

# Cosmological Solitons

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**PNU-IBS workshop on Axion Physics : Search for axions**  
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# OUTLINE

## Introduction

- **Solitons**
- **The Q-ball**

## Q-Ball Collision Simulations

- **Field Profiles and tunnelling action**
- **Symplectic Integrators**
- **Large Q-Ball Collisions**

## Solitons in Axion Physics

- **Axion + Axion Electrodynamics Solitons**
- **Magnetic Vortices in time dependent axion fields.**

# Introduction

# Solitons

Solitons: Self-reinforcing localised, stable, solutions to wave equations.

Dispersion and non-linearity can interact to produce permanent and localised wave forms.

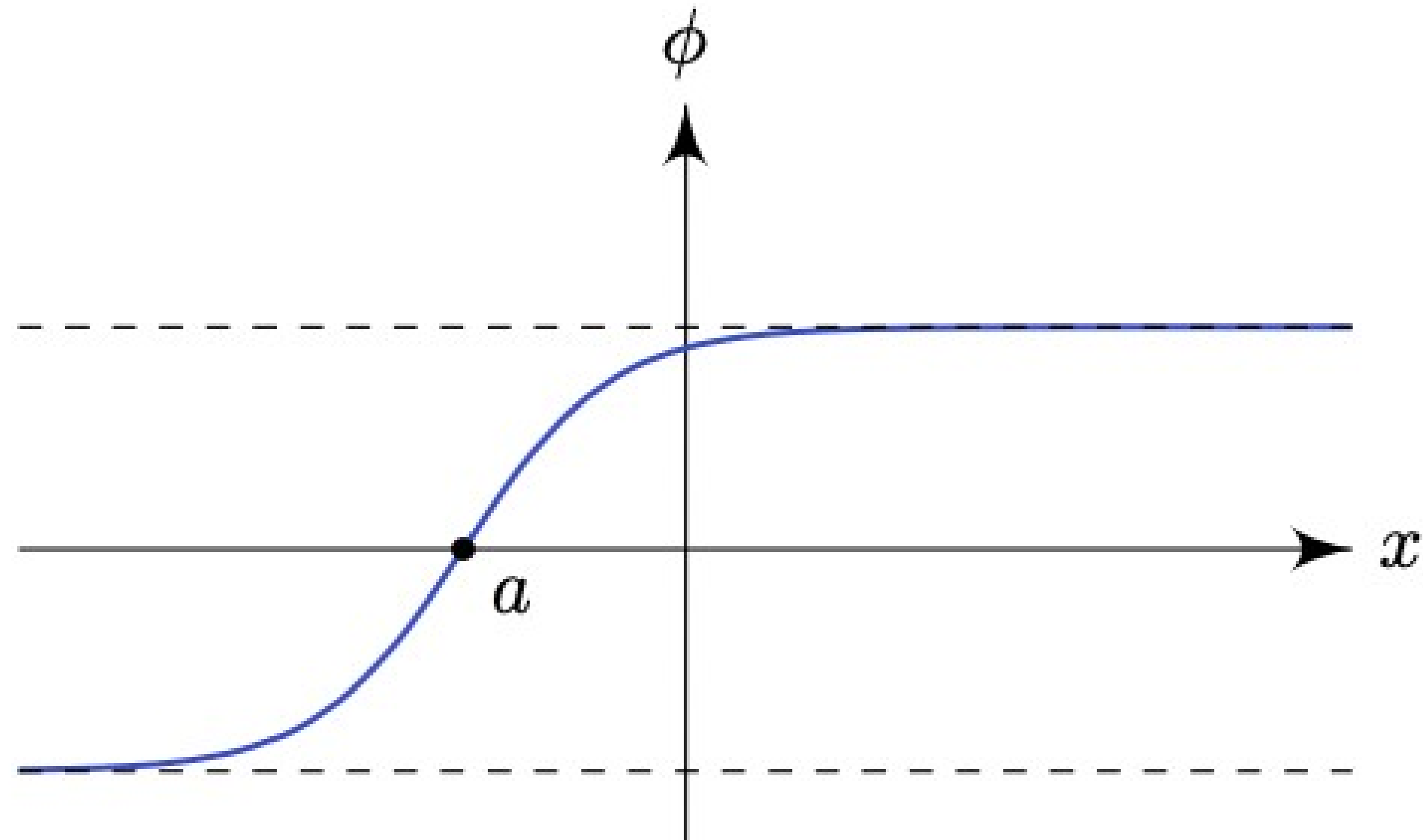
With the ascendancy of field theories over the S-matrix theory, the relevance of solitons to particle physics and cosmology became an important topic of study.

**Scott Russell Aqueduct**

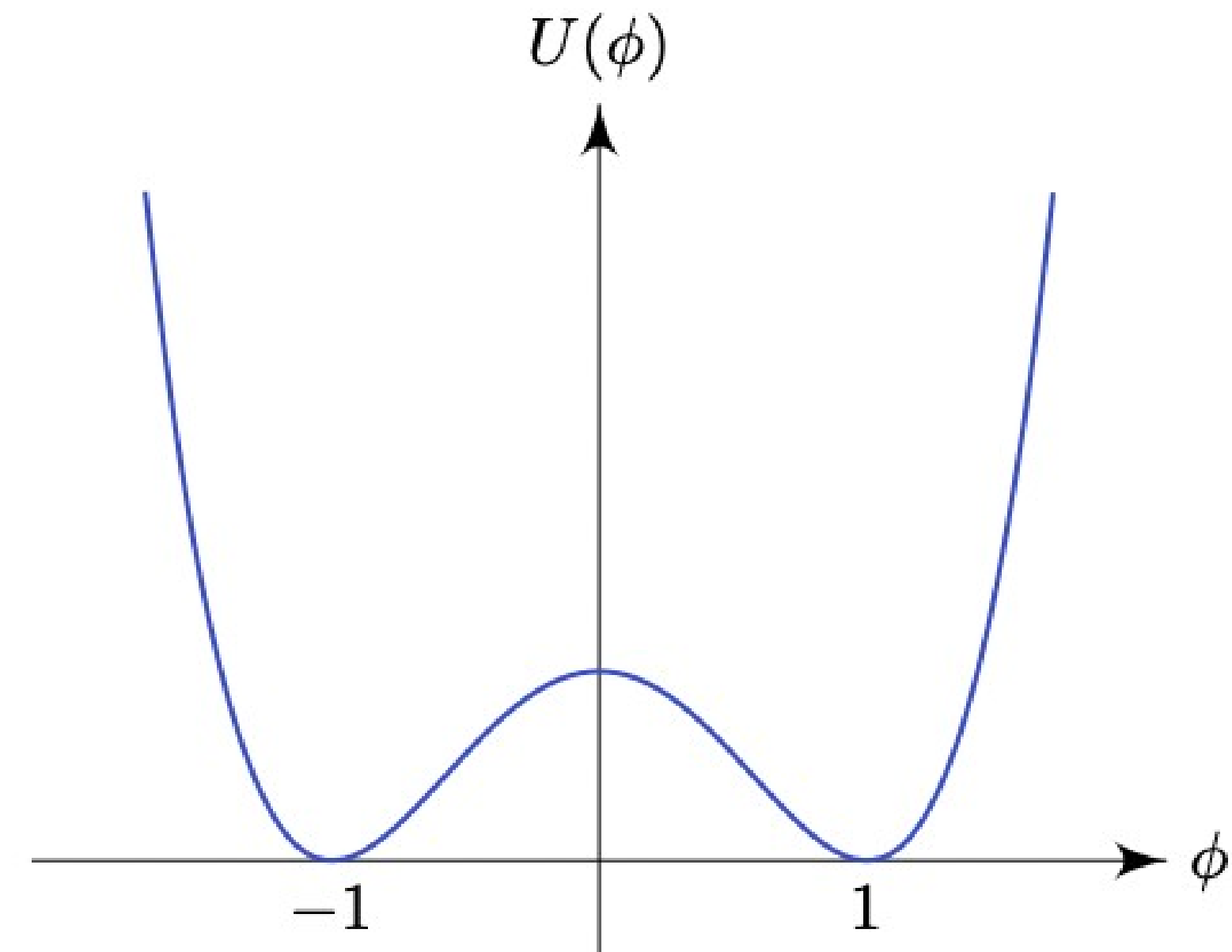


John Scott Russell ‘discovered’ solitons in the Union Canal in Edinburgh.

# A Minimal Example, The 1+1D Kink



$$\phi(x) = \tanh(x - a).$$

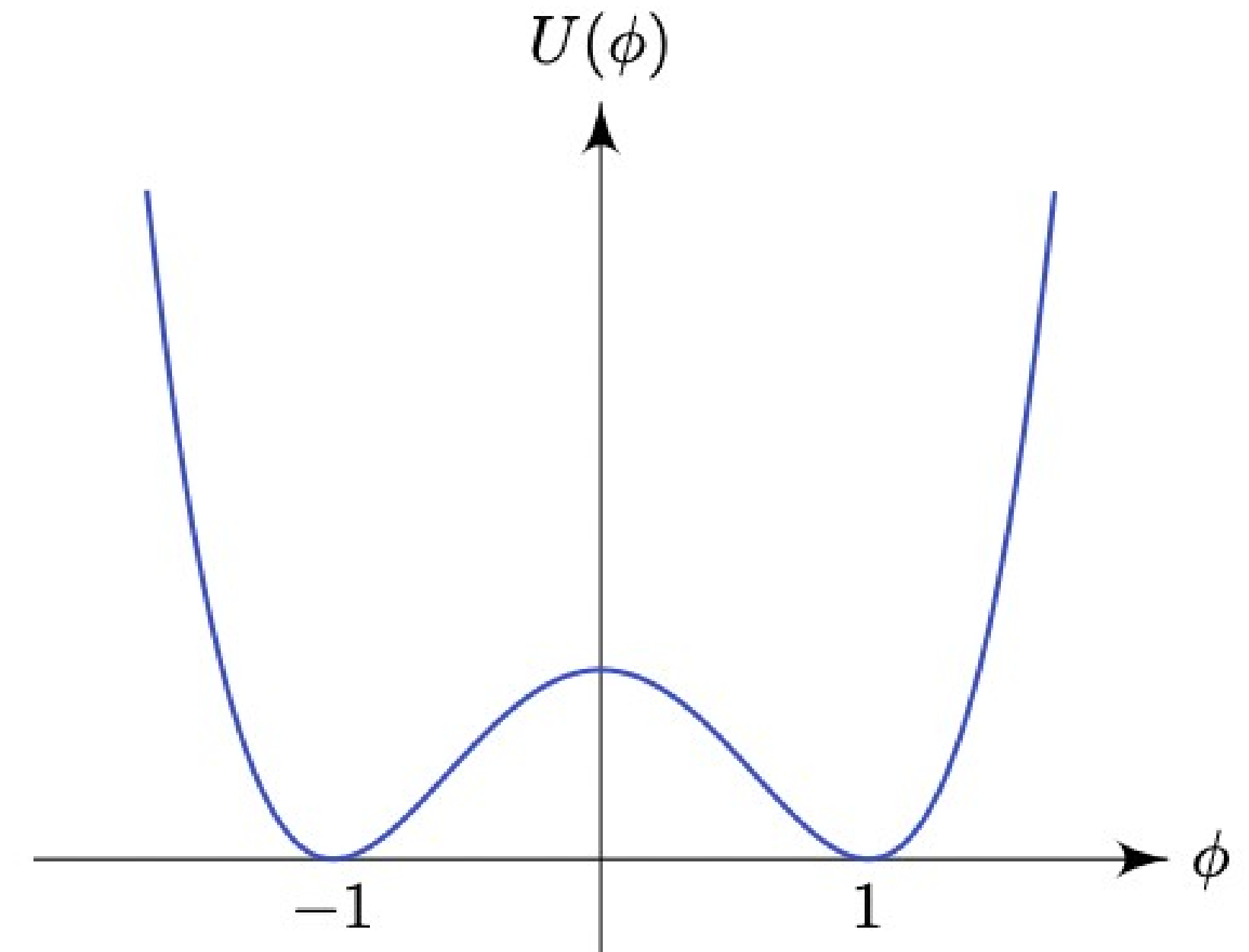
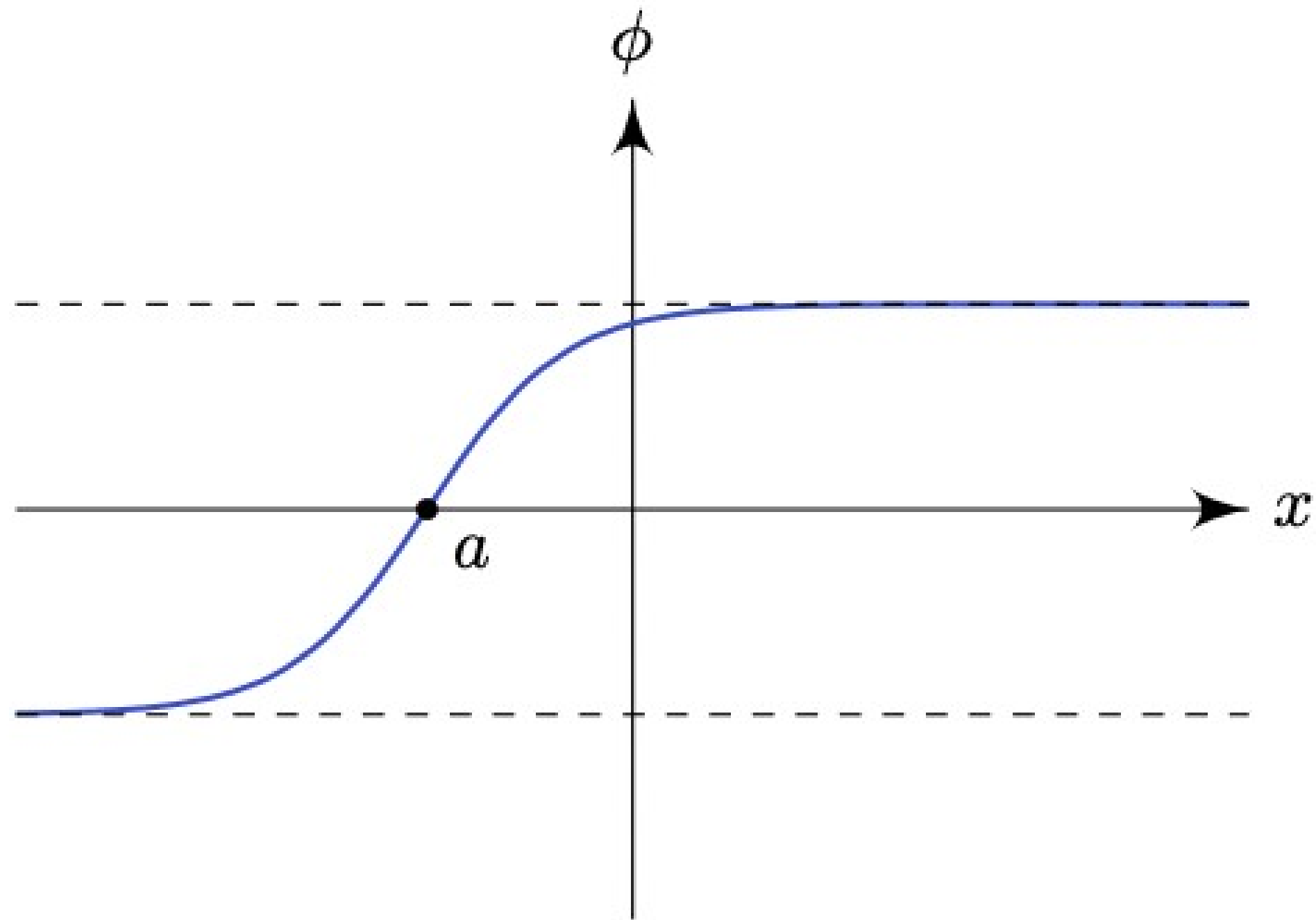


Solution interpolates between two degenerate minima.

Parameter  $a$  is the modulus of the solution, expected by translation invariance of the system and its solutions.

As boundary conditions cannot be changed, solutions are stable as class of solutions satisfying boundary conditions.

# A Minimal Example, The 1+1D Kink



$$H = -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial a^2}$$

$$L = \frac{1}{2} M \dot{a}^2$$

Moduli used to define the action of the soliton.

Can move through space like a particle with particle like properties.

Motion, collisions, bound states etc.

# Derrick's Theorem

Historically, the original obstacle to the soliton model of elementary particles.

In 3 spatial dimensions, non existence of stationary localised solutions to the non-linear Klein-Gordon equation.

$$\nabla^2 \theta - \frac{\partial^2 \theta}{\partial t^2} = \frac{1}{2} f'(\theta), \quad \theta(x, t) \in \mathbb{R}, \quad x \in \mathbb{R}^3,$$

Loopholes include presence of Higgs, gauge fields etc, and also critically the case of localised solutions in space that are separable in time and periodic as a function of time.

# Vakhitov–Kolokolov criterion

In the presence of a field profile that depends on omega, with time dependent phase,

$$\phi_{\omega}(x)e^{-i\omega t}$$

the Noether charge variation with omega having a sign:

$$\frac{d}{d\omega}Q(\omega) < 0$$

Gives the stability condition for a solitary wave equation in a wide class of U(1) invariant Hamiltonian systems.



# The Q Ball

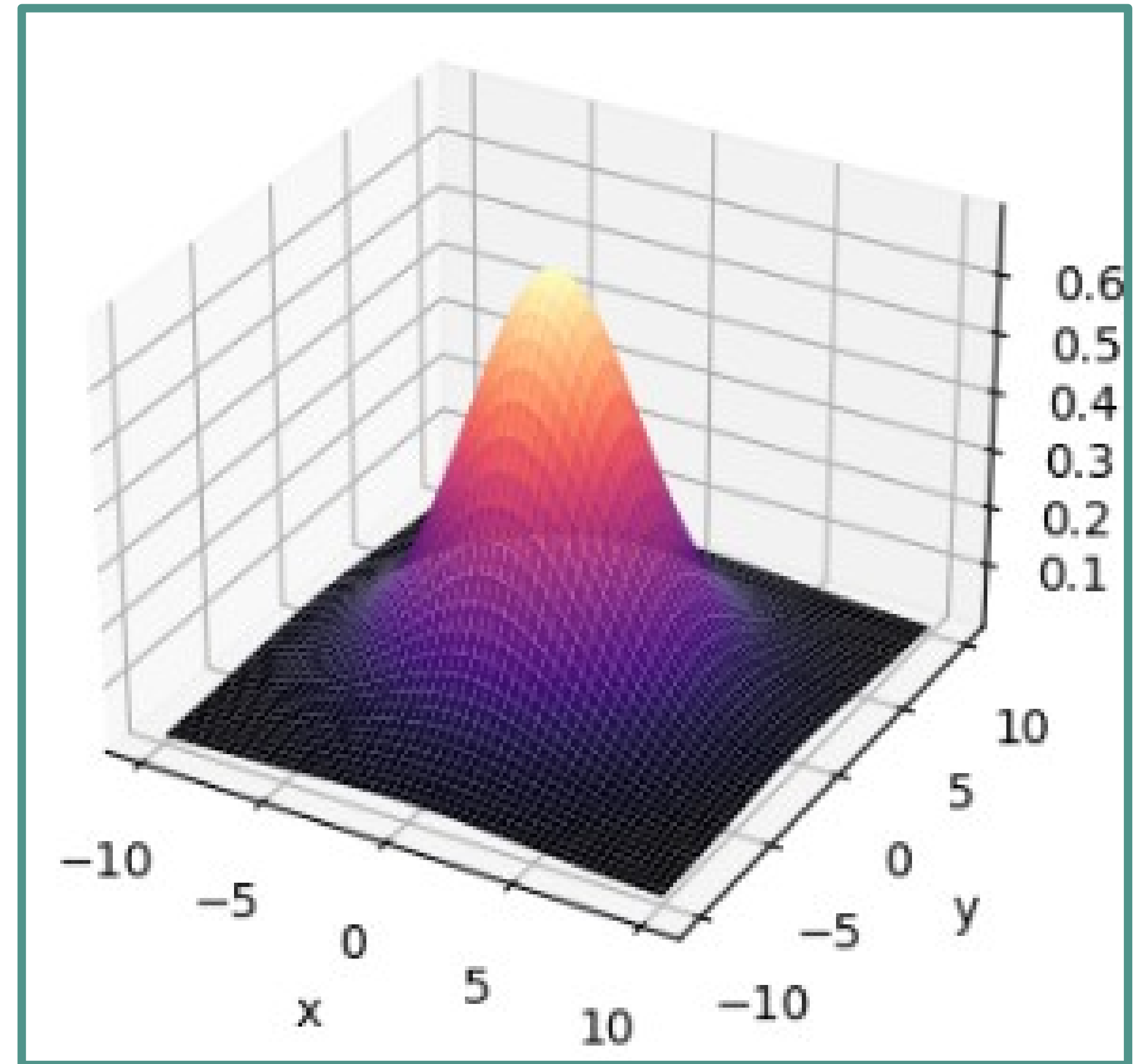
A Q-ball is a non-topological *soliton*, stable via a conserved Noether charge (Q) rather than a topological charge.

Stability against decay into quanta in cases where the Q ball energy is less than an equivalent number of quanta.

Potential must satisfy conditions of some attractive term beyond a mass term that makes 'lumps' in field space lower energy than free states.

Coleman: Proof that a U(1) globally symmetric complex scalar admits solitons that are stable against quantum effects.

Such a minimum energy for given Q, stable against fission into smaller Q-balls.



$$\phi = \sigma(r)e^{i\omega t}$$

# The Q Ball

Local extremum of the Hamiltonian in classical scalar field theory among all configurations that have a *given value*  $Q$  of conserved global  $U(1)$  charge.

Or thin-walled, has a simple step function out to radius  $R$  for spherically symmetric lump.

If the scalar field has Yukawa coupling to fermions Q-balls may decay into fermions transferring charge.

Amounts to finding an extremum of functional with a Lagrange multiplier method.

$$\frac{dE}{d\omega} = \omega \frac{dQ}{d\omega}$$

Time translation broken along with  $U(1)$  but a new expression  $F=H-\omega Q$  is unbroken.

Large Q-Balls?

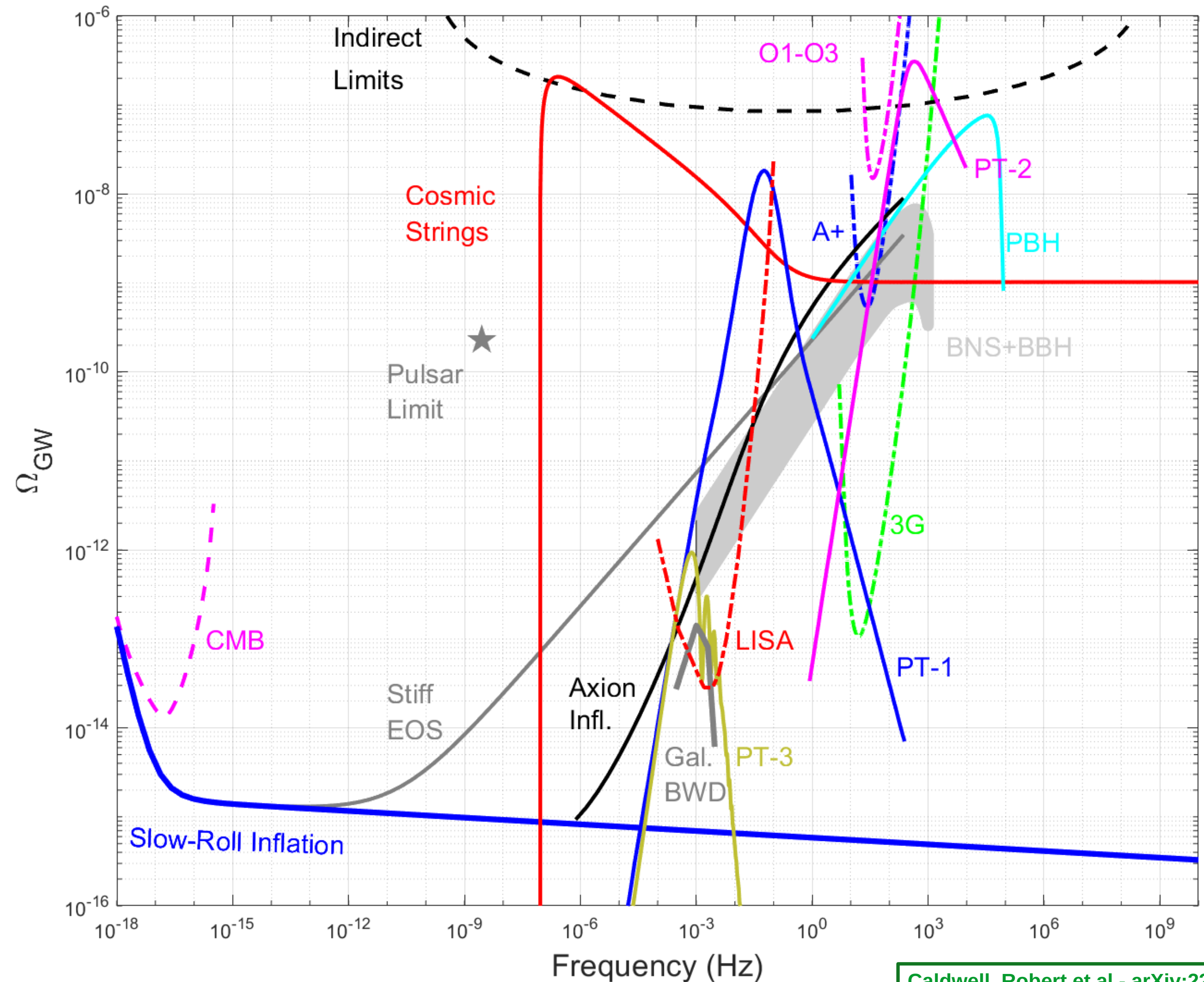
# Gravitational Waves: What's out there?

There is an on going search for dark matter.

We can imagine dark sectors, dark phase transitions, fragmentation...

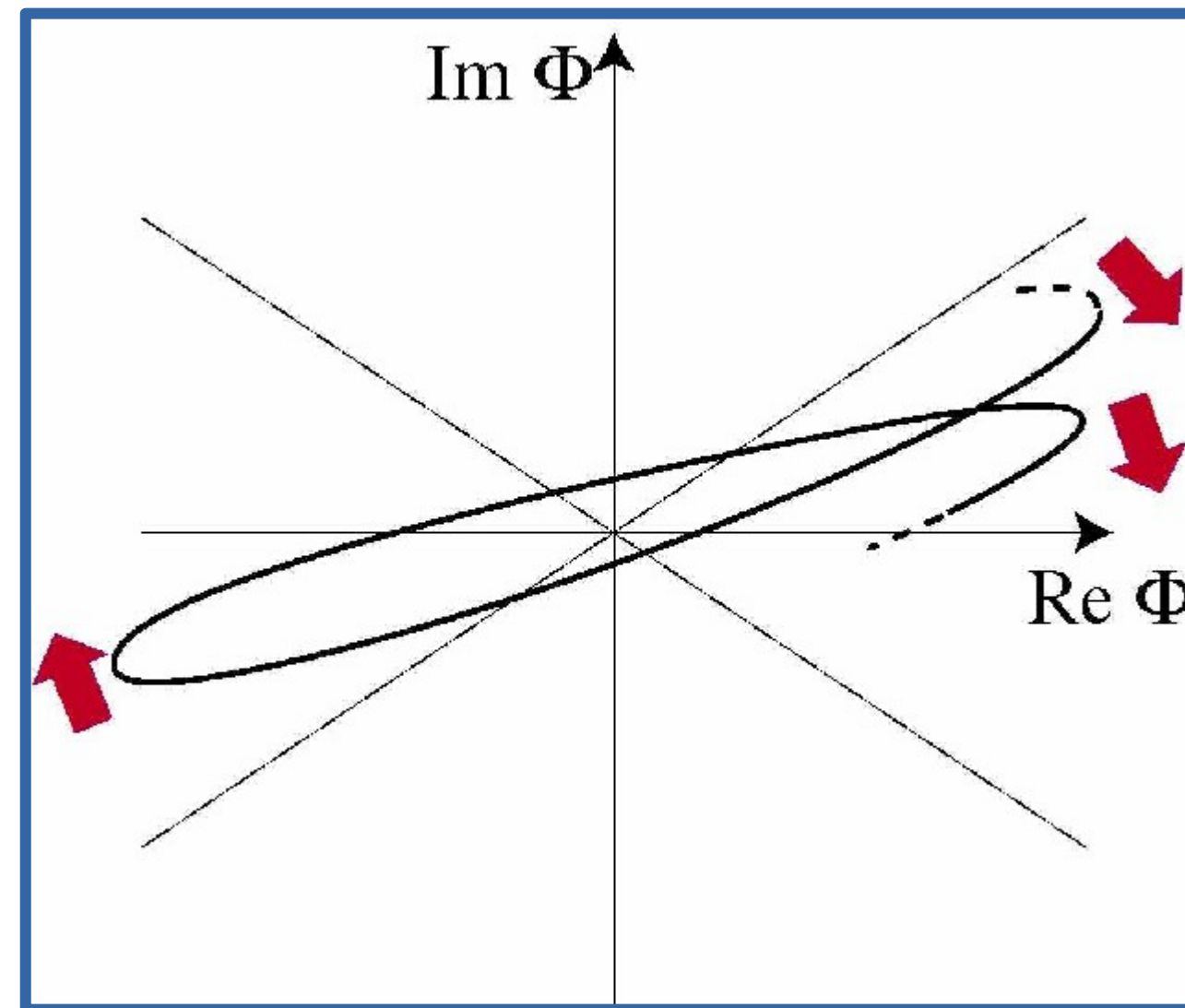
GW astronomy is sector agnostic.

However, we need to understand what sort of things we might be looking at.



# Affleck-Dine Mechanism Review

Q-ball with large charge expected to form in some models of the Affleck Dine mechanism.



Flat directions in e.g SUSY breaking potentials generate large field values with winding phase.

If spatial inhomogeneities in the AD field are induced to grow...

# Fragmentation and Affleck-Dine Mechanism

Formation of Q balls in the early universe.

Fluctuations around the homogenous mode grow non-linearly.

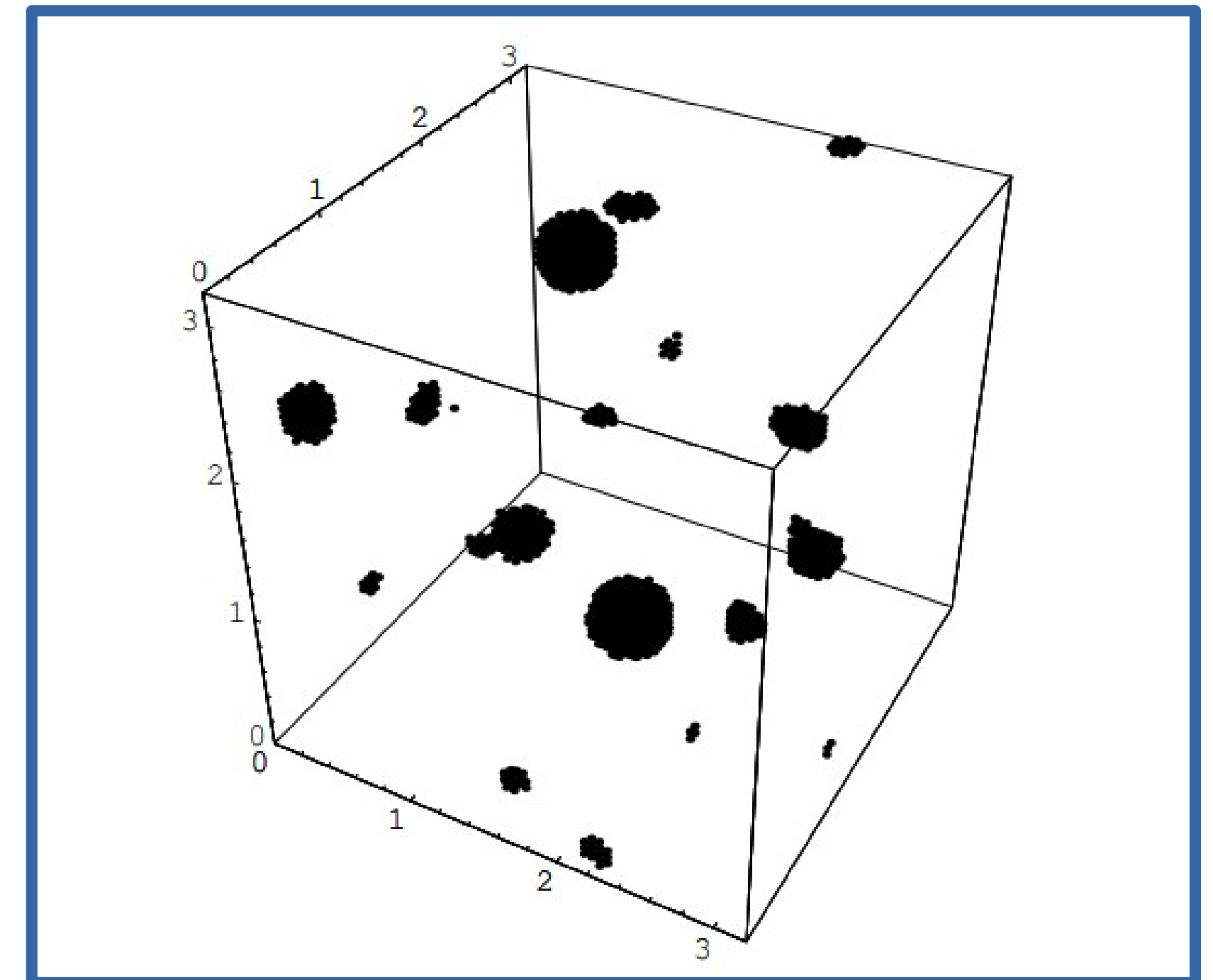
$$V(\Phi) = m^2|\Phi|^2 \left[ 1 + K \log \left( \frac{|\Phi|^2}{M^2} \right) \right] - cH^2|\Phi|^2 + \frac{\lambda^2}{M^2}|\Phi|^6$$

‘Fragmented’ background field devolves into clumps of many Q-balls if  $V/\phi^2$  has a non-zero global minimum.

GWs from Q-Ball formation.

Chiba, Kamada, Yamaguchi, Arxiv: 0912.3585

*Q-Ball formation with velocity can lead to collision events.*



S. Kasuya, M. Kawasaki Arxiv:0002285

# Q Balls in the literature

## As Dark Matter

The interactions of Q-balls can be probed in a number of ways. Q-balls can be stopped by dense objects such as white dwarfs and neutron stars. Q-Balls passing through the earth. Q-Ball accumulation in dense objects.

Kusenko, Shoemaker Arxiv:0905.3929

## Solitosynthesis

Q-Balls continue to collide and grow until critical size where pressure drives the expanding bubble into a phase transition mechanism. Unique GW predictions compared to first order phase transitions.

Croon, Kusenko, Mazumdar, White, Arxiv: 1910.09562

# Q Balls in the literature

## Q-Balls from phase transitions

Q-Ball dark matter formation from shrinking false vacuums packing charge into new stable configurations.



E. Krylov, A. Levin, V. Rubakov, arXiv:1301.0354

## Q-Ball galactic Cores

Troitsky, Arxiv: 1510.07132.

Supermassive compact objects (SMCOs) that sit within galactic centers are typically assumed to be black holes. Some models suggest e.g the Milky Way's core can be a giant non-topological soliton. Models can exist in combination with tiny Q-balls produced in the early Universe that some or all of DM.

# Simulating Collisions



# Q-Ball Physics

Existence conditions for Q balls

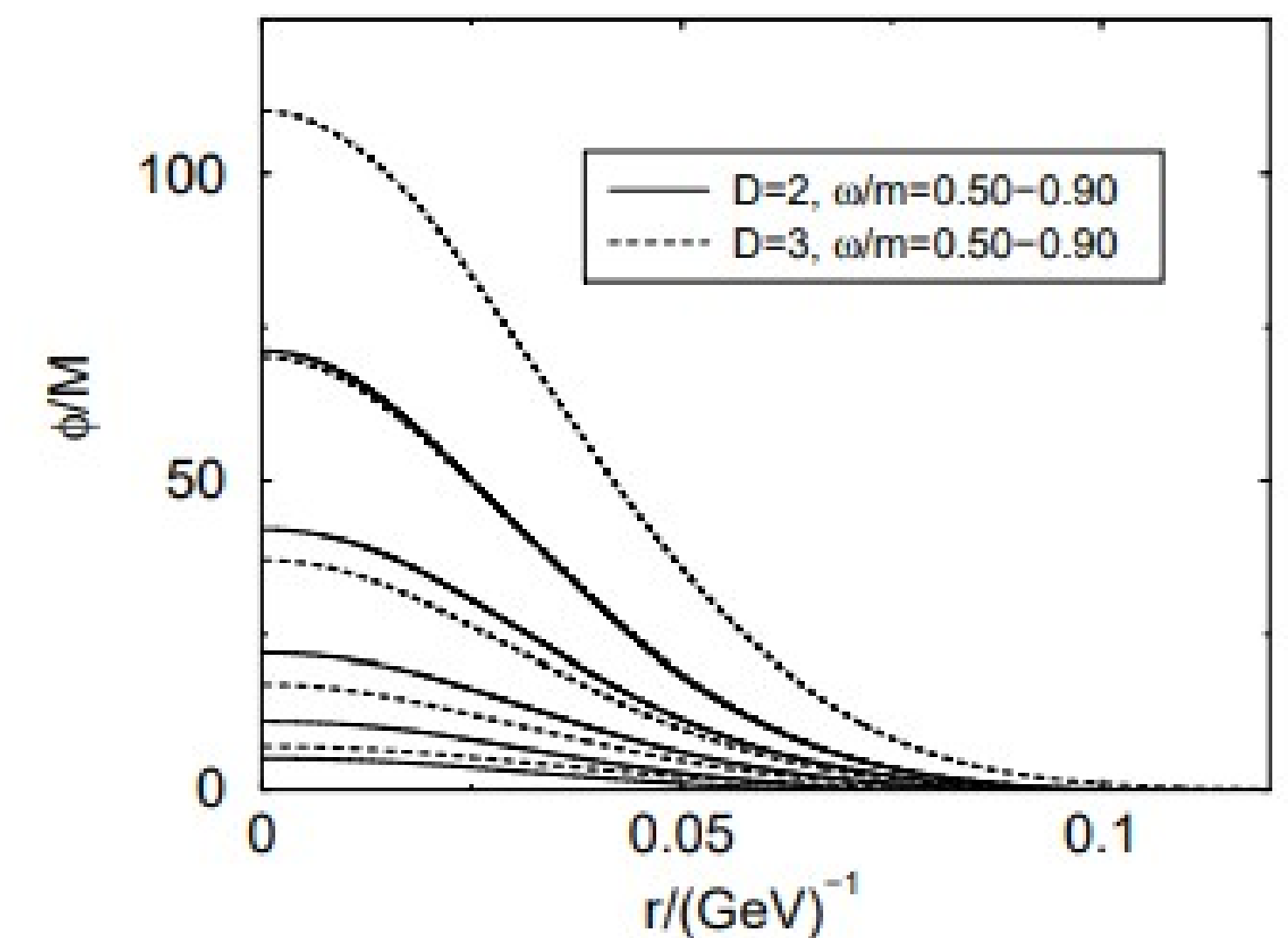
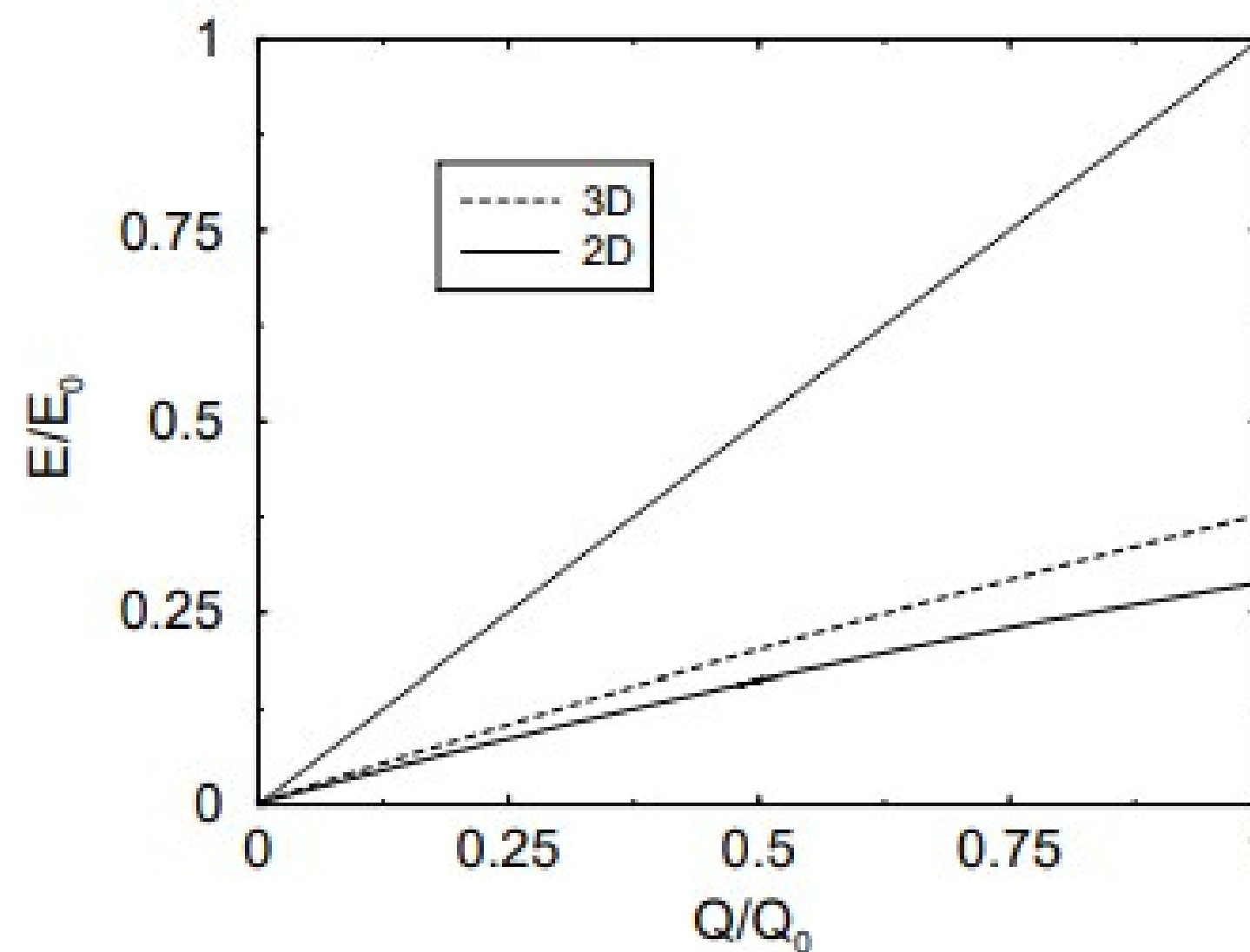
$$\omega_{min} < \omega < \omega_{max} < m$$

For field ansatz,

$$Q = 2\omega \int \phi(r)^2 d^D r.$$

Compare the boundary line for stability in 2D, 3D.

$$Q = \frac{1}{i} \int (\phi^* \partial_t \phi - \phi \partial_t \phi^*) d^D x$$
$$E = \int [|\dot{\phi}|^2 + |\nabla \phi|^2 + U(\phi^* \phi)] d^D x.$$



# Field Profile

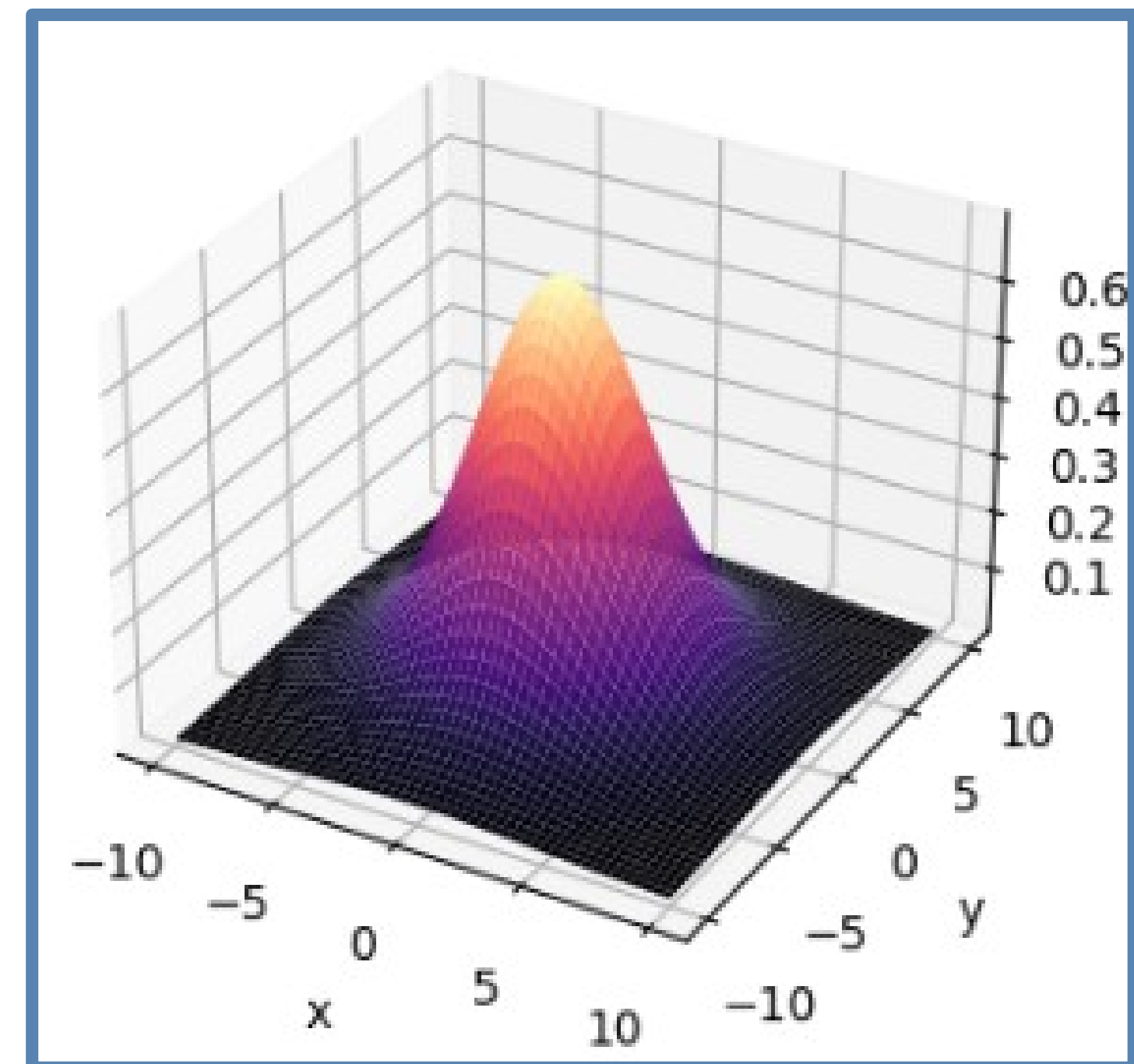
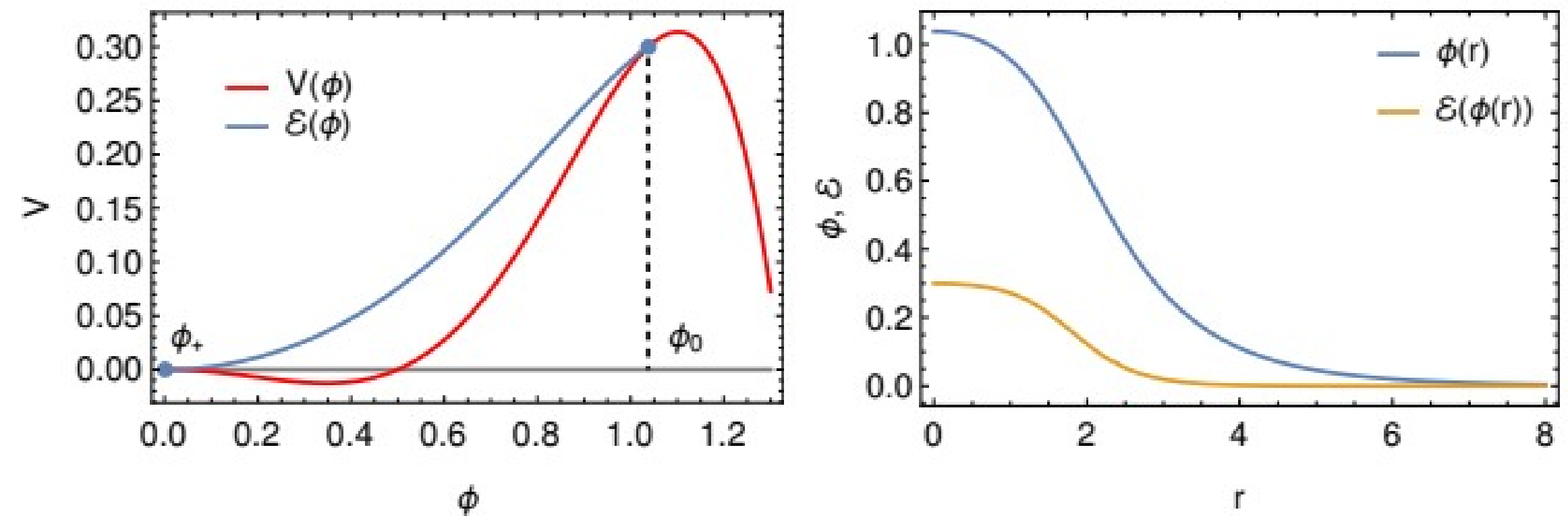
Solving for field profile with radius similar to finding tunnelling action between two vacua.

Some potentials have analytic solutions.

Generally however need to use a shooting method to find numerical radial profile for Q ball as ‘ground state’ for fixed charge.

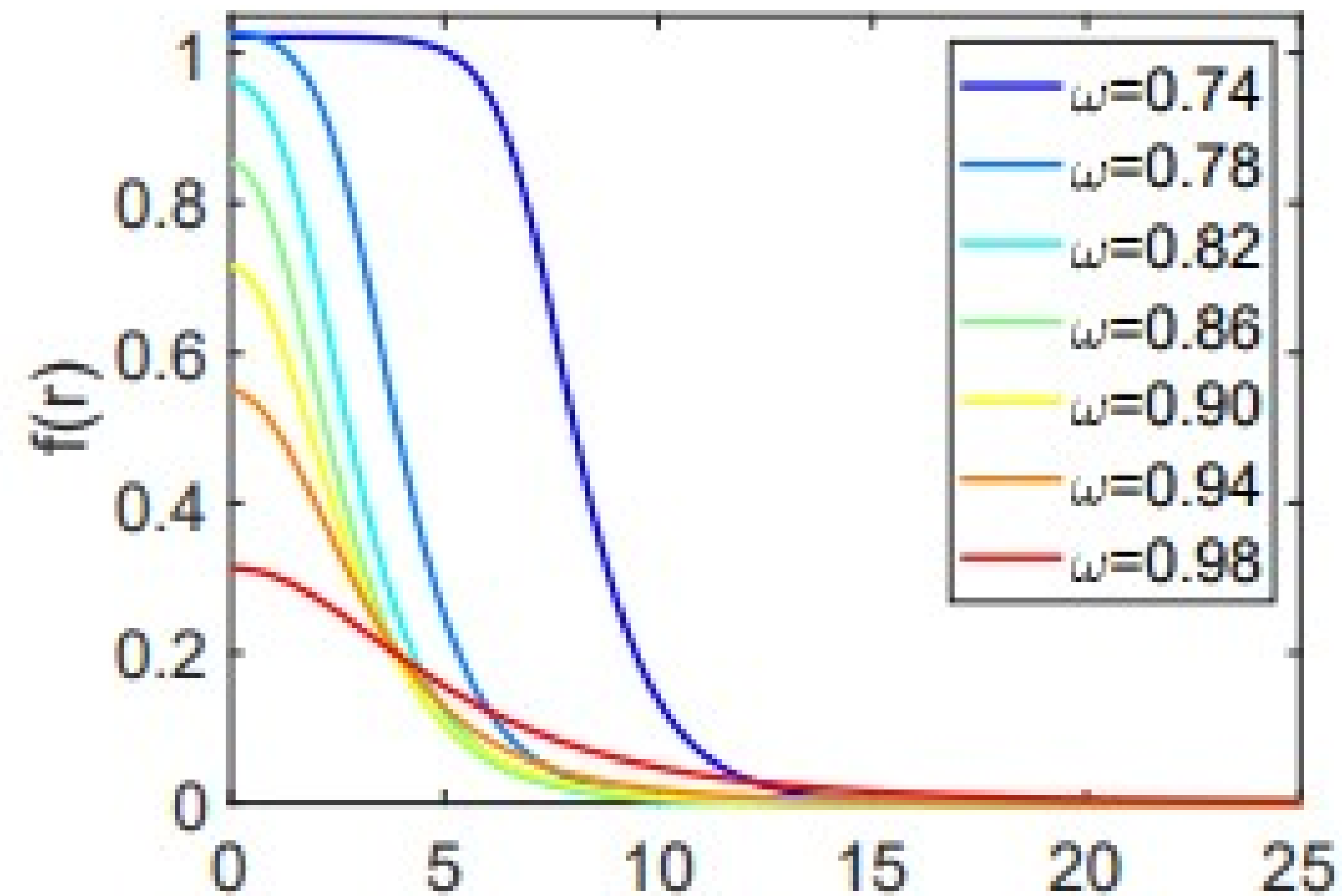
Solve for rest frame profile in flat spacetime.

Espinosa, Heeck, Sokhashvili,  
Arxiv:2307.05667

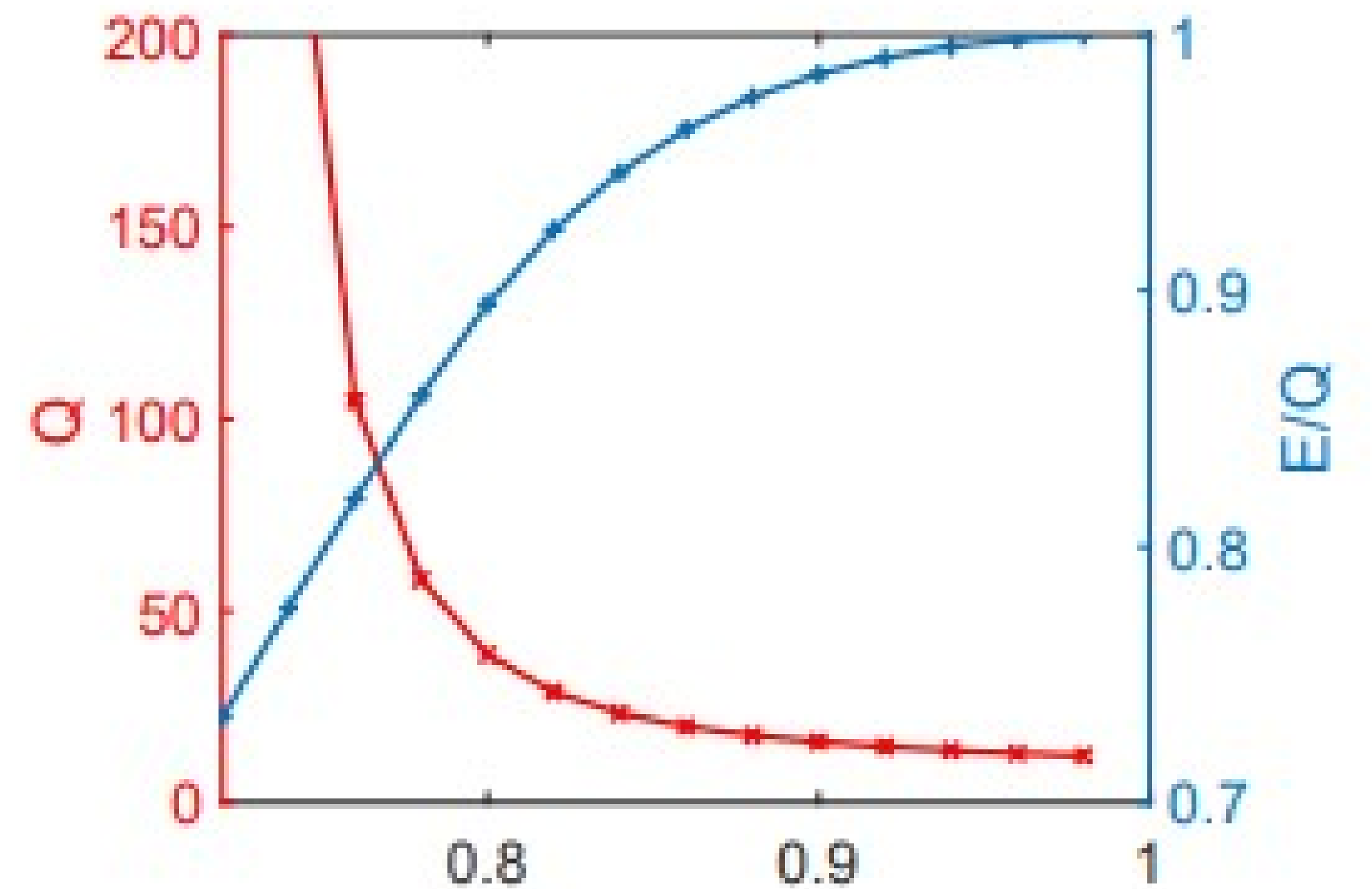


# Field Profiles

Xie, Saffin, Zhou Arxiv: 2101.06988



Thin vs Thick wall



Size, energy, charge, omega.

# Types of Interactions

Large range of possible scenarios, even limited to Q-ball Q-ball interactions.

Examine similar size Q-balls, like velocity.

E.g. Fusion Head on collision for charges  $Q_1, Q_2$ .

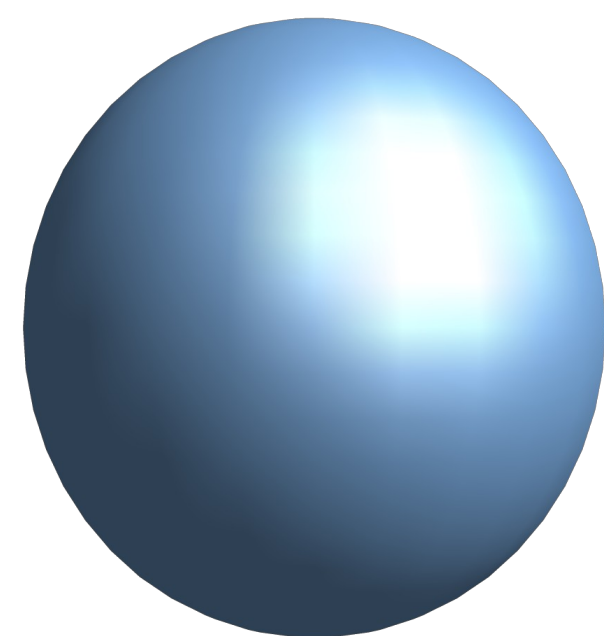
At high velocity can obtain pass through events.

Annihilation eg. for  $Q_2 = -Q_1 \rightarrow$  oscillon/ radiation coupling, boson star collapse? BH?

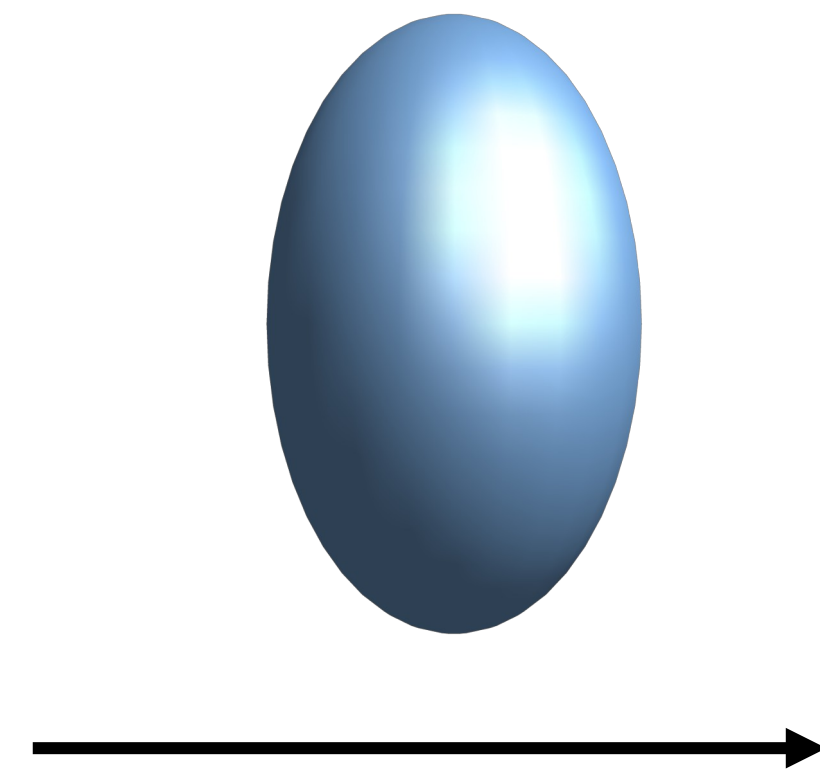
**Oscillons:** unstable but stationary, enduring finite lifetime, or **oscillatons** with gravity included.

**Boson stars:** gravitationally bound field lumps, where Q-ball stability given without gravity effects.

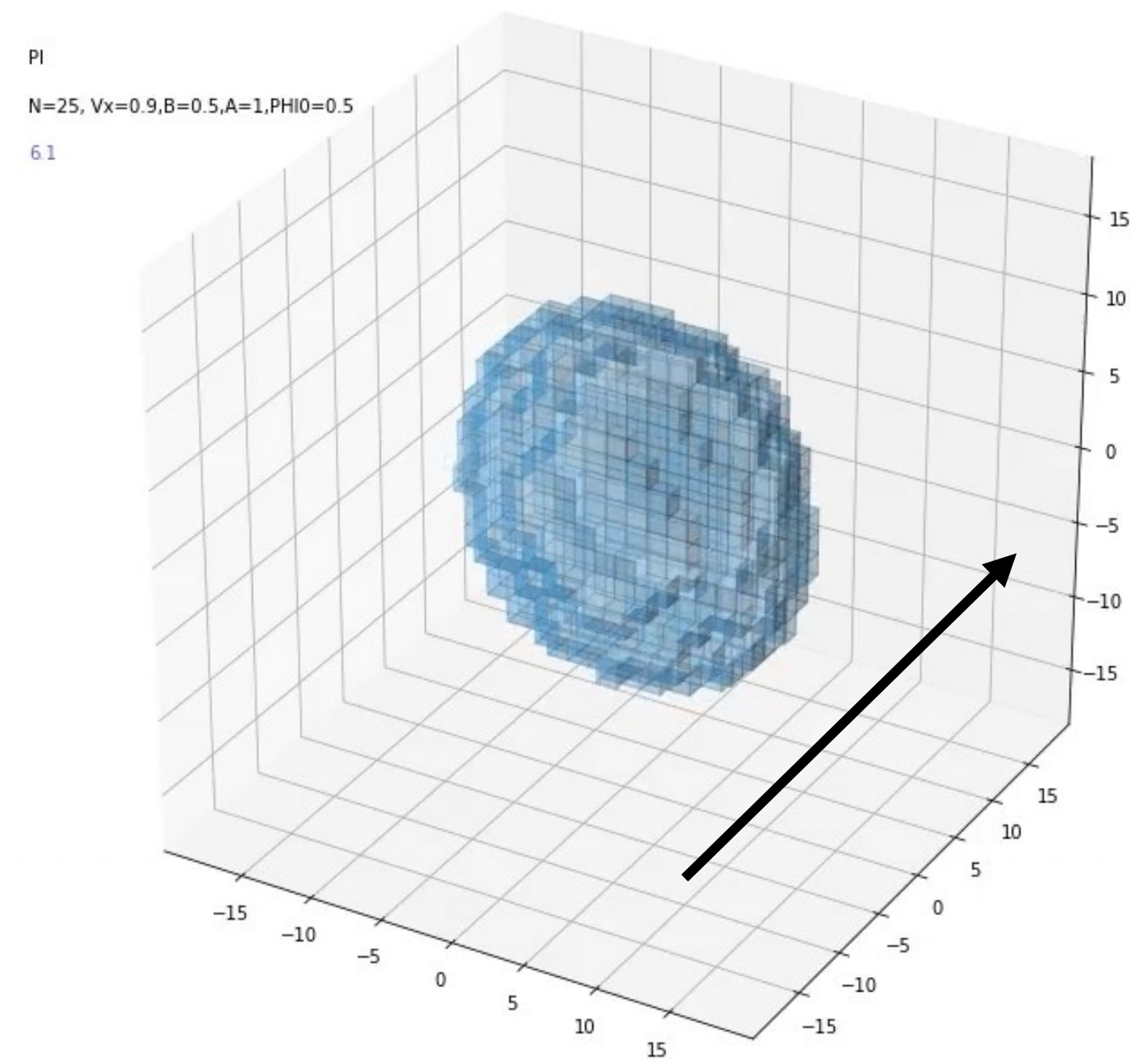
# Moving Q-Balls



# Boosted Q-Balls



# Boosted Q-Balls



# Boosted Q-Balls

Scalar profile solution solved for in stationary frame.

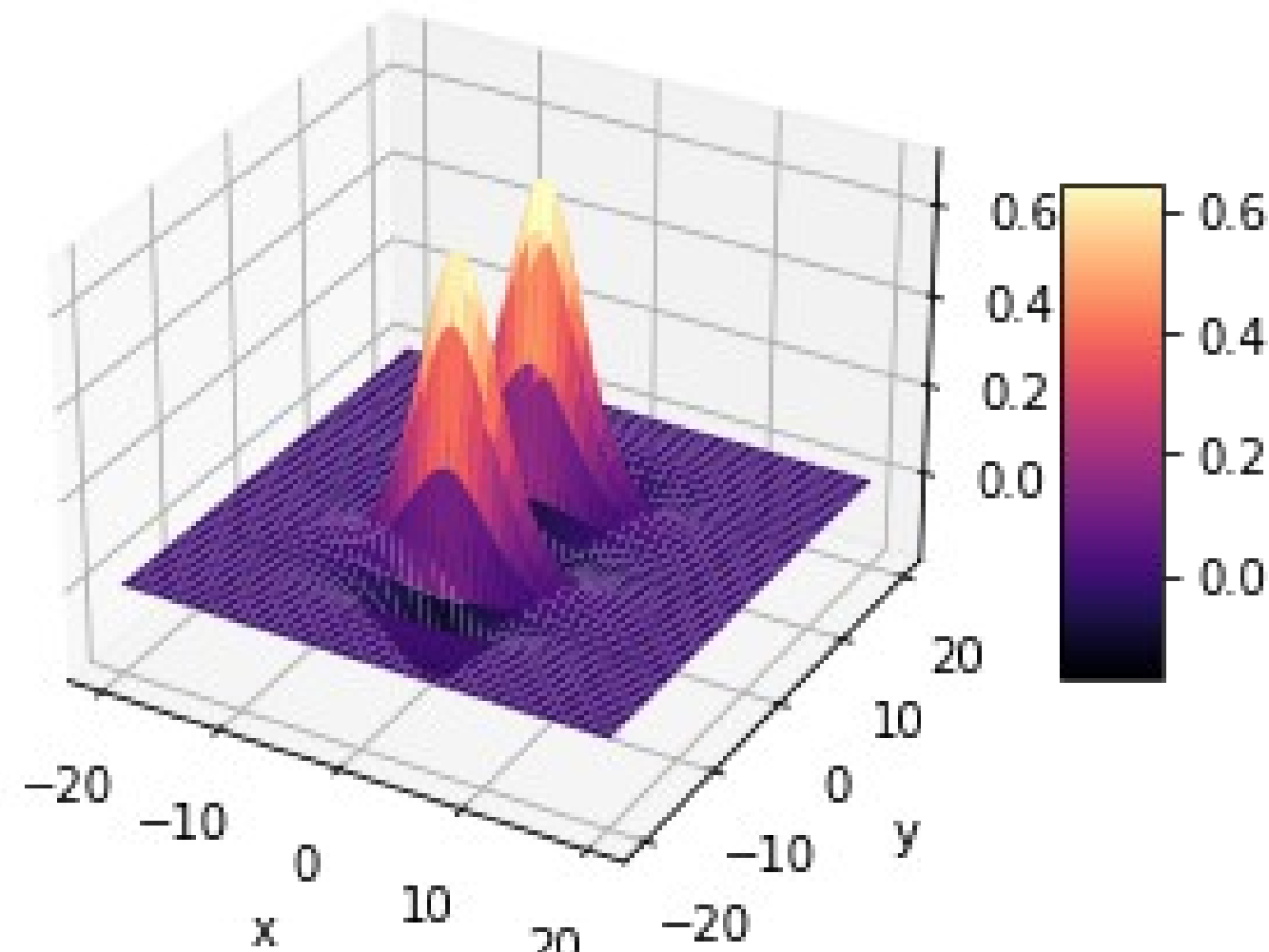
Take the boosted solution for both scalar and conjugate momenta of the fields.

Prepared state pairs can be arranged for input potential and chosen charge Q, velocity.

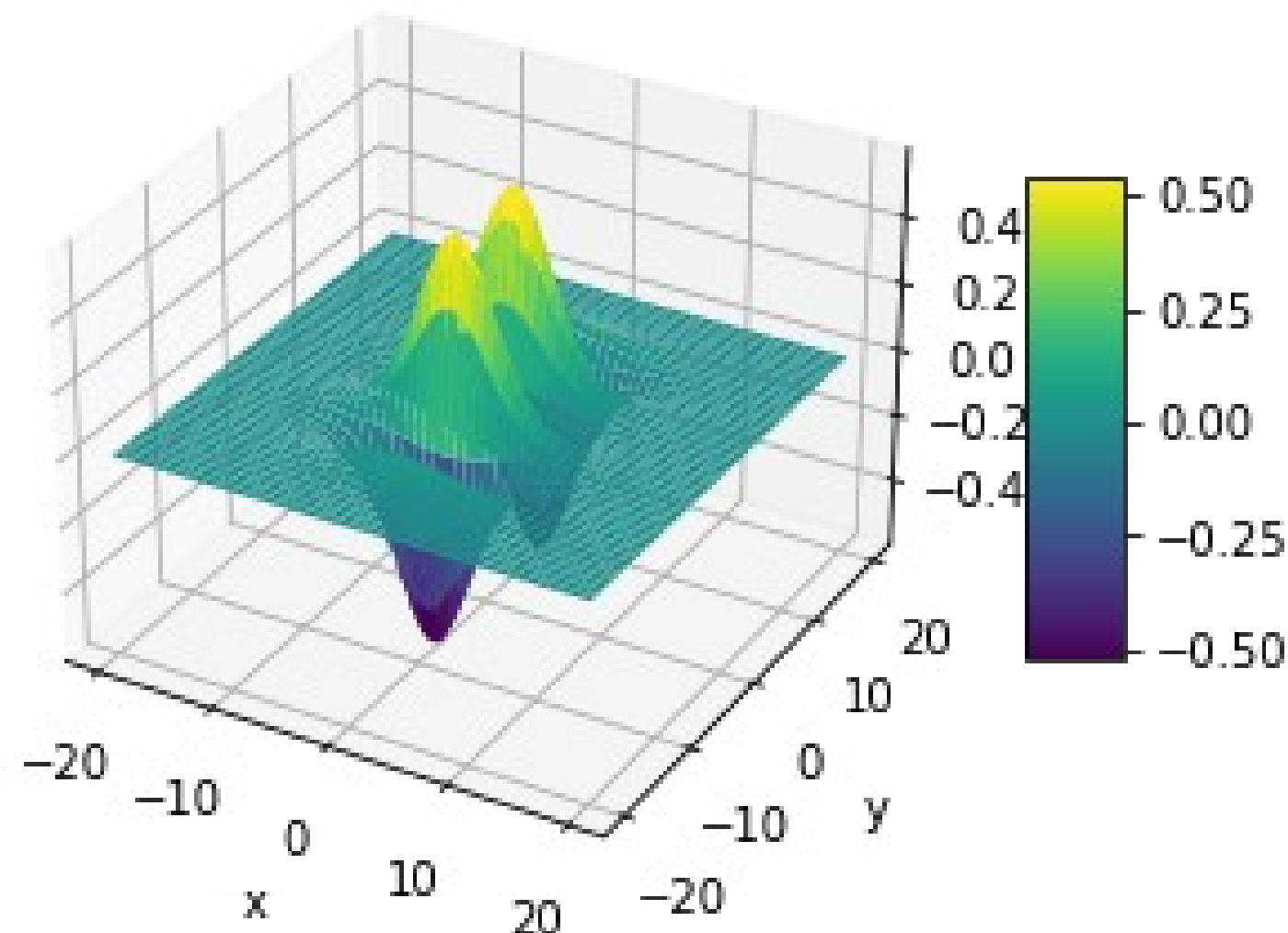
Q-ball, Anti Q-ball,

N-object Q-ball scattering.

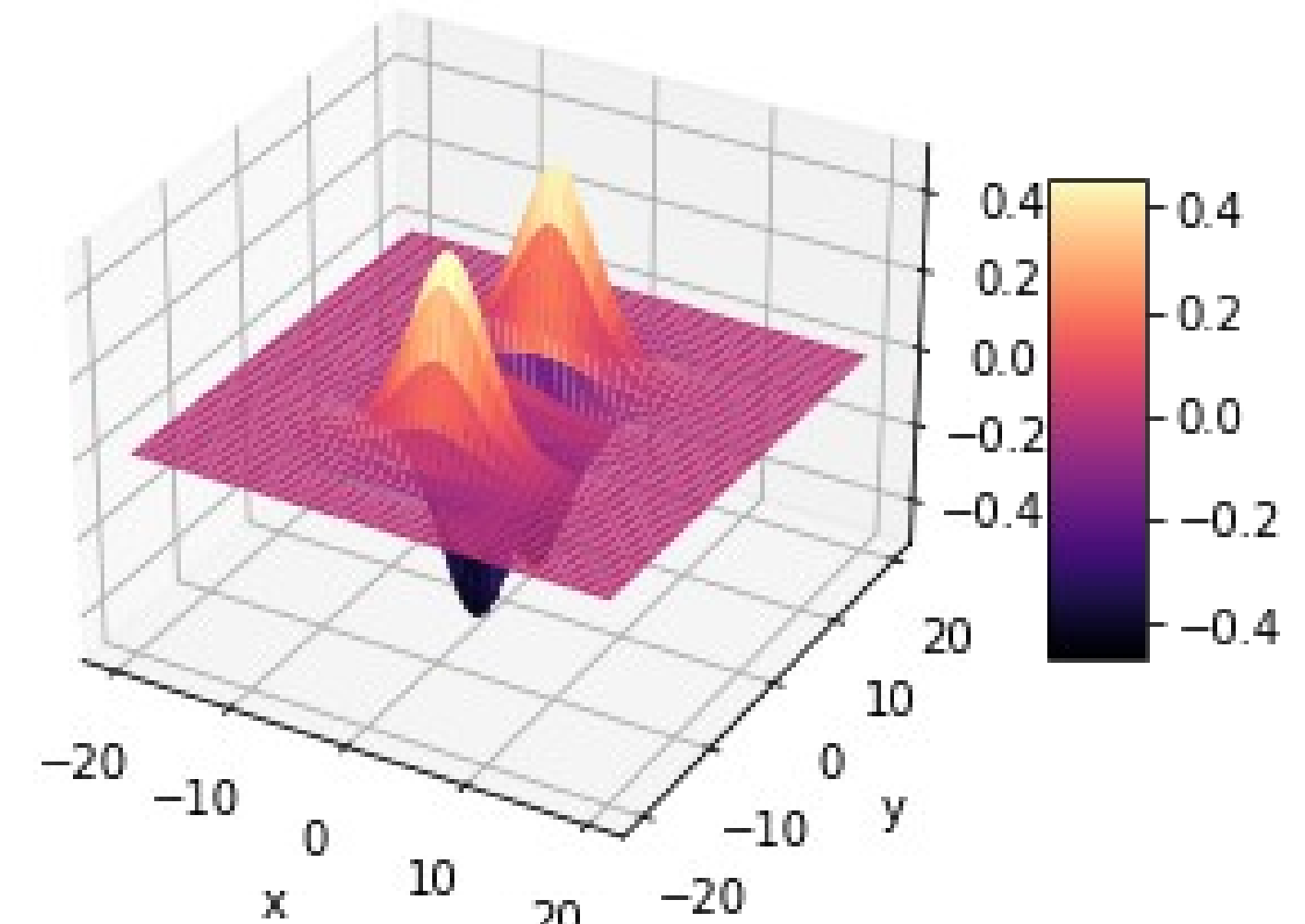
$$\phi_1^0 = \sigma(\gamma x, y) \cos(\omega \gamma v x)$$



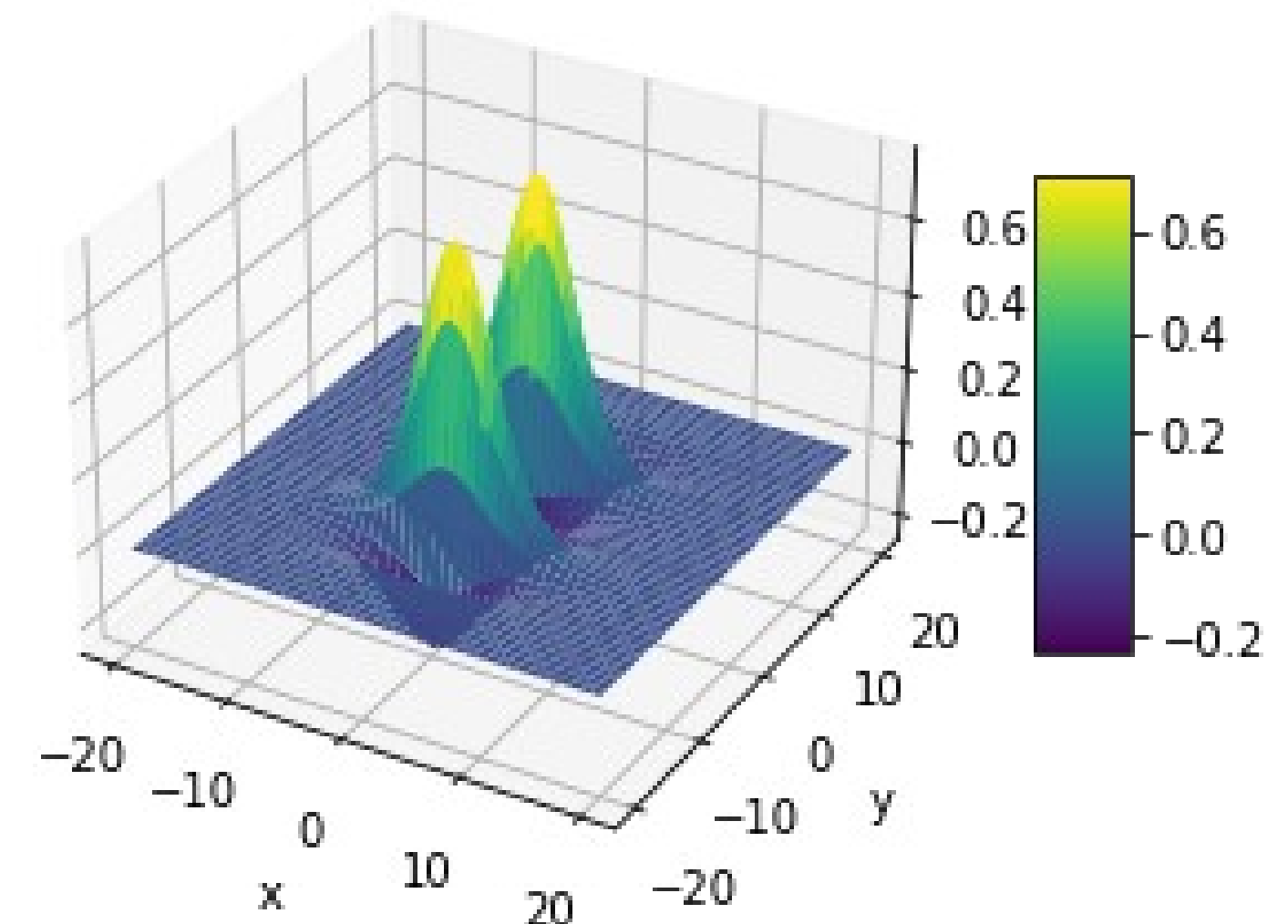
$$\pi_1^0 = v \gamma \cos(\omega \gamma v x) \frac{\partial \sigma(\gamma x, y)}{\partial x} - \omega \gamma \phi_2$$



$$\phi_2^0 = \sigma(\gamma x, y) \sin(\omega \gamma v x)$$



$$\pi_2^0 = v \gamma \sin(\omega \gamma v x) \frac{\partial \sigma(\gamma x, y)}{\partial x} + \omega \gamma \phi_1$$



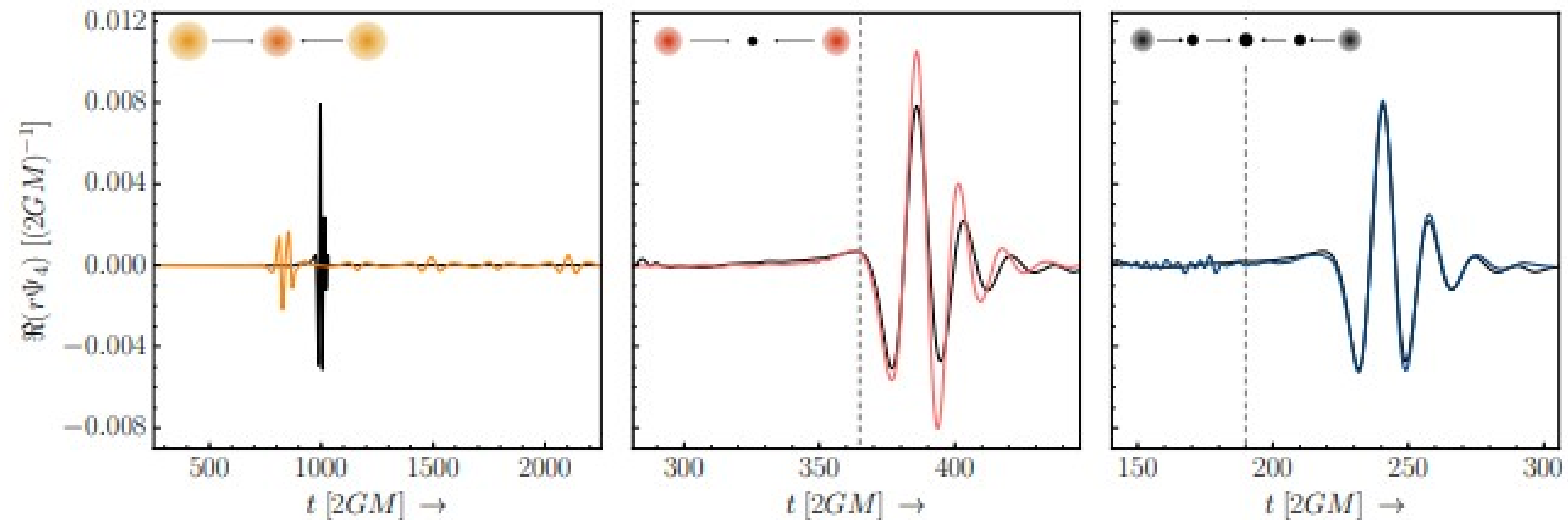


# Simulating Interactions

Charge, size, speed, ..

Of course as size increases the energy per unit charge profile can be calculated for the increasing gravitational influence.

Compare head-on Merger of two equal mass Boson stars.

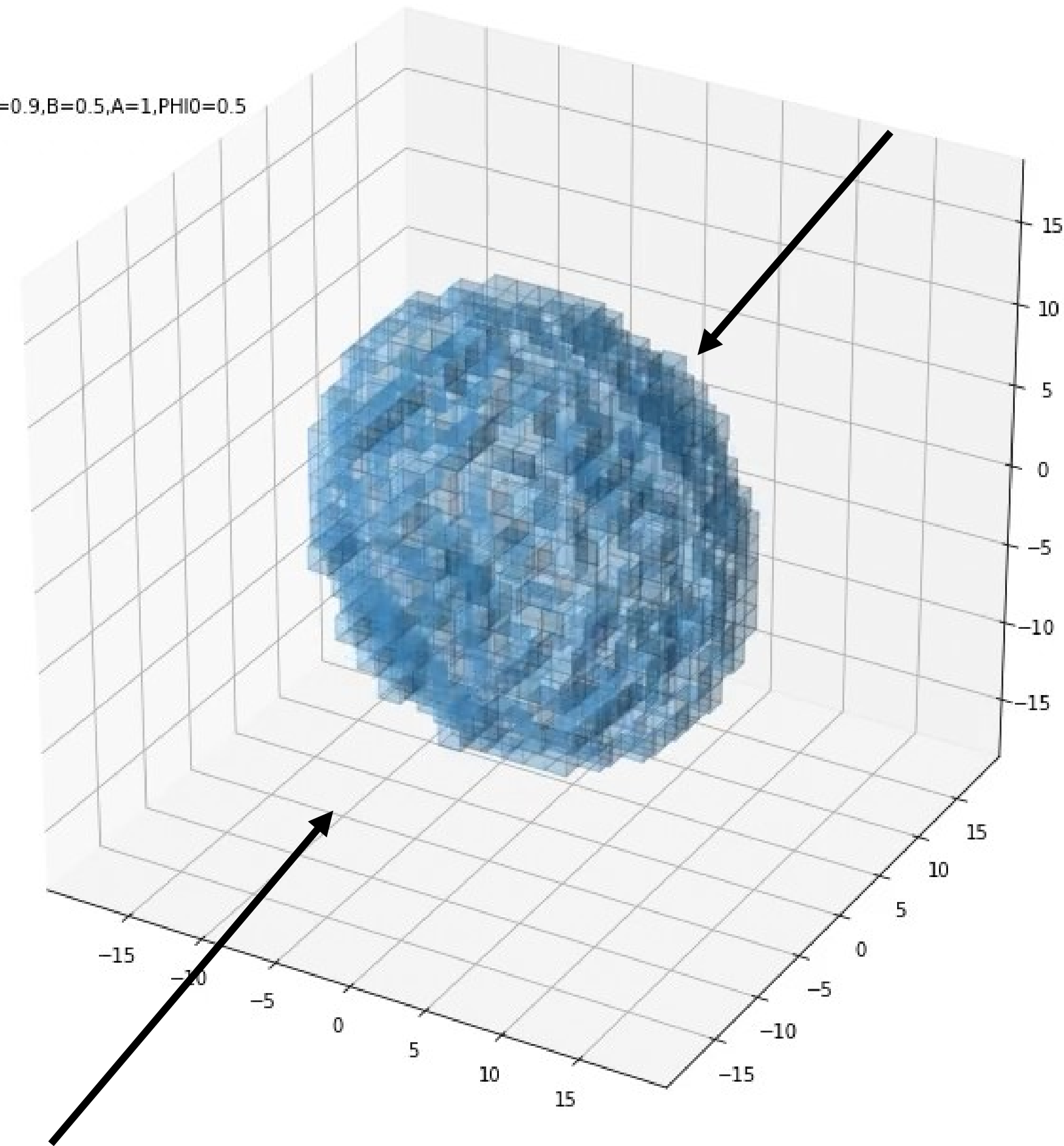


**Helfer, Lim, Garcia, Amin, Arxiv: 1802.06733**

PI

$N=25, V_x=0.9, B=0.5, A=1, \text{PHI0}=0.5$

7.6



# Switching on Gravity

# Coupling to Gravity

Consider cases with weak gravitational field where first order perturbation  $\ll 1$ .

$$S = \int \sqrt{g} d^4x \left( \frac{1}{2} \partial^\mu \phi_\ell \partial_\mu \phi_\ell - V + \frac{M_P^2}{2} R \right)$$

$$\beta_{ij} \equiv (\ln \mathcal{G})_{ij} \quad h_{ij} \approx \beta_{ij} - 2\delta_{ij} \ln a$$

The kinetic and potential energy of scalars is defined as usual with fields and conjugate momenta.

$$S = \int dt (K_f - G_f - V_f + K_g - G_g)$$

“Kinetic” and “Gradient” energy of metric perturbations defined up to second order.

$$\Pi_{\beta_{ij}} \Big|_{(i_1, i_2, i_3)} = \frac{M_P^2}{4} e^{\beta/2} (2 - \delta_{ij}) \left( \dot{\beta}_{ij} - \dot{\beta} \delta_{ij} \right) \Big|_{(i_1, i_2, i_3)}$$

$$K_g \approx \frac{M_P^2}{4} \int e^{\beta/2} d^3x \times \left( \dot{\beta}_{23}^2 + \dot{\beta}_{31}^2 + \dot{\beta}_{12}^2 - \dot{\beta}_{11} \dot{\beta}_{22} - \dot{\beta}_{22} \dot{\beta}_{33} - \dot{\beta}_{33} \dot{\beta}_{11} \right)$$

$$G_g \approx \frac{M_P^2}{4} a(t) \int d^3x \times \left( \beta_{23,1}^2 + \beta_{31,2}^2 + \beta_{12,3}^2 - 2\beta_{23,1}\beta_{31,2} - 2\beta_{31,2}\beta_{12,3} - 2\beta_{12,3}\beta_{23,1} - \beta_{22,1}\beta_{33,1} - \beta_{33,2}\beta_{11,2} - \beta_{11,3}\beta_{22,3} + 2\beta_{23,2}\beta_{11,3} + 2\beta_{31,3}\beta_{22,1} + 2\beta_{12,1}\beta_{33,2} \right)$$

# Symplectic Integrators

State vector on lattice,  $f$ , stores all  $\mathbf{q}$  and conjugate momenta  $\mathbf{p}$ .

$$f(t + dt) = e^{\mathbf{H}dt} f(t)$$

For  $\mathbf{H} = \mathbf{K} + \mathbf{P}$ , nth order ‘leapfrog’ algorithm for non-commuting operators can be found. Using the coefficients for the sixth order integrator as **HLATTICE**.

$$e^{\mathbf{H}dt} = e^{c_1 \mathbf{K}dt} e^{d_1 \mathbf{P}dt} e^{c_2 \mathbf{K}dt} e^{d_2 \mathbf{P}dt} \dots + O(dt^{n+1})$$

$$e^{\mathbf{K}dt} \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} = \begin{pmatrix} \mathbf{p} \\ \mathbf{q} + \frac{\partial \mathbf{K}}{\partial \mathbf{p}} dt \end{pmatrix}$$

$$e^{\mathbf{P}dt} \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} = \begin{pmatrix} \mathbf{p} - \frac{\partial \mathbf{P}}{\partial \mathbf{q}} dt \\ \mathbf{q} \end{pmatrix}$$

# Collision Simulations

Can add additional scalar fields can be added, e.g. light radiation from coupled axion like particle.

Symplectic integrators are noted for long term stability, e.g many body astrophysical simulations.

$$e^{\mathbf{K}_1 dt} \begin{pmatrix} \phi_\ell|_{i_1, i_2, i_3} \\ \Pi_{\phi_\ell}|_{i_1, i_2, i_3} \\ \beta_{11}|_{i_1, i_2, i_3} \\ \beta_{22}|_{i_1, i_2, i_3} \\ \beta_{33}|_{i_1, i_2, i_3} \\ \beta_{23}|_{i_1, i_2, i_3} \\ \beta_{31}|_{i_1, i_2, i_3} \\ \beta_{12}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{11}}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{22}}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{33}}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{23}}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{31}}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{12}}|_{i_1, i_2, i_3} \end{pmatrix} = \begin{pmatrix} (\phi_\ell + e^{-\beta/2} \Pi_{\phi_\ell} dt)|_{i_1, i_2, i_3} \\ \Pi_{\phi_\ell}|_{i_1, i_2, i_3} \\ \beta_{11}|_{i_1, i_2, i_3} \\ \beta_{22}|_{i_1, i_2, i_3} \\ \beta_{33}|_{i_1, i_2, i_3} \\ \left(\beta_{23} + \frac{2e^{-\beta/2}}{M_P^2} \Pi_{\beta_{23}} dt\right)|_{i_1, i_2, i_3} \\ \left(\beta_{31} + \frac{2e^{-\beta/2}}{M_P^2} \Pi_{\beta_{31}} dt\right)|_{i_1, i_2, i_3} \\ \left(\beta_{12} + \frac{2e^{-\beta/2}}{M_P^2} \Pi_{\beta_{12}} dt\right)|_{i_1, i_2, i_3} \\ \Pi_{\beta_{11}} + \frac{\kappa_1 dt}{2}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{22}} + \frac{\kappa_1 dt}{2}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{33}} + \frac{\kappa_1 dt}{2}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{23}}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{31}}|_{i_1, i_2, i_3} \\ \Pi_{\beta_{12}}|_{i_1, i_2, i_3} \end{pmatrix}$$

$$e^{\mathbf{K}_2 dt} \begin{pmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{33} \\ \Pi_{\beta_{11}} \\ \Pi_{\beta_{22}} \\ \Pi_{\beta_{33}} \end{pmatrix} = \begin{pmatrix} \beta_{11} + \frac{2e^{-\beta/2}}{M_P^2} (\Pi_{\beta_{11}} - \Pi_{\beta_{22}} - \Pi_{\beta_{33}}) dt \\ \beta_{22} + \frac{2e^{-\beta/2}}{M_P^2} (\Pi_{\beta_{22}} - \Pi_{\beta_{33}} - \Pi_{\beta_{11}}) dt \\ \beta_{33} + \frac{2e^{-\beta/2}}{M_P^2} (\Pi_{\beta_{33}} - \Pi_{\beta_{11}} - \Pi_{\beta_{22}}) dt \\ \Pi_{\beta_{11}} + \frac{\kappa_2 dt}{2} \\ \Pi_{\beta_{22}} + \frac{\kappa_2 dt}{2} \\ \Pi_{\beta_{33}} + \frac{\kappa_2 dt}{2} \end{pmatrix}$$

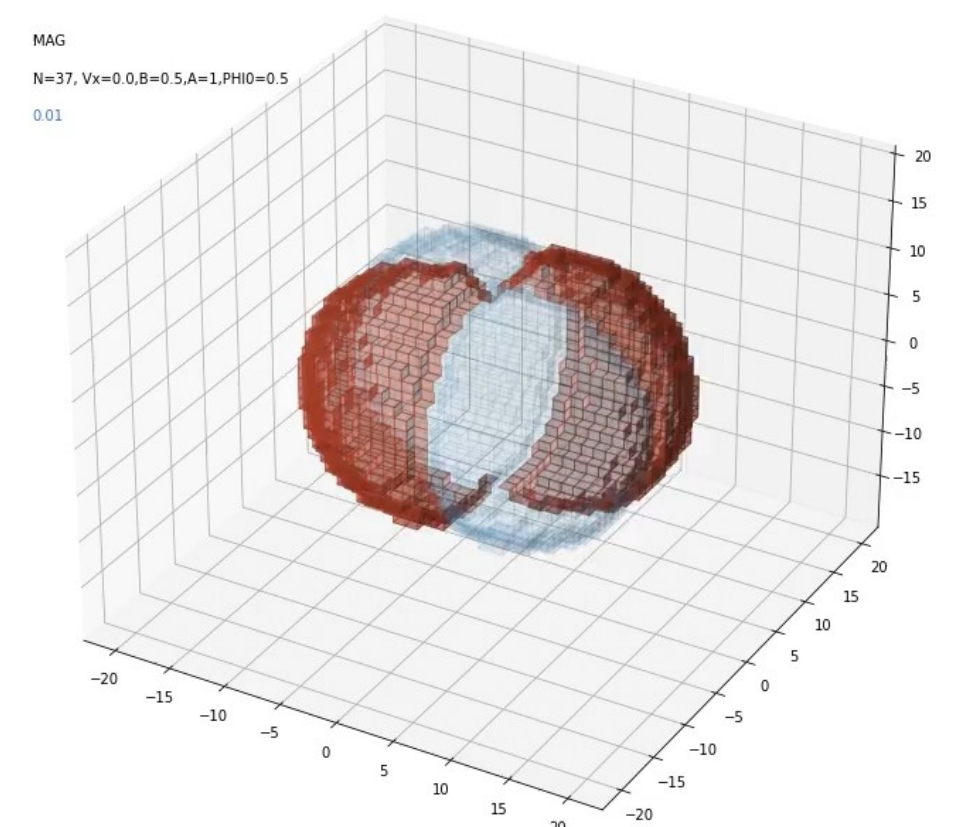
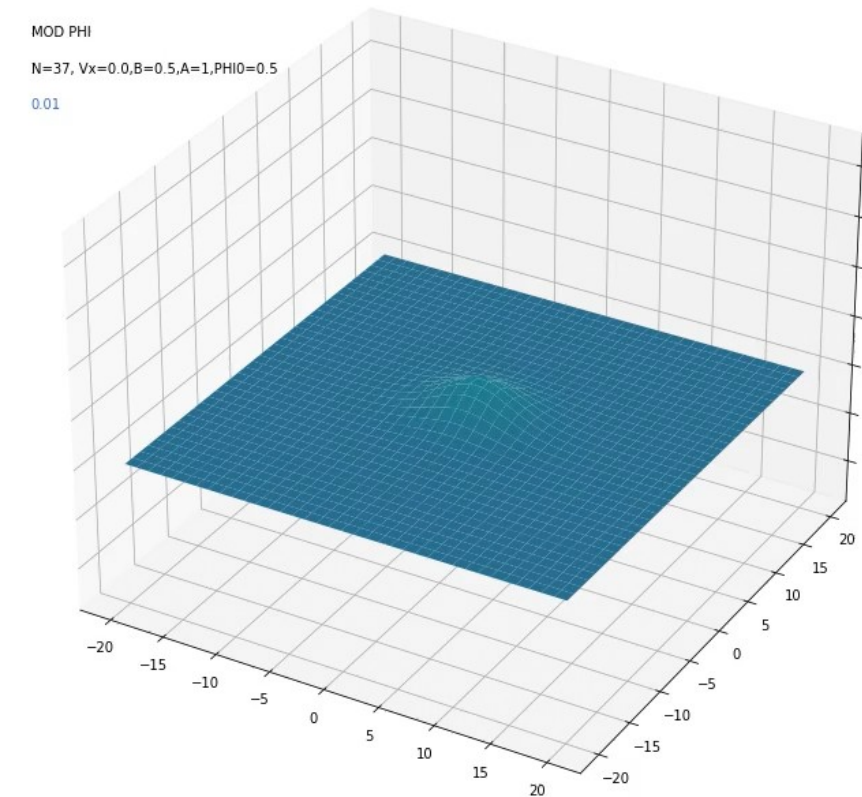
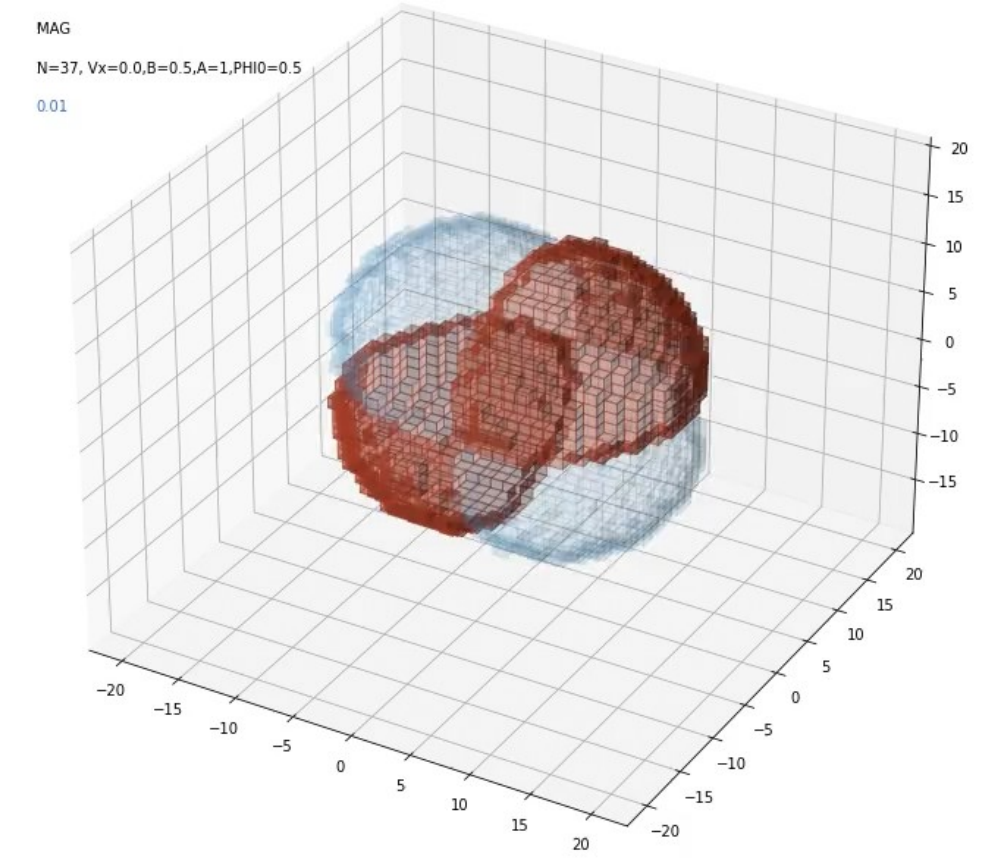
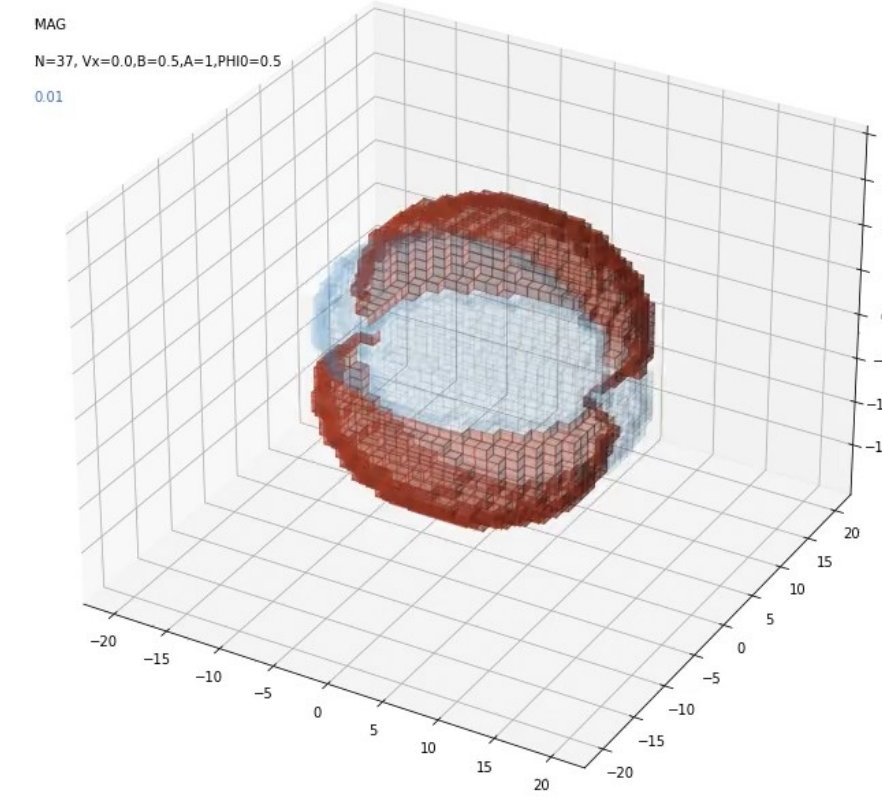
# Collision Simulation

Fusion or merger events with high energy collision.

Choose Q-balls of small enough size that gravity is not important for self-energy, but oscillon-merger creates a spike in perturbations during the collision.

Image the peak of the perturbation, detonation at most turbulent point of collision as oscillon forms.

Energy dissipated may be important for wind-down of 'excited state' in long term evolution.

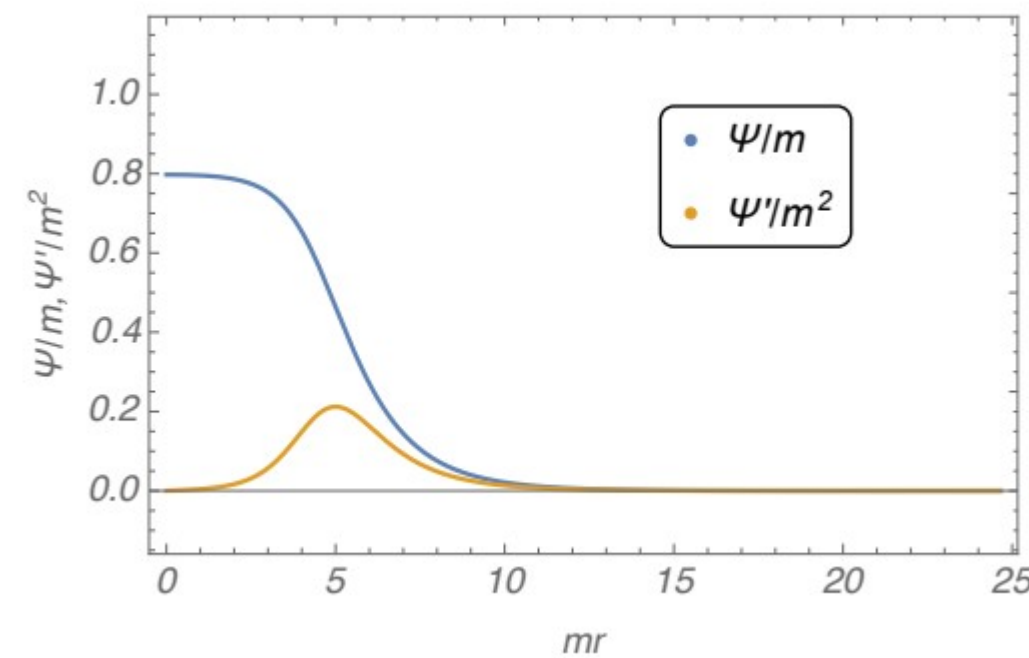


# **Solitons + Axions**



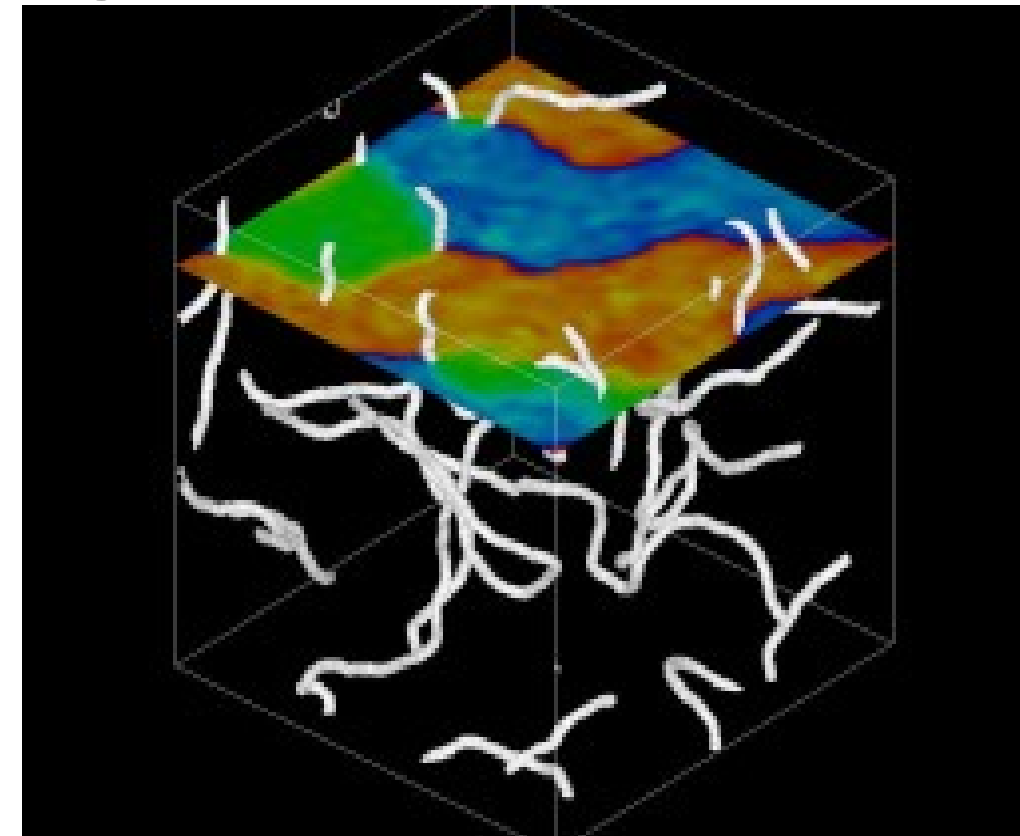
# Other Objects...

Axions/I-Balls/Oscillons  
Pseudo-breather states.



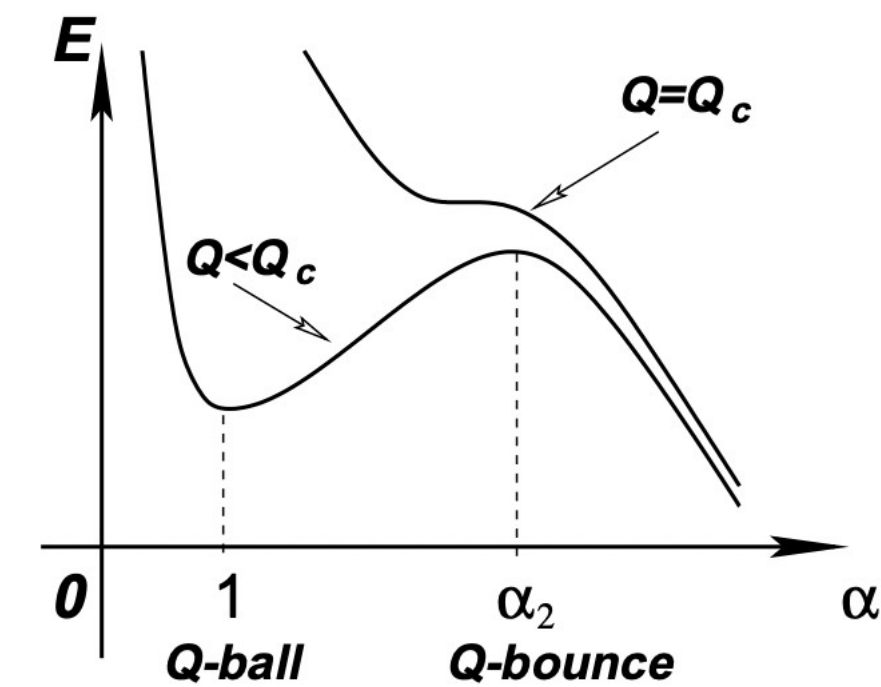
Ibe, Kawasaki, ArXiv:1901.06130

Strings+domain walls:



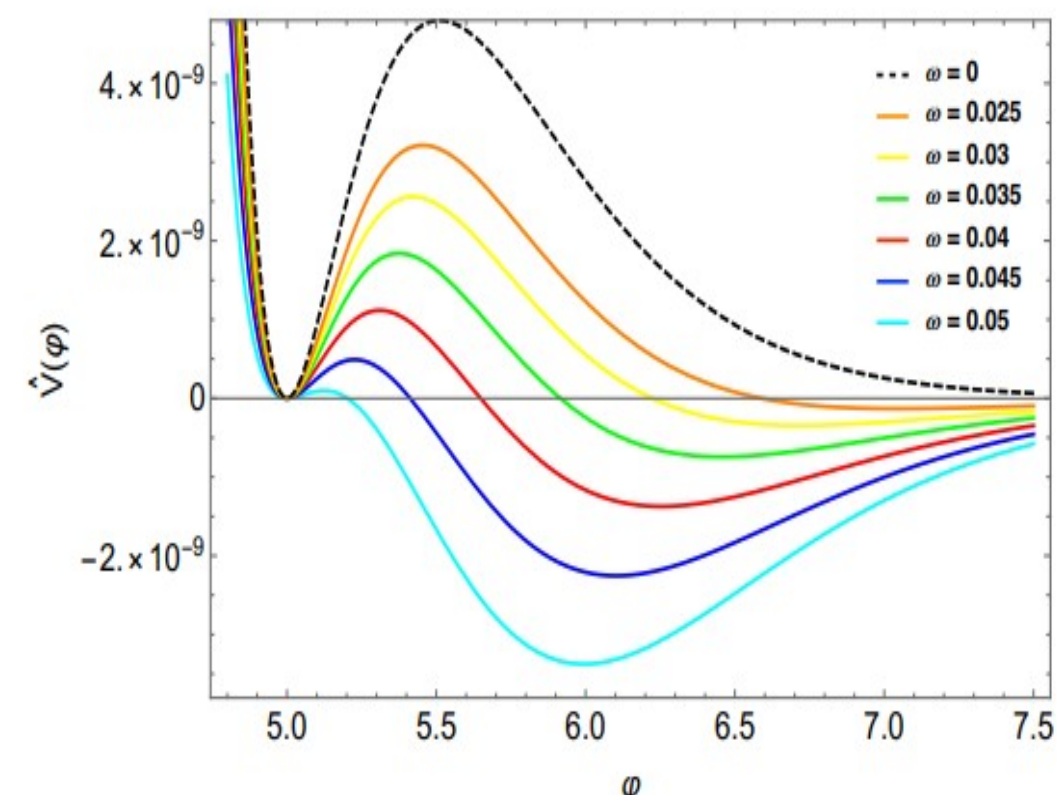
Hiramatsu, Kawasaki, Saikawa,  
Sekiguchi, ArXiv:1207.3166.

Q-Bounce: Critical  
bubbles of true vacuum



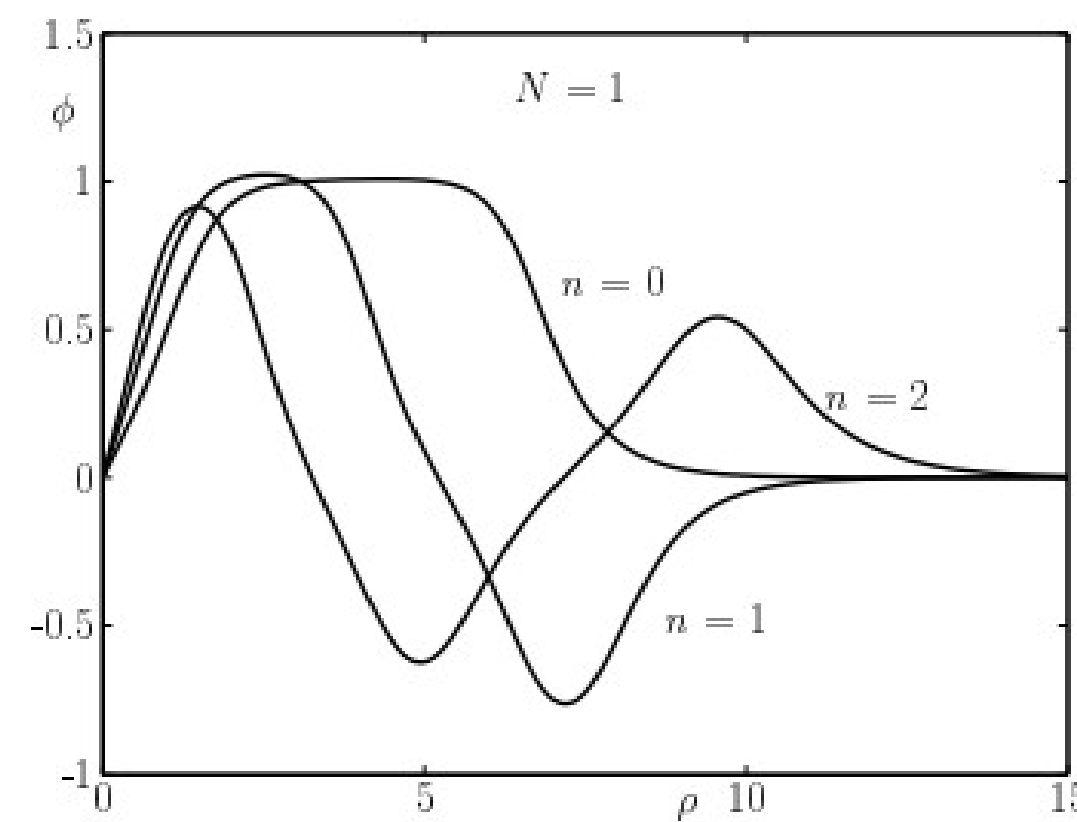
Kusenko, ArXiv:9705361

PQ-Balls:



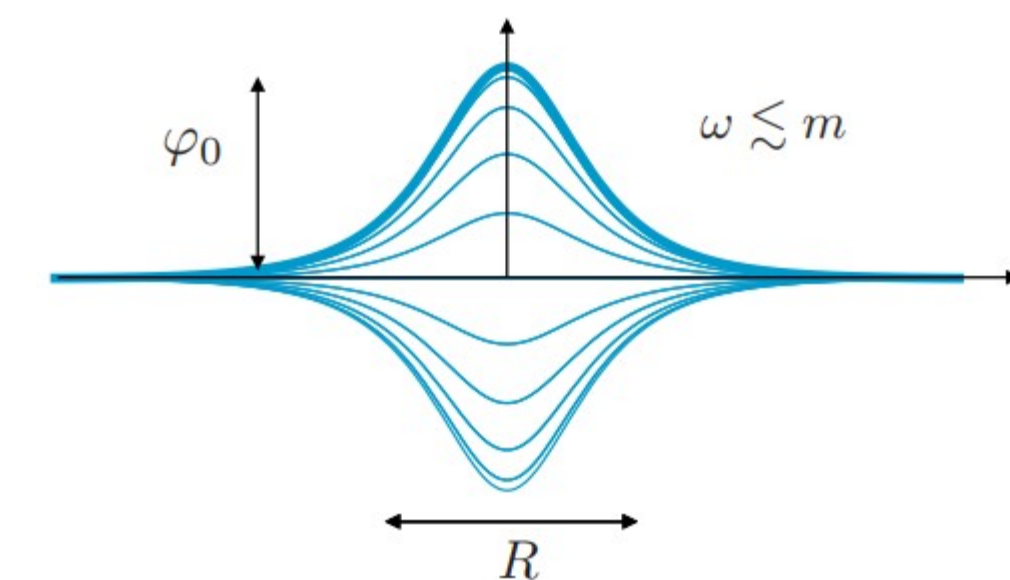
Krippendorf, Muia, Quevedo,  
1806.04690

Q-vortex:



Volkov, Wohnert, ArXiv:0205157

Gravitationally Bound  
Solitons:



Amin, Long, Mou, Saffin  
ArXiv:2103.12082

# Axion Electrodynamics

$$S = \int d^4x \left[ -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{g_{a\gamma}}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \right]$$

Solitons in axion electrodynamics?

$$\begin{aligned} \ddot{\phi} - \nabla^2 \phi + \partial_\phi V &= g_{a\gamma} \mathbf{E} \cdot \mathbf{B}, \\ \dot{\mathbf{E}} &= \nabla \times \mathbf{B} - g_{a\gamma} \left( \dot{\phi} \mathbf{B} + \nabla \phi \times \mathbf{E} \right), \\ \dot{\mathbf{B}} &= -\nabla \times \mathbf{E}, \\ \nabla \cdot \mathbf{E} &= -g_{a\gamma} \nabla \phi \cdot \mathbf{B}, \\ \nabla \cdot \mathbf{B} &= 0. \end{aligned}$$

Spatially or time dependent axion fields act as sources and currents opens up new possibilities for solitons not available in either pure EM gauge theory or scalar field theory.

$$\rho = -g_{a\gamma} \nabla \phi \cdot \mathbf{B} \quad \text{and} \quad \mathbf{J} = g_{a\gamma} \left( \dot{\phi} \mathbf{B} + \nabla \phi \times \mathbf{E} \right)$$

# Linear stability in Complex Solitons

More generally for solution to the EoM and separable perturbation in space and time,

$$\varphi \rightarrow \varphi_s + \varepsilon \delta\varphi \quad \varepsilon \ll 1, \quad \chi(x, t) = e^{i\lambda t} \Phi(x)$$

Sturm-Liouville equation of T.I. schrodinger-like form,

$$-\Phi_{xx} + V_1\Phi = \lambda^2\Phi, \quad \text{with } V_1(x) := \left. \frac{\partial^2 V(\varphi)}{\partial \varphi^2} \right|_{\varphi_s}.$$

For real  $\lambda$ , stable solutions solve,

$$\ddot{\varphi}_s - \varphi_s'' + \left. \frac{\partial V(\varphi)}{\partial \varphi} \right|_{\varphi_s} + \varepsilon \left( \ddot{\chi} - \chi'' + \chi \left. \frac{\partial^2 V(\varphi)}{\partial \varphi^2} \right|_{\varphi_s} \right) + \mathcal{O}(\varepsilon^2) = 0.$$

# Linear stability in Complex Solitons

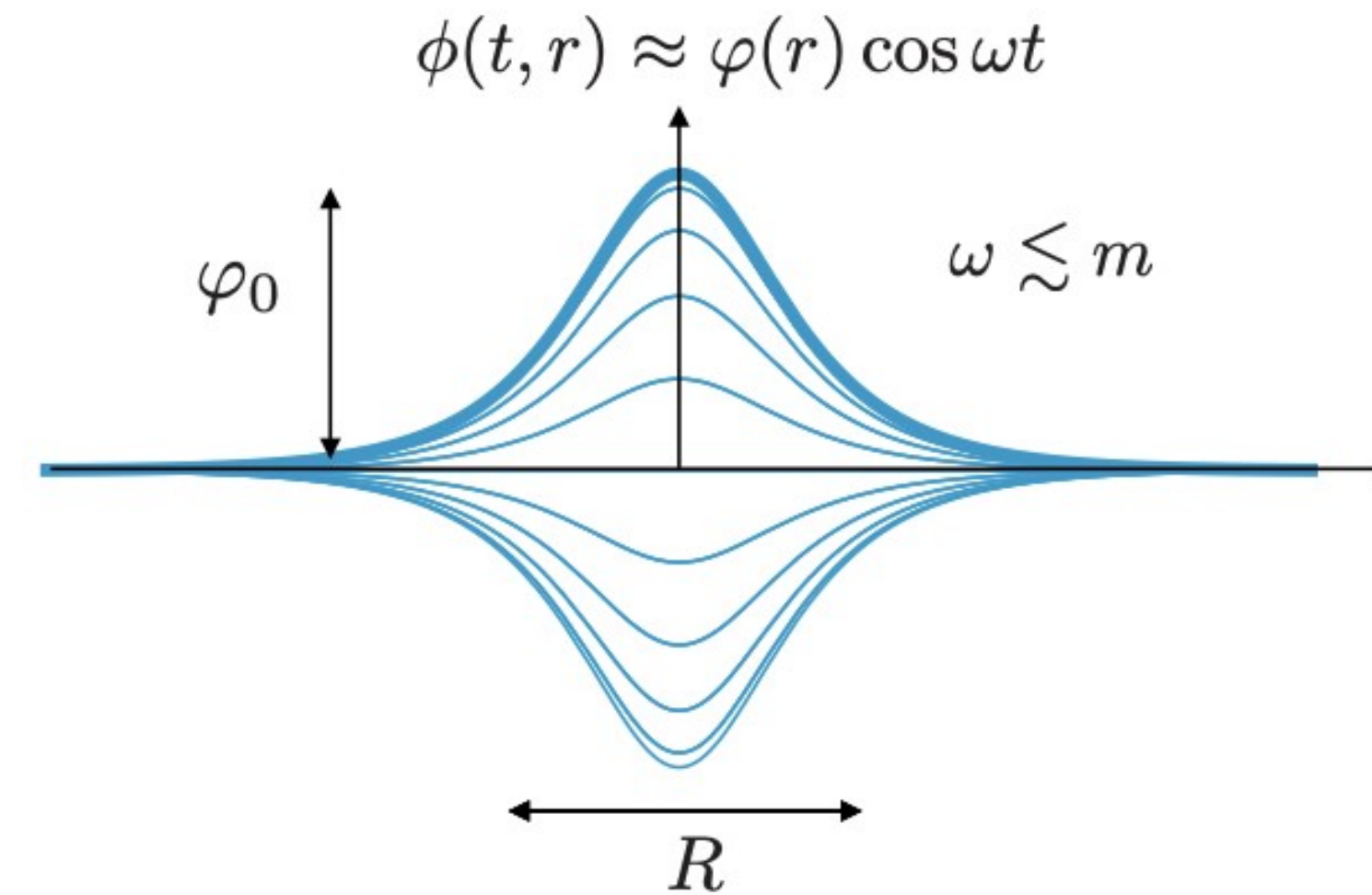
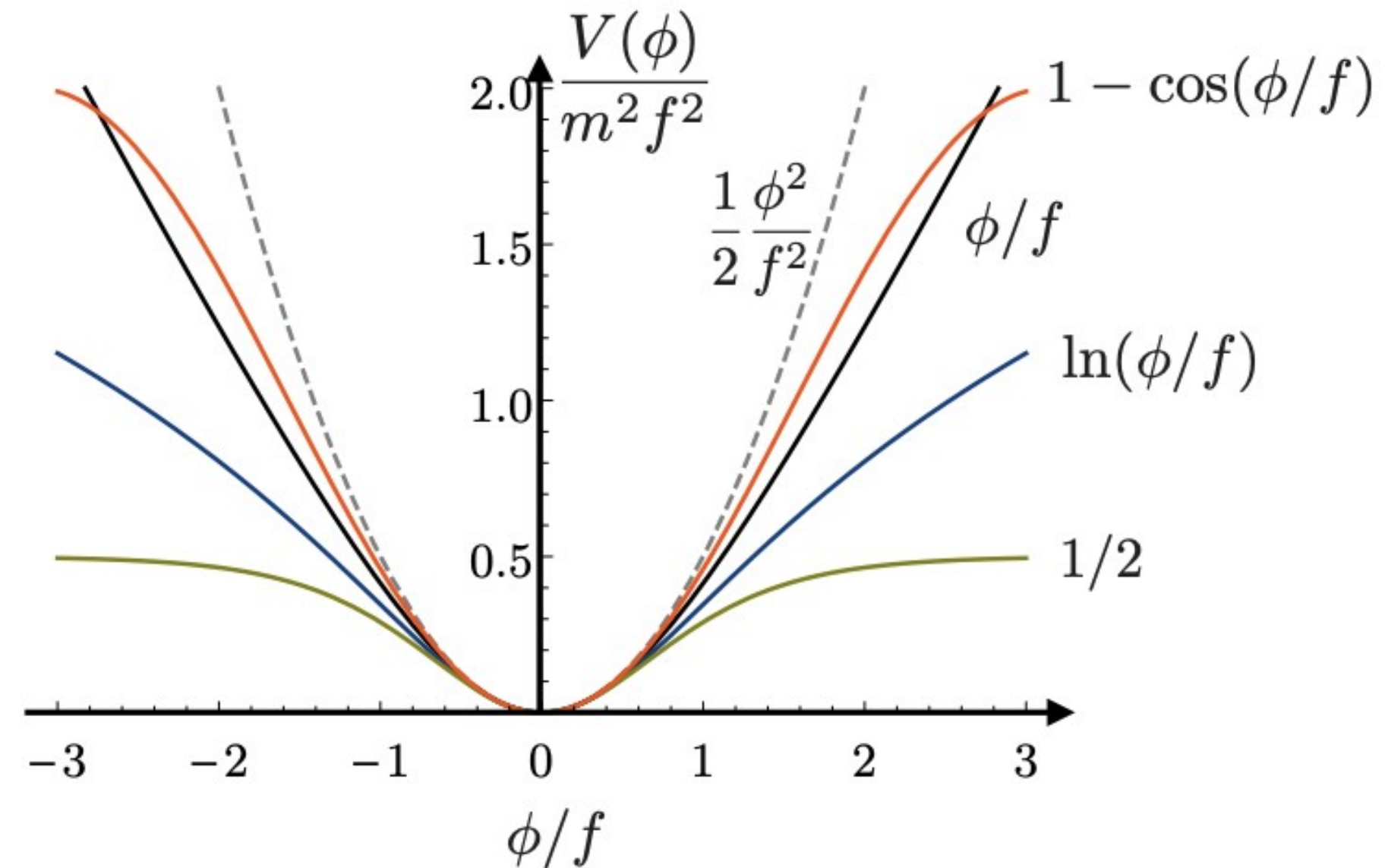
Examine the energy of the perturbed solution,

$$E[\varphi_s + \chi] = E[\varphi_s] + \int_{-\infty}^{\infty} dx \left[ \left( \frac{\partial V(\varphi)}{\partial \varphi} \Big|_{\varphi_s} - \varphi_s'' \right) \chi + \frac{1}{2} \chi \left( \frac{\dot{\chi}^2}{\chi} - \chi'' + \chi \frac{\partial^2 V(\varphi)}{\partial \varphi^2} \Big|_{\varphi_s} \right) \right] + \chi(\chi' + \varphi_s') \Big|_{-\infty}^{\infty} + \mathcal{O}(\varepsilon^3).$$

If the spatial part of the separable solution vanishes at the boundary, the last surface term vanishes and if perturbation solves SL equation then solutions are all of equal energy to the original solution.

# Spherically Symmetric Solitons in B Fields

M.A. Amin, A. J. Long, Z.G. Mou, P. M. Saffin arXiv:2103.12082



$$\phi(t, r) \approx \varphi(r) \cos(\omega t).$$

$$\phi(t, r) = \varphi_0 \operatorname{sech}(r/R) \cos \omega t.$$

Spherically symmetric oscillons in a background B field, e.g. Axion Boson stars.

Radiates EM with characteristic frequency of periodic solution.

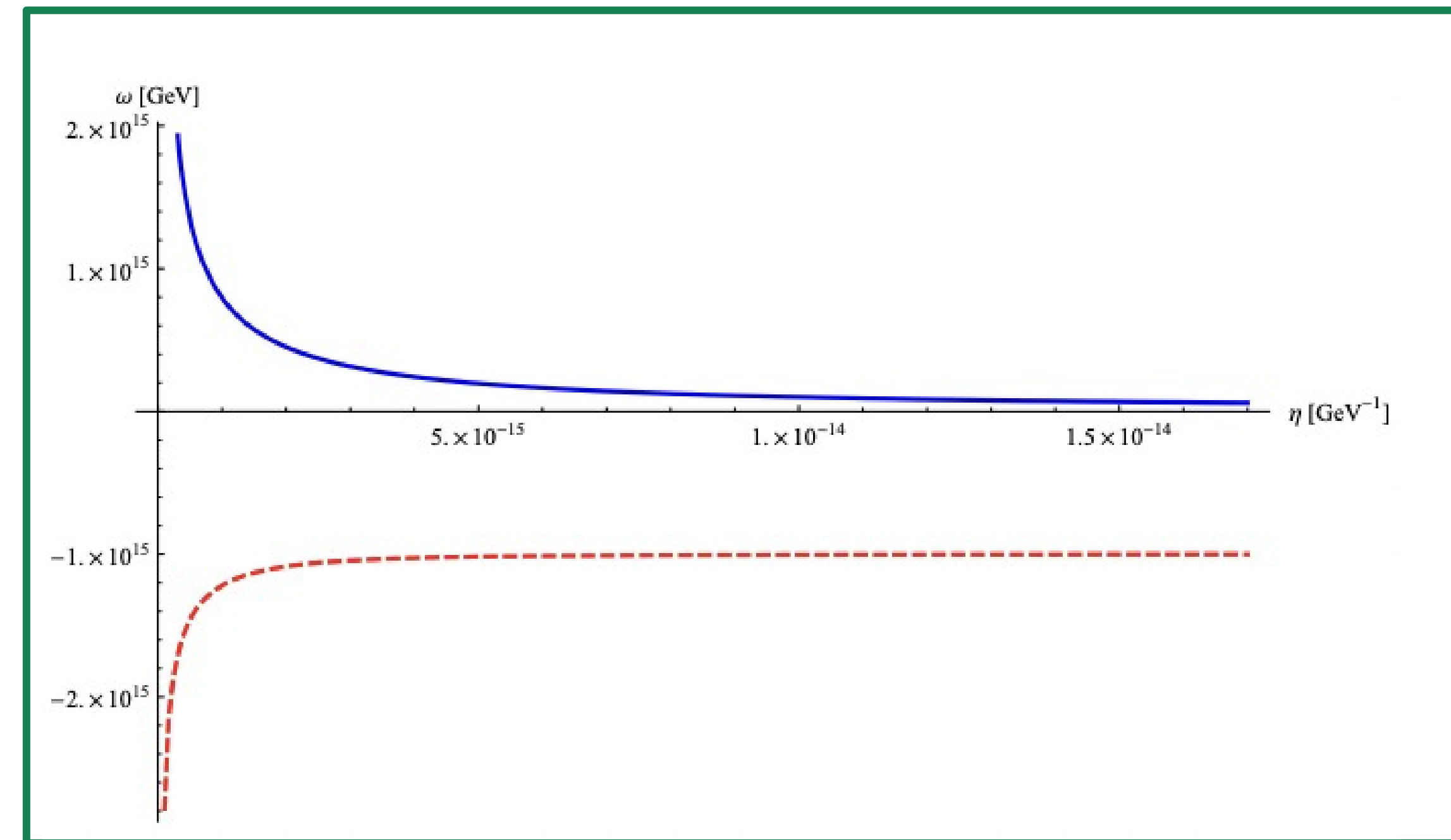
# Axions and Thin Magnetic Flux Strings

In small axion mass limit, what is the EoM for mixed the  $A_z$ -Axion field with background B string in z-direction?

$$[-\vec{\nabla}^2 + gB_0\omega\delta(x)\delta(y)]\psi(\vec{x}) = \omega^2\psi(\vec{x})$$

Unique finite flux of solution compared to original string.

Frequency of periodic solution decreasing with size of the B-field.



Instability of Axions and Photons In The Presence of Cosmic Strings,  
Guendelman, Shilon, arXiv: 0810.4665

# Magnetic Vortex

Large magnetic flux with cylindrical symmetry extending to large radius. Consider flat spacetime.

Look for periods of near constant axion field velocity,

$$a(t) \approx a_0 \sin(m_a t)$$

EoM admits constant field velocity B vortex solution

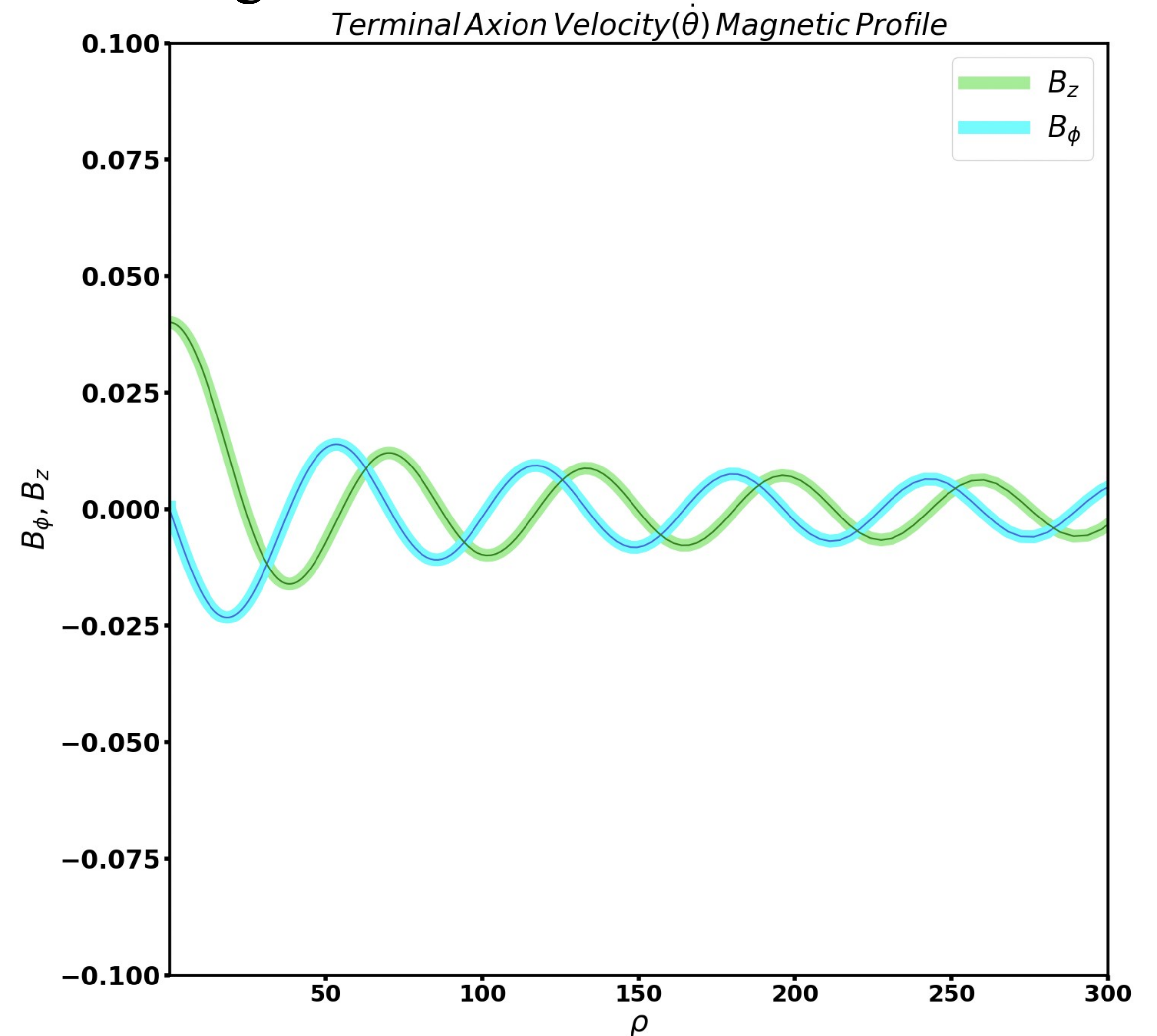
$$B_\phi'' + \frac{1}{\rho} B_\phi' - \left( \frac{1}{\rho^2} - m^2 \right) B_\phi = 0; \quad B_z'' + \frac{1}{\rho} B_z' + m^2 B_z = 0$$

$$B_\phi(\rho) = -m|m| \frac{\Phi}{\mathcal{N}} J_1(|m|\rho), \quad B_z(\rho) = m^2 \frac{\Phi}{\mathcal{N}} J_0(|m|\rho)$$

Perturbative analysis on soliton solution for small accelerations generating Electric fields.

$$\partial^2 a + V'(a) = -g_{a\gamma} \vec{E} \cdot \vec{B}$$

Cutoff at large  $\rho$  for a finite given conserved flux, finite energy per unit length.



# Magnetic Vortex

Take small perturbation around classical solution with spatially dependent perturbation profile,  $f(\rho)$ , symmetric around the centre of the magnetic vortex.

$$a = a_0 + \delta a$$

$$\delta a = \theta(t) f(|m|\rho)$$

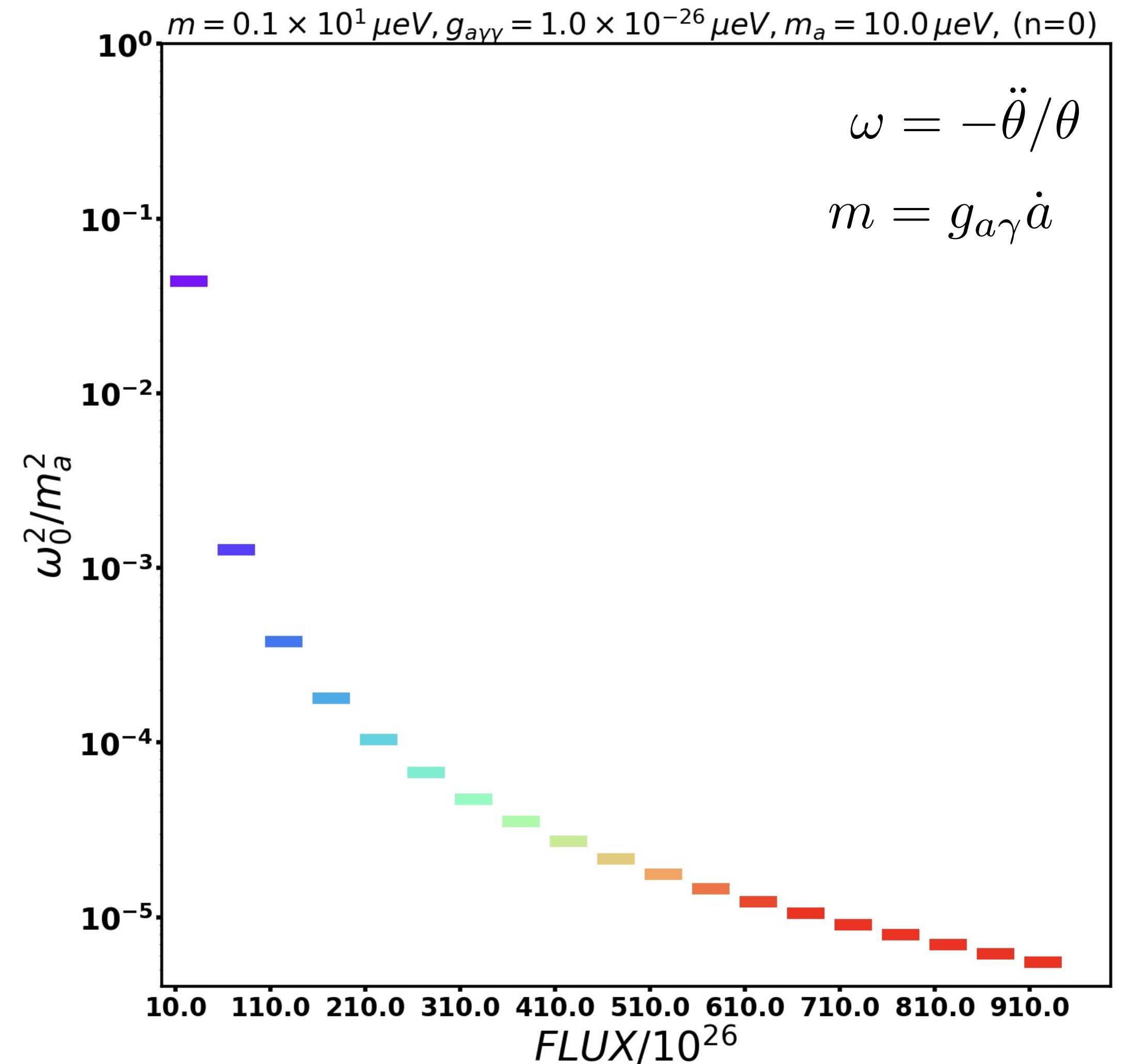
$$H(|m|\rho) = 1 - g_{a\gamma}^2 m|m| \left( \frac{\Phi}{\mathcal{N}} J_0(|m|\rho) \right)^2$$

$$H(|m|\rho) \delta \ddot{a} - \nabla^2 \delta a + m_a^2 \delta a = 0$$

Solutions to schrodinger-like perturbative EoM have discrete solutions for deceleration of the axion field within region of vortex.

Over time scale  $\tau$  compare oscillon's deceleration solutions to typical periodic frequency for continuous solution where  $g_{a\gamma} = 0$ .

Compare to e.g. Ginzberg landau vortex in axion and magnetic field.



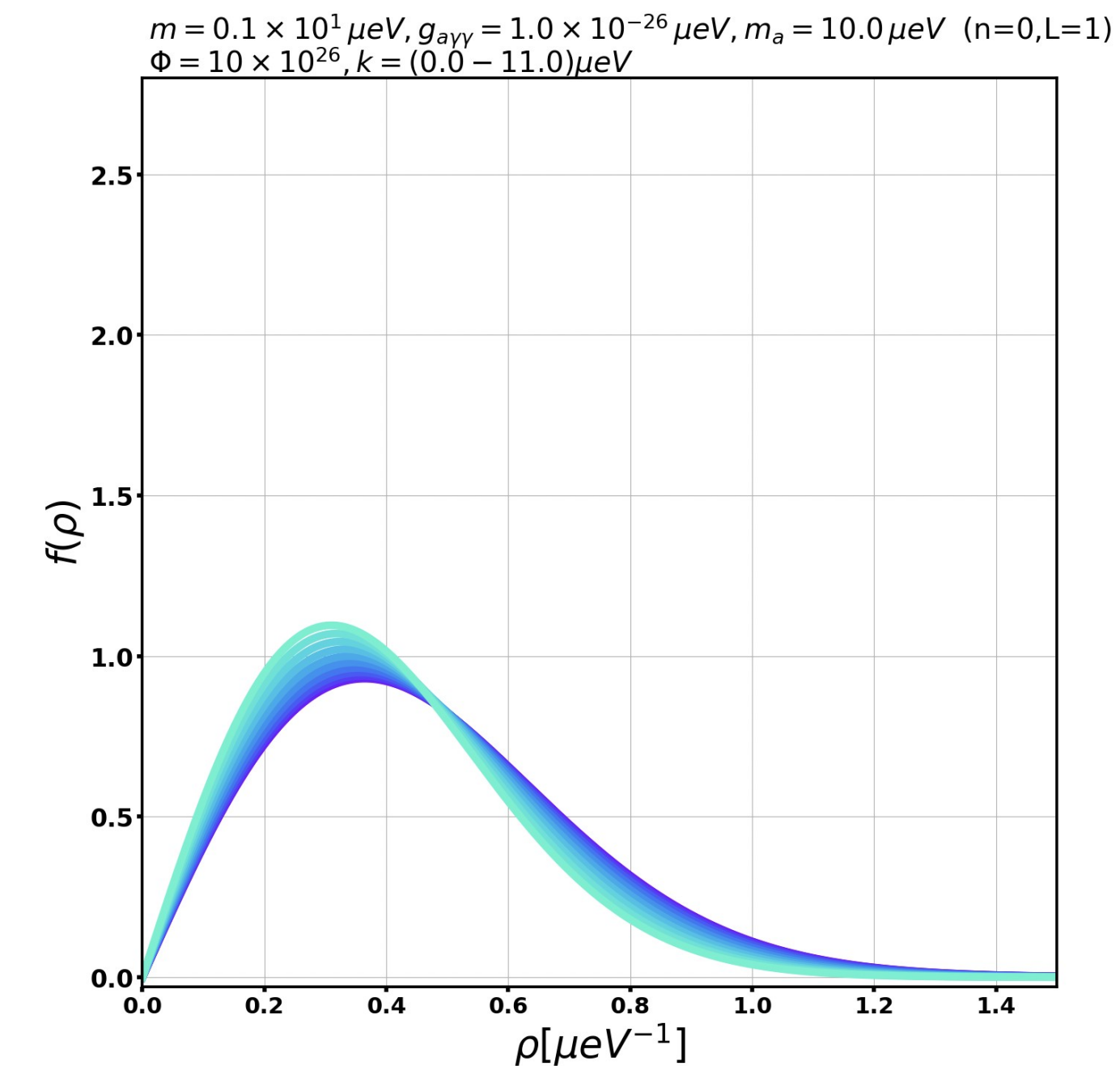
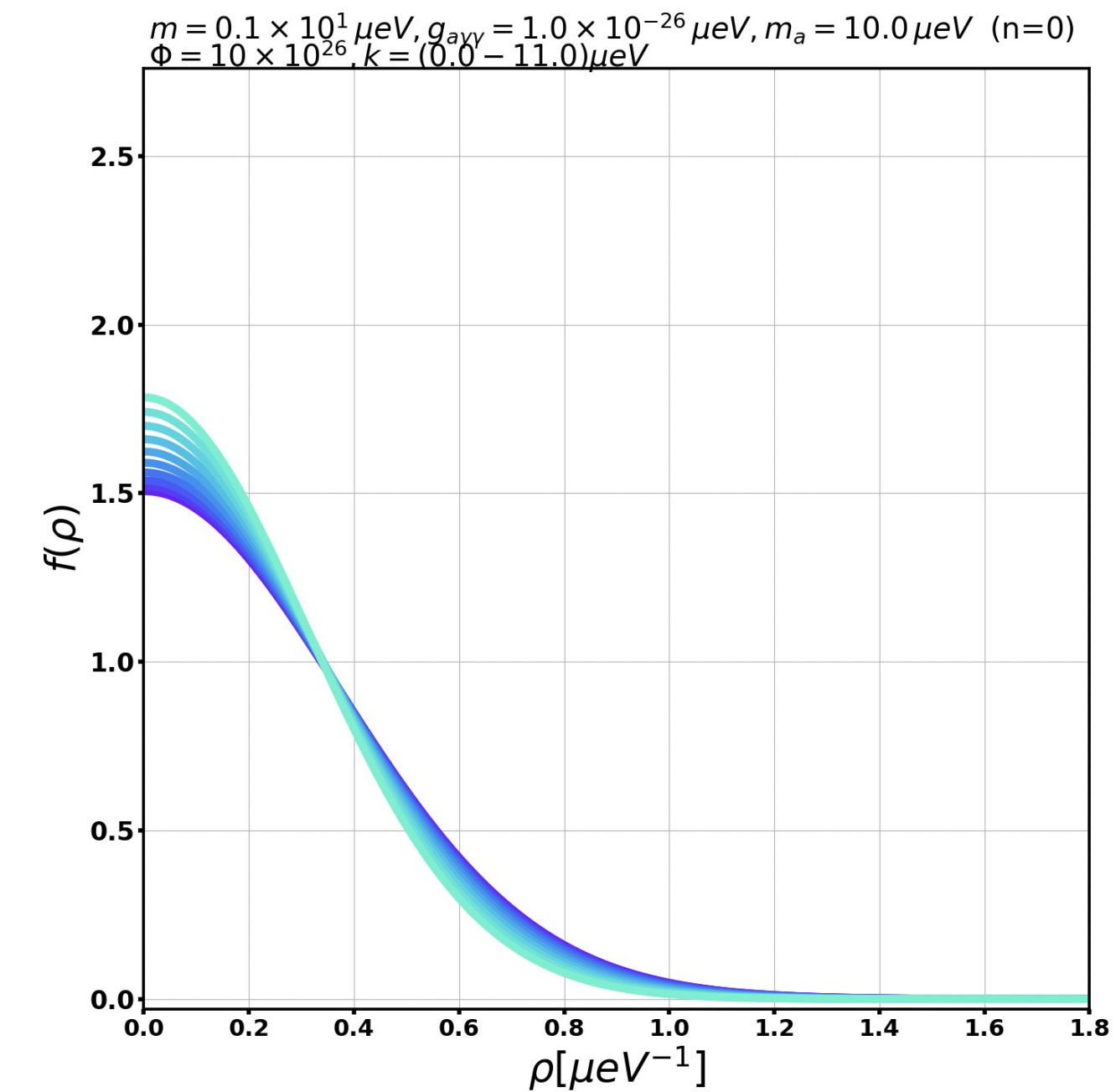


# Magnetic Vortex

In cylindrical system, angular momentum (L-Modes), Z-modes along direction of cylinder comprise more states. For large fluxes, acceleration approaches smaller values.

Consider EM radiation absorption/emission over lifetime  $\tau$  to alternate states of the vortex.

Ongoing simulation of vortex states over short time intervals.



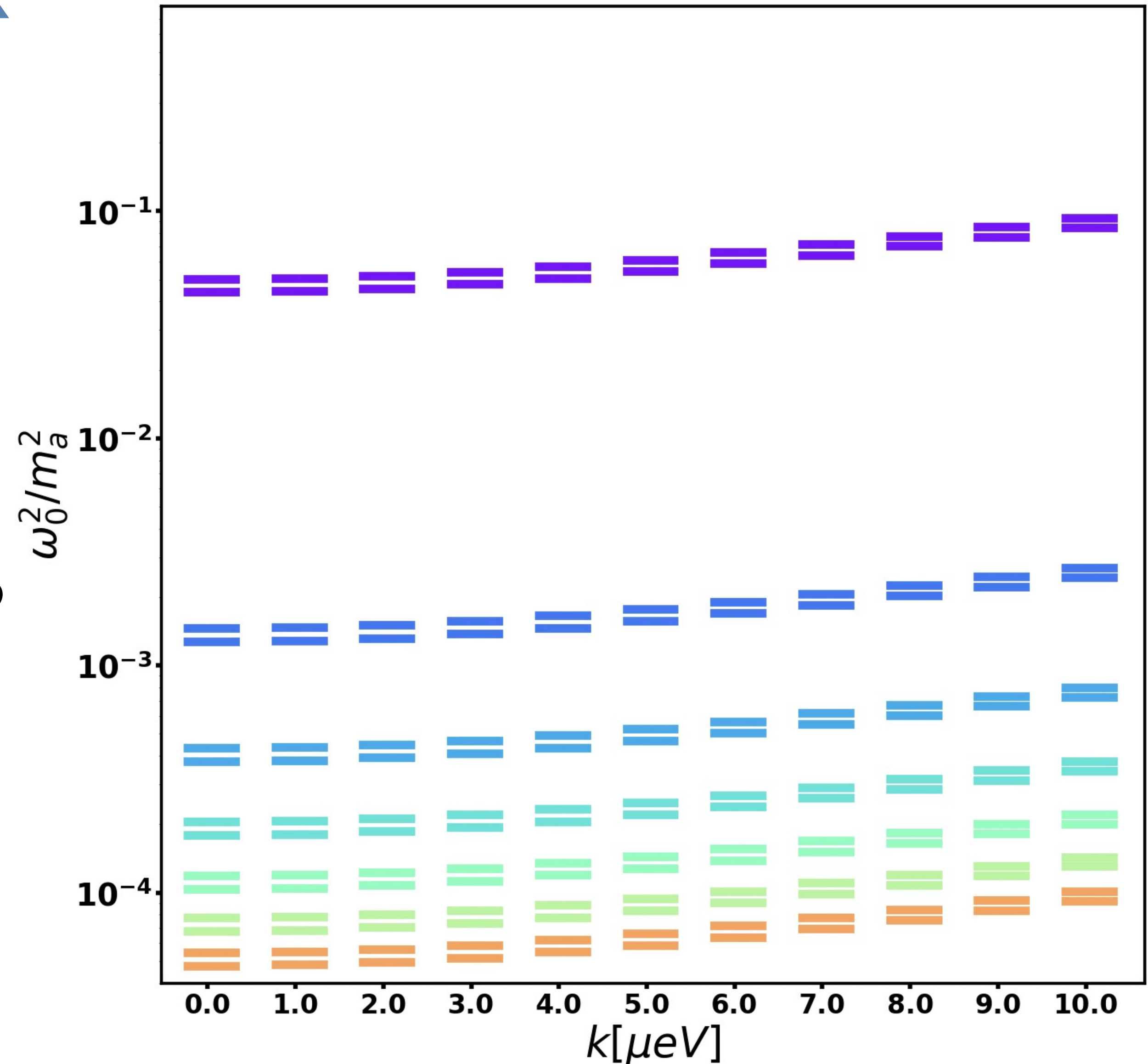
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$$m = 0.1 \times 10^1 \mu\text{eV}, g_{\text{a}\gamma\gamma} = 1.0 \times 10^{-26} \mu\text{eV}, m_a = 10.0 \mu\text{eV}, (n=0, L=0, 1) \\ \Phi = (10 - 310) \times 10^{26}$$



# Summary

Particle-like, atom-like objects emerging from novel solutions in classical field theory are interesting in their own right.

Many models that make use of these objects and their scatterings etc for dark matter, baryogenesis, or predict their possible emergence with either short or long lifetimes.

Thorough understanding of cross-sections and interactions among Q-balls an important part of future Gravitational wave astronomy.

New kinds of axion solitons and oscillons in the presence of background B fields, their evolution and interactions also may be critical in structure formation, observational signals.

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*Thank You*