Axion Domain Walls, Small Instantons and Non-Invertible Symmetry Breaking

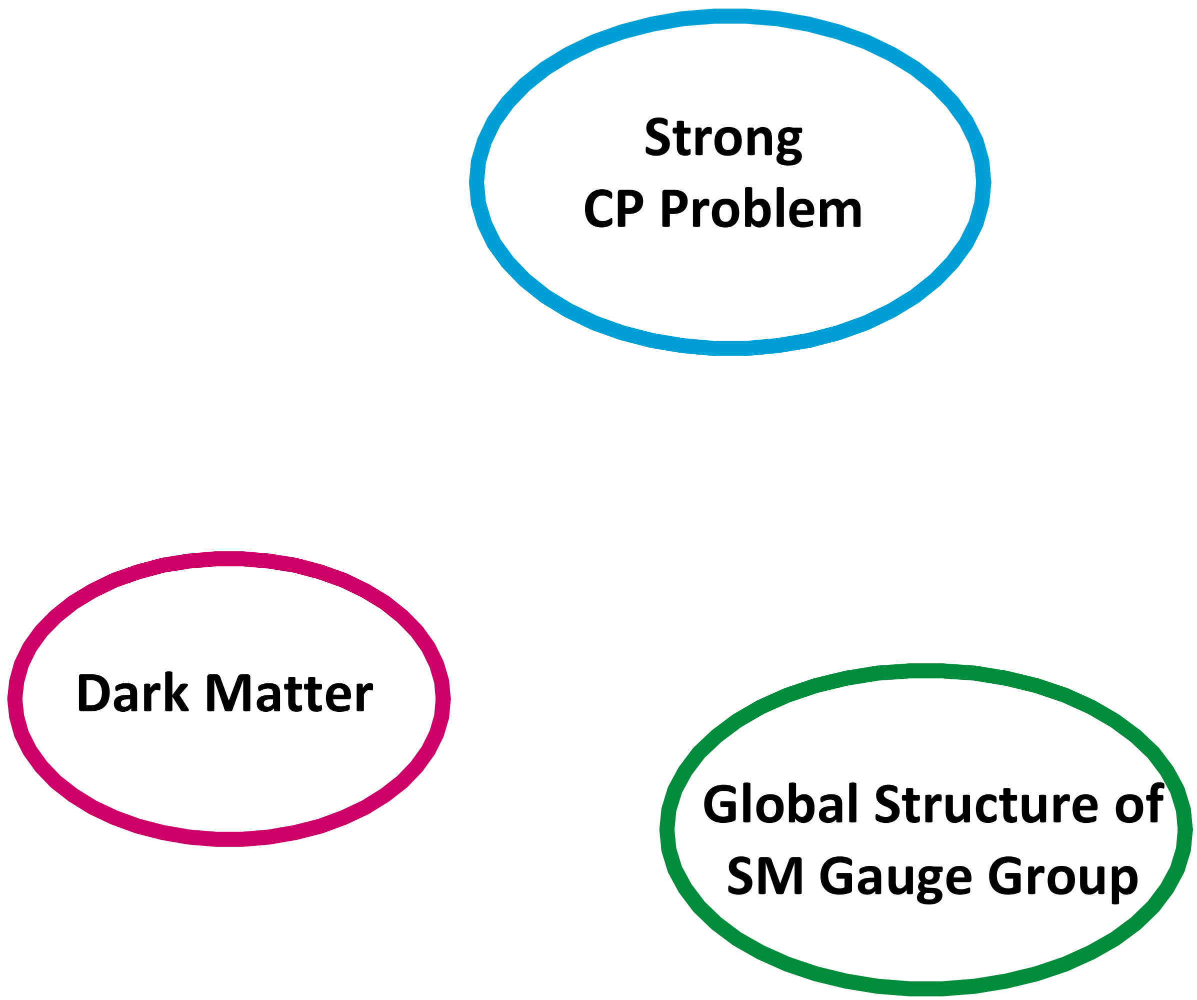
Sungwoo Hong

KAIST

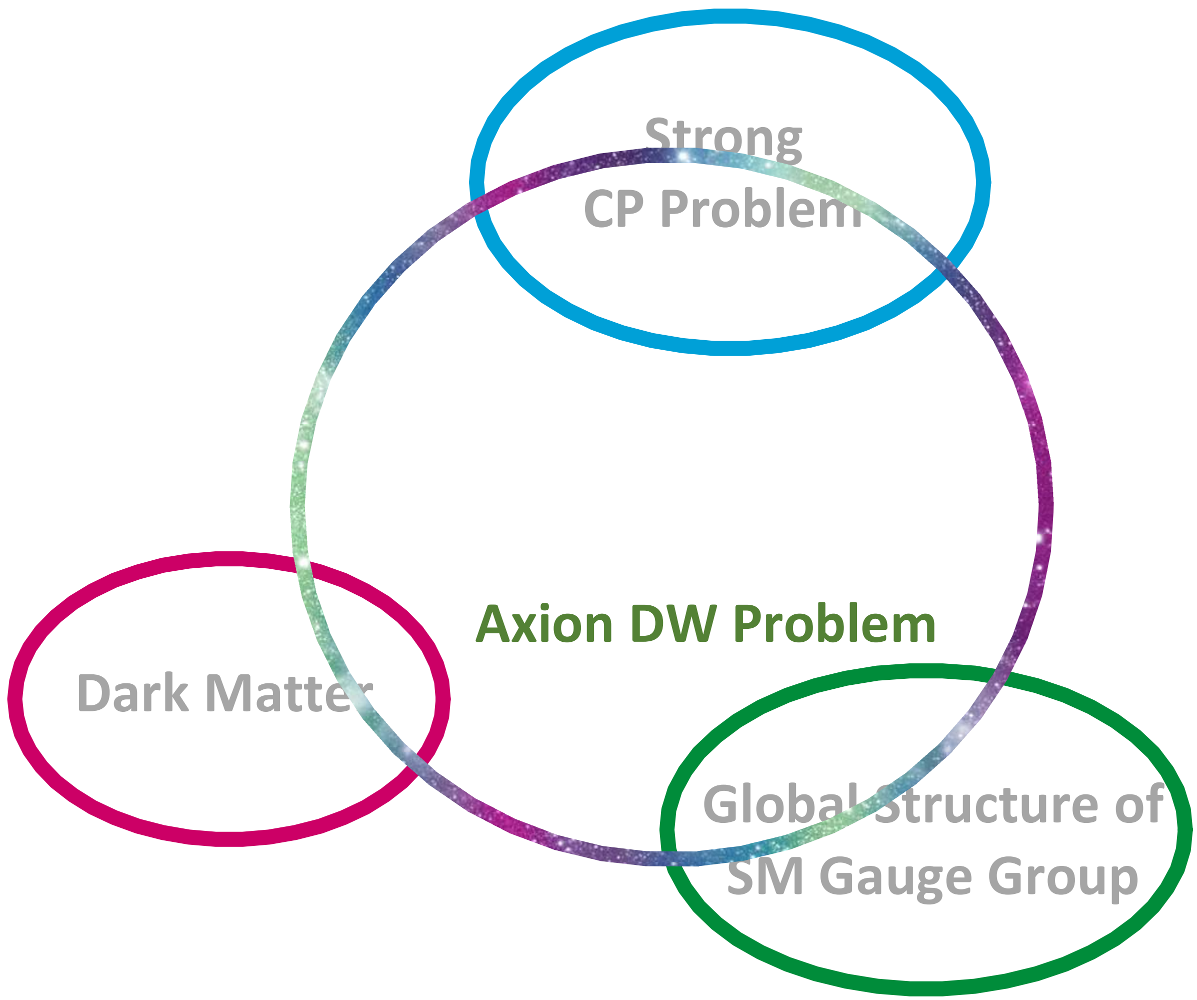
(Based on 2309.05636 with Clay Cordova, Liantao Wang)

PNU-IBS Workshop on Axion Physics: Search for Axions

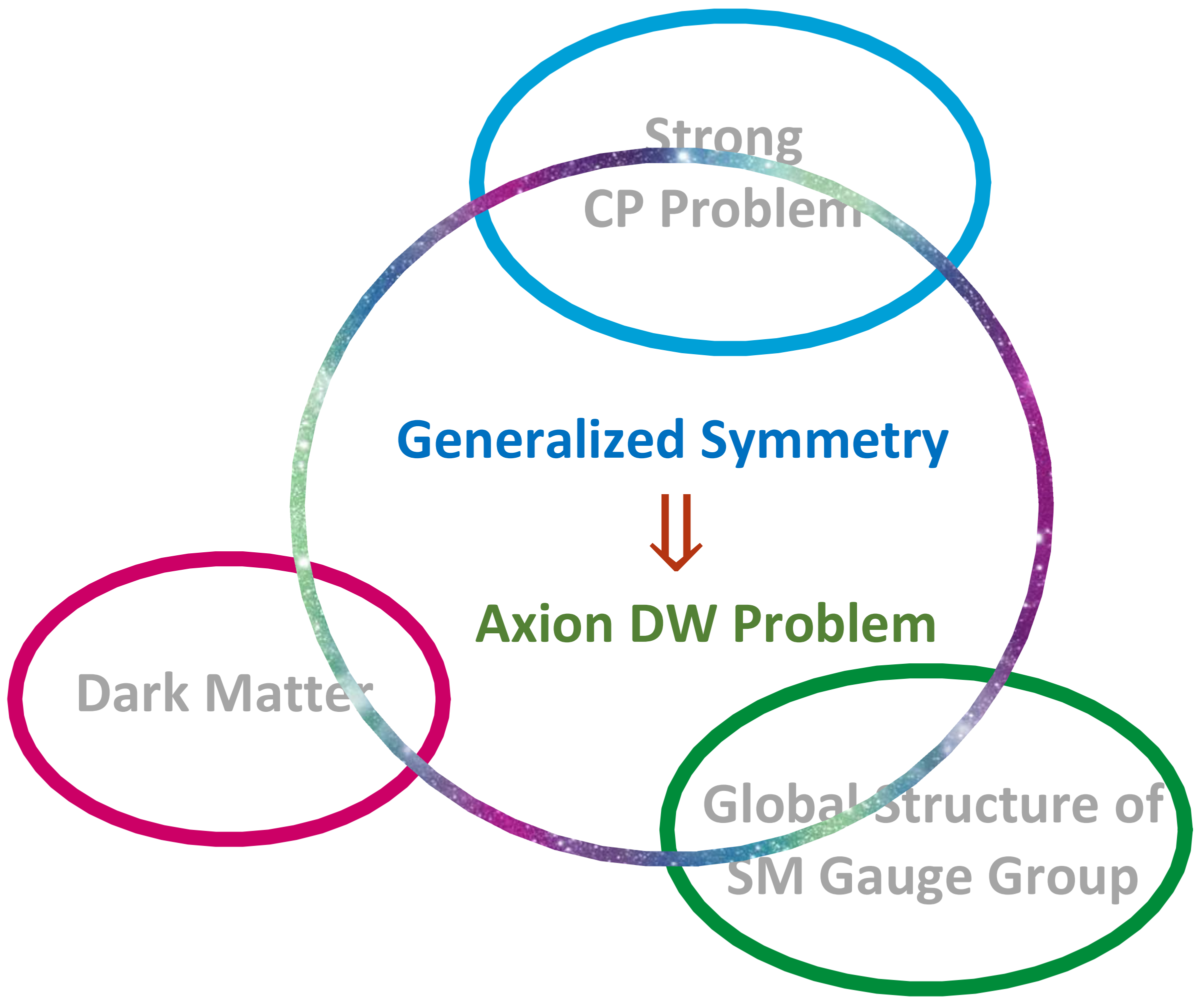
Motivation


Strong
CP Problem
Dark Matter
Global Structure of
SM Gauge Group

Motivation


Strong
CP Problem
Axion DW Problem
Dark Matter
Global Structure of
SM Gauge Group

Motivation


Strong
CP Problem
Generalized Symmetry
⇓
Axion DW Problem
Dark Matter
Global Structure of
SM Gauge Group

Axion Domain Wall Problem

Axion Domain Wall Problem

I. Axion-QCD Theory

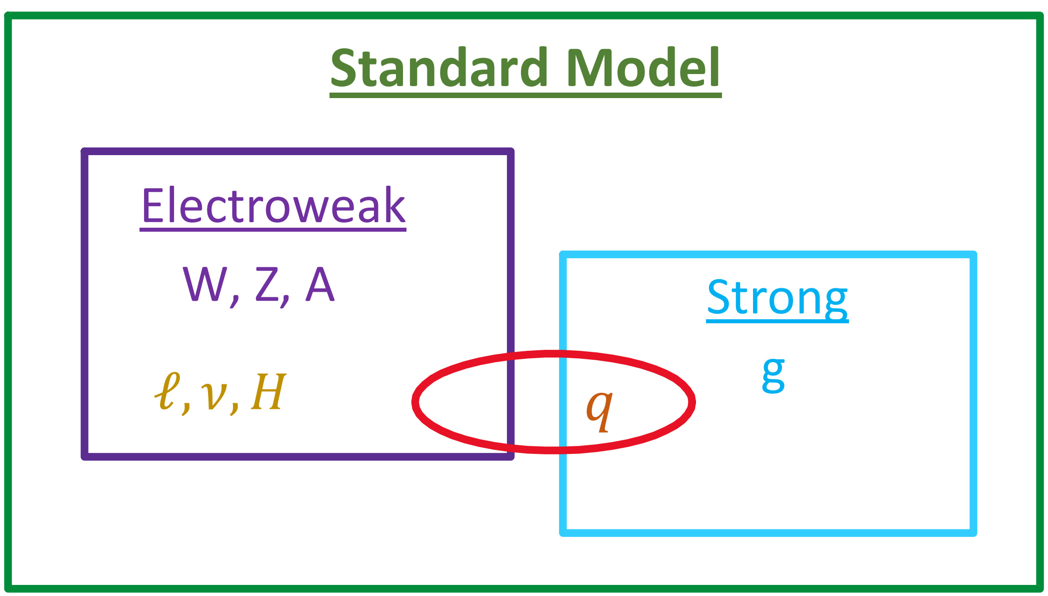
$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

Axion Domain Wall Problem

I. Axion-QCD Theory

$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

(i) Leading Theoretical Proposal to Solve Strong CP Problem

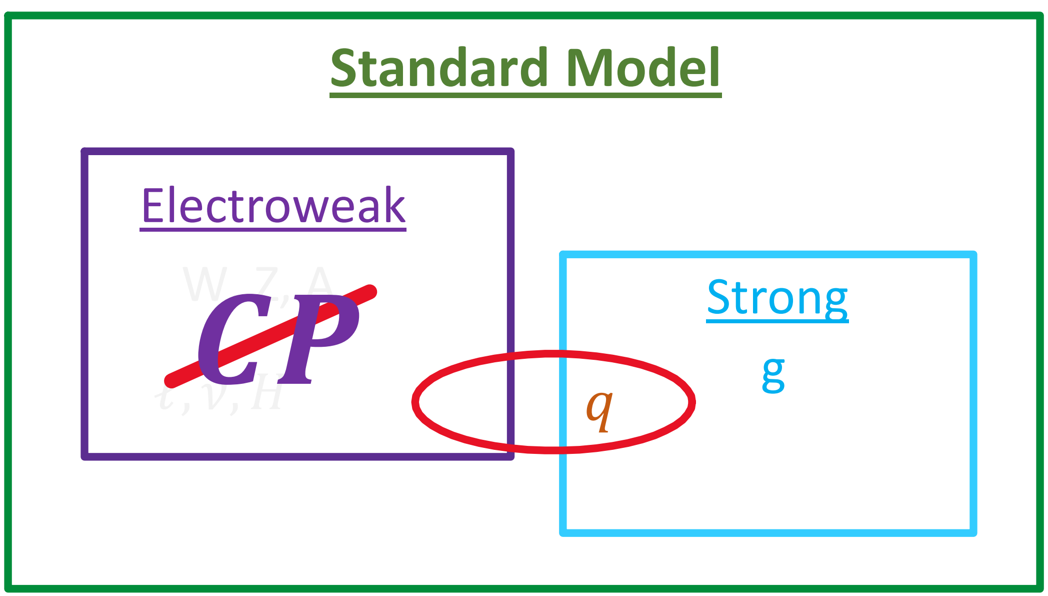


Axion Domain Wall Problem

I. Axion-QCD Theory

$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

(i) Leading Theoretical Proposal to Solve Strong CP Problem

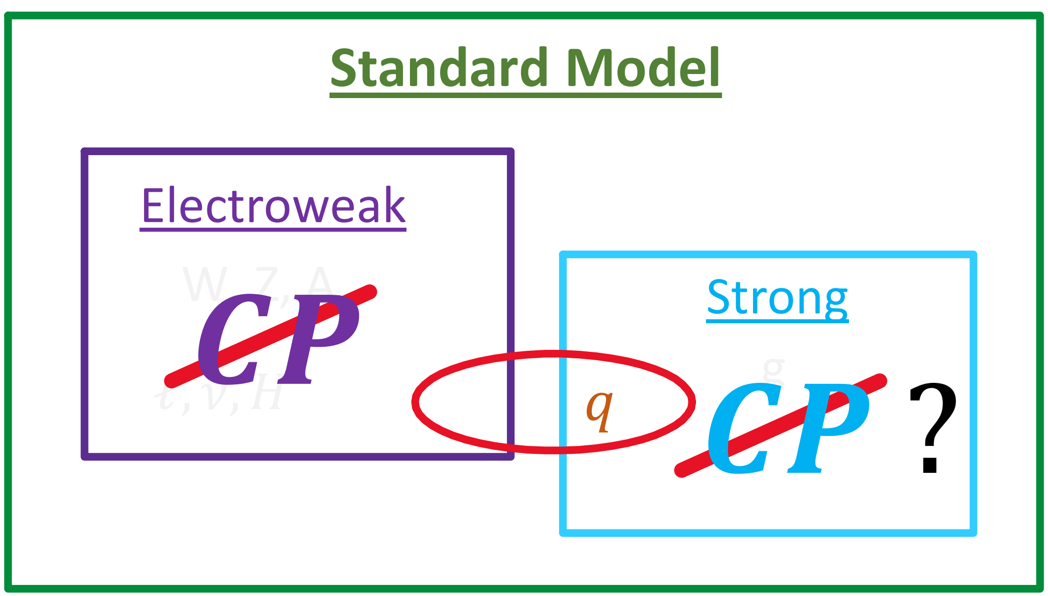


Axion Domain Wall Problem

I. Axion-QCD Theory

$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G)$$

(i) Leading Theoretical Proposal to Solve Strong CP Problem



$$S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G\wedge G)$$

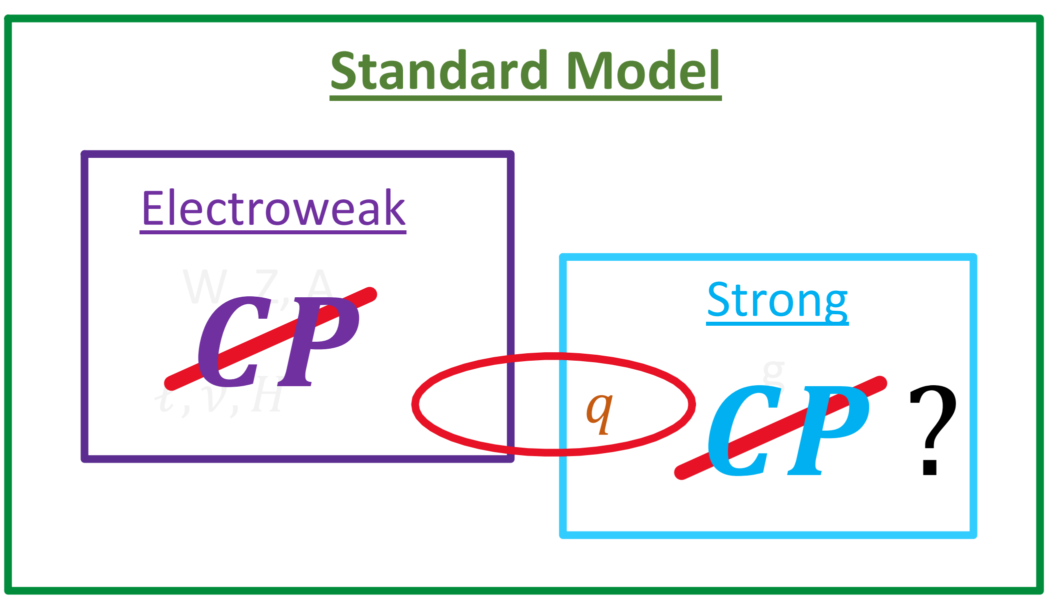
Expectation based on general rules of effective field theory

Axion Domain Wall Problem

I. Axion-QCD Theory

$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

(i) Leading Theoretical Proposal to Solve Strong CP Problem



$$S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$$

Neutron Electric Dipole Moment

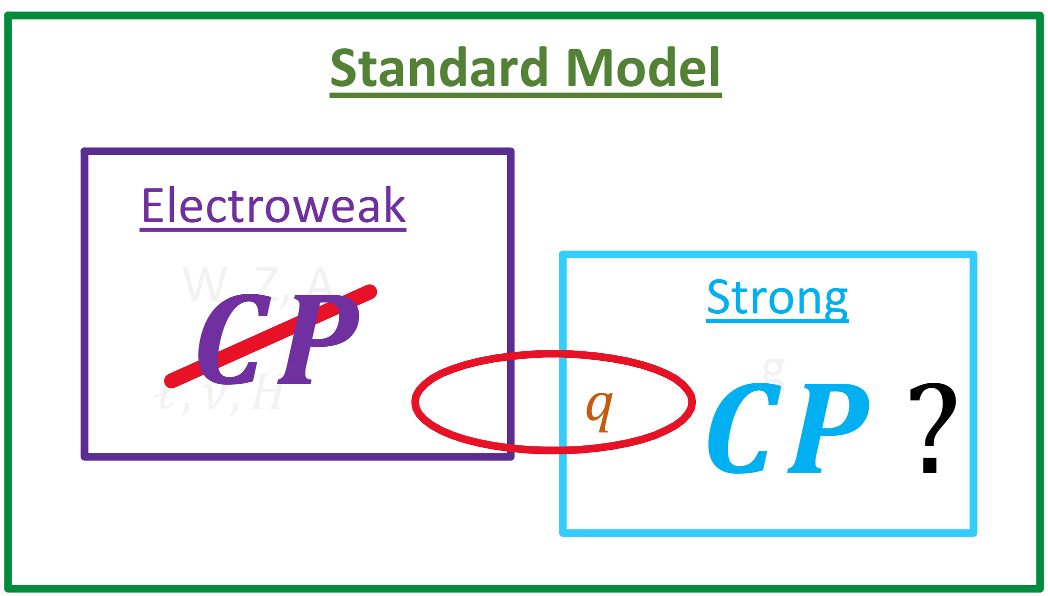
$$d_n \sim 3 \times 10^{16}\,\bar{\theta} \;\rightarrow\; \bar{\theta} < 10^{-10}$$

Axion Domain Wall Problem

I. Axion-QCD Theory

$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

(i) Leading Theoretical Proposal to Solve Strong CP Problem



$$S_{QCD} \supset \frac{i\bar{\boldsymbol{\theta}}}{8\pi^2}\int Tr(G \wedge G)$$

Neutron Electric Dipole Moment

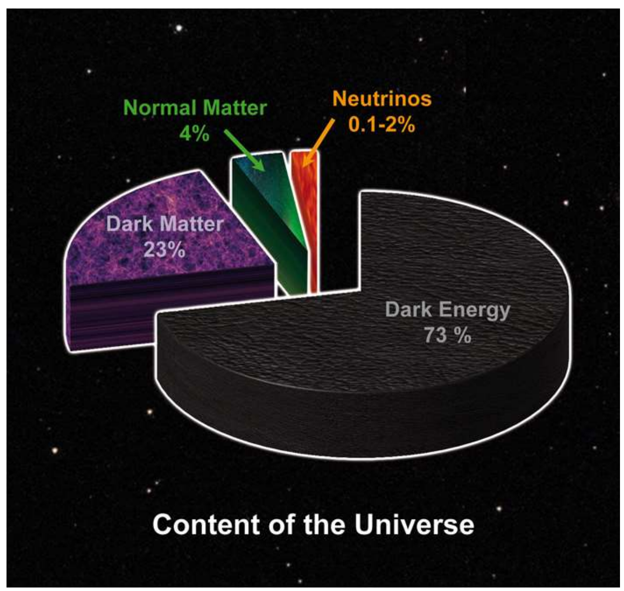
$$d_n \sim 3 \times 10^{16}\, \bar{\boldsymbol{\theta}} \;\rightarrow\; \bar{\boldsymbol{\theta}} < 10^{-10}$$

Axion Domain Wall Problem

I. Axion-QCD Theory

$$S \supset \frac{1}{2}\int \partial_\mu a\,\partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\,Tr(G \wedge G)$$

(i) Leading Theoretical Proposal to Solve Strong CP Problem
(ii) Well-motivated candidate for Dark Matter



Credit: HAP / A. Chantelauze

Axion Domain Wall Problem

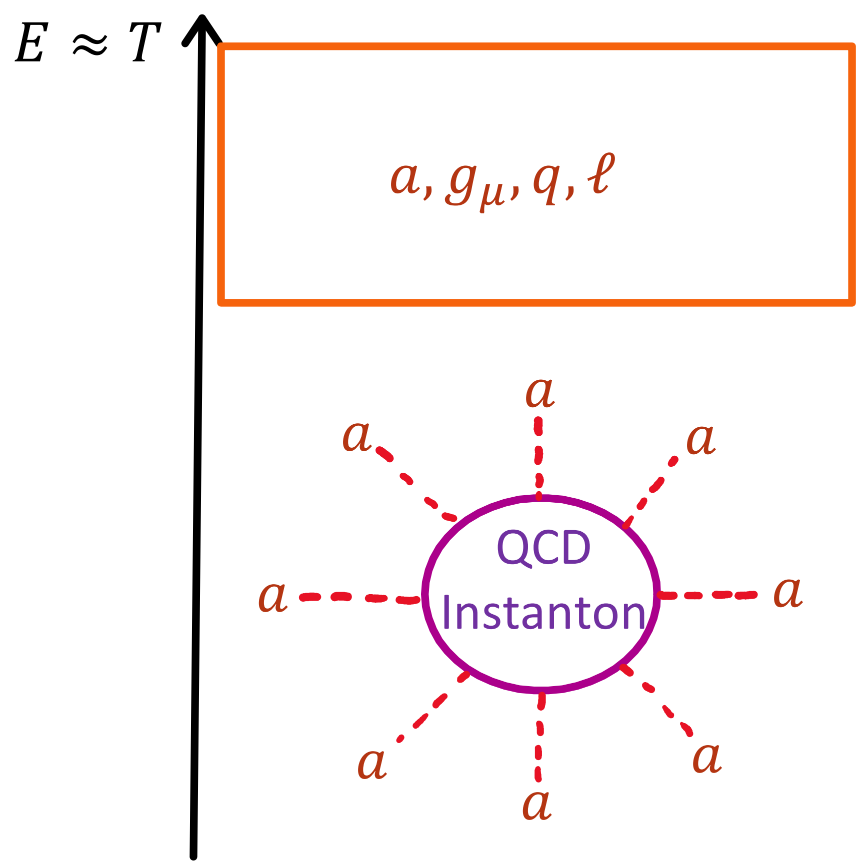
II. Axion Domain Wall Problem

$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G)$$

cf. $S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G\wedge G)$

$U(1)_{PQ} \to Z_K$: K-degenerate vacua

$a \sim a + 2\pi f_a,\qquad a \to a + \frac{2\pi f_a}{K}$



Axion Domain Wall Problem

II. Axion Domain Wall Problem

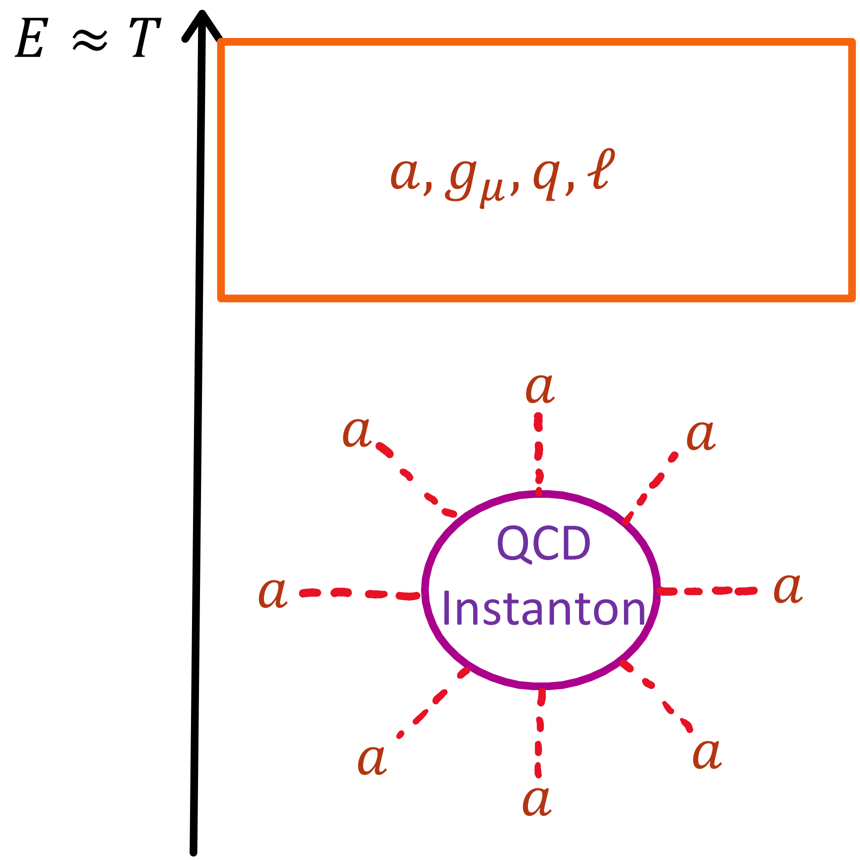
$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

cf. $S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$

$U(1)_{PQ} \to Z_K$: K-degenerate vacua

$a \sim a + 2\pi f_a,\qquad a \to a + \frac{2\pi f_a}{K}$

$$e^{-\frac{8\pi^2}{g^2(\mu)}}$$



Axion Domain Wall Problem

II. Axion Domain Wall Problem

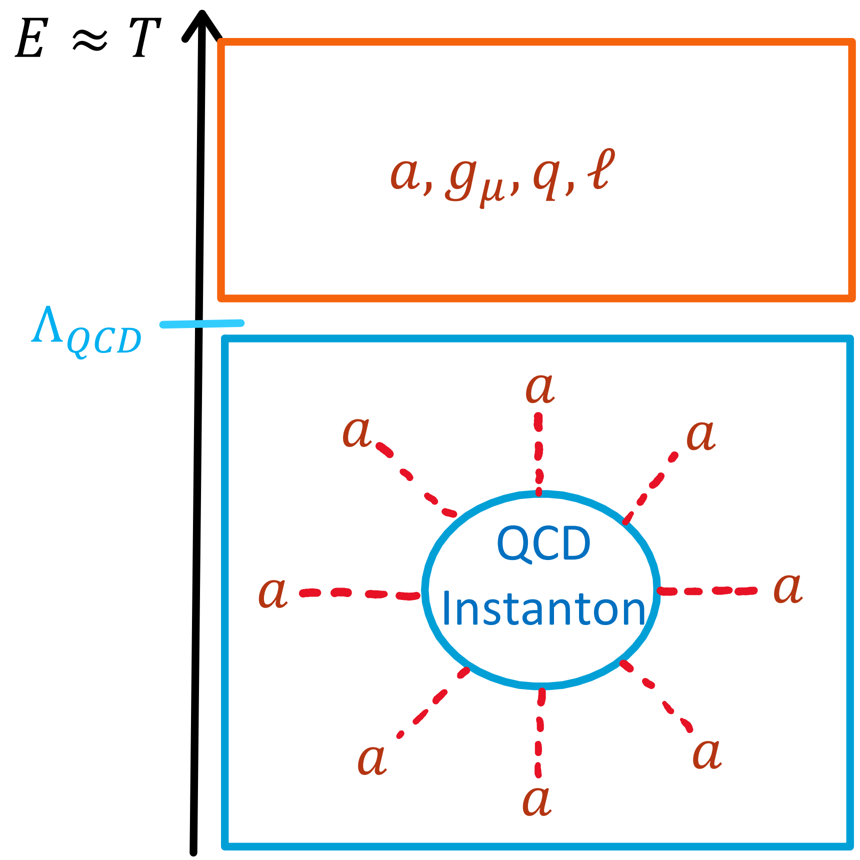
$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

cf. $S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$

$U(1)_{PQ} \to Z_K$: K-degenerate vacua

$a \sim a + 2\pi f_a,\qquad a \to a + \frac{2\pi f_a}{K}$

$$e^{-\frac{8\pi^2}{g^2(\Lambda_{QCD})}}$$



Axion Domain Wall Problem

II. Axion Domain Wall Problem

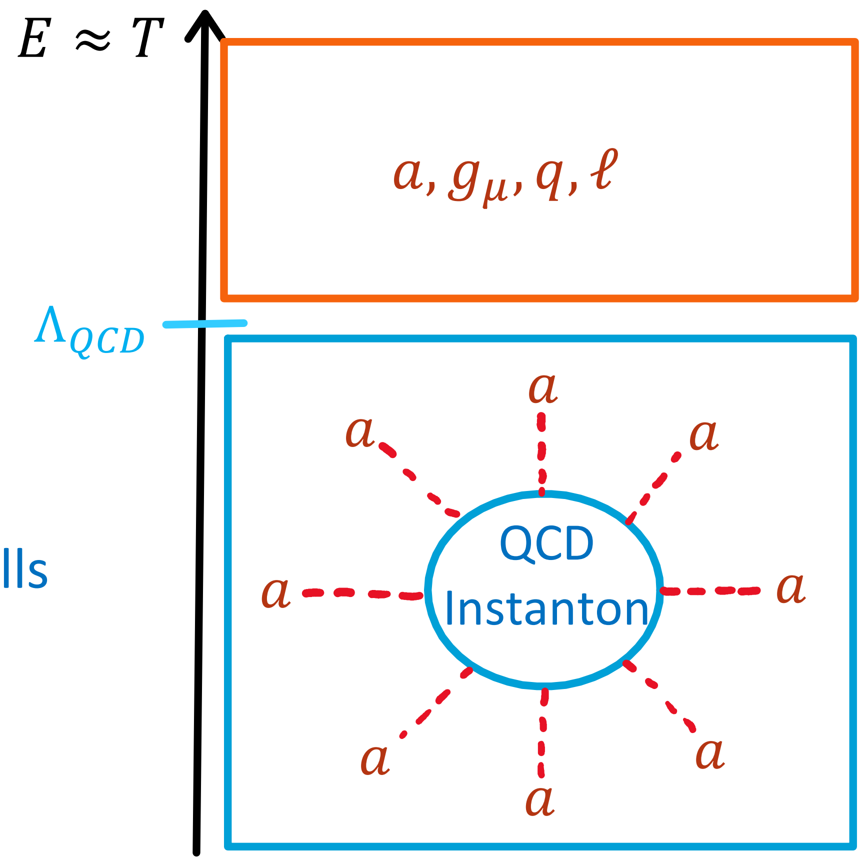
$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

cf. $S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$

$U(1)_{PQ} \rightarrow Z_K$: K-degenerate vacua

$$V(a) = m_u \Lambda_{QCD}^3 \left(1 - \cos\frac{Ka}{f_a}\right)$$

Z_K Spontaneously Broken : K Domain Walls



Axion Domain Wall Problem

II. Axion Domain Wall Problem

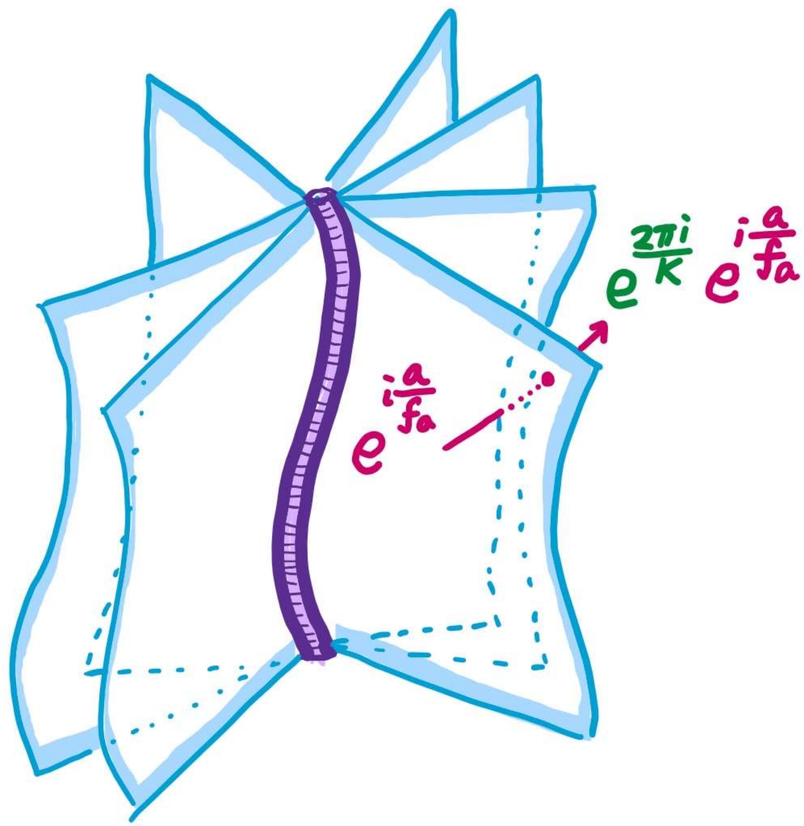
$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

cf. $S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$

$U(1)_{PQ} \to Z_K$: K-degenerate vacua

$$V(a) = m_u \Lambda_{QCD}^3\left(1 - \cos\frac{Ka}{f_a}\right)$$

Z_K Spontaneously Broken : K Domain Walls



Axion Domain Wall Problem

II. Axion Domain Wall Problem

The presence of long-lived axion-Domain-Walls is inconsistent with cosmological observations

- (Dark) matter energy density: $\rho_{\mathrm{DM}} \propto R^{-3} \Rightarrow \ R(t) \propto t^{2/3}$ **(Matter-Dominated)**

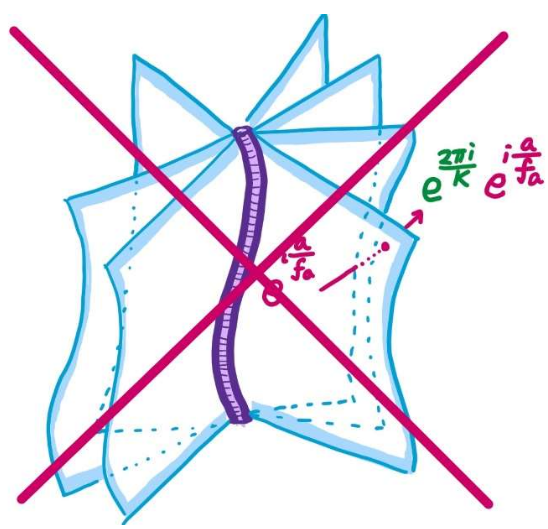
- Domain Wall energy density: $\rho_{\mathrm{DW}} \propto R^{-1} \ \Rightarrow \ R(t) \propto t^{2}$ **(DW-Dominated)**

Axion Domain Wall Problem

II. Axion Domain Wall Problem

The presence of long-lived axion-Domain-Walls is inconsistent with cosmological observations

- (Dark) matter energy density: $\rho_{\mathrm{DM}} \propto R^{-3} \Rightarrow \; R(t) \propto t^{2/3}$ **(Matter-Dominated)**
- Domain Wall energy density: $\rho_{\mathrm{DW}} \propto R^{-1} \; \Rightarrow \; R(t) \propto t^{2}$ **(DW-Dominated)**



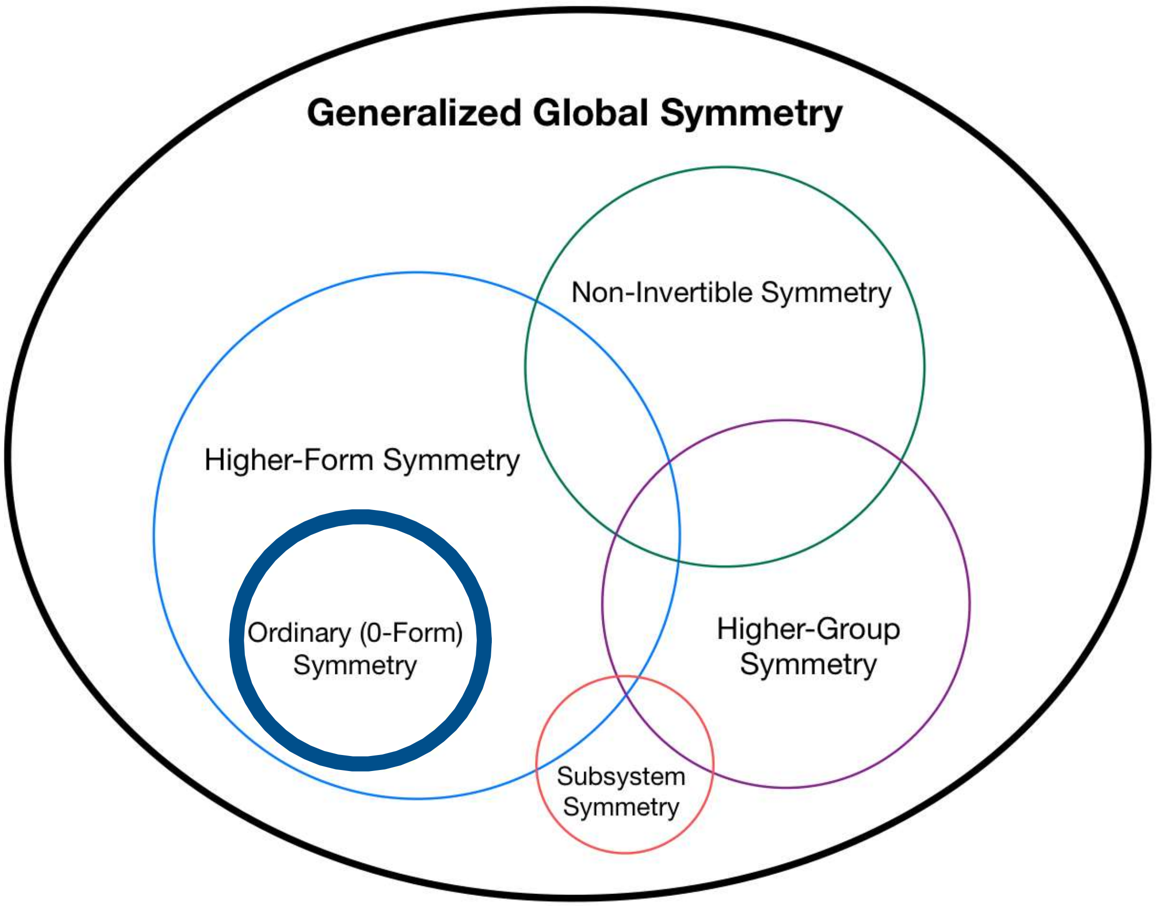
Axion Domain Wall Problem

II. Axion Domain Wall Problem

Success of axion theory hinges on first solving domain wall problem!

Axion Domain Wall Problem

III. Generalized Global Symmetries



Axion Domain Wall Problem

III. Generalized Symmetries ⇒ Axion-DW Problem

The significance of Generalized Symmetry is rather extreme!

Non-Invertible
Axion Domain Wall Problem

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

Consider axion coupled to $G_g = SU(N)/Z_N$

$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$ $G =$ field strength of G_g

1. If $G_g = SU(N)$

   Full axion shift symmetry: $U(1)_{PQ} \to Z_K(\text{invertible})$

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

Consider axion coupled to $G_g = SU(N)/Z_N$

$$S \supset \frac{iK}{8\pi^2} \int \frac{a}{f_a} Tr(G \wedge G)$$

$G =$ field strength of G_g

1. If $G_g = SU(N)$

Full axion shift symmetry: $U(1)_{PQ} \rightarrow Z_K$ (invertible)

$a \sim a + 2\pi f_a$, under $a \rightarrow a + \frac{2\pi f_a}{z}$ with $z \in Z$ (to be determined)

$$S \rightarrow S + \frac{i2\pi K}{z} \int \frac{Tr(G \wedge G)}{8\pi^2}$$

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

Consider axion coupled to $G_g = SU(N)/Z_N$

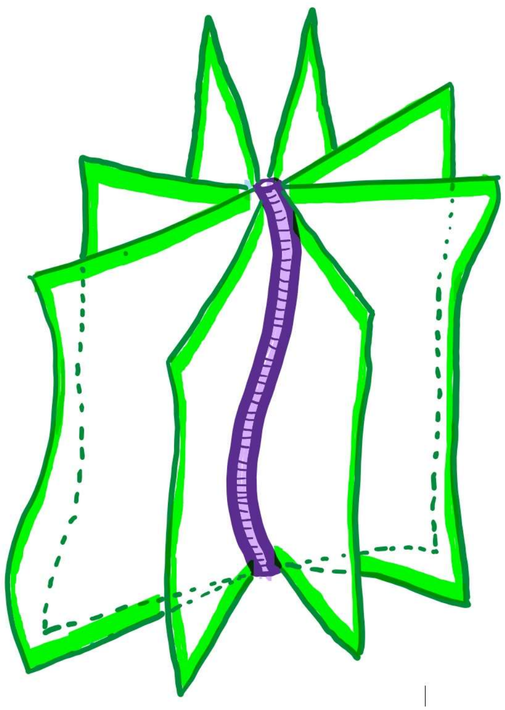
$$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

G = field strength of G_g

1. If $G_g = SU(N)$

   Full axion shift symmetry: $U(1)_{PQ} \to Z_K$ (invertible)

   K degenerate vacua $\to$ K invertible Domain Walls

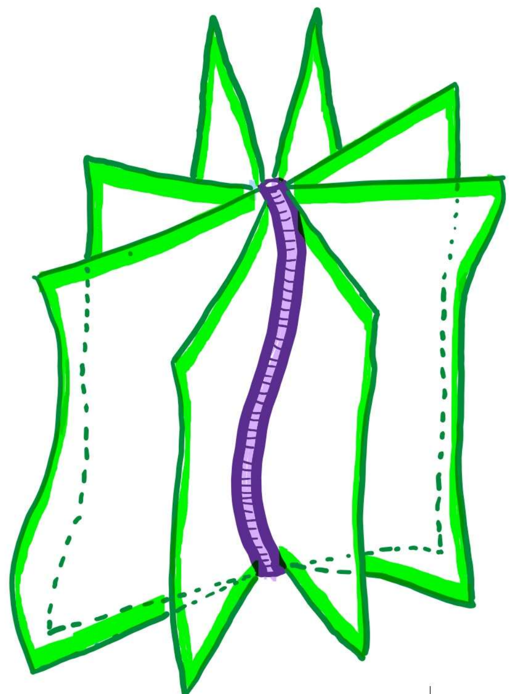


Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

Consider axion coupled to $G_g = SU(N)/Z_N$

$$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G) \qquad G = \text{field strength of } G_g$$



1. If $G_g = SU(N)$

   Full axion shift symmetry: $U(1)_{PQ} \rightarrow Z_K(\text{invertible})$

   K degenerate vacua $\rightarrow$ K invertible Domain Walls

2. If $G_g = SU(N)/Z_N$

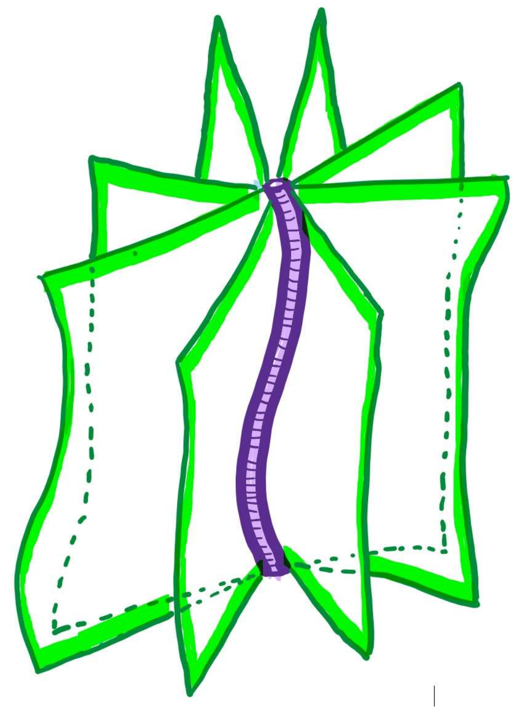
   Expected symmetry: $Z_{K/N}(\text{invertible})$

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

Consider axion coupled to $G_g = SU(N)/Z_N$

$$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G) \qquad G = \text{field strength of } G_g$$



1. If $G_g = SU(N)$

   Full axion shift symmetry: $U(1)_{PQ} \rightarrow Z_K$ (invertible)

   K degenerate vacua $\rightarrow$ K invertible Domain Walls

2. If $G_g = SU(N)/Z_N$

   Enlarged symmetry: $Z_{K/N}$ (invertible) $\subset Z_K$ (non invertible)

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

$S \supset \frac{iK}{8\pi^2} \int \frac{a}{f_a} Tr(G \wedge G)$ $\quad$ $G =$ field strength of G_g

We still fix: $a \sim a + 2\pi f_a$, $\quad$ under $a \to a + \frac{2\pi f_a}{z}$ with $z \in Z$ (to be determined)

$\Rightarrow \delta S = \frac{2\pi i K}{z} \int \frac{Tr(G \wedge G)}{8\pi^2} = \frac{2\pi i K}{z} n + \frac{2\pi i K}{z} \frac{N-1}{N} \int \frac{w \wedge w}{2} = 2\pi i Z$

$w \in H^2(M_4, Z_N)$: $\oint_{\Sigma_2} w = Z/NZ = Z_N$

2. If $G_g = SU(N)/Z_N$

Enlarged symmetry: $Z_{K/N}$(invertible) $\subset$ Z_K(non invertible)

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a} Tr(G \wedge G)$ G = field strength of G_g

We still fix: $a \sim a + 2\pi f_a$, under $a \to a + \frac{2\pi f_a}{z}$ with $z \in Z$ (to be determined)

$$\Rightarrow \delta S = \frac{2\pi i K}{z}\int \frac{Tr(G\wedge G)}{8\pi^2} = \frac{2\pi i K}{z} n + \frac{2\pi i K}{z}\frac{N-1}{N}\int \frac{w\wedge w}{2} = 2\pi i Z$$

$w \in H^2(M_4, Z_N)$: $\oint_{\Sigma_2} w = Z/NZ = Z_N$

$\Rightarrow n$: $U(1)_{PQ} \to Z_K$, $\frac{N-1}{N}\int \frac{w\wedge w}{2}$: $U(1)_{PQ} \to Z_{K/N}$

$\Rightarrow Z^I_{K/N} \subset Z^{NI}_K$

2. If $G_g = SU(N)/Z_N$

Enlarged symmetry: $Z_{K/N}$(invertible) $\subset Z_K$(non invertible)

Non-Invertible Symmetry with Fractional Instanton

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)$

electric 1-form: Z_N

magnetic 1-form: none

Non-Invertible Symmetry with Fractional Instanton

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

Non-Invertible Symmetry

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$$U(1)_A \text{ with } \alpha = \frac{2\pi}{k}, \qquad S \to S + \frac{2\pi A i}{k} \boxed{\int_{M_4} \frac{G \wedge G}{8\pi^2}} + \frac{2\pi A i}{k} \boxed{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}$$

$\in Z$ $\qquad\qquad\qquad$ $\in Z_L$

Non-Invertible Symmetry with Fractional Instanton

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

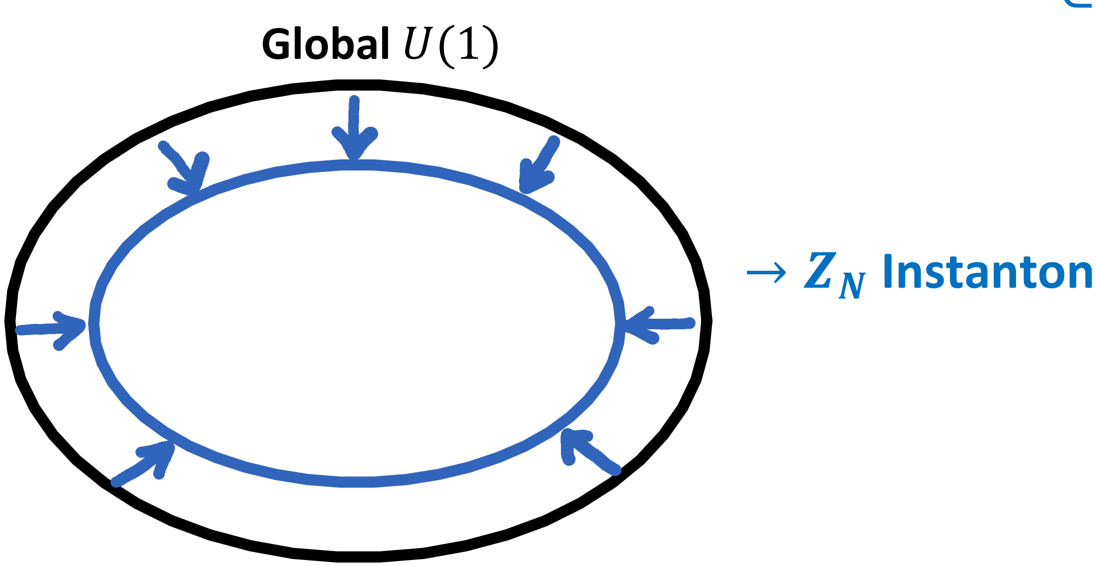
e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{k}$, $\quad S \to S + \frac{2\pi A i}{k} \boxed{\int_{M_4} \frac{G \wedge G}{8\pi^2}} + \frac{2\pi A i}{k} \boxed{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}$

$\in Z$ $\qquad\qquad$ $\in Z_L$



Non-Invertible Symmetry with Fractional Instanton

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)/Z_L$

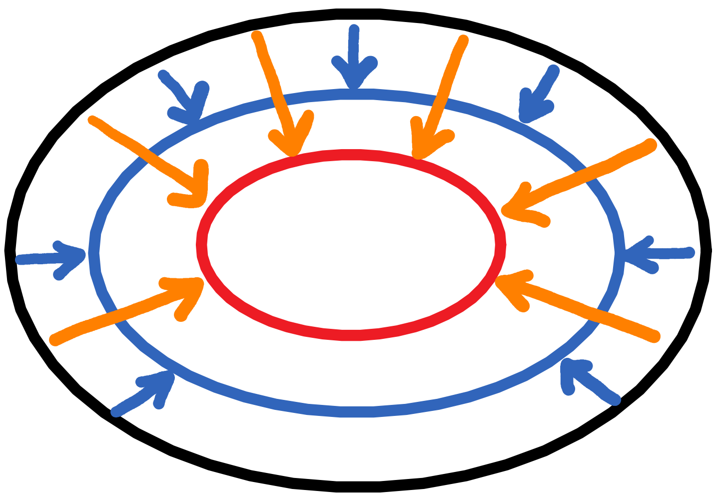
electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{k}$, $\quad S \rightarrow S + \frac{2\pi A i}{k} \boxed{\int_{M_4} \frac{G \wedge G}{8\pi^2}} + \frac{2\pi A i}{k} \boxed{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}$

$\in Z$ $\qquad \in Z_L$

Global $U(1)$



$\rightarrow Z_N$ Instanton

$\rightarrow Z_L$ (fractional) Instanton

Non-Invertible Symmetry with Fractional Instanton

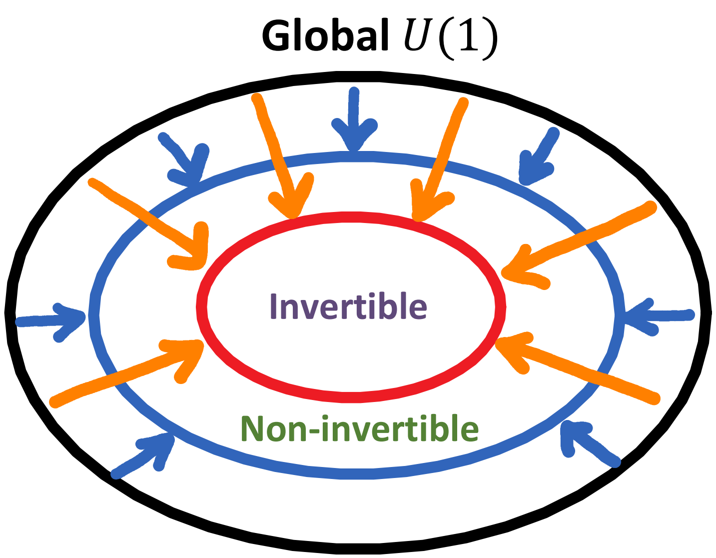
NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)/Z_L$ electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{k}$, $S \to S + \frac{2\pi A i}{k} \boxed{\int_{M_4} \frac{G \wedge G}{8\pi^2}} + \frac{2\pi A i}{k} \boxed{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}$

$\in Z$ $\in Z_L$



$\to Z_N$ Instanton

$\to Z_L$ (fractional) Instanton

Non-Invertible Symmetry with Fractional Instanton

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

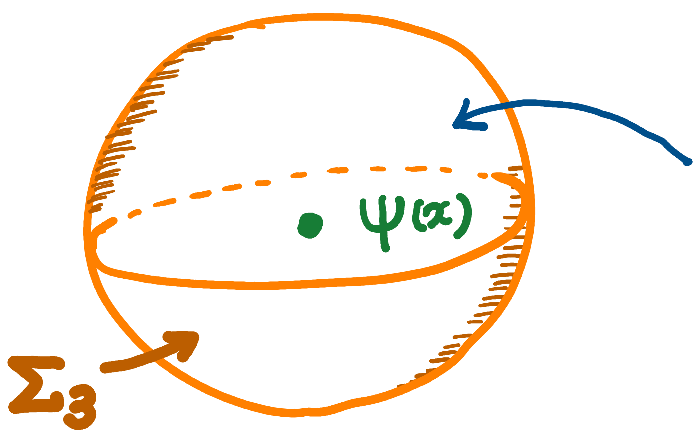
e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{k}$, $\quad S \to S + \frac{2\pi A i}{k} \boxed{\int_{M_4} \frac{G\wedge G}{8\pi^2}} + \frac{2\pi A i}{k} \boxed{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}$

$\in Z \qquad \in Z_L$



$$S_{defect} = \frac{iN}{4\pi} \int_{\Sigma_3} C \wedge dC$$

$$+ \frac{i}{2\pi} \int_{\Sigma_3} C \wedge w_2$$

Non-Invertible Symmetry with Fractional Instanton

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{k}$, $\quad S \to S + \frac{2\pi A i}{k} \underbrace{\int_{M_4} \frac{G \wedge G}{8\pi^2}}_{\in Z} + \frac{2\pi A i}{k} \underbrace{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}_{\in Z_L}$

$$U\left(\frac{2\pi}{k}, \Sigma_3\right) \to D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}\,(w_2)\ with\ \frac{p}{N} = \frac{A}{k}$$

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a} Tr(G \wedge G)$ G = field strength of G_g

We still fix: $a \sim a + 2\pi f_a$, under $a \to a + \frac{2\pi f_a}{z}$ with $z \in Z$ (to be determined)

$$\Rightarrow \delta S = \frac{2\pi i K}{z}\int \frac{Tr(G \wedge G)}{8\pi^2} = \frac{2\pi i K}{z} n + \frac{2\pi i K}{z}\frac{N-1}{N}\int \frac{w \wedge w}{2} = 2\pi i Z$$

$w \in H^2(M_4, Z_N)$: $\oint_{\Sigma_2} w = Z/NZ = Z_N$

$\Rightarrow n$: $U(1)_{PQ} \to Z_K$, $\frac{N-1}{N}\int \frac{w \wedge w}{2}$: $U(1)_{PQ} \to Z_{K/N}$

$\Rightarrow Z^I_{K/N} \subset Z^{NI}_K$

2. If $G_g = SU(N)/Z_N$

Enlarged symmetry: $Z_{K/N}$(invertible) $\subset Z_K$(non invertible)

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

$S \supset \frac{iK}{8\pi^2} \int \frac{a}{f_a} Tr(G \wedge G)$ G = field strength of G_g

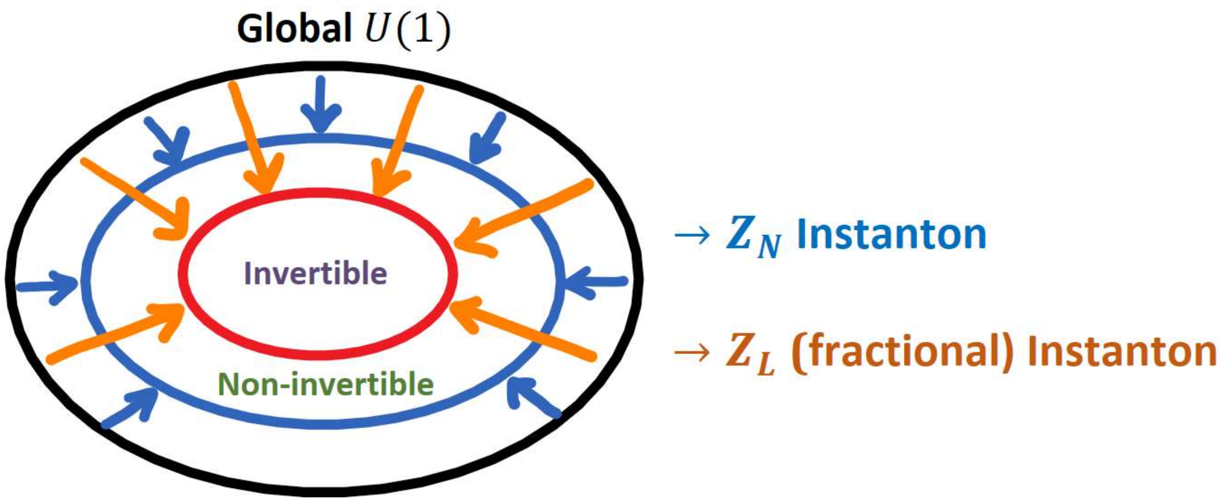
We still fix: $a \sim a + 2\pi f_a$, under $a \to a + \frac{2\pi f_a}{z}$ with $z \in Z$ (to be determined)

$$\Rightarrow \delta S = \frac{2\pi i K}{z} \int \frac{Tr(G \wedge G)}{8\pi^2} = \frac{2\pi i K}{z} n + \frac{2\pi i K}{z} \frac{N-1}{N} \int \frac{w \wedge w}{2} = 2\pi i Z$$

$w \in H^2(M_4, Z_N)$: $\oint_{\Sigma_2} w = Z/NZ = Z_N$

$\Rightarrow n$: $U(1)_{PQ} \to Z_K$, $\frac{N-1}{N} \int \frac{w \wedge w}{2}$: $U(1)_{PQ} \to Z_{K/N}$

$\Rightarrow Z^{I}_{K/N} \subset Z^{NI}_{K}$



Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

Consider axion coupled to $G_g = SU(N)/Z_N$

$$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G) \qquad G = \text{field strength of } G_g$$

1. If $G_g = SU(N)$

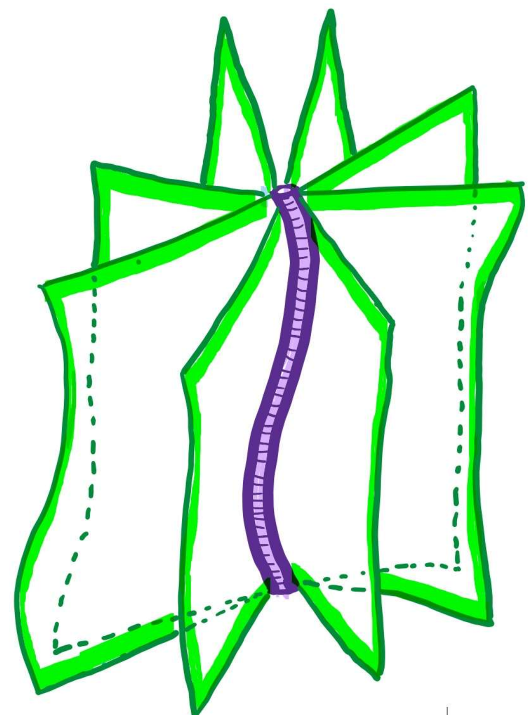
   Full axion shift symmetry: $U(1)_{PQ} \to Z_K(\text{invertible})$

   K degenerate vacua $\rightarrow$ K invertible Domain Walls

2. If $G_g = SU(N)/Z_N$

   Enlarged symmetry: $Z_{K/N}(\text{invertible}) \subset Z_K(\text{non invertible})$

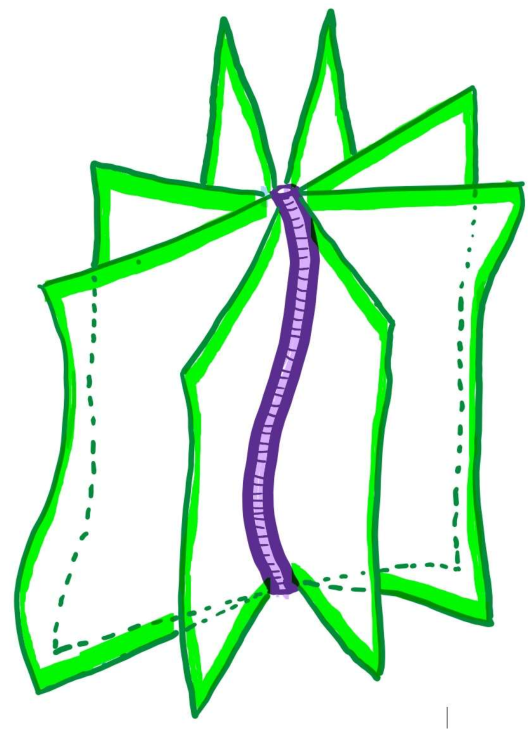
   Invertible $Z_{K/N}$ $\Rightarrow$ K/N degenerate vacua



Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

Consider axion coupled to $G_g = SU(N)/Z_N$



$$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G) \qquad G = \text{field strength of } G_g$$

1. If $G_g = SU(N)$

   Full axion shift symmetry: $U(1)_{PQ} \rightarrow Z_K\text{(invertible)}$

   K degenerate vacua $\rightarrow$ K invertible Domain Walls

2. If $G_g = SU(N)/Z_N$

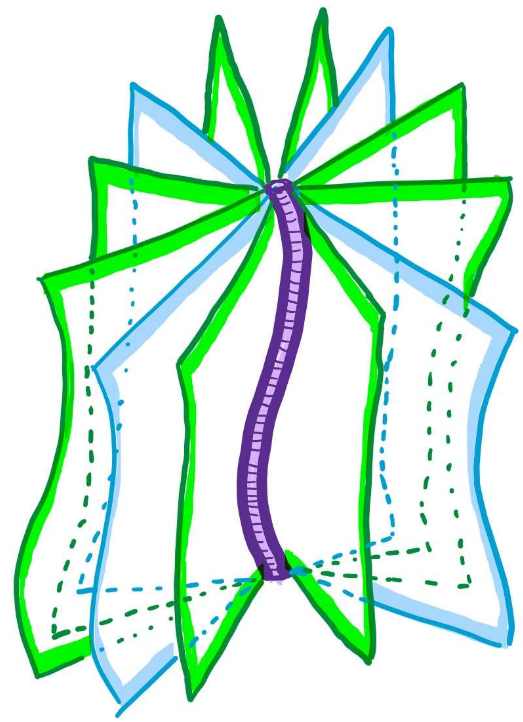
   Enlarged symmetry: $Z_{K/N}\text{(invertible)} \subset Z_K\text{(non invertible)}$

   ~~Invertible $Z_{K/N}$ $\Rightarrow$ K/N degenerate vacua~~

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

Consider axion coupled to $G_g = SU(N)/Z_N$



$S \supset \frac{iK}{8\pi^2} \int \frac{a}{f_a} Tr(G \wedge G)$ $\qquad$ G = field strength of G_g

1. If $G_g = SU(N)$

   Full axion shift symmetry: $U(1)_{PQ} \to Z_K$ (invertible)

   K degenerate vacua $\to$ K invertible Domain Walls

2. If $G_g = SU(N)/Z_N$

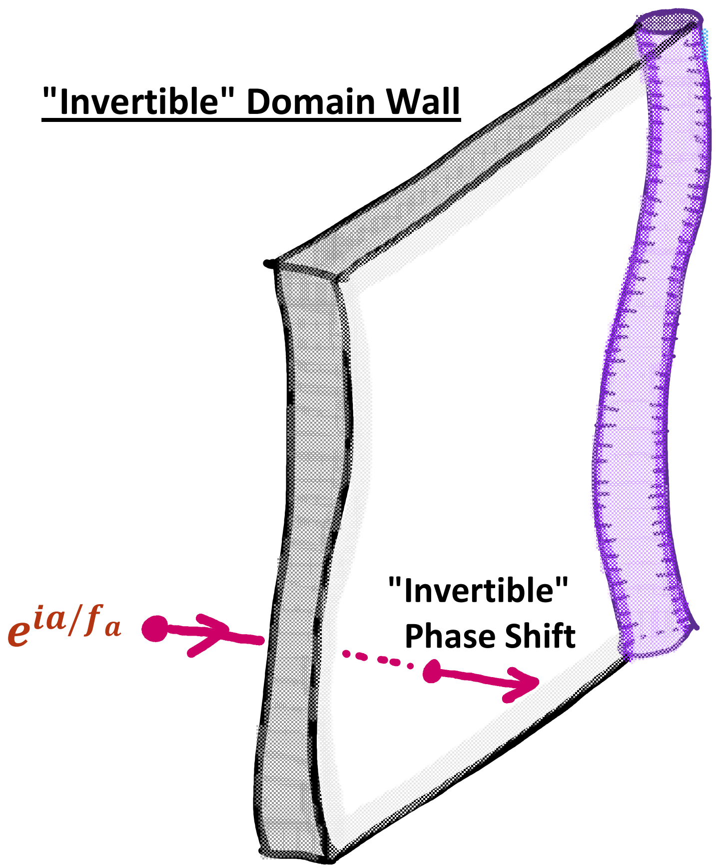
   Enlarged symmetry: $Z_{K/N}$ (invertible) $\subset Z_K$ (non invertible)

   ~~Invertible $Z_{K/N}$ $\Rightarrow$ K/N degenerate vacua~~

   K degenerate vacua $\to$ invertible + non-invertible Domain Walls

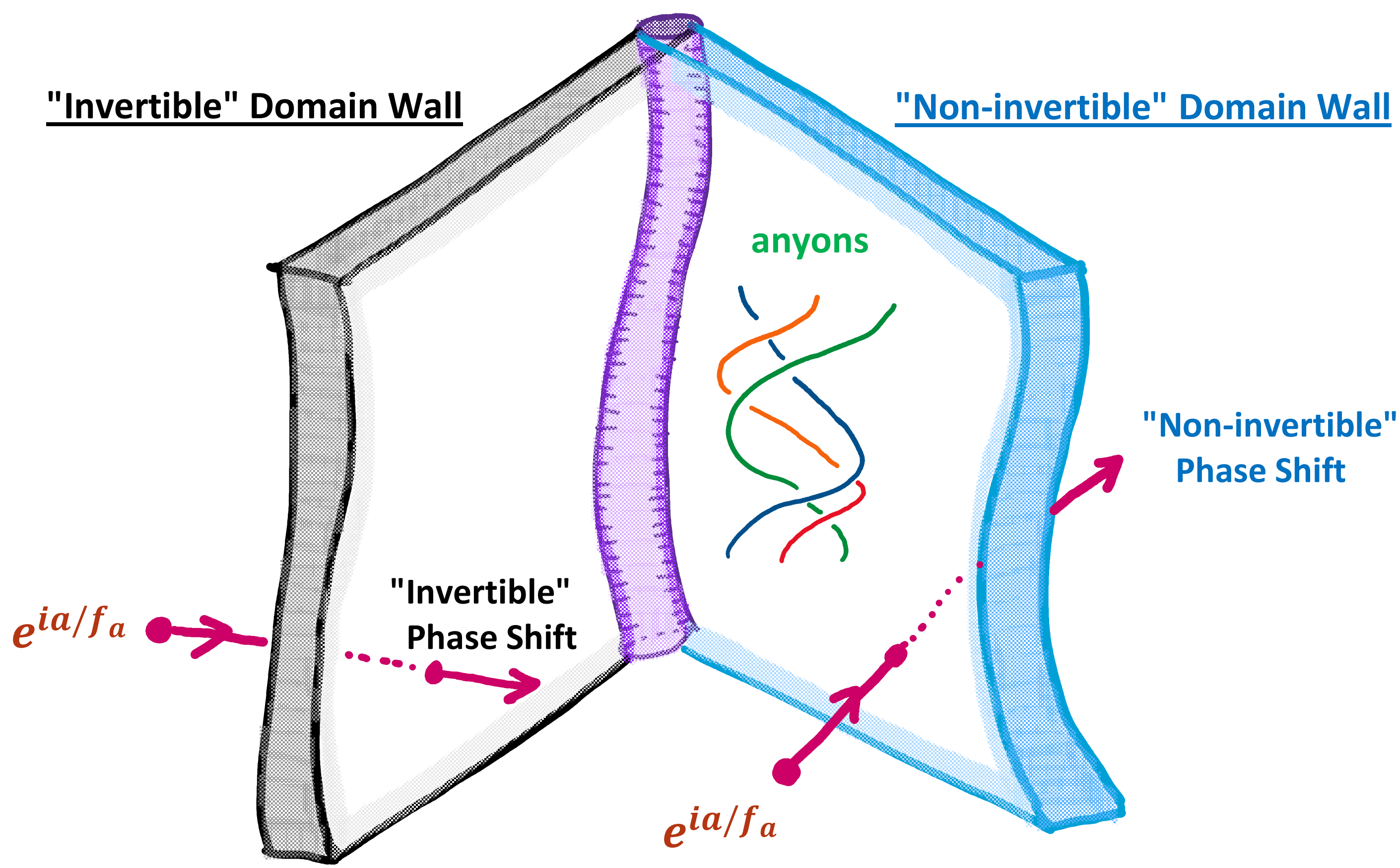
Non-invertible Axion Domain Wall Problem

Invertible DW vs Non-Invertible DW



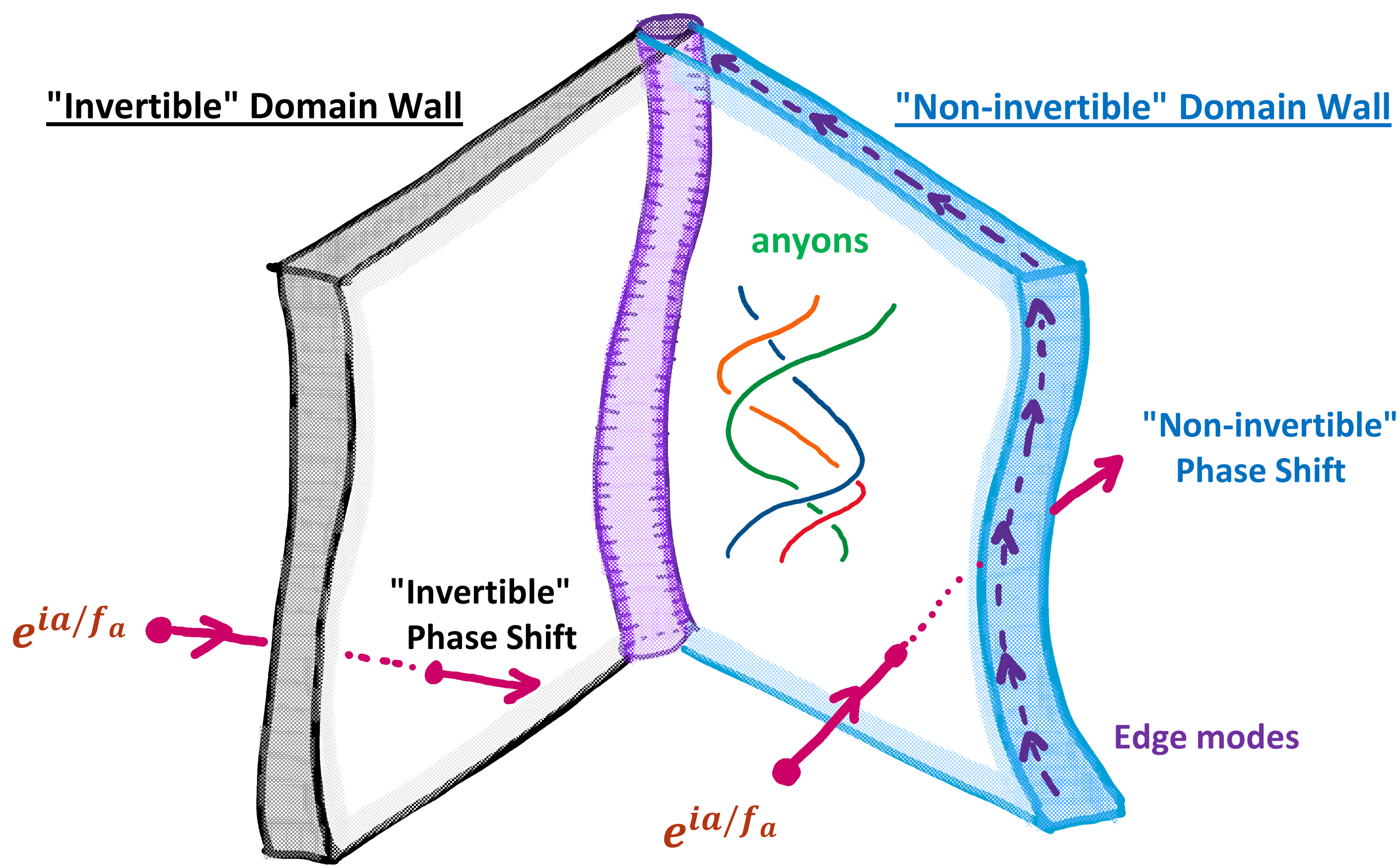
Non-invertible Axion Domain Wall Problem

Invertible DW vs Non-Invertible DW



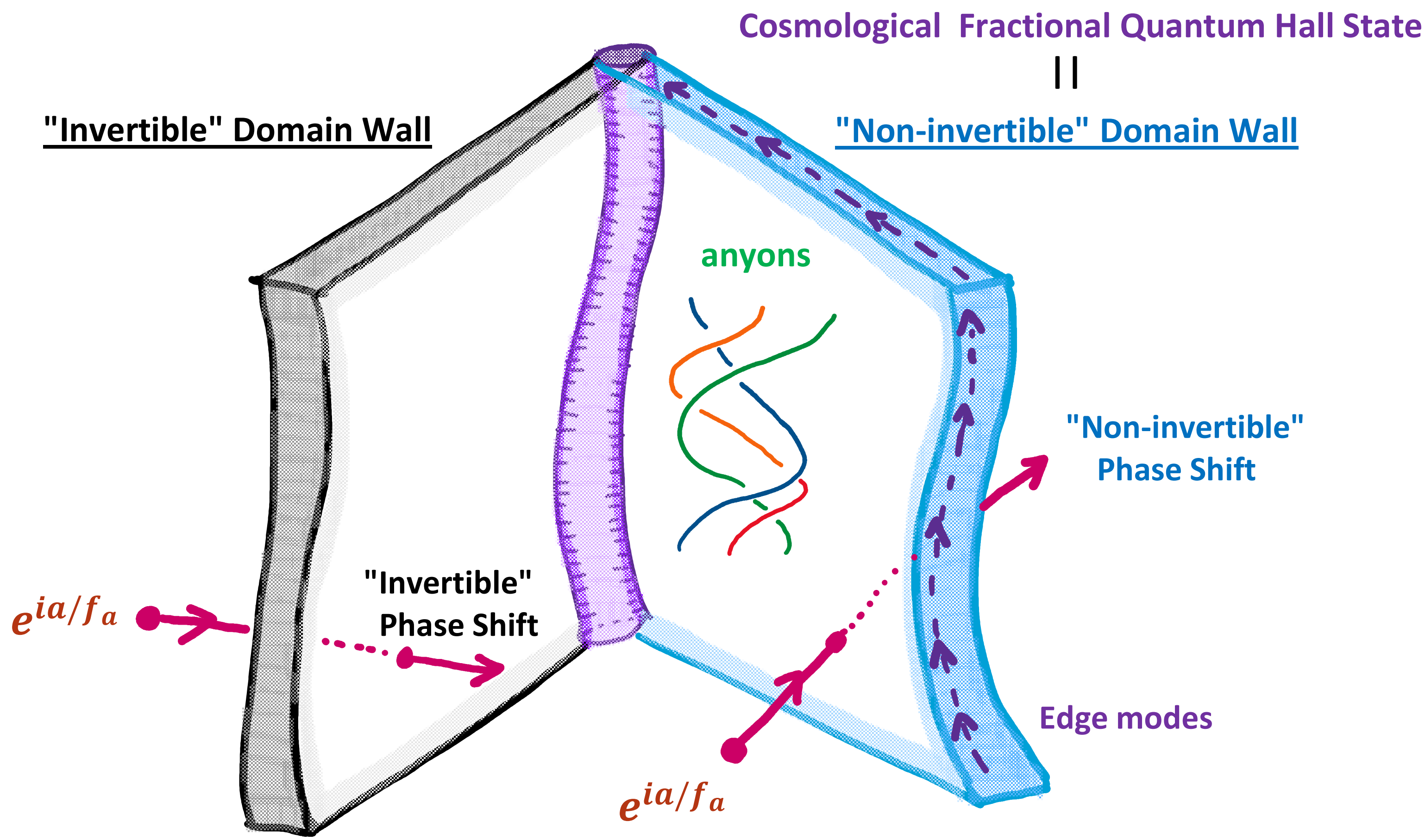
Non-invertible Axion Domain Wall Problem

Invertible DW vs Non-Invertible DW

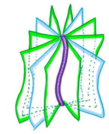


Non-invertible Axion Domain Wall Problem

Invertible DW vs Non-Invertible DW



Non-invertible Axion Domain Wall Problem

II. "Non-invertible Axion Domain Wall Problem" 

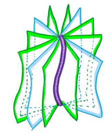
(Clay Cordova, Sungwoo Hong, Liantao Wang '23)

For given choice of the global structure of G_{SM} and anomaly coefficients, axion-SM theory can contain non-invertible-type as well as regular (invertible-type) domain wall defects. Any of these topological defects in the early universe is inconsistent with cosmological observations, and therefore should be made unstable or removed from the spectrum.

$$G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}, \quad \Gamma = 1, Z_2, Z_3, Z_6$$

$$S \supset \frac{i\ell_3}{8\pi^2} \int \frac{a}{f_a} Tr(G \wedge G) + \frac{i\ell_2}{8\pi^2} \int \frac{a}{f_a} Tr(W \wedge W) + \frac{i\ell_1}{8\pi^2} \int \frac{a}{f_a} B \wedge B$$

Non-invertible Axion Domain Wall Problem

II. "Non-invertible Axion Domain Wall Problem" 

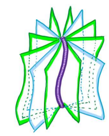
(Clay Cordova, Sungwoo Hong, Liantao Wang '23)

For given choice of the global structure of G_{SM} and anomaly coefficients, axion-SM theory can contain non-invertible-type as well as regular (invertible-type) domain wall defects. Any of these topological defects in the early universe is inconsistent with cosmological observations, and therefore should be made unstable or removed from the spectrum.

(i) The first realization of the true or the most precise version of the domain wall problem

(ii) Understanding of full generalized global symmetry is indispensable to this!

Non-invertible Axion Domain Wall Problem

II. "Non-invertible Axion Domain Wall Problem" 

(Clay Cordova, Sungwoo Hong, Liantao Wang '23)

For given choice of the global structure of G_{SM} and anomaly coefficients, axion-SM theory can contain non-invertible-type as well as regular (invertible-type) domain wall defects. Any of these topological defects in the early universe is inconsistent with cosmological observations, and therefore should be made unstable or removed from the spectrum.

(i) The first realization of the true or the most precise version of the domain wall problem

(ii) Understanding of full generalized global symmetry is indispensable to this!

(iii) All existing solutions address only the old, invertible, version of the problem
(hence would not solve the real problem is $\Gamma = \pi_1(G_{SM})$ is non-trivial)

(iv) correct formulation of the problem
⇒ "Systemtatic" path to solve the problem via non-invertible symmetry breaking
⇒ intrinsic connection between domain wall spectrum and global structure of G_{SM}

Axion-Standard Model

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W \wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B \wedge B$$

1. $G, W, B =$ field strength of $SU(3)_C$, $SU(2)_L, U(1)_Y$, respectively.
2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W\wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B\wedge B$$

1. G, W, B = field strength of $SU(3)_C$, $SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

i. The entire SM matter fields are neutral under Z_6 transformation generated by

$$e^{\frac{2\pi i}{3}\lambda_8} = e^{\frac{2\pi i}{3}} I_3 \in SU(3)_C, \quad e^{\frac{2\pi i}{2}T_3} = -I_2 \in SU(2)_L, \quad e^{\frac{2\pi i}{6}Q_Y} \in U(1)_Y$$

ii. Z_6 Wilson lines are not screened $\Rightarrow$ $Z_6^{(1)}$ electric 1-form center symmetry

iii. Gauging $\Gamma = Z_p^{(1)} \subset Z_6^{(1)} \Rightarrow \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $Z_{6/p}^{(1)}(e) \times Z_p^{(1)}(m)$

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W\wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B\wedge B$$

1. $G, W, B =$ field strength of $SU(3)_C,\ SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = 1$: $\ell_{1,2,3} \in Z$
$\Gamma = Z_2$: $\ell_1 \in 2Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 2\ell_2 \in 4Z$
$\Gamma = Z_3$: $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$
$\Gamma = Z_6$: $\ell_1 \in 6Z$, $\ell_{2,3} \in Z$, $\ell_1 + 2\ell_2 \in 4Z$, and $\ell_1 + 6\ell_3 \in 9Z$

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W\wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B\wedge B$$

1. G, W, B = field strength of $SU(3)_C$, $SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = 1$: $\ell_{1,2,3} \in Z$

$\Gamma = Z_2$: $\ell_1 \in 2Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 2\ell_2 \in 4Z$

$\Gamma = Z_3$: $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$

$\Gamma = Z_6$: $\ell_1 \in 6Z$, $\ell_{2,3} \in Z$, $\ell_1 + 2\ell_2 \in 4Z$, and $\ell_1 + 6\ell_3 \in 9Z$

e.g. Experimental observation: $\ell_1 = 0$, $\ell_2 = 2$, $\ell_3 = 2$

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W\wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B\wedge B$$

1. G, W, B = field strength of $SU(3)_C,\ SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C\times SU(2)_L\times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = 1$: $\ell_{1,2,3} \in Z$
$\Gamma = Z_2$: $\ell_1 \in 2Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 2\ell_2 \in 4Z$
$\Gamma = Z_3$: $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$
$\Gamma = Z_6$: $\ell_1 \in 6Z$, $\ell_{2,3} \in Z$, $\ell_1 + 2\ell_2 \in 4Z$, and $\ell_1 + 6\ell_3 \in 9Z$

e.g. Experimental observation: $\ell_1 = 0,\ \ \ell_2 = 2,\ \ \ell_3 = 2 \ \ \Rightarrow\ \ \Gamma = 1$ or Z_2

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W\wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B\wedge B$$

1. G, W, B = field strength of $SU(3)_C$, $SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = 1$: $\ell_{1,2,3} \in Z$
$\Gamma = Z_2$: $\ell_1 \in 2Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 2\ell_2 \in 4Z$
$\Gamma = Z_3$: $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$
$\Gamma = Z_6$: $\ell_1 \in 6Z$, $\ell_{2,3} \in Z$, $\ell_1 + 2\ell_2 \in 4Z$, and $\ell_1 + 6\ell_3 \in 9Z$

e.g. Experimental observation: $\ell_1 = 0$, $\ell_2 = 2$, $\ell_3 = 2$ $\Rightarrow$ $\Gamma = 1$ or Z_2

$\Gamma = Z_6$: $SU(5)$, $SO(10)$, E_6
$\Gamma = Z_3$: $SU(4)_C \times SU(2)_L \times SU(2)_R$ [Pati-Salam]
$\Gamma = Z_2$: $SU(3)_C \times SU(3)_L \times SU(3)_R$ [Trinification]

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W \wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B \wedge B$$

1. G, W, B = field strength of $SU(3)_C$, $SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = 1$: $\ell_{1,2,3} \in Z$
$\Gamma = Z_2$: $\ell_1 \in 2Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 2\ell_2 \in 4Z$
$\Gamma = Z_3$: $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$
$\Gamma = Z_6$: $\ell_1 \in 6Z$, $\ell_{2,3} \in Z$, $\ell_1 + 2\ell_2 \in 4Z$, and $\ell_1 + 6\ell_3 \in 9Z$

e.g. Experimental observation: $\ell_1 = 0$, $\ell_2 = 2$, $\ell_3 = 2$ $\Rightarrow$ $\Gamma = 1$ or Z_2

~~$\Gamma = Z_6$: $SU(5)$, $SO(10)$, E_6~~
~~$\Gamma = Z_3$: $SU(4)_C \times SU(2)_L \times SU(2)_R$ [Pati-Salam]~~
$\Gamma = Z_2$: $SU(3)_C \times SU(3)_L \times SU(3)_R$ [Trinification]

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W\wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B\wedge B$$

1. G, W, B = field strength of $SU(3)_C,\ SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

3. Quantization of Axion-Gauge Couplings from non-trivial global form

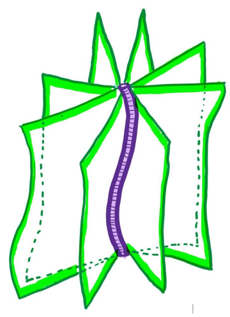
$\Gamma = 1$: $\ell_{1,2,3} \in Z$
$\Gamma = Z_2$: $\ell_1 \in 2Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 2\ell_2 \in 4Z$
$\Gamma = Z_3$: $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$
$\Gamma = Z_6$: $\ell_1 \in 6Z$, $\ell_{2,3} \in Z$, $\ell_1 + 2\ell_2 \in 4Z$, and $\ell_1 + 6\ell_3 \in 9Z$

4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3$

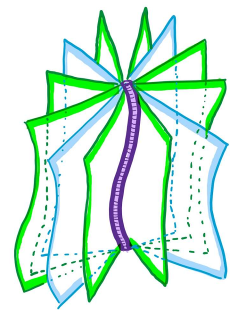
$$Z_K^I \subset Z^{NI}_{\mathrm{gcd}(\ell_2,\ell_3)} \approx Z^{NI}_{\ell_3} \quad \text{where} \quad K = \mathrm{gcd}\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1 + 6\ell_3}{9}\right)$$

Axion-Standard Model

III. Axion-SM



VS



3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = 1 \;:\quad \ell_{1,2,3} \in Z$

$\Gamma = Z_2 :\quad \ell_1 \in 2Z\,,\ \ell_{2,3} \in Z\,,$ and $\ell_1 + 2\ell_2 \in 4Z$

$\Gamma = Z_3 :\quad \ell_1 \in 3Z\,,\ \ell_{2,3} \in Z\,,$ and $\ell_1 + 6\ell_3 \in 9Z$

$\Gamma = Z_6 :\quad \ell_1 \in 6Z\,,\ \ell_{2,3} \in Z\,,\ \ell_1 + 2\ell_2 \in 4Z,$ and $\ell_1 + 6\ell_3 \in 9Z$

4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3\ \ (\ell_1 = 18, \ell_2 = 0, \ell_3 = 3)$

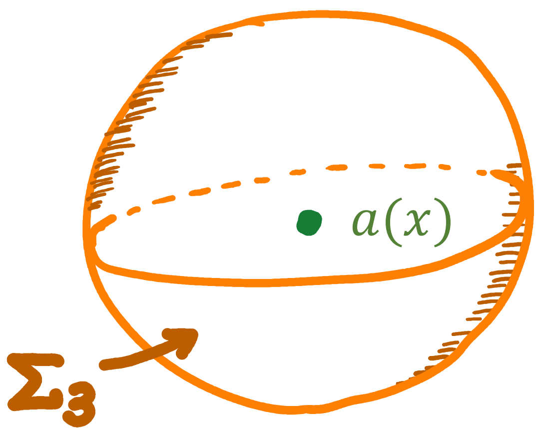
$Z^{I}_{K=1} \subset Z^{NI}_{\ell_3=3}$ where $K = \gcd\left(\frac{\ell_1}{3}, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$

Solving DW Problem by
Non-Invertible Symmetry Breaking

Solving Axion Domain Wall Problem

I. Non-invertible Symmetry Breaking

Non-invertible axion shift symmetry = 0-form + magnetic 1-form composite



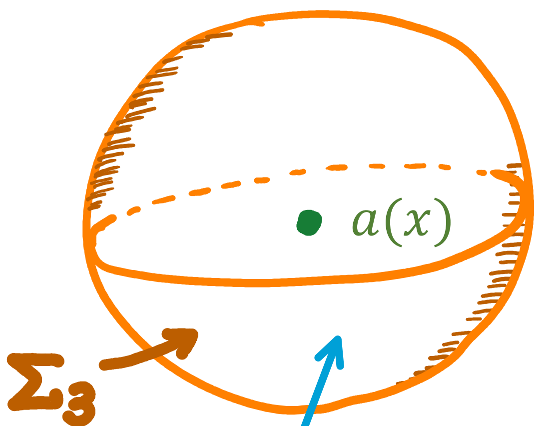
$$U(\alpha, \Sigma_3) = e^{i\alpha Q(\Sigma_3)}$$

$$Q(\Sigma_3) = \int_{\Sigma_3} d^3x\, J^0 = \int_{\Sigma_3} * J_1$$

Solving Axion Domain Wall Problem

I. Non-invertible Symmetry Breaking

Non-invertible axion shift symmetry = 0-form + magnetic 1-form composite



$$S_{defect} = \frac{iN}{4\pi}\int_{\Sigma_3} C \wedge dC$$

$$+ i \int_{\Sigma_3} C \wedge w$$

$$U(\alpha, \Sigma_3) = e^{i\alpha Q(\Sigma_3)}$$

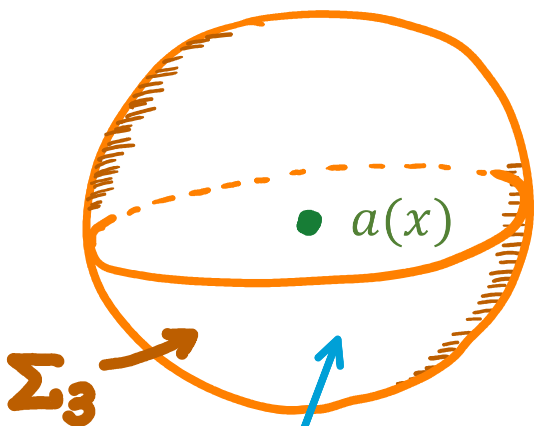
$$Q(\Sigma_3) = \int_{\Sigma_3} d^3x\, J^0 = \int_{\Sigma_3} * J_1$$

$$U\left(\frac{2\pi}{k}, \Sigma_3\right) \to D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}(w)$$

Solving Axion Domain Wall Problem

I. Non-invertible Symmetry Breaking

Non-invertible axion shift symmetry = 0-form + magnetic 1-form composite



$$S_{defect} = \frac{iN}{4\pi}\int_{\Sigma_3} C \wedge dC$$

$$+ i\int_{\Sigma_3} C \wedge w$$

$$U(\alpha, \Sigma_3) = e^{i\alpha Q(\Sigma_3)}$$

$$Q(\Sigma_3) = \int_{\Sigma_3} d^3x\, J^0 = \int_{\Sigma_3} * J_1$$

$$U\left(\frac{2\pi}{k}, \Sigma_3\right) \to D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}(w)$$

$$D \times \bar{D} \sim \sum_{\Sigma_2 \subset \Sigma_3} \exp\left(\frac{2\pi i}{N}\oint_{\Sigma_2} w\right)$$

Solving Axion Domain Wall Problem

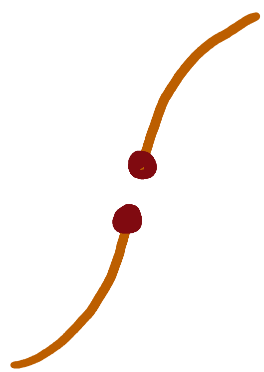
I. Non-invertible Symmetry Breaking

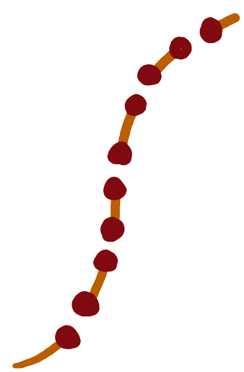
Non-invertible axion shift symmetry = 0-form + magnetic 1-form composite

Breaking of non-invertible symmetry by breaking magnetic 1-form symmetry

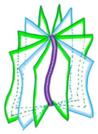
Monopole in UV → 1-form magnetic symm broken

→ non-invertible symm in IR broken

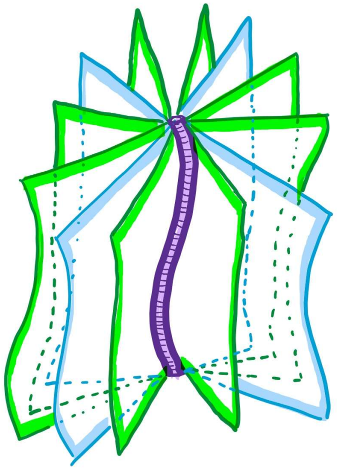




Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking 

(Clay Cordova, Sungwoo Hong, Liantao Wang)

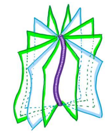


Consider axion coupled to $G_g = SU(N)/Z_N$

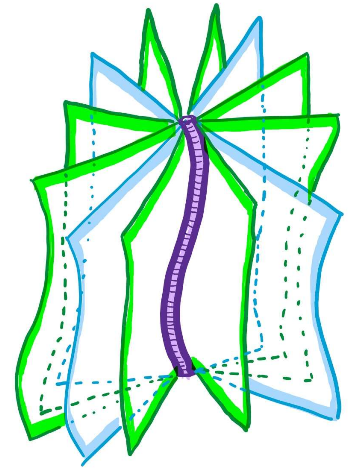
$$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a} Tr(G \wedge G) \qquad G = \text{field strength of } G_g$$

Enlarged symmetry: $Z_{K/N}(\text{invertible}) \subset Z_K(\text{non invertible})$

Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking 

(Clay Cordova, Sungwoo Hong, Liantao Wang)



Consider axion coupled to $G_g = SU(N)/Z_N$

$S \supset \frac{iK}{8\pi^2} \int \frac{a}{f_a} Tr(G \wedge G)$ $\qquad$ $G =$ field strength of G_g

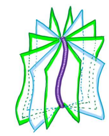
Enlarged symmetry: $Z_{K/N}(\text{invertible}) \subset Z_K(\text{non invertible})$

G_g (IR) Instanton effects $\Rightarrow$ Z_K^{NI} invariant $V(a)$

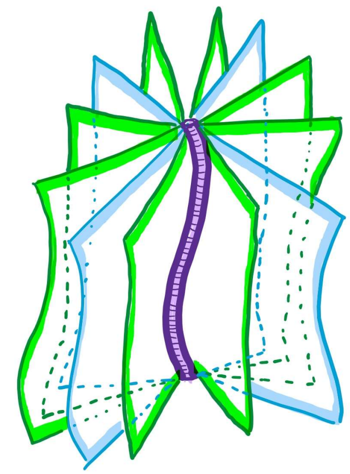
$$V(a) = \sum_{n=NZ} \alpha_n \cos\frac{Kn}{N}\frac{a}{f_a} \ , \qquad \alpha_n \propto e^{-\frac{8\pi^2 n}{g_{IR}^2}}$$

Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking



(Clay Cordova, Sungwoo Hong, Liantao Wang)



Consider axion coupled to $G_g = SU(N)/Z_N$

$S \supset \frac{iK}{8\pi^2} \int \frac{a}{f_a} Tr(G \wedge G)$ $\qquad$ G = field strength of G_g

Enlarged symmetry: $Z_{K/N}(\text{invertible}) \subset Z_K(\text{non invertible})$

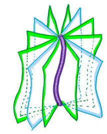
G_g (IR) Instanton effects $\Rightarrow$ Z_K^{NI} invariant $V(a)$

$$V(a) = \sum_{n=NZ} \alpha_n \cos \frac{Kn}{N}\frac{a}{f_a} \quad , \qquad \alpha_n \propto e^{-\frac{8\pi^2 n}{g_{IR}^2}}$$

$G_g \to G_{\text{uv}} = SU(N^2-1) \supset G_g$ with $\pi_1(G_{\text{uv}}) = 0$ $\Rightarrow$ Z_K^{NI} violating $\Delta V(a)$

$$\Delta V(a) = \sum_{m=0}^{N-1} \beta_m \cos \frac{Km}{N}\frac{a}{f_a} \quad , \quad \beta_m \propto e^{-\frac{8\pi^2 m}{g_{uv}^2}}$$

Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking 

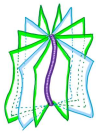
(Clay Cordova, Sungwoo Hong, Liantao Wang)

$$V(a) = V(a) + \Delta V(a)$$

$V(a) = \sum_{n=NZ} \alpha_n \cos\frac{Kn}{N}\frac{a}{f_a}$ and $\Delta V(a) = \sum_{m=0}^{N-1} \beta_m \cos\frac{Km}{N}\frac{a}{f_a}$

($\widehat{G_{IR}}$ - instantons) (G_{UV}/G_{IR} - instantons)

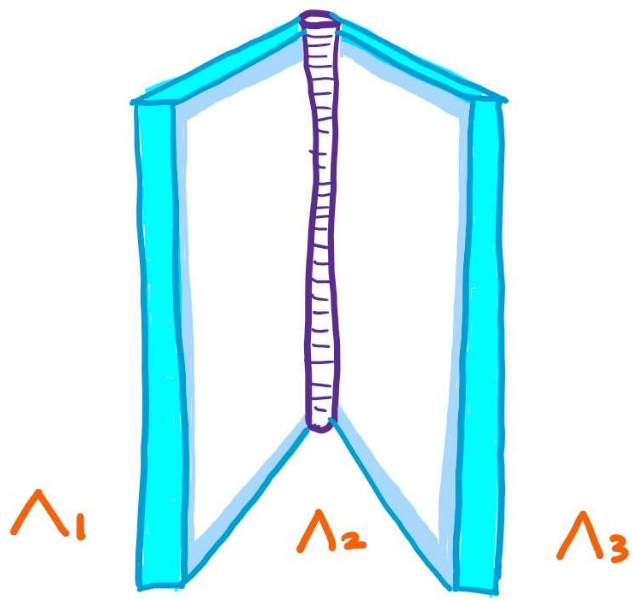
Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking 

(Clay Cordova, Sungwoo Hong, Liantao Wang)

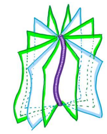
$$V(a) = V(a) + \Delta V(a)$$

$$V(a) = \sum_{n=NZ} \alpha_n \cos \frac{Kn}{N} \frac{a}{f_a}$$



$$\Lambda_1 = \Lambda_2 = \Lambda_3$$

Solving Axion Domain Wall Problem

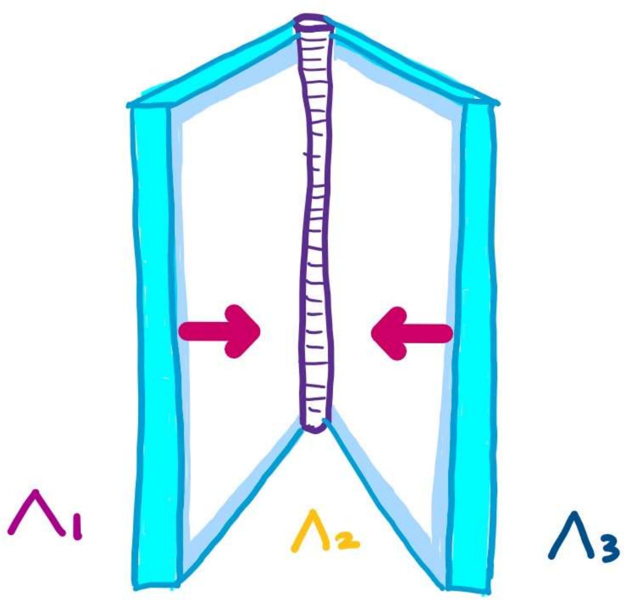
II. Solution by Non-invertible Symmetry Breaking 

(Clay Cordova, Sungwoo Hong, Liantao Wang)

$$V(a) = V(a) + \Delta V(a)$$

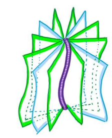
$V(a) = \sum_{n=NZ} \alpha_n \cos\frac{Kn}{N}\frac{a}{f_a}$ and $\Delta V(a) = \sum_{m=0}^{N-1} \beta_m \cos\frac{Km}{N}\frac{a}{f_a}$

($\widehat{G_{IR}}$ - instantons) (G_{UV}/G_{IR} - instantons)



Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking

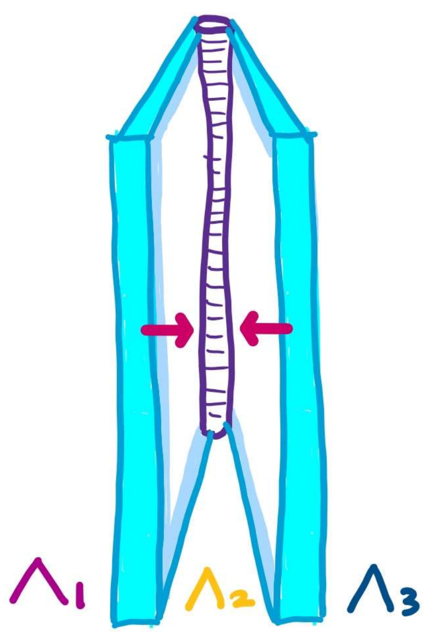


(Clay Cordova, Sungwoo Hong, Liantao Wang)

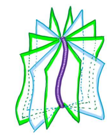
$$V(a) = V(a) + \Delta V(a)$$

$V(a) = \sum_{n=NZ} \alpha_n \cos\frac{Kn}{N}\frac{a}{f_a}$ and $\Delta V(a) = \sum_{m=0}^{N-1} \beta_m \cos\frac{Km}{N}\frac{a}{f_a}$

($\widehat{G_{IR}}$ - instantons) (G_{UV}/G_{IR} - instantons)



Solving Axion Domain Wall Problem

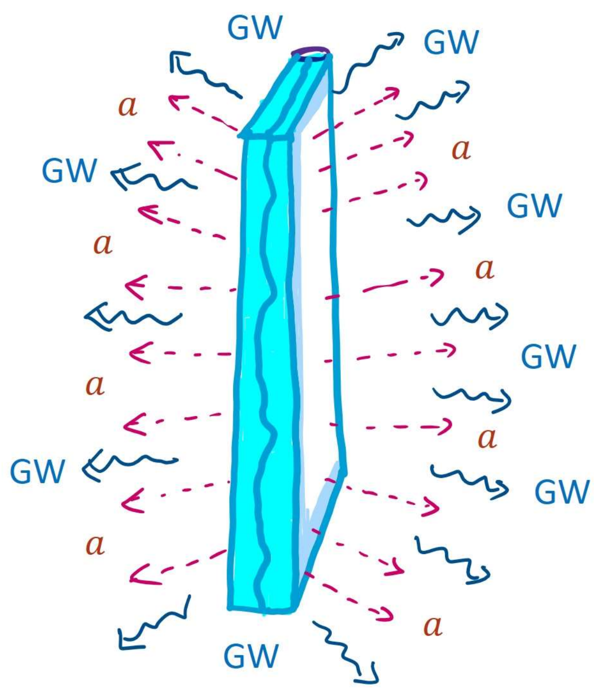
II. Solution by Non-invertible Symmetry Breaking 

(Clay Cordova, Sungwoo Hong, Liantao Wang)

$$V(a) = V(a) + \Delta V(a)$$

$V(a) = \sum_{n=NZ} \alpha_n \cos\frac{Kn}{N}\frac{a}{f_a}$ and $\Delta V(a) = \sum_{m=0}^{N-1} \beta_m \cos\frac{Km}{N}\frac{a}{f_a}$

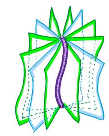
($\widehat{G_{IR}}$ - instantons) (G_{UV}/G_{IR} - instantons)



Solving Axion Domain Wall Problem

III. Constraints on GUT Theories

(Clay Cordova, Sungwoo Hong, Liantao Wang)



1. We found that the most well-known GUTs can not lift the vacuum degeneracy

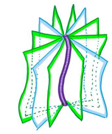
$\Gamma = \mathrm{Z}_6$: $SU(5)$, $SO(10)$, E_6

$\Gamma = Z_3$: $SU(4)_C \times SU(2)_L \times SU(2)_R$ [Pati-Salam]

$\Gamma = Z_2$: $SU(3)_C \times SU(3)_L \times SU(3)_R$ [Trinification]

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories



(Clay Cordova, Sungwoo Hong, Liantao Wang)

1. We found that the most well-known GUTs can not lift the vacuum degeneracy

$\Gamma = Z_6$: $SU(5)$, $SO(10)$, E_6
$\Gamma = Z_3$: $SU(4)_C \times SU(2)_L \times SU(2)_R$ [Pati-Salam]
$\Gamma = Z_2$: $SU(3)_C \times SU(3)_L \times SU(3)_R$ [Trinification]

index of embedding in $G_{\mathrm{UV}} \to G_{\mathrm{IR}}$: 1-IR-instanton = n-UV-instanton

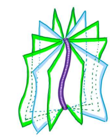
If $n \neq 1$, then $\exists$ G_{UV}-instantons not gauge-equvalent to G_{IR}-instanton

"Small Instantons"

These small instantons can break non-invertible symmetries.

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories



(Clay Cordova, Sungwoo Hong, Liantao Wang)

1. We found that the most well-known GUTs can not lift the vacuum degeneracy

$\Gamma = \mathrm{Z}_6 :\ \ SU(5)\,,\ SO(10)\,,\ E_6$
$\Gamma = Z_3 :\ \ SU(4)_C \times SU(2)_L \times SU(2)_R$ [Pati-Salam]
$\Gamma = Z_2 :\ \ SU(3)_C \times SU(3)_L \times SU(3)_R$ [Trinification]

index of embedding in $G_{\mathrm{UV}} \to G_{\mathrm{IR}}$: 1-IR-instanton = n-UV-instanton

If $n \neq 1$, then $\exists\ G_{\mathrm{UV}}$-instantons not gauge-equvalent to G_{IR}-instanton

"Small Instantons"

These small instantons can break non-invertible symmetries.

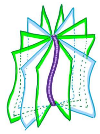
In all these cases, **"index of embedding (n)" = 1**

So, either anomaly coefficients "ℓ" in GUT should be $\boldsymbol{\ell_{\mathrm{UV}} = 1}$
or extra structures have to be supplemented to cure the DW problem

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories

(Clay Cordova, Sungwoo Hong, Liantao Wang)



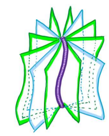
2. GUT constraints on Axion-Gauge Couplings

$$S_{SU(5)} \supset \frac{i\ell_{\mathrm{uv}}}{8\pi^2}\int \frac{a}{f_a}\, Tr(g \wedge g)$$

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories

(Clay Cordova, Sungwoo Hong, Liantao Wang)



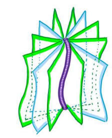
2. GUT constraints on Axion-Gauge Couplings

$$S_{SU(5)} \supset \frac{i\ell_{\text{uv}}}{8\pi^2}\int \frac{a}{f_a}\, Tr(g \wedge g)$$

$$\rightarrow \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W \wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B \wedge B$$

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories



(Clay Cordova, Sungwoo Hong, Liantao Wang)

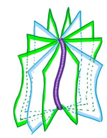
2. GUT constraints on Axion-Gauge Couplings

$$S_{SU(5)} \supset \frac{i\ell_{\mathrm{uv}}}{8\pi^2}\int \frac{a}{f_a}\, Tr(g \wedge g)$$

$$\rightarrow \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W \wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B \wedge B$$

$$\boldsymbol{\ell_3 = \ell_{\mathrm{uv}},\ \ell_2 = \ell_{\mathrm{uv}},\ \ell_1 = 30\ell_{\mathrm{uv}}}$$

Solving Axion Domain Wall Problem



III. Constraints on GUT Theories

(Clay Cordova, Sungwoo Hong, Liantao Wang)

2. GUT constraints on Axion-Gauge Couplings

$$S_{SU(5)} \supset \frac{i\ell_{\rm uv}}{8\pi^2}\int \frac{a}{f_a}\, Tr(g\wedge g)$$

$$\rightarrow \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W\wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B\wedge B$$

$$\boldsymbol{\ell_3 = \ell_{\rm uv},\ \ell_2 = \ell_{\rm uv},\ \ell_1 = 30\ell_{\rm uv}}$$

This is consistent with

$\Gamma = 1$: $\ell_{1,2,3} \in Z$

$\Gamma = Z_2$: $\ell_1 \in 2Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 2\ell_2 \in 4Z$

$\Gamma = Z_3$: $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$

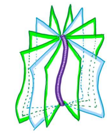
$\Gamma = Z_6$: $\ell_1 \in 6Z$, $\ell_{2,3} \in Z$, $\ell_1 + 2\ell_2 \in 4Z$, and $\ell_1 + 6\ell_3 \in 9Z$

but provides more stringent constraints.

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories

(Clay Cordova, Sungwoo Hong, Liantao Wang)



2. GUT constraints on Axion-Gauge Couplings

$SU(5)\ [Z_6]$: $\ell_3 = \ell_{\mathrm{uv}},\ \ell_2 = \ell_{\mathrm{uv}},\ \ell_1 = 30\ell_{\mathrm{uv}}$

$SU(4)_C \times SU(2)_L \times SU(2)_R\ [Z_3]$: $\ell_3 = \ell_4,\ \ell_2 = \ell_{\mathrm{L}},\ \ell_1 = 12\ell_4 + 18\ell_R$

$SU(3)_C \times SU(3)_L \times SU(3)_R\ [Z_2]$: $\ell_3 = \ell_{\mathrm{c}},\ \ell_2 = \ell_{\mathrm{L}},\ \ell_1 = 6\ell_{\mathrm{L}} + 24\ell_R$

Thank You
For
Your Attention!

Back-up-1

(Quantization of Axion-Gauge Couplings)

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = Z_3 :\quad \ell_1 \in 3Z\,,\ \ell_{2,3} \in Z\,,$ and $\ell_1 + 6\ell_3 \in 9Z$

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = Z_3 : \quad \ell_1 \in 3Z\,,\ \ell_{2,3} \in Z\,,$ and $\ell_1 + 6\ell_3 \in 9Z$

(i) $G_{SM} = \frac{SU(3)_C \times U(1)_Y}{\mathrm{Z}_3} \times SU(2)_L$

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = Z_3 :\quad \ell_1 \in 3Z\,,\ \ \ell_{2,3} \in Z\,,$ and $\ell_1 + 6\ell_3 \in 9Z$

(i) $G_{SM} = \frac{SU(3)_C \times U(1)_Y}{Z_3} \times SU(2)_L$

(ii) Fractional Instanton of $\frac{SU(N)}{Z_N}$: $\quad \int \frac{Tr(G \wedge G)}{8\pi^2} \supset \frac{N-1}{N} \int \frac{w \wedge w}{2}$

(iii) $\oint_{\Sigma_2} \frac{F}{2\pi} = \frac{1}{3} \oint_{\Sigma_2} w(A_3) \ \ \Rightarrow \ \ \frac{F}{2\pi} = \frac{1}{3} w(A_3) + X, \ \ X \in H^2(M_4, Z)$

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = Z_3 : \quad \ell_1 \in 3Z\,,\ \ell_{2,3} \in Z\,,$ and $\ell_1 + 6\ell_3 \in 9Z$

(i) $G_{SM} = \frac{SU(3)_C \times U(1)_Y}{Z_3} \times SU(2)_L$

(ii) Fractional Instanton of $\frac{SU(N)}{Z_N}$: $\quad \int \frac{Tr(G \wedge G)}{8\pi^2} \supset \frac{N-1}{N} \int \frac{w \wedge w}{2}$

(iii) $\oint_{\Sigma_2} \frac{F}{2\pi} = \frac{1}{3} \oint_{\Sigma_2} w(A_3) \quad \Rightarrow \quad \frac{F}{2\pi} = \frac{1}{3} w(A_3) + X, \quad X \in H^2(M_4, Z)$

(iv) Then, the axion periodicity $a \sim a + 2\pi f_a$ is respected if

$$\frac{\ell_1}{2} \int \left(\frac{w(A_3)}{3} + X\right)^2 + \ell_2\, n_2 + \ell_3 \left(n_3 + \frac{2}{3} \int \frac{w(A_3)^2}{2}\right) \in Z$$

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = Z_3 :\quad \ell_1 \in 3Z\,,\ \ell_{2,3} \in Z\,,$ and $\ell_1 + 6\ell_3 \in 9Z$

(i) $G_{SM} = \frac{SU(3)_C \times U(1)_Y}{Z_3} \times SU(2)_L$

(ii) Fractional Instanton of $\frac{SU(N)}{Z_N}$: $\quad \int \frac{Tr(G \wedge G)}{8\pi^2} \supset \frac{N-1}{N} \int \frac{w \wedge w}{2}$

(iii) $\oint_{\Sigma_2} \frac{F}{2\pi} = \frac{1}{3} \oint_{\Sigma_2} w(A_3) \quad \Rightarrow \quad \frac{F}{2\pi} = \frac{1}{3} w(A_3) + X, \quad X \in H^2(M_4, Z)$

(iv) Then, the axion periodicity $a \sim a + 2\pi f_a$ is respected if

$$\frac{\ell_1}{2} \int \left(\frac{w(A_3)}{3} + X\right)^2 + \ell_2\, n_2 + \ \ell_3 \left(n_3 + \frac{2}{3} \int \frac{w(A_3)^2}{2}\right) \in Z$$

(1) Consider first the case when $w(A_3)^2$-term vanishes

$$\frac{\ell_1}{3} \int w(A_3) \wedge X + \ell_2 n_2 + \ell_3 n_3 \in Z \ \Rightarrow \quad \ell_1 \in 3Z,\ \ \ell_2, \ell_3 \in Z$$

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = Z_3: \quad \ell_1 \in 3Z\,,\ \ell_{2,3} \in Z\,,$ and $\ell_1 + 6\ell_3 \in 9Z$

(i) $G_{SM} = \frac{SU(3)_C \times U(1)_Y}{Z_3} \times SU(2)_L$

(ii) Fractional Instanton of $\frac{SU(N)}{Z_N}$: $\quad \int \frac{Tr(G \wedge G)}{8\pi^2} \supset \frac{N-1}{N} \int \frac{w \wedge w}{2}$

(iii) $\oint_{\Sigma_2} \frac{F}{2\pi} = \frac{1}{3} \oint_{\Sigma_2} w(A_3) \quad \Rightarrow \quad \frac{F}{2\pi} = \frac{1}{3} w(A_3) + X, \quad X \in H^2(M_4, Z)$

(iv) Then, the axion periodicity $a \sim a + 2\pi f_a$ is respected if

$$\frac{\ell_1}{2} \int \left(\frac{w(A_3)}{3} + X\right)^2 + \ell_2\, n_2 + \ \ell_3 \left(n_3 + \frac{2}{3} \int \frac{w(A_3)^2}{2}\right) \in Z$$

(1) Consider first the case when $w(A_3)^2$-term vanishes

$$\frac{\ell_1}{3} \int w(A_3) \wedge X + \ell_2 n_2 + \ell_3 n_3 \in Z \ \Rightarrow \quad \ell_1 \in 3Z,\ \ \ell_2, \ell_3 \in Z$$

(2) $w(A_3)^2$: $\left(\frac{\ell_1}{9} + \frac{2\ell_3}{3}\right) \int \frac{w(A_3)^2}{2} \in Z \ \Rightarrow \ \ell_1 + 6\ell_3 \in 9Z$

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W\wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B\wedge B$$

1. $G, W, B =$ field strength of $SU(3)_C,\ SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = 1\ :\quad \ell_{1,2,3} \in Z$

$\Gamma = Z_2 :\quad \ell_1 \in 2Z\ ,\ \ell_{2,3} \in Z\ ,$ and $\ell_1 + 2\ell_2 \in 4Z$

$\Gamma = Z_3 :\quad \ell_1 \in 3Z\ ,\ \ell_{2,3} \in Z\ ,$ and $\ell_1 + 6\ell_3 \in 9Z$

$\Gamma = Z_6 :\quad \ell_1 \in 6Z\ ,\ \ell_{2,3} \in Z\ ,\ \ell_1 + 2\ell_2 \in 4Z,$ and $\ell_1 + 6\ell_3 \in 9Z$

Back-up-2

(Non-invertible Axion Shift Symmetry)

Axion-Standard Model

4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3$

$Z_K^I \subset Z_{\mathrm{gcd}(\ell_2,\ell_3)}^{NI} \approx Z_{\ell_3}^{NI}$ where $K = \mathrm{gcd}\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$

Under $a \to a + 2\pi f_a/z$ (recall: $F/2\pi = w/3 + X$)

$$\delta S = \frac{2\pi i}{z}\frac{\ell_1}{2}\int\left(\frac{w}{3} + X\right)^2 + \frac{2\pi i}{z}\,\ell_2 n_2 + \frac{2\pi i}{z}\,\ell_3\left(n_3 + \frac{2}{3}\int\frac{w^2}{2}\right) \in 2\pi i Z$$

Axion-Standard Model

4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3$

$$Z_K^I \subset Z^{NI}_{\gcd(\ell_2,\ell_3)} \approx Z^{NI}_{\ell_3} \quad \text{where} \quad K = \gcd\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$$

Under $a \to a + 2\pi f_a/z$ (recall: $F/2\pi = w/3 + X$)

$$\delta S = \frac{2\pi i}{z}\frac{\ell_1}{2}\int\left(\frac{w}{3}+X\right)^2 + \frac{2\pi i}{z}\,\ell_2 n_2 + \frac{2\pi i}{z}\,\ell_3\left(n_3 + \frac{2}{3}\int\frac{w^2}{2}\right) \in 2\pi i Z$$

(1) For $w^2 = 0$: $\frac{\ell_1/3}{z}, \frac{\ell_2}{z}, \frac{\ell_3}{z} \in Z$

Axion-Standard Model

4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3$

$$Z_K^I \subset Z^{NI}_{\mathrm{gcd}(\ell_2,\ell_3)} \approx Z^{NI}_{\ell_3} \quad \text{where} \quad K = \mathrm{gcd}\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$$

Under $a \to a + 2\pi f_a/z$ (recall: $F/2\pi = w/3 + X$)

$$\delta S = \frac{2\pi i}{z}\frac{\ell_1}{2}\int\left(\frac{w}{3}+X\right)^2 + \frac{2\pi i}{z}\,\ell_2 n_2 + \frac{2\pi i}{z}\,\ell_3\left(n_3 + \frac{2}{3}\int\frac{w^2}{2}\right) \in 2\pi i Z$$

(1) For $w^2 = 0$: $\frac{\ell_1/3}{z}, \frac{\ell_2}{z}, \frac{\ell_3}{z} \in Z$

(2) From w^2-term : $\frac{\ell_1+6\ell_3}{9z} \in Z$

Axion-Standard Model

4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3$

$$Z_K^I \subset Z^{NI}_{\gcd(\ell_2,\ell_3)} \approx Z^{NI}_{\ell_3} \quad \text{where} \quad K = \gcd\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$$

Under $a \to a + 2\pi f_a/z$ (recall: $F/2\pi = w/3 + X$)

$$\delta S = \frac{2\pi i}{z}\frac{\ell_1}{2}\int\left(\frac{w}{3}+X\right)^2 + \frac{2\pi i}{z}\,\ell_2 n_2 + \frac{2\pi i}{z}\,\ell_3\left(n_3 + \frac{2}{3}\int\frac{w^2}{2}\right) \in 2\pi i Z$$

(1) For $w^2 = 0$: $\frac{\ell_1/3}{z}, \frac{\ell_2}{z}, \frac{\ell_3}{z} \in Z$

(2) From w^2-term : $\frac{\ell_1+6\ell_3}{9z} \in Z$

$\Rightarrow$ Invertible Symmetry: $Z_{\gcd(\ell_1/3,\ell_2,\ell_3,(\ell_1+6\ell_3)/9)}$

Axion-Standard Model

4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3$

$Z_K^I \subset Z^{NI}_{\gcd(\ell_2,\ell_3)} \approx Z^{NI}_{\ell_3}$ where $K = \gcd\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$

Under $a \to a + 2\pi f_a/z$ (recall: $F/2\pi = w/3 + X$)

$$\delta S = \frac{2\pi i}{z}\frac{\ell_1}{2}\int\left(\frac{w}{3}+X\right)^2 + \frac{2\pi i}{z}\,\ell_2 n_2 + \frac{2\pi i}{z}\,\ell_3\left(n_3 + \frac{2}{3}\int\frac{w^2}{2}\right) \in 2\pi i Z$$

(1) For $w^2 = 0$: $\frac{\ell_1/3}{z}, \frac{\ell_2}{z}, \frac{\ell_3}{z} \in Z$

(2) From w^2-term : $\frac{\ell_1+6\ell_3}{9z} \in Z$

$\Rightarrow$ Invertible Symmetry: $Z_{\gcd(\ell_1/3,\ell_2,\ell_3,(\ell_1+6\ell_3)/9)}$

(3) Regular Z-valued instanton effects: $Z_{\gcd(\ell_2,\ell_3)} \approx Z_{\ell_3}$