ultra-low emittance tuning in storage rings.
- Data collection (including calibration) and analysis is very fast: a **few minutes** at CesrTA.
- The technique is **insensitive to BPM gain and alignment errors**.
- The technique can be applied **as easily to a large ring as to a small ring**.
- Initial results at CesrTA look encouraging, but there is still some work needed to understand the full practical potential.
- Simulations for other machines (i.e. KEK-ATF) look similarly promising: it would be interesting to explore application to a collider… *SuperB*!