

# Probing axion coupling to electrons by Chiral Magnetic Effects

Sang Hui Im (IBS-CTPU)

DK Hong, SHI, KS Jeong, D Yeom, 2207.06884

PNU-IBS workshop on Axion Physics: Search for axions

Dec 6th, 2023, Busan, Korea

# Outline

- QCD axion and ALPs
- Experimental test of underlying UV physics for axions and importance of measuring the axion-electron coupling
- Chiral Magnetic Effect (CME)
- Probing the axion-electron coupling by CMEs

# Strong CP problem and QCD axion

$$y_u H Q_L u_R^c + y_d H^* Q_L d_R^c + \frac{g_s^2}{32\pi^2} \theta G \tilde{G}$$



$$\bar{\theta} = \theta + \arg \det (y_u y_d) < 10^{-10}$$

Non-observation  
of neutron EDM

[Abel et al '20]

CP violation in the QCD

$$\text{while } \delta_{\text{CKM}} = \arg \det [y_u y_u^\dagger, y_d y_d^\dagger] \sim \mathcal{O}(1)$$

The QCD vacuum energy is minimized at the CP-conserving point ( $\bar{\theta} = 0$ ).

[Vafa, Witten '84]

$$V_{\text{QCD}} \simeq -m_\pi^2 f_\pi^2 \cos \bar{\theta}$$

Promote  $\bar{\theta}$  to a dynamical field (=QCD axion) :  $\frac{g_s^2}{32\pi^2} \left( \theta + \frac{a}{f_a} \right) G \tilde{G}$

[Peccei, Quinn '77, Weinberg '78, Wilczek '78]

# QCD axion lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{g_s^2}{32\pi^2} c_G \frac{a}{f_a} G^{\mu\nu} \tilde{G}_{\mu\nu} \\ + \frac{a}{f_a} \sum_{A=W,B,\dots} \frac{g_A^2}{32\pi^2} c_A F^{A\mu\nu} \tilde{F}_{\mu\nu}^A + \frac{\partial_\mu a}{f_a} \left( \sum_{\psi=q,\ell,\dots} c_\psi \psi^\dagger \bar{\sigma}^\mu \psi + \sum_{\phi=H,\dots} c_\phi \phi^\dagger i \overleftrightarrow{D}^\mu \phi \right)$$

dictated by i) approximate  $U(1)_{PQ}$  :  $a(x) \rightarrow a(x) + \alpha$

ii) periodicity :  $\frac{a(x)}{f_a} \equiv \frac{a(x)}{f_a} + 2\pi$

$U(1)_{PQ}$  dominantly broken by  $c_G$  (QCD instantons)  $\rightarrow m_a^2 \simeq c_G^2 \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$

By the periodicity, the size of axion couplings are mostly determined by  $f_a$  up to model-dependent constants  $c_G, c_A, c_\psi, c_\phi$



# Axion-Like Particles (ALPs)

- Share the same symmetric properties with QCD axion, but do not necessarily couple to gluons.
- Particularly motivated in string theory.

[Arvanitaki, Dimopoulos, Dubovsky, Kaloper, Marsh-Russell, '09]

$$\frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 + \frac{a}{f_a} \sum_A \frac{g_A^2}{32\pi^2} c_A F^{A\mu\nu} \tilde{F}_{\mu\nu}^A + \frac{\partial_\mu a}{f_a} \left( \sum_\psi c_\psi \psi^\dagger \bar{\sigma}^\mu \psi + \sum_\phi c_\phi \phi^\dagger i \overleftrightarrow{D}^\mu \phi \right)$$

i) approximate  $U(1)_{PQ}$   $a(x) \rightarrow a(x) + c$  ( $c \in \mathbb{R}$ )

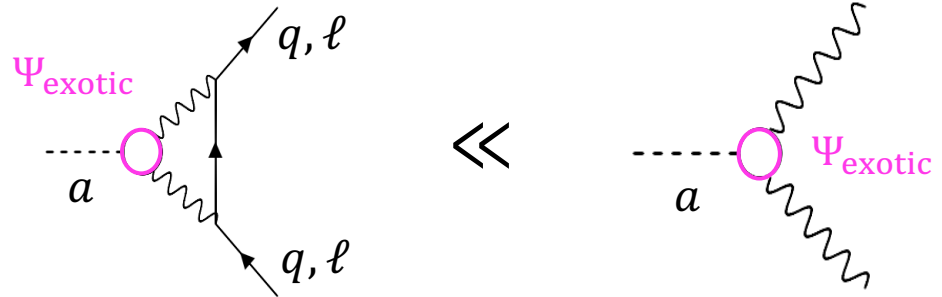
: ALP can be naturally light.

ii) periodicity  $\frac{a(x)}{f_a} \equiv \frac{a(x)}{f_a} + 2\pi$

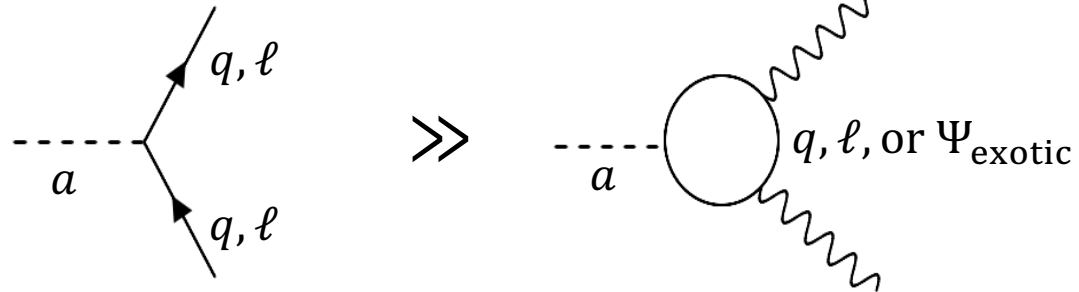
:  $f_a$  characterizes typical size of ALP couplings  
up to model-dependent constants  $c_A, c_\psi, c_\phi$ .

# Characteristic patterns of axion couplings to the SM depending on classes of axion models

## KSVZ-like models

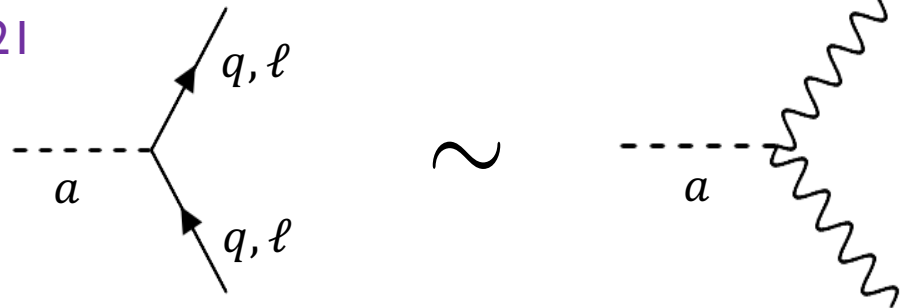


## DFSZ-like models



K Choi, SHI, HJ Kim, H Seong '21

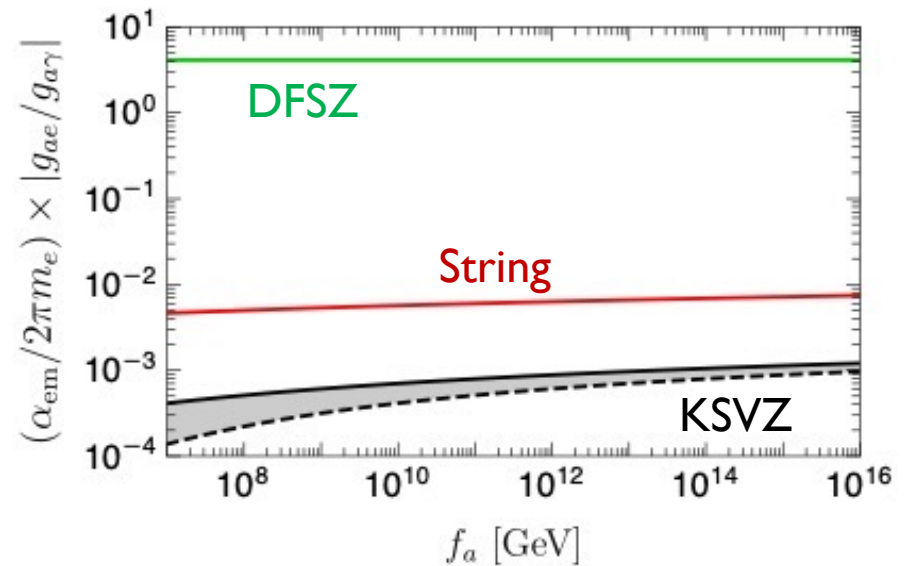
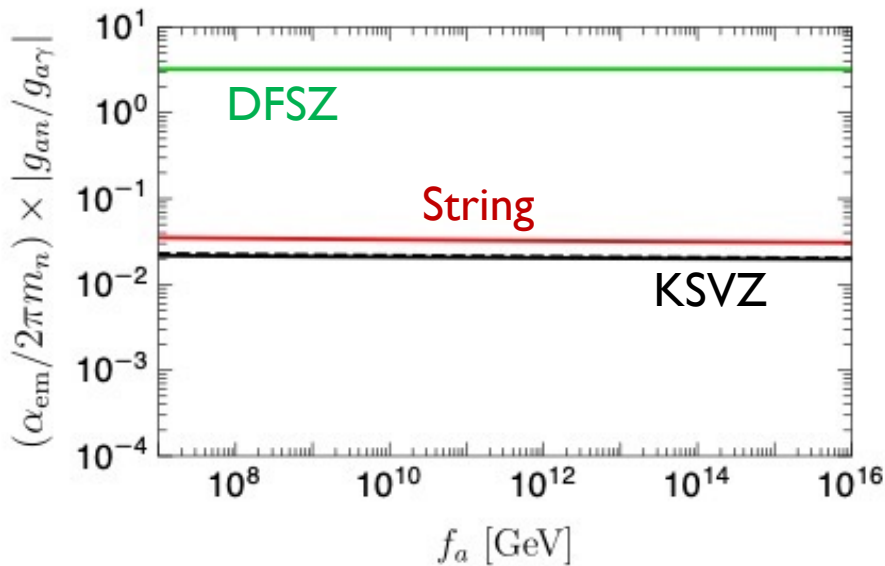
## String-theoretic models



# Experimental test of the classes of axion models

K Choi, SHI, HJ Kim, H Seong '21

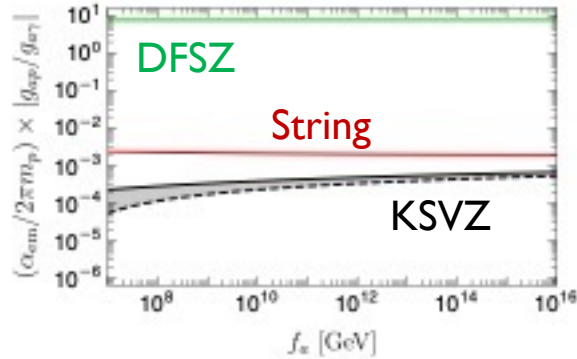
For QCD axion ( $c_G \neq 0$ ),  $g_{ap} \sim \frac{m_p}{f_a}$  regardless of the classes of models



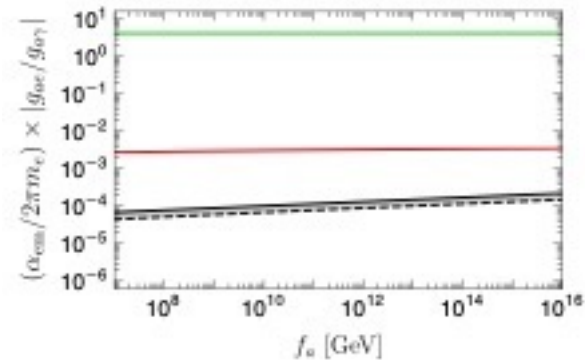
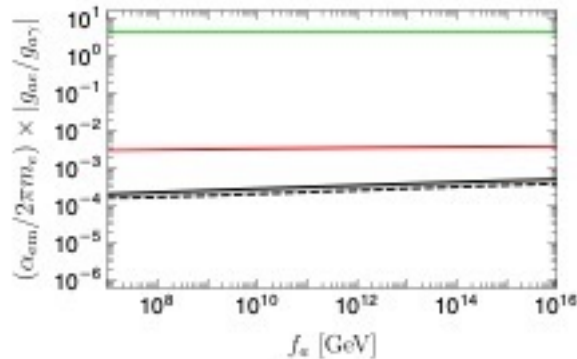
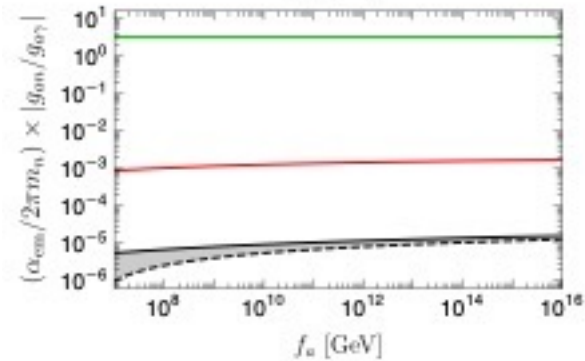
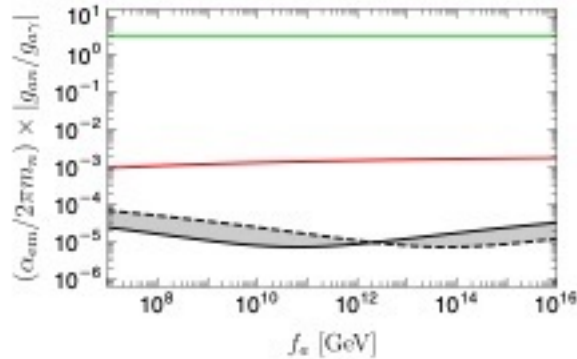
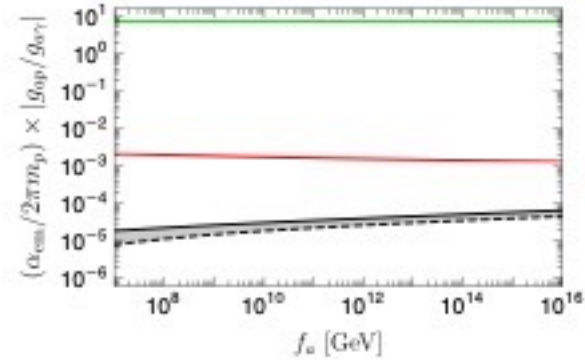
We need to measure the axion-electron coupling to determine the underlying UV physics for QCD axion.

For ALPs with ( $c_G = 0$ ), the axion-nucleon couplings can do the test as well.

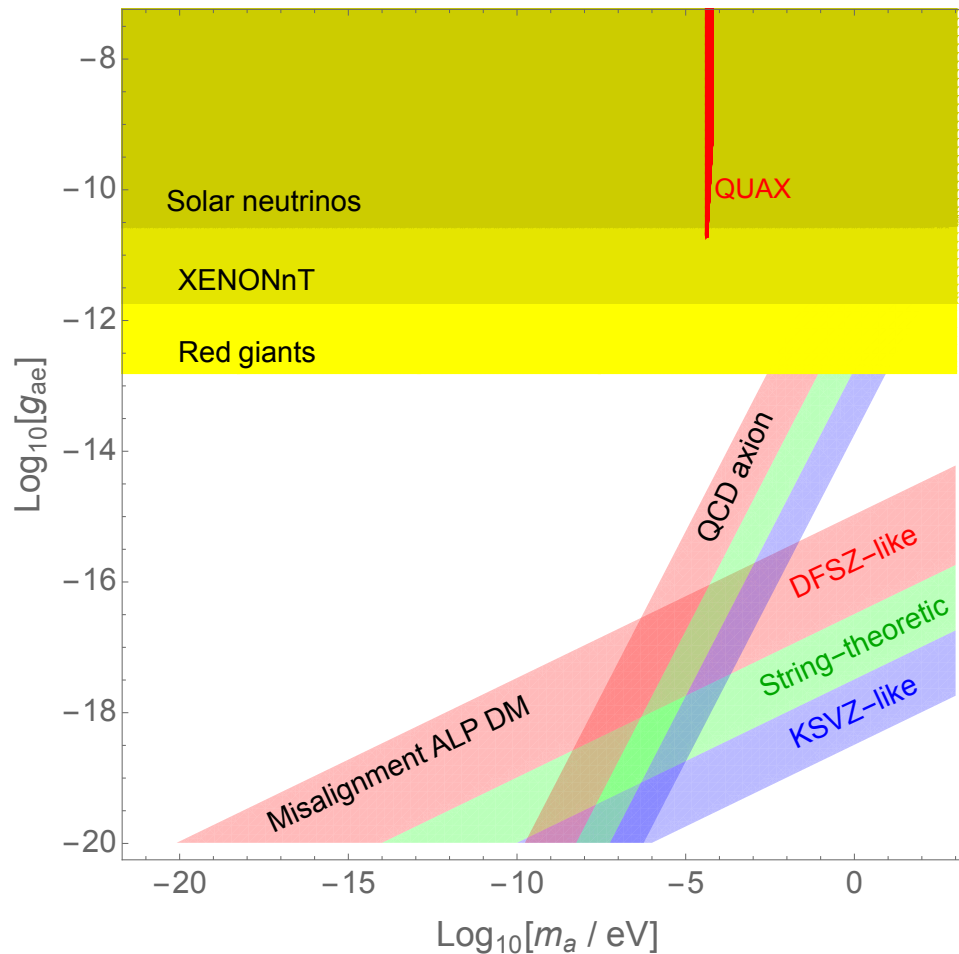
$$c_W = 1 \quad (c_G = c_B = 0)$$



$$c_B = 1 \quad (c_G = c_W = 0)$$



# Experimental test of the axion-electron coupling?

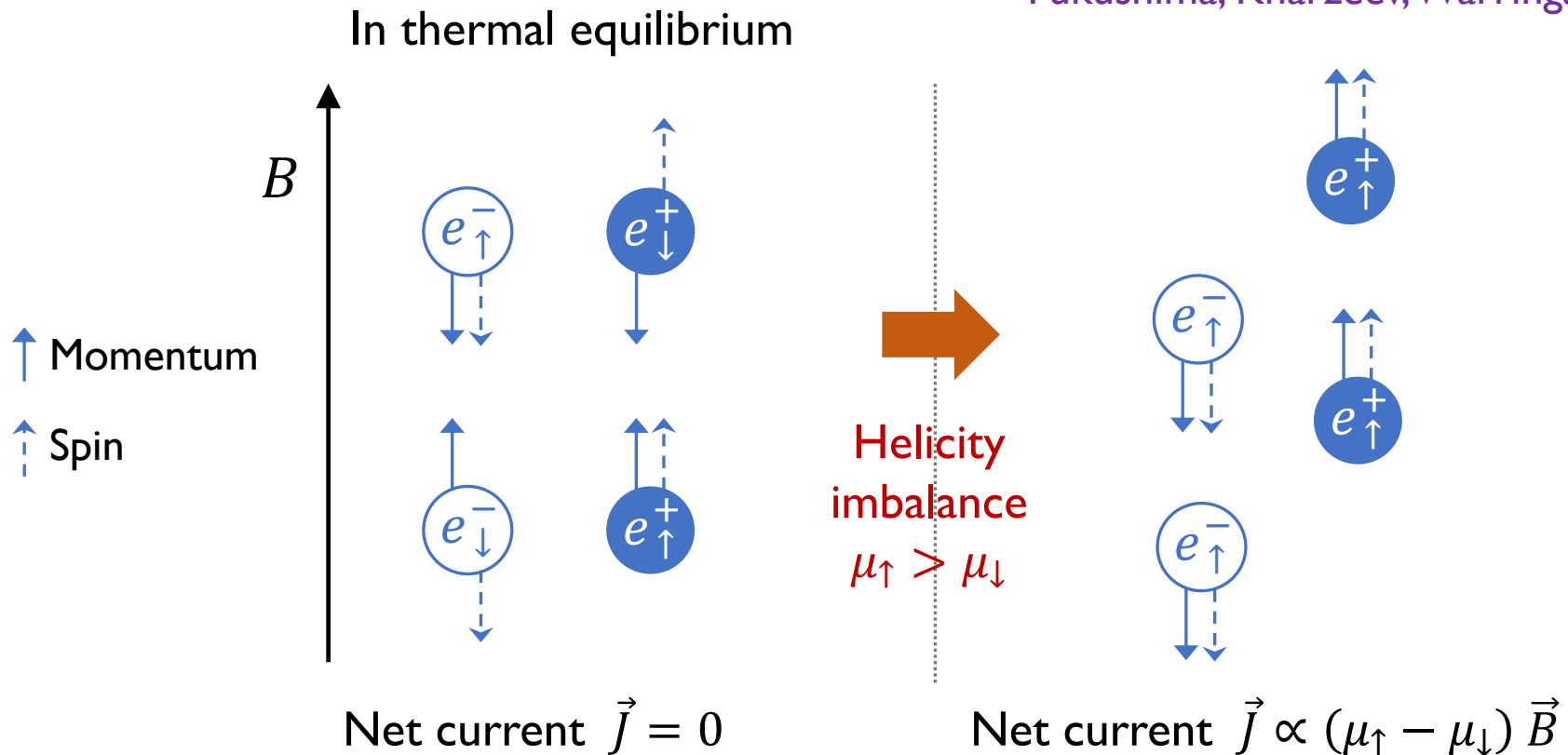


See also So Chigusa's talk

Currently only the QUAX experiment can cover a limited parameter space.

# Chiral Magnetic Effect (CME) in a nutshell

Kharzeev, McLerran, Warringa '08  
Fukushima, Kharzeev, Warringa '08



- The *magnetic field* aligns the spin directions depending on EM charge.
- On top of that, if there is a *helicity imbalance*, a non-zero electric current is generated along the B-field direction.

- Chiral chemical potential

$$\begin{aligned}\mathcal{L} \supset \mu_5(n_R - n_L) &= \mu_5 \left( \psi_R^\dagger \psi_R - \psi_L^\dagger \psi_L \right) & \Psi &= \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \\ &= \mu_5 \bar{\Psi} \gamma^0 \gamma^5 \Psi\end{aligned}$$

It makes *chiral imbalance*, leading to *helicity imbalance*.

- (Vector) chemical potential

$$\begin{aligned}\mathcal{L} \supset \mu(n_R + n_L) &= \mu \left( \psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L \right) \\ &= \mu \bar{\Psi} \gamma^0 \Psi\end{aligned}$$

It makes *charge imbalance* (i.e. particles vs antiparticles).

The *charge imbalance alone does not induce a current*. However,  $\mu$  might be still relevant for the magnitude of the current for a given helicity imbalance.

# Axion background as $\mu_5$

Suppose that we have an axion-electron coupling with time-varying axion field background.

$$c_\Psi \frac{\partial_\mu a}{f_a} \bar{\Psi} \gamma^\mu \gamma^5 \Psi \quad \longrightarrow \quad c_\Psi \frac{\dot{a}}{f_a} \bar{\Psi} \gamma^0 \gamma^5 \Psi$$
$$\mu_5(t) = c_\Psi \frac{\dot{a}}{f_a}$$

On the other hand,

$$c_\Psi \frac{\nabla a}{f_a} \cdot \bar{\Psi} \vec{\gamma} \gamma^5 \Psi \approx \underbrace{2c_\Psi \frac{\nabla a}{f_a}} \cdot \vec{S}_e$$

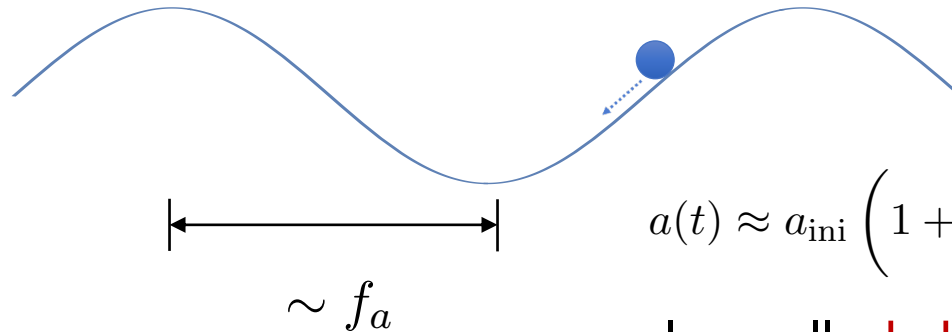
QUAX and (NV Center) are looking for  $\vec{B}_{\text{eff}}$



# Cosmological evolution of an axion field

$$\ddot{a}(t) + 3H\dot{a}(t) + m_a^2 a(t) = 0$$

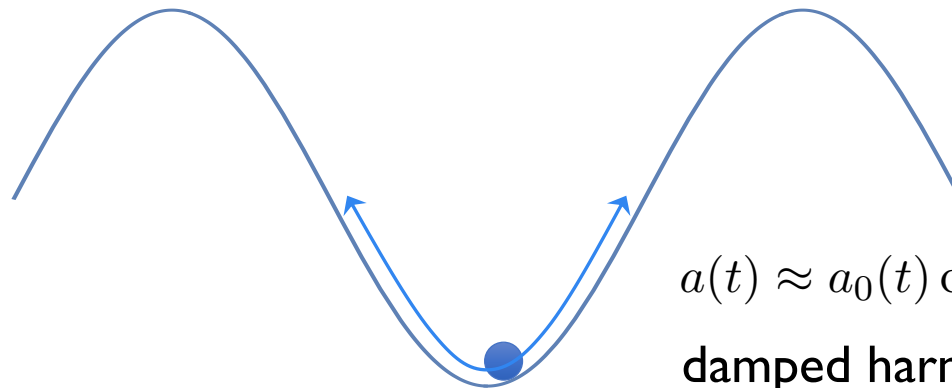
$$m_a \ll H \sim \frac{T^2}{M_{Pl}}$$



$$a(t) \approx a_{\text{ini}} \left( 1 + \frac{1}{20} \left[ \frac{m_a^2}{H_{\text{ini}}^2} - \frac{m_a^2}{H(t)^2} \right] \right)$$

slow-roll : **dark energy**

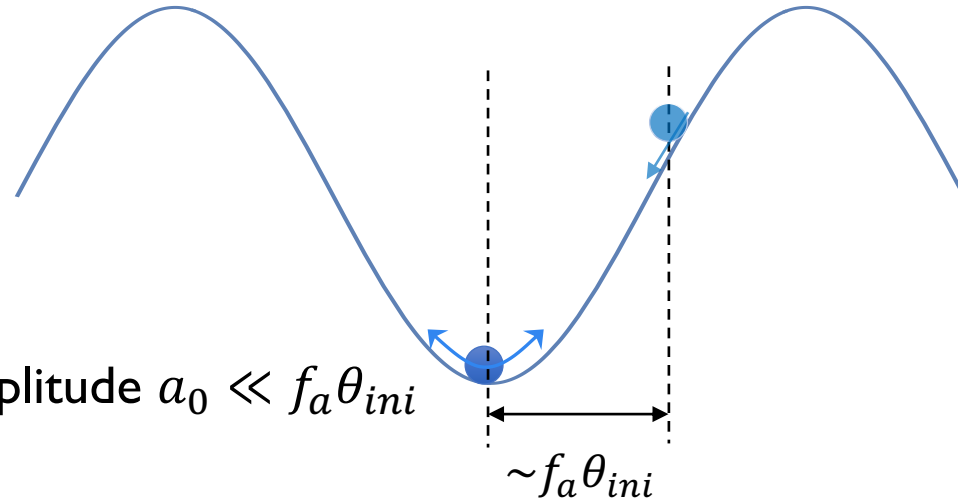
$$m_a \gg H \sim \frac{T^2}{M_{Pl}}$$



$$a(t) \approx a_0(t) \cos(m_a t)$$

damped harmonic oscillator : **cold dark matter**

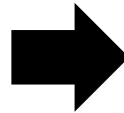
# Misalignment production of axion dark matter and chiral chemical potential



Present oscillation amplitude  $a_0 \ll f_a \theta_{ini}$

$$a(t, \vec{x}) \approx a_0 \cos m_a t$$

$$\rho_a = \frac{1}{2} m_a^2 a_0^2$$



$$\mu_5(t) \approx -c_\Psi \frac{\sqrt{2\rho_a}}{f_a} \sin m_a t$$

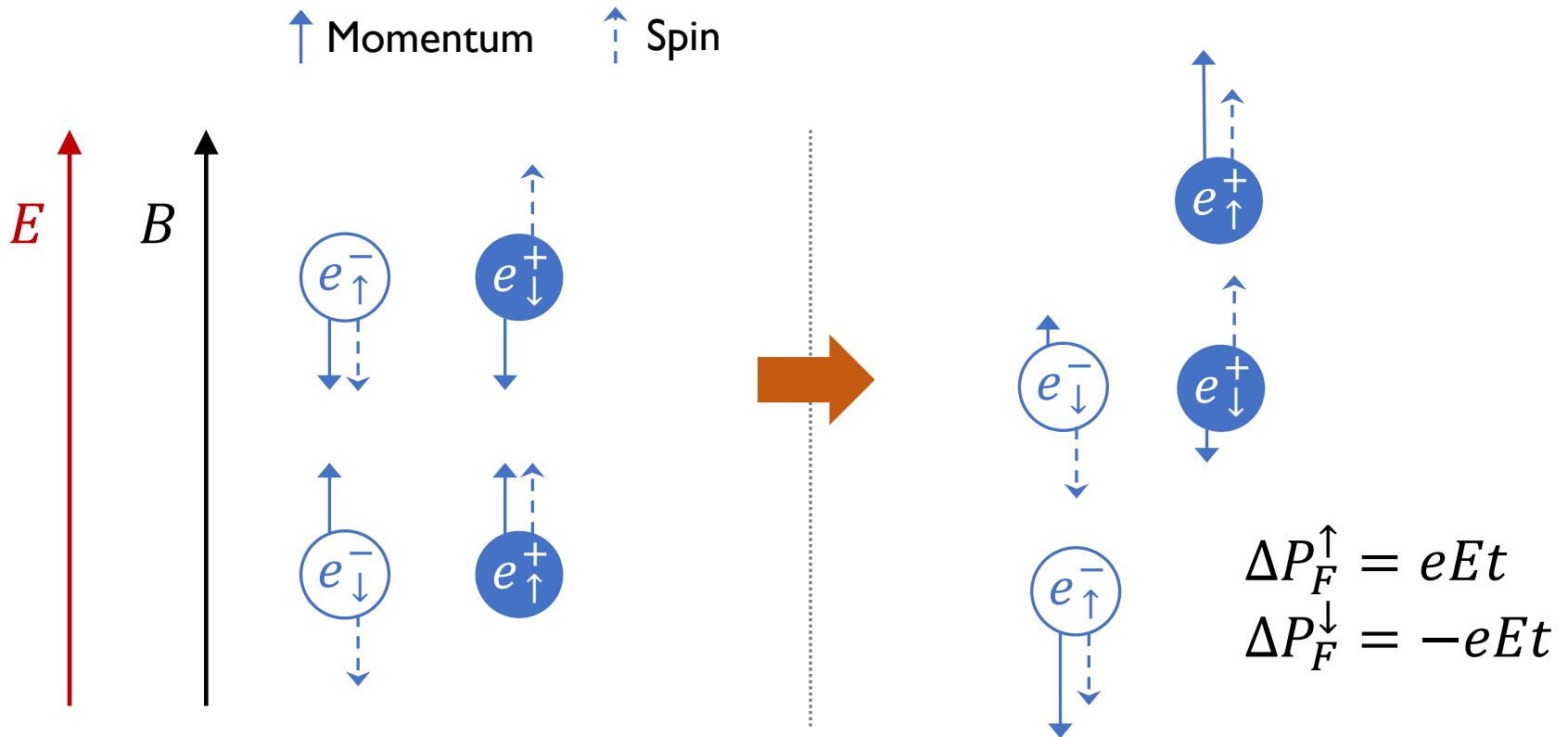
$$\mu_5(t) = c_\Psi \frac{\dot{a}}{f_a}$$

The axion dark matter background gives rise to an oscillating chiral chemical potential for fermions coupled to the axion.

# CME-induced current

## Energy balance argument

Nielsen and Ninomiya '83  
Fukushima, Kharzeev, Warringa '08



Density change of  
helicity-up fermion states

$$\underbrace{\frac{\Delta P_F^\uparrow}{2\pi}}_{\text{Longitudinal number density}} \cdot \underbrace{\frac{eB}{2\pi}}_{\text{Transverse number density in the lowest Landau level}} = \frac{e^2}{4\pi^2} \vec{E} \cdot \vec{B} t$$

Longitudinal number  
density

Transverse number density  
in the *lowest* Landau level

Aharonov and Casher '79

Density change of  
helicity-down fermion states

$$\frac{\Delta P_F^\downarrow}{2\pi} \cdot \frac{eB}{2\pi} = -\frac{e^2}{4\pi^2} \vec{E} \cdot \vec{B} t$$

$$\blackrightarrow \frac{d}{dt} (n_\uparrow - n_\downarrow) = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B}$$

which reproduces the chiral anomaly equation in massless limit

$$\partial_\mu (\bar{\Psi} \gamma^\mu \gamma^5 \Psi) = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B} + 2m \bar{\Psi} i \gamma^5 \Psi$$

$$\frac{d}{dt}(n_{\uparrow} - n_{\downarrow}) = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B}$$

➔

$$\underbrace{\frac{\mathcal{E}}{2} \frac{d}{dt}(N_{\uparrow} - N_{\downarrow})}_{\text{Energy cost per unit time needed for making helicity imbalance}} = \frac{e^2}{4\pi^2} \mathcal{E} \int d^3x \vec{E} \cdot \vec{B}$$

$\mathcal{E}$  : Energy cost for flipping a helicity-down state to a helicity-up state

Energy cost per unit time needed for making helicity imbalance

||

$$\int d^3x \vec{j} \cdot \vec{E}$$

This energy cost has to be supplied by the electric power.

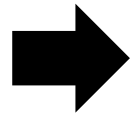
➔

$$\boxed{\vec{j} = \frac{e^2}{4\pi^2} \mathcal{E} \vec{B}}$$

Alternatively

$$\begin{aligned} j &= \int d\rho v \overbrace{dn^\uparrow} \overbrace{v} \\ &= e \cdot \frac{eB}{2\pi} \cdot \left( \int_0^{p_F^\uparrow} \frac{dp_z}{2\pi} \frac{p_z}{\sqrt{p_z^2 + m^2}} - \int_0^{p_F^\downarrow} \frac{dp_z}{2\pi} \frac{p_z}{\sqrt{p_z^2 + m^2}} \right) \\ &= \frac{e^2}{4\pi^2} \cdot \underbrace{\left( \sqrt{(p_F^\uparrow)^2 + m^2} - \sqrt{(p_F^\downarrow)^2 + m^2} \right)}_{\varepsilon} \cdot B \end{aligned}$$

Dirac eq.  $\begin{pmatrix} -m & E - \vec{p} \cdot \vec{\sigma} - \mu_5 \\ E + \vec{p} \cdot \vec{\sigma} + \mu_5 & -m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$   $h = \hat{p} \cdot \vec{\sigma}$  helicity  
 $\vec{p} \cdot \vec{\sigma} = |p|h$

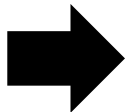


$$E = \sqrt{(|p|h + \mu_5)^2 + m^2}$$

$$p_F^\uparrow = p_F + \mu_5$$

$$p_F^\downarrow = p_F - \mu_5$$

where  $p_F$  is the Fermi momentum when  $\mu_5 = 0$ .



$$\mathcal{E} = \sqrt{(p_F^\uparrow)^2 + m^2} - \sqrt{(p_F^\downarrow)^2 + m^2} \simeq 2\mu_5 v_F$$

$$\vec{j} \simeq \frac{e^2}{2\pi^2} \mu_5 v_F \vec{B}$$

In the original formula by (FKW '08) the  $v_F$  dependence is missing (Hong, SHI, Jeong, Yeom '22).

From DK Hong's slide

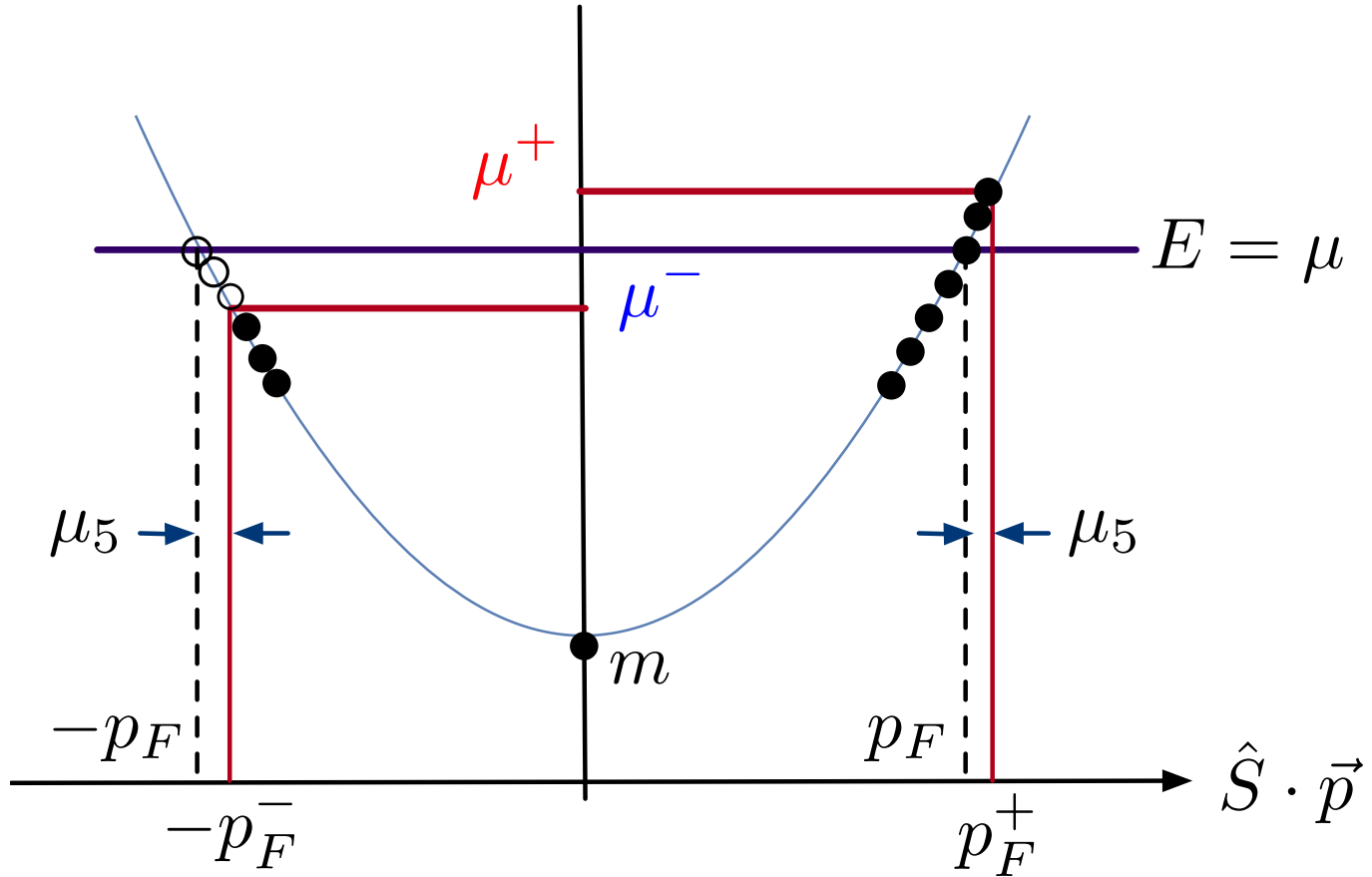


Figure: chiral medium



# CME-induced current

## Field-theoretic calculation



$$\langle j^\mu \rangle = e \langle \bar{\Psi} \gamma^\mu \Psi \rangle = -e \int \frac{d^4 p}{(2\pi)^4} \text{Tr} [\gamma^\mu S_F^{n=0}(p, \mu, \mu_5)]$$

$$S_F^{n=0}(p, \mu, \mu_5) = \left[ \frac{2i (\not{\tilde{p}}_{\parallel} + m) P_- H_+ e^{-p_{\perp}^2/|eB|}}{[(1+i\epsilon) p_0 + \mu_+]^2 - p_z^2 - m^2} + \frac{2i (\not{\tilde{p}}_{\parallel} + m) P_- H_- e^{-p_{\perp}^2/|eB|}}{[(1+i\epsilon) p_0 + \mu_-]^2 - p_z^2 - m^2} \right]$$

: the electron propagator in the lowest Landau level in a dense medium

$P_-$ : spin projection operator

$H_{\pm}$ : helicity projection operator

$$\mu_{\pm} = \sqrt{p_F^{\pm 2} + m^2}$$

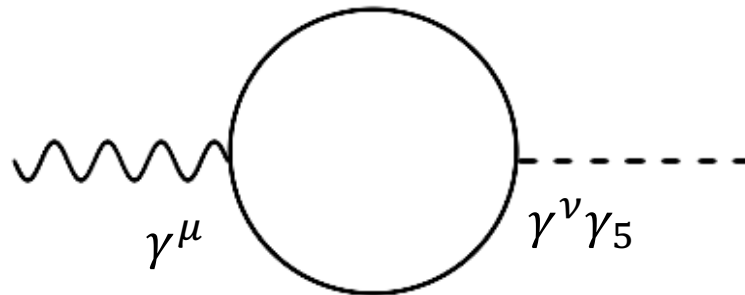
: chemical potential for  
the helicity eigenstates

The field-theoretic calculation reproduces the CME formula obtained by the energy balance argument :

$$\begin{aligned}
 \langle j^3 \rangle &= \frac{e^2 B}{4\pi^2} \left[ \int_0^{\mu_+} dp_0 \int_{p_z > 0} |p_z| \delta(p_0^2 - p_z^2 - m^2) - \int_0^{\mu_-} dp_0 \int_{p_z > 0} |p_z| \delta(p_0^2 - p_z^2 - m^2) \right] \\
 &= \frac{e^2 B}{4\pi^2} \left[ \underbrace{\sqrt{(p_F + \mu_5)^2 + m^2} - \sqrt{(p_F - \mu_5)^2 + m^2}}_{\varepsilon} \right] = \frac{e^2 B}{2\pi^2} \mu_5 v_F [1 + \mathcal{O}(v_F^2, r^2)] ,
 \end{aligned}$$

# CME and chiral anomaly in (1+1)D

Since only the electrons in the lowest Landau level (i.e. transverse zero modes) contribute to the CME current, the physics may be understood in terms of electrons moving in (1+1)D spacetime.



$$\Gamma^{\mu\nu}(q_1)\delta^{(2)}(q_1 + q_2) \equiv \int \prod_i d^2x_i e^{iq_i \cdot x_i} \langle 0 | T j^\mu(x_1) j_5^\nu(x_2) | 0 \rangle$$

In the vacuum,

$$\Gamma_{\text{vac}}^{\mu\nu}(q) = \frac{eB}{4\pi^2} (\epsilon^{\nu\alpha} q_\alpha q^\mu + \epsilon^{\mu\nu} q^2) H(q^2, m^2)$$

$$H(q^2, m^2) = \frac{1}{q^2} \left( 1 - \frac{1}{\sqrt{\tau(1-\tau)}} \tan^{-1} \sqrt{\frac{\tau}{1-\tau}} \right) \simeq -\frac{1}{4m^2} (q \rightarrow 0)$$

$$\tau \equiv \frac{q^2}{4m^2} \quad : \text{No pole at } q^2 = 0 \text{ for massive electrons}$$

The chiral anomaly diagram vanishes as  $q \rightarrow 0$  when there is no massless charged fermion. [Coleman and Grossman '82](#)

In a medium,

$$\Gamma^{\mu\nu}(q) = \frac{eB}{4\pi^2} (\epsilon^{\nu\alpha} q_\alpha q^\mu + \epsilon^{\mu\nu} q^2) F(q^2, m^2, \mu)$$

$$F(q^2, m^2, \mu) = \frac{1}{q^2} \left( \mathcal{O} \left( \frac{q^2}{m^2} \right) + \frac{\mathcal{P}F}{\mu} + \mathcal{O} \left( \frac{q}{\mu} \right) \right)$$

: a pole at  $q^2 = 0$   
even for massive electrons

➔

$$\langle \partial_\nu j_5^\nu \rangle_A = ie \int \frac{d^2q}{4\pi^2} \lim_{q_0 \rightarrow 0} \lim_{q_3 \rightarrow 0} e^{iq \cdot x} q_\nu A_\mu(q) \Gamma^{\mu\nu}(q) = \frac{e^2 B}{4\pi^2} v_F \epsilon^{\mu\nu} F_{\mu\nu}$$

$$\langle j^3 \rangle = -e\mu_5 \lim_{q_0 \rightarrow 0} \lim_{q_3 \rightarrow 0} \Gamma^{30}(q) = \frac{e^2 B}{2\pi^2} v_F \mu_5$$

The CME can be understood as (1+1)D chiral anomaly in a medium induced by *gapless modes at the Fermi surface*.

# Detecting axion dark matter via the CME

$$C_e \frac{\partial_\mu a}{f} \bar{\psi} \gamma^\mu \gamma_5 \psi \quad \longrightarrow \quad \mu_5 = C_e \frac{\dot{a}}{f}$$

$$a(t) = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \sin(m_a t)$$

The axion dark matter field induces an oscillating chiral chemical potential if the axion couples to the electrons.

$$\mu_5 = C_e \frac{\sqrt{\rho_{\text{DM}}}}{f} \cos(m_a t) \sim 0.25 \times 10^{-23} \text{ eV} \cdot \left( \frac{\rho_{\text{DM}}}{0.4 \text{ GeV cm}^{-3}} \right)^{1/2} \cdot \left( \frac{10^{12} \text{ GeV}}{f/C_e} \right)$$

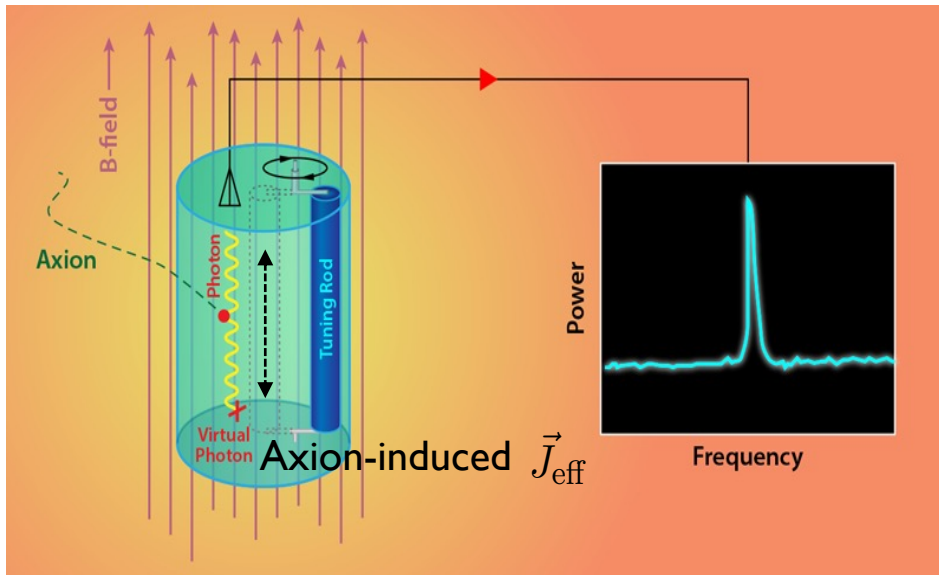
$$\longrightarrow j^3 = \frac{e^2}{2\pi^2} \mu_5 v_F B = 6.8 \times 10^{-15} \text{ A m}^{-2} \cos(m_a t)$$

CME  $\times \left( \frac{v_F}{10^{-2}} \right) \cdot \left( \frac{\rho_{\text{DM}}}{0.4 \text{ GeV cm}^{-3}} \right)^{1/2} \cdot \left( \frac{10^{12} \text{ GeV}}{f/C_e} \right) \cdot \left( \frac{B}{10 \text{ Tesla}} \right)$

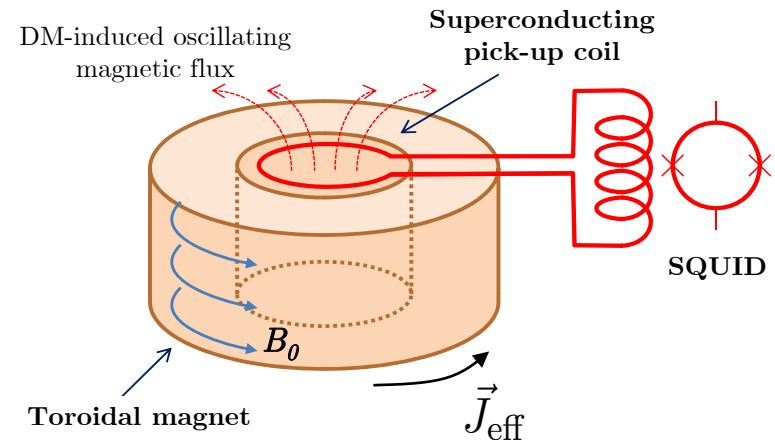
# CME current measurement by modified axion haloscopes

Existing haloscopes for the axion-photon coupling may be used for detecting the CME current by replacing the “vacuum” with a “conductor”.

$$J_{\text{eff}} = g_{a\gamma} \dot{a} B \quad \longleftrightarrow \quad J_{\text{CME}} = \frac{e^2}{2\pi^2} \frac{c_e}{f_a} v_F \dot{a} B$$



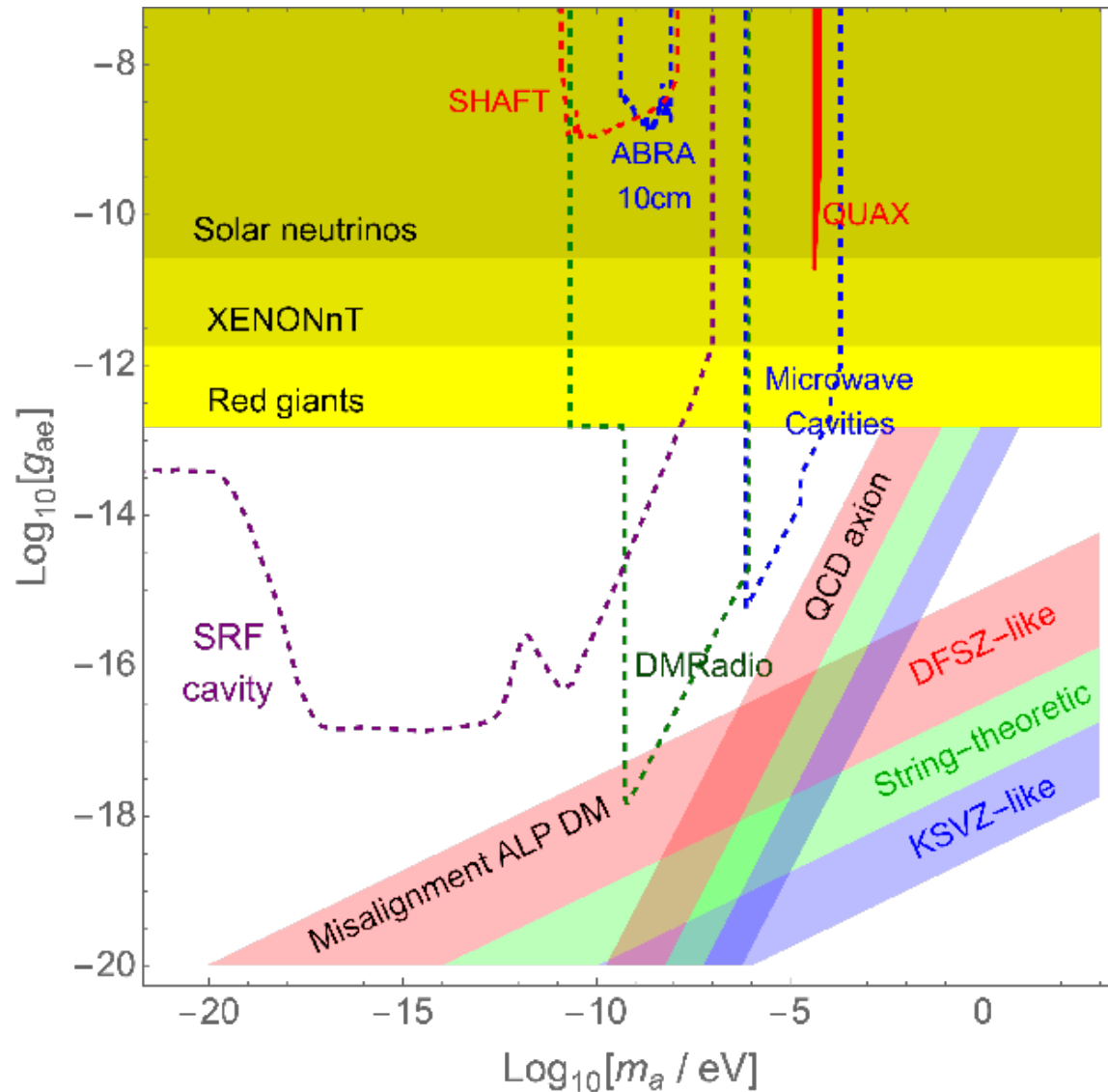
Microwave resonant cavity



ABRACADABRA

Kahn, Safdi, Thaler '16

# Projected sensitivity from existing axion haloscopes





# Conclusions

- Axions are theoretically well-motivated new particles which may be an important clue for underlying UV physics when they are discovered.
- The measurement of the axion-electron coupling is particularly important for determining the microscopic origin of axions.
- The chiral magnetic effect (CME) offers an intriguing possibility for the measurement of the axion-electron coupling, when the axion comprises a major fraction of dark matter.
- We have newly computed the CME-induced current and claim that it is proportional to the Fermi velocity of the electrons.

# Back-up slides

# Laboratory searches for axion DM -photonic probes

$$\frac{g_{a\gamma}}{4} a F \tilde{F} \quad \longrightarrow \quad \nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} \underbrace{-g_{a\gamma} \vec{B} \partial_t a}_{\vec{J}_{\text{eff}}} \quad \text{effective current}$$

Background axion DM field

$$a \approx a_0 \cos [m_a (t - \vec{v} \cdot \vec{x})]$$

$$\rho_a = \frac{1}{2} m_a^2 a_0^2 \quad |\vec{v}| \sim 10^{-3} c$$

$$\vec{J}_{\text{eff}} \approx g_{a\gamma} \sqrt{2\rho_a} \vec{B} \sin m_a t$$

The best experimental sensitivity on  $g_{a\gamma}$  is obtained when  $\rho_a = \rho_{DM}$ .

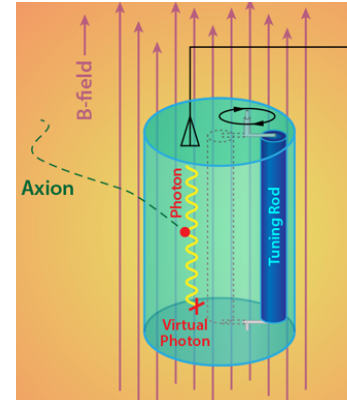
Misalignment axion DM

$$f_a \simeq 10^{17} \text{ GeV} \left( \frac{10^{-22} \text{ eV}}{m_a} \right)^{1/4} \sqrt{\frac{\rho_a}{\rho_{DM}}} \quad \longrightarrow \quad g_{a\gamma} = \frac{e^2}{8\pi^2} \frac{1}{f_a} c_{a\gamma}$$

Given axion DM mass,  
 $g_{a\gamma}$  is determined for  $c_{a\gamma} \sim O(1)$ .

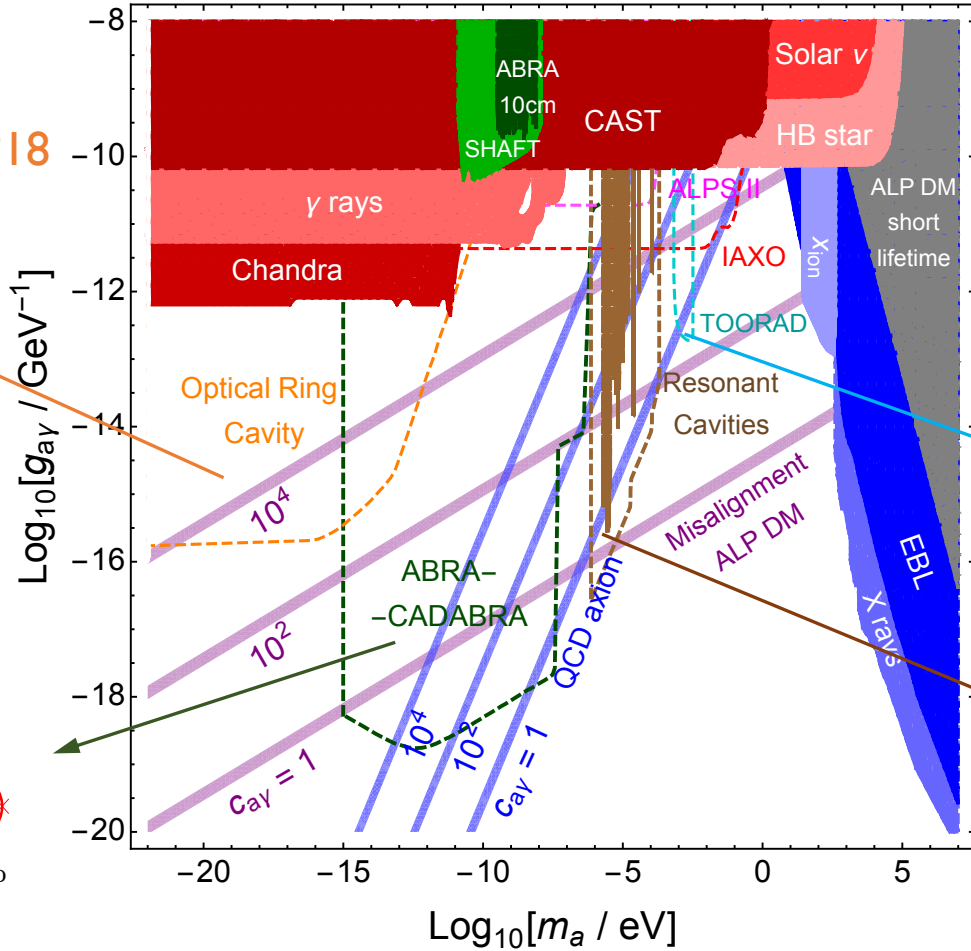
# Current and future limits on $g_{a\gamma}$

Choi, SHI, Shin '20

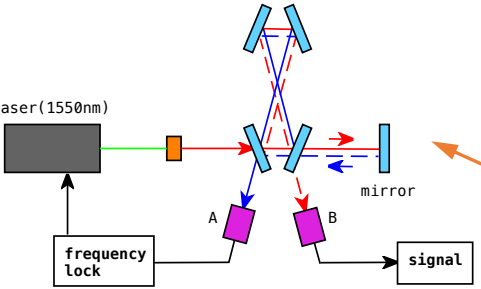


Marsh, Fong, Lenz, Smejkal, Ali '18

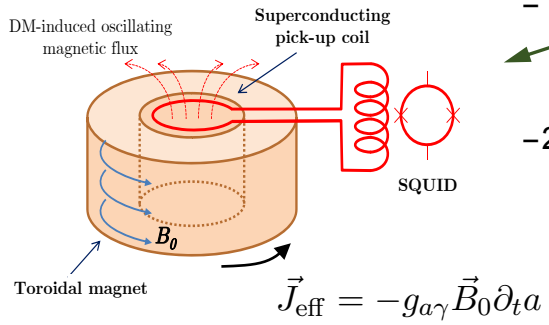
ADMX, IBS-CAPP, MADMAX...



Obata, Fujita, Michimura '18



Kahn, Safdi, Thaler '16



# Laboratory searches for axion DM -nucleonic probes

$$g_{aN} \frac{\partial_\mu a}{2m_N} \bar{N} \gamma^\mu \gamma^5 N \quad \longrightarrow \quad \underbrace{g_{aN} \frac{\nabla a}{\gamma_N m_N}}_{\vec{B}_{\text{eff}}} \cdot \gamma_N \vec{S}_N \quad \gamma_N : \text{nucleon gyromagnetic ratio}$$

Background axion DM field

$$a \approx a_0 \cos [m_a (t - \vec{v} \cdot \vec{x})]$$

$$\rho_a = \frac{1}{2} m_a^2 a_0^2 \quad |\vec{v}| \sim 10^{-3} c$$

$$\vec{B}_{\text{eff}} \approx g_{aN} \frac{\sqrt{2\rho_a}}{\gamma_N m_N} \vec{v}_a \sin m_a t$$

The best experimental sensitivity on  $g_{aN}$  is obtained when  $\rho_a = \rho_{DM}$ .

Misalignment axion DM

$$f_a \simeq 10^{17} \text{ GeV} \left( \frac{10^{-22} \text{ eV}}{m_a} \right)^{1/4} \sqrt{\frac{\rho_a}{\rho_{DM}}} \quad \longrightarrow \quad g_{aN} = \frac{m_N}{f_a} c_{aq} \times \mathcal{O}(1)$$

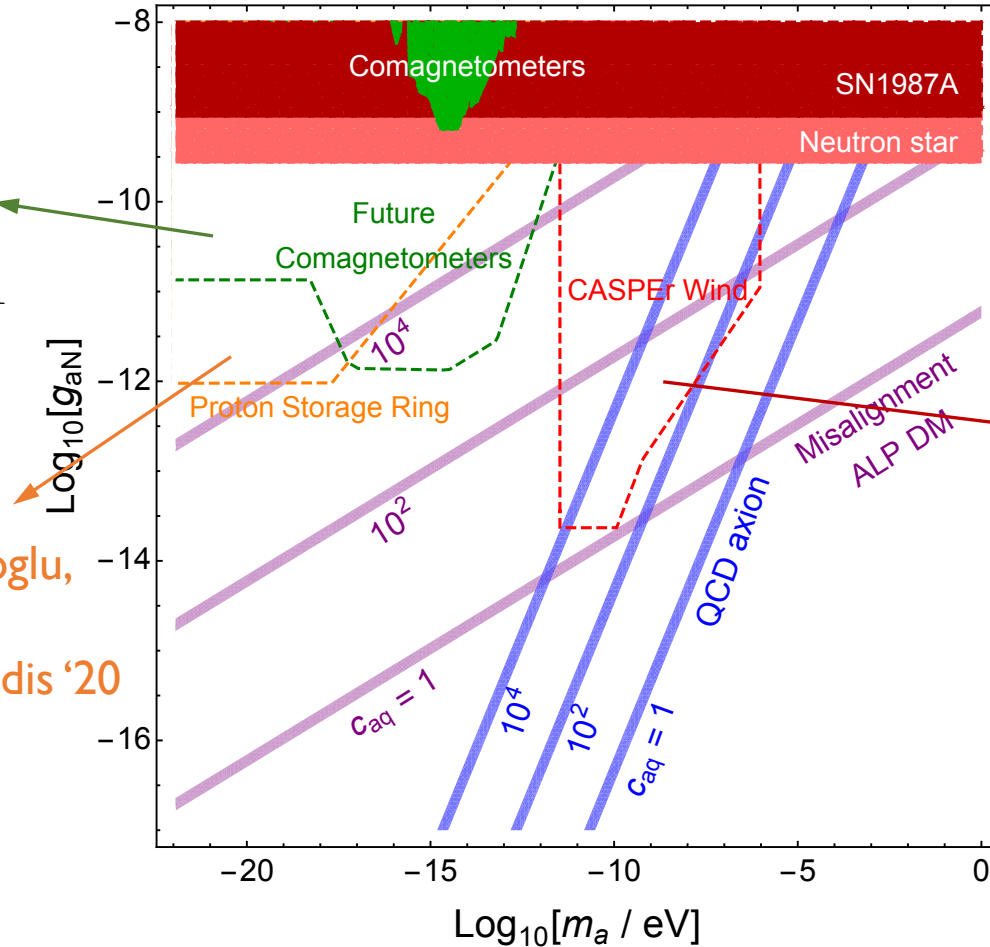
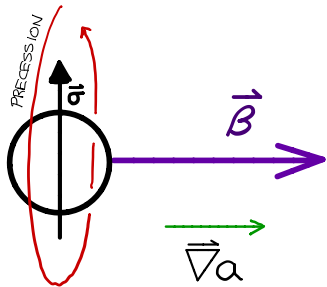
Given axion DM mass,  
 $g_{aN}$  is determined for  $c_{aq} \sim \mathcal{O}(1)$ .

# Current and future limits on $g_{aN}$

Bloch, Hochberg,  
Kuflik, Volansky '19

$$\frac{B_{\text{eff}}^e}{B_{\text{eff}}^N} \sim \frac{c_{ae} m_e}{c_{aN} m_N} \neq 1$$

Graham, Hacıomeroglu,  
Kaplan, Omarov,  
Rajendran, Semertzidis '20



Choi, Shi, Shin '20

Kimball et al '17

