# Effective Theory Approach for Axion Wormholes

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Based on arXiv:2310.11260

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**@ PNU-IBS workshop on Axion Physics : Search for axions**

# Outline

Introduction to QCD Axion

- Strong CP problem, Properties of Axion

Axion quality issue and Giddings-Strominger wormhole solution

- Euclidean wormhole solution and its interpretation

Axion wormhole with axion radial scalar partners

- Complex scalar models
- Numerical results

Effective theory approach for axion wormholes

- General formula
- Analytic results for single axion models

  : Minimal/non-minimal couple to gravity, Metric/Palatini formalism, R2-term

Discussion

# Strong CP problem

In the SM, the gauge symmetry allows P & T violating "theta" term for strong sector

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \frac{g_s^2}{32\pi^2} \theta_0 G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + \cdots \qquad \left( \tilde{G}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma} \right)$$

c.f.) EM analogy: From Maxwell's equations, we know

Under Parity: $\vec{E} \to -\vec{E}$, $\vec{B} \to \vec{B}$ Under Charge Conjugation: $\vec{E} \to -\vec{E}$, $\vec{B} \to -\vec{B}$

Under Time reversal: $\vec{E} \to \vec{E}$, $\vec{B} \to -\vec{B}$ → $F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\,\vec{E} \cdot \vec{B}$ breaks P & T

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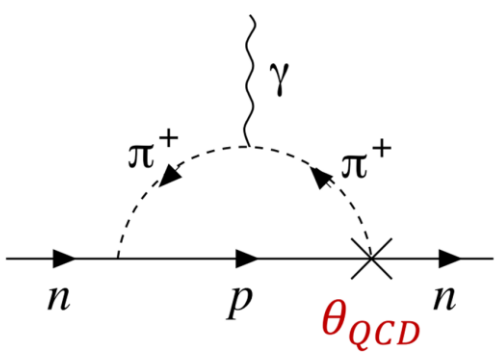
The low energy consequence: **Nucleon Electric Dipole Moment (EDM)**

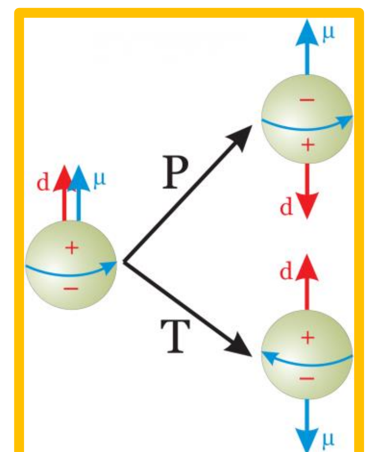
$$L_{eff} = \bar{N}(i\gamma^\mu D_\mu - m_N)N + \bar{N}\sigma^{\mu\nu}N\left(\mu_N F^{\mu\nu} + d_N \tilde{F}^{\mu\nu}\right) + \cdots \to H_{eff} \ni -\vec{S}_N \cdot \left( \frac{g_N e}{2m_N} \vec{B} + d_N \vec{E} \right)$$

$$d_n \simeq 2.4 \times 10^{-16} \theta_{QCD}\, e\, cm \sim \frac{\theta_{QCD}}{8\pi^2} \frac{e}{m_n} \qquad \left( \theta_{QCD} = \theta_0 + \underline{\mathrm{Arg}\,\det(Y_u Y_d)} \right)$$




$$|d_n^{exp}| < 1.8 \times 10^{-26} e\, cm$$
$$\to |\theta_{QCD}| < 10^{-10}$$

expected as O(1)



# Strong CP problem and Axion

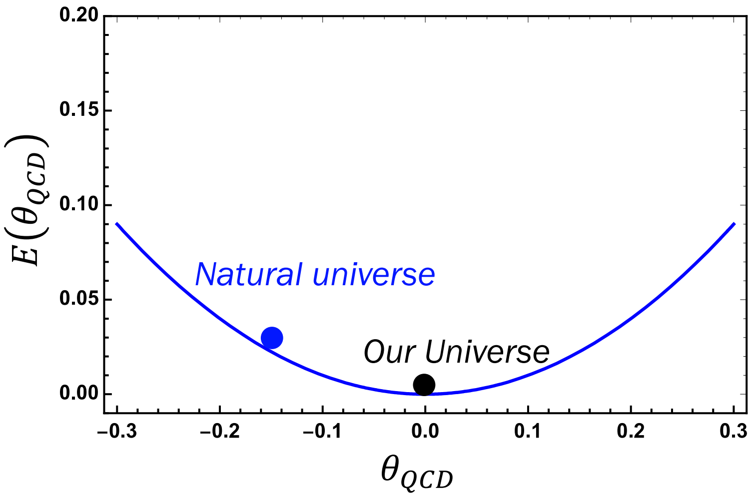
**Strong CP problem:** Why are P & T violating effects from the strong sector too small?

$$\left|\theta_{QCD} = \theta_0 + \mathrm{Arg}\,\det(Y_u Y_d)\right| < 10^{-10}$$

The axion is an elegant solution to this problem.

**Basic idea:** For the strong interaction with the theta term

The ground state energy depends on $\theta_{QCD}$ as $E(\theta_{QCD}) = E_0 + E_1\theta_{QCD}^2 + O(\theta_{QCD}^4)$




There is no reason why our Universe is at the strong CP invariant vacuum $(\theta_{QCD} = 0)$

Promoting the theta angle to the field (axion)!

$$\theta_{QCD} \to \frac{a(x)}{f_a} \equiv \frac{a(x)}{f_a} + 2\pi$$

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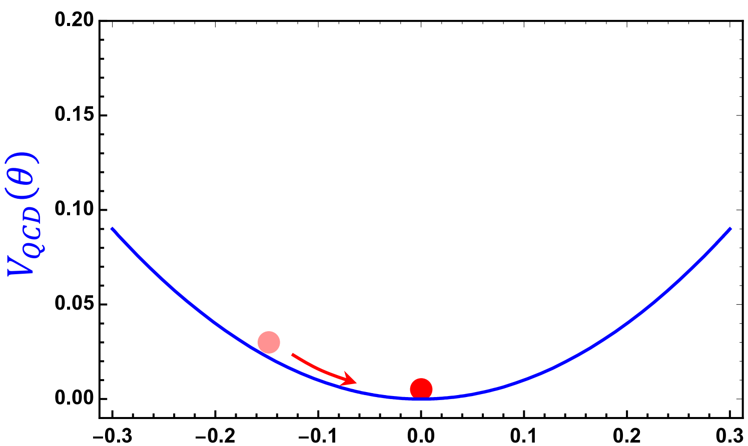



$\theta(x) = \dfrac{a(x)}{f_a}$

The theta dependent ground state energy becomes a scalar potential of the axion

$$\theta_{QCD} \to \theta(x) = \frac{a(x)}{f_a}: \quad E(\theta_{QCD}) \to V_{QCD}(\theta)$$

➔ Dynamically evolves to the strong CP conserving vacuum: $\theta_{QCD} = \langle\theta\rangle = 0$

*Peccei, Quinn PRL 38:1440,*
*Weinberg PRL 40:223,*
*Wilczek PRL 40:279*

# Properties of QCD Axion

The QCD axion not only solves the strong CP problem, but also becomes a promising dark matter candidate, leading to a rich axion phenomenology.

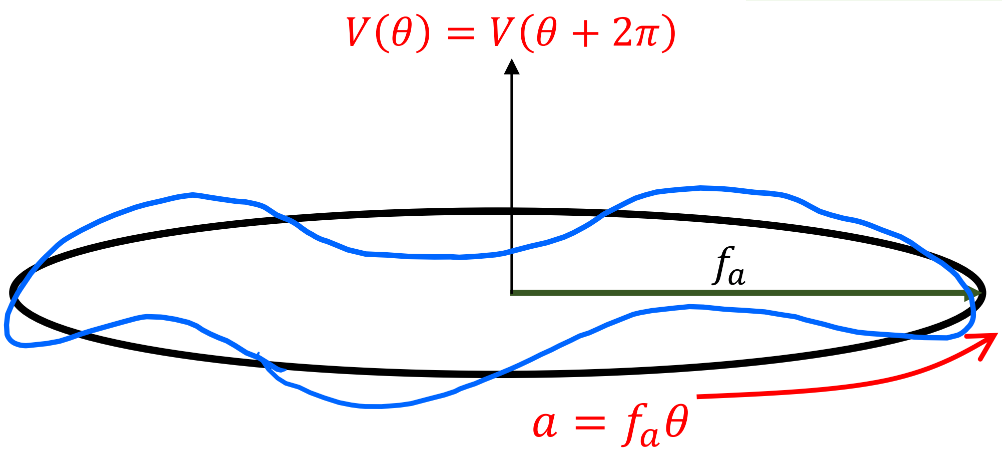
$$\mathcal{L}\left(\mu > \Lambda_{QCD}\right) = \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{c_\gamma e^2}{16\pi^2}\frac{a}{f_a}\vec{E}\cdot\vec{B} + \cdots$$

$$\Downarrow$$

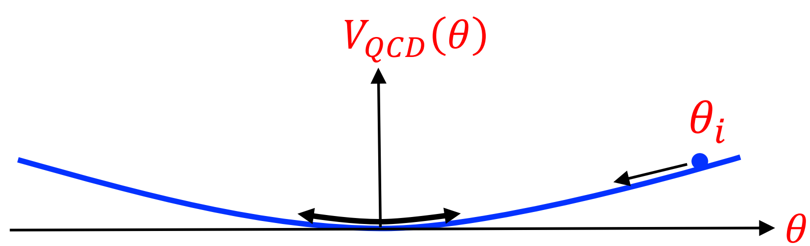
$$\mathcal{L}_{eff}\left(\mu \ll \Lambda_{QCD}\right) = \frac{1}{2}\left(\partial_\mu a\right)^2 - \frac{1}{2}m_a^2 a^2 + \frac{\tilde{c}_\gamma e^2}{16\pi^2}\frac{a}{f_a}\vec{E}\cdot\vec{B} + \cdots$$

$$m_a \simeq \mu\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)$$

*JE Kim PRL 43:103,*
*Shifman et.al. NPB 166:493*
*Dine, et.al. PLB 104:199m,*
*Zhitnitsky Sov.J.Nucl.Phys. 31:260*
*Abbott, Sikivie, PLB 120 (1–3): 127–132*
*...*




**Axion DM**




$$\Omega_a \simeq 0.25\left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{\frac{7}{6}}\left(\theta_i^2\right)$$

# Axion Quality Issue

**However**, the axion solution to the strong CP problem only works when there is no additional source of axion potential (i.e. the breaking terms for shift symmetry of the axion : $\theta \to \theta + \mathbb{R}$).

$$\mathcal{L}\left(\mu > \Lambda_{QCD}\right) = \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G_{\mu\nu}\tilde{G}^{\mu\nu} + \Delta V_{new}(\theta) + \frac{c_\gamma e^2}{16\pi^2}\frac{a}{f_a}\vec{E}\cdot\vec{B} + \cdots$$

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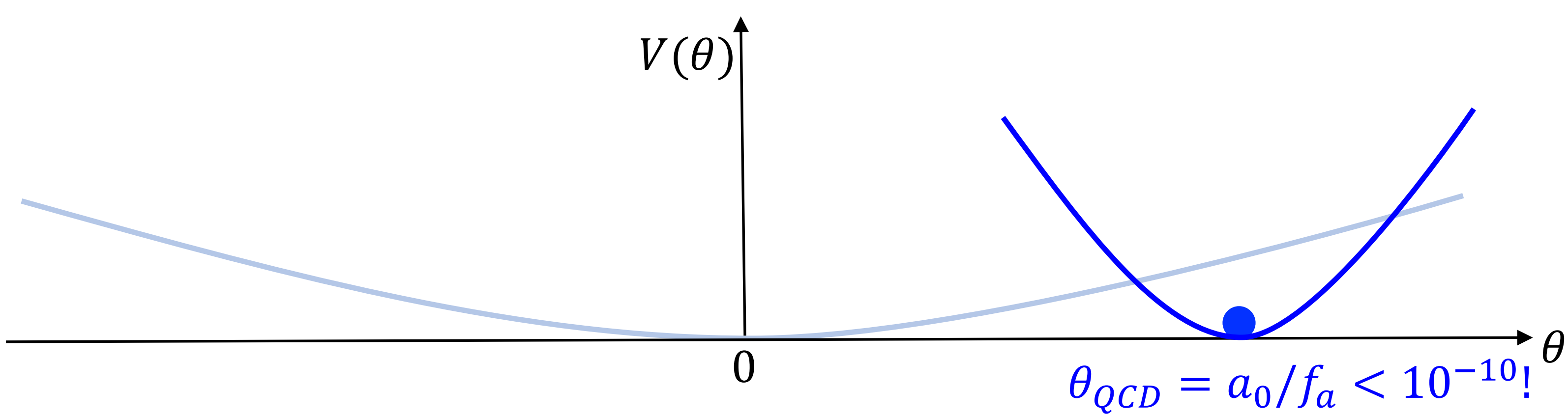
$$m_a \simeq \mu\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right) + \Delta_{new} m_a$$



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Why is it the problem?

Because it is expected that **quantum gravity** would not allow any global symmetry like

Peccei-Quinn Symmetry $U(1)_{PQ}$: $e^{i\theta} \rightarrow e^{i\mathbb{R}}e^{i\theta}$

But it is also difficult to quantify the effect of quantum gravity. For instance, any perturbative quantum field calculations that include gravitons do not give the axion potential.

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**One may argue that existence of instanton (e.g. from axion-gluon-gluon coupling) that satisfies the condition**

$$S_{inst} = \frac{8\pi^2}{g_s^2} < c\,\frac{M_P}{f_a} \qquad (c = O(1))$$

**itself is the consequence of quantum gravity (weak gravity conjecture). Here we will only focus on more explicit effects (calculable part) of gravity in 4D theory.**

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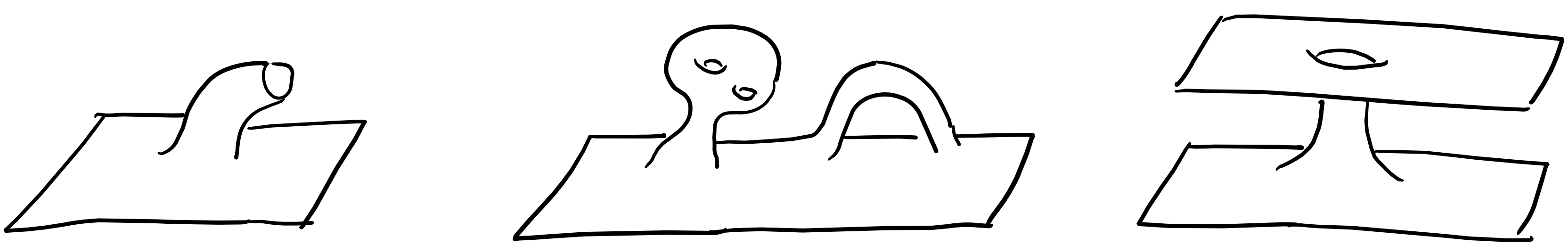
**The nonperturbative effects related to gravity are important.**



# Giddings-Strominger Wormhole

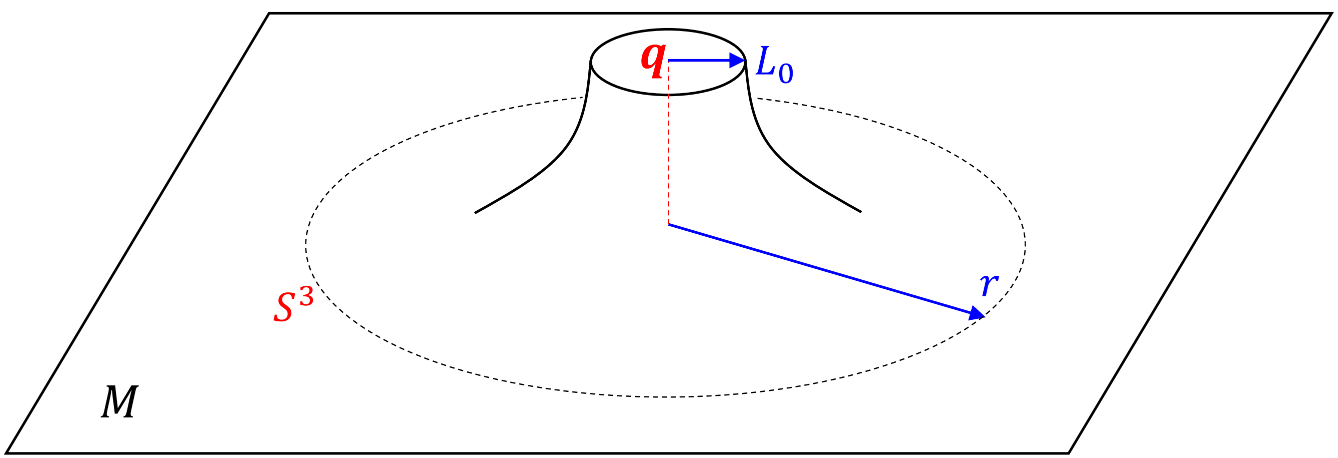
*Giddings, Strominger, NPB 306, 890 (1988)*

The interesting example where the axion's PQ charge is taken away by the Euclidean wormhole

$$ds_{GS}^2 = g_{rr}dr^2 + r^2 d\Omega^2 = \left(\frac{1}{1-(L_0/r)^4}\right)dr^2 + r^2 d\Omega_3^2$$

**The PQ charge of half Euclidean wormhole**

$$\boldsymbol{q} = \int_{\partial_{out}M} f_a^2 \star d\theta_E$$



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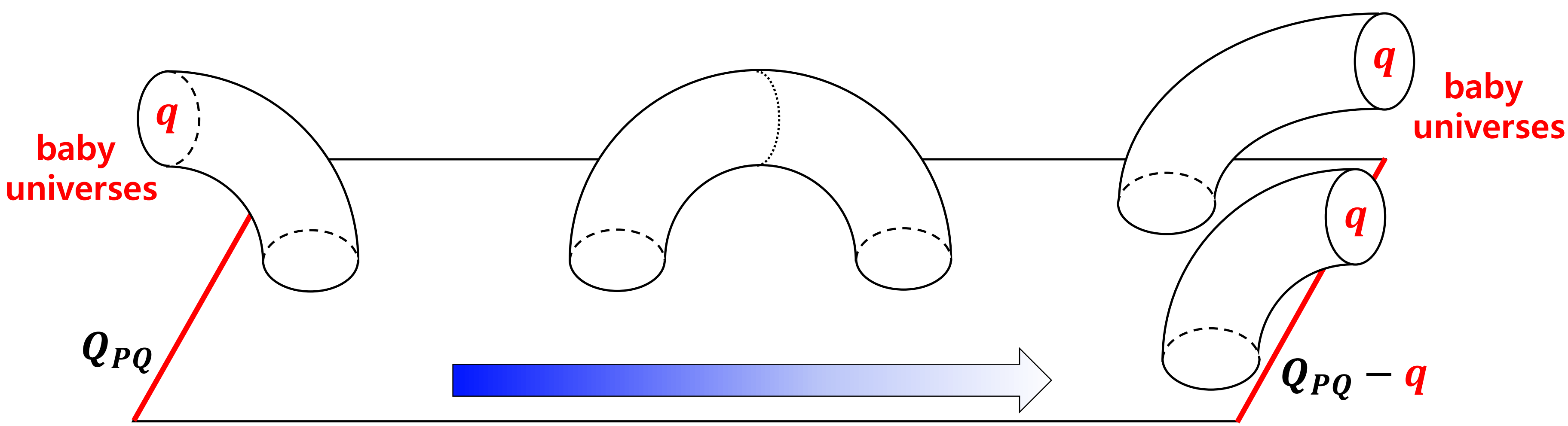
This may lead to the PQ breaking axion potential like instantons, whose contribution is suppressed by the Euclidean action of the half wormhole $S_{wh}$:

$$\Delta V_{wh}(\theta) \sim e^{-S_{wh}[q]} \cos(q\theta + \delta_0)$$

The contribution can be safely ignored for the strong CP problem when $S_{wh} > O(200)$.




*Review about Euclidean wormholes and baby universes : Hebecker, Mikhail, Soler [1807.00824]*

# Giddings-Strominger Wormhole

The Lagrangian density of the axion and gravitational field with the PQ symmetry:

$$\mathcal{L} = \sqrt{-g}\left(-\frac{M_P^2}{2}R + \frac{1}{2}f_a^2\left(\partial_\mu\theta\right)^2\right)$$

One can consider the axion as a gauge field ($\theta \to \theta + 2\pi\mathbb{Z}$). The gauge field nature is more transparent in dual picture using a 3-form gauge field strength $H_{\mu\nu\rho}$ like the EM duality

$$\mathcal{L} = \sqrt{-g}\left(-\frac{M_P^2}{2}R + \frac{1}{12f_a^2}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{1}{6}\theta\epsilon^{\sigma\mu\nu\rho}\partial_\sigma H_{\mu\nu\rho}\right)$$

$$H = dB = f_a^2 \star d\theta \quad \Rightarrow \quad H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} = f_a^2\partial_\gamma\theta\, g^{\gamma\sigma}\,\epsilon_{\sigma\mu\nu\rho}$$

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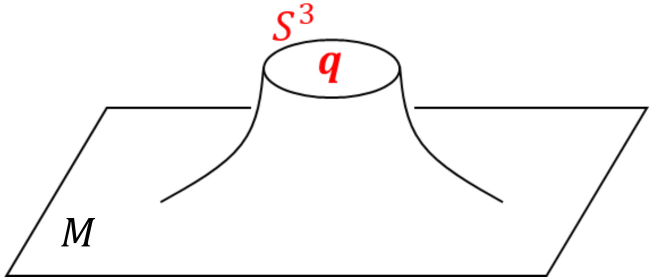
In the wormhole metric with the Euclidean signature, the instanton effects become

$$\exp(-S_{wh}[q] + i\theta q) = \exp\left(-\int d^4x\sqrt{g_E}\left(-\frac{M_P^2}{2}R + \frac{1}{12f_a^2}H_{\mu\nu\rho}H^{\mu\nu\rho}\right) + i\theta\,\frac{1}{6}\int_{\partial_{out}M} d^3s_E^{\mu\nu\rho}H_{\mu\nu\rho}\right)$$

$$\boxed{q = \int_{\partial_{out}M} H = \int_{S^3} f_a^2 \star d\theta_E}$$



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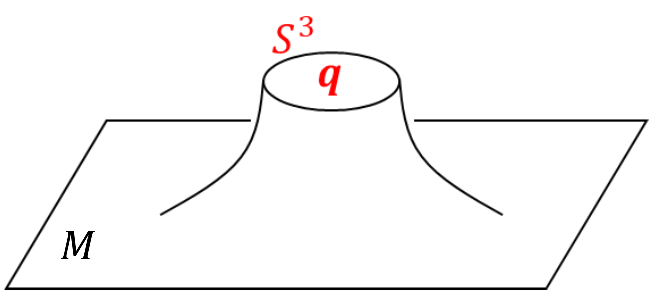
**The PQ charge** plays a crucial role in **maintaining the wormhole throat with a finite size**.



# Giddings-Strominger Wormhole

The Euclidean half wormhole action is given by

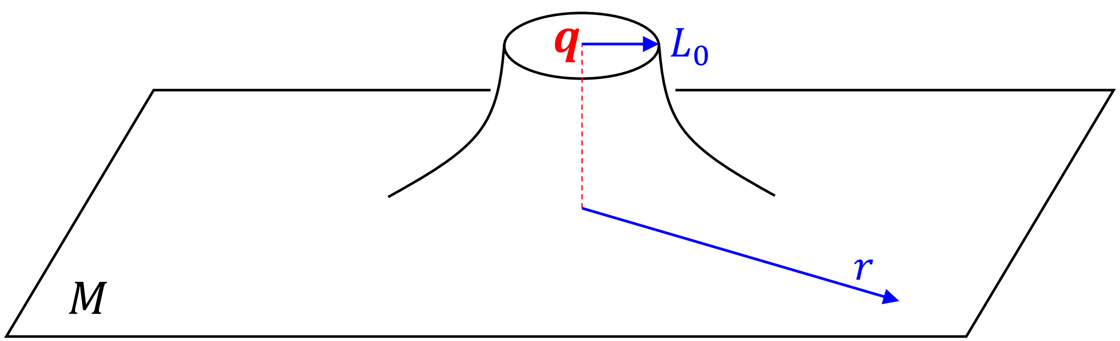
$$S_{wh}[q] = 3\pi^3 M_P^2 L_0^2 = \frac{\pi\sqrt{6}}{4}\frac{qM_P}{f_a}$$

(+the Gibbons-Hawking-York boundary term)

where the wormhole throat radius $L_0$ is

$$L_0 = \left(\frac{q^2}{24\pi^3}\right)^{1/4} \frac{1}{\sqrt{M_P f_a}}$$

This implies when $f_a < 10^{16}\mathrm{GeV}$, the QCD axion quality seems well maintained.



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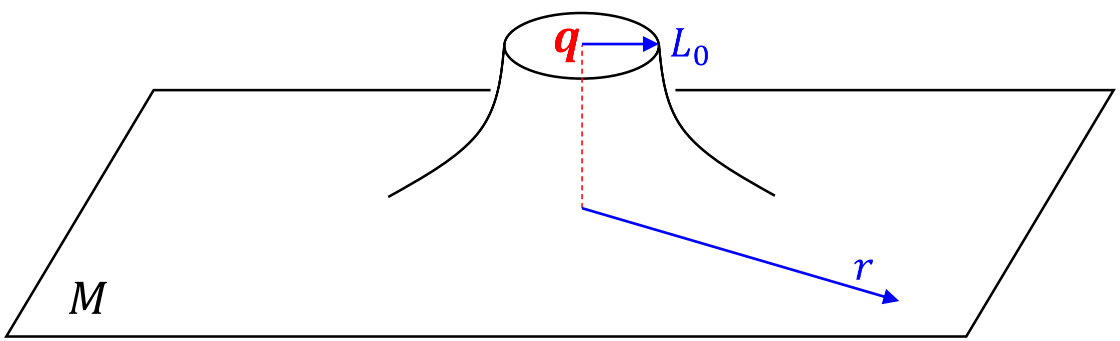



HOWEVER, this may not be realistic for axion models that have radial scalar partners with masses much smaller than $\sqrt{M_P f_a}$. This is because **their dynamics can change the axion decay constant around the wormhole throat significantly from vacuum expectation**.

# Complex Scalar Models

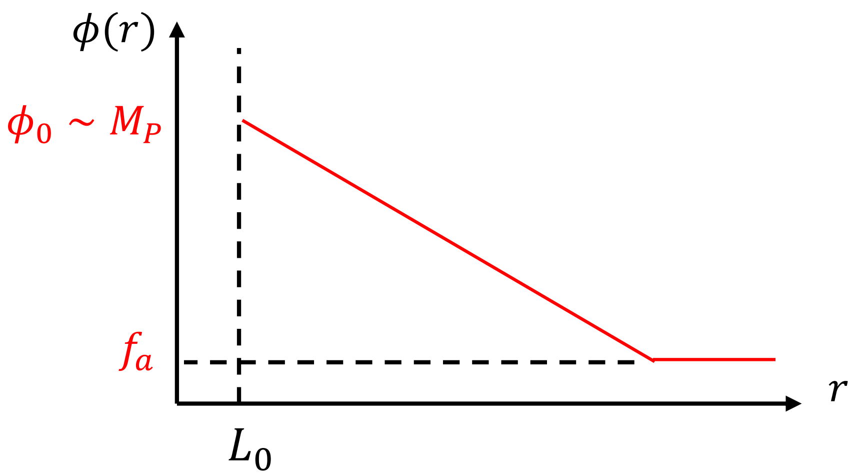
*K.-M. Lee PRL 61, 263 (1988), Kallosh, Linde, Linde, Susskind PRD 52, 912 (1995)*

When the axion is coming from the phase field of the complex scalar

$$\Phi(x) = \frac{1}{\sqrt{2}}\phi(x)e^{i\theta(x)}$$

$$\mathcal{L} = \left|\partial_\mu \Phi\right|^2 - \lambda\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2 = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{1}{2}f_a^2(\phi)\left(\partial_\mu \theta\right)^2 - V(\phi)$$

The field dependent axion decay constant is $f_a(\phi) = \phi$ and $m_\phi = \sqrt{\lambda}f_a \ll \sqrt{M_P f_a}$, $f_a = \langle\phi\rangle$




the wormhole solution of $\phi(r)$

$$L_0 \sim \frac{1}{\sqrt{M_P f_a(\phi_0)}} \sim \frac{1}{\sqrt{M_P \phi_0}} \sim \frac{1}{M_P}$$

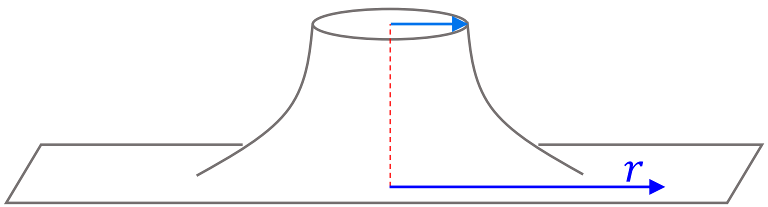
Equations of motions give $f_a(\phi_0) = \phi_0 \sim M_P \gg f_a$, so the throat radius is dramatically reduced.

# Complex Scalar Models

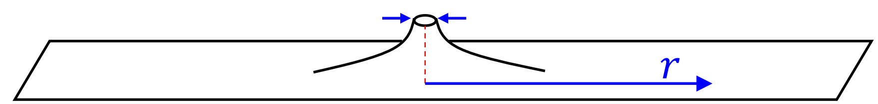
*K.-M. Lee PRL 61, 263 (1988), Kallosh, Linde, Linde, Susskind PRD 52, 912 (1995)*

The corresponding wormhole action is also quite suppressed.

$$L_{GS} \sim \frac{1}{\sqrt{M_P f_a}}$$




$$L_0 \sim \frac{1}{\sqrt{M_P f_a(\phi_0)}} \sim \frac{1}{M_P} \ll L_{GS}$$




$$S_{GS} = S_{wh}^{UV} \sim \frac{M_P}{f_a} \gg 1$$

$$S_{wh} = S_{wh}^{UV} + S_{wh}^{IR} \sim O(1) + \ln \frac{M_P}{f_a}$$

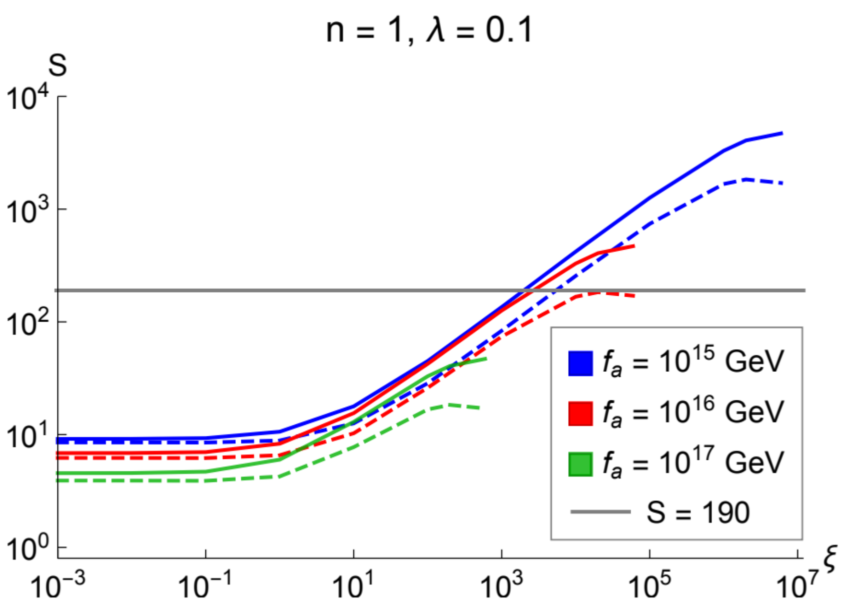
A small wormhole action implies that global symmetry can be easily disrupted by quantum gravity effects. But it also implies that the contribution is highly dependent on the UV physics.

# Nonminimally Coupled Complex Scalar

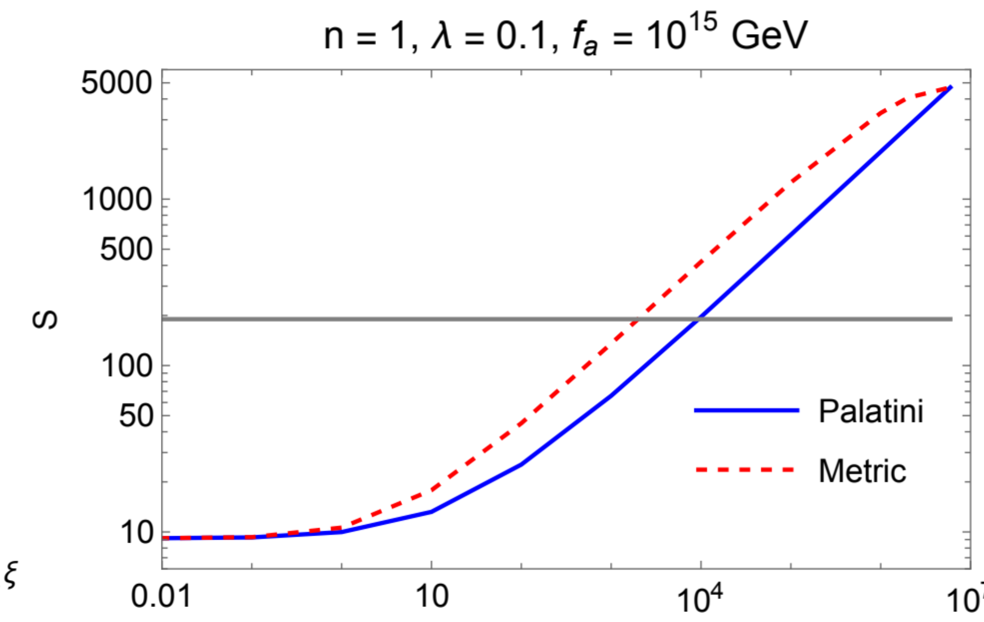
One interesting idea focusing on a role of a large nonminimal coupling to gravity

$$\Delta\mathcal{L} = -\xi|\Phi|^2 R$$

is proposed by K. Hamaguchi et.al. [Metric] (2108.13245) and D.Y. Cheong et.al. [Palatini] (2210.11330). By solving the equations of motion & Einstein field equations numerically, it is shown that a wormhole with a large throat size are obtained. For $\xi > O(10^4)$, $S_{wh} > O(200)$.




**2108.13245**




ξ **2210.11330**

Why does such $\xi$-dependence appear in the wormhole action?

# Nonminimally Coupled Complex Scalar

The transformation from Jordan frame to Einstein frame gives

$$\mathcal{L}_J = -\frac{1}{2}M_P^2 R - \xi|\Phi|^2 R + \left|\partial_\mu \Phi\right|^2 - V_J(\Phi)$$

$$\Rightarrow \quad \mathcal{L}_E = -\frac{1}{2}M_P^2 R + \frac{\alpha \xi^2 \left(\partial_\mu |\Phi|^2\right)}{2(1+2\xi|\Phi|^2/M_P^2)^2} + \frac{\left|\partial_\mu \Phi\right|^2}{(1+2\xi|\Phi|^2/M_P^2)} - \frac{V_J(\Phi)}{(1+2\xi|\Phi|^2/M_P^2)^2}$$

$$\alpha = 6 \quad \left(\text{Metric formalism: } \mathcal{L} = \mathcal{L}\left(g_{\mu\nu}, \partial g_{\mu\nu}, \partial^2 g_{\mu\nu}\right)\right)$$

$$= 0 \quad \left(\text{Palatini formalism: } \mathcal{L} = \mathcal{L}\left(g_{\mu\nu}, \partial \Gamma^{\mu}_{\nu\rho}\right)\right)$$

# Nonminimally Coupled Complex Scalar

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Using $\Phi = \phi e^{i\theta}/\sqrt{2}$

$$\mathcal{L}_E = -\frac{1}{2}M_P^2 R + \frac{1}{2}G(\phi)\left(\partial_\mu\phi\right)^2 + \frac{1}{2}f_a^2(\phi)\left(\partial_\mu\theta\right)^2 - V_E(\phi)$$

where

$$f_a(\phi) = \frac{\phi}{\sqrt{1+\xi\phi^2/M_P^2}} \qquad \rightarrow \ \lim_{\phi\to\infty} f_a(\phi) = \frac{M_P}{\sqrt{\xi}}$$

Around the wormhole throat, $\phi_0 \gtrsim M_P/\sqrt{\xi}$, so $f_a(\phi_0) \simeq M_P/\sqrt{\xi}$.

This implies that the dynamics of radial scalar is **decoupled** from the axion in the UV regime, and the UV contribution $S_{wh}^{UV} \sim M_P/f_a(\phi_0)$ becomes

$$S_{wh}^{UV} \sim \sqrt{\xi}$$

# Effective Theory Approach

[D.Y. Cheong, S.C. Park, CSS 2310.11260]

Note that the UV contribution (near the throat) is quite independent from IR physics for complex scalar models, i.e. independent from the potential of the radial scalar. Therefore, it is more natural to think that the wormhole contribution gives **Peccei-Quinn Violating Effective Local Operators of $\Phi$** in the IR regime, $\mu_{IR} \ll 1/L_0$.

$$S_{wh}[q, f_a] \to S_{wh}[q, \phi] \qquad \phi = \phi_{IR} \equiv \phi(r \gg L_0), \qquad \nabla\phi_{IR} \ll \frac{\phi_{IR}}{L_0}$$

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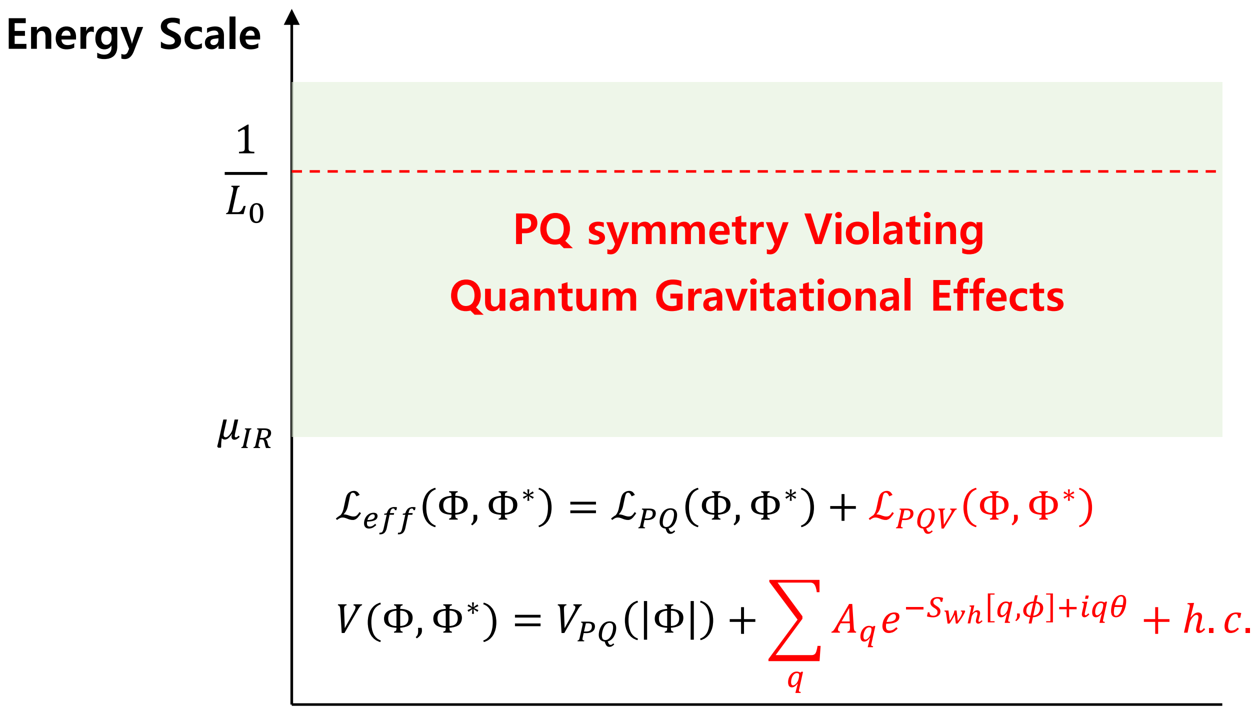

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Solving the equations of motion for an axion wormhole in the massless limit of a radial scalar leads to a significantly simplified analysis. This allows us to **derive analytic expressions**, applicable for $f_a$ much smaller than $M_P$.

In the Einstein frame, the general action

$$S = \int dx\sqrt{|g|}\left(-\frac{1}{2}M_P^2 R + \frac{1}{2}G_{AB}(\phi)\partial_\mu\phi^A\partial^\mu\phi^B + \frac{1}{2}\left(f_a^2(\phi)\right)_{IJ}\left(\partial_\mu\theta^I\right)\left(\partial^\mu\theta^J\right)\right)$$

$$S_{\text{dual}} = \int dx\sqrt{|g|}\left(-\frac{1}{2}M_P^2 R + \frac{1}{2}G_{AB}(\phi)\partial_\mu\phi^A\partial^\mu\phi^B + \frac{1}{12}\left(f_a^{-2}(\phi)\right)^{IJ} H_{I\mu\nu\rho}\, H_J^{\mu\nu\rho}\right)$$

$$H_{I\mu\nu\rho} = \left(f_a^2(\phi)\right)_{IJ}\partial_\alpha\theta^J g^{\alpha\beta}\,\epsilon_{\beta\mu\nu\rho}$$

# Effective Theory Approach

[D.Y. Cheong, S.C. Park, CSS 2310.11260]

For the O(4) symmetric Euclidean wormhole metric

$$ds^2 = \frac{1}{(1 - L_0^4/r^4)} dr^2 + r^2 d\Omega^2 \qquad (R = -6L_0^4/r^6)$$

the Euclidean action

$$S_E = \int d^4x \sqrt{g_E} \left( -\frac{1}{2} M_P^2 R + \frac{1}{2} G_{AB}(\phi) \partial_\mu \phi^A \partial^\mu \phi^B + \frac{1}{12} \left( f_a^{-2}(\phi) \right)^{IJ} H_{I\mu\nu\rho} H_J^{\mu\nu\rho} \right)$$

with the PQ charge quantization for the wormhole

$$\int_{S^3} H_I = q_I$$

and the new variable $\tau$ ($r = L_0 \Rightarrow \tau = 0,\ r = \infty \Rightarrow \tau = \tau_\infty = 1/2\pi L_0^2$)

$$\tau = \frac{1}{4\pi^2 L_0^2} \tan^{-1}\left( \sqrt{r^4/L_0^4 - 1} \right)$$

becomes

$$S_{wh} = \int_0^{\tau_\infty} d\tau \left( 12\pi^4 M_P^2 L_0^4 + \frac{1}{2} G_{AB}(\phi) \frac{d\phi^A}{d\tau} \frac{d\phi^B}{d\tau} + \frac{1}{2} \left( f_a^{-2}(\phi) \right)^{IJ} q_I q_J \right)$$

# Effective Theory Approach

[D.Y. Cheong, S.C. Park, CSS 2310.11260]

The general formula of the half wormhole action can be written as

$$S_{wh}[q,\phi_{IR}] = \int_0^{\tau_\infty} d\tau \left(f_a^{-2}(\phi)\right)^{IJ} q_I q_J = 3\pi^3 M_P^2 L_0^2 + \int_{\phi_0}^{\phi_{IR}} d\phi^A p_A(\phi)$$

where $\phi_0 = \phi(0), \phi_{IR} = \phi(\tau_\infty)$, and $p_A = G_{AB}(\phi)\, d\phi^B/d\tau$.

For various examples, we can identify **UV** and **IR** contributions of the wormhole action, and their analytic structure from the equations of motion for $\phi^A$

$$G_{AB}(\phi)\frac{d^2\phi^B}{d\tau^2} + \Gamma_{ABC}(\phi)\frac{d\phi^B}{d\tau}\frac{d\phi^C}{d\tau} = \frac{\partial}{\partial\phi^A}\left(\frac{1}{2}(f_a^{-2}(\phi))^{IJ} q_I q_J\right)$$

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The UV boundary condition at the throat ($\tau = 0$) gives a relation between the throat size and the axion decay constant at that position

$$12\pi^4 M_P^2 L_0^4 = \frac{1}{2}\left(f_a^{-2}(\phi_0)\right)^{IJ} q_I q_J$$

The IR field dependence ($\tau = \tau_\infty$) of the wormhole action can be clarified in our approach

$$S_{wh}[q, \phi_{IR}]$$

# A Complex Scalar with a Nonminimal Coupling

[D.Y. Cheong, S.C. Park, CSS 2310.11260]

The more simplified analytic formula can be obtained for single axion models. For instance,

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} M_P^2 R - \xi |\Phi|^2 R + \left| \partial_\mu \Phi \right|^2 \right)$$

the wormhole action can be organized by the form

$$S_{wh}[q, \phi] = q \left( S_\xi + \ln \frac{\Lambda_\xi}{\phi} \right)$$

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the wormhole action can be organized by the form

$$S_{wh}[q,\phi] = q\left(S_\xi + \ln\frac{\Lambda_\xi}{\phi}\right)$$

where the near throat contribution $S_\xi = c_0 + c_1\sqrt{\xi}$ is given by

$$\begin{aligned} S_\xi &= \ln\left(\pi\sqrt{3/2}\right) && (\xi \ll 1) \\ &= \ln(\sqrt{2/3}) + \frac{\pi\sqrt{30}}{4}\sqrt{\xi} && (\xi \gg 1, \mathrm{Metric}) \\ &= \ln 2 + \frac{\pi\sqrt{6}}{4}\sqrt{\xi} && (\xi \gg 1, \mathrm{Palatini}) \end{aligned}$$

and the IR contribution is managed by **the perturbative cut-off in the vacuum, $\Lambda_\xi$**:

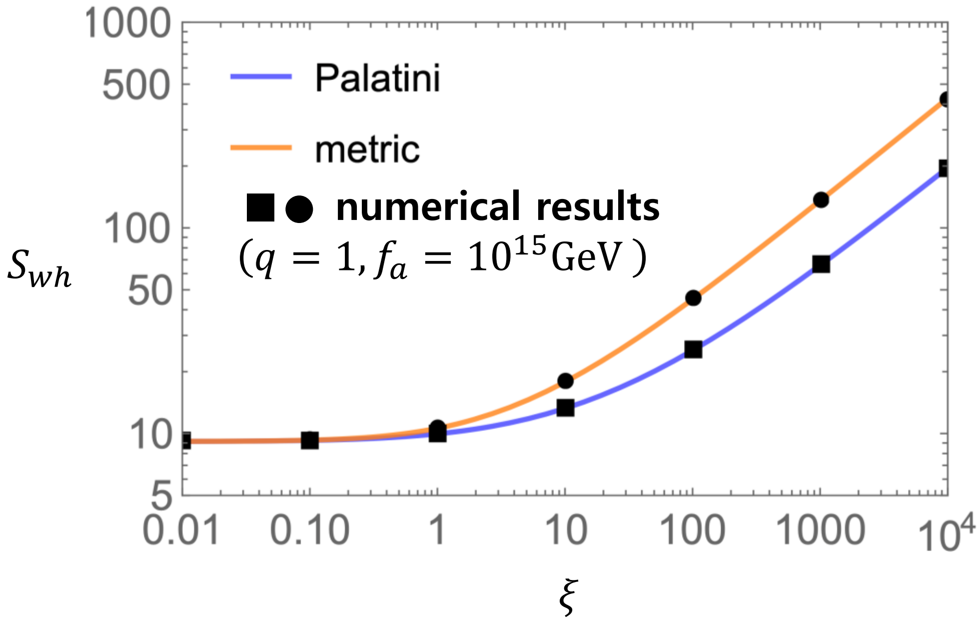
$$\Lambda_\xi = M_P \ (\xi \ll 1) \qquad \frac{M_P}{\xi}\ (\xi \gg 1, \mathrm{Metric}) \qquad \frac{M_P}{\sqrt{\xi}}\ (\xi \gg 1, \mathrm{Palatini})$$

# Analytic Form of the Axion Wormhole Action

[D.Y. Cheong, S.C. Park, CSS 2310.11260]

The analytic formula precisely reproduces the numerical results.

$$S_{wh}[q,\phi] = q\left(S_\xi + \ln\frac{\Lambda_\xi}{\phi}\right)$$




This gives the effective PQV potential at IR regime

$$\Delta V_{PQV}(\Phi,\Phi^*) = A_q \exp(-S_{wh}[q,\phi] + iq\theta) + h.c. = A_q\left(e^{-S_\xi}\frac{\Phi}{\Lambda_\xi}\right)^q + A_q^*\left(e^{-S_\xi}\frac{\Phi^*}{\Lambda_\xi}\right)^q$$

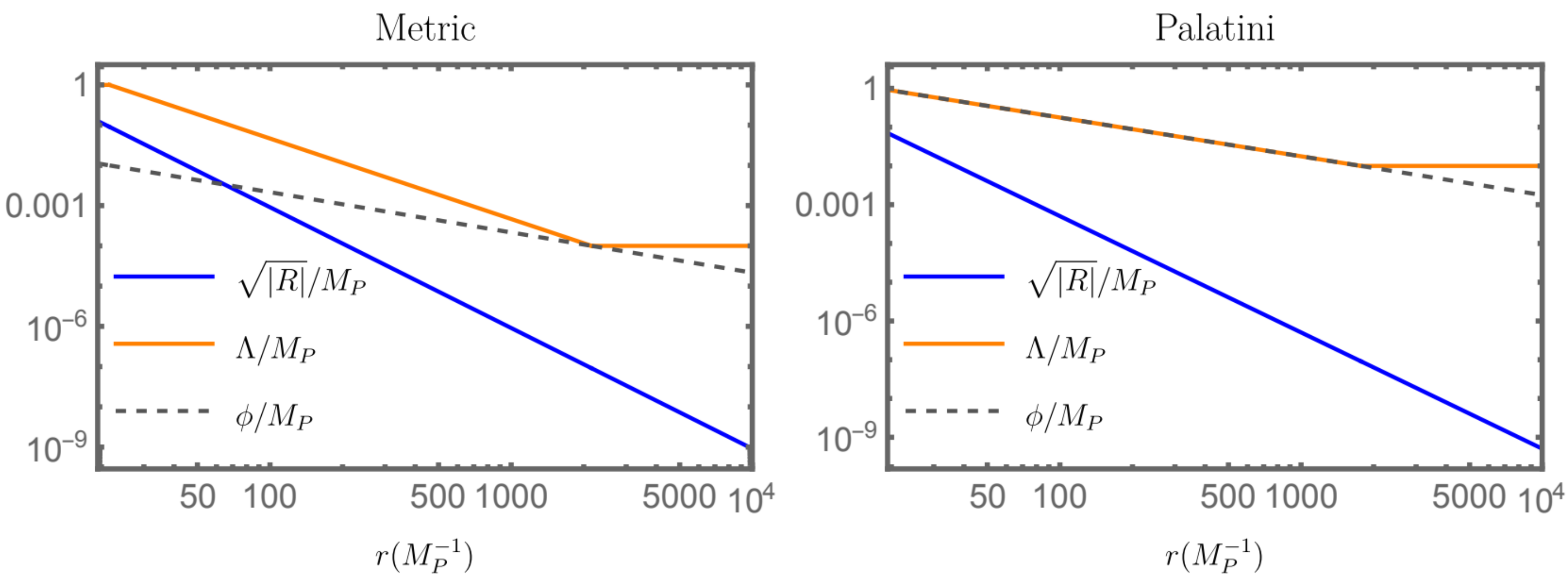
**The** **holomorphic** **nature of the wormhole contribution to local operators** becomes manifest.

# Local Perturbative Cut-off Scales

[D.Y. Cheong, S.C. Park, CSS 2310.11260]

For a large value of $\xi$, the perturbative cut-off depends on the background field value.

We can compare the typical energy scale (curvature scale) of the wormhole background with the perturbative cut-off at each position for Metric and Palatini formalisms




The solutions are safe from the perturbative Unitarity criteria (U(1) symmetries for axions)

# A Complex Scalar + $R^2$ term (Scalaron)

[D.Y. Cheong, S.C. Park, CSS 2310.11260]

One can also study the case that $R^2$ term is added.

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}M_P^2 R - \xi|\Phi|^2 R + \left|\partial_\mu \Phi\right|^2 + \frac{\xi_s}{4}R^2\right)$$

In metric formalism, $R^2$ introduces another scalar degree of freedom $\chi$. In the Einstein frame,

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}M_P^2 R + e^{2\beta\chi/M_P}\left|\partial_\mu \Phi\right|^2 + \frac{1}{2}\left(\partial_\mu \chi\right)^2 - U(\chi, |\Phi|)\right)$$

with $\beta = 1/\sqrt{6}$.

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with $\beta = 1/\sqrt{6}$. The potential

$$U(\chi,|\Phi|) = \frac{M_P^4}{4\xi_s}\left[1 - e^{2\beta\chi/M_P}\left(1 + \frac{2\xi|\Phi|^2}{M_P^2}\right)\right]^2$$

gives a mass of $\chi$ as $m_\chi \sim M_P/\sqrt{\xi_s}$ in the vacuum.

**When $\xi_s \to 0$, $\chi$ is frozen $\partial_\chi U = 0$, i.e. it becomes a complex scalar model.**

$$S_{wh} \simeq q\left(\sqrt{\xi} + \ln\frac{\Lambda_{\xi_\phi}}{\phi}\right) \;\Rightarrow\; S_{wh}^{UV} = O\left(\sqrt{\xi}\right)$$

**While when $\xi_s \to \infty$, $\chi$ becomes massless $U = 0$,**

$$S_{wh} \simeq q\ln\left(\frac{M_P}{e^{\beta\chi/M_P}\phi}\right) \;\Rightarrow\; S_{wh}^{UV} = O(1)$$

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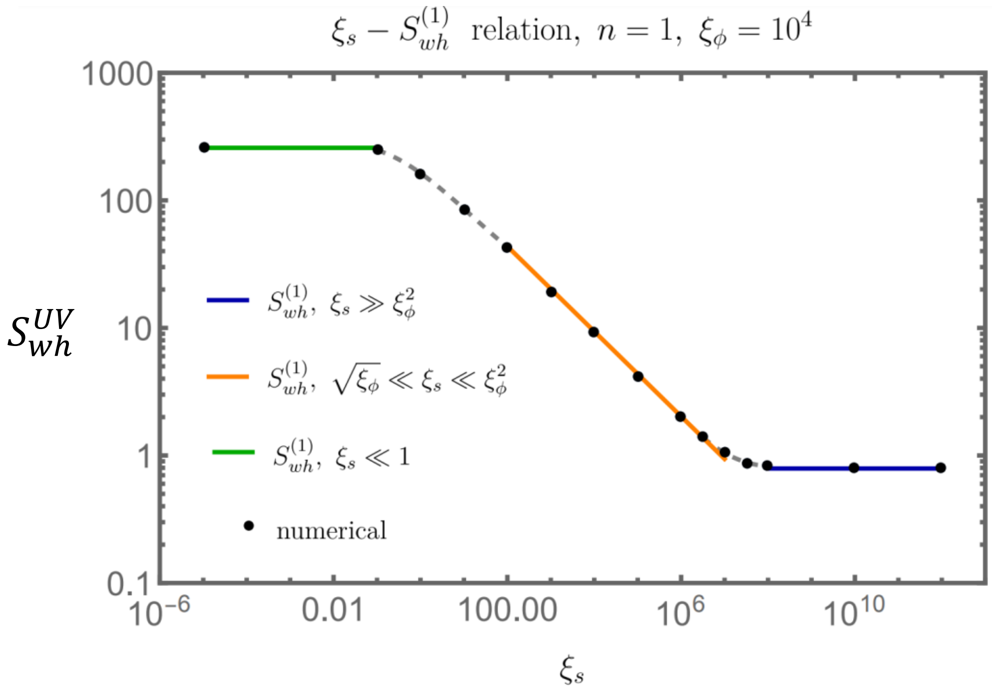
Depending on the relative hierarchy between $\xi_s$ and $\xi_\phi^2$, the wormhole action values are very different. **The effect of $\xi$ becomes screened for a large value of $\xi_s$.**



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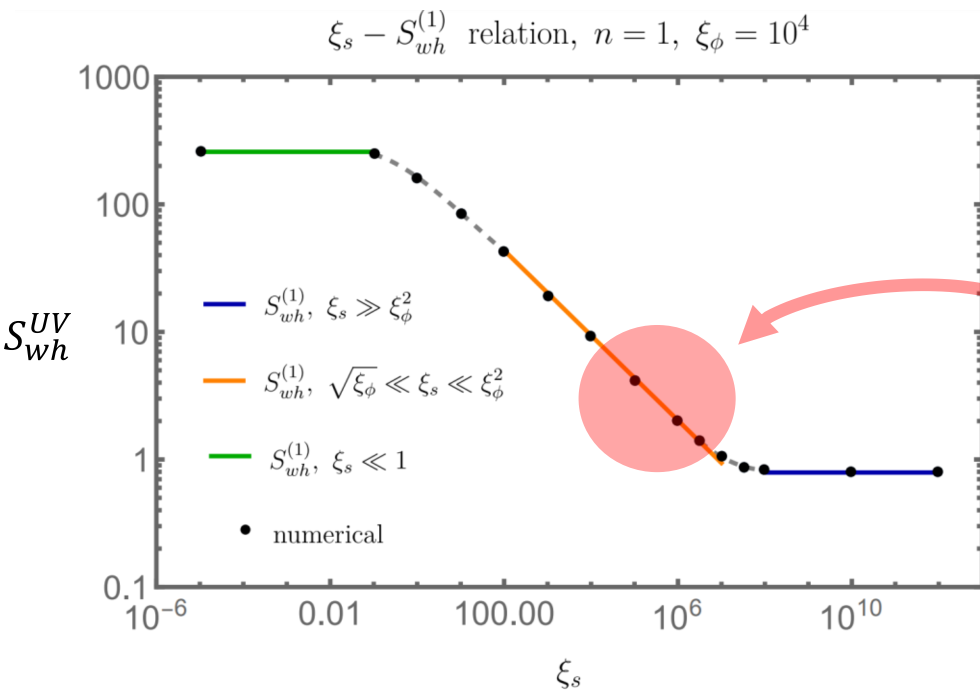
Depending on the relative hierarchy between $\xi_s$ and $\xi_\phi^2$, the wormhole action values are very different. **The effect of $\xi$ becomes screened for a large value of $\xi_s$.** In more realistic cases, $\xi_s$ is generated as proportional to $\xi^2$, so it may be challenging to solve the axion quality problem by introducing a large nonminimal coupling to gravity only.




$$\mu\frac{d\xi_s}{d\mu} = -\frac{1}{4\pi^2}\left(\xi + \frac{1}{6}\right)^2$$

$$\Delta\xi_s \sim 0.02\,\xi^2$$

# Discussion

The axion wormhole action serves as a fascinating illustration that directly reveals quantum gravitational effects of global symmetry breaking through semi-classical calculations.

The actual value of wormhole action can be very different depending on the axion's radial partner and their interactions. In our analysis, we adopt the effective theory approach for the axion wormhole, which yields analytic formulae allowing us to probe more realistic parameter space of axion.

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The actual value of wormhole action can be very different depending on the axion's radial partner and their interactions. In our analysis, we adopt the effective theory approach for the axion wormhole, which yields analytic formulae allowing us to probe more realistic parameter space of axion.

Of particular significance is the role played by nonminimal coupling to gravity, which results in a substantial amplification of the wormhole action. This amplification offers a potential solution to the axion quality problem and could also be relevant to inflationary models. But it still has a potential problem due to the R2 corrections.

Here, we didn't consider any 4D theory cut-off dependence of the wormhole action that can provide the uncertainty of $\Delta S_{wh} \sim M_P^2/\Lambda_{4D}^2$. The axion quality issue could be related with the low cut-off of 4D theory, although it is not predictable at this moment.