부산대학교
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Axion Magnetic Resonance

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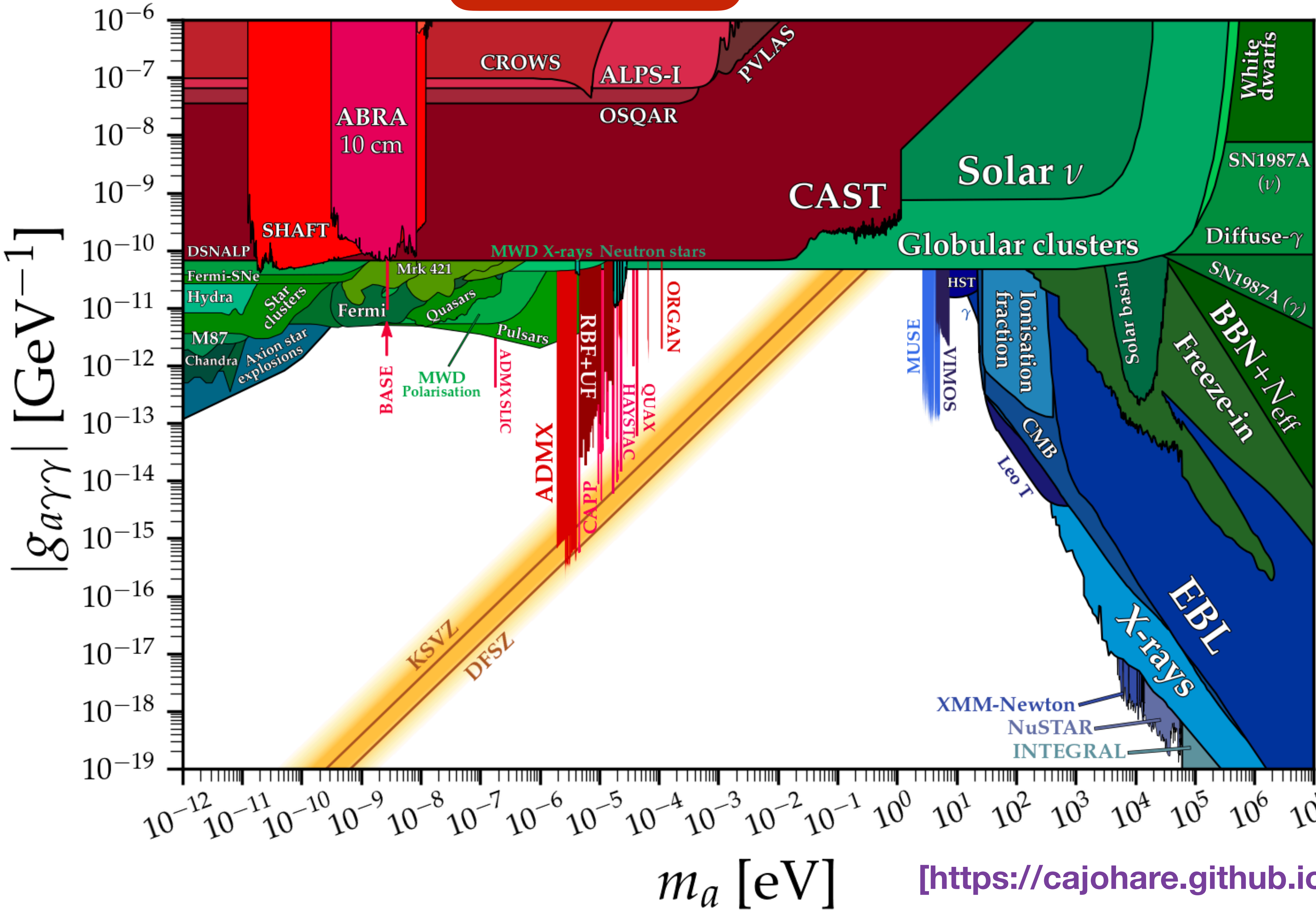
In collaboration with Hyeonseok Seong (DESY), and Chen Sun (Los Alamos)

arXiv:2308.10925, arXiv:2312.XXXXX, and more

Current status on $\frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$

Experimental



Astrophysical

Cosmological

[https://cajohare.github.io/AxionLimits/]

Outline

01 Axion-photon conversion

- *in constant $\overrightarrow{B}$ background*
  - Conventional setup
  - Linear vs non-linear
- *in varying $\overrightarrow{B}$ background*
  - temporal/spatial dependent Hamiltonian
  - Parametric resonance

  "Axion magnetic resonance"

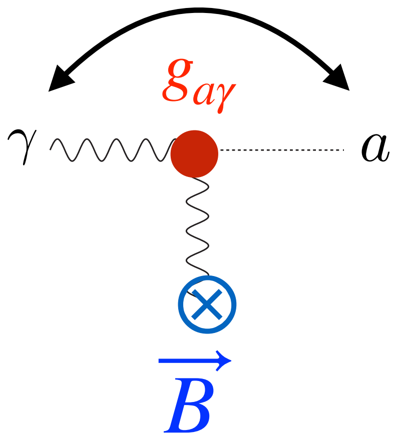
02 Implications

- Experimental - LSTW, helioscope, etc
- Astrophysical & Cosmological

03 Conclusion

Axion-photon oscillation

(a.k.a. Primakoff process)

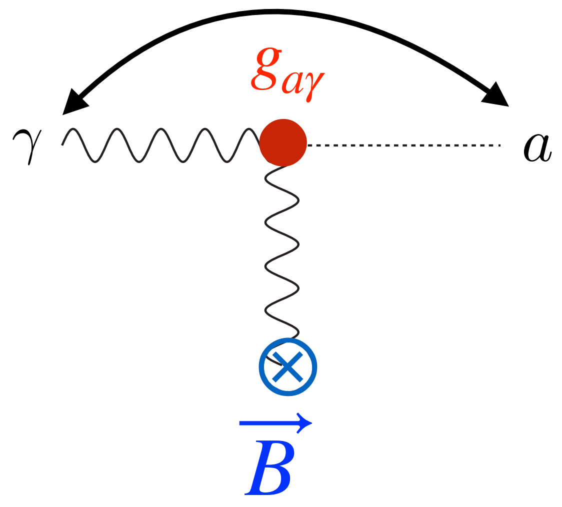


- Conversion in a magnetic background
- Linear vs non-linear regime
- Constant vs spatial/temporal varying B background

$\Rightarrow$ **Axion magnetic resonance**

[G. Raffelt and L. Stodolsky, PRD 37, 1237 (1988)]

Axion-Photon oscillation



"Primakoff" process

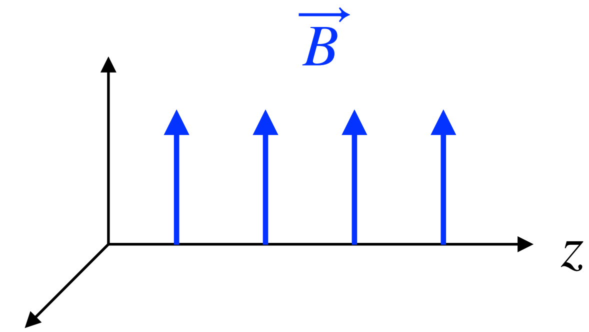
$$\mathscr{L}_{a\,\&\,\gamma} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 - \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$= g_{a\gamma}\vec{E}\cdot\vec{B}\,a$$

- Equation of motion in the presence of a background (transverse) magnetic field $\vec{B}$

$$\partial_\mu F^{\mu\nu} = g_{a\gamma}\tilde{F}^{\mu\nu}\partial_\mu a \qquad \partial_\mu \partial^\mu \vec{A} = g_{a\gamma}\vec{B}\,\partial_t a$$

$$\partial_\mu \partial^\mu a = -m_a^2 a - \frac{1}{4}g_{a\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} \quad ☞ \quad \partial_\mu \partial^\mu a = -m_a^2 a - g_{a\gamma}\vec{B}\cdot\partial_t\vec{A}$$



Axion-Photon oscillation

- "Constant" $\vec{B}$ & propagation in z-direction with $\psi = e^{-i\omega t}\psi(z)$
- For relativistic axions and photons (i.e., $\omega \gg m_a$) ☞ $\omega^2 + \partial_z^2 \approx 2\omega\left(\omega + i\partial_z\right)$

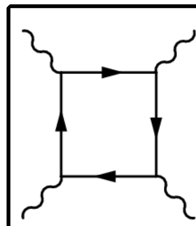
$$\left[\omega + i\partial_z + \begin{pmatrix} \omega\left(n_\perp - 1\right) & 0 & 0 \\ 0 & \omega\left(n_\parallel - 1\right) & g_{a\gamma}B/2 \\ 0 & g_{a\gamma}B/2 & -m_a^2/2\omega \end{pmatrix}\right]\begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix} = 0$$

✓ The only $\vec{A}_\parallel \parallel \vec{B}$ oscillates with axions ($F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\vec{E}\cdot\vec{B}$ & $\vec{E} = \partial_t\vec{A}$)

- Vacuum refractive indices leading to the QED birefringence

"Cotton-Mouton effect"

Euler-Heisenberg

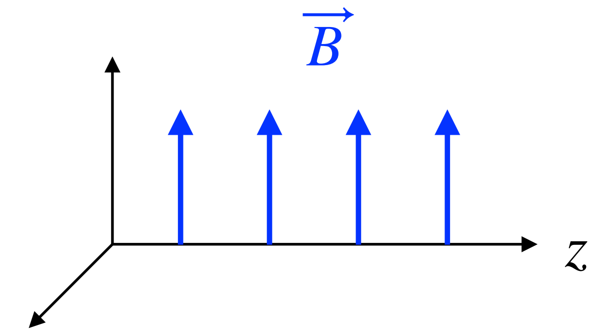


$$\frac{\alpha^2}{90m_e^4}\left[\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{7}{4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right]$$

$$n_\perp - 1 = 4\frac{2\alpha}{45}\frac{B^2}{m_e^4}, \qquad n_\parallel - 1 = 7\frac{2\alpha}{45}\frac{B^2}{m_e^4}$$

with $\frac{2\alpha}{45}\frac{B^2}{m_e^4} = 1.32 \times 10^{-24}\left(\frac{B}{\mathrm{T}}\right)^2$

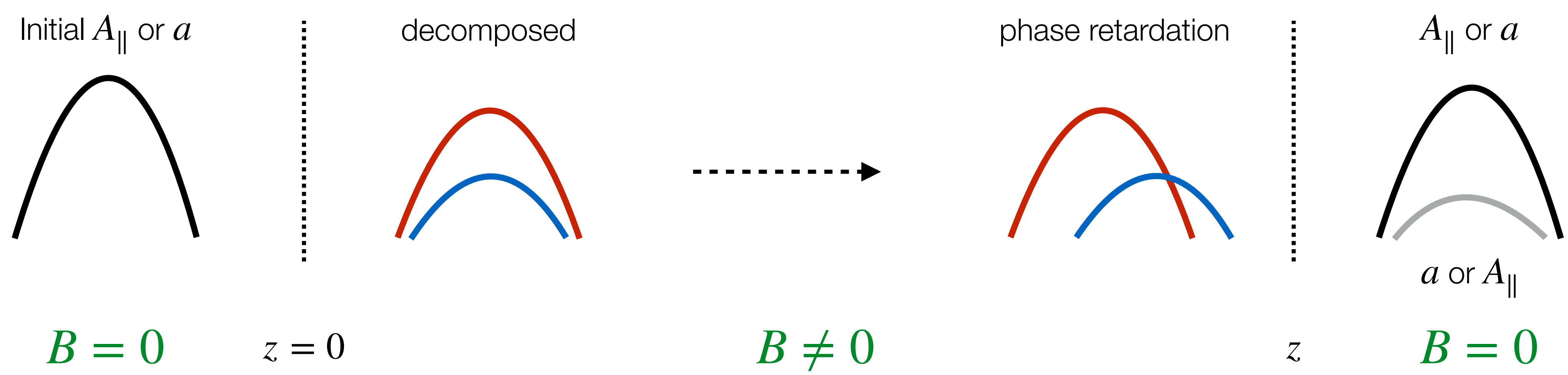
[In constant B]

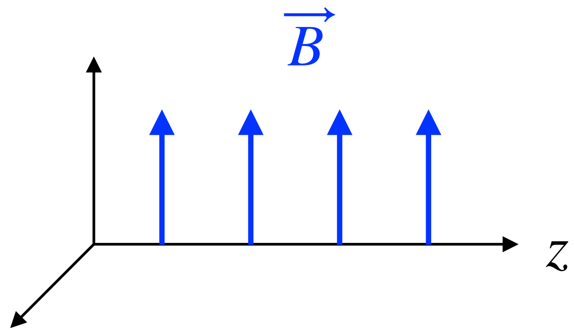


Axion-Photon oscillation

$$\begin{pmatrix} A_\parallel \\ a \end{pmatrix}_z = \exp\left[i \begin{pmatrix} \Delta_\parallel & \Delta_{a\gamma} \\ \Delta_{a\gamma} & \Delta_a \end{pmatrix} z \right] \begin{pmatrix} A_\parallel \\ a \end{pmatrix}_{z=0}$$

$$\Delta_\parallel = \omega \left(n_\parallel - 1 \right), \quad \Delta_a = -\, m_a^2/2\omega, \quad \Delta_{a\gamma} = g_{a\gamma} B/2$$





Axion-Photon oscillation

$$P_{A_{\|}\leftrightarrow a} = \sin^2 2\theta_{a\gamma} \sin^2\left(\frac{\Delta_{\rm osc}}{2} z\right)$$

$$\tan 2\theta_{a\gamma} = \frac{2\Delta_{a\gamma}}{\Delta_a - \Delta_{\|}}$$

and

$$\Delta_{\rm osc} = \sqrt{\left(\Delta_a - \Delta_{\|}\right)^2 + 4\Delta_{a\gamma}^2}$$

The mixing angel between $A_{\|}$ & a

The oscillation frequency corresponding to the phase velocity difference

Condition in experimental setups

$\text{Gauss} = 1.95 \times 10^{-2}\,\text{eV}^2$

$$\omega \sim 1\,\text{eV}, \quad B \sim 10\,\text{T} = 10^5\,\text{Gauss}, \quad L_B \sim (1\text{-}100)\ \text{m}$$

$$\Delta_{\parallel} = 4.7 \times 10^{-15}\,\text{m}^{-1} \left(\frac{\omega}{1\,\text{eV}}\right)\left(\frac{B}{10\,\text{T}}\right)^2$$

$$\Delta_a = -\,2.5 \times 10^{-6}\,\text{m}^{-1} \left(\frac{m_a}{\mu\text{eV}}\right)^2 \left(\frac{\omega}{1\,\text{eV}}\right)^{-1}$$

$$\Delta_{a\gamma} = 4.9 \times 10^{-7}\,\text{m}^{-1} \left(\frac{g_{a\gamma}}{10^{-7}\,\text{GeV}^{-1}}\right)\left(\frac{B}{10\,\text{T}}\right)$$

Linear & non-linear regime

$$P_{A_\parallel \leftrightarrow a} = \sin^2 2\theta_{a\gamma} \sin^2\left(\frac{\Delta_{\rm osc}}{2} L_B\right)$$

$$\tan 2\theta_{a\gamma} = \frac{2\Delta_{a\gamma}}{\Delta_a}$$
$$\Delta_{\rm osc} = \sqrt{\Delta_a^2 + 4\Delta_{a\gamma}^2}$$

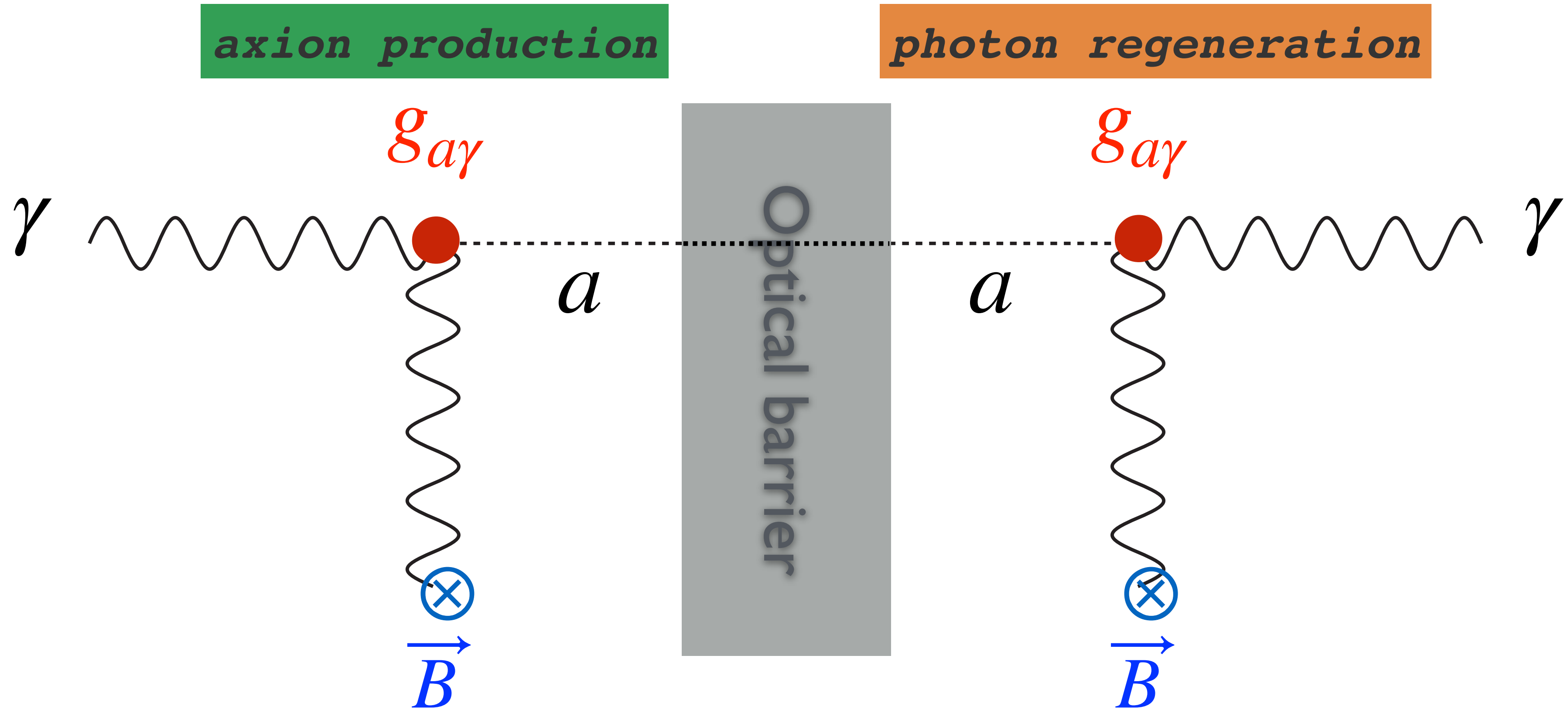
- When $\Delta_{\rm osc} L_B \ll 1$, so-called 'linear regime',

$$P_{A_\parallel \leftrightarrow a} \simeq \left(\Delta_{a\gamma} L_B\right)^2 = 2.4 \times 10^{-13} \left(\frac{g_{a\gamma}}{10^{-7}\,{\rm GeV}^{-1}}\right)^2 \left(\frac{B}{10\,{\rm T}}\right)^2 \left(\frac{L_B}{1\,{\rm m}}\right)^2$$

- When $\Delta_{\rm osc} L_B \gg 1$, so-called 'non-linear regime', typically $\Delta_a \gg \Delta_{a\gamma}$ and $\Delta_{\rm osc} \approx \Delta_a$

$$P^{\rm max}_{A_\parallel \leftrightarrow a} \simeq \sin^2 2\theta_{a\gamma} = 1.5 \times 10^{-13} \left(\frac{g_{a\gamma}}{10^{-7}\,{\rm GeV}^{-1}}\right)^2 \left(\frac{B}{10\,{\rm T}}\right)^2 \left(\frac{\omega}{1\,{\rm eV}}\right)^2 \left(\frac{m_a}{\rm meV}\right)^{-4}$$

Light-Shining-Through-Walls

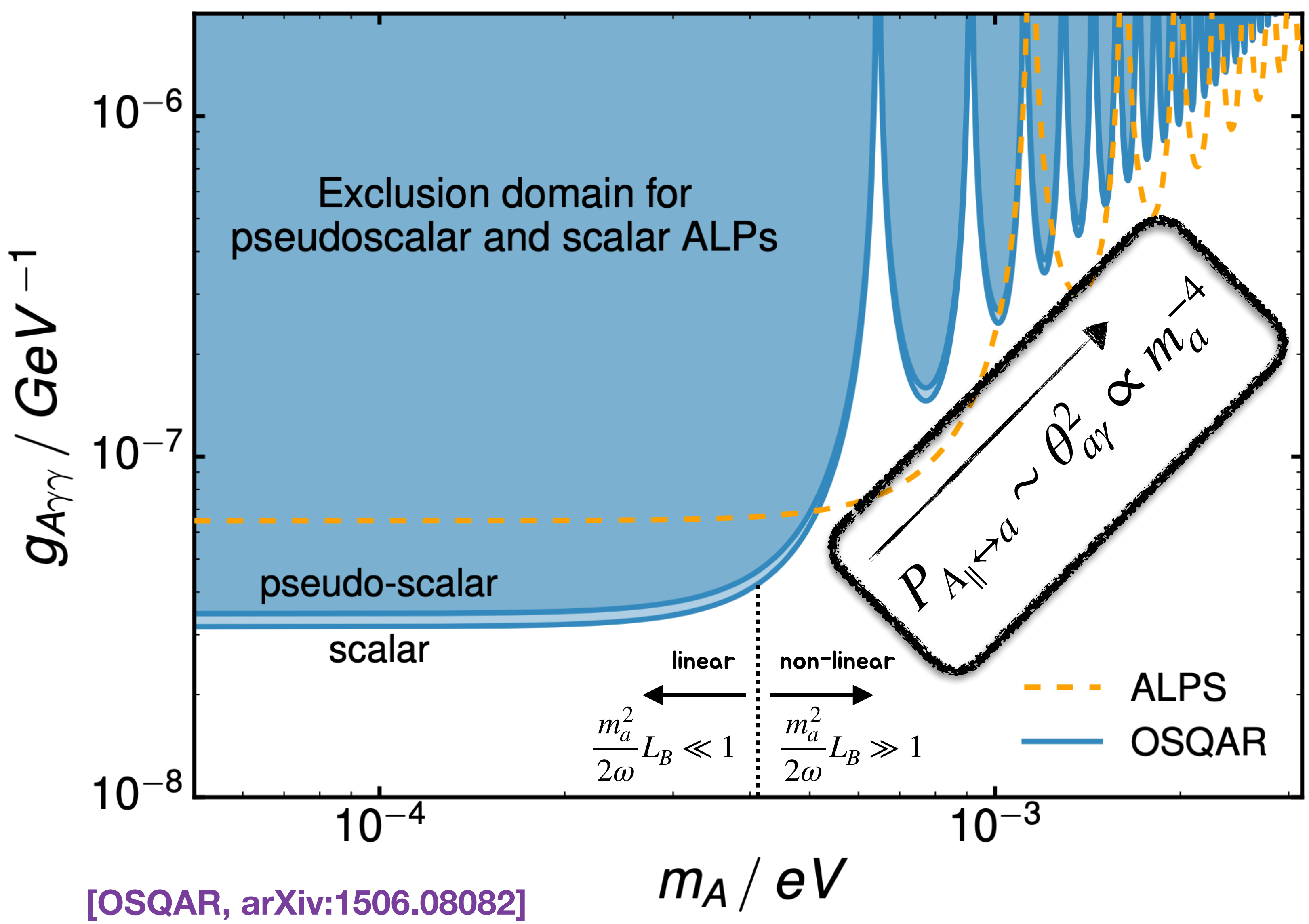


$$\frac{dN_\gamma^{\mathrm{reg}}}{dt} = W_\gamma \times P_{A_\parallel \to a} \times P_{a \to A_\parallel}$$

$$= 3.6 \times 10^{-2}\,\mathrm{s}^{-1} \left(\frac{W_\gamma}{10\,\mathrm{Watt}}\right) \left(\frac{\omega}{1\,\mathrm{eV}}\right)^{-1} \left(\frac{g_{a\gamma}}{10^{-7}\,\mathrm{GeV}^{-1}}\right)^4 \left(\frac{B}{10\,\mathrm{T}}\right)^4 \left(\frac{L_B}{10\,\mathrm{m}}\right)^4$$

"linear regime"

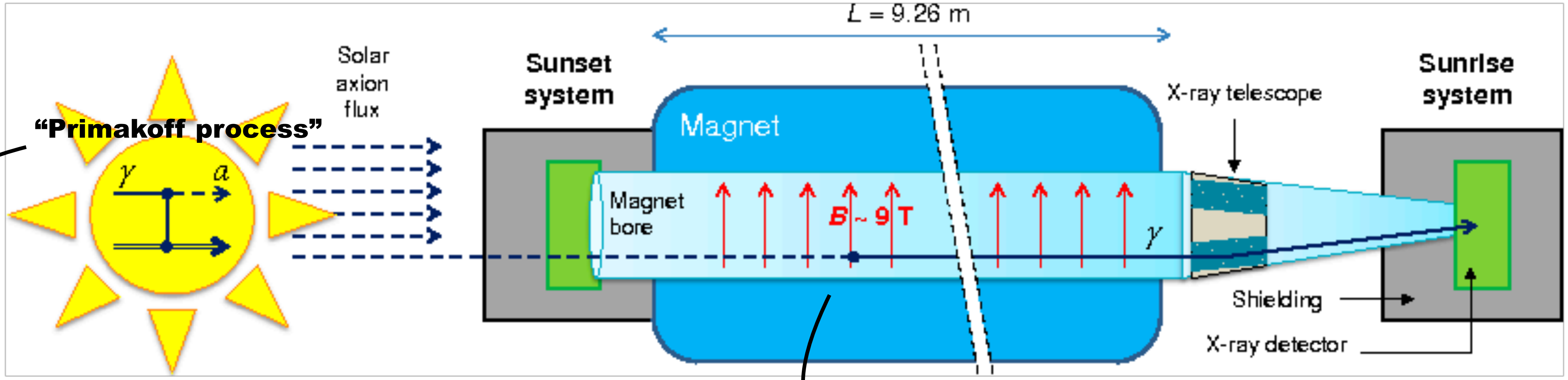
Light-Shining-Through-Walls



[OSQAR, arXiv:1506.08082]

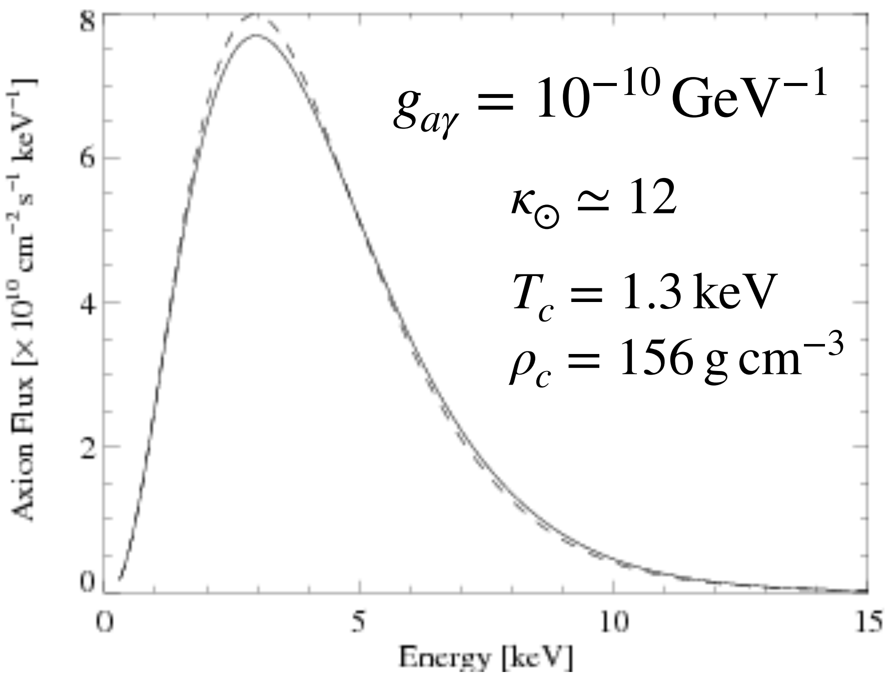
Axion helioscope

[CAST, arXiv:1705.02290]



- Solar axion flux

[CAST, hep-ex/0702006]



$$g_{a\gamma} = 10^{-10}\,\mathrm{GeV}^{-1}$$

$$\kappa_\odot \simeq 12$$

$$T_c = 1.3\,\mathrm{keV}$$

$$\rho_c = 156\,\mathrm{g\,cm^{-3}}$$

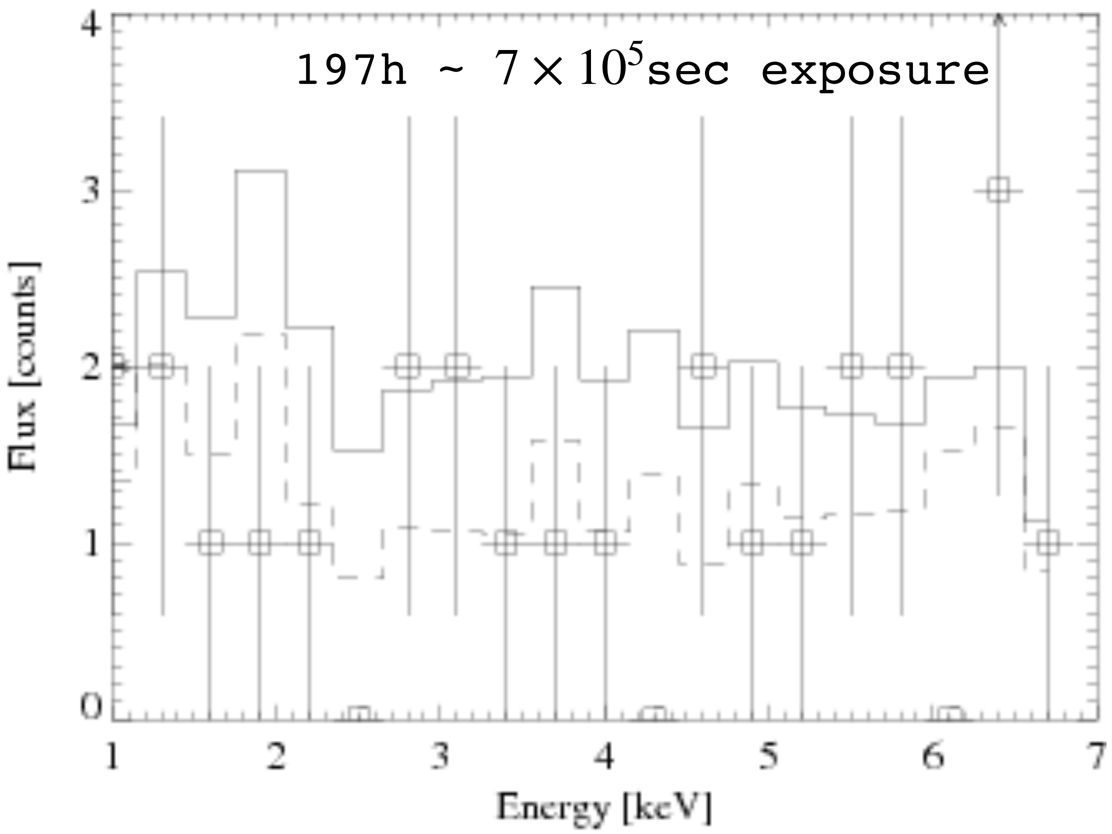
- Axion detection via γ-conversion

$$P_{A_\parallel \to a} \simeq \left(\Delta_{a\gamma} L_B\right)^2 = 2.4 \times 10^{-17} \left(\frac{g_{a\gamma}}{10^{-10}\,\mathrm{GeV}^{-1}}\right)^2 \left(\frac{B}{10\,\mathrm{T}}\right)^2 \left(\frac{L_B}{10\,\mathrm{m}}\right)^2$$

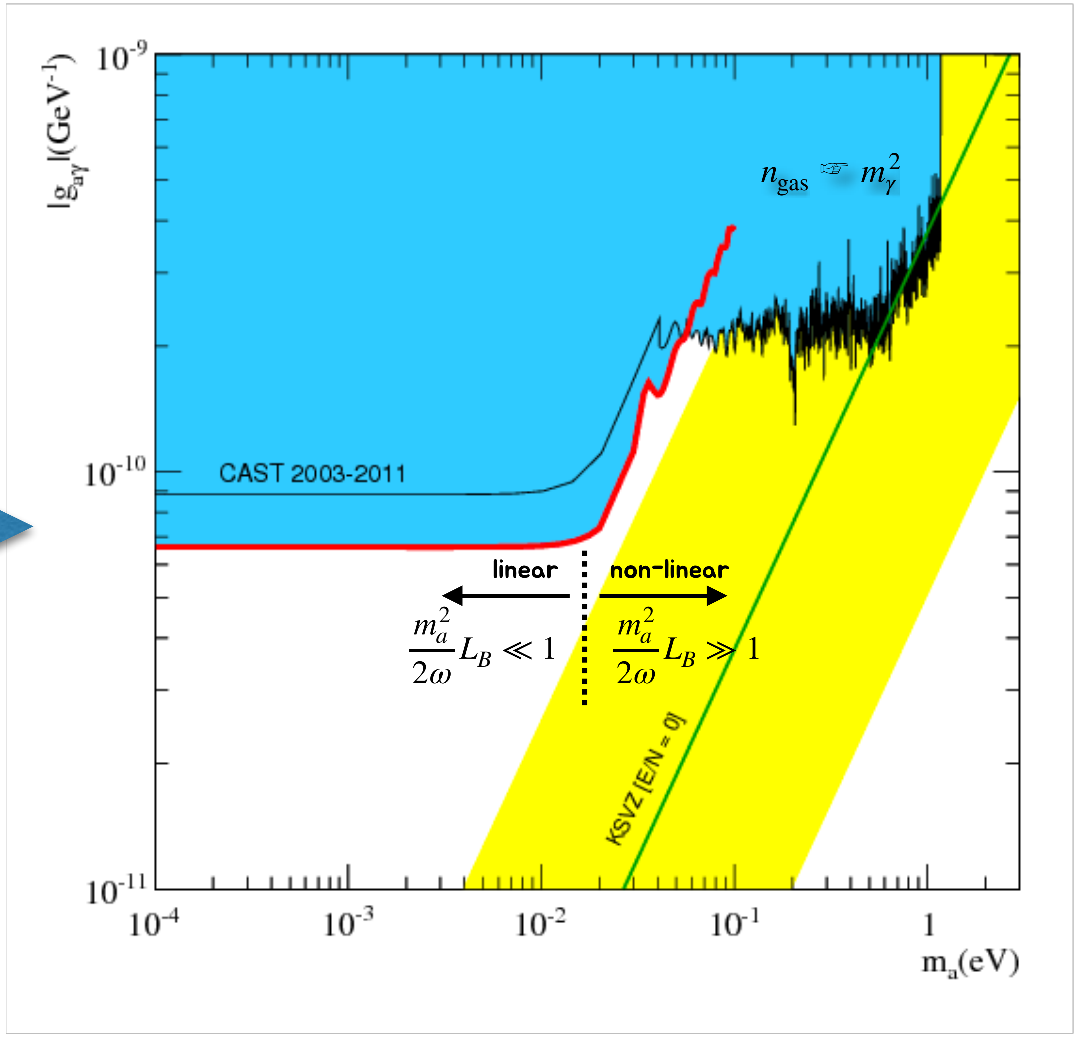
"linear regime"

Axion helioscope

[CAST, hep-ex/0702006]

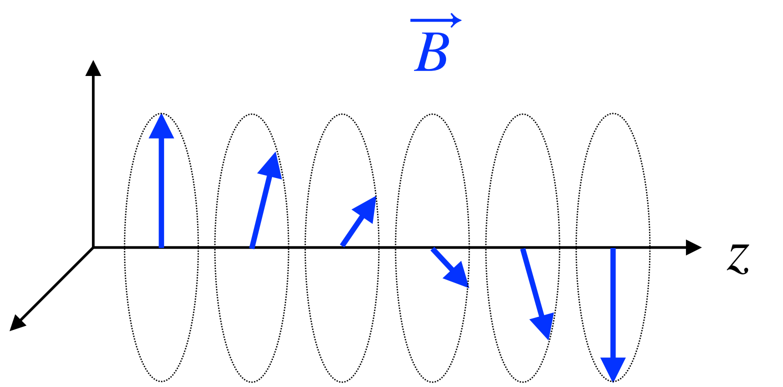


[CAST, arXiv:1705.02290]



Axion magnetic resonance

- Enhancement of axion-photon conversion in a spatially/temporally ***rotating*** magnetic field
- Applications to LSTW & Helioscope



Axion-Photon oscillation

- Equation motion in "spatially varying" $\overrightarrow{B}$ & assuming negligible refractive index

$$\left[\omega + i\partial_z + \begin{pmatrix} 0 & 0 & g_{a\gamma}B\sin\theta/2 \\ 0 & 0 & g_{a\gamma}B\cos\theta/2 \\ g_{a\gamma}B\sin\theta/2 & g_{a\gamma}B\cos\theta/2 & -m_a^2/2\omega \end{pmatrix}\right]\begin{pmatrix} A_x \\ A_y \\ a \end{pmatrix} = 0$$

- In the co-rotate basis with respect to $\overrightarrow{B}$ (i.e., $A_\perp$ & $A_\parallel$)

$$\left[\omega + i\partial_z + \begin{pmatrix} 0 & -i\dot{\theta} & 0 \\ i\dot{\theta} & 0 & g_{a\gamma}B/2 \\ 0 & g_{a\gamma}B/2 & -m_a^2/2\omega \end{pmatrix}\right]\begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix} = 0$$

Frequency of B variation
from $-iU^\dagger\partial_z U$ with the co-rotation U

Axion-Photon oscillation

- In the circular-polarized basis as diagonalized in the photon states

$$\left[\omega + i\partial_z + \begin{pmatrix} -\dot{\theta} & 0 & g_{a\gamma}B/2\sqrt{2} \\ 0 & \dot{\theta} & g_{a\gamma}B/2\sqrt{2} \\ g_{a\gamma}B/2\sqrt{2} & g_{a\gamma}B/2\sqrt{2} & -m_a^2/2\omega \end{pmatrix}\right] \begin{pmatrix} A_- \\ A_+ \\ a \end{pmatrix} = 0$$

- Enhancement of conversion probability occurs when $|\dot{\theta}| = m_a^2/2\omega \approx \Delta_{\rm osc}$
  - Effectively modified photon dispersion due to the co-rotating frame

    ☞ **Compensation of the momentum transfer** $m_a^2/2\omega$

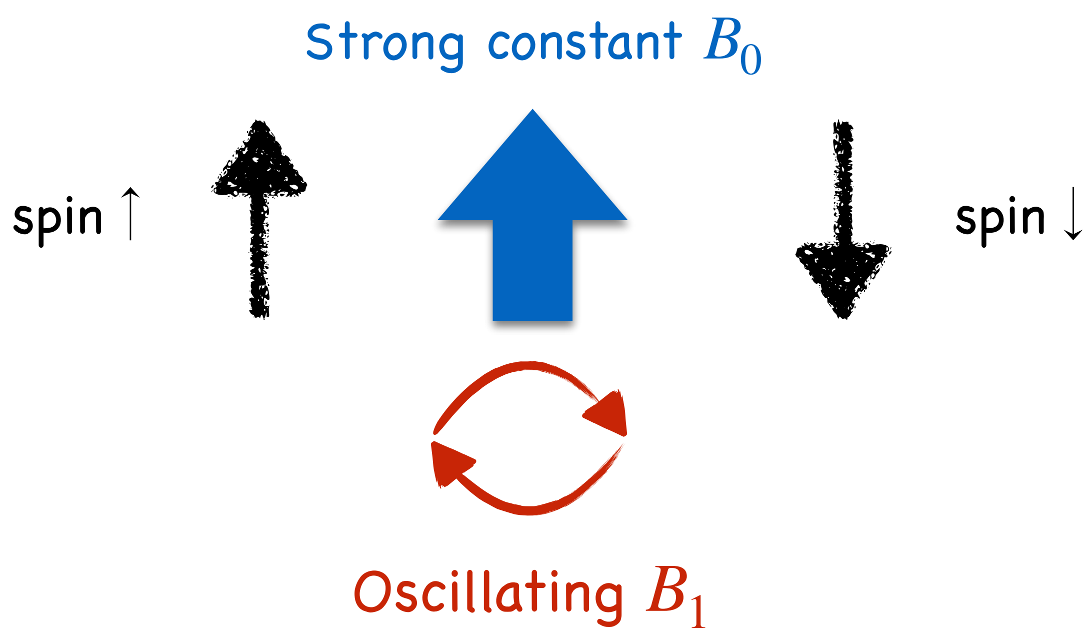
    Cavity setup is valid

  - Parametric resonance

    ☞ **Oscillation frequency** $\frac{\Delta_{\rm osc}}{2} \times 2$ = **System's frequency** $\dot{\theta}$

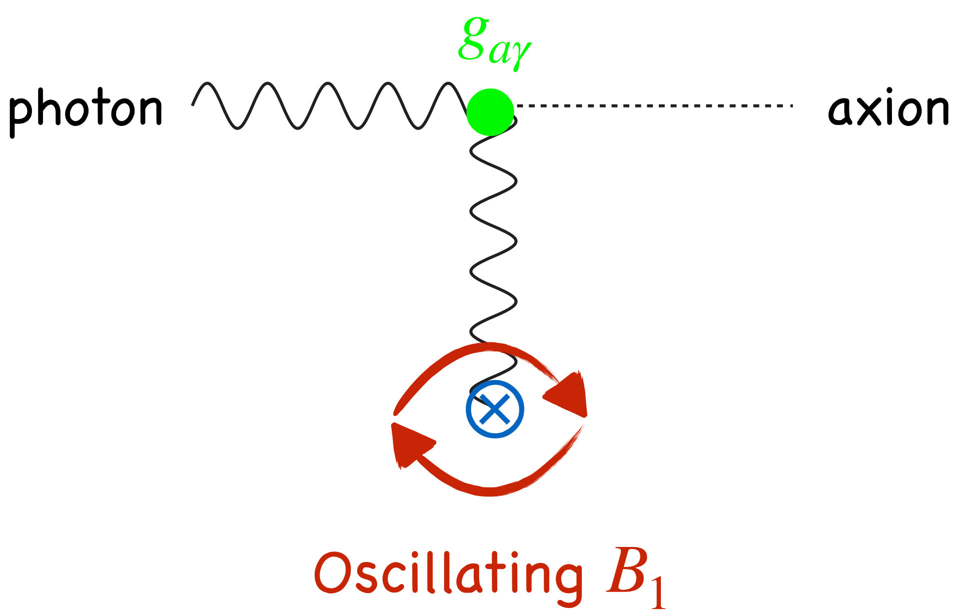
Axion magnetic resonance (AMR)

Nuclear magnetic resonance



- Two states: spin ↑↓
- Larmor precession frequency $\omega_0 = \mu B_0$
- Transition in oscillating B_1 with $\dot{\theta} = d\hat{B}_1/dt$
- Rabi frequency $\sqrt{(\omega_0 - \dot{\theta})^2 + (\mu B_1)^2}$

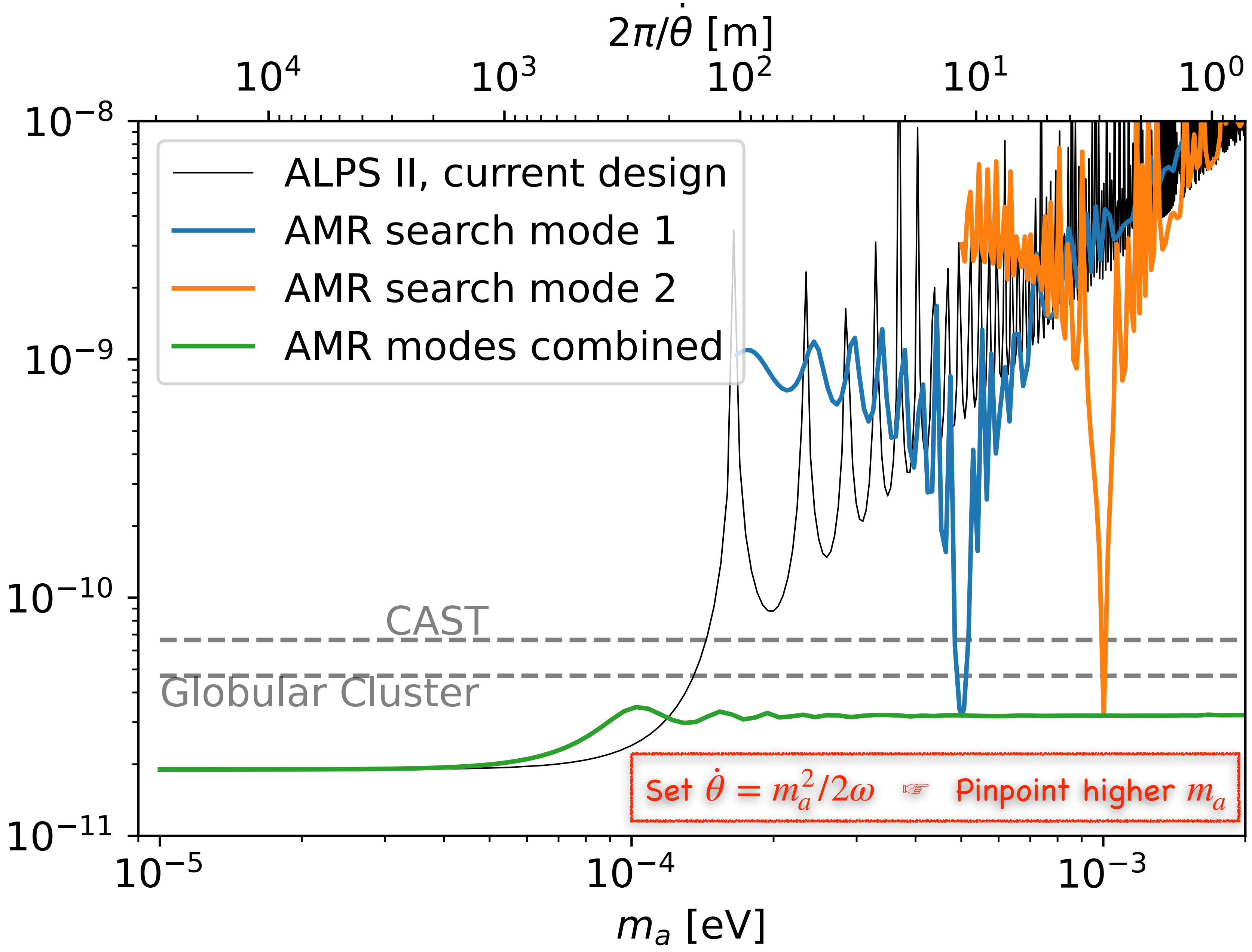
Axion magnetic resonance



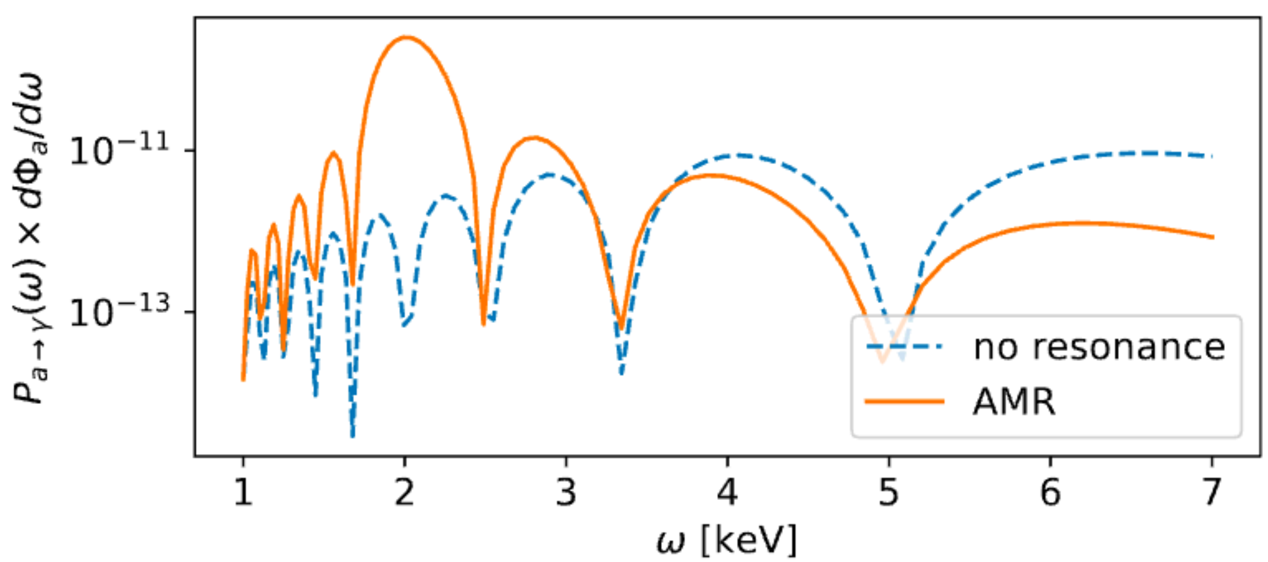
- Two state: photon & axion
- Axion momentum transfer $\omega_a = m_a^2/2E$
- Transition in oscillating B_1 with $\dot{\theta} = d\hat{B}_1/dt$
- Rabi frequency $\sqrt{(\omega_a - \dot{\theta})^2 + (g_{a\gamma}B/2)^2}$

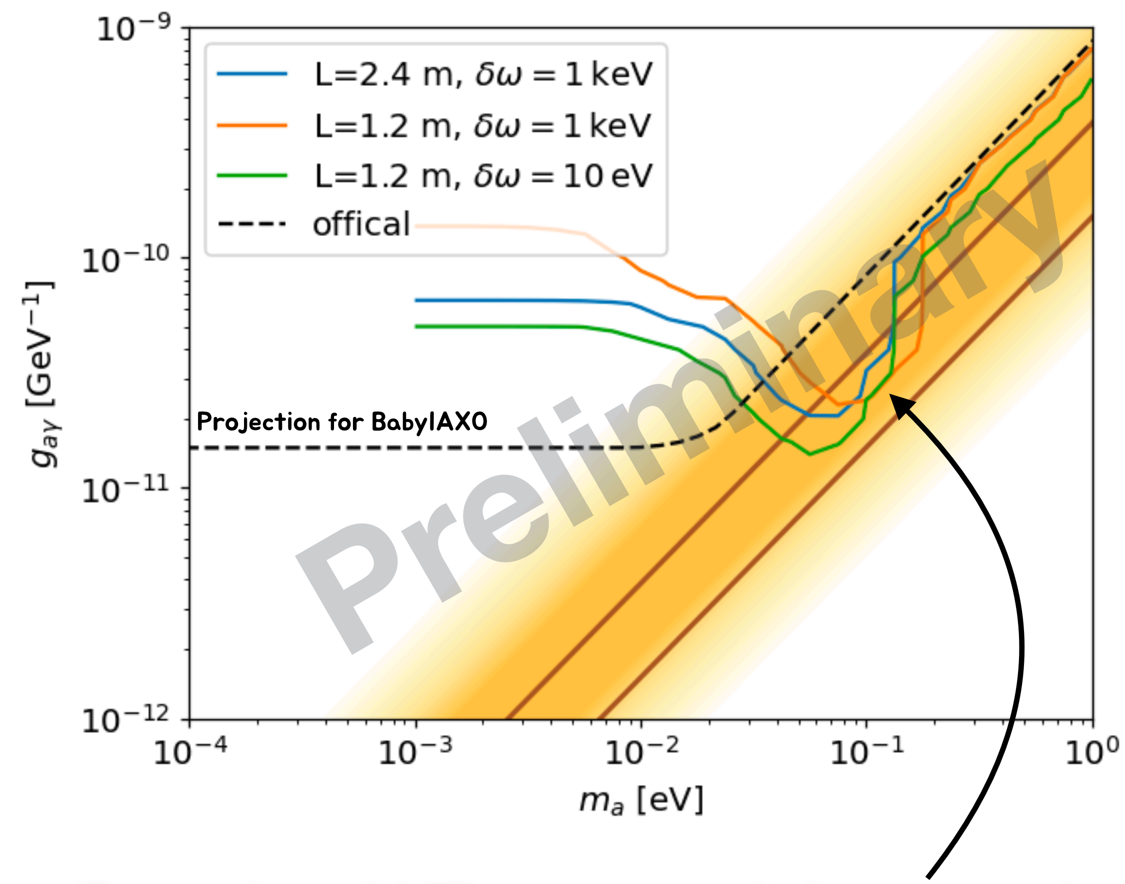
AMR in LSTW

[C. Sun, H. Seong, SY, 23]



AMR in helioscope





- Solar axion via Primakoff $\gamma + (N, Z) \to a + (N, Z)$
- Continuum spectrum in $1\text{-}10\,\mathrm{keV}$ range

☞ **Broader AMR parametric scanning**

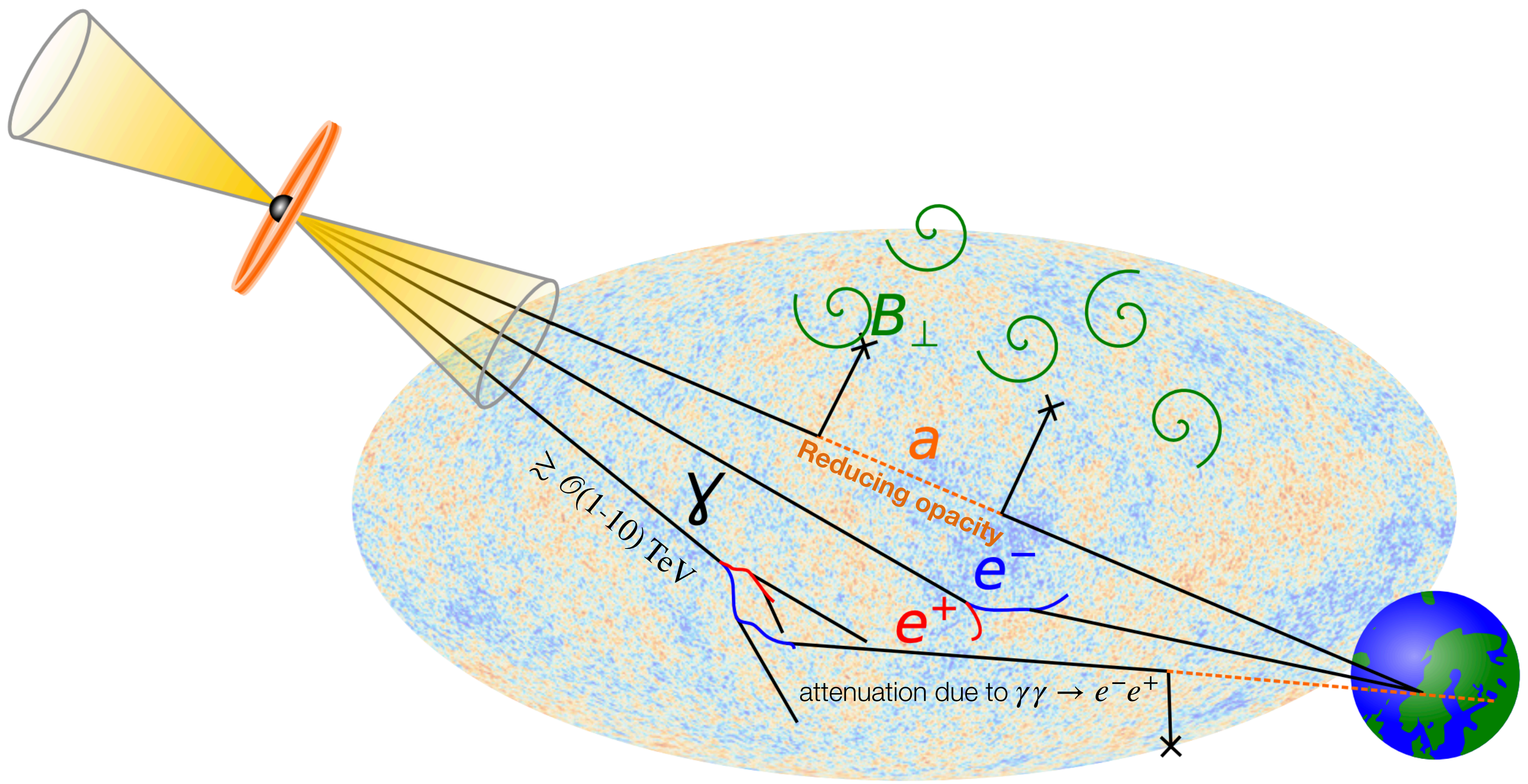
Other applications?

- Impact on axion-photon (resonant) oscillation
- Astrophysical & Cosmological implications

TeV transparency & Axion wiggles

γ-ray opacity

γ-ray spectrum

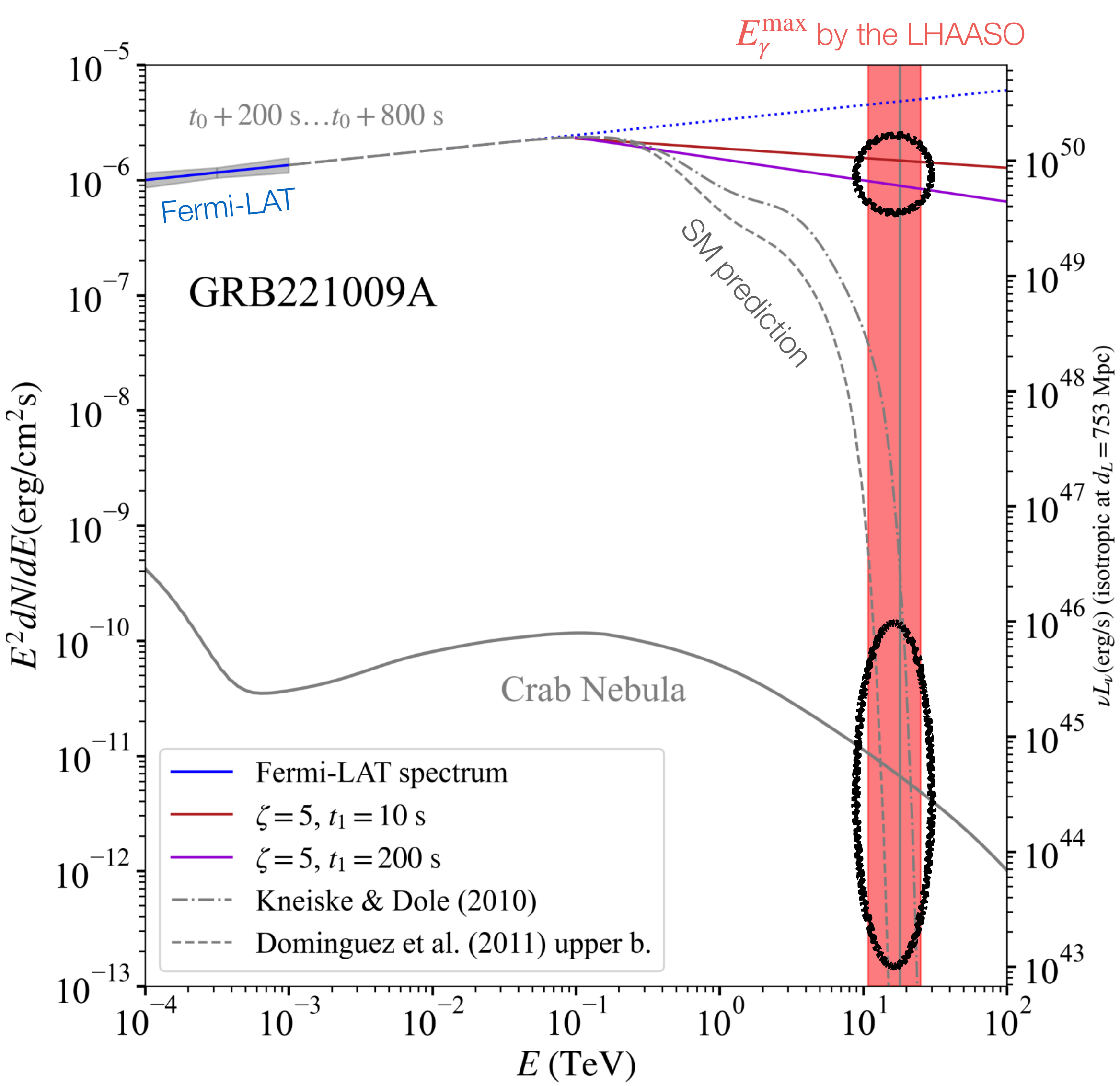


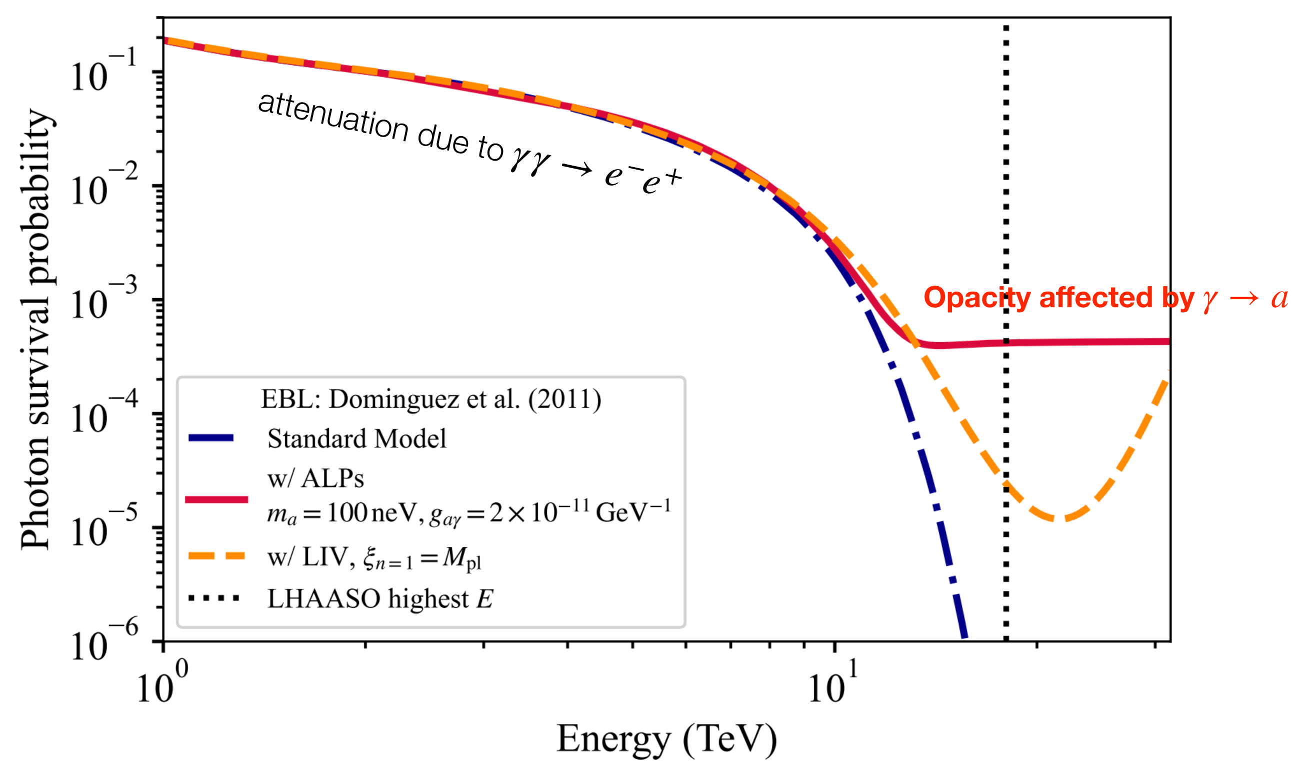
[Michael Kachelrieß's slide @ 1st General Meeting of COST Action COSMIC WISPers (CA21106)]

TeV transparency & Axion wiggles

γ-ray opacity

γ-ray spectrum



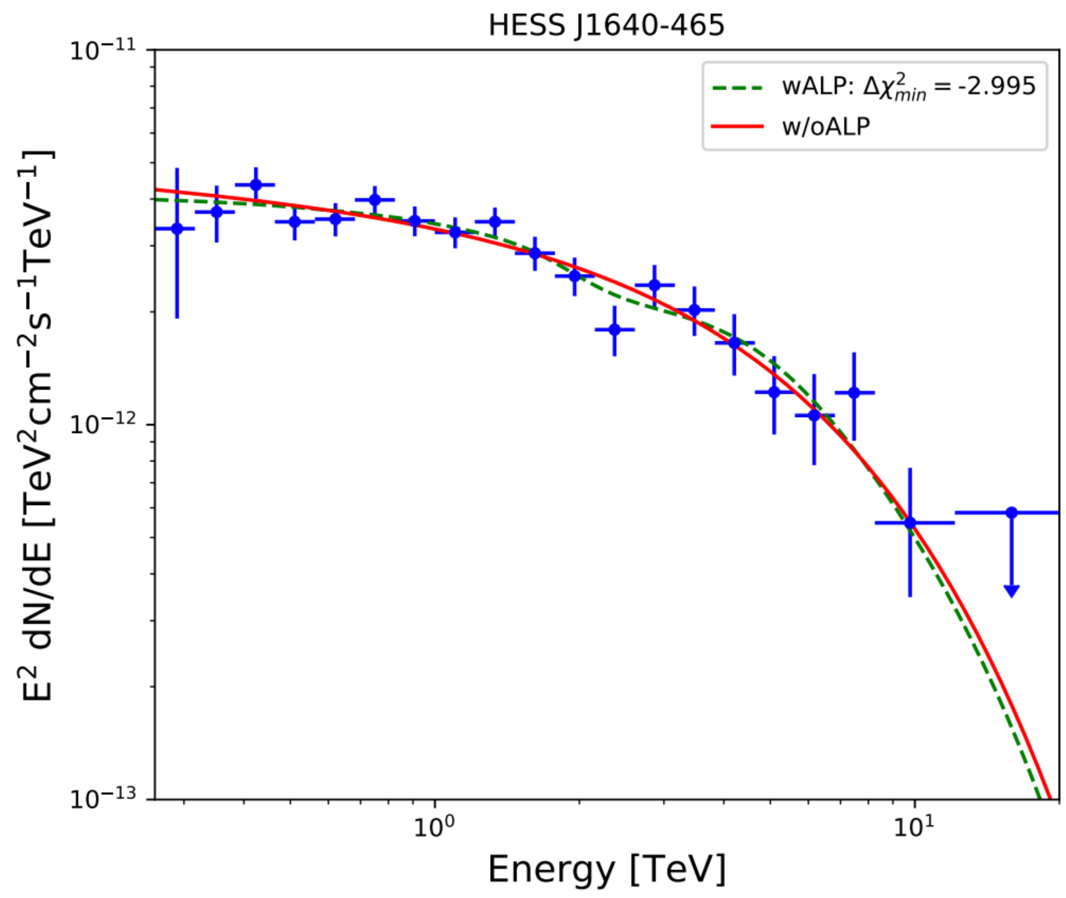


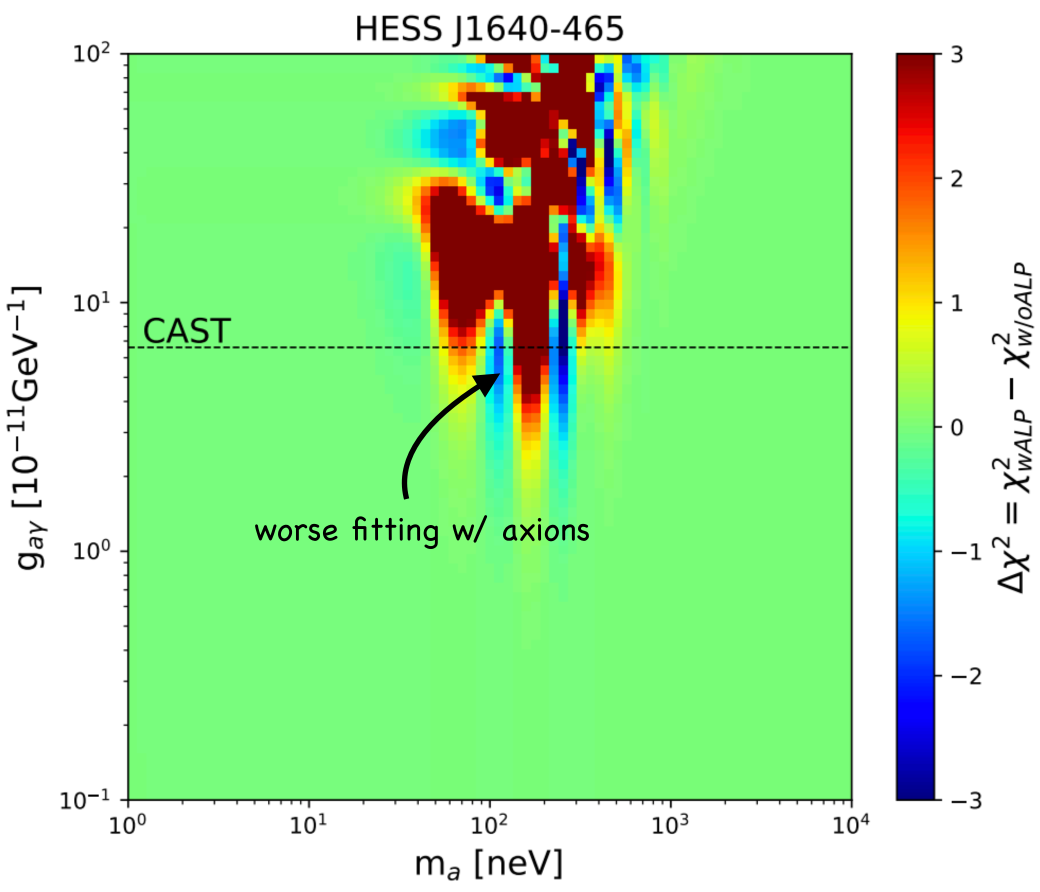
[A. Baktash et al., 23]

TeV transparency & Axion wiggles

γ-ray opacity

γ-ray spectrum





[Y. Liang et al., 18]

Condition in astrophysical searches

Galactic $\vec{B}$

$$B \sim \mu\text{G}, \quad l_{\text{coh}} \sim 10\,\text{kpc}$$

$$|\dot{\theta}| = \mathcal{O}(0.1)\,\text{kpc}^{-1}$$

$$\Delta_{\parallel} = 0.8 \times 10^{-4}\,\text{kpc}^{-1}\left(\frac{\omega}{\text{TeV}}\right)$$

$$\Delta_a = -\,0.8 \times 10^{-4}\,\text{kpc}^{-1}\left(\frac{m_a}{\text{neV}}\right)^2\left(\frac{\omega}{\text{TeV}}\right)^{-1}$$

$$\Delta_{a\gamma} = 1.5 \times 10^{-2}\,\text{kpc}^{-1}\left(\frac{g_{a\gamma}}{10^{-11}\,\text{GeV}^{-1}}\right)\left(\frac{B}{\mu\text{G}}\right)$$

Intergalactic $\vec{B}$

$$B \sim \text{nG}, \quad l_{\text{coh}} \sim \text{Mpc}$$

$$|\dot{\theta}| = \mathcal{O}(1)\,\text{Mpc}^{-1}$$

$$\Delta_{\parallel} = 0.8 \times 10^{-1}\,\text{Mpc}^{-1}\left(\frac{\omega}{\text{TeV}}\right)$$

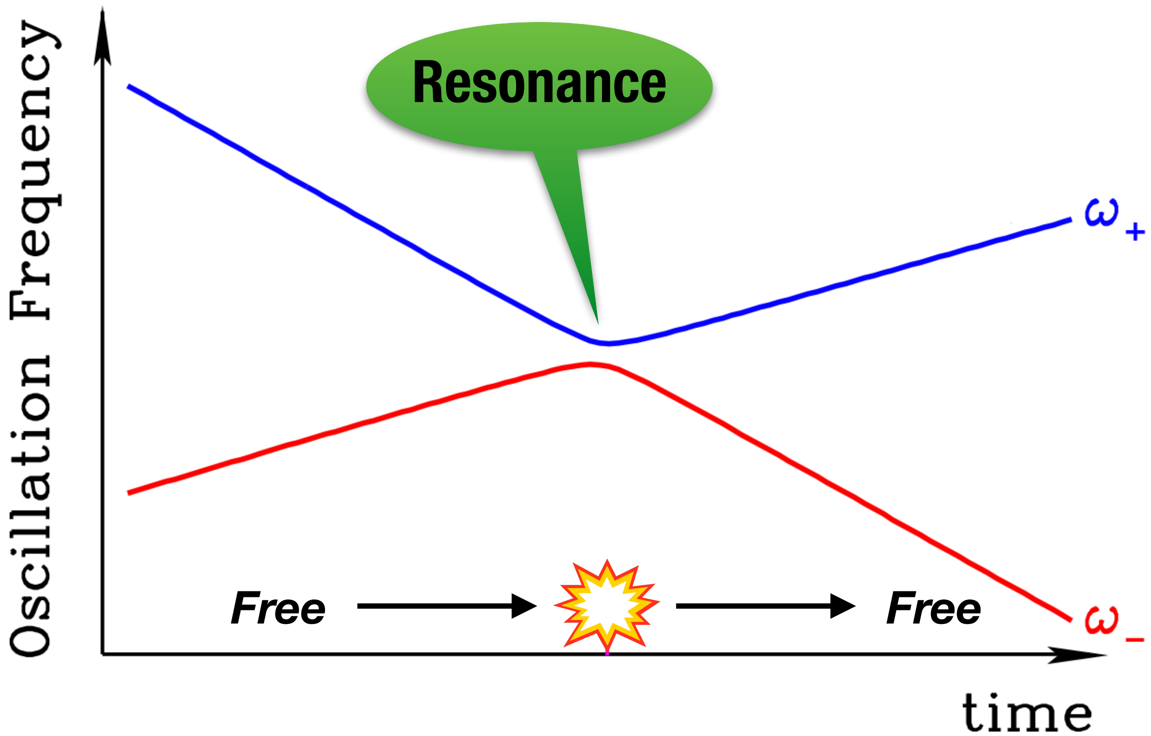
$$\Delta_a = -\,0.8 \times 10^{-1}\,\text{Mpc}^{-1}\left(\frac{m_a}{\text{neV}}\right)^2\left(\frac{\omega}{\text{TeV}}\right)^{-1}$$

$$\Delta_{a\gamma} = 1.5 \times 10^{-2}\,\text{Mpc}^{-1}\left(\frac{g_{a\gamma}}{10^{-11}\,\text{GeV}^{-1}}\right)\left(\frac{B}{\text{nG}}\right)$$

Suppression of $\gamma \leftrightarrow a$?

Resonant conversion

In the basis of mass eigenstates,

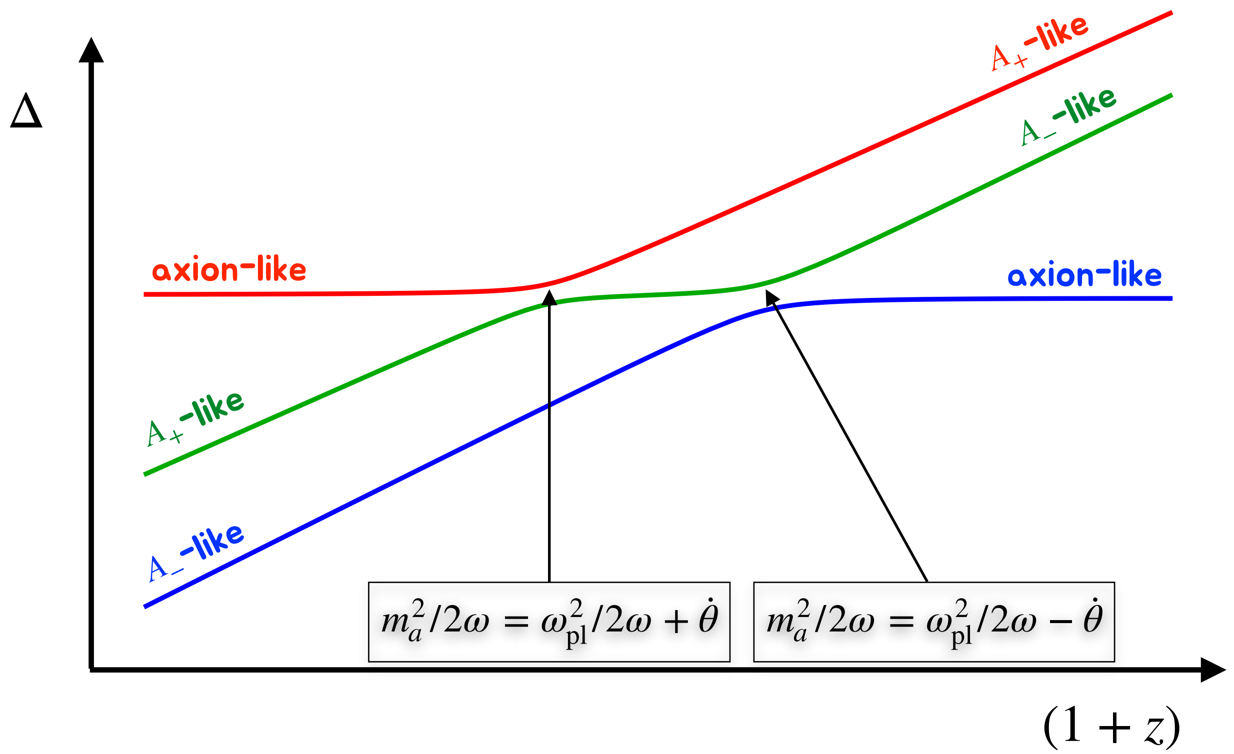


1. Adiabatic condition for eigenstates except the resonance point
2. Resonance period short enough

Can be derived analytically!

Resonant a-γ conversion in cosmo

$$\left[i\partial_z + \begin{pmatrix} -\omega_{\rm pl}^2/2\omega & \dot\theta & 0 \\ \dot\theta & -\omega_{\rm pl}^2/2\omega & g_{a\gamma}B/2 \\ 0 & g_{a\gamma}B/2 & -m_a^2/2\omega \end{pmatrix} \right] \begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix} = 0$$



- 3-state system, not 2 due to $\dot\theta$
- δt between the two resonance points

$$\delta t \sim \frac{\dot\theta}{d(\omega_{\rm pl}^2/2\omega)/dt} \sim \frac{\dot\theta}{\omega_{\rm pl}^2/2\omega} H_{\rm res}^{-1} \ll H_{\rm res}^{-1}$$

☞ **no oscillation pattern by $\dot\theta$**

Conclusion

- Axion-photon oscillations are available in a spin-1 background
- Hamiltonian to describe $a \leftrightarrow \gamma$ must involve a directional information of a spin-1 background: parametrized by $\dot{\theta}$
- Parametric resonance due to system's variation

$\Rightarrow$ **Axion magnetic resonance**

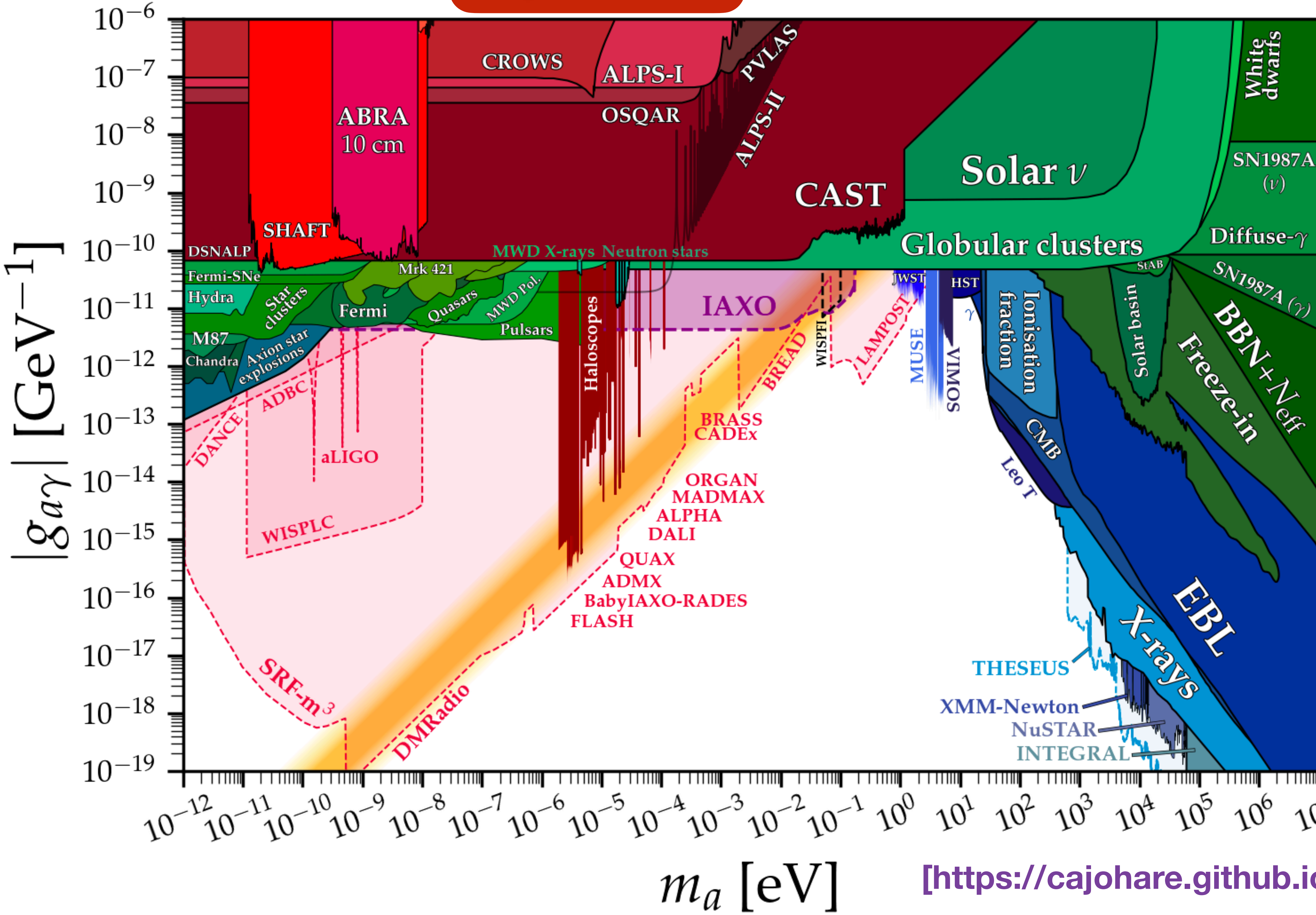
- More interesting on astrophysical & cosmological implications

Thank you!

Back up

Proejction on $\frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$

Experimental

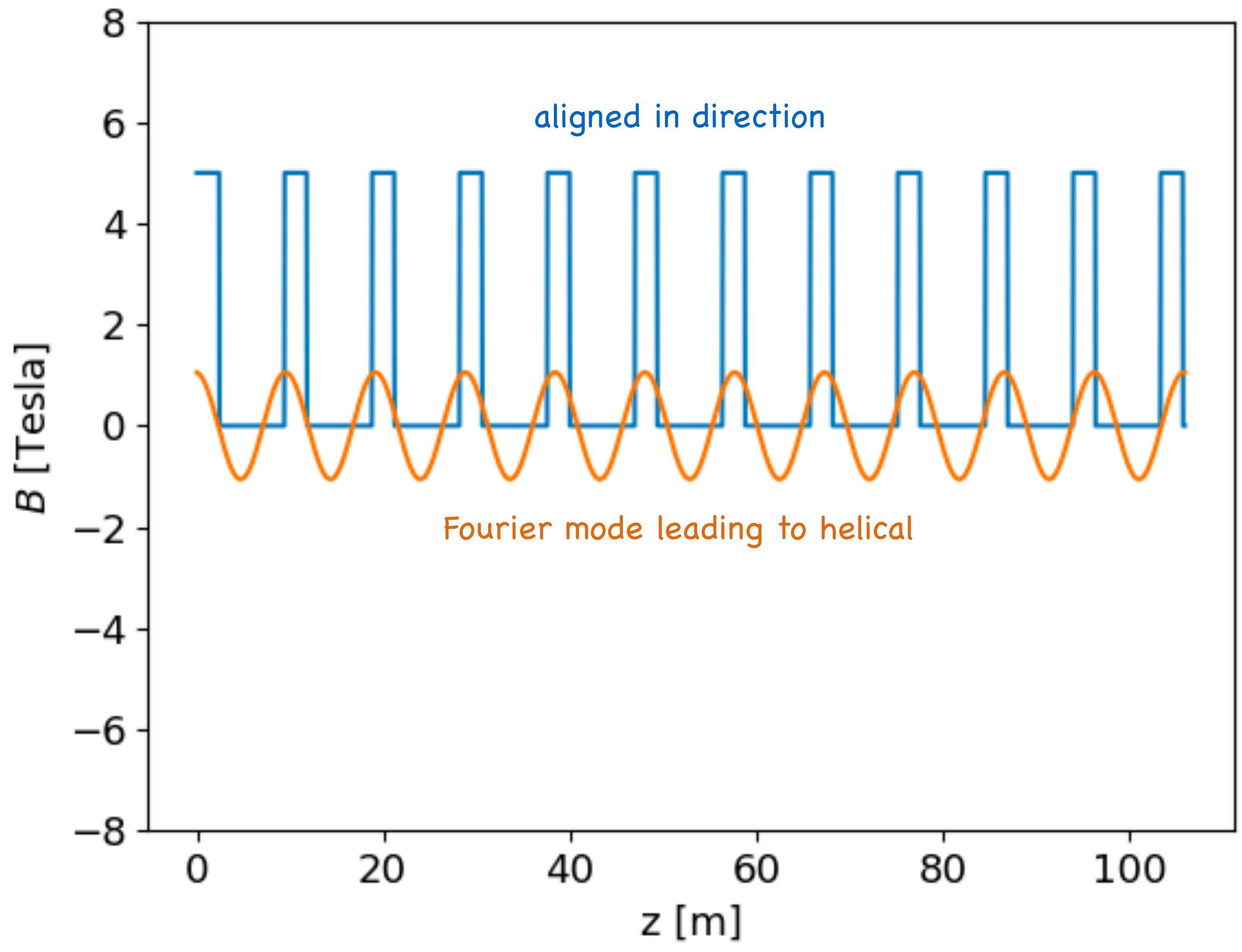


Astrophysical

Cosmological

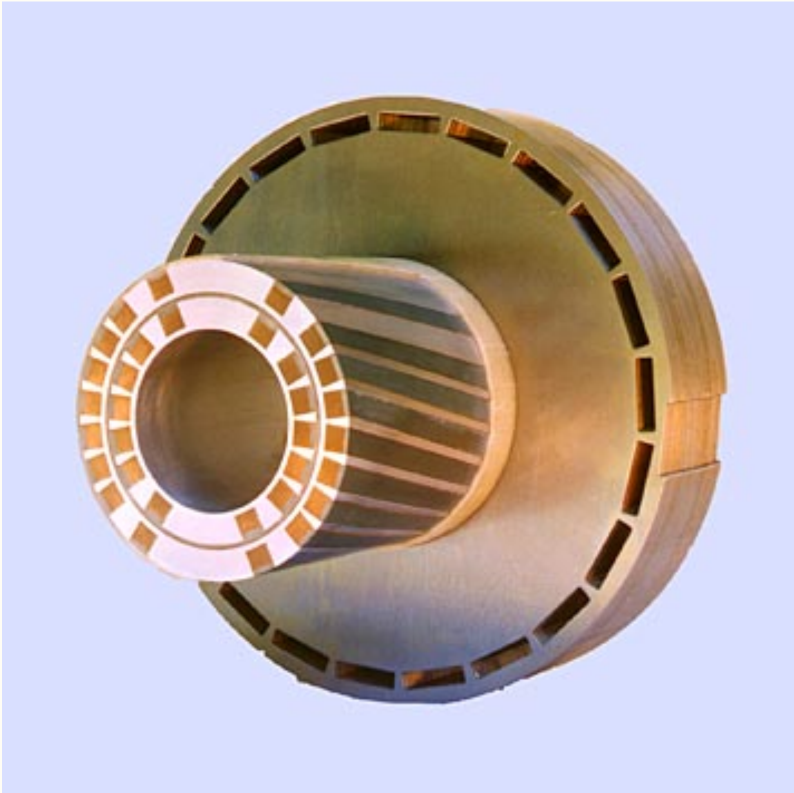
[https://cajohare.github.io/AxionLimits/]

Wiggler configuration



Experimental feasibility

Experimental Implications – B regularity (cont'd)



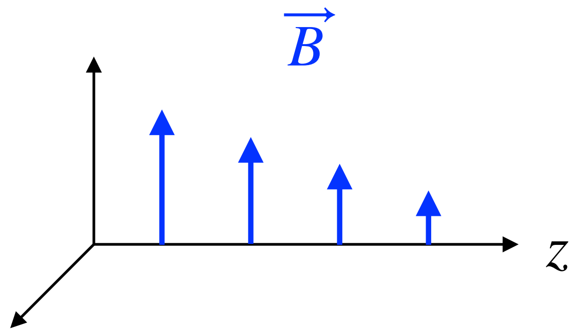
Magnets at Relativistic Heavy Ion Collider (RHIC), BNL:

- superconducting dipole magnet ~ 5 T
- 1740 magnets adopted by RHIC
- 30-/36-strand SC cable for...
  ... 80-100/130-180 mm apertures
- **B** field rotates 360 degrees in 2.4 meters
- designed to control proton spin for polarized proton colliding
- sub-percent error in field irregularity easily achieved:
  $\int |\mathbf{B}| dz \approx 10\,\mathrm{T}\cdot\mathrm{m}$
  $\left[(\int B_x(z)dz)^2 + (\int B_y(z)dz)^2\right]^{1/2} < 0.05\,\mathrm{T}\cdot\mathrm{m}$

10.1016/S0168-9002(02)01940-X

[In harmonic B]

Axion-Photon oscillation



- Equation motion in "harmonic" $\vec{B}$ & assuming negligible refractive index

$$\left[i\partial_z + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g_{a\gamma}B(z)/2 \\ 0 & g_{a\gamma}B(z)/2 & -m_a^2/2\omega \end{pmatrix} \right] \begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix} = 0$$

- Enhancement of conversion probability occurs when $|\dot{B}/B| = m_a^2/2\omega \approx \Delta_{\rm osc}$
  - Harmonic as mixture of the two opposite rotating background (e.g., $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$)

    ☞ **Compensation of the momentum transfer** $m_a^2/2\omega$

  - Parametric resonance

    ☞ **Oscillation frequency** $\dfrac{\Delta_{\rm osc}}{2} \times 2$ **=** **System's frequency** $\dot{\theta}$

Resonant conversion probability

Level-crossing transition at resonance

- The resonant conversion probability is

Initial mixing angle | Final mixing angle

$$P_{\rm res}^{\gamma\leftrightarrow\phi} \simeq \frac{1}{2} + \left(p - \frac{1}{2}\right)\cos 2\theta_0 \cos 2\theta_i$$

$$\simeq 1 - p$$

- The level-crossing transition rate is determined by adiabaticity at the resonance

[S. J. Parke, 86], [C.Zener, 32]

$$p \simeq \exp\left(-2\pi r k \sin^2\theta_0\right)_{t=t_{\rm res}} \qquad r = \left|\frac{d\ln\omega_{\rm pl}^2/m_\phi^2}{dt}\right|^{-1}$$

- The resonant conversion probability in non-adiabatic level-crossing case $p \approx 1$

$$P_{\rm res}^{\gamma\leftrightarrow\phi} \simeq r\frac{\pi m_{\rm mix}^4}{\omega m_\phi^2}$$