Peccei-Quinn Inflation at the Pole and Axion Kinetic Misalignment

Hyun Min Lee

Chung-Ang University, Korea

Ref. HML, A. Menkara, M-J. Seong, J-H. Song, 2310.17710 [hep-ph]

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Outline

- Introduction
- Pole inflation with Higgs or PQ fields
- Reheating and axion kinetic misalignment
- Conclusions

Introduction

Inflation @ Planck

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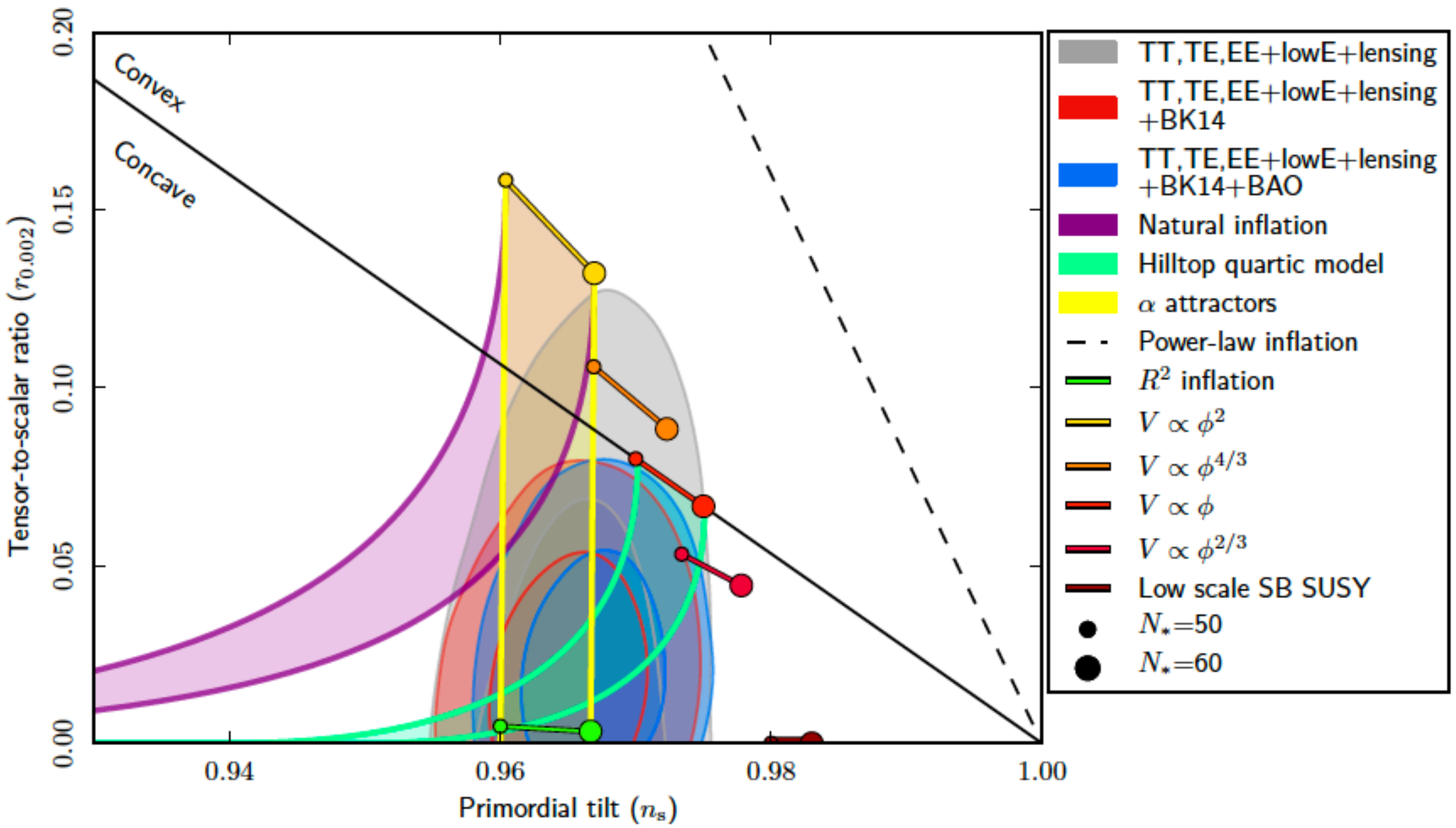
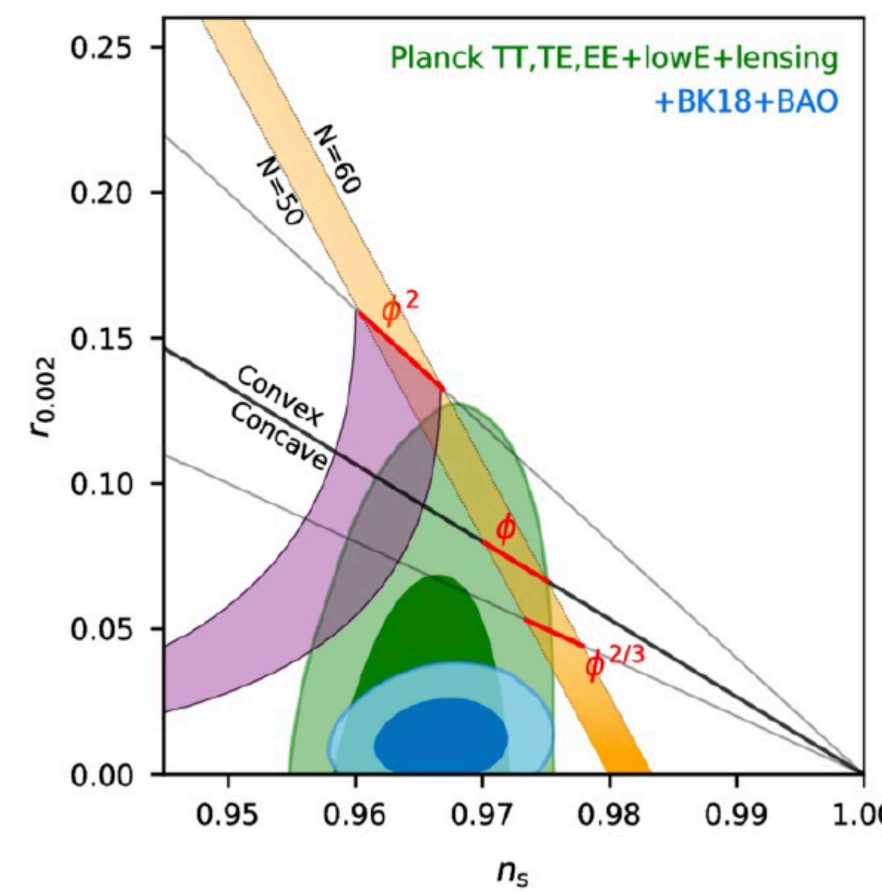




Inflation explains horizon, homogeneity, isotropy, flatness, etc.

Red-tilt CMB anisotropies show a strong indication for inflation:

n_s=0.9665±0.0038 [Planck+BAO]

Bunch of inflation models are ruled out by tensor-to-scalar ratio:
r<0.035 [Planck18+BK18+BAO]

Models with small r are testable by Bicep3, Simon, CMB S4, LiteBird.

PQ symmetry & axion

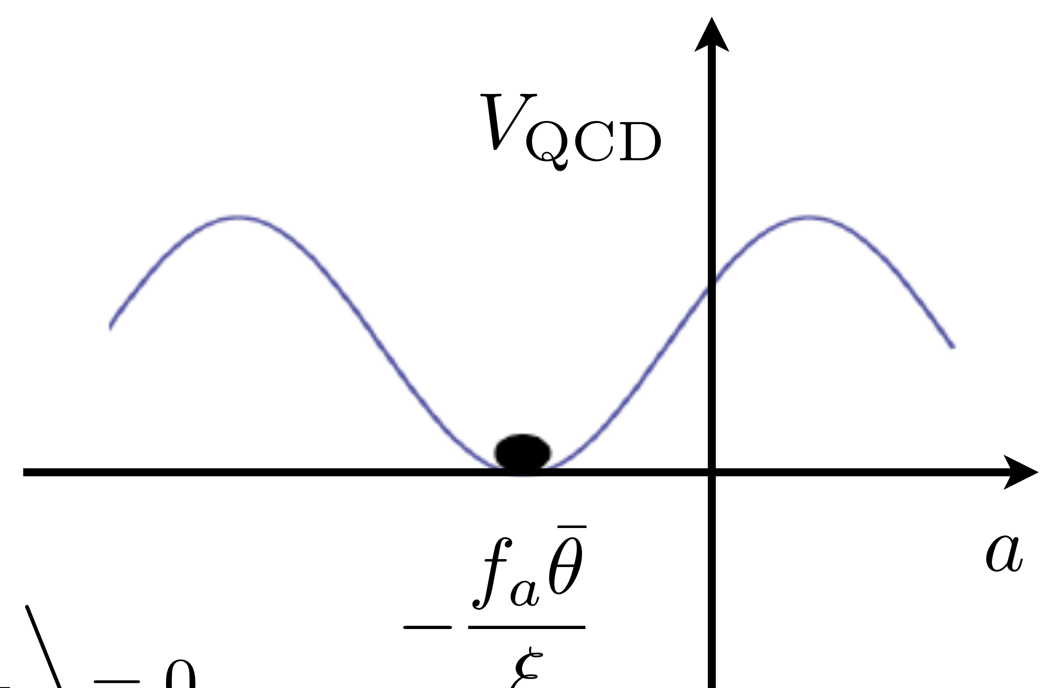
- Axion is a pseudo-Nambu-Goldstone coming from the spontaneous breakdown of U(1) Peccei-Quinn symmetry.

$$\Phi(x) = \frac{1}{\sqrt{2}} f_a \, e^{ia(x)/f_a}, \qquad a(x) \to a(x) + 2\pi f_a$$

- U(1) PQ symmetry becomes anomalous due to QCD anomalies (KSVZ or DFSZ): axion-gluon couplings solve the strong CP problem via QCD instantons.

$$\mathcal{L}_\theta = \frac{g_s^2}{32\pi^2} \left(\bar{\theta} + \xi \, \frac{a}{f_a} \right) G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

$$\longrightarrow \quad V_{\rm QCD} = -\Lambda^4_{\rm QCD} \cos \left(\bar{\theta} + \xi \, \frac{a}{f_a} \right)$$



Minimum of the potential: $\left\langle \bar{\theta} + \xi \, \frac{a}{f_a} \right\rangle = 0$

Axion quality

- Bounds from neutron EDM constrain the effective θ:

$$d_n = \frac{e}{\Lambda_{\rm QCD}^2}\frac{m_u m_d}{m_u+m_d}\,\theta_{\rm eff} < 1.8\times 10^{-26} e\,{\rm cm} \longrightarrow \theta_{\rm eff} = \bar{\theta} + \xi\frac{\langle a\rangle}{f_a} < 10^{-10}$$

- A continuous global symmetry is broken explicitly by quantum gravity, but whose effect must be small enough.

"Axion quality problem"

Explicit PQ breaking at Planck scale: $V_{\rm PQV} = \frac{c_n}{M_P^{n-4}}\,\Phi^n + {\rm h.c.}$

$$\Phi(x) = \frac{1}{\sqrt{2}}\,f_a\,e^{ia(x)/f_a} \longrightarrow V_{\rm PQV} = \frac{c_n}{2^{n/2-1}}\left(\frac{f_a}{M_P}\right)^n M_P^4\cos\left(n\,\frac{a}{f_a}+\alpha\right)$$

$$\theta_{\rm eff} < 10^{-10} \longrightarrow n \gtrsim 12 \ {\rm for}\ f_a = 10^{12}\,{\rm GeV}$$

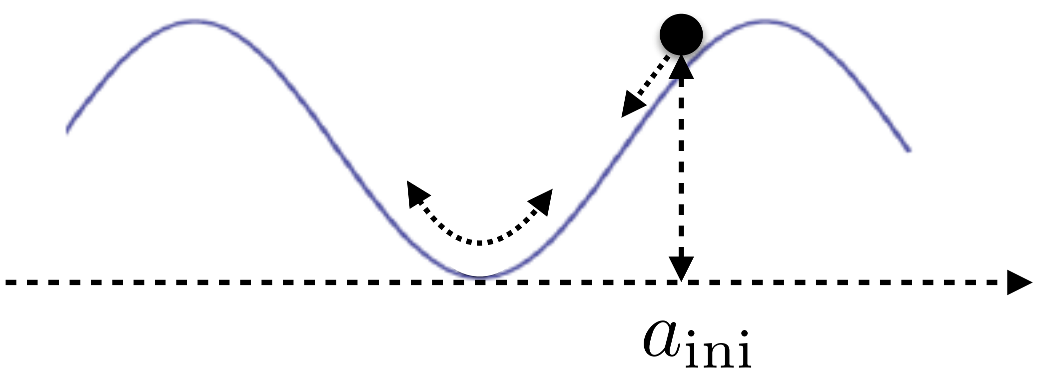
e.g. higher order discrete R symmetries

[HML et al (2011)]

Axion dark matter

- Axion can be a candidate for cold dark matter.
- Axion abundance is determined by the initial misalignment before QCD phase transition.

[Preskill et al; Abbott et al; Dine et al (1983)]



"Axion misalignment mechanism"

$m_a \simeq 3H$: coherent oscillation

$\longrightarrow$ Axion cold dark matter

Axion abundance:

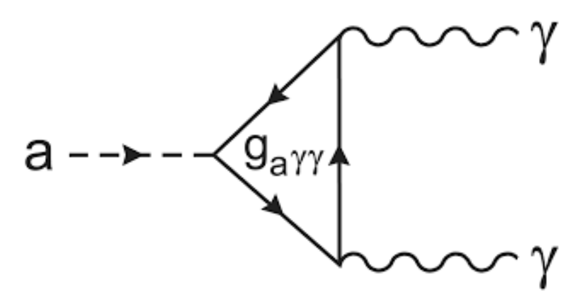
$$\Omega_a h^2 = \frac{\rho_a(a_{\rm ini})}{\rho_c/h^2}\,\frac{m_a(0)}{m_a(T_{\rm osc})}\left(\frac{g_{s*}(T_0)}{g_{s*}(T_{\rm osc})}\right)^{1/3}\left(\frac{T_0}{T_{\rm osc}}\right)^3$$

$$\simeq 0.12\left(\frac{f_a}{9\times 10^{11}\,{\rm GeV}}\right)^{1.165}\left(\frac{a_{\rm ini}}{f_a}\right)^2$$

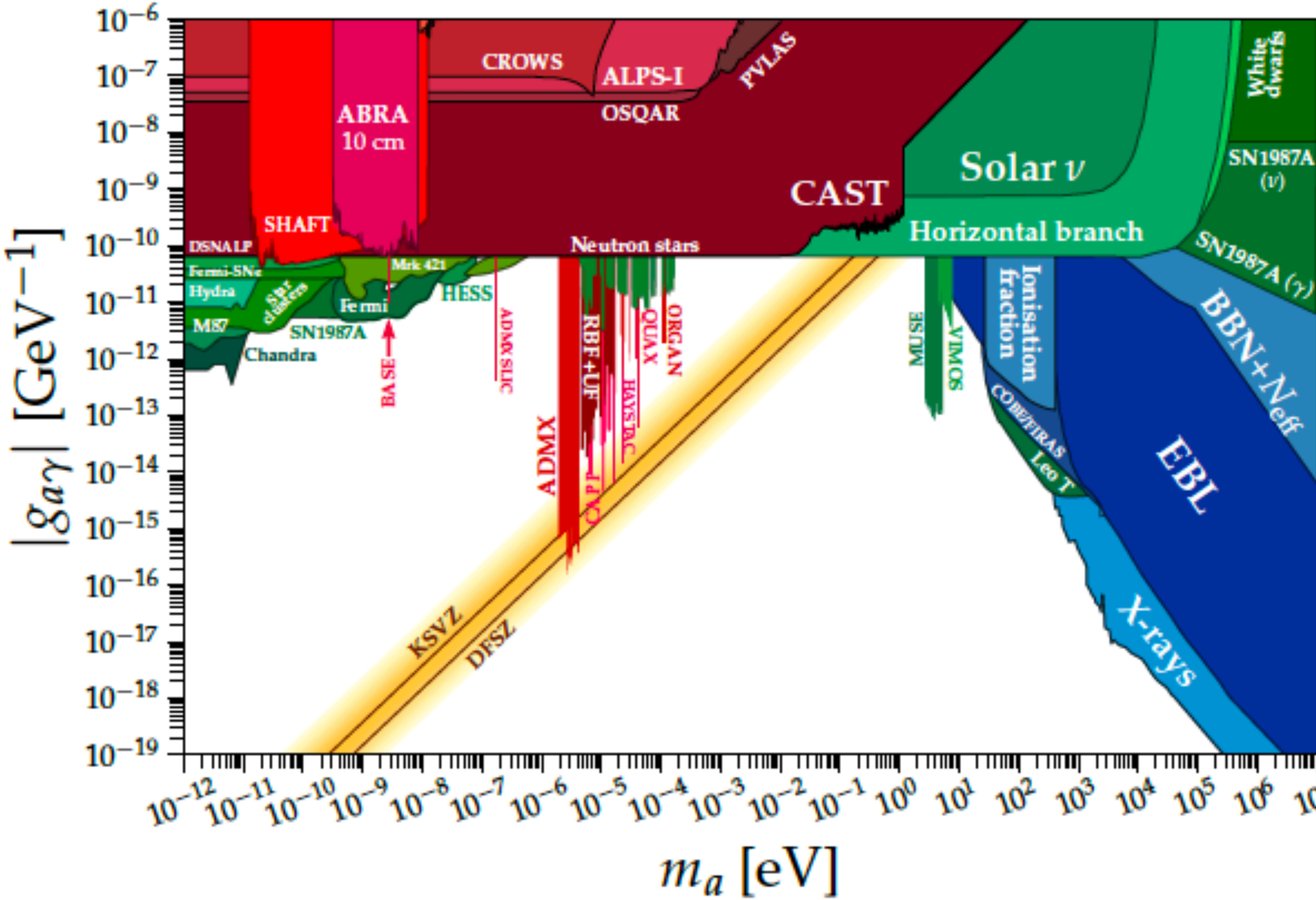
Axion window:

$$10^8\,{\rm GeV} < f_a < 10^{12}\,{\rm GeV}$$

Supernova (lower bound) — Relic density (upper bound)



Bounds on axions



Axion rotation

- Explicit PQ breaking becomes more significant when the saxion (PQ modulus) is not at the minimum.

$$V_{\rm PQV} = \frac{c_n}{2^{n/2-1}}\left(\frac{\rho}{M_P}\right)^n M_P^4 \cos\left(n\,\frac{a}{f_a}+\alpha\right), \quad \rho \gg f_a$$

⟶ Velocity kick for axion before QCD phase transition.

[Co, Hall, Harigaya (2019)] "Axion kinetic misalignment"

- The saxion field sets the initial condition for axion velocity and rolls down to the minimum before QCD PT.

Noether charge conservation: $J_\theta = f_a^2\dot{\theta}, \quad a^3 J_\theta = {\rm const}$

After QCD PT, the axion starts to oscillate about the minimum with $\rho_{a,{\rm rot}} \simeq m_a J_\theta > \rho_{a,{\rm pot}}$

Smaller axion decay constant is favored: $f_a < 1.5\times 10^{11}\,{\rm GeV}$

Pole inflation with Higgs or PQ fields

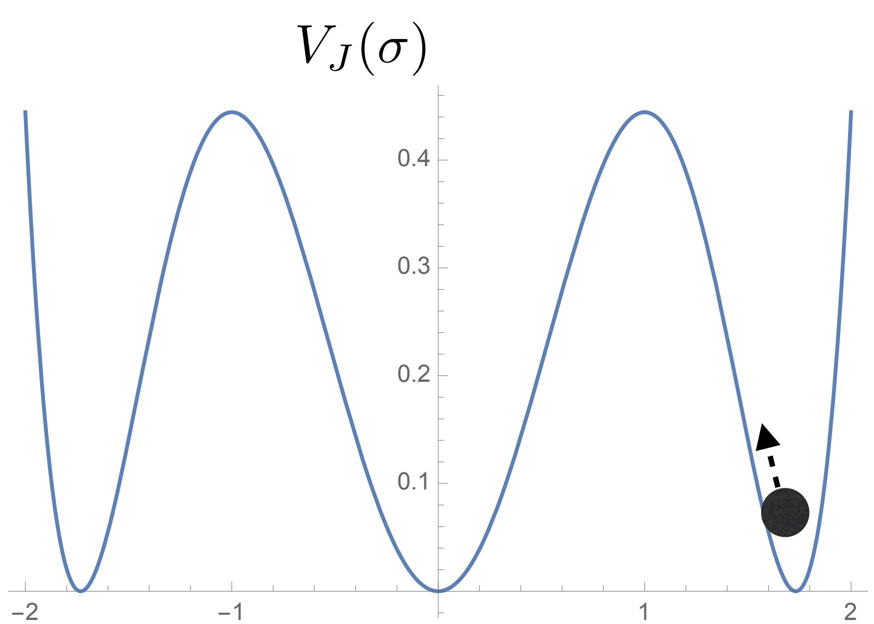
Pole inflation

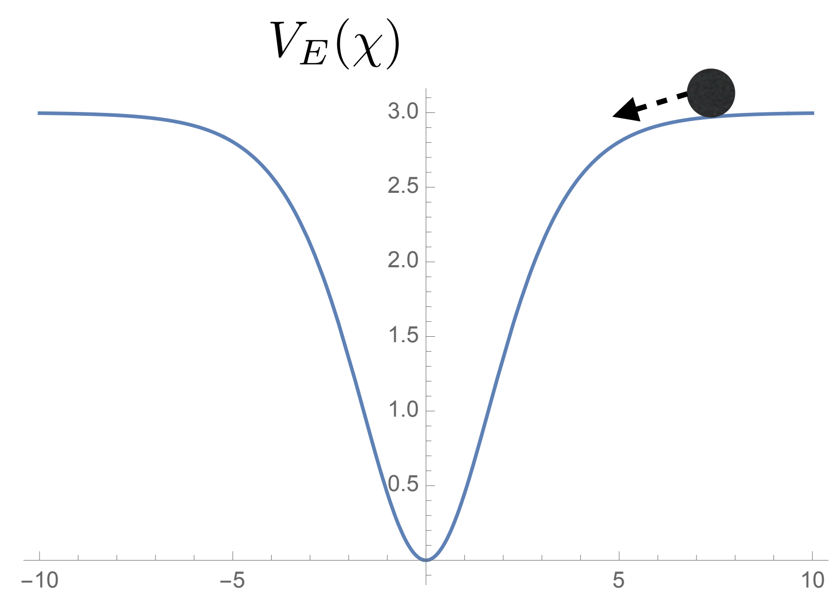
- Singlet scalar field σ with a conformal coupling:

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}\left(1-\frac{1}{6}\sigma^2\right)R + \frac{1}{2}(\partial_\mu\sigma)^2 - V_J(\sigma)\ , \quad V_J = \frac{1}{2}M^2\sigma^2\left(1-\frac{1}{6}\sigma^2\right)^2$$

$$\Rightarrow \frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}R + \boxed{\frac{1}{2}\frac{(\partial_\mu\sigma)^2}{\left(1-\frac{1}{6}\sigma^2\right)^2}} - \frac{1}{2}M^2\sigma^2$$

$$= -\frac{1}{2}R + \frac{1}{2}(\partial_\mu\chi)^2 - 3M^2\tanh^2\left(\frac{\chi}{\sqrt{6}}\right)$$





Inflation occurs near the pole. Complex scalar = inflaton ?

e.g. Kallosh, Linde, Roest (2013)

Higgs pole inflation

- Consider the Higgs field near conformal coupling:

Jordan-frame: $$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\,\Omega(H)R(g_J) + |D_\mu H|^2 - V_J(H)$$

$\Omega = 1 - \frac{1}{3M_P^2}|H|^2$: "conformal limit" [S. Clery, HML, A. Menkara (2023)]

Einstein-frame: $$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}M_P^2 R(g_E) + \frac{|D_\mu H|^2}{\left(1-\frac{1}{3M_P^2}|H|^2\right)^2} - \frac{1}{3M_P^2}\left(|H|^2|D_\mu H|^2 - \frac{1}{4}\partial_\mu|H|^2\partial^\mu|H|^2\right) - \frac{V_J(H)}{\left(1-\frac{1}{3M_P^2}|H|^2\right)^2}$$

Unitary gauge: $H^T = (0,h)/\sqrt{2} \longrightarrow |H|^2|D_\mu H|^2 - \frac{1}{4}\partial_\mu|H|^2\partial^\mu|H|^2 = 0$

$$\Rightarrow \frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}M_P^2 R + \frac{1}{2}\frac{(\partial_\mu h)^2}{\left(1-\frac{1}{6M_P^2}h^2\right)^2} - \frac{V_J(\frac{1}{\sqrt{2}}h)}{\left(1-\frac{1}{6M_P^2}h^2\right)^2}$$

: Pole inflation type!

Inflationary predictions

- Take the canonical Higgs potential in Einstein frame.

$$V_E(H) = \underline{c_m \Lambda^{4-2m} |H|^{2m}}$$

$\longleftrightarrow$ Jordan frame: $V_J(H) = \underline{c_m \Lambda^{4-2m} |H|^{2m} \left(1 - \frac{1}{3M_P^2}|H|^2\right)^2}$

$|H|^{2m}, |H|^{2m+2}, |H|^{2m+4}$: equally important in Jordan frame.

Canonical inflaton: $h = \sqrt{6} M_P \tanh\left(\frac{\phi}{\sqrt{6} M_P}\right)$

$\longrightarrow$ $V_E(\phi) = 3^m c_m \Lambda^{4-2m} M_P^{2m} \left[\tanh\left(\frac{\phi}{\sqrt{6} M_P}\right)\right]^{2m}$ "Higgs pole inflation"

- Inflationary predictions: $N = 60 \longrightarrow$ $n_s = 0.966,\ r = 0.0033$ insensitive to m

Spectral index

$$n_s = 1 - \frac{4N+3}{2(N^2 - \frac{9}{16m^2})}$$

Tensor-to-scalar ratio

$$r = \frac{12}{N^2 - \frac{9}{16m^2}}$$

CMB normalization

$$3^m c_m \left(\frac{\Lambda}{M_P}\right)^{4-2m} = (3.1 \times 10^{-8})\, r$$

CMB energy bound

CMB energy bound: $\frac{3\mu_H^2}{M_P^2} + 9\lambda_H + \sum_{m=3}^{\infty} 3^m c_m \left(\frac{\Lambda}{M_P}\right)^{4-2m} = 1.0 \times 10^{-10}$

$m = 0$: Vacuum energy dominance → Graceful exit problem

$V_E = V_0$

Small constant vacuum energy for present.

$m = 1$: $\mu_H = 1.4 \times 10^{13}\,\mathrm{GeV}$ → Positive Higgs mass.

$V_E = \mu_H^2 |H|^2$

Inconsistent with EWSB unless Higgs mass becomes positive after inflation.

$m = 2$: $\lambda_H = 1.1 \times 10^{-11}$ → Need of positive but small quartic coupling during inflation.

$V_E = \lambda_H |H|^4$

Consistent with EWSB and dark energy.

$m > 2$: Small coefficient of higher order term;

$V_E \sim |H|^6, |H|^8, \cdots$

Higgs quartic coupling bounded, $\lambda_H \lesssim 1.1 \times 10^{-11}$

PQ pole inflation

- Consider the PQ field near conformal coupling and introduce the PQ conserving and violating potentials.

[HML, A. Menkara, M-J, Seung, J-H, Song (2023)]

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\,\Omega(\Phi)R(g_J) + |\partial_\mu\Phi|^2 - \Omega^2(\Phi)V_E(\Phi)$$

$$\Omega(\Phi) = 1 - \frac{1}{3M_P^2}|\Phi|^2,$$

$$V_E(\Phi) = V_0' + \frac{\beta_m}{M_P^{2m-4}}|\Phi|^{2m} - m_\Phi^2|\Phi|^2 + \left(\sum_{k=0}^{[n/2]}\frac{c_k}{2M_P^{n-4}}|\Phi|^{2k}\Phi^{n-2k} + \text{h.c.}\right)$$

PQ conserving → PQ inflation, SSB of U(1) PQ

PQ violating → Axion kinetic misalignment

PQ conserving potential

- Bounds on PQ conserving potential.

$$m = 2: \quad V_{\rm PQ} = V_0 + \lambda_\Phi \left(|\Phi|^2 - \frac{f_a^2}{2} \right)^2, \qquad f_a = \sqrt{-m_\Phi^2/\lambda_\Phi}$$

$$\Phi = \frac{1}{\sqrt{2}} \rho\, e^{i\theta}, \qquad \rho = \sqrt{6} M_P \tanh\left(\frac{\phi}{\sqrt{6} M_P} \right),$$

$$\longrightarrow \quad \frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2} M_P^2 R + \frac{1}{2} (\partial_\mu \phi)^2 + 3 M_P^2 \sinh^2 \left(\frac{\phi}{\sqrt{6} M_P} \right) (\partial_\mu \theta)^2 - V_E(\phi, \theta),$$

Inflaton potential:

$$V_E(\phi,\theta) = V_{\rm PQ}(\phi) + V_{\rm PQV}(\rho,\theta),$$

$$V_{\rm PQ}(\phi) = V_0 + \frac{1}{4} \lambda_\Phi \left(6 M_P^2 \tanh^2 \left(\frac{\phi}{\sqrt{6} M_P} \right) - f_a^2 \right)^2, \qquad \lambda_\Phi = 1.1 \times 10^{-11}$$

- General PQ conserving potential.

$$V_{\rm PQ} = \frac{\beta_m}{M_P^{2m-4}} |\Phi|^{2m} - m_\Phi^2 |\Phi|^2, \qquad \langle \rho \rangle \sim \langle \phi \rangle \sim f_a = (m_\Phi^2 M_P^{2m-4})^{1/(2m-2)}$$

$$\longrightarrow \quad V_{\rm PQ}(\phi) = V_0' + 3^m \beta_m M_P^4 \left[\tanh \left(\frac{\phi}{\sqrt{6} M_P} \right) \right]^{2m} - 3 m_\Phi^2 M_P^2 \tanh^2 \left(\frac{\phi}{\sqrt{6} M_P} \right), \qquad 3^m \beta_m = 1.0 \times 10^{-10}$$

PQ violating potential

- Bounds on PQ violating potential.

$$V_{\rm PQV}(\rho,\theta) = 3^{n/2} M_P^4 \tanh^n\left(\frac{\phi}{\sqrt{6}M_P}\right)\sum_{k=0}^{[n/2]} |c_k| \cos\Big((n-2k)\theta + A_k\Big)$$

Axion quality: $|\theta_{\rm eff}| = \left|\bar{\theta} + \xi\frac{\langle a\rangle}{f_a}\right| < 10^{-10}$

$$V_{\rm eff}(a) = V_0 - \Lambda_{\rm QCD}^4 \cos\left(\bar{\theta} + \xi\frac{a}{f_a}\right) + V_{\rm PQV} \longrightarrow \left(\frac{f_a}{M_P}\right)^n \lesssim \frac{2^{n/2}\xi}{(n-2k)|c_k|}\left(\frac{\Lambda_{\rm QCD}}{M_P}\right)^4$$

$f_a = 10^{12}\,{\rm GeV},\ \xi = 1,\ |c_k| = \mathcal{O}(1),$ $n \gtrsim 12$ In general, $n \propto 1/\ln(M_P/f_a)$

PQ inflation leads to stringent bounds on PQ violation:

$$V_n(\theta_i)/M_P^4 = 3^{n/2}\sum_{k=0}^{[n/2]} |c_k|\cos\Big((n-2k)\theta_i + A_k\Big) < 1.0\times 10^{-10}, \qquad 3^{n/2}|c_k| \lesssim 10^{-10} \text{ each}$$

$\longrightarrow$ $n \gtrsim 10(5)$ for $f_a = 10^{12}(10^6)\,{\rm GeV},\ \xi = 1$ and $|c_k| \lesssim 10^{-12}$

Axion rotation during inflation

- PQ violating potential leads to nonzero axion velocity.

PQ modulus: $\ddot{\phi} + 3H\dot{\phi} - \sqrt{6}M_P \sinh\left(\frac{\phi}{\sqrt{6}M_P}\right)\cosh\left(\frac{\phi}{\sqrt{6}M_P}\right)\dot{\theta}^2 = -\frac{\partial V_E}{\partial \phi}$

$$\phi \sim \sqrt{6}M_P, \qquad \dot{\phi} \simeq -\frac{1}{3H}\frac{\partial V_E}{\partial \phi} = -\sqrt{2\epsilon_\phi}\, M_P H$$

Slow-roll inflation

Axion: $6M_P^2 \sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)\left[\ddot{\theta} + 3H\dot{\theta} + \frac{2}{\sqrt{6}M_P}\coth\left(\frac{\phi}{\sqrt{6}M_P}\right)\dot{\phi}\,\dot{\theta}\right] = -\frac{\partial V_E}{\partial \theta}$

$$\dot{\theta} \simeq -\frac{1}{3H}\frac{\frac{\partial V_E}{\partial \theta}}{6M_P^2 \sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)} = -\frac{\sqrt{2\epsilon_\theta}\, H}{6\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)} \ll H$$

Slow-rolling, but sub-dominant for inflation

→ Nonzero axion velocity at the end of inflation

Bound on axion velocity during inflation:

$$3^{n/2}|c_k| \lesssim 10^{-10} \quad \longrightarrow \quad |\dot{\theta}_{\rm end}| \lesssim \frac{M_P \tanh^n\left(\frac{\phi_{\rm end}}{\sqrt{6}M_P}\right)}{18 H_I \sinh^2\left(\frac{\phi_{\rm end}}{\sqrt{6}M_P}\right)} \cdot 10^{-10} \simeq 0.9^n \times 6 \times 10^{-7} M_P$$

Reheating and axion kinetic misalignment

Saxion/axion after inflation

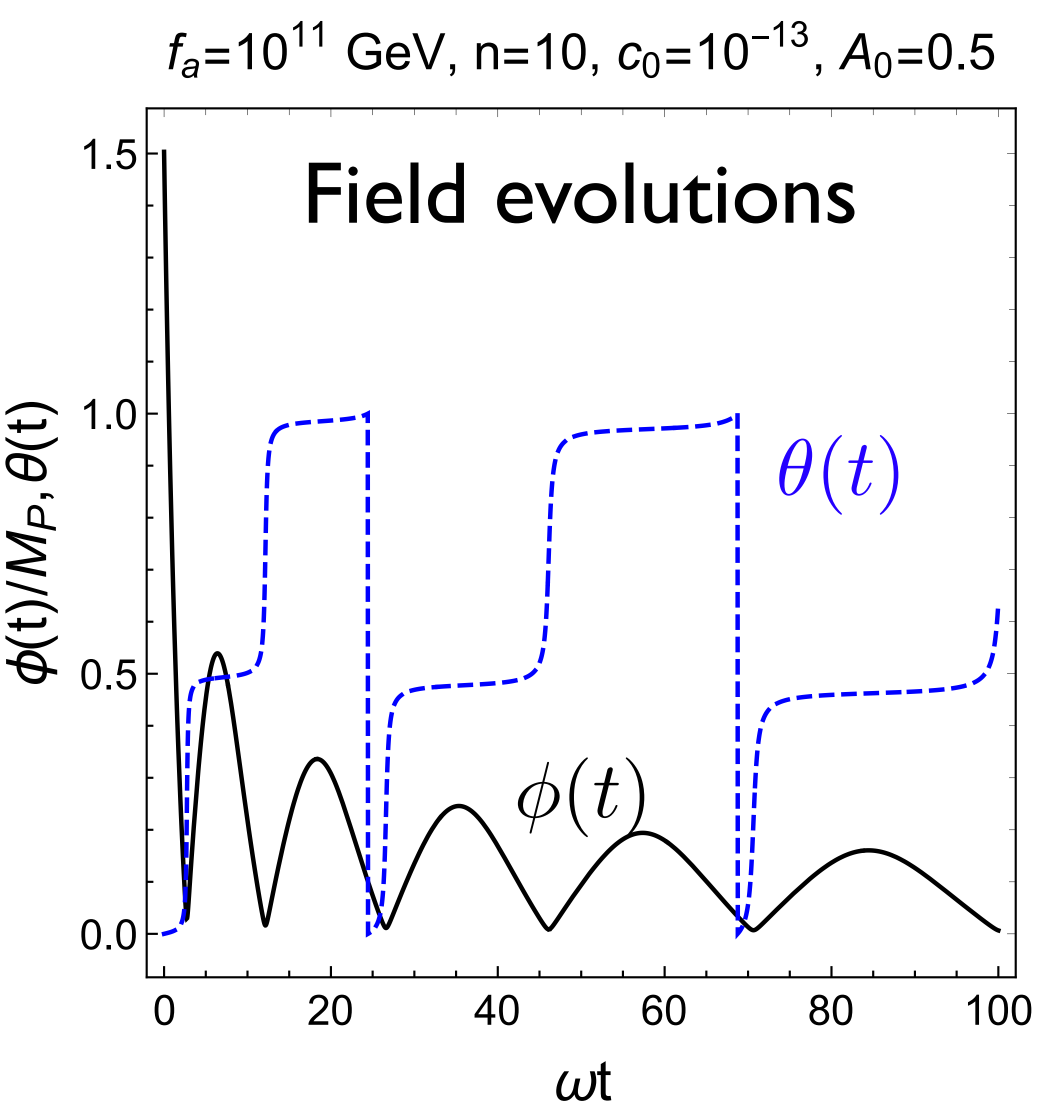
Post-inflationary dynamics: [HML, A. Menkara, M-J, Seung, J-H, Song (2023)]

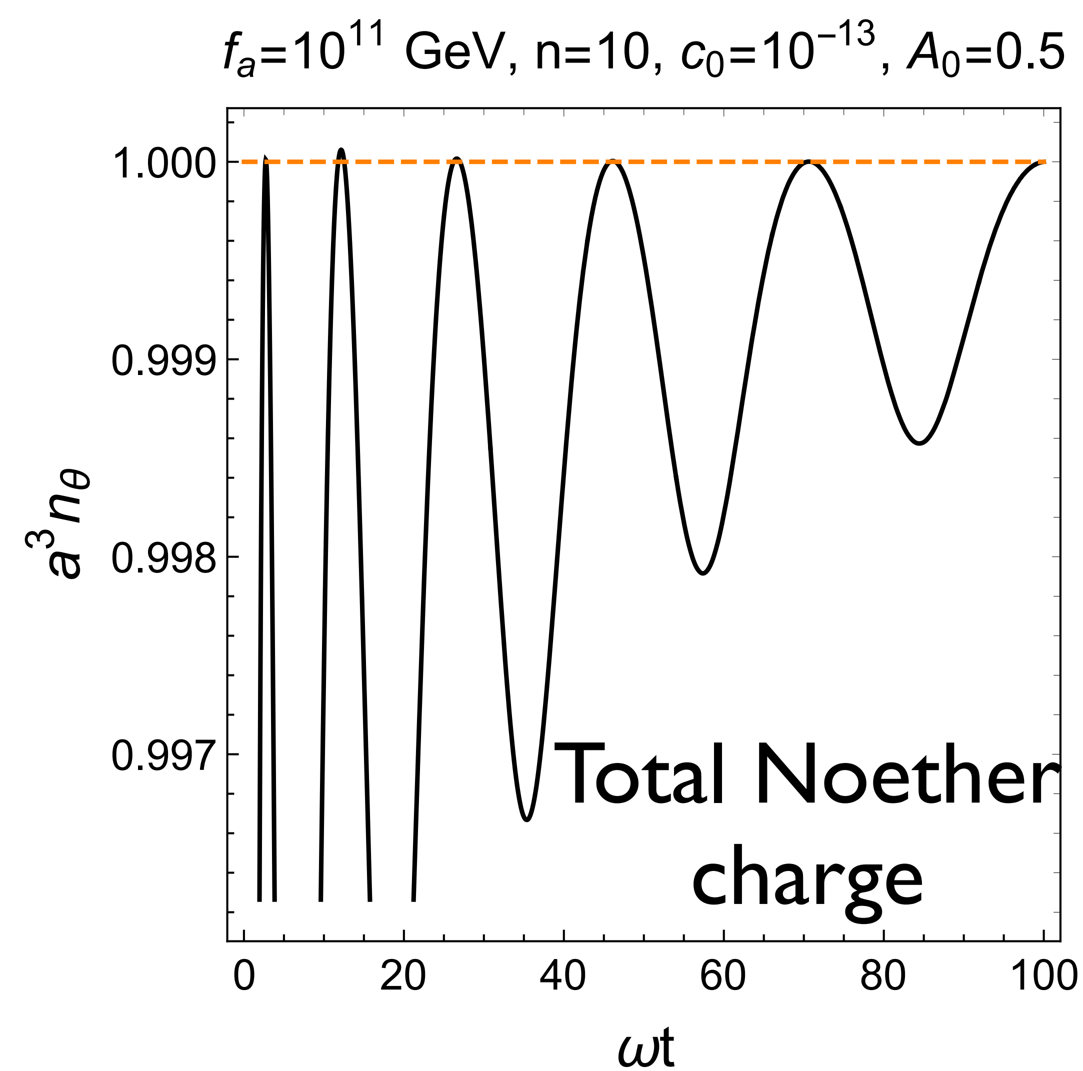
$$\phi \ll \sqrt{6}M_P, \qquad \ddot{\phi} + 3H\dot{\phi} - \phi\,\dot{\theta}^2 \;\simeq\; -\frac{\partial V_E}{\partial \phi}, \qquad \phi^2(\ddot{\theta} + 3H\dot{\theta}) + 2\phi\dot{\phi}\,\dot{\theta} \;\simeq\; -\frac{\partial V_E}{\partial \theta}$$

Approximately conserved Noether charge: $n_\theta = \phi^2\dot{\theta}\,, \quad \frac{d}{dt}(a^3 n_\theta) = 0$

$$\ddot{\phi} + 3H\dot{\phi} \simeq \frac{C^2}{a^6\phi^3} - \lambda_\Phi \phi^3$$

Centrifugal term diluted;
PQ conserving potential dominates!

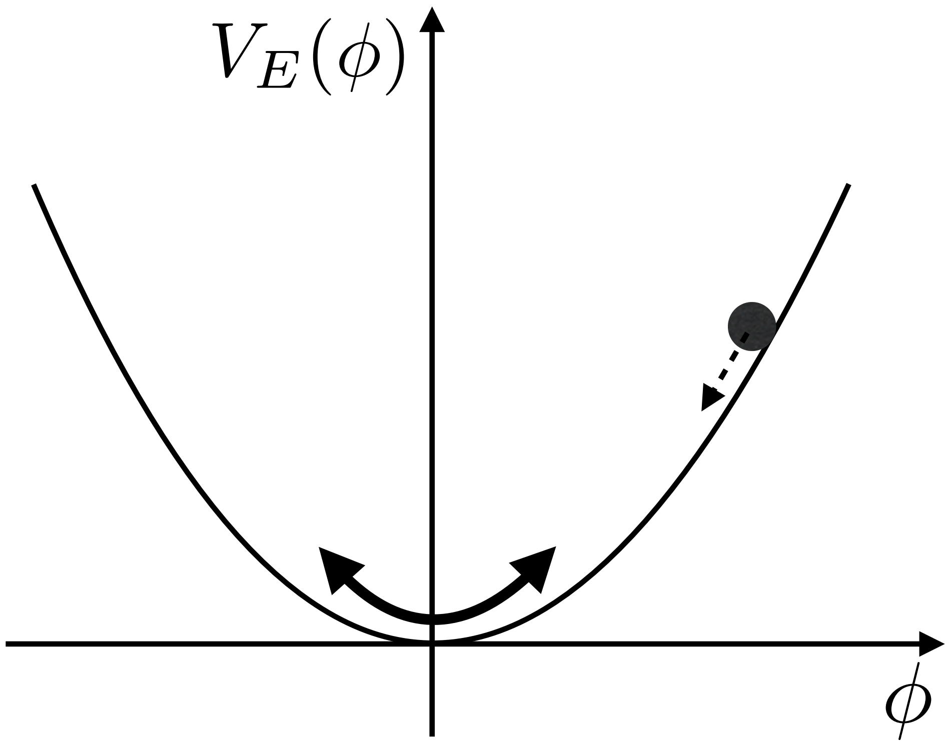




Inflaton condensate

- PQ field oscillates around zero after inflation.

$$V_E(\phi) \simeq \frac{c_m}{2^m}\Lambda^{4-2m}\phi^{2m} \equiv \alpha_m \phi^{2m}$$



Virial theorem: $\left\langle \frac{dS}{dt} \right\rangle = 0, \quad S \equiv \phi\dot{\phi}$

$$\rightarrow \left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle = -\left\langle \frac{1}{2}\phi\ddot{\phi} \right\rangle = m\langle V_E(\phi)\rangle$$

(Hubble friction ignored)

$$\begin{aligned} \rho_\phi &= \left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle + \langle V_E(\phi)\rangle = (m+1)\langle V_E(\phi)\rangle, \\ p_\phi &= \left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle - \langle V_E(\phi)\rangle = (m-1)\langle V_E(\phi)\rangle. \end{aligned}$$

$$\Rightarrow \langle w_\phi \rangle = \frac{p_\phi}{\rho_\phi} = \frac{m-1}{m+1}$$

"general equation of state(e.o.s)"

With Hubble friction, Inflaton = "damped" harmonic oscillator.

$$\boxed{\phi = \phi_0(t)\mathcal{P}(t)}$$

$\phi_0(t)$: inflaton amplitude,

$\mathcal{P}(t) = \sum_{n=-\infty}^{\infty} \mathcal{P}_n e^{-in\omega t}$: oscillating part, $\omega = m_\phi \sqrt{\frac{\pi m}{2m-1}} \frac{\Gamma\left(\frac{1}{2}+\frac{1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)}, \quad m_\phi^2 = V_E''(\phi_0)$

Perturbative reheating

- Perturbative reheating from inflaton decays.

$$\ddot{\phi} + (3H + \Gamma_\phi)\dot{\phi} + V_E' = 0 \longrightarrow \dot{\rho}_\phi + 3(1+w_\phi)H\rho_\phi \simeq -\Gamma_\phi(1+w_\phi)\rho_\phi$$

Boltzmann equation for inflaton

plus $\dot{\rho}_R + 4H\rho_R = \Gamma_\phi(1+w_\phi)\rho_\phi\,,\qquad H^2 = \dfrac{\rho_\phi + \rho_R}{3M_P^2}$

$$\rho_R = \rho_{\rm SM} + \rho_a\,,\qquad \Gamma_\phi = \sum_f \Gamma_{\phi\to f\bar{f}} + \Gamma_{\phi\phi\to HH} + \Gamma_{\phi\phi\to aa}$$

PQ inflaton decays/annihilates into axions, heavy quarks (in KSVZ), Higgs pair, etc.

$$\langle\Gamma_{\phi\phi\to aa}\rangle = \frac{\lambda_\Phi^2\phi_0^2\omega}{2\pi m_\phi^2}(m+1)(2m-1)\Sigma_m^a\left\langle\left(1-\frac{m_a^2}{\omega^2 n^2}\right)^{1/2}\right\rangle$$

$\phi \gg f_a,\ m_a^2 = \lambda_\Phi\phi^2$

$\Sigma_m^a = \sum_{n=1}^\infty n|(\mathcal{P}^2)_n|^2$

$$\langle\Gamma_{\phi\phi\to HH}\rangle = \frac{\lambda_{H\Phi}^2\phi_0^2\omega}{\pi m_\phi^2}(m+1)(2m-1)\Sigma_m^H\left\langle\left(1-\frac{m_H^2}{\omega^2 n^2}\right)^{1/2}\right\rangle$$

$\Sigma_m^H = \sum_{n=1}^\infty n|(\mathcal{P}^2)_n|^2$

$$\langle\Gamma_{\phi\to f\bar{f}}\rangle = \frac{N_c y_f^2\omega^3}{8\pi m_\phi^2}(m+1)(2m-1)\Sigma_m^f\left\langle\left(1-\frac{4m_f^2}{\omega^2 n^2}\right)^{3/2}\right\rangle$$

$\Sigma_m^f = \sum_{n=1}^\infty n^3|\mathcal{P}_n|^2$

Reheating temperature

- Inflation/radiation evolution depends on inflaton e.o.s.

$$\rho_\phi \simeq \rho_{\rm end}\left(\frac{a_{\rm end}}{a}\right)^{3(1+\omega_\phi)} = \rho_{\rm end}\left(\frac{a_{\rm end}}{a}\right)^{\frac{6m}{m+1}}, \quad \rho_R \sim \begin{cases} a^{-4}, & 7-2m<0 \\ a^{-\frac{3(2m-1)}{m+1}}, & 7-2m>0 \end{cases}$$

Boltzmann equations ⟶ $\rho_R = \rho_\phi$ ⟶ Reheating temperature

$$T_{\rm RH} = \begin{cases} \left(\frac{30}{\pi g_*(T_{\rm RH})}\right)^{1/4}\left[\frac{2m}{4+m-6mk}\left(\sqrt{3}M_P^{2(1-2k)}\gamma_\phi\right)\right]^{\frac{1}{2(1-2k)}}, & 4+m-6mk>0, \\ \left(\frac{30}{\pi g_*(T_{\rm RH})}\right)^{1/4}\left[\frac{2m}{6mk-4-m}\left(\sqrt{3}M_P^{2(1-2k)}\gamma_\phi\right)(\rho_{\rm end})^{\frac{6mk-m-4}{6m}}\right]^{\frac{3m}{4(m-2)}}, & 4+m-6mk<0 \end{cases}$$

$\Gamma_\phi = \gamma_\phi \rho_\phi^k / M_P^{4k-1}$ decay: $k=1,$ scattering: $k=2$

Reheating temperature for quartic potential:

A small Higgs-portal coupling is efficient!

$$T_{\rm RH}|_{\rm decay} \simeq 2.9\times 10^4\,{\rm GeV}\left(\frac{100}{g_*(T_{\rm reh})}\right)^{1/4}\left(\frac{y_f}{10^{-4}}\right)\left(\frac{\lambda_\Phi}{10^{-11}}\right)^{1/4},$$

$$T_{\rm RH}|_{\rm scattering} \simeq 6.0\times 10^{11}\,{\rm GeV}\left(\frac{100}{g_*(T_{\rm reh})}\right)^{1/4}\left(\frac{\max[\lambda_{H\Phi}, \sqrt{4N_c}y_f^2 m_f/\omega]}{10^{-7}}\right)^2\left(\frac{10^{-11}}{\lambda_\Phi}\right)^{3/4}$$

Axion dark radiation

- Axions produced during inflation:

$$\frac{\Gamma_{\phi\phi\to aa}}{\Gamma_{\phi\phi\to HH}} \simeq \frac{\lambda_\Phi^2}{2\lambda_{H\Phi}^2} \lesssim 1$$: sub-dominant radiation

$$\longrightarrow \quad \Delta N_{\rm eff} = 0.02678\left(\frac{Y_a}{Y_a^{\rm eq}}\right)\left(\frac{106.75}{g_{*s}(T_{\rm reh})}\right)^{4/3}$$ "Dark Radiation"

$$Y_a = {\rm BR}(\phi\phi\to aa)\,\frac{\rho_\phi}{\omega s}, \quad \rho_\phi = \rho_R \quad \longrightarrow \quad \frac{Y_a}{Y_a^{\rm eq}} = \frac{3\lambda_\Phi^{3/2} g_{*s}(T_{\rm reh})T_{\rm reh}}{2.2\lambda_{H\Phi}^2\phi_0}$$

<u>High reheating temperature:</u> Axions get thermalized!

$$T_{\rm reh} \gtrsim 1.7\times 10^9\,{\rm GeV}\left(\frac{f_a}{10^{11}\,{\rm GeV}}\right)^{2.246} \equiv T_{a,\rm eq}$$

After axion decoupling, $\Delta N_{\rm eff} = \frac{4}{7}\left(\frac{T_{a,0}}{T_{\nu,0}}\right)^4 = \frac{4}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{g_{*s}(T_0)}{g_{*s}(T_{\rm reh})}\right)^{4/3}$

e.g. $\Delta N_{\rm eff} = 0.02363$ (KSVZ) , detectable in future CMB experiments

<u>Low reheating temperature:</u>

$$T_{\rm reh} < T_{a,\rm eq}\,, \quad \frac{Y_a}{Y_a^{\rm eq}} = 0.025\left(\frac{T_{\rm reh}}{T_{a,\rm eq}}\right)\left(\frac{10^{-11}}{\lambda_{H\Phi}}\right)^2\left(\frac{f_a}{10^{11}\,{\rm GeV}}\right)^{2.246}$$ suppressed DR!

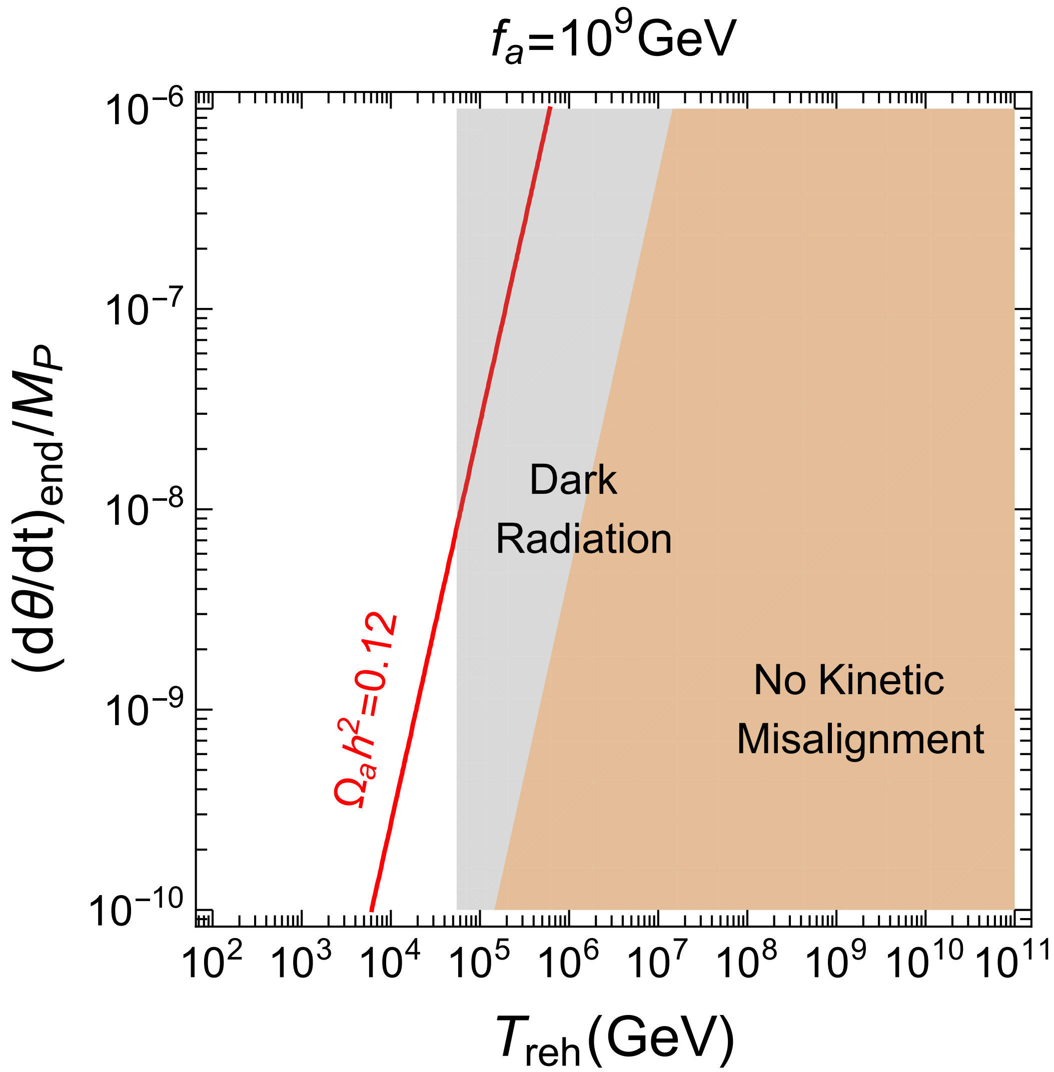
Axion velocity & abundance

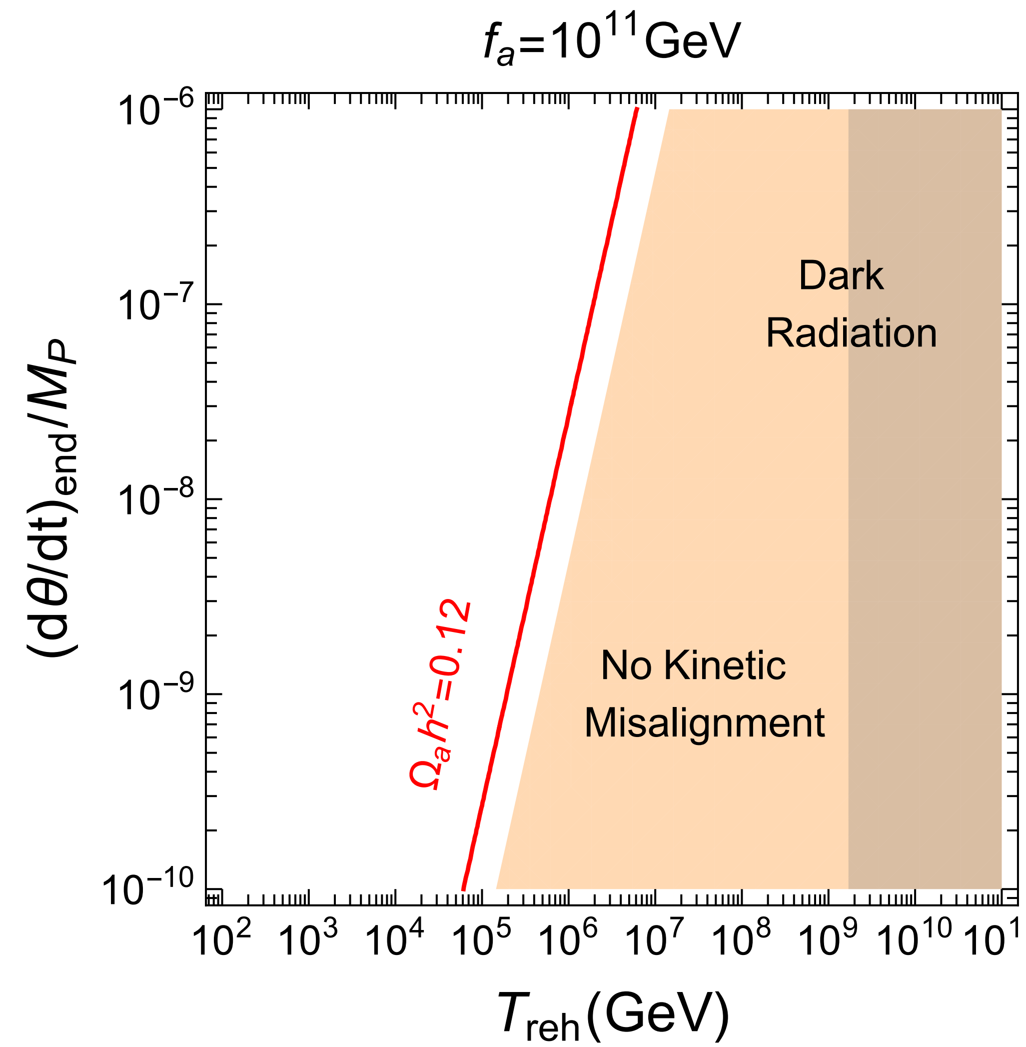
$$a^3 J = a^3\phi^2\dot\theta \simeq \text{const} \quad \begin{cases} \text{Reheating:} & \phi \propto a^{-1}\ (m=2) \longrightarrow \dot\theta \propto a^{-1} \\ \text{Post-reheating:} & \phi = f_a \longrightarrow \dot\theta \propto a^{-3} \end{cases}$$

$$\dot\theta(t) = \dot\theta_{\rm end}\left(\frac{a_{\rm end}}{a_{\rm RH}}\right)\left(\frac{a_{\rm RH}}{a(t)}\right)^3 \longrightarrow \dot\theta(T_*) = \dot\theta_{\rm end}\left(\frac{\pi^2 g_*(T_{\rm RH})T_{\rm RH}^4}{45 V_E(\phi_{\rm end})}\right)^{1/4}\left(\frac{g_*(T_*)}{g_*(T_{\rm RH})}\right)\left(\frac{T_*}{T_{\rm RH}}\right)^3$$

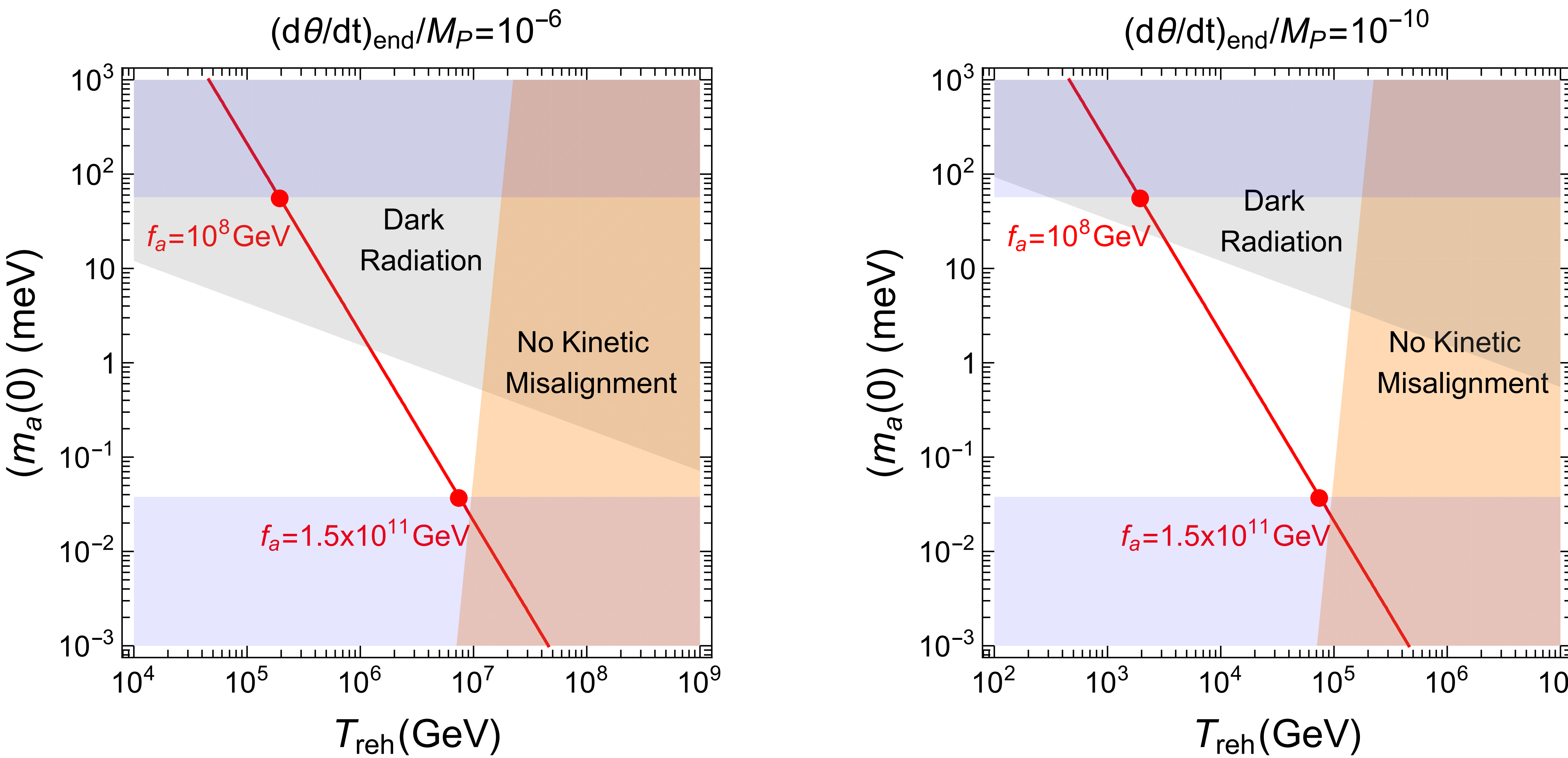
[HML, A. Menkara, M-J, Seung, J-H, Song (2023)]

@ axion oscillation





Axion velocity & abundance



[HML, A. Menkara, M-J, Seung, J-H, Song (2023)]

Even for a relatively small axion decay constant, $f_a < 1.5 \times 10^{11}\,\mathrm{GeV}$

Axion kinetic misalignment dominant for axion DM production.

PQ inflation leads to the initial nonzero velocity for axion!

Conclusions

- PQ pole inflation is the perturbative expansion about conformal coupling, being distinguishable from Higgs or Starobinsky inflation by the general equation of state during reheating.

- A small quartic coupling during PQ inflation is maintained for small couplings of the PQ field. A relatively low reheating temperature is achieved for PQ field couplings to Higgs and extra heavy quarks.

- PQ breaking potential, which is consistent with axion quality, sets the initial velocity for the axion during inflation, which accounts for the correct relic density even for a small axion decay constant.