

Precision physics in SMEFT with multiboson d

1st COMETA Meeting

Izmir, Turkey

28 February 2024

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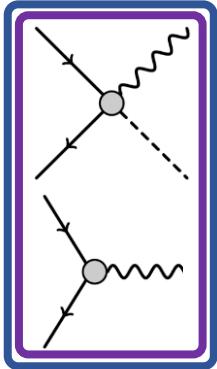
MANCHESTER
1824

The University of Manchester

Wilson Coefficients and how to bound them

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \frac{c_i^{(8)}}{\Lambda^4} \mathcal{O}_i^{(8)} + \mathcal{O}\left(\frac{1}{\Lambda^5}\right)$$

Same particles and gauge symmetries as the SM



$$\mathcal{O}_{uW} = (\bar{q}_L \sigma^{\mu\nu} u_R) \tau^I \tilde{H} W_{\mu\nu}^I$$

$$\mathcal{O}_{dW} = (\bar{q}_L \sigma^{\mu\nu} d_R) \tau^I H W_{\mu\nu}^I$$

$$\mathcal{O}_{uB} = (\bar{q}_L \sigma^{\mu\nu} u_R) \tilde{H} B_{\mu\nu}$$

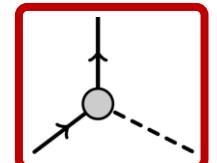
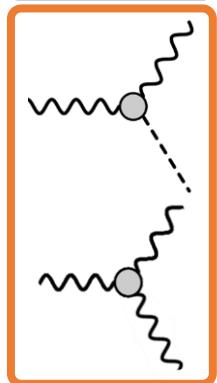
$$\mathcal{O}_{dB} = (\bar{q}_L \sigma^{\mu\nu} d_R) H B_{\mu\nu}$$

Dipoles

$$\mathcal{O}_{u\varphi} = H^\dagger H (\bar{q}_L \tilde{H} u_R)$$

$$\mathcal{O}_{d\varphi} = H^\dagger H (\bar{q}_L H d_R)$$

Yukawa



$$\mathcal{O}_{\varphi q}^{(3)} = (\bar{Q}_L \sigma^a \gamma^\mu Q_L) \left(i H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H \right)$$

$$\mathcal{O}_{\varphi q}^{(1)} = (\bar{Q}_L \gamma^\mu Q_L) \left(i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right)$$

$$\mathcal{O}_{\varphi u} = (\bar{u}_R \gamma^\mu u_R) \left(i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right)$$

$$\mathcal{O}_{\varphi d} = (\bar{d}_R \gamma^\mu d_R) \left(i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right)$$

$$\mathcal{O}_{\varphi ud} = (\bar{u}_R \gamma^\mu d_R) \left(i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right)$$

Fermion current

$$\mathcal{O}_W = \epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$$

$$\mathcal{O}_{\varphi D} = (H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$$

$$\mathcal{O}_{\varphi W} = H^\dagger H W^{a,\mu\nu} W_{\mu\nu}^a$$

$$\mathcal{O}_{\varphi \widetilde{W}} = H^\dagger H W^{a,\mu\nu} \widetilde{W}_{\mu\nu}^a$$

$$\mathcal{O}_{\varphi B} = H^\dagger H B^{\mu\nu} B_{\mu\nu}$$

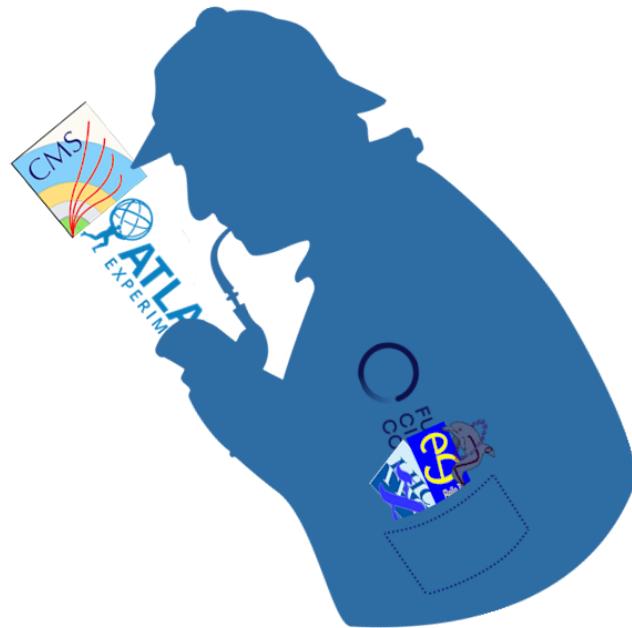
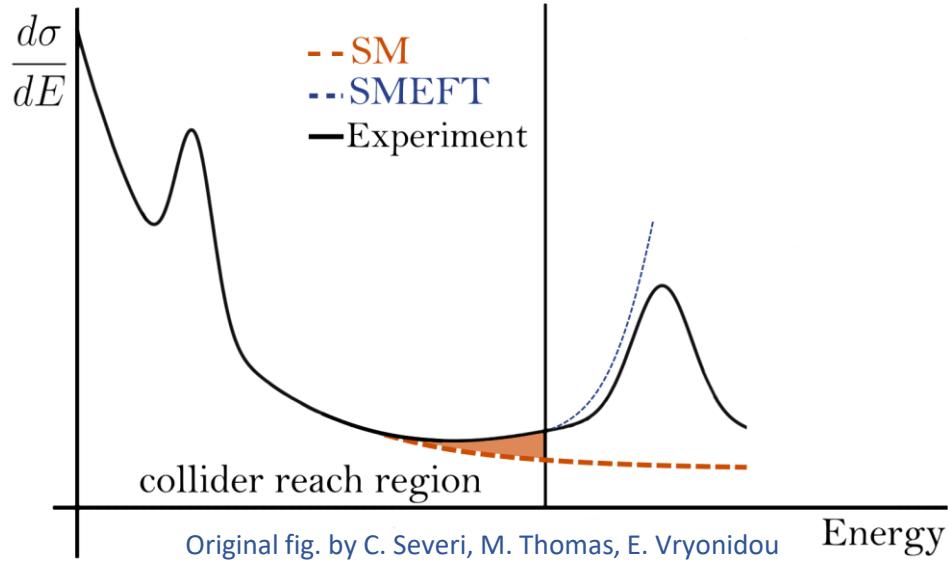
$$\mathcal{O}_{\varphi \widetilde{B}} = H^\dagger H B^{\mu\nu} \widetilde{B}_{\mu\nu}$$

$$\mathcal{O}_{\varphi WB} = H^\dagger \sigma^a H B^{\mu\nu} W_{\mu\nu}^a$$

$$\mathcal{O}_{\varphi \widetilde{WB}} = H^\dagger \sigma^a H B^{\mu\nu} \widetilde{W}_{\mu\nu}^a$$

Bosonic

Why Diboson?: Its effect on the tails



	SM	BSM
$q_{L,R}\bar{q}_{L,R} \rightarrow V_L V_L(h)$	~ 1	$\sim E^2/M^2$
$q_{L,R}\bar{q}_{L,R} \rightarrow V_{\pm} V_L(h)$	$\sim m_W/E$	$\sim m_W E/M^2$
$q_{L,R}\bar{q}_{L,R} \rightarrow V_{\pm} V_{\pm}$	$\sim m_W^2/E^2$	$\sim E^2/M^2$
$q_{L,R}\bar{q}_{L,R} \rightarrow V_{\pm} V_{\mp}$	~ 1	~ 1

Franceschini et al [1712.01310]

Vh: A diboson case study

Based on: Bishara, Englert, Grojean, Panico, ANR [2208.11134]

Many dim-6 operators affect $Vh\dots$

$$\mathcal{O}_{\varphi D} = (H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$$

$$\mathcal{O}_{\varphi W} = H^\dagger H W^{a,\mu\nu} W_{\mu\nu}^a$$

$$\mathcal{O}_{\varphi \widetilde{W}} = H^\dagger H W^{a,\mu\nu} \widetilde{W}_{\mu\nu}^a$$

$$\mathcal{O}_{\varphi B} = H^\dagger H B^{\mu\nu} B_{\mu\nu}$$

$$\mathcal{O}_{\varphi \widetilde{B}} = H^\dagger H B^{\mu\nu} \widetilde{B}_{\mu\nu}$$

$$\mathcal{O}_{\varphi WB} = H^\dagger \sigma^a H B^{\mu\nu} W_{\mu\nu}^a$$

$$\mathcal{O}_{\varphi \widetilde{WB}} = H^\dagger \sigma^a H B^{\mu\nu} \widetilde{W}_{\mu\nu}^a$$

$$\mathcal{O}_{uW} = (\bar{q}_L \sigma^{\mu\nu} u_R) \tau^I \tilde{H} W_{\mu\nu}^I$$

$$\mathcal{O}_{dW} = (\bar{q}_L \sigma^{\mu\nu} d_R) \tau^I H W_{\mu\nu}^I$$

$$\mathcal{O}_{uB} = (\bar{q}_L \sigma^{\mu\nu} u_R) \tilde{H} B_{\mu\nu}$$

$$\mathcal{O}_{dB} = (\bar{q}_L \sigma^{\mu\nu} d_R) H B_{\mu\nu}$$

$$\mathcal{O}_{\varphi ud} = (\bar{u}_R \gamma^\mu d_R) \left(i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right)$$

$$\mathcal{O}_{\varphi q}^{(3)} = (\bar{Q}_L \sigma^a \gamma^\mu Q_L) \left(i H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H \right)$$

$$\mathcal{O}_{\varphi q}^{(1)} = (\bar{Q}_L \gamma^\mu Q_L) \left(i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right)$$

$$\mathcal{O}_{\varphi u} = (\bar{u}_R \gamma^\mu u_R) \left(i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right)$$

$$\mathcal{O}_{\varphi d} = (\bar{d}_R \gamma^\mu d_R) \left(i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right)$$

$$\mathcal{O}_{u\varphi} = H^\dagger H (\bar{q}_L \tilde{H} u_R)$$

$$\mathcal{O}_{d\varphi} = H^\dagger H (\bar{q}_L H d_R)$$

— MFV suppressed

— Sub-leading energy growth

— No interference with SM for massless quarks

...but just a few are well probed.

A trick of the tails

$V = W, Z$

$q\bar{q}' \rightarrow Vh$

V polarization	SM	$\mathcal{O}_{\varphi f}$	$\mathcal{O}_{\varphi W}$	$\mathcal{O}_{\varphi \widetilde{W}}$
$\lambda = 0$	1	$\frac{\hat{s}}{\Lambda^2}$	$\frac{M_W^2}{\Lambda^2}$	0
$\lambda = \pm$	$\frac{M_W}{\sqrt{\hat{s}}}$	$\frac{\sqrt{\hat{s}} M_W}{\Lambda^2}$	$\frac{\sqrt{\hat{s}} M_W}{\Lambda^2}$	$\frac{\sqrt{\hat{s}} M_W}{\Lambda^2}$

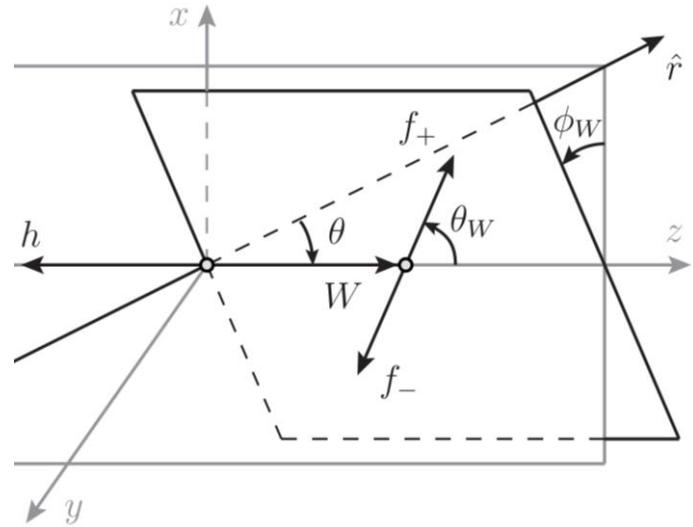
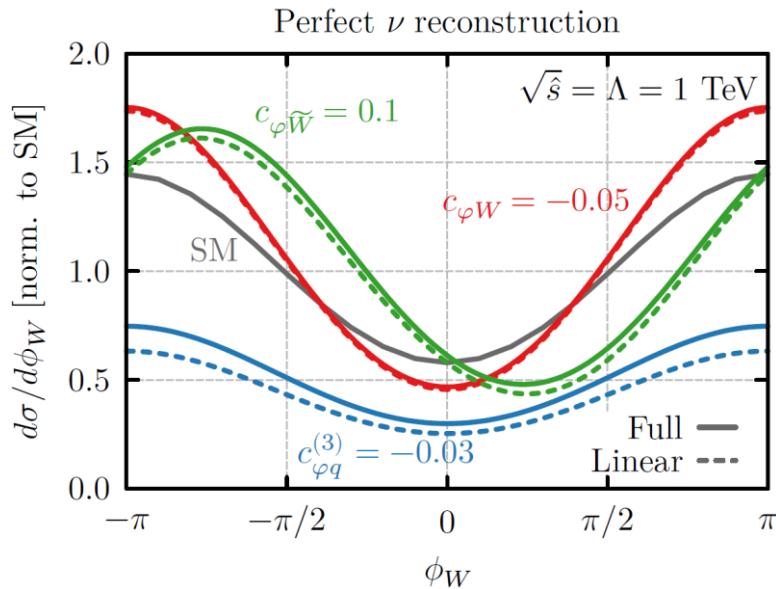
$$\mathcal{O}_{\varphi f} = \mathcal{O}_{\varphi q}^{(3)}, \mathcal{O}_{\varphi q}^{(1)}, \mathcal{O}_{\varphi u}, \mathcal{O}_{\varphi d}$$

Differential in p_T  Same-polarization Interference

Resurrecting the interference in diboson

Angular observables improve sensitivity

Azatov et al [1707.08060]; Panico et al [1708.07823]; Azatov et al [1901.04821];
 Banerjee et al [1905.02728, 1912.07628]; Bishara, ANR et al [2004.06122]...



$$\sigma_{\mathcal{O}_{\varphi q}^{(3)}}^{int} \sim \frac{\hat{s}}{\Lambda^2}$$

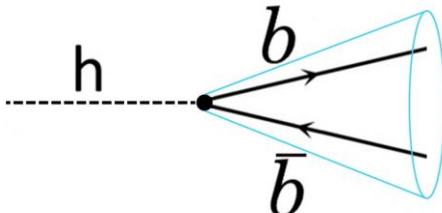
$$\sigma_{\mathcal{O}_{\varphi W}}^{int} \sim \frac{\sqrt{\hat{s}} M_W}{\Lambda^2} \cos(\phi_W)$$

$$\sigma_{\mathcal{O}_{\varphi \widetilde{W}}}^{int} \sim \frac{\sqrt{\hat{s}} M_W}{\Lambda^2} \sin(\phi_W)$$

Bonus: CP-odd observables!

The power of combining regimes

Boosted



ATLAS [2008.02508]

DOI: [10.1016/j.physletb.2021.136204](https://doi.org/10.1016/j.physletb.2021.136204)

28th April 2021

Measurement of the associated production of a Higgs boson decaying into b -quarks with a vector boson at high transverse momentum in $p\ p$ collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector

The ATLAS Collaboration

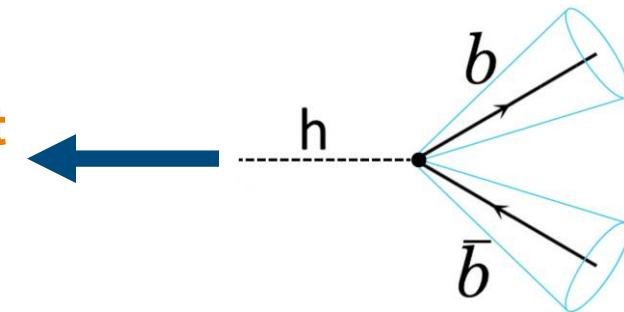
Scale-invariant
tagging

Gouzevitch et al [1303.6636]
Bishara et al [1611.03860]

With use of
Mass-drop tagging

Butterworth et al, 0802.2470

Resolved



ATLAS, 2007.02873

DOI: [10.1140/epjc/s10052-020-08677-2](https://doi.org/10.1140/epjc/s10052-020-08677-2)

9th March 2021

Measurements of WH and ZH production in the $H \rightarrow b\bar{b}$ decay channel in $p\ p$ collisions at 13 TeV with the ATLAS detector

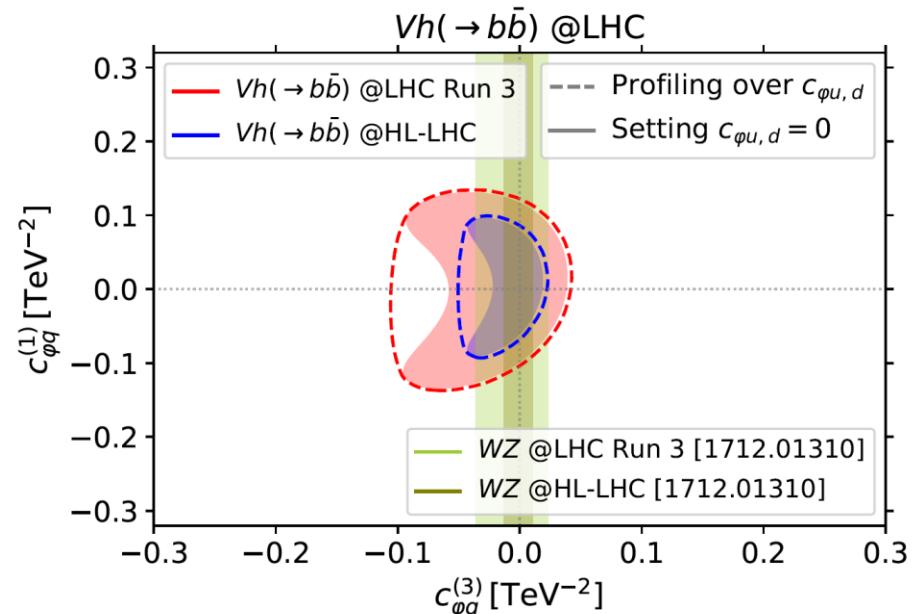
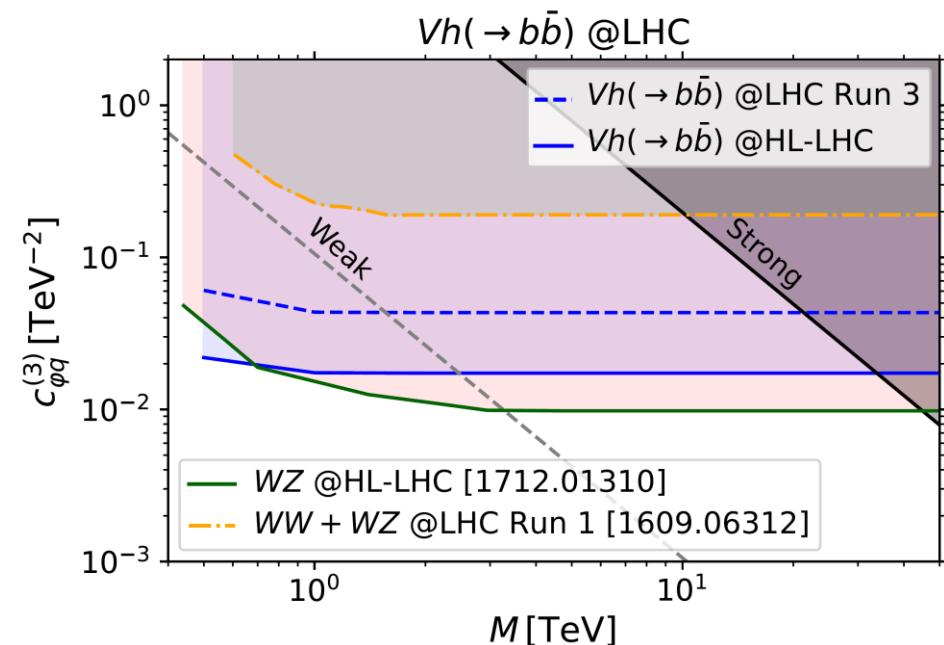
The ATLAS Collaboration

Categories	Variable	(HL-)LHC
0-lepton	boosted	$p_{T,\min} [\text{GeV}]$
	resolved	$\{0, 300, 350, \infty\}$ $\{0, 160, 200, 250, \infty\}$
1-lepton	boosted	$p_T^h [\text{GeV}]$
	resolved	$\{0, 175, 250, 300, \infty\}$ $\{0, 175, 250, \infty\}$
2-lepton	boosted	$p_{T,\min} [\text{GeV}]$
	resolved	$\{250, \infty\}$ $\{175, 200, \infty\}$

Adding Resolved category: 10-20% improvement at LHC

8.

All of diboson at (HL-)LHC



LHC Run 3 $\xrightarrow{\sim 2x}$ HL-LHC

LHC Run 3 is limited by statistics

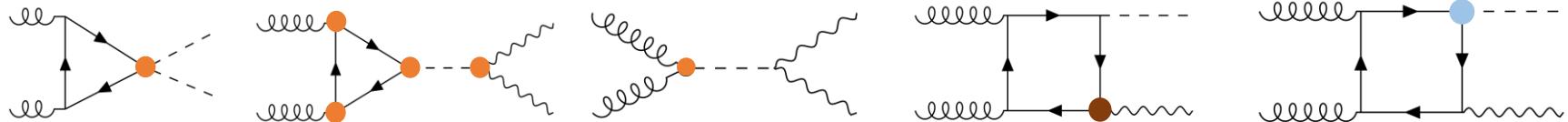
Combining diboson channels closes flat directions

Diboson: Beyond the Basics

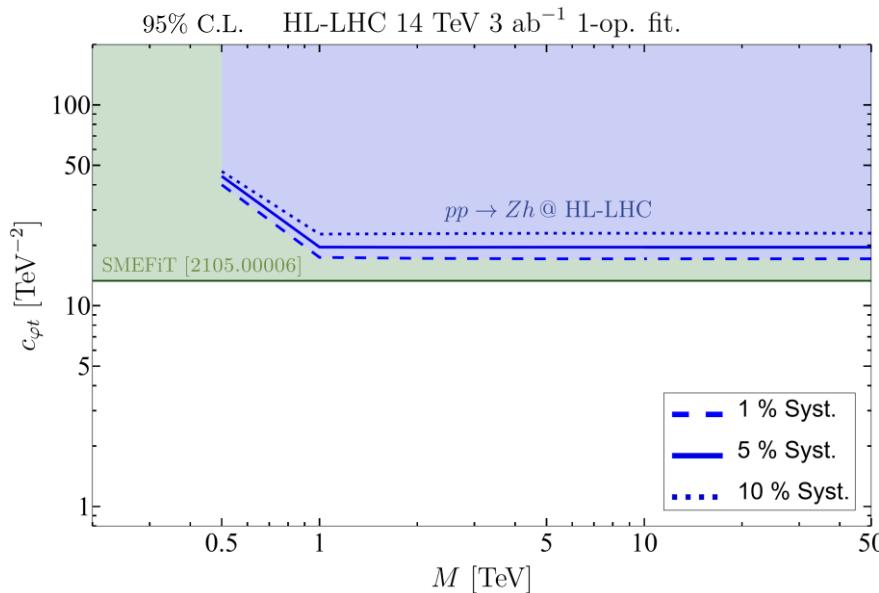
Loop-induced processes matter

ANR, M. Thomas, E. Vryonidou [2306.18215]

Dim-6 SMEFT effects in $gg \rightarrow HH, ZH, ZZ, WW$



$$\mathcal{M}_{++00} \sim \begin{array}{|c|c|c|} \hline \mathcal{O}_{\varphi t} & \mathcal{O}_{\varphi Q}^{(-)} & \mathcal{O}_{t\varphi} \\ \hline \frac{m_t^2 v e g_s^2}{32\pi^2 m_Z c_w s_w} \left[\log\left(\frac{s}{m_t^2}\right) - i\pi \right]^2 & \frac{m_t^2 v e g_s^2}{32\pi^2 m_Z c_w s_w} \left[\log\left(\frac{s}{m_t^2}\right) - i\pi \right]^2 & \frac{m_t v^2 e g_s^2}{32\sqrt{2}\pi^2 m_Z c_w s_w} \left[\log\left(\frac{s}{m_t^2}\right) - i\pi \right]^2 \\ \hline \end{array} gg \rightarrow ZH$$



Interesting interplay with quark channel at NNLO

R. Gauld, U. Haisch, L. Schnell [2311.06107]

Dimension-8 effects in WW, WZ, WH and ZH

C. Degrande, H. Li [2303.10493]

T. Corbett, A. Martin [2306.00053]

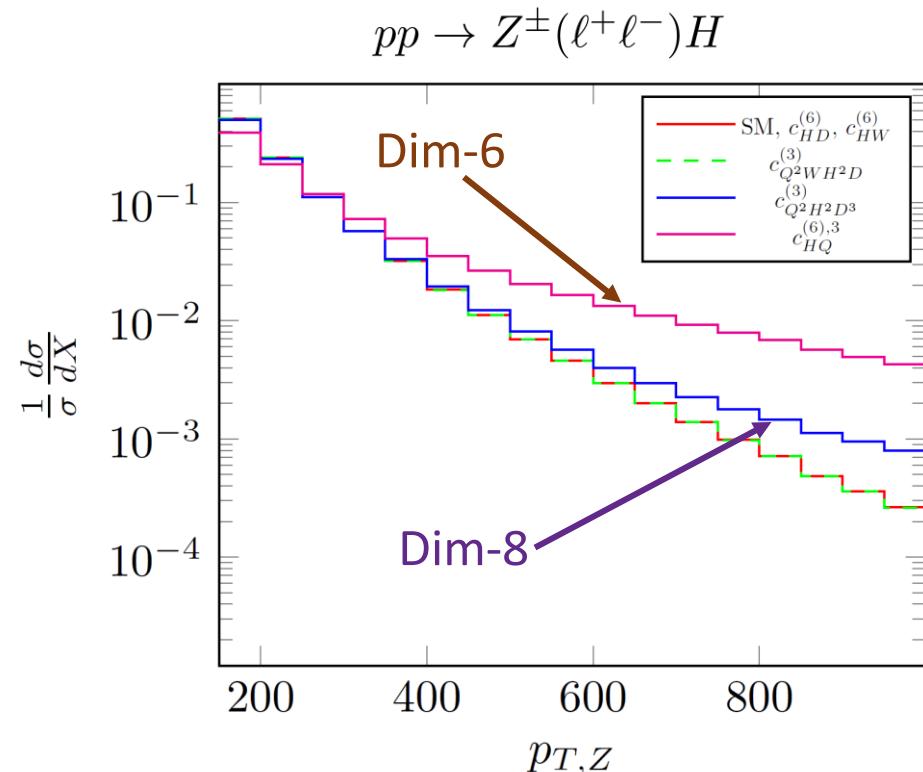
$$q\bar{q} \rightarrow WZ$$

Operator	$2 \operatorname{Re}(\mathcal{A}^{\text{SM}} \mathcal{A}^{\text{NP}*})$	$2 \int d\Omega \operatorname{Re}(\mathcal{A}^{\text{SM}} \mathcal{A}^{\text{NP}*})$
\mathcal{O}_4	$u\bar{d} : e_4 S^2 + f_4 S + g_4$	$\bar{e}_4 S^2 + \bar{f}_4 S + \bar{g}_4$
\mathcal{O}_5	$u\bar{d} : \frac{\Gamma_W}{M_W} (f_5 S + g_5)$	$\frac{\Gamma_W}{M_W} (\bar{f}_5 S + \bar{g}_5)$
\mathcal{O}_6	$u\bar{d} : e_6 S^2 + f_6 S + g_6$	$\bar{e}_6 S^2 + \bar{f}_6 S + \bar{g}_6$
\mathcal{O}_7	$u\bar{d} : \frac{\Gamma_W}{M_W} (f_7 S + g_7)$	$\frac{\Gamma_W}{M_W} (\bar{f}_7 S + \bar{g}_7)$
\mathcal{O}_{11}	$u\bar{d} : g_{11} \frac{\Gamma_W}{M_W}$	0
\mathcal{O}_{12}	$u\bar{d} : e_{12} S^2 + f_{12} S + g_{12}$	$\bar{e}_{12} S^2 + \bar{f}_{12} S + \bar{g}_{12}$
\mathcal{O}_{13}	$u\bar{d} : e_{13} S^2 + f_{13} S + g_{13}$	$\bar{e}_{13} S^2 + \bar{f}_{13} S + \bar{g}_{13}$
\mathcal{O}_{18}	$u\bar{d} : 0$	0

$$\mathcal{O}_4 = i W^{I\mu}{}_\lambda B^{\nu\lambda} \left(\bar{q}_{Lp}^i \gamma_\nu \left(\tau^I \right)_i {}^j \overleftrightarrow{D}_\mu q_{Lrj} \right)$$

$$\mathcal{O}_{12} = i \epsilon^{IJK} \tilde{W}^{I\mu}{}_\nu W^{J\nu}{}_\lambda \left(\bar{q}_{Lp}^i \gamma^\lambda \left(\tau^K \right)_i {}^j \overleftrightarrow{D}_\mu q_{Lrj} \right)$$

Several operators generate
maximal energy growth



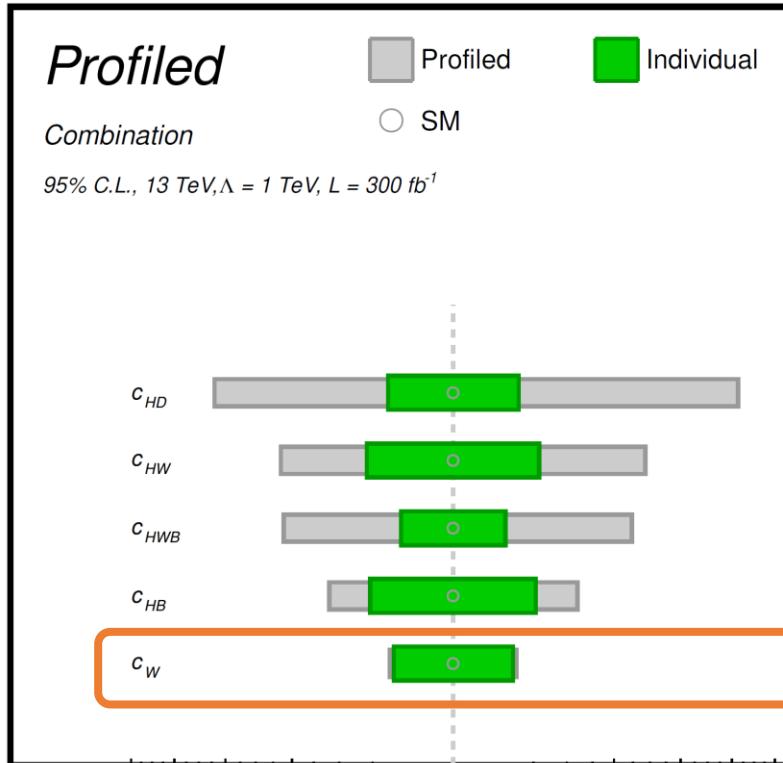
$$c_i^{(6),(8)} = 1/(3 \text{ TeV})^{2,4}$$

Mild effect due to SM-suppressed
interference

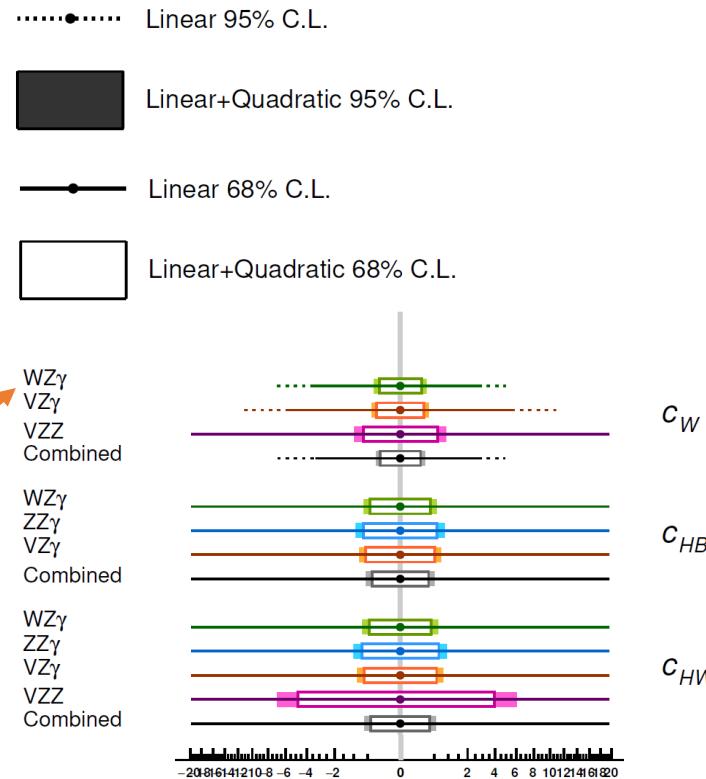
Extra bosons for extra fun: Triboson

R. Bellan, S. Bhattacharya, G. Boldrini, F. Cetorelli, P. Govoni [2303.18215]

WZZ, ZZZ, WZ γ , ZZ γ



$\Lambda=1 \text{ TeV} \quad 300 \text{ fb}^{-1} \quad (13 \text{ TeV})$



$$c_W \varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$$

Stay tuned, more to come!

See E. Celada's talk tomorrow

E. Celada et al [24XX.ZZZYY]

WH goes CP-odd at LHC

R. Barrué, P. Conde-Muñó, V. Dao, R. Santos [2308.02882]

$$\tilde{\mathcal{O}}_{HW} = \frac{c_{HW}}{\Lambda^2} H^\dagger H \tilde{W}_{\mu\nu}^I W^{I\mu\nu} = \frac{c_{HW}}{\Lambda^2} H^\dagger H \epsilon_{\mu\nu\rho\sigma} W^{I\rho\sigma} W^{I\mu\nu}$$

Observable	Linearized limits	Full limits
$Q_\ell \cos \delta^+ \in [-1.0, -2/3, -1/3, 0., 1/3, 2/3]$	[-0.227, 0.227]	[-0.264, 0.216]
$m_T^{\ell\nu b\bar{b}} \in [0, 400, 800] \text{ GeV} \otimes Q_\ell \cos \delta^+ \in [-1.0, -2/3, -1/3, 0., 1/3, 2/3]$	[-0.093, 0.093]	[-0.096, 0.096]
$p_T^W \in [0, 75, 150, 250, 400, 600] \text{ GeV} \otimes Q_\ell \cos \delta^+ \in [-1.0, -2/3, -1/3, 0., 1/3, 2/3]$	[-0.088, 0.088]	[-0.096, 0.072]
SALLY, w/ ν 4-vector	[-0.056, 0.056]	[-0.072, 0.144]
SALLY, w/ detector-level observables	[-0.067, 0.067]	[-0.144, 0.096]

↑
MadMiner

Not a surprise!



$$\sigma_{\mathcal{O}_{\varphi\widetilde{W}}}^{int} \sim \frac{\sqrt{\hat{s}} M_W}{\Lambda^2} \sin(\phi_W)$$

Bishara, ANR, et al [2004.06122]

NLO effects? Coming soon...

ANR, E. Vryonidou [24XX.ZZZYY]

Other diboson channels are also great CPV probes

N. Clarke Hall, et al [2209.05143]

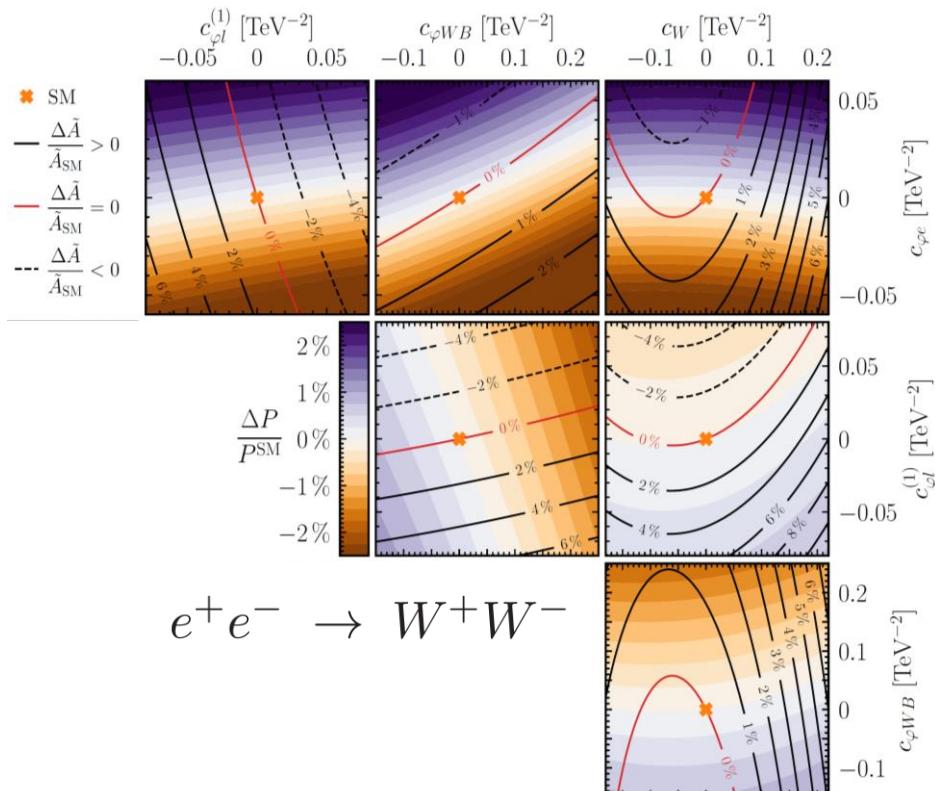
14.

Multiboson in the future

Diboson at FCC-ee

Entanglement in W^+W^- , W^+Z , ZZ

R. Aoude, et al [2307.09675]

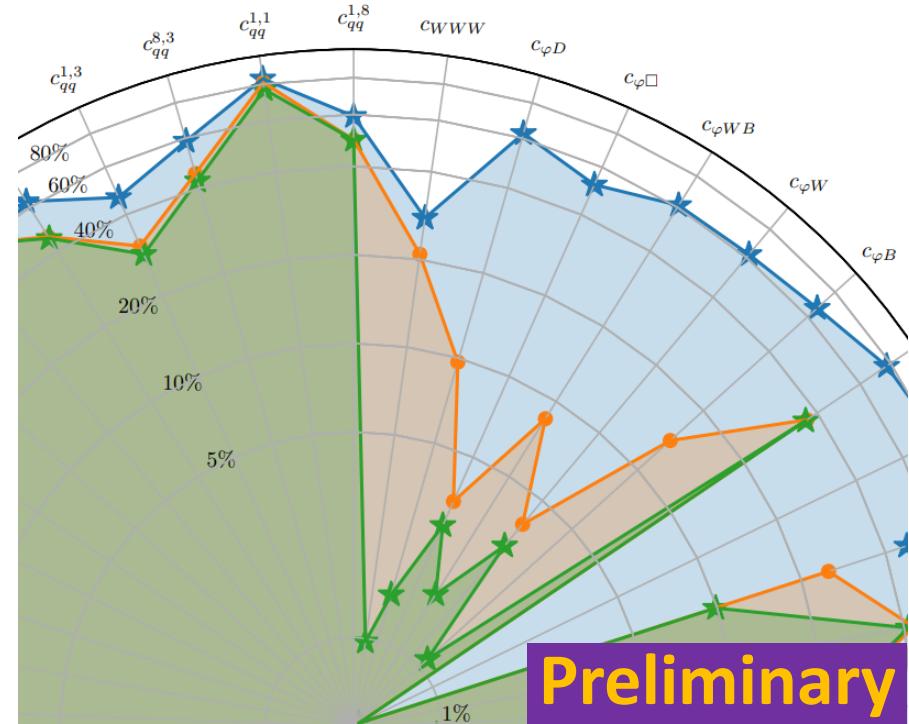


J. De Blas, et al [1907.04311, 2206.08326]

SMEFiT (w/ E. Celada, J. ter Hoeve, ANR) [24XX.YYYZZ]

- HLLHC
- HLLHC + FCCee w/o OO
- HLLHC + FCCee

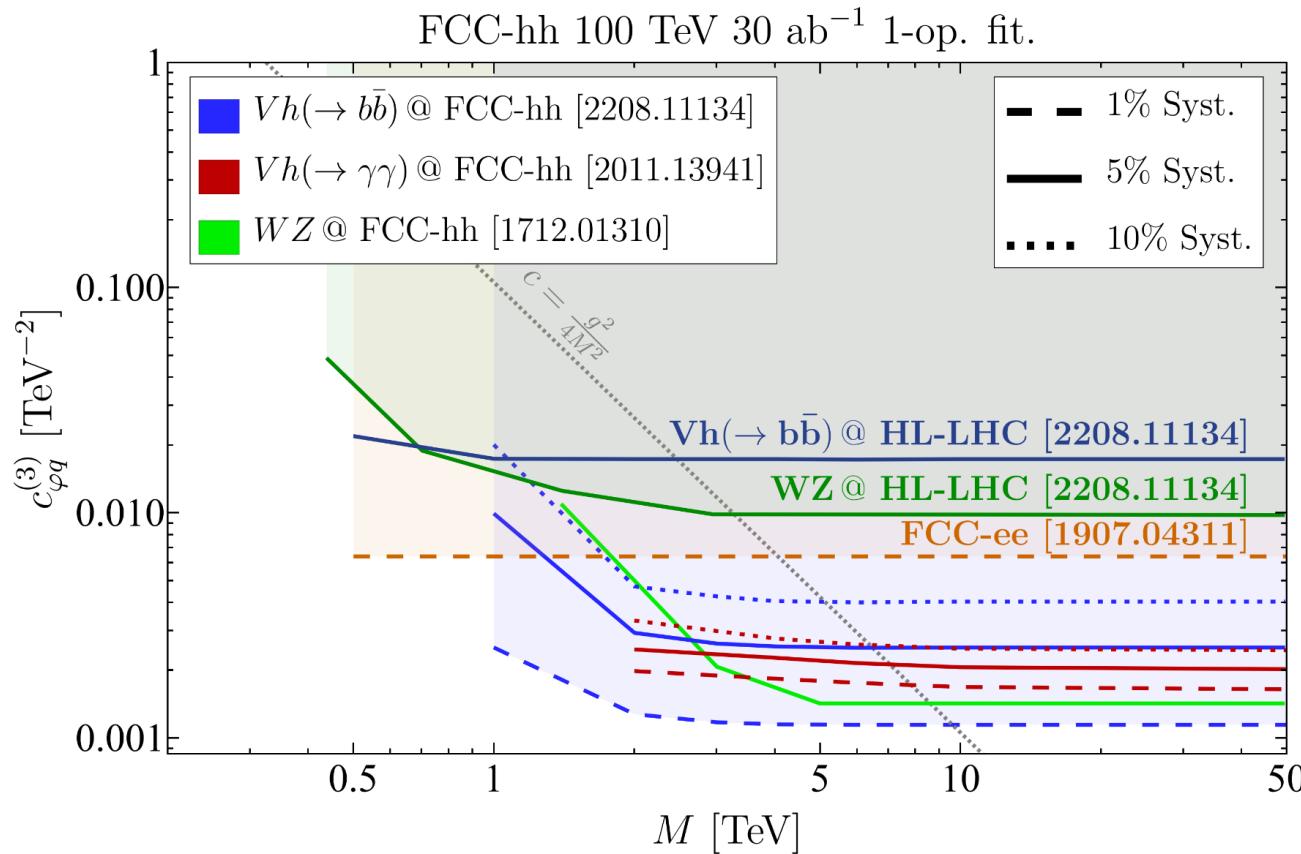
Ratio of Uncertainties to Global Fit Baseline, NLO $\mathcal{O}(\Lambda^{-2})$



All the W^+W^- power

Diboson at FCC-hh

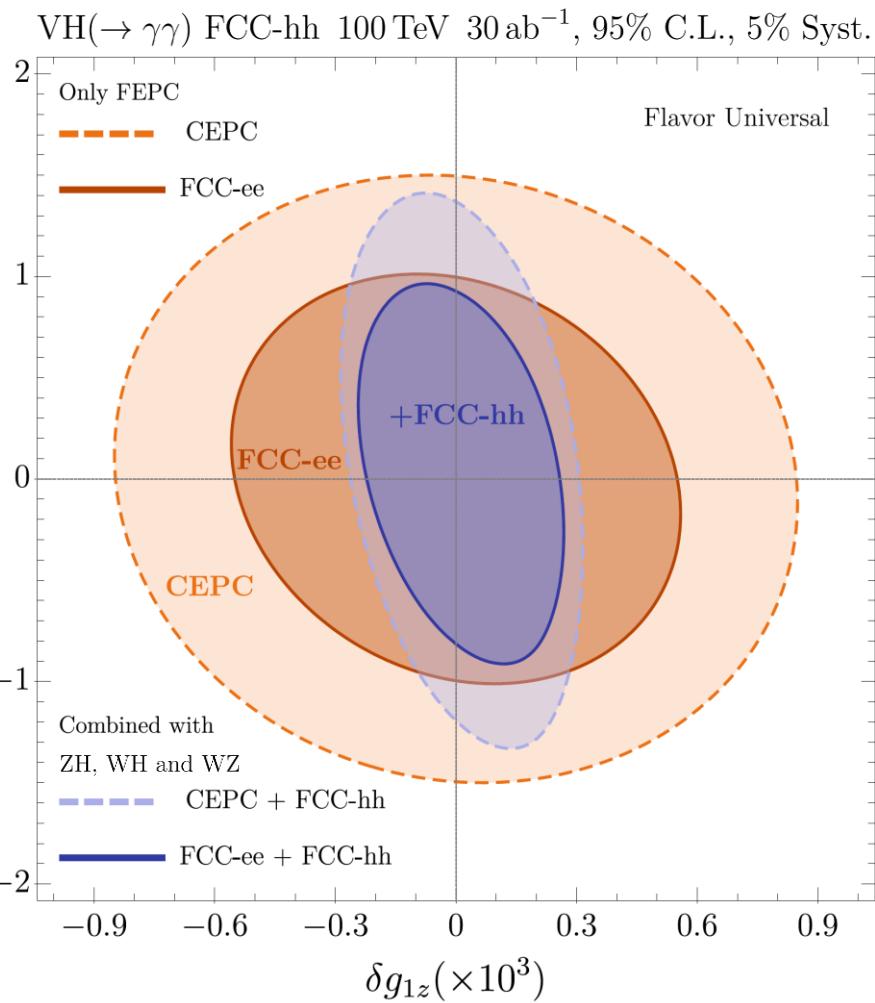
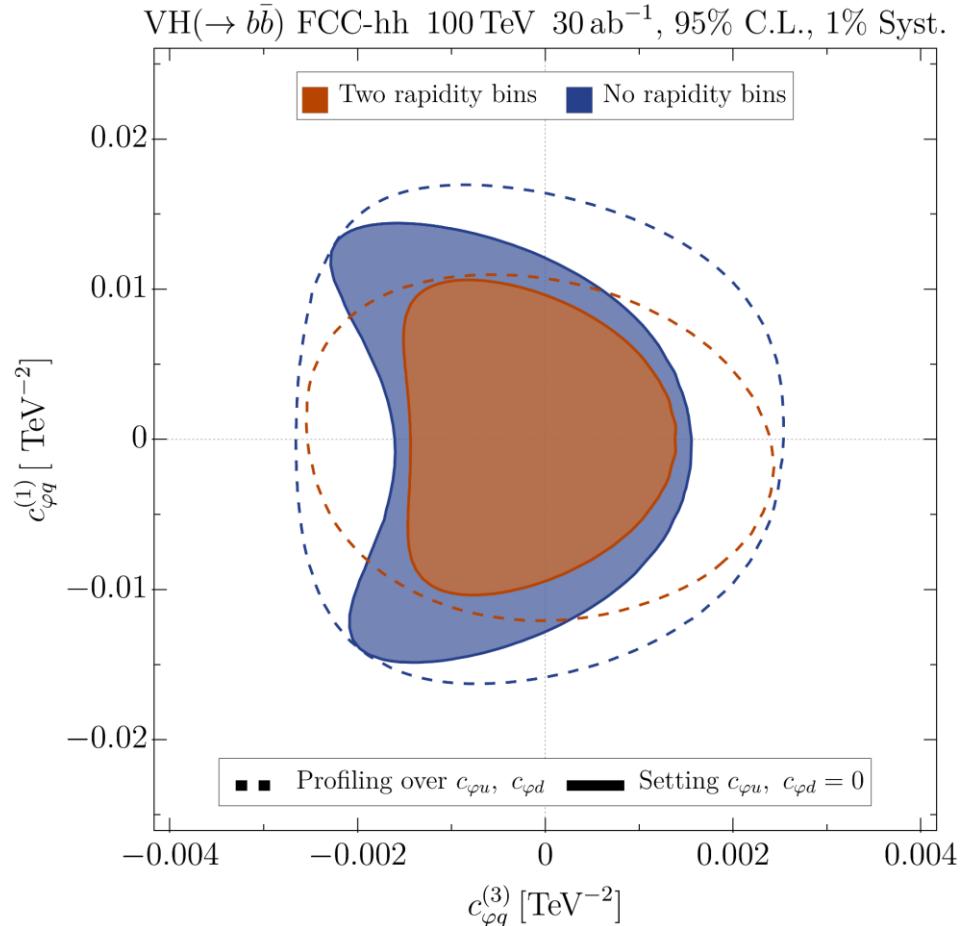
Improves the old, adds new channels



HL-LHC $\xrightarrow{\sim 5x}$ FCC-hh

$h \rightarrow \gamma\gamma \approx h \rightarrow b\bar{b}$ @FCC-hh

The double-binning advantage at FCC-hh



Complementarity
FCC-ee + FCC-hh

Concluding Multiremarks

- Diboson is a key process in the precision program at (HL-)LHC.
- Its several incarnations offer a very rich Physics landscape.
- NLO QCD effects are understood in CP-even cases. NLO EW and CP-odd are the next frontier and WIP.
- Dimension-8 and Triboson have just started being explored. Much to do!
- LHC has exceeded expectations, HH seems within reach.
- Multiboson will remain relevant for decades to come.

Thanks for your attention
and happy workshop!

Contact:

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<http://www.hep.man.ac.uk/>

Appendix

Growing helicity amplitudes in $gg \rightarrow ZH$

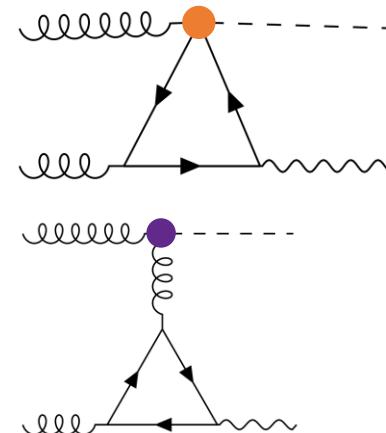
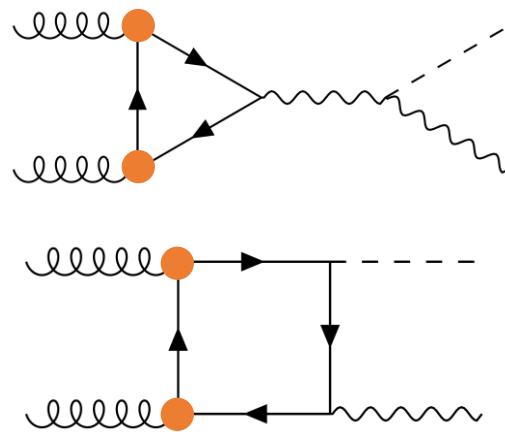
$\lambda_{g_1}, \lambda_{g_2}, \lambda_H, \lambda_Z$	\mathcal{O}_{tG}	$\mathcal{O}_{\varphi G}$
$+, +, 0, +$	$\sqrt{s} \frac{m_t g_s^2 g_{t,A}^Z}{\pi^2} \log\left(\frac{s}{m_t^2}\right)$	—
$+, +, 0, -$	$\sqrt{s} \frac{m_t g_s^2 g_{t,A}^Z}{\pi^2} \log\left(\frac{s}{m_t^2}\right)$	—
$+, +, 0, 0$	$\frac{m_t m_Z g_s^2 g_{t,A}^Z}{\pi^2} \log^2\left(\frac{s}{m_t^2}\right)$	$\frac{m_t^2 v g_s^2 g_{t,A}^Z}{\pi^2 m_Z} \log^2\left(\frac{s}{m_t^2}\right)$
$+, -, 0, +$	$\sqrt{s} \frac{m_t g_s^2 g_{t,A}^Z}{\pi^2}$	—
$+, -, 0, 0$	$s \frac{m_t g_s^2 g_{t,A}^Z}{\pi^2 m_Z}$	$\frac{m_t^2 v g_s^2 g_{t,A}^Z}{\pi^2 m_Z} \log^2\left(\frac{s}{m_t^2}\right)$

Tightly constrained

$$-0.019 \text{ TeV}^{-2} < c_{\varphi G} < 0.003 \text{ TeV}^{-2}$$

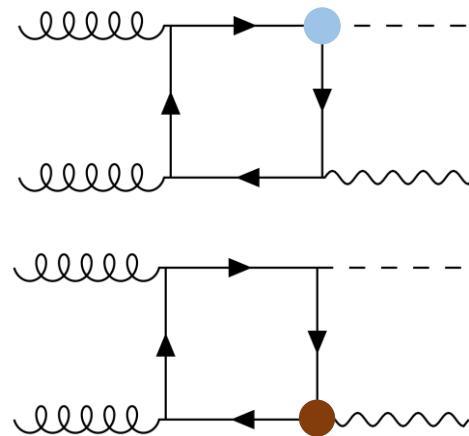
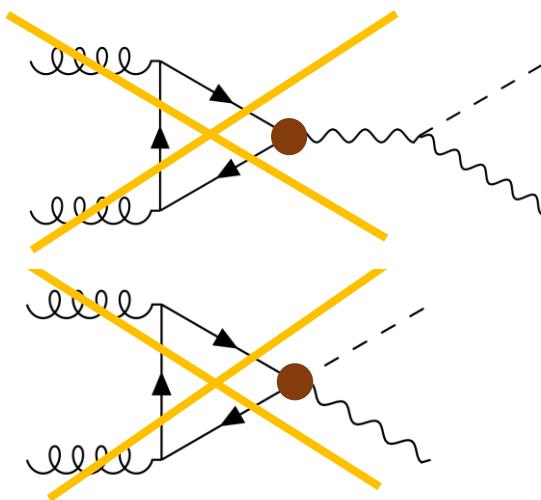
$$0.062 \text{ TeV}^{-2} < c_{tG} < 0.24 \text{ TeV}^{-2}$$

SMEFiT Collab. [[arXiv:2105.00006](https://arxiv.org/abs/2105.00006)]



Helicity amplitudes in $gg \rightarrow ZH$

$\lambda_{g_1}, \lambda_{g_2}, \lambda_H, \lambda_Z$	$\mathcal{O}_{\varphi t}$	$\mathcal{O}_{\varphi Q}^{(-)}$	$\mathcal{O}_{t\varphi}$
$+, +, 0, 0$	$\frac{m_t^2 v e g_s^2}{32\pi^2 m_Z c_w s_w} \left[\log\left(\frac{s}{m_t^2}\right) - i\pi \right]^2$	$\frac{m_t^2 v e g_s^2}{32\pi^2 m_Z c_w s_w} \left[\log\left(\frac{s}{m_t^2}\right) - i\pi \right]^2$	$\frac{m_t v^2 e g_s^2}{32\sqrt{2}\pi^2 m_Z c_w s_w} \left[\log\left(\frac{s}{m_t^2}\right) - i\pi \right]^2$



Sensitive only to

$$c_{\varphi Q}^{(-)} - c_{\varphi t} + \frac{c_{t\varphi}}{y_t}$$

→ exact degeneracy

The triangles cancel each other out

Bounds on Higgs and top operators from the tails of $pp \rightarrow ZH$

Third-generation operators

$$\mathcal{O}_{\varphi Q}^{(1)} \quad c_{\varphi Q}^{(1)}$$

$$i(\varphi^\dagger \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{Q} \gamma^\mu Q)$$

$$\mathcal{O}_{\varphi Q}^{(3)} \quad c_{\varphi Q}^{(3)}$$

$$i(\varphi^\dagger \overset{\leftrightarrow}{D}_\mu \tau_I \varphi)(\bar{Q} \gamma^\mu \tau^I Q)$$

$$\mathcal{O}_{\varphi Q}^{(-)} \quad c_{\varphi Q}^{(-)}$$

$$c_{\varphi Q}^{(1)} - c_{\varphi Q}^{(3)}$$

$$\mathcal{O}_{\varphi t} \quad c_{\varphi t}$$

$$i(\varphi^\dagger \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{t} \gamma^\mu t)$$

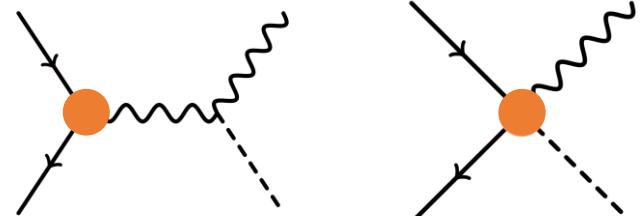
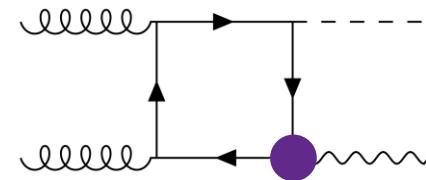
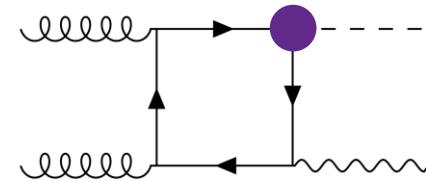
$$\mathcal{O}_{t\varphi} \quad c_{t\varphi}$$

$$\left(\varphi^\dagger \varphi - \frac{v^2}{2}\right) \bar{Q} t \tilde{\varphi} + \text{h.c.}$$

Probed by $gg \rightarrow ZH$

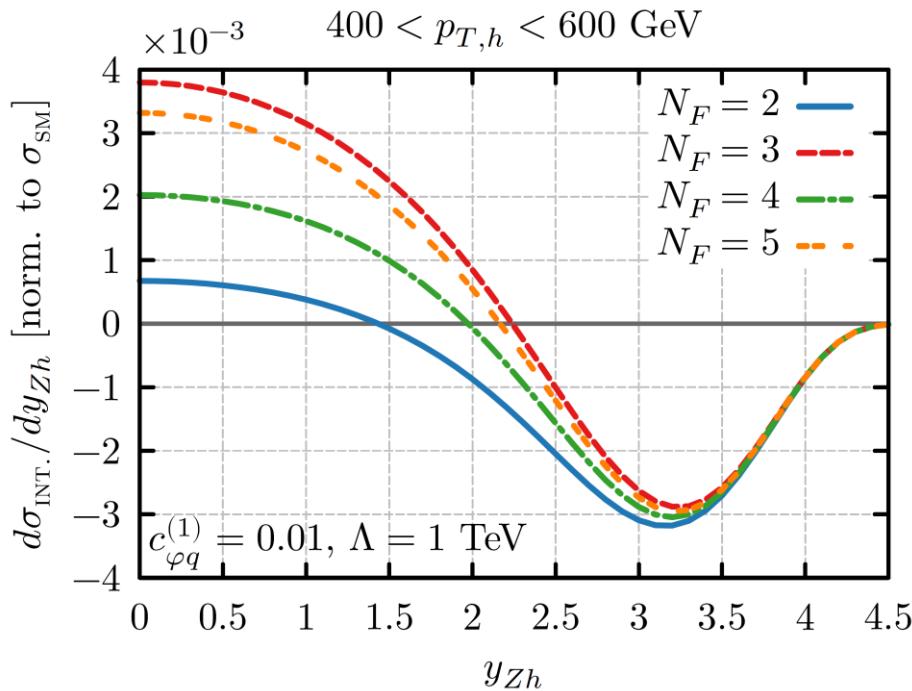
Probed by $qq \rightarrow ZH$

Quark and gluon channels interplay



A3.

Lifting up cancellations



$$\sigma_{\mathcal{O}_{\varphi q}^{(1)}}^{\text{int}} \propto s_W^2 Q - T_3$$

Cancellation of up and down contributions

Differential in p_T and rapidity

$\text{Min}\{p_T^h, p_T^Z\} \in \{200, 400, 600, 800, 1000, \infty\} \text{ GeV}$

$|y_{Zh}| \in [0, 2), [2, 6]$

(Slightly different rapidity binning for $Z \rightarrow \nu\bar{\nu}$)